



# LUND UNIVERSITY

## Wide band characteristic mode tracking utilizing far-field patterns

Miers, Zachary; Lau, Buon Kiong

*Published in:*  
IEEE Antennas and Wireless Propagation Letters

*DOI:*  
[10.1109/LAWP.2015.2417351](https://doi.org/10.1109/LAWP.2015.2417351)

2015

*Document Version:*  
Peer reviewed version (aka post-print)

[Link to publication](#)

*Citation for published version (APA):*  
Miers, Z., & Lau, B. K. (2015). Wide band characteristic mode tracking utilizing far-field patterns. *IEEE Antennas and Wireless Propagation Letters*, 14, 1658-1661. <https://doi.org/10.1109/LAWP.2015.2417351>

*Total number of authors:*  
2

### General rights

Unless other specific re-use rights are stated the following general rights apply:  
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# Wide Band Characteristic Mode Tracking Utilizing Far-Field Patterns

Zachary Miers, *Student Member*, Buon Kiong Lau, *Senior Member, IEEE*

**Abstract**— The Theory of Characteristic Modes provides a convenient tool for designing multi-antennas for MIMO applications, as it enables orthogonal radiation patterns to be excited in a given antenna structure. Moreover, the frequency behavior of the modes reveals interesting wideband properties of the structure. However, the tracking of characteristic modes over frequency remains a challenge, especially when differences between modes is limited to high currents in small regions of the structure. The common approach of tracking characteristic modes is through correlating the modal currents over frequency, this leads to multiple eigenvalues being mapped to the same eigencurrent. In this work, we propose a new approach to track characteristic modes by means of cross correlating far-field patterns, which effectively eliminates the mode mapping ambiguity.

**Index Terms**—Antenna array, antenna design, characteristic modes, MIMO systems, mutual coupling, mobile antenna.

## I. INTRODUCTION

THE Theory of Characteristic Modes (CMs) [1] has led to an increased understanding of the fundamental properties of designing antennas in small Multiple-Input Multiple-Output (MIMO) terminals. CM analysis computes the radiation and scattering properties of any structure through solving a generalized eigenvalue problem which exploits the method of moments (MoM) impedance matrix. When the problem is solved, multiple eigenvalues ( $\lambda_n$ 's) are produced; eigenvalues over frequency provide information on the excitability as well as the obtainable bandwidth of a specific eigencurrent ( $J_n$ ) [2]. Eigencurrents are useful to antenna design engineers as each eigencurrent provides information on how to adapt a structure to support multiple orthogonal antennas as well as how to excite each mode efficiently. CMs provide further insight when computed across a wide frequency band, allowing for the excitation of one or more CMs at multiple frequencies using a single feed element. Computing all modes over a wide frequency band becomes challenging as eigenvalues are not generally sorted between one frequency point and the next [3]. Furthermore, modes are not always maintained across frequency points, as modes can become unstable and cease to exist, whereas other modes can appear and become stable

without having any relation to previous modes [4].

Different approaches have been studied for tracking eigenmodes across wide frequency bands. The simplest tracking method is based on correlating the eigenvalues across frequency and sorting the modes in ascending order from the lowest frequency to the highest frequency [4]. This approach to tracking and classifying eigenvalues produces errors with symmetric geometries [3] and large frequency step sizes [5], [6]. In contrast, tracking eigenvalues through eigenvectors, or eigencurrents, is a widely adopted technique in antenna-centric CM research [3]-[6]. Although there are many types of current-based CM tracking, the modal assurance criterion method (MACM) is the most popular one [3]. In principle, MACM computes the correlation matrix of the eigencurrents over two distinct frequency points. If the correlation between two eigencurrents of separate frequencies exceeds a user-defined threshold (e.g., 0.9), then they are considered to be of the same eigenmode. In this way, a permutation matrix that maps (or links) eigencurrents of the same eigenmodes is formed from the correlation matrix by setting correlation values exceeding the threshold to 1 and otherwise to 0. MACM can be adapted in different ways to offer solutions to some but not all of the problems which have been associated with MACM tracking, e.g. [6]. Even though some of the adaptations to MACM allow for the tracking of modes which spontaneously begin or end, here on referred to as degenerate modes, MACM may not be suitable for use with complex structures or large frequency steps. In an effort to address the problems associated with MACM, different approaches have recently been implemented. The only commercially available software offering CM analysis, FEKO [7], implements a bounded linear correlation function that improves upon MACM. In [6], this specialized linear correlation function is extended by adding extrapolation of the eigenvalues to create a hybrid tracking algorithm that outperforms the previous outlined approaches.

