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Control of Liquid Slosh in an Industrial Packaging Machine

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Abstract Linear movement of open containers containing liquid is considered. The design is based on a simple linearized slosh model. An open-loop acceleration trajectory is calculated using optimal control techniques. The calculated acceleration profiles are evaluated using experiments with a laser-based sensor and recordings by a video camera. The performance is better than previous ad-hoc controllers.

1. Introduction

This paper considers movement of open containers containing liquid, which is a common operation in the packaging industry. Improved movement methods are crucial for optimizing packaging cost and profit. Today it must also be possible to use the same packaging machine for quite different products and it should be possible to change production quickly. Optimization of movement profiles is therefore an important and demanding challenge.

The operation of a packaging machine can be divided into three independent sub tasks: folding, filling and sealing. These tasks are performed simultaneously on three different packages, see Fig. 1.

The folded package is placed in a holder that carries the package through the machine. The movement is performed stepwise, the number of steps between the different sub tasks depends on the machine type.

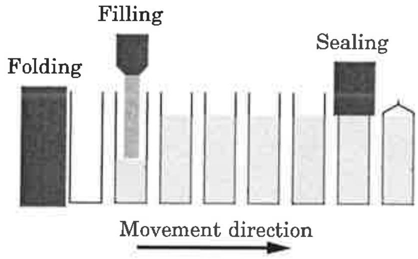

Figure 1 Schematic picture of the packaging machine.

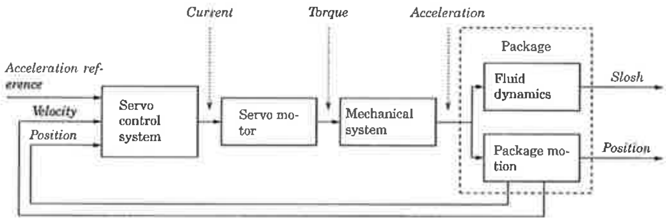

Figure 2 Block diagram of the motion control system.

The same movement is applied in every step on all packages. The time it takes to produce a package is determined by the filling time and the time it takes to move the package one step.

The packages contain liquid when they are moved between the filling and the sealing stations. The movement induces liquid motion in the package. This is what we refer to as slosh. The amount of slosh depends on how the package is accelerated and on the properties of the liquid. There are large differences between skim milk and yoghurt.

If there is too much slosh the liquid might splash on the surfaces that should be be sealed. This can result in packages that are not properly sealed and possibly not airtight. If the package is not airtight the storage time is decreased considerably. This is particularly critical for aseptic packages, which are supposed to have a very long storage time.

A block diagram of the motion control system is shown in Fig. 2. It is (today) not realistic to measure the slosh on-line on the final commercial system. The reference is specified as an acceleration profile which is integrated twice to generate a position reference. The only measurements that are available to the servo system are the position and velocity of the package. Therefore, the only way to control the slosh is through the acceleration reference.

The problem is to calculate an open-loop acceleration profile that moves the package one step on minimum time with acceptable slosh. During the movement the slosh has to be bounded below a certain level. The same acceleration profile is applied at each step. Therefore, the acceleration must be such that the slosh constraint is not violated when the acceleration profile is repeated. One way to achieve this is to ensure that the slosh is in the same state at the beginning of each movement step. The natural choice of initial state of the slosh is that the liquid is at rest, since this is the state after the package has been filled.

An industrial testbed has been used where a carriage is mounted on a belt driven by a servo system similar to the one in the packaging machine. The container, an infrared laser displacement sensor, used to measure the surface elevation, and a small video camera is mounted on the carriage. Photographs of the testbed is shown in Fig. 3.

The problem is solved by first deriving a model of the slosh and then applying optimal control techniques to calculate the acceleration profile for the container. The control performance is then evaluated by experiments. A more detailed description of the problem and solutions to the movement problem when the package is moved only one step is found in [7]. An early version of the results in the present paper can also be found in [8].

2. Slosh modeling

Modeling of the sloshing behavior of liquids in externally excited containers has been studied in several applications, e.g. fuel slosh in aerospace applications, vehicle and ship dynamics, earthquake engineering and movement of containers. The choice of slosh model is nontrivial. The most advanced detailed model is described by a set of three dimensional nonlinear partial differential equations (PDE). Such models are hard to use in controller design, but can be useful for simulation and analysis. The nonlinear effects are important for very rapid movements when the surface elevation is large.

