LUND UNIVERSITY

Practical Evaluation of a Novel Multivariable Relay Autotuner with Short and Efficient Excitation

Berner, Josefin; Soltesz, Kristian; Åström, Karl Johan; Hägglund, Tore

Practical Evaluation of a Novel Multivariable Relay Autotuner with Short and Efficient Excitation

Josefin Berner, Kristian Soltesz, Karl Johan Åström, Tore Hägglund

Abstract— In this paper we propose an autotuning method that combines a setup for decentralized relay autotuning of two-input two-output systems with an identification method that uses short experiments to estimate up to second-order time-delayed systems. A small modification of the experiment gives better low-frequency excitation and improved models. The method is successfully demonstrated in simulations and on a quadruple tank process.

I. INTRODUCTION

PID controllers are, and will probably remain, the most common controller type used in industry. Since the commonly used PID tuning rules rely on dynamic models of the processes to be controlled, automatic methods of obtaining these models are of practical value. For single-input single-output (SISO) systems, the introduction of the relay autotuner [2] proved to be a successful way to do this. The relay autotuner has since then been improved and modified in e.g. [12], [13], [9], [3]. A review of identification from relay experiments, written a few years ago, is found in [10].

Not all industrial systems are, or can be approximated by, SISO systems. This motivates an extension of automatic tuning methods to handle multivariable systems. The literature on this subject is not as extensive as for the SISO case, but there are some papers written on multivariable autotuning, of which [14] should be mentioned as it proposed the decentralized relay experiments that we will use in this paper. In [7] this method was the basis for a method to find FOTD or SOTD models from the experiments. Other papers have been published where each sub-loop is tuned individually, or where a sequential tuning procedure is used [16], [11]. All these methods require several experiments, which is a drawback. A decentralized method that only needs one short experiment, without any assumptions on coupling-level of the subsystems, was proposed in [5]. In this paper we extend and improve that method. This is done by modifying the experiment to increase the low-frequency excitation, and by using an improved identification method from [4] where both first-order time-delayed (FOTD) and second-order time-delayed (SOTD) models are found, even in the presence of non-stationary starting conditions. The proposed method is evaluated by simulations, and is also tested on a physical two-input two-output (TITO) process, a quadruple tank process available in our control laboratory.
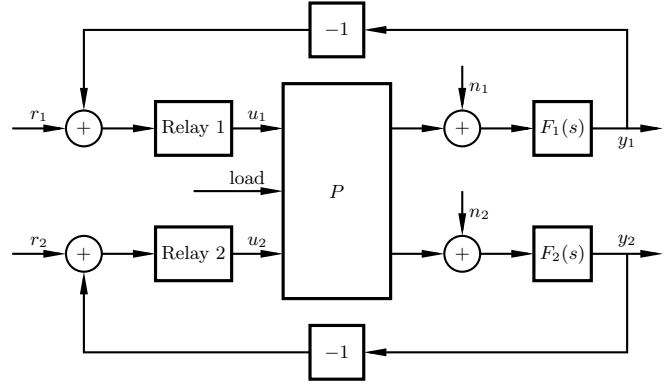
Fig. 1: Setup of the relay experiment for the two-input two-output process P, with optional noise filters F_1 and F_2.

II. METHOD

The proposed method consists of combining the SISO-autotuner from [4] with the decentralized framework from [5] to get a well-functioning autotuner for TITO-systems that provides FOTD and SOTD models from a short experiment. The experiment is slightly modified to increase low-frequency excitation, as will be described in Sec. IV. From the obtained models, a centralized PID controller is designed, based on the optimization method described in [6]. The obtained controller performance is evaluated in Sec. V.

A. Decentralized asymmetric relay experiment

The experiment setup used in this paper is shown in Fig. 1. In the decentralized relay experiment both loops are closed with relay feedback simultaneously. The relays used are asymmetric, with different asymmetry levels, and more extensively described in [3]. The relay feedback will cause both loops to oscillate, exciting the output signals in frequency intervals relevant for PID control. The original decentralized method proposed in [14] was modified in [5] to use an identification method that do not require limit cycle convergence of the experiment, which allowed a significantly shorter experiment time.

