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Gislén, Lars

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SOLAR, LUNAR AND UMBRAL SIZES IN THE AL-KHWĀRIZMĪ AND PEURBACH ECLIPSE TABLES

Lars Gislén
Lund University, Dala 7163m 24297, Hörby, Sweden.
E-Mail: LarsG@vasterstad.se

Abstract: We analyze and compare the structure of two tables, the tables of al-Khwārizmī and George Peurbach's *Tabulae Eclypsium*, looking at the syzygy angular velocities of the Sun and the Moon and the angular radii of the Sun, the Moon and the lunar eclipse umbra. We show that part of the underlying structure has a mediaeval Indian origin. Finally, we compare recomputed tables with Peurbach's tables.

Keywords: al-Khwārizmī, Peurbach, solar and lunar velocities, solar, lunar and umbral size, solar and lunar models, syzygy, Classical Indian astronomy.

1 INTRODUCTION

Several Islamic and medieval astronomical works contain tables giving the apparent angular radius of the Sun, the Moon and the lunar eclipse umbra. Such tables can be found for instance in the tables of al-Khwārzimī (Suter, 1914), the Toledan Tables (Pedersen, 2002), and in the *Tabulae Eclypsium* (Peurbach, 1514) just to mention a few (Tables 1 and 2). The tables in the Toledan tables (Pedersen, 2002: 1416) are essentially identical to those of al-Khwārizmī. In later editions (1518, and 1524) of the Alfonsine Tables there are tables identical to the tables in the *Tabulae Eclypsium* although with a 6° spacing for the argument instead of 5°. These tables are a later addition to the original Alfonsine Tables, possibly adapted from Peubach's tables.

The fundamental input parameters for all these tables are the syzygy true angular velocities of the Sun and the Moon. Solar, lunar and umbral radii and solar and lunar velocities are further necessary components for computing eclipse size and duration as can be seen in for example in the *Tabulae Eclypsium* and Peter Apian's eclipse volvelle in the *Astronomicum Caesareum* (Gislén, 2016). The tables of al-Khwārizmī have earlier been analyzed by Neugebauer (1962) in his commentary to Suter's (1914) edition, where they point out the influence from Indian astronomy.

The leftmost columns of the tables show the solar or lunar argument in signs and degrees. The argument is the mediaeval term for the angular distance of the mean longitude from the apogee of a luminary. In the table from al-Khwārizmī then follow columns for the solar and lunar velocities. The next columns show solar, and lunar angular radii. The final pair of columns are used to calculate the size of the umbra, which depends on both the solar and lunar argument, but these are treated differently by al-Khwārizmī and Peurbach.

2 ANGULAR VELOCITIES

2.1 The Sun

In Ptolemy's model of the Sun the true longitude λ of the Sun is given by

$$\lambda = \lambda_{mean} - q(\gamma) \qquad (1)$$

where the last term is the equation of center which is a function of the solar argument, γ. The equation of center is in the *Almagest* calculated from

$$\tan q(\gamma) = e \sin \gamma / (1 + e \cos \gamma) \qquad (2a)$$

Pedersen (Pedersen,1974:150) gives an equiv--lent formula for $\sin q(\gamma)$.

The angular velocity is the time derivative of (1), which gives

$$v = v_{mean} - v_{\gamma}\, e\, (\cos \gamma + e)/(1 + e^2 + 2e \cos \gamma)$$
$$= v_{mean}\, (1 - e\, (\cos \gamma + e)/(1 + e^2 + 2e \cos \gamma) \qquad (3)$$

where v_{mean} is the mean velocity in longitude and v_{γ} is the velocity in argument that for the Sun is the same as v_{mean}.

2.2 The Moon

For the Moon there is a choice between Ptolemy's first lunar model that is a simple eccentric model and Ptolemy's final lunar model where the center of the epicycle is moved back and forth with a crank mechanism with its position being a function of twice the elongation of the Moon from the Sun (Gislén, 2017: 156–157). For the first lunar model there is also a possibility to neglect second order terms in the eccentricity. This essentially the approximation used in the Indian *Sūryasiddhānta*. The lunar velocity is computed by taking the time derivative of expressions for the true longitude. In the final lunar model the crank mechanism does not influence the true syzygy longitudes as the elongation of the Moon is zero but will introduce a correction to the lunar argument and modify the lunar velocity. This results in three options for the lunar syzygy angular velocity:

Table 1: Part of a table from al-Khwārizmī (after Suter, 1914: 175).