In this work, a method for tracking the CMs of complex structures through far-field analysis is presented. Specifically, characteristic (or eigen-) electric and magnetic fields can be calculated from eigencurrents for any given frequency [1]. The characteristic electric (and magnetic) far-field patterns at a given frequency are mutually orthogonal, a property that is ideally suited for both MIMO antenna design [8] and mode-tracking. Utilizing the orthogonality property, every individual mode is unique and has no relation (or correlation) to any other mode. Even though the eigencurrents are orthogonal,

Manuscript received January 15, 2015. This work was supported by Vetenskapsrådet under Grants No. 2010-468 and No. 2012-4859.

The authors are with the Department of Electrical and Information Technology, Lund University, 221 00 Lund, Sweden (e-mail: Zachary.Miers@eit.lth.se; Buon\_Kiong.Lau@eit.lth.se).

just as the eigen far-fields, the variability in the eigencurrent distributions increases with structure complexity and frequency, whereas the far-fields remain relatively stable. This behavior enables the proposed far-field based mode-tracking to outperform the eigencurrent-based MACM.

## II. CHARACTERISTIC MODE TRACKING

CMs are constructed through optimizing the currents in any given structure, minimizing the stored power and maximizing the radiated power. The fundamental basis equation for CMs is built from the MoM impedance matrix and defined in [1] as

$$F(J) = \frac{\langle J, XJ \rangle}{\langle J, RJ \rangle} = \frac{\text{power stored}}{\text{power radiated}}. \quad (1)$$

The theory states that there exists a set of eigencurrents which minimize (1). This problem can be solved using a weighted eigen equation, commonly called the base equation for CMs

$$XJ_n = \lambda RJ_n. \quad (2)$$

When properly solved, (2) provides a set of eigenvalues and eigencurrents that correspond to those obtained through optimizing the radiated far-field of any given structure. As mentioned, MACM and its different adaptations, utilize the eigencurrents to organize and track each mode as they progress in frequency. For most applications, this technique has been found to yield accurate tracking of individual CMs. However, both this technique and the more evolved hybrid model [6] can break down under three situations: The primary one is described by differences in the fine features of the eigencurrents (over frequency) within one mode, which can decorrelate the currents enough to give the illusion of one mode ending and a new mode beginning; The second is that the mathematical computation of CMs for a symmetric structure can result in multiple eigencurrents with mirror symmetry across the structure (i.e., with flipped currents) even though the characteristic patterns are nearly identical. This causes the tracking algorithm to separate a single mode into two modes at different frequencies; the third is the use of large frequency step size between the adjacent frequency points, which can also lead to tracking errors. The majority of the tracking errors resulting from these three situations can be solved by substantially decreasing the frequency step size. However, this brute force approach significantly increases the computational time, as the eigen equation (2) is solved for each frequency.

To mitigate the problems in current-based modal tracking, a new mode-tracking algorithm based on characteristic far-field patterns is proposed. The proposed approach utilizes far-field pattern tracking through applying the standard envelope correlation coefficient (ECC) equation [9] on the characteristic far-field patterns of two adjacent frequencies instead of the far-field patterns of two antenna ports, i.e.,

$$\rho_{e,mn}^{CM} \approx \frac{|\oint_{4\pi} [E_{\phi,m}(\Omega) E_{\phi,n}^*(\Omega) + E_{\theta,m}(\Omega) E_{\theta,n}^*(\Omega)] d\Omega|}{|\oint_{4\pi} G_m(\Omega) d\Omega| |\oint_{4\pi} G_n(\Omega) d\Omega|} \quad (3)$$

