



# LUND UNIVERSITY

## Estimation, Tracking, and Bounds in Close-Range Radar Systems

Jansson, Andreas

2026

[Link to publication](#)

*Citation for published version (APA):*

Jansson, A. (2026). *Estimation, Tracking, and Bounds in Close-Range Radar Systems*. Centre for Mathematical Sciences, Lund University.

*Total number of authors:*

1

### General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00



# Estimation, Tracking, and Bounds in Close-Range Radar Systems

---

ANDREAS JANSSON

Lund University  
Faculty of Engineering  
Centre for Mathematical Sciences  
Mathematics





# Estimation, Tracking, and Bounds in Close-Range Radar Systems



# Estimation, Tracking, and Bounds in Close-Range Radar Systems

Andreas Jansson



**LUND**  
UNIVERSITY

Doctoral dissertation for the degree of Doctor of Philosophy at the Faculty of Engineering at Lund University to be publicly defended on Friday the 23rd of October 2026 at 9:00 in the lecture hall MH:R at the Centre for Mathematical Sciences, Lund. Faculty opponent will be Fredrik Gustafsson.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |                                                                                             |       |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---------------------------------------------------------------------------------------------|-------|
| Organization<br><b>LUND UNIVERSITY</b><br>Centre for Mathematical Sciences<br>Mathematical Statistics<br>Box 118<br>SE-221 00 Lund, Sweden                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |  | Document name<br><b>DOCTORAL DISSERTATION</b>                                               |       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  | Date of disputation<br><b>2026-10-23</b>                                                    |       |
| Author(s)<br>Andreas Jansson                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |  | Sponsoring organization<br>Wallenberg AI, Autonomous Systems<br>and Software Program (WASP) |       |
| Title and subtitle<br>Estimation, Tracking, and Bounds in Close-Range Radar Systems                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |  |                                                                                             |       |
| Abstract<br><p>This thesis investigates the use of performance bounds in radar signal processing, with particular emphasis on the Cramér–Rao bound (CRB) and its misspecified and Bayesian variants, the MCRB and the BCRB. These bounds characterize fundamental limits on estimation accuracy and can therefore be used to assess the performance attainable under a given signal model. However, they can also be used prospectively, as design objectives for selecting sensors or planning future measurements.</p> <p>Simplified models and assumptions are vital when modeling radar systems in the real world, as an exact model may be too complicated to use in practice. Such simplifications introduce a trade-off between computational tractability and estimation accuracy. Paper A studies this trade-off for close-range radar reflections in the presence of time-varying clock offsets. The MCRB is used to quantify the effects of neglecting wavefront curvature and clock jitter, and a computationally efficient estimator is proposed to mitigate these effects.</p> <p>The remaining papers consider how performance bounds can be used to design the radar system itself. Paper B addresses sensor selection for the identification of deceptive jammers. The sensors are selected to improve the estimation of the deception range, which provides a means of distinguishing a retransmitted jamming signal from a genuine target reflection. Paper C extends the idea of sensor selection to drones by considering path planning for a MIMO radar drone swarm. The drone trajectories are planned using the BCRB to improve target localization while considering constraints on motion and resource usage. Finally, Paper D considers target tracking by a drone swarm in a GPS-denied environment, where the drone locations themselves are uncertain. Direct-path and target-reflected measurements are both used to estimate the sensor geometry and track the target.</p> <p>Taken together, the papers demonstrate two complementary uses of performance bounds. First, they provide a means of understanding how modeling assumptions, hardware imperfections, and uncertain sensor locations limit estimation accuracy. Second, they provide a framework for selecting sensors and controlling drones so that future measurements are as informative as possible.</p> |  |                                                                                             |       |
| Key words<br>Radar signals, Estimation theory, Target tracking, Sensor selection, Estimation bounds, Model uncertainty.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |                                                                                             |       |
| Classification system and/or index terms (if any)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  |                                                                                             |       |
| Supplementary bibliographical information                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |  | Language<br>English                                                                         |       |
| ISSN and key title<br>1404-0034<br>Doctoral Theses in Mathematical Sciences 2026:10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |  | ISBN<br>978-91-6858-042-5 (print)<br>978-91-6858-043-2 (pdf)                                |       |
| Recipient's notes                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |  | Number of pages<br>xx+146                                                                   | Price |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  | Security classification                                                                     |       |

I, the undersigned, being the copyright owner of the abstract of the above-mentioned dissertation, hereby grant to all reference sources the permission to publish and disseminate the abstract of the above-mentioned dissertation.

Signature \_\_\_\_\_

Date 2026-09-14

# Estimation, Tracking, and Bounds in Close-Range Radar Systems

Andreas Jansson



**LUND**  
UNIVERSITY

Thesis advisor: Andreas Jakobsson

## Copyright

p. i–27 © Andreas Jansson

Paper A © The authors

Paper B © The authors

Paper C © The authors

Paper D © The authors

## Published by:

Centre for Mathematical Sciences

Mathematical Statistics

Faculty of Engineering

Lund University

Lund 2026

ISBN: 978-91-6858-042-5 (print)

ISBN: 978-91-6858-043-2 (pdf)

ISSN: 1404-0034

LUTFMS-1054-2026

Doctoral Theses in Mathematical Sciences 2026:10

Printed in Sweden by Media-Tryck, Lund University, Lund 2026



Media-Tryck is a Nordic Swan Ecolabel  
certified provider of printed material.  
Read more about our environmental  
work at [www.mediatryck.lu.se](http://www.mediatryck.lu.se)

**MADE IN SWEDEN** 

For those who come after.  
— *Clair Obscur: Expedition 33*



# Contents

|                                                                                                        |           |
|--------------------------------------------------------------------------------------------------------|-----------|
| Abstract . . . . .                                                                                     | xi        |
| List of publications . . . . .                                                                         | xii       |
| Popular summary in English . . . . .                                                                   | xv        |
| Populärvetenskaplig sammanfattning på svenska . . . . .                                                | xvii      |
| Acknowledgements . . . . .                                                                             | xix       |
| <b>Estimation, Tracking, and Bounds in Close-Range Radar Systems</b>                                   | <b>1</b>  |
| 1 Introduction . . . . .                                                                               | 3         |
| 2 Radar Fundamentals . . . . .                                                                         | 6         |
| 3 On Cramér–Rao Bounds . . . . .                                                                       | 9         |
| 4 On Sensor Selection . . . . .                                                                        | 12        |
| 5 On Clock Offsets . . . . .                                                                           | 14        |
| 6 On Deceptive Jammers . . . . .                                                                       | 16        |
| 7 On Path Planning . . . . .                                                                           | 18        |
| 8 Conclusion and topics of future research . . . . .                                                   | 19        |
| <b>References</b>                                                                                      | <b>22</b> |
| <b>Scientific publications</b>                                                                         | <b>29</b> |
| Paper A: Locating Close-Range Radar Reflections Measured using Time-Varying<br>Clock Offsets . . . . . | 31        |
| Paper B: Optimal Sensor Selection for Jammer Identification in MIMO Radar<br>Systems . . . . .         | 63        |
| Paper C: On MIMO Radar Drone Swarm Path Planning . . . . .                                             | 87        |
| Paper D: Localization of Drones in a GPS-denied environment using a Drone Swarm                        | 115       |



## Abstract

This thesis investigates the use of performance bounds in radar signal processing, with particular emphasis on the Cramér–Rao bound (CRB) and its misspecified and Bayesian variants, the MCRB and the BCRB. These bounds characterize fundamental limits on estimation accuracy and can therefore be used to assess the performance attainable under a given signal model. However, they can also be used prospectively, as design objectives for selecting sensors or planning future measurements.

Simplified models and assumptions are vital when modeling radar systems in the real world, as an exact model may be too complicated to use in practice. Such simplifications introduce a trade-off between computational tractability and estimation accuracy. Paper A studies this trade-off for close-range radar reflections in the presence of time-varying clock offsets. The MCRB is used to quantify the effects of neglecting wavefront curvature and clock jitter, and a computationally efficient estimator is proposed to mitigate these effects.

The remaining papers consider how performance bounds can be used to design the radar system itself. Paper B addresses sensor selection for the identification of deceptive jammers. The sensors are selected to improve the estimation of the deception range, which provides a means of distinguishing a retransmitted jamming signal from a genuine target reflection. Paper C extends the idea of sensor selection to drones by considering path planning for a MIMO radar drone swarm. The drone trajectories are planned using the BCRB to improve target localization while considering constraints on motion and resource usage. Finally, Paper D considers target tracking by a drone swarm in a GPS-denied environment, where the drone locations themselves are uncertain. Direct-path and target-reflected measurements are both used to estimate the sensor geometry and track the target.

Taken together, the papers demonstrate two complementary uses of performance bounds. First, they provide a means of understanding how modeling assumptions, hardware imperfections, and uncertain sensor locations limit estimation accuracy. Second, they provide a framework for selecting sensors and controlling drones so that future measurements are as informative as possible.

## List of publications

This thesis is based on the following papers:

- Paper A    Andreas Jansson and Andreas Jakobsson, “Locating Close-Range Radar Reflections Measured using Time-Varying Clock Offsets”. *Signal Processing*, vol. 242, art.no. 110418, June 2026.
- Paper B    Andreas Jansson and Andreas Jakobsson, “Optimal Sensor Selection for Jammer Identification in MIMO Radar Systems”. *Submitted*.
- Paper C    Andreas Jansson and Andreas Jakobsson, “On MIMO Radar Drone Swarm Path Planning”. *Submitted*.
- Paper D    Andreas Jansson and Andreas Jakobsson, “Localization of Drones in a GPS-denied environment using a Drone Swarm”. *Submitted*.