B. Identification of models

As was described in [5], each of the output ports consist of a sum of the subsystem outputs. For the TITO case this means that the output signals are $y_1 = P_{11}u_1 + P_{12}u_2$, and $y_2 = P_{21}u_1 + P_{22}u_2$. Thus, we can identify the subsystems related to one output separately from the subsystems related to the other output. The sub-models estimated in this work

are FOTD and SOTD models of the form

$$P_{i,j}(s) = \frac{b}{s^n + a_1 s^{n-1} + \ldots + a_n} e^{-sL}, \qquad (1)$$

where $1 \leq n \leq 2$. The model parameters are estimated along with the initial state(s) x_0 to get the parameter vector $\theta = \begin{bmatrix} b & a & L & x_0 \end{bmatrix}$, from an output error minimization described in [4]. If desired, stability of the model can be enforced by introducing constraints on the parameters in the minimization method. Since the output data is the sum of the output of two subsystems it means that both θ_1 and θ_2 should be found simultaneously. The strategy used in this paper is to first find an FOTD or SOTD model $\hat{P}_{11}$, assuming that $y_1 = P_{11}u_1$, and a model $\hat{P}_{12}$ assuming that $y_1 = P_{12}u_2$, where the model selection is done according to the Akaike Information Criteria [1]. A second set of models is then estimated from the alternative assumption that $y_1 - \hat{P}_{12}u_2 = P_{11}u_1$ and $y_1 - \hat{P}_{11}u_1 = P_{12}u_2$. Combinations of these separate models are used as starting values for the estimation of the combined parameter vector $\theta = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}$.

C. MIMO PID tuning

In this paper we use the multi-input multi-output (MIMO) PID tuning method described in [6], with some small modifications. The idea of the method is to find controller matrices K_P, K_I and K_D by solving an optimization problem, where

$$||(P(0)K_I)^{-1}||_\infty \qquad (2)$$

is minimized subject to constraints on the maximum sensitivities $||S||_\infty$ and $||T||_\infty$, as well as a constraint on the control signal. In the proposed version in the referred paper, a first order filter is applied to the derivative part of the controller. We have chosen to instead use one of the suggested modifications, and use a second order filter on the full controller. Hence our controller structure is

$$C(s) = \frac{1}{1 + s\tau + (s\tau)^2/2} \left(K_P + \frac{1}{s}K_I + sK_D \right), \quad (3)$$

where τ is the filter time constant, in this paper chosen as

$$\tau = \frac{1}{k \max(\omega_c)}, \qquad (4)$$

where k is a factor chosen to be 5, and $\max(\omega_c)$ is the largest crossover frequency of the obtained sub-models. By introducing this filter, and sending the filtered process output to the optimization method, we no longer need the restriction on the control signal, and that constraint is hence removed.

For a TITO (or general MIMO) system there could be a lot of questions asked about the controller structure. For example, should it be centralized like this? Should you really have integral parts on all entries in the controller matrix? Should the same filter be used for all input-output-pairings? Since the focus of this paper is not MIMO PID design, but rather how to obtain good enough models for PID design, these questions will not be answered here. We merely use the described MIMO PID design as an example of what results could be obtained from our autotuning method. The models we obtain could just as well be used to tune decentralized
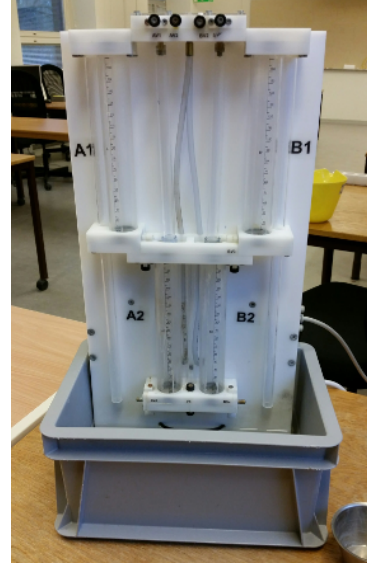

Fig. 2: The quadruple tank used for experiments. The inputs are the signals to the two pumps that pump water to both the upper and the lower tanks in different proportions depending on the configuration. The output signals, y_1 and y_2, are the water levels in the two lower tanks.

PID controllers with or without decoupling, or by any other method preferred by the user.