where  $G_i(\Omega) = |E_{\phi,i}(\Omega)|^2 + |E_{\theta,i}(\Omega)|^2$ ,  $d\Omega = \sin\theta d\phi d\theta = (\phi, \theta)$ .  $E_{\phi,m}(\Omega)$  and  $E_{\theta,m}(\Omega)$  are respectively the  $\phi$ - and  $\theta$ -polarized

characteristic electric far-field patterns of mode  $m$  at a frequency point  $f_i$ , whereas  $E_{\phi,n}(\Omega)$  and  $E_{\theta,n}(\Omega)$  are the  $\phi$ - and  $\theta$ -polarized characteristic electric far-field patterns of mode  $n$  at the next frequency point  $f_{i+1}$ . A rule of thumb for the pattern discretization is that it should be less than the 3 dB beamwidth of the modal pattern with highest directivity.

The first step in the proposed approach is to sort the eigenvalues in ascending order from the lowest eigenvalue to the highest eigenvalue. The second step is to individually cross correlate each far-field pattern with the far-field patterns at subsequent frequency steps using (3). When a new mode is found, it is assigned a new mode number. Finally, as modes become corrupt or degenerate over frequency, these modes are disregarded at subsequent frequencies. Nevertheless, the disregarded modes are continually tested against new or resurging modes, forbidding any mode from obtaining multiple mode numbers. The algorithm for the proposed tracking method is provided as a flowchart in Fig. 1.

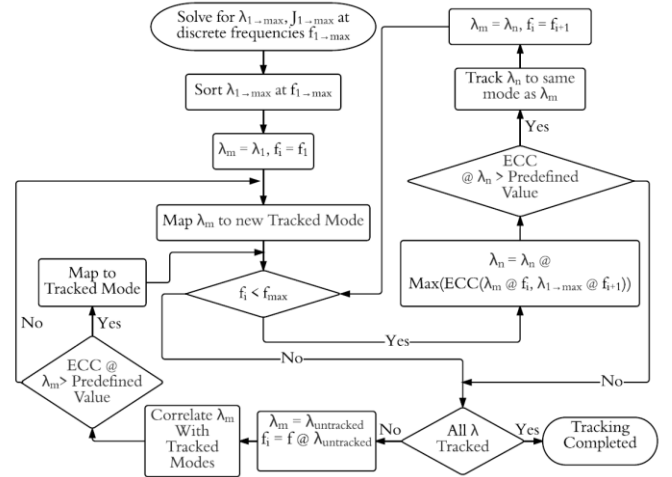


Fig. 1: Algorithm for far-field tracking of eigenmodes across frequency.

Tracking eigenvectors across frequency through the use of far-field patterns limits the susceptibility of mode swapping and false mode detection in many other MACM and hybrid models, as will be demonstrated in a numerical example below. The main advantage to this technique is derived from the slow evolution of far-field patterns in comparison to the much faster localized evolution of modal currents. This allows for larger frequency step sizes to be taken while reducing errors derived from minute changes in currents as well as image currents in symmetric structures.

## III. NUMERICAL EXAMPLE

To demonstrate the effectiveness of the proposed approach, a similar structure to that used in [6] was designed and analyzed. The structure consists of a flat chassis measuring 60 mm × 120 mm, situated 2 mm above the ground plane is a 2 mm wide rectangular metal ring (e.g., around the display screen) with a size of 59 mm × 119 mm. The metal ring and the physical nature of the flat chassis allow for the formation of modes where the differences between the modes can be limited to high currents in small regions of the structure.

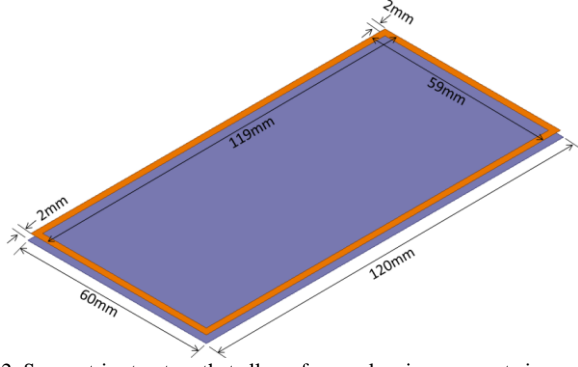


Fig. 2: Symmetric structure that allows for overlapping resonant eigenmodes.

