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2017

Document Version:

Peer reviewed version (aka post-print)

[Link to publication](#)

Citation for published version (APA):

Kristensson, G., Stratis, I., Wellander, N., & Yannacopoulos, A. (2017). *On the analysis of the exterior Calderón operator for a non-spherical geometry*. Paper presented at XXXIth URSI General Assembly and Scientific Symposium, 2017, Montréal, Canada.

Total number of authors:

4

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On the analysis of the exterior Calderón operator for a non-spherical geometry

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Let Ω be an open, bounded, domain in \mathbb{R}^3 with simply connected Lipschitz boundary Γ . The outward pointing unit normal is denoted $\hat{\mathbf{v}}$, and denote the exterior of the domain Ω by $\Omega_e = \mathbb{R}^3 \setminus \bar{\Omega}$. See Figure 1 for a typical geometry.

Consider the following exterior problem:

$$\begin{aligned}
 & 1) (\mathbf{E}_{sc}, \mathbf{H}_{sc}) \in H_{loc}(\text{curl}, \bar{\Omega}_e) \times H_{loc}(\text{curl}, \bar{\Omega}_e) \\
 & 2) \begin{cases} \nabla \times \mathbf{E}_{sc}(\mathbf{x}) = ik\mathbf{H}_{sc}(\mathbf{x}) \\ \nabla \times \mathbf{H}_{sc}(\mathbf{x}) = -ik\mathbf{E}_{sc}(\mathbf{x}) \end{cases} \quad \mathbf{x} \in \Omega_e \\
 & 3) \begin{cases} \hat{\mathbf{x}} \times \mathbf{E}_{sc}(\mathbf{x}) - \mathbf{H}_{sc}(\mathbf{x}) = o(1/x) \\ \text{or} \\ \hat{\mathbf{x}} \times \mathbf{H}_{sc}(\mathbf{x}) + \mathbf{E}_{sc}(\mathbf{x}) = o(1/x) \end{cases} \quad \text{as } x \rightarrow \infty \\
 & 4) \hat{\mathbf{v}} \times \mathbf{E}_{sc} \in H^{-1/2}(\text{div}, \Gamma)
 \end{aligned} \quad (\text{Problem (E)})$$

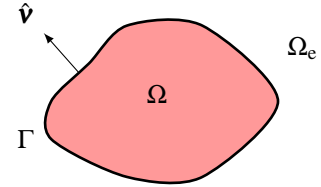


Figure 1. Typical geometry of the scattering problem in this paper.

where $x = |\mathbf{x}|$. This problem has a unique solution [1].

The exterior Calderón operator C^e is defined as $C^e : \hat{\mathbf{v}} \times \mathbf{E}_{sc} \mapsto \hat{\mathbf{v}} \times \mathbf{H}_{sc}$, from the Sobolev space $H^{-1/2}(\text{div}, \Gamma)$ onto itself, where the fields \mathbf{E}_{sc} and \mathbf{H}_{sc} satisfy Problem (E). The exterior Calderón operator is strongly related to the scattering problem for a PEC scatterer. The norm of this operator, which is finite in the space $H^{-1/2}(\text{div}, \Gamma)$ but not in $L^2(\Gamma; \mathbb{C}^3)$, quantifies the largest amplification factor of the surface current for a given incident field on a PEC surface.

In this paper, we prove:

- There exists an intrinsic orthonormal basis on Γ . This basis generalizes the concept of vector spherical harmonics on a spherical surface to a general Lipschitz surface, and constitutes a natural basis for the analysis of the exterior scattering problem.
- Expressed in the expansion coefficients of this intrinsic basis, we find a representation map of the exterior Calderón operator. We prove that this map is invertible and we also give a simple expression of its inverse.
- As an operator in $H^{-1/2}(\text{div}, \Gamma)$, the norm of the Calderón operator is finite. We find a simple way of computing this norm as the largest eigenvalue of a quadratic form using the representation map.
- The connection between the transition matrix (T-matrix) for a PEC obstacle and the corresponding Calderón operator.

References

- [1] M. Cessenat, *Mathematical Methods in Electromagnetism*, ser. Series on Advances in Mathematics for Applied Sciences — Vol. 41. Singapore: World Scientific Publisher, 1996.