III. EXAMPLE PROCESSES

We will evaluate the proposed method on the Wood-Berry distillation column [15], commonly used as a benchmark process for TITO control, and a modified version of the quadruple tank, described in [8], used in many control laboratories. The dynamics of the Wood-Berry distillation column are given by

$$G_{WB} = \begin{pmatrix} \dfrac{12.8e^{-s}}{1 + 16.7s} & \dfrac{-18.9e^{-3s}}{1 + 21s} \\[2ex] \dfrac{6.6e^{-7s}}{1 + 10.9s} & \dfrac{-19.4e^{-3s}}{1 + 14.4s} \end{pmatrix}. \qquad (5)$$

The quadruple tank in our control lab, shown in Fig. 2, has been modified from the one in [8] to get faster dynamics. The linearized minimum-phase configuration for this tank process can be modeled by

$$G_{QT} = \begin{pmatrix} \dfrac{0.14e^{-s}}{s + 0.043} & \dfrac{0.0088e^{-s}}{s^2 + 0.19s + 0.0061} \\[2ex] \dfrac{0.0088e^{-s}}{s^2 + 0.19s + 0.0061} & \dfrac{0.14e^{-s}}{s + 0.043} \end{pmatrix}, \qquad (6)$$

where the time delay is a simplification of the dynamics in the pumps and sensors.

Some characteristics of the example processes are worth mentioning. The Wood-Berry distillation column has only FOTD entries and should be well-estimated by the method in [5], while the quadruple tank is a mix of first-order and second-order systems, and could therefore be improved

by the SOTD estimations from [4]. G_{WB} has high and negative gains, which puts requirements on the generality of the autotuner implementation. The quadruple tank model is linearized around a working point at half the tank height, so in order to do experiments it is first required to bring the system up to the operating point. This startup raises the question of when the system is in steady-state, and the estimation of initial states from [4] is useful to avoid that problem. The quadruple tank model is symmetric, which means that if the asymmetry levels in the two relays were the same, the input-output data from the two loops would be identical. That would reduce the information for the identification process, so the different asymmetry levels used by this method are very beneficial for this process.

A. Experiment settings

For the quadruple tank the working point was chosen as half-full tanks, and the nominal control signals were adjusted to achieve this. In all simulation plots, these working points has been subtracted from the result to get a system oscillating around zero. The Wood-Berry column was assumed to oscillate around a working point normalized to zero. The data sequences used by the identification method only contain the actual experiments, and the plots in this paper only show this, and not the part where the system is brought to its working point or where the noise level is measured. The sample time was $t_s = 0.005$ s, and the hysteresis level in noise-free simulations was $h = 0.4$. The asymmetry levels of the two relays were set to $\gamma_1 = 2$ and $\gamma_2 = 1.5$.

IV. MODIFICATIONS TO EXPERIMENT

The examples in [5] showed a very good fit of the model output data from short experiments consisting of three relay switches. However, additional experiments indicated that the obtained models sometimes gave poor estimates of the static gains. For the classic SISO relay autotuner the static gain is irrelevant since it only uses the critical point. However, a good estimate of the static gain matrix is needed for the multivariable design used in this paper, see (2). This motivated a slight modification of the experiment. Simply increasing the experiment length did not help much, as illustrated for the quadruple tank in Fig. 3, where the obtained model output fits the data extremely well, but the models are not that satisfactory. Instead we decided to increase the low-frequency excitation of the experiment by changing the on and off amplitudes of the relay in order to induce a step in u_{ref}. By doing this small modification we get the results in Fig. 4. As can be seen the experiment lengths are more or less the same, the process output still oscillates around the same level, but the obtained models are much better.
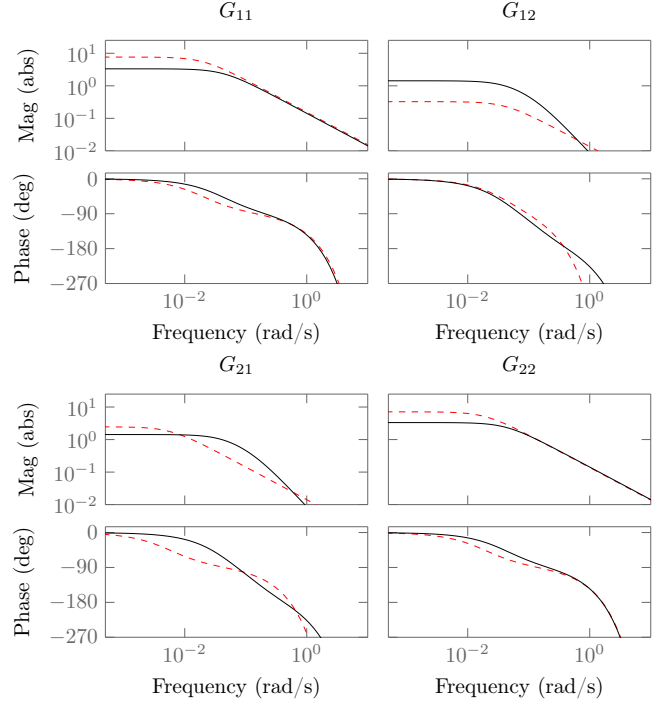
V. RESULTS