The unique structure creates multiple resonant modes which intersect one another at around 2.6 GHz. The multiple resonant modes resemble high order dipole and loop currents which form both on the ring as well as the chassis. The intersection of these modes results in modes which are difficult to track across frequency due to the narrow bandwidth of the resonant modes. The structure was analyzed in 10 MHz steps from 2.5 GHz to 2.7 GHz, with the 20 most significant eigenmodes (eigenvalues with lowest magnitudes) calculated at each frequency step.

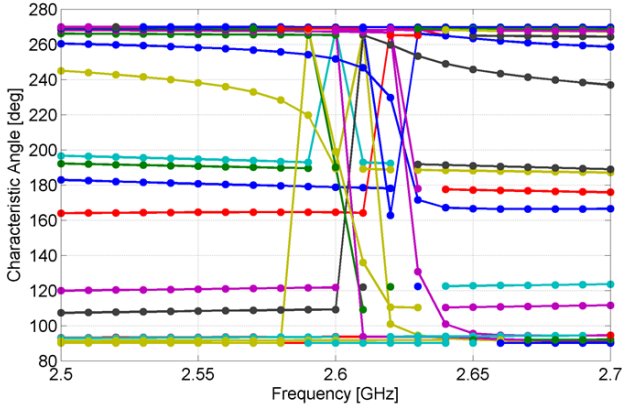


Fig. 3: Mode-tracking using MACM for a 120 mm  $\times$  60 mm PEC structure with a detached metal ring.

The MACM tracking algorithm presented in [3] was used as a baseline tracking algorithm. The plotting of the characteristic angle of the tracked eigen curves in Fig. 3 shows a significant number of mode swappings as well as a substantial number of degenerate modes. A total of 17 significant mode swaps occur between 2.58 GHz and 2.63 GHz, where the phase of the eigenvalues can change by up to  $200^\circ$  in a single frequency. The MACM tracking algorithm produced 51 distinct modes allowing for the identification of 31 degenerate modes within the calculated frequency band.

Significant improvement was made to the MACM tracking algorithm through the inclusion of a current based bounded linear correlation function as described in [6], this hybrid technique produces the eigenmodes as seen in Fig. 4. Between 2.58 GHz and 2.63 GHz, only one significant mode swapping occurred. However, the hybrid tracking algorithm produced 66 distinct modes, allowing for identification of 46 degenerate modes within the calculated frequency band.

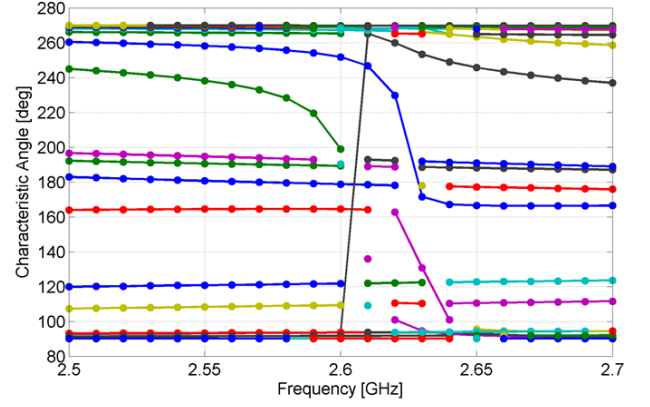


Fig. 4: Mode-tracking using current based hybrid method for a 120 mm  $\times$  60 mm PEC structure with a detached metal ring.

The proposed far-field tracking algorithm is fundamentally different than current based solutions and as such produces mode tracking with fewer degenerate modes and no observable mode swappings. The tracked modes produced by means of the far-field algorithm are presented in Fig. 5. When analyzing the structure using current based solutions such as the MACM and hybrid method small current fluctuations between Rao-Wilton-Glisson (RWG) edge elements cause algorithms to evaluate the small changes as new orthogonal modes. However, the presented far-field algorithm averages the small changes between edge elements allowing for modes to be mapped correctly without compromising orthogonality when utilizing larger frequency step sizes. This effect can be directly observed as there are significantly less degenerate modes and no swapped modes. The tracking algorithm produced 28 distinct modes, allowing for identification of 8 degenerate modes within the frequency band. Due to the fundamentally different mathematical approaches to tracked eigenmodes several of the modes are further analyzed below.