Additional papers not included in the thesis:

1.    Andreas Jansson, Filip Elvander, Peter Almers, and Andreas Jakobsson, “Range-based Radar Model Structure Selection”, *28th European Signal Processing Conference*, pp. 2269–2273. Amsterdam, Netherlands, January 18–22, 2021.
2.    Andreas Jansson and Andreas Jakobsson, “Robust Localization of Close-Range Radar Reflections”, *57th Asilomar Conference on Signals, Systems, and Computers*, pp. 549–553. Pacific Grove, California, USA, October 29–November 1, 2023.
3.    Andreas Jansson and Andreas Jakobsson, “Close-Range Direction of Arrival Estimation in the Presence of Clock Jitter”, *49th IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 8671–8675. Seoul, Republic of Korea, April 14–19, 2024.
4.    Andreas Jansson and Andreas Jakobsson, “Optimal Sensor Selection for Jammer Identification”, *33rd European Signal Processing Conference*, pp. 2297–2301. Palermo, Italy, September 8–12, 2025.

## Author contributions

- Paper A    Andreas Jansson and Andreas Jakobsson, “Locating Close-Range Radar Reflections Measured using Time-Varying Clock Offsets”. *Signal Processing*, vol. 242, art.no. 110418, June 2026.  
**Joint main idea, derivations, numerical simulations, and write-up.**
- Paper B    Andreas Jansson and Andreas Jakobsson, “Optimal Sensor Selection for Jammer Identification in MIMO Radar Systems”. *Submitted*.  
**Joint main idea, derivations, numerical simulations, and write-up.**
- Paper C    Andreas Jansson and Andreas Jakobsson, “On MIMO Radar Drone Swarm Path Planning”. *Submitted*.  
**Joint main idea, derivations, numerical simulations, and write-up.**
- Paper D    Andreas Jansson and Andreas Jakobsson, “Localization of Drones in a GPS-denied environment using a Drone Swarm”. *Submitted*.  
**Joint main idea, derivations, numerical simulations, and write-up.**



## Popular summary in English

There are many methods for locating targets of interest with a radar system, but it is commonly viewed as a *parameter estimation* problem. Given a model and the parameters of a problem, such as the location and velocity of the target, the parameters can be inserted into the model to simulate the data. Parameter estimation is essentially the inverse of this. In other words, given a model and observed data, the objective is to find the parameters that best fit that data.

In principle, this could be done by testing every single possible parameter value to find which one fits the data best. In reality, however, this is unrealistic for the simple reason that there are too many possible parameter values; as the number of parameters grows, the search space grows exponentially and quickly becomes intractable.

This is made even more difficult by the fact that the data is never quite as pure as the model suggests. As the statistician George Box famously said: “All models are wrong, but some are useful.” Some things are completely infeasible to model in detail, such as the interaction of individual air molecules with the radar signal, or the background noise from outer space. These effects are commonly collected into what is called *noise*, meaning random disturbances added on top of the modeled data. One of the major analysis tools used throughout this thesis is called the Cramér–Rao bound (CRB). In simple terms, the CRB describes how accurately the parameters of a problem, such as the location of a target, can possibly be estimated from noisy data.

Some other things are feasible to model, but one may still decide not to. It is often easier or faster to find the sought-after parameters if the model is simplified. The results will usually be worse, but that difference is sometimes negligible. Whether this difference really is negligible can be determined with another tool used frequently in this thesis, namely the misspecified CRB (MCRB).

One practical problem in radar systems that is commonly ignored in models is that of faulty or unsynchronized clocks. Every radar system needs a clock to measure the range to targets, and when employing multiple sensors, these need to use the same time to work efficiently. But this is not necessarily true in practice; their time measurements may contain small, rapidly changing errors. The effects of jointly using several of these faulty clocks can be major; this phenomenon is explored in Paper A. Another point of interest that this thesis considers is optimal sensor placement: where should sensors be placed to yield the best results? This is of interest as it allows for improved performance, but also because it can reduce the number of sensors needed without much loss in accuracy. This is especially important for industrial applications, where it can significantly reduce costs with very little loss in performance. This concept is investigated in Paper B. The related problem of path planning for a drone swarm is considered in Paper C, whereas Paper D considers target and drone localization when the locations of the drones are uncertain.



# Populärvetenskaplig sammanfattning på svenska

Det finns många metoder för att lokalisera mål av intresse med ett radarsystem, men problemet ses ofta som ett *parameterskattningsproblem*. Om man har en modell och känner till parametrarna i ett problem, till exempel målets position och hastighet, kan parametrarna sättas in i modellen för att simulera data. Parameterskattning är i princip det omvända. Med andra ord: givet en modell och observerade data är målet att hitta de parametrar som passar bäst till dessa data.

I princip skulle detta kunna göras genom att testa varje möjligt parametervärde och se vilket som passar bäst till data. I praktiken är detta dock orealistiskt, av den enkla anledningen att det finns för många möjliga parametervärden. När antalet parametrar ökar så växer sökutrymmet exponentiellt och blir snabbt omöjligt att söka igenom.

Detta blir ännu svårare eftersom data aldrig är riktigt så ren som modellen antyder. Som statistikern George Box uttryckte det: "Alla modeller är fel, men vissa är användbara." Vissa saker är helt enkelt omöjliga att modellera i detalj, till exempel hur enskilda luftmolekyler påverkar radarsignalen, eller bakgrundsbruset från yttre rymden. Sådana effekter samlas ofta i det som kallas *brus*, alltså slumpmässiga störningar som läggs ovanpå de modellerade data. Ett av de viktigaste analysverktygen som används i den här avhandlingen kallas Cramér–Rao-gränsen (CRB). Enkelt uttryckt beskriver CRB hur noggrant parametrarna i ett problem, till exempel ett måls position, kan skattas från brusiga data.

Andra saker är möjliga att modellera, men man kan ändå välja att inte göra det. Det är ofta enklare eller snabbare att hitta de sökta parametrarna om modellen förenklas. Resultaten blir vanligtvis sämre, men ibland är skillnaden försumbar. Om skillnaden verkligen är försumbar kan undersökas med ett annat verktyg som används ofta i den här avhandlingen, nämligen den misspecificerade CRB:n (MCRB).

Ett praktiskt problem som ofta ignoreras i modeller är felaktiga klockor. Varje radarsystem behöver en klocka för att mäta avstånd till mål. I praktiken är dessa klockor inte perfekta; deras tidsmätningar innehåller små fel som förändras snabbt. Effekterna av att använda flera sådana felaktiga klockor tillsammans kan vara stora. Detta fenomen undersöks i Artikel A.

Ett annat ämne som behandlas i den här avhandlingen är optimal sensorplacering: var bör sensorer placeras för att ge bäst resultat? Detta är intressant eftersom det kan ge bättre prestanda, men också eftersom det kan minska antalet sensorer som behövs utan att noggrannheten försämras särskilt mycket. Detta är extra viktigt för industriella tillämpningar, där det kan sänka kostnaderna betydligt med mycket liten förlust i prestanda. Detta undersöks i Artikel B. Det närliggande problemet att planera rörelserna för en drönarsvärm behandlas i Artikel C, medan Artikel D behandlar lokalisering av både drönarna och ett mål när drönarnas egna positioner är osäkra.



## Acknowledgements

First and foremost, I would like to thank my supervisor, Prof. Andreas Jakobsson. The immense effort you have put into always being available and supporting me throughout my PhD studies is greatly appreciated. Discussing the ideas and logic behind a problem, rather than only the dry equations, is always fun and inspiring. Thank you also for bringing some much-needed humor and personality to the world of mathematics!

I would also like to thank all my colleagues at the Division of Mathematical Statistics. In particular, thank you Niloofar, David, Tadas, Malin, and Klara, for the many group presentations and discussions. And thank you to my coauthors, Filip and Peter, for the enjoyable and rewarding collaboration at the early stage of my PhD.

I am grateful to Acconeer for supporting my PhD, and especially to my two supervisors there, Peter and Rikard.

I would also like to thank my high-school physics teacher, Karin, for putting in extra effort to challenge me. This played a major role in developing my genuine enjoyment of studying mathematics and physics.

A special thank you to my mom and dad for sparking my interest in how mathematics can be used to understand and solve real-world problems. Without that, none of this would have even begun.

Thank you to my brother, Daniel, for giving me a chance to unwind through our evening chats and games.

Finally, I would like to thank my wonderful girlfriend and global optimum, Sofia, for keeping me sane during the stressful times. Thank you for listening to me rant about the misspecified Cramér–Rao lower bound!

## Funding

This thesis was supported in part by the Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundations.



# Estimation, Tracking, and Bounds in Close-Range Radar Systems



# 1 Introduction

The underlying principle of radar systems, transmitting an electromagnetic wave and interpreting its reflection, has remained essentially unchanged since the technology's inception in the early 1900s. Despite this, the range of problems radar is asked to solve continues to expand. In the early days of radar, its purpose was only to detect whether a target is present or not and potentially estimate the range to that target. Then, the technology evolved to estimate the so-called direction-of-arrival (DOA), i.e., the angle to the target. Today, radar is used to guide autonomous vehicles through traffic, to monitor a patient's breathing without physical contact, to map the subsurface structure of soil and ice, to steer a robotic lawnmower around a garden, and many more things. This thesis is concerned with some of the practical problems that arise across many of these applications, and especially how those problems affect the performance of the system. In all papers, the central question is essentially the same: How well can the parameters of interest be estimated? And how does this depend on the sensor geometry or the model used?