A. Simulations

In the simulation study we compare the controller performance for the autotuner, to a PID controller tuned by the same method, but from the true process model. To make the simulations a bit more realistic we added band-limited white
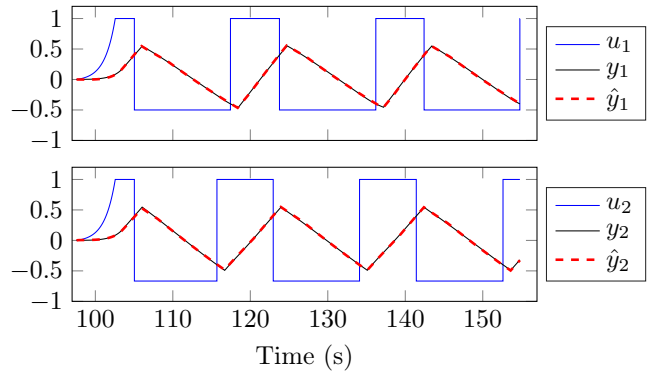
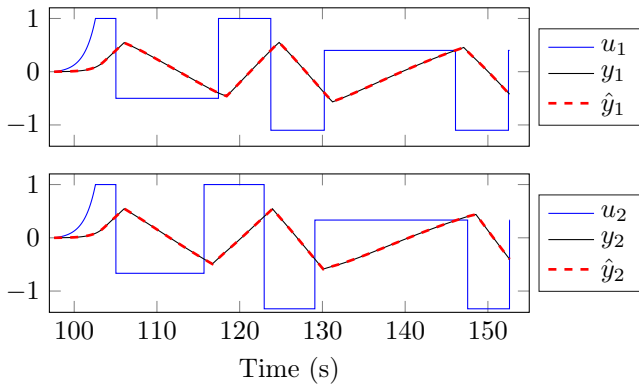(a) Simulation data and obtained model output data $\hat{y}$.

(b) Bode plots for the true model in solid black and estimated model in dashed red.

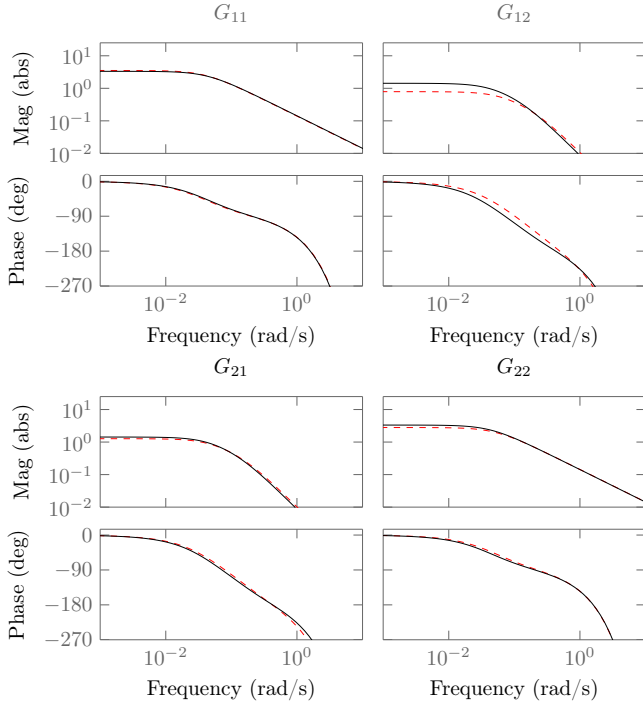Fig. 3: Simulation results for G_{QT} from original experiment.

noise with the standard deviation $\sigma_n = 0.1$ for Wood-Berry, and $\sigma_n = 0.035$ for the quadruple tank. The reason for the different levels are that the processes are not normalized to the same scale and hence have very different gains.

1) The Wood-Berry Distillation Column: The simulation data for G_{WB} is shown in Fig. 5. The output data fit is very good, and the obtained model is essentially identical to the true model. Since the models are identical, so are the optimized controllers. The response from the controllers to setpoint changes is seen in Fig. 6.