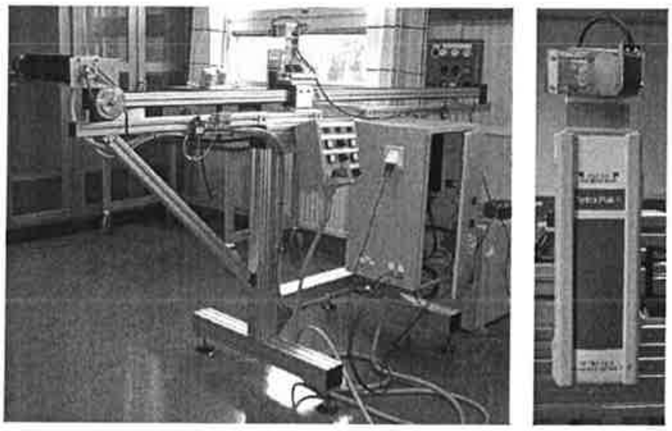

Figure 3 Photographs of the testbed. (left) Overview of the testbed, (right) close-up on on the container an the laser displacement sensor.

Numerical solutions to the 2D problem using the finite element method (FEM) is given in [1] and [9], and using the boundary element method (BEM) in [10]. Analytical solutions to the slosh problem can be obtained by approximations and is presented in [3] and [11]. These solutions are only valid for oscillation amplitudes much smaller than the wave length.

Two nonlinear phenomena can be observed in experiments with large slosh amplitudes: the oscillation is asymmetric and the oscillation frequency is slightly amplitude dependent (the oscillation frequency increases with decreasing amplitude). More complicated nonlinear models must be used to capture these effects, this is not discussed here. In Fig. 4 a video recording of the liquid is shown when the following acceleration is applied.

$$a(t) = \begin{cases} 6 & 0 \geq t < 0.05 \\ -6 & 0.05 \geq t < 0.1 \\ 0 & 0.1 \geq t \end{cases} \tag{1}$$

The video recording shows the complex behavior of the liquid.

A simple linear model with four states is used which captures the behavior for small slosh amplitudes well. The slosh model is given by the state space model

$$\dot{x} = \underbrace{\begin{bmatrix} -2\zeta\omega & -\omega & 0 & 0 \\ \omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} a\omega/2g \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{B} u \tag{2}$$

where x_2 is the surface elevation in meters, x_1 is the time derivative of the surface elevation divided by ω, x_3 is the container velocity and x_4 the container position.

For a rectangular container with liquid depth h and width a the oscillation frequency of the first harmonic is given by the expression below, see [3] and [11].

$$\omega = \sqrt{\frac{g\pi}{a} \tanh \frac{h\pi}{a}} \tag{3}$$

Throughout this paper a container with $h = 0.2$ m and $a = 0.07$ m is studied, which gives the theoretical value $\omega = 21.0$ rad/s.