These questions motivate the three themes in the title of this thesis: estimation, tracking, and bounds. Estimation refers to the problem of inferring unknown parameters from radar measurements, and to the practical difficulties that may arise from issues such as imperfect clocks, simplified propagation models, and uncertain sensor locations. Tracking extends estimation from a single snapshot to following that estimate over time, where a system combines new measurements with previously gathered information. Bounds, finally, refers to a fundamental limit on the performance of estimators based on the specified model. Throughout this thesis, variants of the Cramér–Rao lower bound (CRB) are used for this purpose, but the main purpose of these bounds in this thesis is one step further. Instead of only using the CRB as a bound on the optimal estimation performance, it is here often used as a metric to be optimized. This allows, for instance, the placement of sensors to be optimized to yield the best possible CRB.

## Motivation

Much of classical radar theory is built around a set of convenient simplifications. Targets are often assumed to lie in the far field of the sensor array, allowing the wavefront arriving at the array to be treated as planar rather than curved. Sensors are generally assumed to share a common, perfectly synchronized clock, so that measured time delays can be directly related to physical distances.

These simplifications are reasonable in many settings, but it is important to realize that these assumptions are often implicitly made and to understand when those assumptions may no longer hold. For instance, the far-field assumption becomes increasingly strained as targets get closer to the radar system, potentially introducing both bias and variance to parameter estimates. Similarly, radar systems with cheap electronics may not have a reliable clock. Depending

on what measurements are used from this system, this may be irrelevant, causing merely insignificant changes, or it may cause catastrophic failures, posing a substantial problem.

A second, related challenge arises with the use of distributed and multiple-input multiple-output (MIMO) radar systems, in which several transmitters and receivers observe a target from different viewing angles. Systems with such spatial diversity provide clear benefits: improved localization accuracy, reduced sensitivity to unfavorable viewing angles, and robustness against weak reflections in any single channel. These benefits, however, come with a design question: given a set of candidate sensor locations, and given constraints on cost, power, or the number of sensors that can be deployed, which sensors should actually be used, and where should they be placed? A variant of this question arises when the sensors themselves are mobile, as in a swarm of drones each with a radar transmitter and/or receiver.

## Performance bounds

The problems outlined above, namely clock jitter, model mismatch, sensor selection, and swarm-based tracking, may seem rather unrelated at first. In this thesis, we will examine the use of the CRB as a tool for addressing these aspects in different ways.

In its classical form, the CRB provides a lower bound on the covariance of any unbiased estimator. As mentioned earlier, this means that it is a fundamental limit on how well parameters can be estimated, based on the model used and quality of the measurements received. Because the CRB depends explicitly on the sensor geometry, the signal model, and the noise statistics, the bound can be computed and analyzed before any data is collected or any estimator is designed. This makes it a valuable tool for system design: rather than asking how well a specific estimator performs, one can ask how a system should be configured, e.g., where to place the sensors or what waveform to transmit, in order for a statistically efficient estimator to achieve optimal performance.

Two extensions of the classical bound play a central role in this thesis. The first, the misspecified CRB (MCRB), concerns the case when the model used for estimation does not match the model that actually generated the data, i.e., a misspecified (or mismatched) model. This is often the situation when simplifying assumptions no longer hold, such as when a far-field model is used for a target that is too close to the radar system. Sometimes, it is a deliberate decision to use a simpler misspecified model in order to allow for a computationally more efficient solution, thereby voluntarily accepting some performance loss in order to speed up the estimation process. By comparing the MCRB with the classical CRB, one may then evaluate how much of the potential performance is lost due to the model mismatch. The second extension is the Bayesian CRB (BCRB), which addresses the scenario where there is some prior information about an unknown parameter. This is often the case for tracking problems, as the tracker has gathered information from previous time steps.

A central theme in this thesis is that these bounds can be evaluated, and optimized, before the

measurement is even collected. This is what allows performance bounds to be used not merely to describe performance, but also to optimize performance. A bound that can be evaluated for a hypothetical future sensor configuration can also be optimized over the space of such configurations, turning this analysis tool into a planning and design tool. This idea of using the CRB as the objective function to optimize over candidate configurations is explored in different ways throughout the thesis.

## Outline and contributions of the thesis

The remainder of this thesis is organized as follows. The introductory chapters 2–7 provide background for the four papers presented in the thesis, and several concepts mentioned above are also expanded on in more mathematical detail. The scientific contributions of the thesis are then presented in four appended papers.

Paper A considers the problem of locating close-range radar reflections in the presence of time-varying clock offsets between transmitters and receivers. The paper derives the CRB and MCRB that are used to quantify the degradation caused both by such clock jitter and by the commonly used far-field approximation. Furthermore, it proposes a computationally efficient, robust DOA estimator that partially compensates for both effects, yielding substantially improved localization accuracy for close-range targets compared to conventional estimators.

Paper B addresses the problem of optimal sensor selection for identifying deception jammers in a MIMO radar system. By exploiting the deception range that a jammer necessarily introduces when retransmitting an intercepted signal, the paper formulates a sensor selection scheme based on a CRB weighted to emphasize the importance of this deception range parameter. Together with this, an efficient algorithm for estimating both the deception range and deception Doppler is introduced.

Paper C turns to the problem of path planning for a drone-swarm MIMO radar system tracking a moving target. The target is tracked with a Kalman filter, and the information from this tracker is then used to compute a BCRB, which in turn is used as the performance metric in the objective. Further, penalties for transmitting pulses and accelerating are added to the objective to allow for balancing the importance of localization accuracy against resource usage. A planner then optimizes this objective, subject to realistic constraints on the acceleration, velocity, and proximity to the target.

Paper D addresses the related problem of jointly localizing the drones in a swarm and tracking a target when external positioning information such as GPS is unavailable or uncertain. By exploiting both the direct signal path between drones and the paths reflected off the target, the paper proposes estimators with different trade-offs between accuracy and computational cost. The results illustrate the importance of using the near-field model in order to explicitly capture the position offset, and not model it as a common delay for all drones.

Finally, in Chapter 8, we conclude on the work by examining what these contributions suggest about the broader use of performance bounds as design tools in radar systems, and to outline directions in which this work might be extended.

## 2 Radar Fundamentals

Radar, originally an acronym for radio detection and ranging, dates back to the early 1900s, but experienced major developments in the 1930s and 1940s, prior to and during World War II. It is a method of using electromagnetic waves, specifically radio waves, to remotely sense the position, velocity, and other characteristics of a target [1, 2]. Radio signals cover a wide range of wavelengths, all of which are significantly longer than that of visible light; typically, radar applications use signals in the microwave range, corresponding to wavelengths approximately between 0.1 mm and 30 cm. In this chapter, we will in some further detail examine the measurement setup and the signal model that results from the transmission of the radar pulse, the reflection on some object in the path of the signal waveform (generally denoted a target in radar terminology), and then backscattered to the receiver, which may be the same system that transmitted the pulse, or a separate receiving unit.

### 2.1 Propagation Geometry

Consider a transmitter (TX) located at position  $\mathbf{p}^{\mathcal{T}}$ , a receiver (RX) located at  $\mathbf{p}^{\mathcal{R}}$ , and a point target located at  $\mathbf{x}$ . The transmitted signal will then first travel the distance

$$d^{\mathcal{T}}(\mathbf{x}) = \|\mathbf{x} - \mathbf{p}^{\mathcal{T}}\|_2 \quad (1)$$

from the transmitter to the target and then, after being reflected on the target, the distance

$$d^{\mathcal{R}}(\mathbf{x}) = \|\mathbf{x} - \mathbf{p}^{\mathcal{R}}\|_2 \quad (2)$$

from the target to the receiver. The corresponding propagation delay is then [3]

$$\tau(\mathbf{x}) = \frac{d^{\mathcal{T}}(\mathbf{x}) + d^{\mathcal{R}}(\mathbf{x})}{c}, \quad (3)$$

where  $c$  denotes the speed of light. When the transmitter and receiver are located at different positions, the resulting delay, termed a bistatic delay, restricts the target to an ellipsoid with the transmitter and receiver as its foci. From one such transmission, one may thus not determine a unique location of the target; in order to do so, several different transmit and/or receive locations are generally required.

### 2.2 Pulsed Radar Signal Model

A pulsed radar periodically transmits pulses, whereas its receiving units listen for returning reflections to measure the time delay introduced from the pulse traveling from the transmitter

to the target and then on to the receiver. Let  $\phi(t)$  denote the complex baseband signal of one transmitted pulse. To form the transmitted radar signal, the baseband signal is modulated onto a carrier with angular frequency  $\omega$ , yielding

$$s(t) = \phi(t)e^{j\omega t}. \quad (4)$$

The corresponding physical waveform is given by the real part of  $s(t)$ , but the complex representation is retained as it is a more convenient model. The received signal may be modeled as a scaled and delayed copy of the transmitted waveform,

$$x(t) = \alpha s(t - \tau(\mathbf{x})) + n(t) = \alpha \phi(t - \tau(\mathbf{x})) e^{j\omega(t - \tau(\mathbf{x}))} + n(t), \quad (5)$$

where the complex coefficient  $\alpha$  accounts for the target reflectivity, propagation loss, and other constant scaling and phase factors, whereas  $n(t)$  denotes an additive measurement noise. Demodulating the received signal by multiplying it by  $e^{-j\omega t}$  and low-pass filtering yields the baseband representation [4]

$$\tilde{x}(t) = \alpha \phi(t - \tau(\mathbf{x})) e^{-j\omega\tau(\mathbf{x})} + v(t), \quad (6)$$

where  $v(t)$  denotes the demodulated measurement noise. The delay thus affects both the pulse envelope and the phase of the carrier. Although the envelope is typically used to resolve the delay over a larger interval, the carrier phase can provide considerably finer information when its ambiguity can be resolved.

