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Red giant depletion in globular cluster cores

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ABSTRACT

We investigate the observed depletion of red giants in the cores of post-core-collapse globular clusters. In particular, the evolutionary scenario we consider is a binary consisting of two low-mass stars which undergoes two common-envelope phases. The first common-envelope phase occurs when the primary is a red giant resulting in a helium white dwarf and main-sequence star in a detached binary. The second common-envelope phase occurs shortly after the secondary becomes a red giant. During the second common-envelope phase, the degenerate helium cores merge resulting in a core mass greater than the helium burning limit and the formation of a horizontal branch star. We show that stellar encounters enhance this evolutionary route in post-core-collapse clusters. These encounters increase the population of binary secondaries which would have evolved on to the red giant branch in the recent past.

Key words: binaries: close – stars: evolution – stars: general – globular clusters: general.

1 INTRODUCTION

Observations by Djorgovski et al. (1991) have shown that clusters with central cusps (presumably collapsed cores) have colour and population gradients. Their observations show that a number of post-core-collapse (PCC) clusters become bluer toward their centres, which is an effect of the demise of the red giant and/or sub-giant populations, and possibly an increase in the number of faint, blue objects. The *B–I* colour gradients are 0.1–0.3 mag per decade in radius in arcsec and extend to 20–30 arcsec.

The cores of PCC clusters have been imaged using the high spatial resolution of the *Hubble Space Telescope* (Bailyn 1994; Shara et al. 1998). Their observations show a depletion of the red giant branch stars relative to the horizontal branch. In addition, a population of supra-horizontal-branch stars (SHBs) has been observed. These are stars which lie in the Hertzsprung–Russell (HR) diagram as a distinct population brighter than normal horizontal branch stars. Bailyn (1994) found SHBs in the core of 47 Tuc which were about one magnitude brighter, while Shara et al. (1998), in images of NGC 6522, found SHBs which were one and a half magnitudes brighter in the *B* band. The formation of these objects is not well understood.

In PCC clusters, more stellar encounters are expected to occur than for the non-PCC clusters. These encounters could deplete the red giant population through two methods. First, collisions between red giants and binaries could either eject the red giant core from the surrounding envelope or result in a common-envelope phase (Adams, Davies & Sills 2003). This effect could at most deplete a few per cent of the red giant population. Secondly, stellar encounters would modify the binary population through hardening (encounters making the binary tighter) and exchanges (the third star is exchanged with the lower mass binary component if it is more massive). This would result in a different binary population to the non-PCC clusters. This modified population evolves such that the red giants, which would be observed today, undergo mergers with their binary companions.

As red giants evolve into horizontal branch stars, any evolutionary model which is considered needs to explain why the blue objects are not depleted in the same manner as the red giants. We investigate whether such an evolutionary scenario is possible. Such a scenario would require binary interactions to turn/accelerate the evolution of the red giants into becoming horizontal branch stars (see Djorgovski et al. 1991). In this paper we discuss an evolutionary channel which naturally explains both the observed depletion of red giants and the non-depletion of horizontal branch/faint blue stars in PCC clusters. In this channel, the binary consists of two low-mass (below helium-flash limit) stars and evolves through one or two common-envelope phases. In the first common-envelope phase, the primary is a red giant and loses its envelope to leave a helium white dwarf–main-sequence star binary. Depending on the separation after the first common-envelope phase, the secondary star can evolve to fill its Roche lobe as either a main-sequence star or as a red giant. If it fills its Roche lobe as a main-sequence star, a merger occurs resulting in a substantially nuclear evolved red giant. If it fills its Roche lobe as a red giant, a second common-envelope phase occurs. The two degenerate helium cores coalesce during the second common-envelope phase resulting in a helium burning core and the remainder of the common envelope forming an envelope around the merged object.

We examine the required parameter space (initial masses and separation) to explain why the depletion is more likely to occur in PCC clusters. Stellar encounters are expected to play a significant role in PCC clusters. After a number of encounters, the original binary will have had its separation ground down and its components will

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have higher masses due to exchanges (see Davies 1995). The initial mass range which would produce red giants today is between 0.6 and 1.0 $M_\odot$ depending on the age of the cluster, the metallicity and hence the turn-off mass. We show that without encounters this mass range is more likely to be found in single stars and the higher mass binary components (the primaries). After a number of encounters, a larger proportion of this mass range is found in binaries and, in particular, as the secondaries of binaries. It is these secondaries which lose their envelopes in the second common-envelope phase, explaining why red giant depletion is only observed in PCC clusters.