Gedval elbuht (CO)
[Tabula motus veri Solis et Lunae]

C 76v, O 133v.

Semitae numerorum				Harket elscems fil zahat Motus Solis in hora[1])		Harket[2]) Lunae in hora		Dimidium rotae Solis (O)		Dimidia quantitas rotae Lunae[3]) (O)		Primum dimidium rotae Draconis (O)		Secundum dimidium rotae Draconis (O)	
Sig.	Gr.	Sig.	Gr.	Min.[4])	Sec.[5])	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
0	1	11	29	2	22	30	12	15	40	14	38	11	52	48	19
0	2	11	28	2	22	30	12	15	40	14	38	11	52	48	20
0	3	11	27	2	22	30	12	15	40	14	39	11	52	48	20
0	4	11	26	2	22	30	13	15	40	14	39	11	52	48	20
0	5	11	25	2	22	30	13	15	40	14	39	11	52	48	20
0	6	11	24	2	22	30	13	15	40	14	39	11	52	48	21
0	7	11	23	2	22	30	13	15	40	14	39	11	52	48	21
0	8	11	22	2	22	30	14	15	40	14	39	11	52	48	22
0	9	11	21	2	22	30	14	15	40	14	39	11	52	48	22
0	10	11	20	2	22	30	14	15	40	14	39	11	52	48	23
0	11	11	19	2	22	30	15	15	40	14	40	11	52	48	24
0	12	11	18	2	22	30	15	15	40	14	40	11	52	48	25
0	13	11	17	2	22	30	16	15	41	14	40	11	52	48	26
0	14	11	16	2	22	30	17	15	41	14	40	11	53	48	27
0	15	11	15	2	23	30	17	15	41	14	41	11	53	48	28
0	16	11	14	2	23	30	18	15	41	14	41	11	53	48	29
0	17	11	13	2	23	30	19	15	41	14	42	11	53	48	30
0	18	11	12	2	23	30	20	15	41	14	42	11	53	48	32
0	19	11	11	2	23	30	21	15	41	14	43	11	53	48	34
0	20	11	10	2	23	30	22	15	41	14	43	11	53	48	35
0	21	11	9	2	23	30	23	15	42	14	44	11	53	48	37
0	22	11	8	2	23	30	24	15	42	14	44	11	54	48	39
0	23	11	7	2	23	30	25	15	42	14	45	11	54	48	41
0	24	11	6	2	23	30	27	15	42	14	45	11	54	48	43
0	25	11	5	2	23	30	28	15	43	14	46	11	54	48	44[b])
0	26	11	4	2	23	30	29	15	43	14	47	11	54	48	47
0	27	11	3	2	23	30	30	15	43	14	47	11	55	48	49
0	28	11	2	2	23	30	32	15	44	14	48	11	55	48	51
0	29	11	1	2	23	30	33	15	44	14	48	11	55	48	53
1[a])	0	11	0	2	23	30	34	15	44	14	49	11	55	48	55

[1]) Harket .... hora] O, Motus solis in hora. harket elihems ficaht C — [2]) Harket] O, motus C — [3]) Lunae] Solis O — [4]) Min.] O, dak. C, *itemque in cet. colum.* — [5]) Sec.] O, theniae C, *itemque in cet. colum.*

[a]) 1] 0 CO — [b]) 44] O, 64 C.

a) Exact first model

$v = v_{mean} - v_\gamma\, e\, (\cos\gamma + e)/(1 + e^2 + 2e\cos\gamma)$ (4a)

where now the velocity in argument, $v\gamma$, is different from the mean velocity

b) Approximate first lunar model

$v = v_{mean} - v_\gamma\, e \cos\gamma$ (4b)

c) Final lunar model

$v = v_{mean} - (v_\gamma + 0.3\, v_\eta)\, e\, (\cos\gamma + e)/(1 + e^2 + 2e\cos\gamma)$ (4c)

where v_η is the velocity in elongation. The term $0.3v_\eta$ is generated by the crank mechanism (Gislén, 2017: 157).