The pulses are commonly repeated with a pulse repetition interval  $T_r$ . Samples within a pulse interval are referred to as fast-time samples and are primarily used to estimate the time delay between the signal transmission and reception, whereas the sequence of pulses forms the slow-time dimension and may be used to observe changes in the target state and estimate Doppler shifts.

### 2.3 Matched Filtering

The received baseband signal is commonly passed through a so-called matched filter, i.e., the signal is matched to the transmitted pulse. In additive white Gaussian noise, the matched filter maximizes the output signal-to-noise ratio (SNR) at the delay for which the received and transmitted waveforms are aligned. This delay may equivalently be viewed as the distance the waveform has propagated, creating the notion of range bins. For the model in (6), and omitting the noise term, the matched-filter output may be expressed as [4]

$$y(\tau) = \int \tilde{x}(t) \phi^*(t - \tau) dt = \alpha e^{-j\omega\tau(\mathbf{x})} \Lambda(\tau(\mathbf{x}) - \tau), \quad (7)$$

where

$$\Lambda(\Delta\tau) = \int \phi(t) \phi^*(t + \Delta\tau) dt \quad (8)$$

is the autocorrelation function of the transmitted pulse. The magnitude of (7) is maximized when the tested delay  $\tau$  agrees with the propagation delay  $\tau(\mathbf{x})$ , i.e., when  $\tau = \tau(\mathbf{x})$ . The location of the peak can therefore be used to estimate the propagation distance.

If the target or sensors move, the propagation distance varies with time. Over a sufficiently short observation interval, this variation may be represented by a Doppler shift  $\nu$ , yielding the approximate baseband model

$$\tilde{x}(t) = \alpha \phi(t - \tau) e^{-j\omega\tau} e^{j2\pi\nu t} + v(t). \quad (9)$$

The Doppler shift is proportional to the rate of change of the total path length from TX to target and back to RX. Delay and Doppler estimates thus contain information about the target position and velocity, respectively. With repeated pulses, the delay is primarily resolved using the fast-time samples, whereas the phase progression across slow time may be used to estimate the Doppler shift.

## 2.4 MIMO and Distributed Radar

A multiple-input multiple-output (MIMO) radar uses multiple TX and RX antennas. Provided that the transmitted waveforms can be separated at the receivers, a system with  $M$  transmitters and  $N$  receivers may provide up to  $MN$  distinguishable TX–RX channels. For transmitter  $m$  at location  $\mathbf{p}_m^{\mathcal{T}}$  and receiver  $n$  at  $\mathbf{p}_n^{\mathcal{R}}$ , the time delay for a target at  $\mathbf{x}$  is [5]

$$\tau_{mn}(\mathbf{x}) = \frac{\|\mathbf{x} - \mathbf{p}_m^{\mathcal{T}}\|_2 + \|\mathbf{x} - \mathbf{p}_n^{\mathcal{R}}\|_2}{c}. \quad (10)$$

In a MIMO system, each channel observes the target through a different propagation path. When the antennas are spatially distributed, these paths provide spatial diversity and complementary information about the target. Such diversity can improve localization accuracy and reduce the sensitivity to an unfavorable viewing angle or a weak reflection in an individual channel.

However, there are not only advantages with having a distributed geometry; there are also several practical challenges. The signals measured by the different receivers must be associated with the correct transmitters, and the transmitter and receiver locations must be sufficiently well known. This is a complicated problem that deserves attention in its own right. In order to keep the presentation focused, we have in this work assumed that such an association has already been made, using a method such as the one presented in [6].

In a distributed system, the receivers may also observe the signal propagating directly from a transmitter, in addition to the signal reflected by a target. This direct-path delay depends on the transmitter and receiver positions but not on the target position,

$$\tau_{mn}^{\text{d}} = \frac{\|\mathbf{p}_m^{\mathcal{T}} - \mathbf{p}_n^{\mathcal{R}}\|_2}{c}. \quad (11)$$

Direct-path measurements therefore provide information about the relative sensor geometry, which is especially useful when sensor locations are uncertain.

### 3 On Cramér–Rao Bounds

The Cramér–Rao lower bound (CRB) [7, 8] characterizes the best estimation accuracy that is attainable by an unbiased estimator for a given statistical model, forming a theoretical lower bound on how well a parameter may possibly be estimated. This causes such bounds to have notable usefulness as a benchmark when evaluating an estimator and as a means of studying how the signal model, noise level, and measurement geometry affect the available information. Consider a measurement vector  $\mathbf{y}$  with probability density function (PDF)  $p(\mathbf{y}; \boldsymbol{\theta})$ , where  $\boldsymbol{\theta} \in \mathbb{R}^d$  is an unknown deterministic parameter vector. The Fisher information matrix (FIM) is defined as [9]

$$\mathbf{J}(\boldsymbol{\theta}) = \mathbb{E}_{\mathbf{y};\boldsymbol{\theta}} [\nabla_{\boldsymbol{\theta}} \log p(\mathbf{y}; \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T \log p(\mathbf{y}; \boldsymbol{\theta})], \quad (12)$$

where  $\mathbb{E}_{\mathbf{y};\boldsymbol{\theta}}[\cdot]$  denotes the expected value with respect to  $\mathbf{y}$  at the parameter value  $\boldsymbol{\theta}$ ,  $\nabla_{\boldsymbol{\theta}}$  the gradient with respect to  $\boldsymbol{\theta}$ , and  $(\cdot)^T$  the transpose. Under typical regularity conditions<sup>1</sup>, the FIM may also be written as

$$\mathbf{J}(\boldsymbol{\theta}) = -\mathbb{E}_{\mathbf{y};\boldsymbol{\theta}} [\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^T \log p(\mathbf{y}; \boldsymbol{\theta})]. \quad (13)$$

Notably, for independent observations  $\mathbf{y}_1, \dots, \mathbf{y}_N$ , their information matrices  $\mathbf{J}_1(\boldsymbol{\theta}), \dots, \mathbf{J}_N(\boldsymbol{\theta})$  are additive, yielding the total FIM

$$\mathbf{J}(\boldsymbol{\theta}) = \sum_{n=1}^N \mathbf{J}_n(\boldsymbol{\theta}). \quad (14)$$

Thus, if the observations are also identically distributed, the total information is  $N$  times the information contained in one observation.

Let  $\hat{\boldsymbol{\theta}}$  denote an unbiased estimator of the parameter vector  $\boldsymbol{\theta}$ , i.e.,  $\mathbb{E}[\hat{\boldsymbol{\theta}}] = \boldsymbol{\theta}$ . Its error covariance is then bounded according to [10, 11]

$$\mathbb{E} [(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T] \succeq \mathbf{J}^{-1}(\boldsymbol{\theta}), \quad (15)$$

where  $\mathbf{A} \succeq \mathbf{B}$  denotes that  $\mathbf{A} - \mathbf{B}$  is positive semidefinite. The matrix  $\mathbf{J}^{-1}(\boldsymbol{\theta})$  is referred to as the CRB. A large amount of Fisher information consequently corresponds to a small lower bound on the estimation variance. In the scalar case, this can simply be written as

$$\text{var}(\hat{\theta}) \geq \frac{1}{J(\theta)}. \quad (16)$$

---

<sup>1</sup>Typical sufficient conditions are that the likelihood is twice differentiable with respect to  $\boldsymbol{\theta}$ , that its support does not depend on  $\boldsymbol{\theta}$ , and that differentiation may be interchanged with integration.

Often, only some of the unknown parameters are of interest. In that case, it can be useful to partition the parameter vector as  $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T \boldsymbol{\theta}_2^T]^T$ , where  $\boldsymbol{\theta}_1$  contains the parameters of interest and  $\boldsymbol{\theta}_2$  contains nuisance parameters, and to accordingly partition the FIM as

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}. \quad (17)$$

The quantity of interest is then the upper-left block of the inverse FIM, denoted by  $[\mathbf{J}^{-1}]_{11}$ . Using the block-matrix inversion formula, this block can then be expressed as [12]

$$[\mathbf{J}^{-1}]_{11} = (\mathbf{J}_{11} - \mathbf{J}_{12}\mathbf{J}_{22}^{-1}\mathbf{J}_{21})^{-1}. \quad (18)$$

This construction is important and commonly used in radar localization, where unknown reflection coefficients, clock parameters, or deception parameters (more on these in the following) are often treated as nuisance parameters coupled to the target position.

### 3.1 The Misspecified Cramér–Rao Bound

The classical CRB presumes that the model used by the estimator coincides with the model that generated the measurements. However, in practice, it is not uncommon that estimators instead rely on simplified models, either by making simplifying assumptions or by deliberate omission of model complexities aiming to simplify the estimator. When such a simplification introduces a non-negligible model mismatch, the classical CRB computed from the assumed model no longer accurately describes the achievable estimation performance. In such a case, the misspecified CRB (MCRB) [13] (see also [14]) is instead more fitting to use as it allows for a mismatch between the true data model and the model used for estimation. A typical example is the use of a far-field model for a close-range target. Although the wavefront generated by any target at a finite distance is curved, the far-field approximation is accurate when this curvature is negligible across the sensor array. In practice, this is therefore a very commonly used approximation. As the target approaches the array, however, the curvature becomes increasingly significant, potentially introducing substantial bias and variance into the resulting parameter estimates. In a scenario like this, the MCRB can be compared with the CRB to find a point where the difference between the two models is negligible in terms of estimation performance. This point could then, for instance, be used as a threshold to switch estimation model when the target passes it.