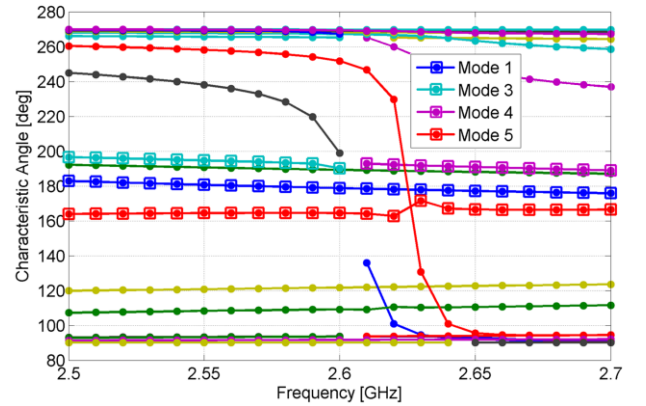


Fig. 5: Mode-tracking using far-field tracking algorithm for a 120 mm  $\times$  60 mm PEC structure with a detached metal ring.

It is commonly reported that current based tracking algorithms deteriorate quickly as the frequency step size is increased [5]-[6]. Therefore the accuracy of current based methods can be significantly improved by mean of increasing the frequency resolution. This unique identity can be used to verify the results of the proposed far-field tracking algorithm. While the far-field results (Fig. 5) are similar to those of both

current based solutions (Figs. 3 and 4) significant differences are observed due to the lower number of degenerate modes produced using the this algorithm. Therefore, to determine the tracking accuracy, four of the most significant modes were analyzed using the hybrid method. The frequency step size was reduced from 10 MHz to 250 kHz. Modes 1, 3, 4, and 5 were tracked and the result is shown in Fig. 6. It should be noted that the modes in Figs. 4 and 6 were produced using the same hybrid algorithm, the differences between the figures are only due to the different frequency resolutions. When solved using a step size of 500 kHz the identical modes 3 and 4 were split into two separate modes. Differences in mesh density were found to have a significant impact on the required frequency resolution.

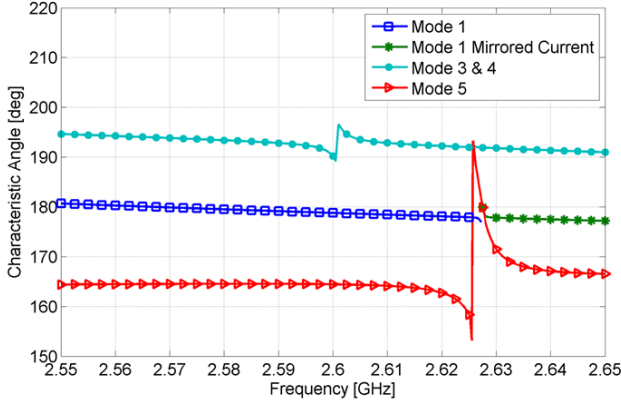


Fig. 6: Mode-tracking using current based hybrid method for described structures using increased frequency resolution (250 kHz frequency step size).

The performance of the hybrid tracking algorithm with fine frequency step closely matches that of the proposed far-field tracking algorithm. The analysis of mode 1 shows an extremely high Q antiresonance at 2.629 GHz. Before and after the antiresonance, the currents are mirrored across the symmetry of the structure, as can be seen in Fig. 7. The mirrored currents produce highly correlated far-field patterns with a slight reduction in current correlation; one degenerate mode was created rather than the two degenerate modes which were created in Fig. 4. However, due to the symmetry of the structure, relatively high current correlation, as well as the orthogonality of the far-field patterns at adjacent frequency points, the mode can be mapped to one constant mode. An antiresonance with a significantly lower Q is seen in modes 3 and 4; this mode is mapped to two separate modes in the far-field analysis due to one point of the analysis falling directly on the antiresonance. This degenerate mode, produced using the far-field algorithm, can be eliminated through increasing the frequency resolution to 2.5 MHz. Mode 5 produces the lowest Q antiresonance of the evaluated modes, allowing for identical mapping between the far-field analysis and the increased frequency resolution hybrid analysis. Using Matlab codes, the total computational time per frequency point was faster in the hybrid algorithm by 1.6 times. However, due to the required decrease in frequency step size needed to accurately track modes using the hybrid algorithm, the far-field algorithm ran 5.9 times faster for the described structure.

