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On Characteristic Eigenvalues of Complex Media in Surface Integral Formulations

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***Abstract*—Although surface integral equations (SIEs) have been extensively used in solving electromagnetic problems of penetrable objects, there are still open issues relating to their application to the Theory of Characteristic Modes. This work demonstrates that when an SIE is used to solve for the characteristic modes (CMs) of a dielectric or magnetic object, the resulting eigenvalues are unrelated to the reactive power of the object, unlike the eigenvalues of perfect electric conductors. However, it is proposed that the classical eigenvalues, which provide useful physical insights, can be extracted from the SIE CM solution using Poynting's theorem. Large discrepancies between the SIE CM eigenvalues and the proposed eigenvalues, as well as eigenvalue-derived characteristic quantities, are highlighted using a numerical example. The modal resonances as predicted by the proposed eigenvalues closely match those obtained for natural resonance modes.**

***Index Terms*—Antenna analysis, characteristic modes, method of moments, Poynting's theorem, Sturm-Liouville theory.**

I. Introduction

Although the Theory of Characteristic Modes (TCM) [1] for conducting bodies is increasingly embraced and utilized by the electromagnetic community, the underlying aspects of analyzing penetrable objects using TCM is not fully understood. Moreover, problems associated with characteristic mode (CM) analysis of such objects, based on either volume integral equation (VIE) or surface integral equation (SIE), have received relatively little attention outside of their original works in [2] and [3]. Two important problems are: 1) the loss of the physical interpretation for the CM eigenvalues in terms of reactive power [2], [4], relative to the classical eigenvalue definition for perfect electric conductors (PECs) [1]; 2) the presence of internal resonances in SIE CM solutions [5].

Recently, there has been a renewed interest in solving for the CMs of penetrable objects, with new formulations being introduced to restore the physical meaning of the eigenvalues by representing the fields with only electric or magnetic surface currents [4], [6], [7]. Furthermore, two of the SIE formulations proposed in [6] are claimed to be free of internal resonances. However, these new formulations are only applicable for single or multiple separate homogeneous [4], [6] or inhomogeneous [7] objects, because only one set of equivalent source (electric or magnetic currents) exists in the PMCHWT equation for a single material body. Furthermore, as the currents are derived in a different manner than other more traditional SIE CM formulations [3], the non-physical surface currents will not be equivalent across formulation types [6].

On the other hand, SIE formulations used for CM analysis, e.g., Poggio-Miller-Chan-Harrington-Wu-Tsai (PMCHWT) in [3], can handle multiple and multi-layer homogeneous objects, such as dielectric-coated objects and composite objects of multiple materials. Furthermore, the internal resonances inherent to some SIE formulations (e.g., the symmetric form of the PMCHWT formulation [3]) can be removed via suitable post-processing techniques [5], [8]. However, the remaining eigenvalues are still unrelated to the reactive power, which means that these eigenvalues cannot be used to obtain the modal quality factor using (21) in [9]. In [2] it was proved that the eigenvalues derived from a VIE impedance matrix do not solve for the reactive power of the object. However, this derivation cannot be directly applied to SIE formulations. This is because in VIE formulations, the incident field is represented by both a scattered field as well as a constitutive relationship between the currents and the material, whereas SIE formulations only relate the currents to an incident field. Furthermore, the original SIE-based CM formulation [3] provides no discussion on the relationship between the SIE eigenvalues and the reactive power. In addition, even though the problem of eigenvalue interpretation has been alluded (e.g., [4], [6]), no explicit proof has been provided.

In this context, this letter provides, for the first time, a detailed proof that the eigenvalues solved by any symmetric or asymmetric SIE CM formulation are unrelated to the reactive power. The proof provides critical insights which are not included in the original SIE CM derivation [3]. Additionally, the imaginary part of Poynting's theorem is used to solve for the proposed reactive power based eigenvalues using both symmetric and asymmetric SIEs. Throughout the paper, no simplification or additional boundary condition are required to solve the SIE's combined field method of moments (MoM) impedance matrix, which is used to calculate the eigenvalues. However, the symmetric version of PMCHWT SIE is used in the provided example. Once the proposed eigenvalues are obtained, they can be used to correctly calculate eigenvalue-based characteristic quantities including modal resonances,

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characteristic angle, modal bandwidth and modal quality factor. The modal resonances found using the eigenvalues closely match the natural resonances found by means of both a MoM decomposition and the finite element method (FEM).