Because the true distribution does not necessarily belong to the assumed model family, there may be no parameter value for which the two distributions coincide. One therefore typically introduces the pseudo-true parameter  $\boldsymbol{\theta}_0$ , defined as the parameter value for which the assumed distribution  $p_a(\mathbf{y}; \boldsymbol{\theta})$  is closest to the true distribution  $p_t(\mathbf{y})$ , i.e., [13, 14]

$$\boldsymbol{\theta}_0 = \arg \min_{\boldsymbol{\theta}} D_{\text{KL}}(p_t(\mathbf{y}) \parallel p_a(\mathbf{y}; \boldsymbol{\theta})) \quad (19)$$

$$= \arg \max_{\boldsymbol{\theta}} \mathbb{E}_{p_t} [\log p_a(\mathbf{y}; \boldsymbol{\theta})], \quad (20)$$

where  $D_{\text{KL}}(p\|q)$  denotes the Kullback–Leibler divergence between the PDFs  $p$  and  $q$ . Define

$$\mathbf{A}(\boldsymbol{\theta}_0) = \mathbb{E}_{p_t} [\nabla_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}}^T \log p_a(\mathbf{y}; \boldsymbol{\theta})]_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}, \quad (21)$$

$$\mathbf{B}(\boldsymbol{\theta}_0) = \mathbb{E}_{p_t} [\nabla_{\boldsymbol{\theta}} \log p_a(\mathbf{y}; \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T \log p_a(\mathbf{y}; \boldsymbol{\theta})]_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}. \quad (22)$$

The MCRB then has the form [13]

$$\mathbf{C}_{\text{MCRB}}(\boldsymbol{\theta}_0) = \mathbf{A}^{-1}(\boldsymbol{\theta}_0) \mathbf{B}(\boldsymbol{\theta}_0) \mathbf{A}^{-1}(\boldsymbol{\theta}_0). \quad (23)$$

For estimators that are unbiased with respect to  $\boldsymbol{\theta}_0$ , it bounds the covariance of the estimation error  $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0$ . If  $\boldsymbol{\theta}_t$  denotes the physical parameter associated with the true model, the estimation error relative to that parameter may also contain a bias,

$$\mathbf{b} = \boldsymbol{\theta}_0 - \boldsymbol{\theta}_t. \quad (24)$$

Consequently, the corresponding mean squared error (MSE) is bounded by

$$\mathbb{E} [(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_t)(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_t)^T] \succeq \mathbf{C}_{\text{MCRB}}(\boldsymbol{\theta}_0) + \mathbf{b}\mathbf{b}^T. \quad (25)$$

If the assumed model is correctly specified, then  $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_t$  and  $\mathbf{B} = -\mathbf{A}$ , which reduces the MCRB to the classical CRB. The MCRB therefore separates two effects of model mismatch: a change in the estimator variance and a bias caused by the pseudo-true parameter differing from the physical parameter.

### 3.2 Bayesian Cramér–Rao Bound

Another limitation with the classical CRB is that it must be evaluated at a specific value of the unknown parameter  $\boldsymbol{\theta}$ . However, in tracking and adaptive sensing applications, the true parameter is rarely known exactly. Often, an earlier position estimate may be used in forming a subsequent estimate, but such an estimate always comes with some degree of uncertainty. When the unknown parameter vector contains both deterministic and random components, the two formulations may be combined in the hybrid Cramér–Rao bound (HCRB) [15]. In the fully Bayesian setting considered here, the Bayesian Cramér–Rao bound (BCRB) [16] uses the prior information about the target state and treats the parameter as a random variable with a prior density  $p(\boldsymbol{\theta})$ . Averaging over both the measurements and the uncertain parameter, the relevant density for the BCRB is then the joint density

$$p(\mathbf{y}, \boldsymbol{\theta}) = p(\mathbf{y} | \boldsymbol{\theta}) p(\boldsymbol{\theta}). \quad (26)$$

The Bayesian FIM (BFIM) is then defined as [17]

$$\mathbf{J}_B = \mathbb{E}_{\mathbf{y}, \boldsymbol{\theta}} [\nabla_{\boldsymbol{\theta}} \log p(\mathbf{y}, \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T \log p(\mathbf{y}, \boldsymbol{\theta})], \quad (27)$$

which, under appropriate regularity conditions, can be decomposed as

$$\mathbf{J}_B = \mathbb{E}_{\boldsymbol{\theta}} [\mathbf{J}_D(\boldsymbol{\theta})] + \mathbf{J}_P, \quad (28)$$

$$\mathbf{J}_D(\boldsymbol{\theta}) = \mathbb{E}_{\mathbf{y}|\boldsymbol{\theta}} [\nabla_{\boldsymbol{\theta}} \log p(\mathbf{y} | \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T \log p(\mathbf{y} | \boldsymbol{\theta})], \quad (29)$$

$$\mathbf{J}_P = \mathbb{E}_{\boldsymbol{\theta}} [\nabla_{\boldsymbol{\theta}} \log p(\boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T \log p(\boldsymbol{\theta})]. \quad (30)$$

Here,  $\mathbf{J}_D$  is the FIM resulting from the signal model and  $\mathbf{J}_P$  is the FIM resulting from the prior information. For a Gaussian prior with covariance  $\mathbf{P}$ , the prior FIM is  $\mathbf{J}_P = \mathbf{P}^{-1}$ . The BCRB then bounds the MSE as [16]

$$\mathbb{E}_{\mathbf{y}, \boldsymbol{\theta}} [(\hat{\boldsymbol{\theta}}(\mathbf{y}) - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}}(\mathbf{y}) - \boldsymbol{\theta})^T] \succeq \mathbf{J}_B^{-1}. \quad (31)$$

The two terms  $\mathbb{E}_{\boldsymbol{\theta}} [\mathbf{J}_D(\boldsymbol{\theta})]$  and  $\mathbf{J}_P$  in the BFIM clearly represent complementary sources of information. Evidently, an informative prior may then reduce the achievable MSE, particularly when the measurements provide limited information.

## 4 On Sensor Selection

The performance of a radar system depends not only on the number and quality of its sensors, but also on where the sensors are located. This is particularly important in distributed radar systems, where the viewing angle may vary drastically depending on the location of a sensor. Some sensor geometries provide complementary viewing angles and make the unknown parameters easy to estimate, whereas others yield measurements containing largely redundant information. The choice of sensor geometry may therefore have a substantial effect on the resulting estimation performance.

The term sensor selection refers to the problem of selecting a subset of sensors from a larger set of candidate sensor locations, with the objective of optimizing the performance of the system. This problem may arise when only a limited number of sensors can be constructed or activated due to cost, energy, or other resource constraints. When sensors may instead be placed anywhere within a continuous region, the resulting problem may be approximated by discretizing the region into a dense grid of potential sensor locations, from which a suitable subset can then be selected.

Consider a set of  $N_c$  candidate sensors, and let  $w_n \in \{0, 1\}$  indicate whether the  $n$ th sensor is selected. Assuming independent measurements, the FIM contributions from the selected sensors are additive, allowing the total FIM to be expressed as

$$\mathbf{J}(\mathbf{w}; \boldsymbol{\theta}) = \sum_{n=1}^{N_c} w_n \mathbf{J}_n(\boldsymbol{\theta}), \quad (32)$$

where  $\mathbf{w} = [w_1, \dots, w_{N_c}]^T$  and  $\mathbf{J}_n(\boldsymbol{\theta})$  denotes the FIM for the  $n$ th candidate. In a MIMO radar system, a candidate may represent either a TX or RX sensor. The total FIM may then

be written as

$$\mathbf{J}(\mathbf{w}^{\mathcal{T}}, \mathbf{w}^{\mathcal{R}}; \boldsymbol{\theta}) = \sum_{m=1}^{M_c} \sum_{n=1}^{N_c} w_m^{\mathcal{T}} w_n^{\mathcal{R}} \mathbf{J}_{mn}(\boldsymbol{\theta}), \quad (33)$$

where  $M_c$  and  $N_c$  are the numbers of candidate TXs and RXs, respectively. The product  $w_m^{\mathcal{T}} w_n^{\mathcal{R}}$  ensures that the FIM for the channel from TX  $m$  to RX  $n$  is only added to the sum when both of its associated sensors are selected.

As discussed in the previous chapter, the inverse FIM bounds the covariance of any unbiased estimator. The CRB can therefore be used to evaluate a candidate sensor configuration before measurements are collected. A common approach is to select the sensors minimizing a scalar function of the resulting CRB [18, 19], such that

$$\underset{\mathbf{w}}{\text{minimize}} \quad \phi(\mathbf{J}^{-1}(\mathbf{w}; \boldsymbol{\theta})) \quad \text{subject to} \quad \sum_{\ell=1}^L w_{\ell} \leq K, \quad w_{\ell} \in \{0, 1\}, \quad (34)$$

where  $K$  denotes the available sensor budget and  $\phi(\cdot)$  is a scalar cost function. In this way, the CRB provides a direct link between the sensor selection and the theoretically achievable estimation accuracy.

Several cost functions may be used to summarize the CRB, none of which fully capture the meaning. The most common alternative, A-optimal design, minimizes its trace,

$$\phi_A(\mathbf{J}^{-1}) = \text{tr}(\mathbf{J}^{-1}), \quad (35)$$

and thereby minimizes the sum of the variance bounds of the considered parameters. As an alternative, D-optimal design minimizes the determinant of the CRB,

$$\phi_D(\mathbf{J}^{-1}) = \det(\mathbf{J}^{-1}), \quad (36)$$

thus reducing the volume of the corresponding uncertainty ellipsoid. E-optimal design instead minimizes the largest eigenvalue of the CRB,

$$\phi_E(\mathbf{J}^{-1}) = \lambda_{\max}(\mathbf{J}^{-1}), \quad (37)$$

emphasizing the least accurately estimated parameter. The choice of cost function depends on the intended use of the system, and different choices may yield different sensor configurations.