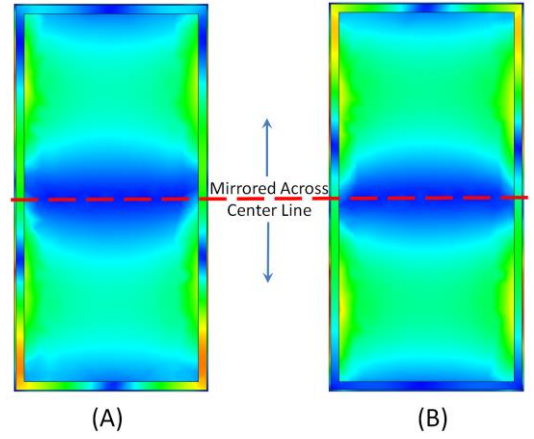


Fig. 7: Eigencurrents of Mode 1 at 2.62 GHz (A) and 2.64 GHz (B).

#### IV. CONCLUSION

CM analysis can be used to reduce the time of developing new and innovative mobile antennas as well as identifying the radiation potential of any given structure. Moreover, the benefit of CM aided antenna design can be further increased through accurate tracking of CMs across wide frequency bands. Previous tracking methods were based on the correlation of the eigencurrents across frequency. These current based solutions can fail, and become less accurate as the step size between subsequent frequencies is increased. In this work, it was shown that characteristic far-field patterns can be used to accurately track CM across wide frequency bands. This technique can greatly reduce the computational time required to accurately track eigenmodes across wide frequency bands, since much larger step sizes can be used as compared to the existing current based tracking algorithms.

#### ACKNOWLEDGMENT

The authors would like to thank FEKO for donating the commercial EM software used in this study.

#### REFERENCES

- [1] R. F. Harrington and J. R. Mautz, "Theory of characteristic modes for conducting bodies," *IEEE Trans. Antennas Propag.*, vol. 19, no. 5, pp. 622-628, Sep. 1971.
- [2] R. F. Harrington and J. R. Mautz, "Control of radar scattering by reactive loading," *IEEE Trans. Antennas Propag.*, vol. 20, no. 4, pp. 446-454, Jul. 1972.
- [3] B. D. Raines and R. G. Rojas, "Wideband tracking of characteristic modes," in *Proc. 5th Europ. Conf. Antennas Propag. (EUCAP'2011)*, Rome, Italy, Apr. 11-15, 2011, pp. 1978-1981.
- [4] M. Capek, P. Hazdra, P. Hamouz, and J. Eichler, "A method for tracking characteristic numbers and vectors," *Progress in Electromagnetics Research B*, vol. 33, pp. 115-134, 2011.
- [5] B. Raines and R. Rojas, "Wideband characteristic mode tracking," *IEEE Trans. Antennas Propag.*, vol. 60, no. 7, pp. 3537-3541, 2012.
- [6] D. J. Ludick, et al., "A hybrid tracking algorithm for characteristic mode analysis," in *Proc. Int. Conf. Electromagnetics in Advanced Appl. (ICEAA'2014)*, Aug. 3-8, 2014, pp. 455-458.
- [7] FEKO Suite 7.0, EM Software & Systems-S.A. (Pty) Ltd, Stellenbosch, South Africa, 2015, <http://www.feko.info>.
- [8] R. Tian, B. K. Lau, and Z. Ying, "Multiplexing efficiency of MIMO antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 10, no. 1, pp. 183-186, 2011.
- [9] R. G. Vaughan and J. B. Andersen, "Antenna diversity in mobile communications," *IEEE Trans. Veh. Technol.*, vol. 36, no. 4, pp. 149-172, Nov. 1987.