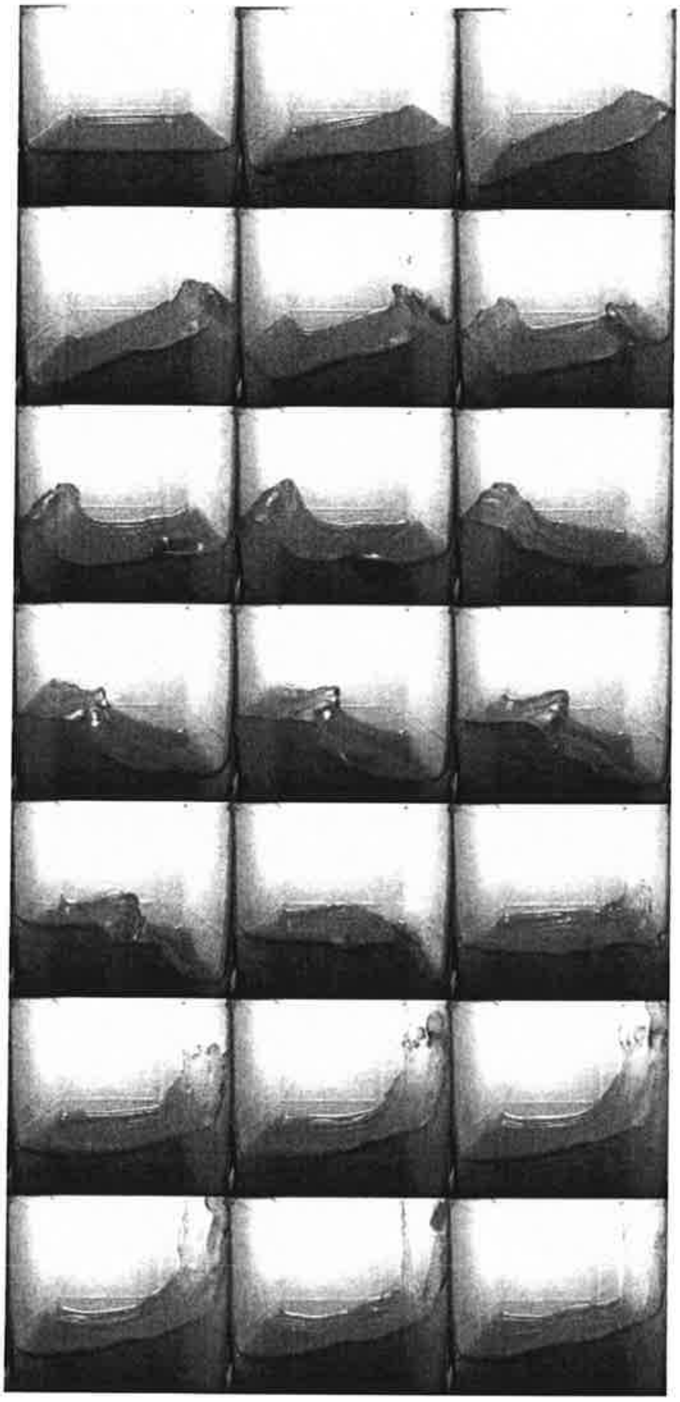

Figure 4 Video recording when the acceleration in (1) is applied. The recording starts in the upper left image and continues left to right each row downwards. There is 20 ms between each image. Notice the complex behavior of the liquid.

3. Calculation of acceleration profiles

The industrial, practical solution to the movement problem has been to use ad-hoc guessing to determine the structure of the acceleration profile. The development engineers then use experiments and experience to tune the parameters. This procedure is very time consuming and therefore it is of great interest to develop methods to calculate good acceleration profiles.

A solution is presented in [6]. The idea is to decrease the excitation of the resonance by a second order notch filter. Another approach is given in [4] where the problem is solved using optimal control techniques and a linear second order slosh model. In the optimization the control signal is discretized and the cost function

$$\frac{\alpha^2}{2}[s^2(T) + \dot{s}^2(T)] + \frac{\beta^2}{2}\int_0^T u^2(T)\, dt$$

is minimized for different values of α and β and with various constraints on $u(t)$ and $s(t)$. In [5] the problem is approached using flat systems.

In this paper optimal control techniques are applied to calculate the acceleration profile. The first approach was based on minimum-time control but was found to be non-robust against model errors, see [7] and [8]. The minimum-time approach resulted in a theoretical movement time of 0.38 s, but in practice the achievable movement time was 0.7 s. Here we present a minimum-energy based approach with better results.

3.1 Constraints

The following constraints have been used when solving the optimal control problems:

1. Acceleration: $|u(t)| \leq u_{max} = 9.81$ m
2. Slosh: $|x_2(t)| \leq s_{max} = 0.035$ m
3. Initial state: $x(0) = [0 \quad 0 \quad 0 \quad 0]^T$
4. Terminal state: $x(T) = [0 \quad 0 \quad 0 \quad L]^T$

The movement distance is $L = 0.2$ m.

3.2 Minimum-energy based optimization

The cost function

$$J = \int_0^T u^2(t)\, dt$$

is minimized subject to constraints 1-4 and the slosh model. The movement time T is assumed fixed in the optimization. The movement time T should be treated as the main tuning parameter. The choice of T involves a compromise between control signal smoothness and the production rate.