When the parameters have different units, scales, or levels of importance, minimizing the unweighted trace may cause an imbalance between the priority of the parameters. A weighted A-optimal cost may then be formed as

$$\phi_W(\mathbf{J}^{-1}) = \text{tr}(\mathbf{W}\mathbf{J}^{-1}), \quad (38)$$

where  $\mathbf{W}$  is typically a diagonal matrix with non-negative weights [20, 21]. By choosing these weights appropriately, the importance of parameters may be rebalanced; but similarly, the weights may be chosen to purposely emphasize the parameters most relevant to the considered task. For instance, as in Paper B, when the objective is to distinguish a deceptive jammer from a true target, the weight associated with the deception range may be increased (more on this topic in the following). Parameters assigned zero weight do not directly affect the objective, although they may still need to be estimated accurately because of their coupling to the parameters of interest.

The binary constraints on whether a TX and/or RX candidate is included make the above sensor selection problem a combinatorial optimization problem, which quickly becomes computationally infeasible as the number of candidates grows beyond a modest size. A practical concern is therefore how to approximate the formulation to obtain a computationally feasible solution. This is commonly done by using greedy selection [22], convex relaxation [18, 23], or iterative swapping procedures [24].

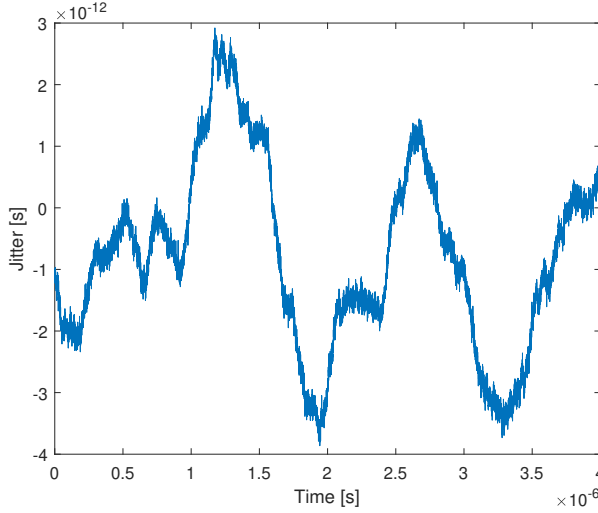
## 5 On Clock Offsets

Distributed radar systems are based on measurements collected by spatially separated transmitters and receivers. For practical reasons, it is common that the sensors do not share a common time reference, i.e., their local clocks are not perfectly synchronized. This is especially true for systems employing multiple platforms, such as the here examined drone swarms. A simple clock model for sensor  $p$  is

$$t^p = (1 + \epsilon_p)t + \delta_p, \quad (39)$$

where  $t$  denotes a common reference time,  $\delta_p$  is the clock offset, and  $\epsilon_p$  is the relative clock skew [25]. The offset describes a constant displacement of the clock, whereas the skew causes the clock to drift, i.e., the displacement changes gradually with time. These clock errors essentially enter a radar model as propagation delays. Thus, they may change the apparent range and may be mistaken for a displacement of the target. Additionally, very fast variations known as clock jitter may also arise. An example of such a jitter is illustrated in Figure 1. This jitter is usually very small, but because the carrier frequency multiplies a clock error  $\Delta$  to yield a phase error on the magnitude of  $\omega\Delta$ , phase-based estimators may still be heavily affected by it. The synchronization errors that arise can then reduce coherent processing gain, bias localization and velocity estimates, and impair target detection [26, 27]. These effects are particularly important in distributed MIMO radar, where the system depends on combining measurements from sensors with independent clocks. In practice, the clock parameters themselves may even vary due to oscillator instability, temperature changes, and platform motion [28, 29].

The most direct solution is to synchronize the clocks using a shared reference or sufficiently stable oscillators. Regrettably, this is more difficult for clocks whose offsets vary over time. In



**Figure 1:** An example of a clock jitter realization (see Paper A for further details).

this case, the offset and skew can be included in a state-space model and tracked recursively, for instance with a Kalman filter [28, 29]. In a radar network, direct-path signals between the transmitters and receivers are especially useful for this purpose: their propagation delays depend on the sensor geometry but not on the target position, and they can therefore be used for calibration. Similarly, if the sensor positions are uncertain, instead of the clock offsets, these direct-path measurements can be used to calibrate the locations of the sensors [30].

Sometimes, direct synchronization is not possible or sufficiently accurate. An alternative is then to treat the clock errors as nuisance parameters in the signal model and estimate them together with the actual parameters of interest. With nuisance parameters like this, the CRB expression in (18) is convenient to use to quantify the loss in target information caused by these additional unknowns. Another possibility is to use estimation methods that are robust to these clock errors, although this robustness is often at the expense of losing some information. As is clear, the appropriate method to deal with clock errors depends on the specific type of clock errors, what reference measurements are available, and the specific application.

Paper A investigates the effect of clock jitter between platforms; specifically, it investigates whether the clock jitter can be dismissed from the model or if it is vital to include or otherwise account for. The MCRB is derived and compared to the classical CRB, showing that excluding the jitter from the model yields a dramatically higher bound.

## 6 On Deceptive Jammers

Due to the importance and usefulness of radar systems, several forms of countermeasures aiming to reduce their efficiency have been developed [31]. A common approach is to employ a separate system that aims to jam the radar receivers, causing these to miss, add, or misplace targets. Generally, a jammer aims to prevent a radar from reliably detecting or estimating the state of a target, which may be done in different ways. Noise jamming attempts to mask the target response by raising the interference level [32, 33], whereas deception jamming produces structured signals intended to resemble genuine radar echoes. Modern deceptive jammers commonly intercept a radar signal, store a copy, modify it, and retransmit it toward the radar. By applying controlled delays, frequency shifts, and amplitude changes to the signal, a jammer may generate one or several false targets with prescribed apparent ranges and velocities [34, 35]. In this work, we will focus our attention on this form of jammers.

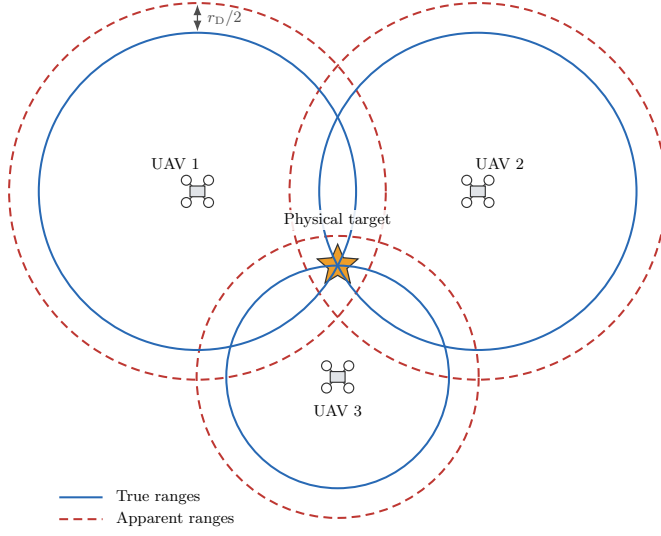
Consider a retransmitted signal to which the jammer adds a time delay  $\Delta_D$ . Relative to an ordinary reflection, this produces an additional apparent propagation distance

$$r_D = c\Delta_D, \quad (40)$$

which is referred to as the deception range. Due to the inherent processing time required by the jammer, this deception range is always strictly positive (if the deceptive signal is retransmitted during the same pulse). Estimating the deception range can therefore provide useful information for discriminating between deceptive signals and true reflections. If the radar system is too predictable, though, a deceptive jammer can take advantage of that to retransmit a deceptive signal with negative or zero deception range the next pulse. The jammer may similarly introduce a deception Doppler  $\nu_D$  by shifting the signal frequency, which emulates an apparent velocity. These deception parameters do not necessarily correspond to the position or velocity of any physical object. Since the retransmission is derived from the radar waveform, however, it can retain much of its structure. It may therefore be difficult to reject a deceptive signal solely based on received power or angle of arrival.

There are many countermeasures that make it difficult for deceptive jammers to predict or reproduce the transmitted waveform [36]. Pulse-to-pulse changes in phase, frequency, coding, or pulse repetition interval (PRI) introduce waveform diversity that a delayed retransmission may fail to follow. The inconsistency can then be exploited to suppress the jammer or separate it from genuine reflections. Random pulse initial phases have, for example, been used to counter velocity-deception signals [34, 35]. Frequency diverse array MIMO (FDA-MIMO) radar provides another form of diversity by transmitting several different frequencies to create jointly range-, angle-, and Doppler-dependent responses, which also aids the discrimination of deceptive interferences from true reflections [37, 38, 39].

Another countermeasure to deceptive jammers is spatial diversity. A reflection from a physical target must be consistent with the propagation geometry of every TX-RX pair. A deceptive



**Figure 2:** The effect of deception range. The circles no longer intersect at a common point.

jammer will struggle to recreate this effect in the retransmitted signal as, for example, a simple deception range will result in a common delay for all sensors, regardless of their location. The measured delays will then no longer make physical sense coming from a single point. This effect is illustrated in Figure 2. The figure shows that for the true reflection delay, the circles representing this intersect at a single point, namely the location of the target. However, when a common delay is added, the circles no longer intersect at a single point, indicating that the signal does not originate from a real target. Similar problems arise with deception Doppler as well. Distributed or multistatic measurements can therefore be combined to test whether the observed delays, Doppler shifts, and directions are compatible with a single reflecting target, although in order to be able to do so, these parameters may need to be estimated with high accuracy. Several methods first estimate the target and deception parameters and subsequently apply a likelihood-ratio or related hypothesis test to distinguish a target from a jammer [40, 41, 42]. Estimating the deception range and Doppler is useful in this setting because nonzero estimates provide direct evidence of waveform manipulation, whereas their variation across channels may reveal the physical jammer location.