Relations (1)–(4) are modern formulae and Peurbach and al-Khwārizmī used other methods to arrive at equivalent results. Ptolemy used the chord function instead of the sine function and in Indian mathematics the tangent function was not used. The derivative of the equation of center, $q(\gamma)$, can be numerically calculated from a table with q tabulated for every degree by using the

Table 2: An eclipse table from the *Tabulae Eclypsium* (after Peurbach, 1514).

Tabula Semidiametroꝝ Luminariũ ꝛ Umbre.

Lineenumeri cõmunes.				Semi diame ter visual Solis		Semi diamè ter visual Lune.		Semi diame ter vmbre		Varia tio vm bre mi nuēda	
Sig	G	Sig	G	mi	sa	mi	sa	mi	sa	mi	sa
0	0	12	0	15	40	14	30	37	42	0	0
0	5	11	25	15	40	14	31	37	44	0	0
0	10	11	20	15	41	14	32	37	47	0	0
0	15	11	15	15	41	14	34	37	51	0	1
0	20	11	10	15	42	14	36	37	56	0	2
0	25	11	5	15	43	14	38	38	3	0	3
1	0	11	0	15	45	14	41	38	11	0	4
1	5	10	25	15	47	14	45	38	21	0	5
1	10	10	20	15	49	14	49	38	32	0	6
1	15	10	15	15	51	14	54	38	45	0	7
1	20	10	10	15	53	14	59	38	59	0	8
1	25	10	5	15	55	15	5	39	14	0	10
2	0	10	0	15	58	15	12	39	31	0	12
2	5	9	25	16	0	15	19	39	49	0	14
2	10	9	20	16	3	15	26	40	8	0	16
2	15	9	15	16	6	15	34	40	28	0	18
2	20	9	10	16	9	15	42	40	49	0	21
2	25	9	5	16	12	15	50	41	11	0	23
3	a	9	0	16	15	15	59	41	33	0	26
3	5	8	25	16	18	16	8	41	56	0	28
3	10	8	20	16	22	16	17	42	21	0	31
3	15	8	15	16	25	16	27	42	47	0	33
3	20	8	10	16	28	16	37	43	13	0	36
3	25	8	5	16	32	16	47	43	38	0	38
4	0	8	0	16	35	16	56	44	2	0	41
4	5	7	25	16	38	17	5	44	26	0	43
4	10	7	20	16	41	17	14	44	49	0	45
4	15	7	15	16	44	17	22	45	11	0	47
4	20	7	10	16	46	17	30	45	31	0	49
4	25	7	5	16	48	17	38	45	50	0	51
5	0	7	0	16	50	17	44	46	7	0	53
5	5	6	25	16	51	17	49	46	22	0	54
5	10	6	20	16	52	17	54	46	34	0	54
5	15	6	15	16	53	17	58	46	44	0	55
5	20	6	10	16	54	18	1	46	51	0	55
5	25	6	5	16	55	18	3	46	55	0	56
6	0	6	0	16	55	18	4	46	57	0	56
										0	

with q tabulated for every degree by using the difference (Goldstein, 1994)

$(q(\gamma + 1°) - q(\gamma - 1°))/2$

The parameters used by Peurbach are v_{mean} = 32.941′/hour, v_γ = 32.662′/hour, and v_η = 30.477′/hour (Gislén, 2017). All of these parameters can be extracted from the Ratdolt (1483).

2.3 The Velocity Tables

In al-Khwārizmī's table the solar velocity is well reproduced by taking e = 2.3/60 (Suter, 1914: 175–180) and in *Tabulae Eclypsium* there is a separate table of solar velocities that is well described by the same value of the solar eccentricity (Peurbach, 1514).

In the tables of al-Khwārizmī (Suter, 1914: 175–180) the lunar velocities are exactly reproduced by using the approximate first lunar model (4b) with the product $v_\gamma \cdot e$ = 2′ 44″. With the mean value of the lunar argument velocity being 32′ 56″ this gives $e \approx 4.98/60$.