2) Quadruple tank model: The simulation data for the quadruple tank model, G_{QT}, is shown in Fig. 7. Bode plots of the obtained models are shown in Fig. 8, the controller performance for a step in setpoint, and a step in input load disturbance, is shown in Fig. 9 and Fig. 10 respectively. The model is good, but the small difference in gain of G_{21}

(a) Simulation data and obtained model output data $\hat{y}$.

(b) Bode plots for the true model in solid black and estimated model in dashed red.

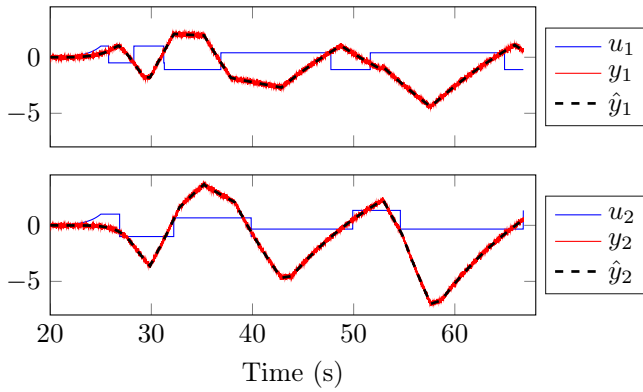Fig. 4: Simulation results for G_{QT} from modified experiment.

Fig. 5: Simulation data for G_{WB}, along with the output data $\hat{y}$ resulting from the obtained model.
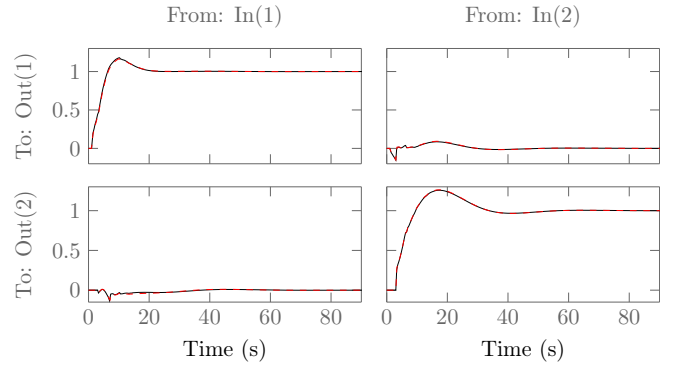

Fig. 6: Controller performance for a step change in setpoint for G_{WB}. The controller tuned from the true model is shown in solid black, and the controller tuned for the estimated model in dashed red.
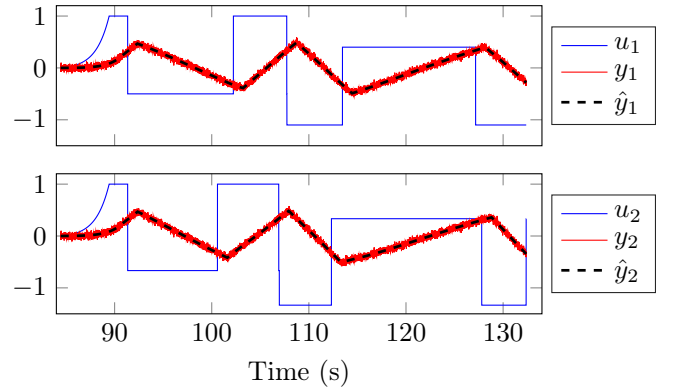

Fig. 7: Simulation data for G_{QT}, along with the output data $\hat{y}$ resulting from the obtained model.

results in somewhat differing behavior to the controller for that subsystem. However, the performance is still satisfactory.

B. Quadruple tank experiment

The proposed method was also evaluated on a real quadruple tank process in the control lab at Lund University. The sensors were too noisy to perform the experiment in a good way, so a low-pass filter

$$F(s) = \frac{1}{0.1s + 1} \tag{7}$$

was added to each output, as in Fig. 1. To get a model of the unfiltered dynamics from the identification, the input identification data was run through the same filter. Hence, the filters do not affect the obtained model, but they slightly change the experiment excitation.