The described optimization problem is solved numerically and the result is shown in Fig. 5 for different values of T. The figure shows that by increasing the movement time slightly we can make the acceleration profile much smoother.

Analytical solutions to the minimum-energy problem can actually be easily obtained with the following modifications of the constraints:

- Remove the control constraint 1
- Replace the slosh inequality constraint 2 with a quadratic penalty on the slosh

This gives the following cost function and constraints

$$\begin{aligned} J &= \frac{1}{2}\int_0^T x^T(t)Qx(t) + Ru^2(t)\,dt \\ x(0) &= [0 \quad 0 \quad 0 \quad 0]^T \\ x(T) &= [0 \quad 0 \quad 0 \quad L]^T \end{aligned} \tag{4}$$

The control $u(t)$ that minimizes J is obtained by simultaneously solving the system equation (2) and the Euler-Lagrange equations, see [2]

$$\dot{\lambda}^T = -\frac{\partial H}{\partial x} \tag{5}$$

$$\frac{\partial H}{\partial u} = 0 \tag{6}$$

with the Hamiltonian

$$H = \frac{1}{2}(x^TQx + Ru^2) + \lambda^T(Ax + Bu)$$

Equations (2), (5) and (6) give

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \underbrace{\begin{bmatrix} A & -\frac{1}{R}BB^T \\ -Q & -A^T \end{bmatrix}}_{M} \begin{bmatrix} x \\ \lambda \end{bmatrix} \tag{7}$$

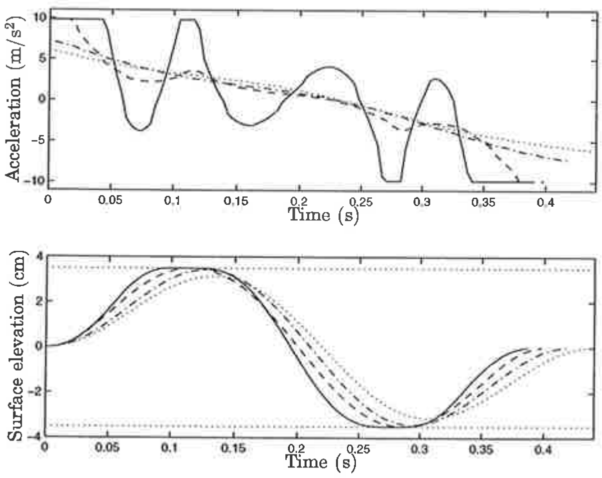

Figure 5 Simulation of the minimum-energy acceleration profiles for different values of T. The acceleration profile is much smoother already for a small increase of the movement time T.

and

$$u = -\frac{1}{R}B^T\lambda \tag{8}$$

The solutions to (7) can be written as

$$\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \Phi(t) \begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} \tag{9}$$

where $\Phi(t) = e^{Mt}$.

Insertion of $x(0) = 0$ in (9) gives

$$x(t) = \Phi_{1:4,5:8}(t)\lambda(0) \tag{10}$$

$$\lambda(t) = \Phi_{5:8,5:8}(t)\lambda(0) \tag{11}$$

where $\Phi_{1:4,5:8}$ means the sub matrix with row 1 to 4 and column 5 to 8 of Φ. Evaluation of (10) at time T gives

$$\lambda(0) = [\Phi_{1:4,5:8}(T)]^{-1}x(T) \tag{12}$$

The optimal control $u(t)$ is hence

$$u(t) = -\frac{1}{R}B^T\Phi_{5:8,5:8}(t)[\Phi_{1:4,5:8}(T)]^{-1}x(T)$$

For the case when there is no penalty on the state trajectory ($Q = 0$, $R = 1$) $\Phi(t)$ is

$$\Phi(t) = \begin{bmatrix} e^{At} & \Phi_{1:4,5:8}(t) \\ 0 & e^{-A^Tt} \end{bmatrix}$$

and with $\zeta = 0$

$$B^T\Phi^T_{5:8,5:8}(t) = \begin{bmatrix} \frac{a\omega}{2g}\cos\omega t & -\frac{a\omega}{2g}\sin\omega t & 1 & -t \end{bmatrix}$$

this gives the optimal solution

$$u(t) = -\frac{a\omega}{2g}(\lambda_1(0)\cos\omega t - \lambda_2(0)\sin\omega t) + \lambda_3(0) - \lambda_4(0)t \tag{13}$$

where $\lambda(0)$ is given by (12).