The table of the true lunar velocities in the *Tabulae Eclypsium* fits the exact first lunar mo-

Table 3: The syzygy lunar velocities used by Peurbach for calculating the lunar radius and the true syzygy times.

Argument	Velocity/hour ′	″
0	29	38
5	29	38
10	29	40
15	29	43
20	29	47
25	29	52
30	29	59
35	30	6
40	30	15
45	30	25
50	30	36
55	30	48
60	31	1
65	31	15
70	31	30
75	31	46
80	32	2
85	32	20
90	32	38
95	32	57
100	33	16
105	33	35
110	33	55
115	34	14
120	34	33
125	34	52
130	35	10
135	35	28
140	35	44
145	35	59
150	36	12
155	36	24
160	36	34
165	36	42
170	36	48
175	36	51
180	36	52

del with e = 5.16/60 but for the calculation of the angular radius of the Moon Peurbach uses the exact final lunar model with the same eccentricity (see Table 3). These lunar velocities show a larger variation with lunar argument than the velcities derived from the simpler models and are not found explicitly anywhere in the *Tabulae Eclypsium*. These lunar velocities are also used for the tables in the *Tabulae Eclypsium* that provide the correction for converting mean syzygy time to true syzygy time and are also used by Peter Apian for his syzygy volvelle in the *Astronomicum Caesareum* (Gislén, 2017).

3 THE RELATION BETWEEN ANGULAR RADIUS AND VELOCITY

In Classical Indian astronomy represented by the *Sūryasiddhānta* (Burgess, 2000: 144) the angular radius of the Sun and the Moon are assumed to be proportional to their respective angular velocities.[1] The assumption can be motivated by the following modern argument. The angular radius ρ is proportional to the inverse of the distance d from the Earth, which in turn in Ptolemy's model is given by

$$d = d_{mean}\sqrt{(1 + e^2 + 2e \cos \gamma)} \approx d_{mean} (1 + e \cos \gamma) \quad (5)$$

neglecting second order powers of the eccentricity. We further have in the same approximation that the angular radius will be proportional to

$$1/d \approx (1 – e \cos \gamma)/d_{mean} \quad (6)$$

Comparing this with (3) we see that in this approximation the angular radius ρ and angular velocity v are proportional for all values of the argument, and we can write

$$\rho = kv \quad (7)$$

with some constant of proportionality k.

For the Sun al-Khwārizmī has k_{sun} = 33/5 = 6.6 (Neugebauer, 1962: 105; Pedersen, 2002: 1415) and in the Toledan Tables the same value (Toomer, 1968: 83), where the solar velocity is expressed in arc minutes per hour and the angular radius in arc minutes. This value is used in the computations. The *Canones Azarchelis* (Pedersen, 1986(II): 223) gets the solar diameter by multiplying the hourly velocity in arc seconds by 2¹⁄₅ and dividing by 10 which is the same recipe as in the Toledan Tables.

For the angular radius of the Moon, the same kind of proportionality is assumed (Neugebauer, 1962: 105–107). Suter (1914: 79) gives k_{moon} = 29/60 ≈ 0.483, Toledan Tables (Pedersen, 2002: 1415) k_{moon} = 120/247 ≈ 0.485, and Neugebauer (1962: 106) gives the value 0,58,10 for the ratio of the lunar diameter in al-Khwārizmī, corresponding to a value of k_{moon} ≈ 0.485. Toomer cites 47/48 for the ratio of the lunar diameter and angular velocity giving a value of k_{moon} ≈ 0.4895 the same value is given in the *Canones Azarchelius* (Pedersen, 1986(II): 223). This is also the value given by al-Battani (Nallino, 1903(I) :97). This value gives the best fit to Peurbach's lunar an-gular radius and is used in the computations.

Using these relations it is now possible to calculate the angular radii of the Sun and the Moon given their velocities as a function of their respective arguments and their mean radii.