To evaluate the method we performed 10 experiments. Data (filtered) for one of these is shown in Fig. 11, and Bode plots for the resulting models are shown for all of them in Fig. 12. A MIMO PID controller was designed for each of the obtained models. The controller performance for the model obtained from the experiment in Fig. 11 (the one showed by a thick red line in Fig. 12) is shown in Fig. 13.
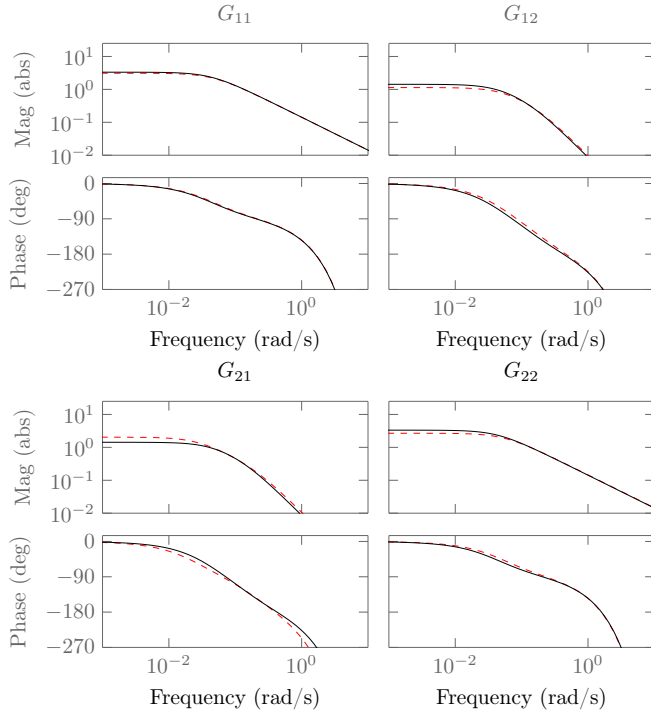
Fig. 8: Bode plots of the obtained models for G_{QT}. The true model is shown in black, the estimated model in dashed red.
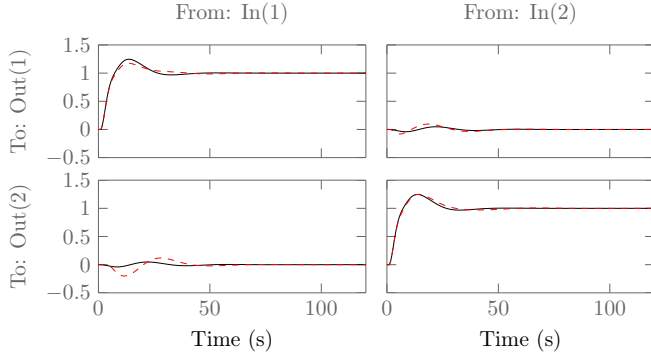

Fig. 9: Controller performance for a step change in setpoint for G_{QT}. The controller tuned from the true model is shown in solid black, and the controller tuned for the estimated model in dashed red.

At time 120 s a step change in the setpoint for y_1 was made, followed by a step change for y_2 at 180 s. At time 250 s, a load disturbance was added by manually pouring 0.5 dl water to the tank with output y_1, and at 350 s a corresponding load was put on y_2.

VI. DISCUSSION

The simulation study shows that good models are obtained for the example systems, using the proposed method with the modified experiment, even with the addition of noise. The need for the extra step more or less doubles the experiment time compared to the SISO experiments in [4] and the first TITO tests in [5]. However, the experiment is still short since
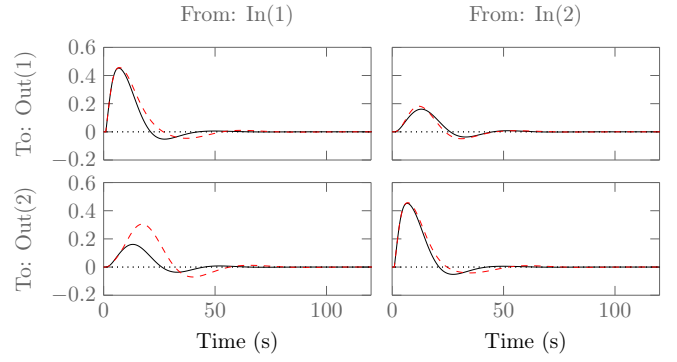

Fig. 10: Controller performance for a step change in input load disturbance for G_{QT}. The controller tuned from the true model is shown in black, and the controller tuned for the estimated model in red.
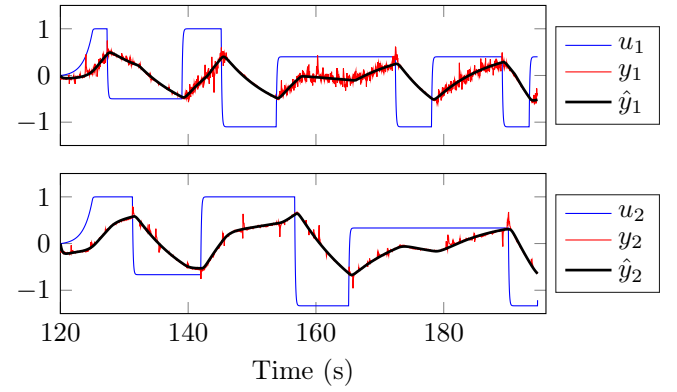

