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On the analysis of the exterior Calderón operator for a non-spherical geometry

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Let Ω be an open, bounded, domain in $\mathbb{R}^3$ with simply connected Lipschitz boundary Γ. The outward pointing unit normal is denoted $\hat{\boldsymbol{\nu}}$, and denote the exterior of the domain Ω by $\Omega_\mathrm{e} = \mathbb{R}^3 \setminus \overline{\Omega}$. See Figure 1 for a typical geometry.

Consider the following exterior problem:

$$
\begin{aligned}
&1)\ (\boldsymbol{E}_\mathrm{sc}, \boldsymbol{H}_\mathrm{sc}) \in H_\mathrm{loc}(\mathrm{curl}, \overline{\Omega}_\mathrm{e}) \times H_\mathrm{loc}(\mathrm{curl}, \overline{\Omega}_\mathrm{e})\\
&2)\ \begin{cases} \nabla \times \boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) = \mathrm{i}k\boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) \\ \nabla \times \boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) = -\mathrm{i}k\boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) \end{cases} \qquad \boldsymbol{x} \in \Omega_\mathrm{e}\\
&3)\ \begin{cases} \hat{\boldsymbol{x}} \times \boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) - \boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) = o(1/x) \\ \text{or} \\ \hat{\boldsymbol{x}} \times \boldsymbol{H}_\mathrm{sc}(\boldsymbol{x}) + \boldsymbol{E}_\mathrm{sc}(\boldsymbol{x}) = o(1/x) \end{cases} \qquad \text{as } x \to \infty \qquad \text{(Problem (E))}\\
&4)\ \hat{\boldsymbol{\nu}} \times \boldsymbol{E}_\mathrm{sc} \in H^{-1/2}(\mathrm{div}, \Gamma)
\end{aligned}
$$

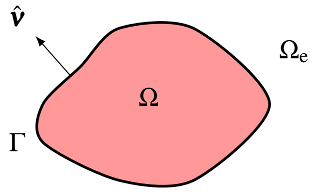

Figure 1. Typical geometry of the scattering problem in this paper.

where $x = |\boldsymbol{x}|$. This problem has a unique solution [1].

The exterior Calderón operator C^e is defined as $C^\mathrm{e} : \hat{\boldsymbol{\nu}} \times \boldsymbol{E}_\mathrm{sc} \mapsto \hat{\boldsymbol{\nu}} \times \boldsymbol{H}_\mathrm{sc}$, from the Sobolev space $H^{-1/2}(\mathrm{div}, \Gamma)$ onto itself, where the fields $\boldsymbol{E}_\mathrm{sc}$ and $\boldsymbol{H}_\mathrm{sc}$ satisfy Problem (E). The exterior Calderón operator is strongly related to the scattering problem for a PEC scatterer. The norm of this operator, which is finite in the space $H^{-1/2}(\mathrm{div}, \Gamma)$ but not in $L^2(\Gamma; \mathbb{C}^3)$, quantifies the largest amplification factor of the surface current for a given incident field on a PEC surface.

In this paper, we prove:

- There exists an intrinsic orthonormal basis on Γ. This basis generalizes the concept of vector spherical harmonics on a spherical surface to a general Lipschitz surface, and constitutes a natural basis for the analysis of the exterior scattering problem.
- Expressed in the expansion coefficients of this intrinsic basis, we find a representation map of the exterior Calderón operator. We prove that this map is invertible and we also give a simple expression of its inverse.
- As an operator in $H^{-1/2}(\mathrm{div}, \Gamma)$, the norm of the Calderón operator is finite. We find a simple way of computing this norm as the largest eigenvalue of a quadratic form using the representation map.
- The connection between the transition matrix (T-matrix) for a PEC obstacle and the corresponding Calderón operator.

References

[1] M. Cessenat, *Mathematical Methods in Electromagnetism*, ser. Series on Advances in Mathematics for Applied Sciences — Vol. 41. Singapore: World Scientific Publisher, 1996.