4 THE UMBRAL RADIUS

The columns for the umbral radius require some more elaborations. In Figure 1, S is the Sun, E, the Earth, D is the Sun–Earth distance. P is the umbral plane where the Moon is located and d the Earth–Moon distance. x is the distance from the umbral plane to the apex of the umbra. R is the solar radius and r the Earth radius. s is the umbral radius.

Referring to Figure 1 we have from similar triangles

$$s/x \approx r/(d + x) \quad (8)$$

where we have used that the umbral apex angle is small, of the order of 4′. We can now calculate the quantity x, the distance from the apex of the umbral cone and the Moon

$$x = s\, d/(r – s) \quad (9)$$

From similar triangles we have again

$$R/(D + d + x) = s / x = R/(D + d + s\, d/(r – s)) = (r – s)/d \quad (10)$$

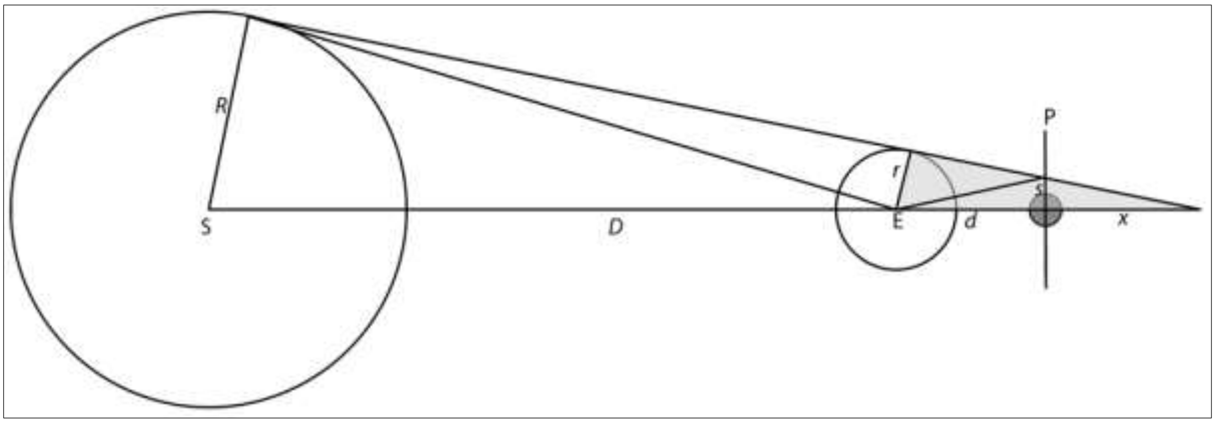

Figure 1: Lunar eclipse geometry (plot: Lars Gislén).

and finally get the angular size of the umbra

$\rho_{umbra} = s/d = r/d - \rho_{sun}\,(1 - r/R)$ (11)

The first term is a function of the lunar argument, the second one of the solar argument. In al-Khwārizmī (Neugebauer, 1962: 105–107) these terms are tabulated in two separate columns computed from the solar and lunar velocites per hour using that the lunar column is computed by $(8/5)v_{moon}$ and the solar column by $5v_{sun}$. We will now show that the factors 8/5 and 5 originate from Indian astronomy. Part of the argument have been given earlier by Suter (1914: 78ff), Neugebauer (1962:106ff), and al-Battānī (Nallino, 1903(I): 97).

The term r/d in (11), can be interpreted as the horizontal parallax of the Moon. In the *Sūrya-siddhānta,* the parallax of the Sun or the Moon is assumed to be given as their daily motion divided by 15. For the Moon, with the velocity expressed arc minutes per hour, we then have $24v_{moon}/ = (8/5)v_{moon}$ which is precisely the rule used.

The solar part of the umbra is

$\rho_{sun}\,(1 - r/R) = (33/5)v_{sun}\,(1 - r/R)$ (12)

The term r/R can be written $(r/D)(D/R)$. The first factor is the parallax of the Sun and again using the Indian parallax rule we get $r/D = (8/5)\,v_{sun}$ and the second factor is the inverse of the solar angular radius with $R/D = \rho_{sun} = (33/5)v_{sun}$. Inserting this in (12) we get

$(33/5)\,v_{sun}\,(1 - r/R) = (33/5)\,v_{sun}\,(1 - 8/33) = 5v_{sun}$ (13)

which again is precisely the rule used. The ratio of the solar radius and the Earth radius, R/r, will be $33/8 \approx 4.12$, different from Ptolemy's value of 5.5 (Pedersen, 1974: 213). It is evident that the rules in al-Khwārizmī's solar and lunar table for the size of the umbra have an Indian origin.