Fig. 11: Experiment data from the quadruple tank process. Both the input and output have been filtered by $F(s)$, and the working point of approximately $u = 5$, $y = 5$, has been subtracted.

it does not have to wait for limit cycle convergence and only runs for a total of 5 switches.

The results were improved by the increased low-frequency excitation. This increased excitation can be achieved in many ways, but by changing the setpoint for u instead of taking a step in y, we do not cause the process to drift away from its working point, which is an advantage. The exact size of the step in u_{ref} could be further investigated, but by moving in the direction from the high relay amplitude towards the low relay amplitude, a larger step can be taken without risking to terminate the oscillations.

From the experiments on the quadruple tank it can be noted that the experiment seems to work very well. All obtained models are similar to each other, even if one or two differ slightly in static gain of G_{21}, or for large phase lags. The obtained controller shows satisfactory responses to both setpoint changes and load disturbances. The obtained model is not symmetric, and neither is therefore the controller. This is due to uneven wear in the process and can be seen by the somewhat different responses in y_1 and y_2. Another thing worth noting from the experiments, is the characteristics of
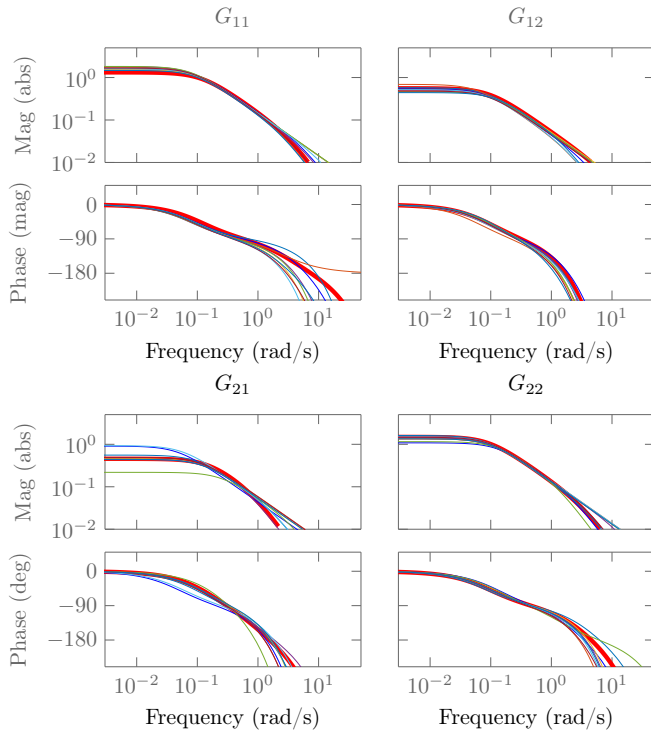
Fig. 12: Bode plots for the models obtained from ten experiments on the quadruple tank. The thick red line is the model from the experiment illustrated in Fig. 11 and used in Fig. 13.
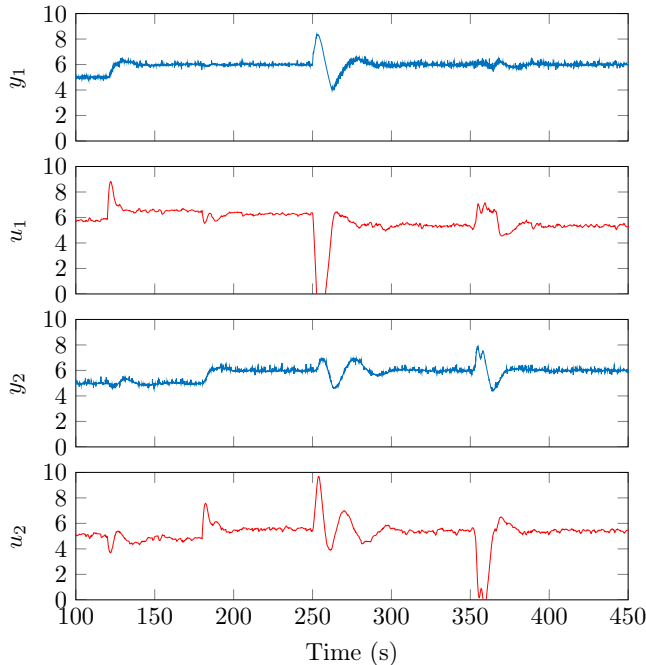