The control strategy in (13) is evaluated for the movement in several steps. The container is moved five times and the movement time is 0.46 s and the filling is 0.44 s. If the nominal value of $\omega = 21.0$ given by (3) is used when calculating the acceleration profile we do not get the expected performance. A way of defining performance is to study the remaining oscillation after one movement is performed. Define the residual slosh as $r(t) = s(t + T)$ where $s(t)$ is the surface elevation

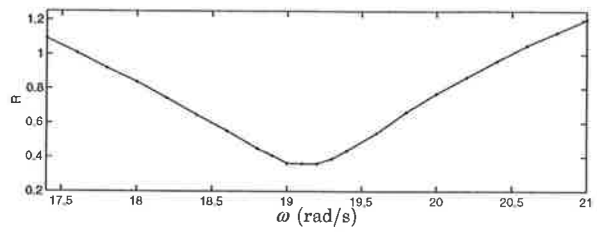

Figure 6 Experimental results of the performance measure R for different values of ω. A minimum is observed at $\omega = 19.1$; the theoretical value for small amplitudes is $\omega = 21.0$.

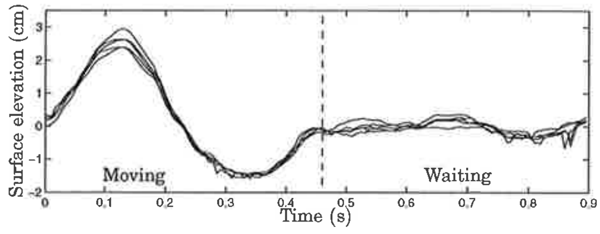

Figure 8 Experiment when the container is moved five times showing the surface elevation, the movement cycles are plotted upon each other. The result is acceptable.

and T is the movement time. A performance measure is then defined as

$$R = \sqrt{\int_0^\infty r^2(t)\, dt}$$

The performance measure R is shown in Fig. 6 for experiments with different values of ω.

A minimum is observed for the acceleration profile calculated for $\omega = 19.1$. The surface elevation for acceleration profiles designed for $\omega = 21.0$ and $\omega = 19.1$ are compared in Fig. 7. The result for $\omega = 19.1$ is preferable. Note that there is still some residual slosh for $t > 0.46$ s. These are due to high-frequency oscillations and oscillations in the direction perpendicular to the movement. Such oscillations are hard to model. A video recording of the liquid when a acceleration profile calculated for $\omega = 19.1$ is shown in Fig.9.

The surface elevation when five consecutive movements are performed is shown in Fig. 8. The figure

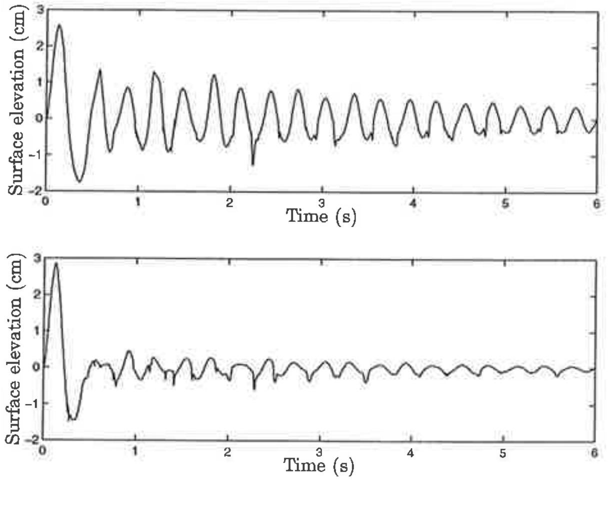

Figure 7 Experimental data: surface elevation for $\omega = 21.0$ (upper) and $\omega = 19.1$ (lower).

shows that the acceleration profile performs very well. Since the residual slosh is small the performance is independent of the time between the movements (i.e. the filling time). Therefore the same acceleration profile works even if the filling time is changed.