4.1 The Umbral Radius in the *Tabulae Eclypsium*

In the *Tabulae Eclypsium* the terms in (11) are reshuffled:

$\rho_{umbra} = r/d - \rho_{sun,0}(1 - r/R) - (\rho_{sun} - \rho_{sun,0})(1 - r/R)$ (14)

The quantity $\rho_{sun,0}$ is the angular radius of the Sun for zero solar argument. The combination of the first two terms is a function of the lunar argument only and the combination of the last two terms is a function of the solar argument only and being a difference between two terms it is small. In the *Tabulae Eclypsium* the combined first two terms is tabulated as 2.6 times the lunar angular radius and the second two terms as $5(v_{sun} - v_{sun,0})$ retaining their Indian origin and used as a correction to the umbra due to the varying solar distance and called *Variatio Umbra*. This reshuffling of the terms is conceptually and pedagogically important. It uses the fact that the size of the umbra is mainly determined by the distance to the Moon and that the dependence on the variation in solar distance just generates a small correction, less than one arc minute. The factor 2.6 originates from Ptolemy's *Almagest* (Pedersen, 1974: 209), given there as the size of the umbra relative to the size of the Moon[2] and by Ptolemy said to be the result of "... a great number of [similar] observations." It is also the value used in the Toledan Tables (Toomer, 1968: 83), by al-Battani (Nallino, 1903(I): 97), and by Canones Azarchelis (Pedersen, 1986(II): 224)

5 RESULTS

Using the eccentricities of the Sun and the Moon listed in the *Tabulae Eclypsium* and rules and formulae (13) and (14), solar, lunar, and umbral radii and *Variatio Umbra* were calculated.

Table 4 shows a comparison between the computed and tabular values in the *Tabulae Eclypsium*. The computed values are rounded to the nearest arc second. The first column is the solar or lunar argument in degrees. The units for the solar, lunar, and umbral radii are arc minutes and seconds except for the Variatio Umbra

Table 4: Comparison between computed and actual values in the *Tabulae Eclypsium*.