II. Eigenvalues From Poynting's Theorem

TCM is a unique amalgamation of two separate theories: Poynting's theorem and the Sturm-Liouville theory. Without understanding both theories it is difficult to identify why TCM -derived eigenvalues are not valid in SIE formulations for penetrable objects.

In electromagnetic theory, Poynting's theorem is a statement of the conservation of energy for electromagnetic fields (in space and time), i.e., the sum of the power leaving a closed surface S (e.g., an object) ($\mathscr{P}_{rad}$), the power dissipated within the corresponding volume V ($\mathscr{P}_d$) and the time rate of increase of energy stored (reactive power) in V ($\Delta\mathscr{P}_{stored}$) must be zero, as given by

$$\underbrace{\oiint_S (\boldsymbol{\mathcal{E}}\times\boldsymbol{\mathcal{H}})\bullet d\boldsymbol{s}}_{\mathscr{P}_{rad}}+\underbrace{\iiint_V \sigma\|\boldsymbol{\mathcal{E}}\|^2\,dv}_{\mathscr{P}_d}+\underbrace{\tfrac{\partial}{\partial t}\iiint_V \tfrac{1}{2}\left[\mu\|\boldsymbol{\mathcal{H}}\|^2+\mu\|\boldsymbol{\mathcal{E}}\|^2\right]dv}_{\Delta\mathscr{P}_{stored}}=0. \tag{1}$$

In (1), $\boldsymbol{\mathcal{E}}\times\boldsymbol{\mathcal{H}}$ is the Poynting vector, $\boldsymbol{\mathcal{E}}$ is the electric field intensity, $\boldsymbol{\mathcal{H}}$ is the magnetic field intensity, μ is the permeability, ε is the permittivity, σ is the conductivity, and $\|\bullet\|$ is the Frobenius norm operator. Moreover, the left hand side of (1) is equal to the total complex source power $\mathscr{P}_s$ [11], which is a sum of the inner products of the source fields and currents. In time-harmonic fields, $\mathscr{P}_s$ can be expressed as

$$P_s=\left\langle \boldsymbol{J}^*,\boldsymbol{E}\right\rangle+\left\langle \boldsymbol{M},\boldsymbol{H}^*\right\rangle=-\tfrac{1}{2}\iiint_V\left(\boldsymbol{H}^*\bullet\boldsymbol{M}+\boldsymbol{E}\bullet\boldsymbol{J}^*\right)dv \tag{2}$$

$$=\underbrace{\oiint_S \tfrac{1}{2}\left(\boldsymbol{E}\times\boldsymbol{H}^*\right)\bullet d\boldsymbol{s}}_{P_{rad}}+\underbrace{\tfrac{1}{2}\iiint_V \sigma|\boldsymbol{E}|^2\,dv}_{P_d}+\underbrace{j\,\omega\iiint_V \tfrac{1}{2}\left(\mu|\boldsymbol{H}|^2-\varepsilon|\boldsymbol{E}|^2\right)dv}_{\Delta P_{stored}}$$

where the real part of (2) equals the radiated P_{rad} and dissipated P_d powers, whereas the imaginary part gives the reactive power ΔP_{stored}, which equals $2\omega\,(W_m - W_e)$, with W_m and W_e being the magnetic and electric energies, respectively [10], [11]. Moreover, $\boldsymbol{J}$ and $\boldsymbol{M}$ represent the electric and magnetic current sources, respectively, and $(\bullet)^*$ is the complex conjugate operator.

A. Sturm-Liouville Theory

To show how the CM eigenvalues are related to the stored and radiated/dissipated powers as described by the imaginary and real parts of (2), the Sturm-Liouville theory, as applied to define TCM, is presented in the following.