The performance of jammer-identification methods depends strongly on the available geometry and on how much information the measurements provide about the parameters. For a given sensor geometry candidate, the CRB can be used to quantify how accurately the target and deception parameters can be estimated. The CRB can then be weighted with (38) to emphasize certain parameters that are more important for discrimination, such as the deception range. This gives an objective measure for comparing sensor configurations before

the measurements are collected.

This topic is explored in Paper B, where the deception range of a signal is estimated to aid in discriminating between a deceptive jammer and a true reflective target. This paper also connects this topic with the topic of sensor selection by optimizing the sensor selection specifically for the estimation of the deception range.

A different form of jamming may target the GPS signals used by mobile sensors to determine their own locations. By denying the sensors reliable positioning information, such interference increases the uncertainty in the sensor geometry and may degrade target localization. This problem is considered in Paper D, where signals directly between drones, without reflection on a target, are exploited to estimate their relative locations. This enables the target to then be tracked using the reflected signals in a GPS-denied environment.

## 7 On Path Planning

A strongly related topic to sensor selection is that of path planning for mobile units. Path planning concerns the selection of a trajectory from an initial state to a desired destination. In this work, we consider the path planning for a swarm of drones, in which case the feasibility of the trajectory can be determined by, for instance, obstacles, limitations on velocity, acceleration, turning rate, flight altitude, and energy consumption. When several drones operate together, collision avoidance is another thing that should typically also be considered.

Classical planning methods primarily seek a short or low-cost path through a known environment. For example, discretizing the environment allows graph-search methods such as A\* to be used, which uses a heuristic to search for a minimum-cost sequence of neighboring states [43]. However, the objective for sensing applications is rarely only to reach a specific destination. Instead, the purpose may be to gather as much information as possible, and the trajectory could therefore preferably be chosen to maximize the gained information. As previously mentioned, the geometry of the system, i.e., the location of the sensors, determines the information available. A path that is short or convenient may provide poor localization accuracy if, for example, the target is viewed from nearly identical directions throughout the observation interval. Path planning in this context therefore attempts to optimize some measure of information, while also taking into account some cost of the physical motion. Common choices of objective functions for information include the predicted state covariance, the FIM, the CRB or some of its variants, detection probability, and information-theoretic quantities such as entropy [44, 45]. These criteria make it possible to reward trajectories that create informative viewing geometries while still accounting for motion, energy, and/or safety constraints.

Since future measurements and target motion are generally unknown, these trajectories are often planned in a closed-loop manner. From measurements, a tracker first produces an estim-

ate of the target location together with its uncertainty; the next step in the trajectory is then planned and executed using this information. The loop then repeats as new measurements enter the system, allowing the sensor geometry to adapt as the target estimate changes. In simpler approaches, the drone may instead follow a predetermined orbit or maintain a desired distance and/or bearing to the target. Such rules require less computation, but they do not explicitly optimize localization performance.

For a drone swarm, the paths may be optimized centrally or in a distributed manner. Centralized optimization uses the combined information of all drones. However, such a formulation may be computationally costly for larger swarms, and will typically be strongly dependent on key nodes in the system, making the system sensitive to faults in these units. Distributed methods divide the problem between the drones, to various degrees. Each drone may, for instance, keep track of its own information and share it with neighboring drones to get an estimate of the total information. This improves scalability and robustness to communication loss, although coordination between the drones is required to avoid redundant geometries and collisions [46, 47, 48].

Path planning for radar drones can therefore be viewed as a form of sensor selection extended over time. Rather than selecting a subset of fixed candidate locations, the system repeatedly selects future sensor locations that are within a specified feasible region.

This is the topic of Paper C, which proposes a planner that maximizes localization accuracy while simultaneously limiting resource usage such as acceleration and pulse transmissions. Furthermore, this planner allows for tuning of the parameters to balance the trade-off between performance and resource usage.

## 8 Conclusion and topics of future research

This thesis examines estimation, tracking, and performance bounds in close-range and distributed MIMO radar systems, with a particular focus on how model mismatch, uncertainty, and sensor geometry affect achievable accuracy. These aspects are studied through the lens of the CRB and its variants, which provide fundamental limits on estimation accuracy. These limits are then used as quantitative objectives for optimizing the performance of the sensing system.

One central conclusion is that simplifying assumptions must be evaluated in relation to the given context. An approximation that is harmless in one scenario may become the dominant source of error in another. As an example, the effects of the far-field approximation and unmodeled clock jitter in Paper A illustrate how model mismatch may introduce both an increase in estimator variance and a systematic estimation bias. Similarly, the results in Paper D show that uncertainty in the sensor locations needs to be accounted for and modeled correctly. These findings emphasize the usefulness of the MCRB, as opposed to the classical CRB, when the assumed model does not match the true signal model.

Another central conclusion is that treating sensor geometry as a design variable can substantially improve the performance of a radar system. Spatial diversity, for example, is often a good heuristic to strive for, but it does not guarantee accurate estimation. Paper B emphasizes that the optimal sensor configuration depends on the specific task at hand, i.e., which parameter is important for the intended purpose. Thus, the scalar criterion used to summarize the matrix-valued CRB should reflect the actual objective of the system.

The same principle extends naturally from selecting fixed sensors to controlling mobile ones, such as drones. In Paper C, the BCRB is used to combine the information expected from future candidate measurements with the uncertainty already present in the target estimate. This allows future sensor positions and pulse transmissions to be selected in a way to optimize the performance bound, while subject to physical motion constraints and extra costs associated with resource usage. Consequently, sensor selection and path planning can be viewed as closely related problems: the former selects among a fixed set of candidate locations, whereas the latter selects future locations from a repeatedly updating set of feasible locations, as the target and the available information evolve.

In conclusion, this thesis shows the vast applications of performance bounds and how they can be used to analyze and optimize the performance of a system. The usefulness of models is also shown to depend greatly on the assumptions made and their validity for the specific context, and the MCRB has been shown to be a powerful tool for this. In the framework presented here, the CRB, MCRB, and BCRB are not merely analysis tools; instead they are used as design tools to optimize performance subject to the available resources.

## Topics of future research

Whereas the present thesis primarily concerns estimating the locations and motion of radar targets, future work could also consider using the received signals to determine the type of target being observed. One possibility is to exploit the micro-Doppler components produced by motions within the target. For a drone, the rotation of the propellers produces characteristic Doppler modulations that differ from those generated by, for instance, the flapping wings of a bird. These signatures may therefore be used to distinguish drones from birds and to infer properties such as the number and rotational speeds of the rotors [49, 50].

The use of micro-Doppler signatures for target classification, however, also introduces a new vulnerability. Similar to the deceptive jammer in Paper B, a deceptive device may intercept and artificially modulate the phase and amplitude on the reflected signal. This may produce a false micro-Doppler signature, causing the radar to mistake one type of target for another. Such micro-Doppler camouflage has been demonstrated experimentally by making a drone appear as a helicopter to the observing radar, and the same principle may be used to make birds appear as drones [51]. Future research could therefore investigate methods for distinguishing genuine micro-Doppler signatures from artificially generated ones.

An important problem that has been left outside of the scope of this thesis is that of association, i.e., determining which measurements correspond to which targets [52]. A related, also overlooked, concern is the separation of closely spaced targets. Paper B and Paper D briefly consider multi-target examples, but then assume oracle association; this is a major simplification that is made to focus the attention on the intended concepts and methods of the papers. Future work could clearly consider these separation and association problems, for example with an optimal mass transport formulation [6, 53].

A further area of interest would be to examine the use of other forms of optimality, such as D- and E-optimality, potentially also using combinations of optimality criteria to gain further flexibility and/or robustness in the formulation. Notably, any form of radar system should preferably exhibit notable robustness to unexpected signal measurements, such as active jamming signals, but also from strong interference signals originating from, for example, telecommunication transmissions. Another issue that we have not explored in this work is how to improve the system's resilience to clutter, which will be a significant concern when aiming to detect surface or ground motion. Especially sea clutter, resulting from reflections on waves, will make it difficult to distinguish slowly moving surface targets from waves, necessitating further improvements.

The methods presented in the appended papers may also be extended in several directions. The close-range estimator in Paper A could be extended to exploit both phase and amplitude information, thereby retaining its robustness to near-field effects and clock jitter without sacrificing far-field performance. Paper B could consider multiple jammers cooperating to construct a more realistic deceptive target. For large drone swarms, the exhaustive optimization over the pulse-transmission vector in Paper C becomes computationally prohibitive; the planner could likely be made more efficient to handle this problem. Finally, it would be interesting to allow for clock offsets together with the position offsets in the GPS-denied localization framework in Paper D. Although both affect the measured propagation delays, they do so in different ways, and an important question is under which geometries and measurement configurations they can be distinguished and jointly estimated.