Argument	Solar Radius					Lunar radius					Umbral radius					Variatio Umbra		
	Computed		Peurbach			Computed		Peurbach			Computed		Peurbach			Computed	Peurbach	
0	15	40	15	40	0	14	30	14	30	0	37	43	37	42	1	0	0	0
5	15	41	15	40	1	14	31	14	31	0	37	44	37	44	0	0	1	–1
10	15	41	15	41	0	14	32	14	32	0	37	46	37	47	–1	0	1	–1
15	15	42	15	41	1	14	33	14	34	–1	37	50	37	51	–1	1	1	0
20	15	42	15	42	0	14	35	14	36	–1	37	55	37	56	–1	1	2	–1
25	15	44	15	43	1	14	38	14	38	0	38	2	38	3	–1	2	3	–1
30	15	45	15	45	0	14	41	14	41	0	38	10	38	11	–1	3	4	–1
35	15	46	15	47	–1	14	44	14	45	–1	38	19	38	21	–2	4	5	–1
40	15	48	15	49	–1	14	49	14	49	0	38	30	38	32	–2	6	6	0
45	15	50	15	51	–1	14	53	14	54	–1	38	43	38	45	–2	7	7	0
50	15	52	15	53	–1	14	59	14	59	0	38	57	38	59	–2	9	8	1
55	15	55	15	55	0	15	5	15	5	0	39	12	39	14	–2	11	10	1
60	15	57	15	58	–1	15	11	15	12	–1	39	29	39	31	–2	13	12	1
65	15	60	16	0	0	15	18	15	19	–1	39	47	39	49	–2	15	14	1
70	16	3	16	3	0	15	25	15	26	–1	40	6	40	8	–2	17	16	1
75	16	6	16	6	0	15	33	15	34	–1	40	26	40	28	–2	19	18	1
80	16	9	16	9	0	15	41	15	42	–1	40	47	40	49	–2	21	21	0
85	16	12	16	12	0	15	50	15	50	0	41	9	41	11	–2	24	23	1
90	16	15	16	15	0	15	59	15	59	0	41	32	41	33	–1	26	26	0
95	16	18	16	18	0	16	8	16	8	0	41	56	41	56	0	29	28	1
100	16	22	16	22	0	16	17	16	17	0	42	20	42	21	–1	31	31	0
105	16	25	16	25	0	16	27	16	27	0	42	45	42	47	–2	34	33	1
110	16	28	16	28	0	16	36	16	37	–1	43	10	43	13	–3	36	36	0
115	16	31	16	32	–1	16	46	16	47	–1	43	35	43	38	–3	39	38	1
120	16	34	16	35	–1	16	55	16	56	–1	43	59	44	2	–3	41	41	0
125	16	37	16	38	–1	17	4	17	5	–1	44	23	44	26	–3	43	43	0
130	16	40	16	41	–1	17	13	17	14	–1	44	46	44	49	–3	45	45	0
135	16	43	16	44	–1	17	22	17	22	0	45	8	45	11	–3	47	47	0
140	16	45	16	46	–1	17	30	17	30	0	45	29	45	31	–2	49	49	0
145	16	48	16	48	0	17	37	17	38	–1	45	48	45	50	–2	51	51	0
150	16	50	16	50	0	17	44	17	44	0	46	5	46	7	–2	52	53	–1
155	16	51	16	51	0	17	49	17	49	0	46	20	46	22	–2	54	54	0
160	16	53	16	52	1	17	54	17	54	0	46	33	46	34	–1	55	54	1
165	16	54	16	53	1	17	58	17	58	0	46	43	46	44	–1	56	55	1
170	16	55	16	54	1	18	1	18	1	0	46	50	46	51	–1	56	55	1
175	16	55	16	55	0	18	3	18	3	0	46	55	46	55	0	57	56	1
180	16	55	16	55	0	18	3	18	4	–1	46	56	46	57	–1	57	56	1

column where it is arc seconds. The last column of each group shows the residuals (computed minus tabular value) in arc seconds. The larger residuals for the umbra are the result of the multiplication factor 2.6. The small residuals, in arc seconds and allowing for rounding errors, give a strong indication that this was how the tables were calculated, confirming their Indian heritage.

6 NOTES

1. Kepler's first law states that the product of the distance from the focus and the angular velocity is constant in an elliptic orbit. Since the angular radius is inversely proportional to the distance, it follows that the angular radius and angular velocity are proportional.
2. By modern astronomy the mean angular radius of the Moon is 15′ 32″ and the mean umbra has an angular radius of 41′ 13″ giving a ratio of umbral radius to the Moon's radius of 2.65 (Gurnette and Woolley, 1961).

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Dr. Lars Gislén was born in Lund (Sweden) in 1938, and received a PhD in High Energy Particle Physics from the University of Lund in 1972. He worked in 1970 and 1971 as a researcher at the Laboratoire de Physique Théorique in Orsay (France) with models of high energy particle scattering. He has also done research on atmospheric optics and with physical modelling of biological systems and evolution.

He has worked as an Assistant Professor (University Lector) at the Department of Theoretical Physics at the University of Lund, where he gave courses on classical mechanics, electrodynamics, statistical mechanics, relativity theory, particle physics, cosmology, solid state physics and system theory. For more than twenty years he was a delegation leader and mentor for the Swedish team in the International Physics Olympiad and the International Young Physicists' Tournament.

Lars retired in 1983, and since then his interests have focused on Medieval European astronomy and on the astronomy and calendars of India and Southeast Asia. He has published more than 35 research papers in this field. He has also made public several spreadsheet tools implementing a number of astronomical models from Ptolemy to Kepler as well as computer tools for the calendars of India and Southeast Asia. He is a member of the IAU./Users/lars/Desktop/IMG_1068.jpeg.