First, the classical Sturm-Liouville problem [13] is defined by a second-order linear homogeneous operator L on a vector space of $C^2\left([a,b]\right)$. This allows the classical problem to be expressed as a specialized weighted eigenvalue problem

$$L\left(y_n\right)=-\frac{d}{dx}\left[p\left(x\right)\frac{dy_n}{dx}\right]+q\left(x\right)y_n=\nu_n r\left(x\right)y_n, \tag{3}$$

where $p(x)$ and $q(x)$ are given and $r(x)$ is a chosen weighting function. This equation is meant to determine the eigenvalues (ν_n) and their corresponding eigenfunctions (y_n) of the L operator, where y_n is required to be real, as defined by Theorem 1.2 of [13]. The Sturm-Liouville theory provides a proof that the eigenfunctions of different eigenvalues are orthogonal with respect to $r(x)$, stated mathematically by [13]

$$\left\langle y_m^*,r\left(x\right)y_n\right\rangle=\int_a^b y_m^* r\left(x\right)y_n\,dx=\delta_{mn}, \tag{4}$$

where δ_{mn} is the Dirac delta function. In (4), the Hermitian transpose has been added as this equation is related to power in TCM. However, since y_n must be real and symmetric in Sturm-Liouville problems, the Hermitian transpose has no impact.

B. Classical TCM as a Specific of Sturm-Liouville Problem

In the following, electromagnetic theory and Poynting's theorem are used to determine the proper weighting function that allows for CMs to be solved. We begin by considering the classical case of a lossless PEC object in vacuum with $\boldsymbol{M}=\boldsymbol{0}$, $\sigma = 0$, $\varepsilon=\varepsilon_0$, $\mu=\mu_0$ [1]. Therefore, (2) simplifies to

$$\begin{aligned}\left\langle \boldsymbol{J}^*,\boldsymbol{E}\right\rangle&=-\tfrac{1}{2}\iiint_V\left(\boldsymbol{E}\bullet\boldsymbol{J}^*\right)dv\\&=\underbrace{\oiint_S \tfrac{1}{2}\left(\boldsymbol{E}\times\boldsymbol{H}^*\right)\bullet d\boldsymbol{s}}_{P_{rad}}+\underbrace{j\omega\iiint_V \tfrac{1}{2}\left(\mu_0|\boldsymbol{H}|^2-\varepsilon_0|\boldsymbol{E}|^2\right)dv}_{\Delta P_{stored}=2\omega(W_m-W_e)}\end{aligned} \tag{5}$$

On the other hand, Maxwell's equations can be used to show that when a surface current density ($\boldsymbol{J}$) is induced on a PEC object of surface S, the object will produce a scattered electric field ($\boldsymbol{E}^s$) through the operator L_1. Specifically, the tangential component of $\boldsymbol{E}^s$ can be expressed in terms of $\boldsymbol{J}$ [12], as given by

$$\left[L_1(\boldsymbol{J})\right]_{\tan}=\left(-\boldsymbol{E}^s\right)_{\tan}=\left(j\omega\boldsymbol{A}+\nabla\Phi\right)_{\tan} \tag{6}$$

where the right hand side is a second-order linear differential equation. $\boldsymbol{A}$ and Φ are vector magnetic potential and scalar electric potential, respectively. Since the tangential part of the L_1 operator must transform a surface current density ($\boldsymbol{J}$ in [A/m]) to an electric-field ($\boldsymbol{E}$ in [V/m]), it has the dimension of impedance (Ω). Thus, (6) becomes an impedance operator

$$Z(\boldsymbol{J})=\left(j\omega\boldsymbol{A}+\nabla\Phi\right)_{\tan} \tag{7}$$

The impedance operator Z must be a symmetric operator due to reciprocity in the L_1 operator, and it is comprised of both a real component (R) and an imaginary component (X), i.e. $Z=R+jX$. Since (7) is a linear second-order operator, as operator L in (3), Z can be set as the Sturm-Liouville problem, i.e., $Z(\boldsymbol{J}_n)=L\left(y_n\right)$. To find the weighting function $r(x)$, it is important to recognize that since the eigenfunctions y_n are required to be real, eigenfunction theory dictates that $r(x)$ should be Hermitian symmetric. Furthermore, the source (or incident) field in (5) can be represented by $\boldsymbol{E}=\boldsymbol{E}^i=Z(\boldsymbol{J}_n)$. Since $\boldsymbol{J}_n$ is real, then the real part of the Poynting's theorem (5), i.e., $\left\langle \boldsymbol{J}_n,R\left(\boldsymbol{J}_n\right)\right\rangle$, corresponds to the radiated power P_{rad} of eigenmode n. Moreover, given that $R\left(\boldsymbol{J}_n\right)$ is Hermitian symmetric, the choice of $r\left(x\right)=R\left(\boldsymbol{J}_n\right)$ and identity (4) ensures that the far-fields are mutually orthogonal. Here, it is noted that (4) also requires the normalization of $\boldsymbol{J}_n$ such as