Fig. 13: The quadruple tank controlled by our obtained controller. Step changes occur at time 120 s and 180 s, and load disturbances enter at time 250 s and 350 s for the two output tanks respectively. Here the working points have not been subtracted, and the plot starts at 100 s when the system has reached its stationary level.

the noise. It is clearly seen from y_1 in Fig. 11 that the noise is much larger on the way up (that is, when more water is pumped in to the tank) than on the way down. It is also clear by comparing the noise in Fig. 11 with the one in Fig. 7 that the real noise is not at all as white and even as in the simulations. This does, however, not seem to deteriorate the results of the proposed autotuner.

VII. CONCLUSION

We have proposed an autotuner for TITO systems that gives FOTD or SOTD models for each sub-model. The experiment is extended by a step in the relay amplitudes, which allows better models to be obtained. The method handles start from non-stationarity, and the experiment duration is short. The results are good for the evaluated simulation examples, as well as for the experiments on the quadruple tank process. This shows that the proposed extended version of the fast and simple relay autotuner can be succesfully used also for TITO systems, and that more advanced, time-consuming system identification methods are unnecessary in this case.

REFERENCES

[1] Hirotugu Akaike. A new look at the statistical model identification. *IEEE transactions on automatic control*, 19(6):716–723, 1974.

[2] Karl Johan Åström and Tore Hägglund. Automatic tuning of simple regulators with specifications on phase and amplitude margins. *Automatica*, 20(5):645–651, September 1984.

[3] Josefin Berner, Tore Hägglund, and Karl Johan Åström. Asymmetric relay autotuning–Practical features for industrial use. *Control Engineering Practice*, 54:231–245, 2016.

[4] Josefin Berner and Kristian Soltesz. Shorter experiments in relay-autotuners. In *Emerging Technologies and Factory Automation, Limassol, Cyprus*. IEEE, 2017. Accepted for publication.

[5] Josefin Berner, Kristian Soltesz, Tore Hägglund, and Karl Johan Åström. Autotuner identification of TITO systems using a single relay feedback experiment. In *IFAC World Congress, Toulouse, France*. IFAC, 2017. Accepted for publication.

[6] Stephen Boyd, Martin Hast, and Karl Johan Åström. MIMO PID tuning via iterated LMI restriction. *International Journal of Robust and Nonlinear Control*, 2015.

[7] M Chidambaram and Vivek Sathe. *Relay Autotuning for Identification and Control*. Cambridge University Press, 2014.

[8] Karl Henrik Johansson. The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on control systems technology*, 8(3):456–465, 2000.

[9] I. Kaya and D.P. Atherton. Parameter estimation from relay autotuning with asymmetric limit cycle data. *J. Process Control*, 11(4):429–439, August 2001.

[10] Tao Liu, Qing-Guo Wang, and Hsiao-Ping Huang. A tutorial review on process identification from step or relay feedback test. *Journal of Process Control*, 23(10):1597–1623, 2013.

[11] Ai P. Loh, Chang C. Hang, Choon K. Quek, and Vinod U. Vasnani. Autotuning of multiloop proportional-integral controllers using relay feedback. *Industrial & Engineering Chemistry Research*, 32(6):1102–1107, jun 1993.

[12] William L. Luyben. Derivation of transfer functions for highly nonlinear distillation columns. *Ind. Eng. Chem. Res.*, 26(12):2490–2495, December 1987.

[13] Shih-Haur Shen, Jiun-Sheng Wu, and Cheng-Ching Yu. Use of biased-relay feedback for system identification. *AIChE J.*, 42(4):1174–1180, April 1996.

[14] Qing-Guo Wang, Biao Zou, Tong-Heng Lee, and Qiang Bi. Autotuning of multivariable PID controllers from decentralized relay feedback. *Automatica*, 33(3):319–330, 1997.

[15] RK Wood and MW Berry. Terminal composition control of a binary distillation column. *Chemical Engineering Science*, 28(9):1707–1717, 1973.

[16] Cheng-Ching Yu. *Autotuning of PID controllers: A relay feedback approach*. Springer Science & Business Media, 2006.