4. Conclusions

An industrially relevant problem has been described where an open container with liquid should be moved quickly without excessive slosh. A linear model has been used to model the slosh and it is shown that some of the nonlinear effects can be compensated by modifying the parameters in the linear model. The problem has been solved using techniques from optimal control both numerically and analytically. The calculated controllers have been implemented and verified in an industrial testbed and give better performance than previous controllers. The main advantage is that this is a systematic way to calculate good acceleration profiles instead of the old ad-hoc methods. Future work includes modeling of the nonlinear effects and application of the method on other liquids and package geometries. The developed methods are now being used in industry with good results.

Acknowledgments

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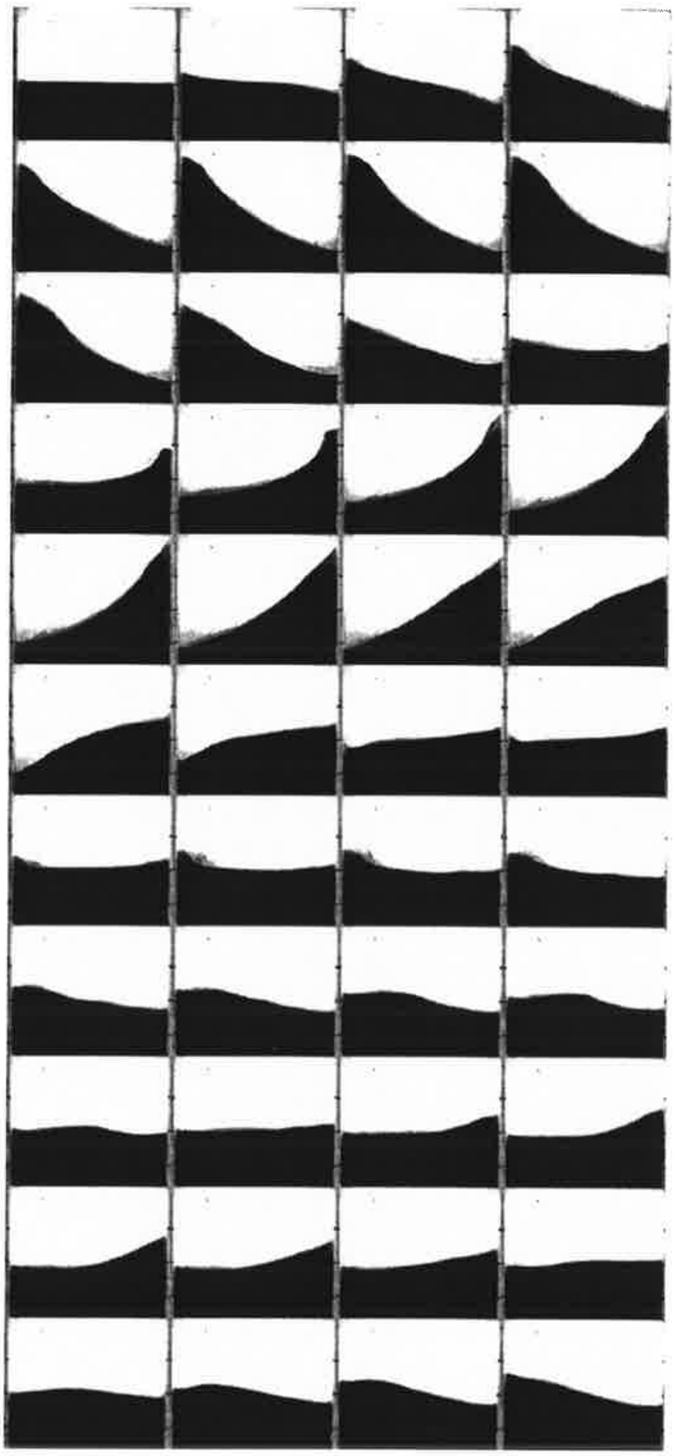

Figure 9 Video recording when an acceleration profile calculated for $\omega = 19.1$ is used, the movement time is 0.46 s. The recording starts in the upper left image and continues each row downwards. There is 20 ms between each image.