$\left\langle \boldsymbol{J}_n, R(\boldsymbol{J}_n)\right\rangle = 1$. Therefore, the classical TCM formulation is a specific form of the Sturm-Liouville problem that provide characteristic currents (eigenfunctions) with orthogonal far-fields, i.e.,

$$Z(J_n) = \nu_n R(J_n) \tag{8}$$

To simplify (8), it is noted that the equation is complex due to the operator Z. Expanding $Z = R + jX$, the real part of ν_n is equal to 1 and the imaginary part can be defined as λ_n, i.e. $\nu_n = 1 + j\lambda_n$. Collecting similar terms, the familiar TCM eigenvalue problem is derived

$$X(J_n) = \lambda_n R(J_n)\,. \tag{9}$$

In (9), the new eigenvalue λ_n relates only to the imaginary part of (5), since $\left\langle \boldsymbol{J}_n, X(\boldsymbol{J}_n)\right\rangle = \lambda_n$ and $\left\langle \boldsymbol{J}_n, R(\boldsymbol{J}_n)\right\rangle = 1$. In general, the Poynting's theorem based definition for a lossless PEC object is given by [1]

$$\lambda_n^P = \frac{\left\langle \boldsymbol{J}_n, X(\boldsymbol{J}_n)\right\rangle}{\left\langle \boldsymbol{J}_n, R(\boldsymbol{J}_n)\right\rangle} = \frac{\oiint_S \boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I \, ds}{\oiint_S \boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R \, ds} = \frac{2\omega(W_m - W_e)}{P_{rad}}\,. \tag{10}$$

Although the derivations leading to (9) consider a lossless PEC object, the same concept can be used to obtain physically relevant eigenvalues CMs for lossy, penetrable objects.

C. Generalized Eigenvalue for Penetrable Objects

For the special case of a lossless PEC object, it has been shown that the CM eigenvalue λ_n relates to the imaginary part of the Poynting's theorem in (5). For a homogeneous penetrable object, the surface equivalence principle stipulates that the fields can be represented by both equivalent electric currents $\boldsymbol{J}$ and magnetic currents $\boldsymbol{M}$ on the boundary surface S. However, when applied to the CM eigenvalue problem, the Sturm-Liouville theory requires that the characteristic currents are real. For symmetric SIE formulations, the currents may be represented by the real vector $[\boldsymbol{J}_n; \boldsymbol{M}_n]$, i.e. $\boldsymbol{J}_n^I = \mathrm{Im}(\boldsymbol{J}_n) = 0$ and $\boldsymbol{M}_n^I = \mathrm{Im}(\boldsymbol{M}_n) = 0$, where $(\bullet)^I$ refers to the imaginary part of the vector quantity. It should be noted that the symmetric SIE case (providing symmetric Z in the original formulation) is interesting but purely conceptual, since no such formulation has yet been demonstrated. Additionally, in the case of an asymmetric SIE formulation (i.e., asymmetric PMCHWT [3]) which requires forced symmetry, the source vector is defined as $[\boldsymbol{J}_n; j\boldsymbol{M}_n]$, which must be real. This results $\boldsymbol{J}_n^I = \mathrm{Im}(\boldsymbol{J}_n) = 0$ and $\boldsymbol{M}_n^R = \mathrm{Re}(\boldsymbol{M}_n) = 0$, where $(\bullet)^R$ represents the real part. Using this information, the real and imaginary parts of Poynting's theorem can be expanded and related directly to the real and imaginary parts of the electric and magnetic field quantities, as in (11) for $[\boldsymbol{J}_n; \boldsymbol{M}_n]$, and (12) for $[\boldsymbol{J}_n; j\boldsymbol{M}_n]$.