# References

- [1] M. I. Skolnik, *Introduction to Radar Systems*. New York: McGraw Hill, 1962.
- [2] —, *Radar Handbook*. The McGraw-Hill Companies, 2008.
- [3] N. J. Willis, *Bistatic Radar*, 2nd ed. SciTech Publishing, 2005.
- [4] M. A. Richards, *Fundamentals of Radar Signal Processing*, 3rd ed. McGraw Hill, 2022.
- [5] V. S. Chernyak, *Fundamentals of Multisite Radar Systems: Multistatic Radars and Multiradar Systems*. Gordon and Breach Science Publishers, 1998.
- [6] C. Yang, Q. Ling, X. Sheng, M. Mu, and A. Jakobsson, “Multi-target sonar localization using an optimal mass transport framework,” *IEEE Transactions on Instrumentation and Measurement*, vol. 73, 2024, art. no. 9517512.
- [7] C. R. Rao, “Information and the accuracy attainable in the estimation of statistical parameters,” *Bull. Calcutta Math. Soc*, vol. 37, no. 3, pp. 81–91, 1945.
- [8] H. Cramér, *Mathematical Methods of Statistics*, ser. Princeton Mathematical Series. Princeton, NJ: Princeton University Press, 1946, no. 9.
- [9] R. A. Fisher, “On the mathematical foundations of theoretical statistics,” in *Breakthroughs in statistics: Foundations and basic theory*. Springer, 1992, pp. 11–44.
- [10] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory*. Englewood Cliffs, N.J.: Prentice-Hall, 1993.
- [11] P. Stoica and R. Moses, *Spectral Analysis of Signals*. Upper Saddle River, N.J.: Prentice Hall, 2005.
- [12] K. M. Abadir and J. R. Magnus, *Matrix algebra*. Cambridge University Press NY, 2005, vol. 1.

- [13] C. D. Richmond and L. L. Horowitz, "Parameter Bounds on Estimation Accuracy Under Model Misspecification," *IEEE Transactions on Signal Processing*, vol. 63, no. 9, pp. 2263–2278, 2015.
- [14] S. Fortunati, F. Gini, M. S. Greco, and C. D. Richmond, "Performance bounds for parameter estimation under misspecified models: Fundamental findings and applications," *IEEE Signal Processing Magazine*, vol. 34, no. 6, pp. 142–157, Nov 2017.
- [15] H. Messer, "The Hybrid Cramér–Rao Lower Bound - From Practice to Theory," in *Fourth IEEE Workshop on Sensor Array and Multichannel Processing, 2006.*, 12–14 July, 2006, pp. 304–307.
- [16] H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*. John Wiley and Sons, Inc., 2002.
- [17] H. L. V. Trees and K. L. Bell, Eds., *Bayesian Bounds for Parameter Estimation and Nonlinear Filtering/Tracking*. Hoboken, NJ: Wiley-IEEE Press, 2007.
- [18] S. Joshi and S. Boyd, "Sensor selection via convex optimization," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 451–462, 2009.
- [19] S. P. Chepuri and G. Leus, "Sparsity-Promoting Sensor Selection for Non-Linear Measurement Models," *IEEE Transactions on Signal Processing*, vol. 63, no. 3, pp. 684–698, February 2015.
- [20] J. Swärd, F. Elvander, and A. Jakobsson, "Designing sampling schemes for multi-dimensional data," *Signal Processing*, vol. 150, pp. 1–10, 2018.
- [21] J. Cheng, M. Juhlin, A. Jakobsson, and W.-Q. Wang, "Designing optimal frequency offsets for frequency diverse array MIMO radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 6, pp. 8104–8118, 2023.
- [22] M. Shamaiah, S. Banerjee, and H. Vikalo, "Greedy sensor selection: Leveraging submodularity," in *Proceedings of the 49th IEEE Conference on Decision and Control*, 2010, pp. 2572–2577.
- [23] M. Juhlin and A. Jakobsson, "Optimal sensor placement for localizing structured signal sources," *Signal Processing*, vol. 202, p. 108679, 2023.
- [24] A. C. Atkinson and A. N. Donev, *Optimum Experimental Designs*, ser. Oxford Statistical Science Series. Oxford: Clarendon Press, 1992, vol. 8.
- [25] R. H. Kenney, J. G. Metcalf, and J. W. McDaniel, "Wireless distributed frequency and phase synchronization for mobile platforms in cooperative digital radar networks," *IEEE Transactions on Radar Systems*, vol. 2, pp. 268–287, 2024.

- [26] H. Song, G. Wen, Y. Liang, L. Zhu, and D. Luo, "Target localization and clock refinement in distributed MIMO radar systems with time synchronization errors," *IEEE Transactions on Signal Processing*, vol. 69, pp. 3088–3103, 2021.
- [27] S. Yang, A. Jakobsson, and W. Yi, "Moving target detection using a distributed MIMO radar system with synchronization errors," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 61, pp. 1–17, 2023.
- [28] H. Kim, X. Ma, and B. R. Hamilton, "Tracking low-precision clocks with time-varying drifts using Kalman filtering," *IEEE/ACM Transactions on Networking*, vol. 20, no. 1, pp. 257–270, 2012.
- [29] B. Luo and Y.-C. Wu, "Distributed clock parameters tracking in wireless sensor network," *IEEE Transactions on Wireless Communications*, vol. 12, no. 12, pp. 6464–6475, 2013.
- [30] K. N. Ramamohan, S. P. Chepuri, D. F. Comesana, and G. Leus, "Self-calibration of acoustic scalar and vector sensor arrays," *IEEE Transactions on Signal Processing*, vol. 71, pp. 61–75, 2023.
- [31] D. C. Schleher, *Electronic Warfare in the Information Age*. Boston, MA, USA: Artech House, 1999.
- [32] R. Poisel, *Modern communications jamming principles and techniques*. Artech house, 2011.
- [33] Y. Pan, D. Xie, Y. Zhao, X. Wang, and Z. Huang, "Overview of radar jamming waveform design," *Remote Sensing*, vol. 17, no. 7, p. 1218, 2025.
- [34] J. Zhang, D. Zhu, and G. Zhang, "New antivelocity deception jamming technique using pulses with adaptive initial phases," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 49, no. 2, pp. 1290–1300, 2013.
- [35] Z. Liu, J. Sui, Z. Wei, and X. Li, "A sparse-driven anti-velocity deception jamming strategy based on pulse-doppler radar with random pulse initial phases," *Sensors*, vol. 18, no. 4, p. 1249, 2018.
- [36] H. Calatrava, S. Tang, and P. Closas, "Advances in anti-deception jamming strategies for radar systems: A survey," *IEEE Aerospace and Electronic Systems Magazine*, vol. 41, no. 4, pp. 66–93, 2025.
- [37] W.-Q. Wang, H. C. So, and A. Farina, "FDA-MIMO signal processing for mainlobe jammer suppression," in *Proceedings of the 27th European Signal Processing Conference (EUSIPCO)*, 2019, pp. 1–5.

- [38] J. Zhu, S. Zhu, J. Xu, and L. Lan, "Discrimination of target and mainlobe jammers with FDA-MIMO radar," *IEEE Signal Processing Letters*, vol. 30, pp. 583–587, 2023.
- [39] Z. Wu, S. Zhu, J. Xu, L. Lan, and M. Zhang, "Interference suppression method with MR-FDA-MIMO radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 5, pp. 6250–6264, 2023.
- [40] X. Song, P. Willett, and S. Zhou, "Jammer detection and estimation with MIMO radar," in *Proceedings of the 46th Asilomar Conference on Signals, Systems and Computers*, 2012, pp. 1312–1316.
- [41] S. Zhao and Z. Liu, "Deception parameter estimation and discrimination in distributed multiple-radar architectures," *IEEE Sensors Journal*, vol. 17, no. 19, pp. 6322–6330, 2017.
- [42] S. Zhao and Y. Zhou, "Discrimination between radar targets and deception jamming in distributed multiple-radar architectures," *IET Radar, Sonar & Navigation*, 2017.
- [43] P. E. Hart, N. J. Nilsson, and B. Raphael, "A formal basis for the heuristic determination of minimum cost paths," *IEEE Transactions on Systems Science and Cybernetics*, vol. 4, no. 2, pp. 100–107, 1968.
- [44] U. Oktel and R. L. Moses, "A bayesian approach to array geometry design," *IEEE Transactions on Signal Processing*, vol. 53, no. 5, pp. 1919–1923, 2005.
- [45] K. Dogancay and H. Hmam, "UAV path optimization for angle-only self-localization and target tracking based on the bayesian fisher information matrix," *Sensors*, vol. 24, no. 10, p. 3120, 2024.
- [46] S. James, R. Raheb, and A. Hudak, "UAV swarm path planning," in *Proceedings of the Integrated Communications Navigation and Surveillance Conference*, 2020, pp. 2G3–1–2G3–12.
- [47] Y. Wu, K. H. Low, B. Pang, and Q. Tan, "Swarm-based 4d path planning for drone operations in urban environments," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 8, pp. 7464–7479, 2021.
- [48] R. J. Myers, S. M. Perera, G. McLewee, D. Huang, and H. Song, "Multi-beam beamforming-based ML algorithm to optimize the routing of drone swarms," *Drones*, vol. 8, no. 2, p. 57, 2024.
- [49] T. Gong, Y. Cheng, X. Li, and D. Chen, "Micromotion detection of moving and spinning object based on rotational Doppler shift," *IEEE Microwave and Wireless Components Letters*, vol. 28, no. 9, pp. 843–845, 2018.

- [50] S. Rahman and D. A. Robertson, “Radar micro-Doppler signatures of drones and birds at K-band and W-band,” *Scientific reports*, vol. 8, no. 1, p. 17396, 2018.
- [51] V. Kozlov, A. Sheleg, D. Vovchuk, R. Melnitsky, A. Glam, A. Boag, and P. Ginzburg, “Micro-doppler radar deception and camouflage of airborne targets,” *Scientific Reports*, vol. 16, no. 1, p. 847, 2025.
- [52] D. B. Reid, “An algorithm for tracking multiple targets,” *IEEE Transactions on Automatic Control*, vol. 24, no. 6, pp. 843–854, 1979.
- [53] S. Kolouri, S. R. Park, M. Thorpe, D. Slepcev, and G. K. Rohde, “Optimal Mass Transport: Signal processing and machine-learning applications,” *IEEE Signal Process. Mag.*, vol. 34, no. 4, pp. 43–59, July 2017.