$$\begin{aligned} P_s &= -\tfrac{1}{2}\oiint_S \underbrace{\boldsymbol{J}_n^*}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{E} + \underbrace{\boldsymbol{M}_n}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{H}^* \, ds \\ &= \underbrace{-\tfrac{1}{2}\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^R\right) ds}_{P_{rad}+P_d} + j\underbrace{\left(-\tfrac{1}{2}\right)\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I - \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^I\right) ds}_{\Delta P_{stored}=2\omega(W_m-W_e)} \end{aligned} \tag{11}$$

$$\begin{aligned} P_s &= -\tfrac{1}{2}\oiint_S \underbrace{\boldsymbol{J}_n^*}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{E} + \underbrace{\boldsymbol{M}_n}_{\mathrm{Re}(\;)=0} \bullet \boldsymbol{H}^* \, ds \\ &= \underbrace{-\tfrac{1}{2}\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^I\right) ds}_{P_{rad}+P_d} + j\underbrace{\left(-\tfrac{1}{2}\right)\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I + \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^R\right) ds}_{\Delta P_{stored}=2\omega(W_m-W_e)} \end{aligned} \tag{12}$$

Applying the classical definition of eigenvalue [1] in (10), the equivalent Poynting's theorem based eigenvalues for symmetric and asymmetric (forced-symmetric) SIEs are given by (13) and (14), respectively.

$$\lambda_n^P = \frac{\iint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I - \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^I\right) ds}{\iint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^R\right) ds} = \frac{2\omega(W_m - W_e)}{P_{rad} + P_d} \tag{13}$$

$$\lambda_n^P = \frac{\iint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I + \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^R\right) ds}{\iint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^I\right) ds} = \frac{2\omega(W_m - W_e)}{P_{rad} + P_d} \tag{14}$$

Having the same physical meaning as the eigenvalues of the PEC case, the eigenvalues of (13) and (14) can be used to correctly determine modal resonances [11] as well as all eigenvalue-derived properties, e.g., characteristic angle, modal significance, modal bandwidth and modal quality factor [9].

III. Eigenvalues From TCM SIE Solutions

Although the Poynting's theorem based definition of the eigenvalue for lossy, penetrable objects, i.e., (13) and (14), retain the desired relationship to the reactive power, they do not correspond to the eigenvalues derived directly from applying the Sturm-Liouville theory to SIE formulations. Additionally, TCM derived eigenvalues cannot be related to the reactive power, and thus cannot be used to give physical insights. This is because the classical CM derivation in [1] considers only PEC objects, which means $\boldsymbol{M}_n = \boldsymbol{0}$. Thus, the inner product of (2) can be directly equated to the Sturm-Liouville inner product of (4), leading to (10). However, in SIEs involving penetrable objects, the magnetic currents are no longer zero and the inner product of (2) is no longer equivalent to that of (5). This can be shown mathematically by (15) for a symmetric SIE ($\boldsymbol{M}_n$ is real), as well as by (16) for an asymmetric SIE where the impedance matrix is forced into symmetry ($\boldsymbol{M}_n$ is imaginary).

$$\begin{aligned} \left\langle [\boldsymbol{J}_n, \boldsymbol{M}_n]^*, Z([\boldsymbol{J}_n, \boldsymbol{M}_n])\right\rangle &= -\tfrac{1}{2}\oiint_S \underbrace{\boldsymbol{J}_n^*}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{E} + \underbrace{\boldsymbol{M}_n^*}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{H} \, ds \\ &= \underbrace{-\tfrac{1}{2}\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^R\right) ds}_{P_{rad}+P_d} + j\underbrace{\left(-\tfrac{1}{2}\right)\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I + \boldsymbol{M}_n^R \bullet \boldsymbol{H}_n^I\right) ds}_{Unknown \neq \Delta P_{stored}} \\ &\neq P_s \end{aligned} \tag{15}$$

$$\begin{aligned} \left\langle [\boldsymbol{J}_n, \boldsymbol{M}_n]^*, Z([\boldsymbol{J}_n, \boldsymbol{M}_n])\right\rangle &= -\tfrac{1}{2}\oiint_S \underbrace{\boldsymbol{J}_n^*}_{\mathrm{Im}(\;)=0} \bullet \boldsymbol{E} + \underbrace{\boldsymbol{M}_n^*}_{\mathrm{Re}(\;)=0} \bullet \boldsymbol{H} \, ds \\ &= \underbrace{-\tfrac{1}{2}\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^R + \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^I\right) ds}_{P_{rad}+P_d} + j\underbrace{\left(-\tfrac{1}{2}\right)\oiint_S \left(\boldsymbol{J}_n^R \bullet \boldsymbol{E}_n^I - \boldsymbol{M}_n^I \bullet \boldsymbol{H}_n^R\right) ds}_{Unknown \neq \Delta P_{stored}} \\ &\neq P_s \end{aligned} \tag{16}$$

It should be noted that even though the CM eigenvalues defined by the ratio of the imaginary part to the real part in (15) and (16) are no longer directly related to the reactive power, the real part still equals that of Poynting's theorem and it also fulfills the orthogonality requirement (4) with respect to the weighting function $r(x)=R(\boldsymbol{J}_n)$. This means that the solved characteristic currents still provide orthogonal far-fields. Moreover, the eigenvalues based on Poynting's theorem can be calculated from these currents as defined in (13) and (14), which can then be applied for CM analysis and design problems that rely on the eigenvalues or eigenvalue-derived quantities.

IV. Numerical Example

An example is provided here to demonstrate the importance of having the correct eigenvalues for the reactive power based on (13) and (14). In this example, the penetrable object is a dielectric cube measuring 2.54 cm × 2.54cm × 2.54cm, which is made of a lossless dielectric with a relative permittivity $\varepsilon_r = 9.4$ The eigenvalues obtained from an impedance matrix of an asymmetric SIE (i.e., symmetric PMCHWT) that was forced into symmetry is compared to the corresponding eigenvalues defined in (14) using Poynting's theorem. The operator form of the equations was implemented in matrix form by using the Rao-Wilton-Glisson (RWG) basis functions in the MoM computations and the internal resonances were removed in post-processing using [5]. The solved object utilized 1152 basis functions. The eigenvalues for the first five CMs (λ_1-λ_5) were calculated using both definitions and shown in Fig. 1. The modal significance, characteristic angle, and modal quality factor for λ_1, λ_2, and λ_3 were calculated at 2.5 GHz, 3 GHz, and 4 GHz, respectively. It is noted that eigenvalue was chosen to be shown in this figure, instead of modal significance (used in [6] and [7]) or the magnitude of the eigenvalue (used in [4]), as the latter two do not provide insights into the type of stored energy (electric or magnetic).

As can be seen in Fig. 1, the zero-crossings of the eigenvalues from the two different definitions are nearly identical for λ_1 and λ_2, but the type of stored energy (capacitive or inductive), modal significance, characteristic angle, and quality factors can vary significantly between the two definitions. Finally, the same structure's natural resonant modes can be computed from FEM using the HFSS software, as well as from a MoM impedance matrix decomposition [14]. The natural resonances were found to be within 1.3% of the characteristic resonances obtained from (14).

Furthermore, it is noted that the same object was used for computing the TCM SIE and VIE eigenvalues in [5]. However, it was mentioned that the plotted eigenvalues should not be related to the reactive power [5].

V. Conclusion

This letter has demonstrated that TCM derived eigenvalues of an object as found using an SIE MoM impedance matrix cannot be related to an object's reactive power. Furthermore, this article has shown a method for simple computation of the correct eigenvalues using the TCM-derived currents and the Poynting's theorem definition of reactive power. The impact of the different definitions are illustrated with an example.

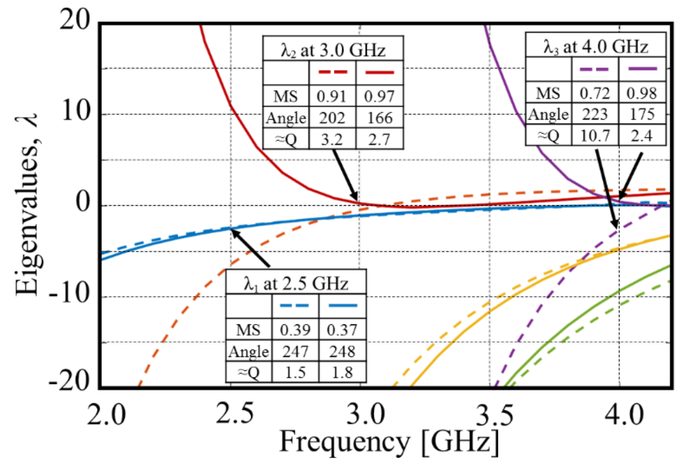

Fig. 1. Eigenvalues of lossless cube structure in [5] based on both Poynting's theorem definition from (14) (solid lines) and TCM definition (dashed lines).

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