Our evolutionary scenario is discussed in Section 2. In Section 3 we investigate the effects stellar encounters have in modifying the binary population of PCC clusters. In particular, we simulate the evolution of a population of stars in which the binaries have undergone varying numbers of encounters. Section 4 contains a discussion of our results, in particular the observational consequences and the other products of our evolutionary scenario. We give our conclusions in Section 5.

2 EVOLUTIONARY SCENARIO

The evolutionary scenario for depleting the red giant branch is shown in Fig. 1. We consider the evolution of binary stars where convective mass transfer occurs when the primary evolves off the main sequence and fills its Roche lobe. A common-envelope phase follows. The secondary and the degenerate helium core spiral together as the envelope is ejected. The secondary then evolves to fill its Roche lobe as either a main-sequence star or as a red giant depending on the separation after the first common-envelope phase.

If the secondary fills its Roche lobe as a main-sequence star, stable mass transfer occurs initially. The secondary is more massive than the degenerate helium core and the binary evolves to shorter periods. Eventually the main-sequence star loses so much mass that it becomes fully convective and unstable mass transfer occurs, resulting in a merger of the main-sequence star and the helium white dwarf. The merged object is a substantially nuclear evolved red giant branch star. This red giant branch star evolves on to the horizontal branch quickly compared to the time it would have taken the secondary to evolve through the red giant branch if it had been isolated.

If instead the secondary fills its Roche lobe as a red giant, convective mass transfer again occurs, forming a second common-envelope phase. During this common-envelope phase, the degenerate helium cores spiral together. If the cores coalesce during the second common-envelope phase, the combined core masses may be enough to ignite helium. The remainder of the common envelope is left around the merged helium burning core and resembles a horizontal branch star. If the hydrogen envelope mass is small, then the product may resemble an sdB star. Alternatively, after the second common-envelope phase a tight double degenerate helium white-dwarf system could be formed. This system could be close enough that inspiral due to gravitational radiation brings them into contact. A merging double degenerate helium white-dwarf system probably becomes an sdB star too.

To model the binary evolution we use the binary evolution code of Hurley, Tout & Pols (2002). This code uses analytical formulae for stellar evolution (Hurley, Pols & Tout 2000) and includes the effects of tides, gravitational radiation, mass transfer and mass loss due to winds. For the common envelope, the code assumes the inspiral is given by an energy balance between the envelope binding energy and the difference in orbital energy of the inspiralling components.

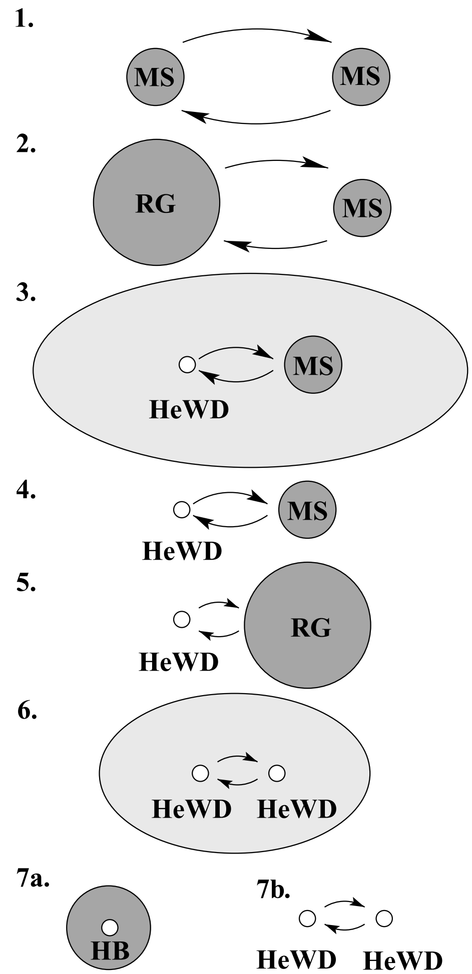

Figure 1. An evolutionary scenario for depleting the red giant branch, some of which may become horizontal branch stars. Beginning with two main-sequence stars (phase 1), the primary evolves into a red giant (phase 2) filling its Roche lobe, leading to the onset of a common-envelope phase (3) where the red giant envelope engulfs the main-sequence secondary star and the degenerate helium core. The post-common-envelope system consists of a tight binary containing the degenerate helium core (a white dwarf) and the main-sequence secondary star (phase 4). The secondary evolves to fill its Roche lobe as either a main-sequence star or as a red giant. If it fills its Roche lobe as a main-sequence star, a merger occurs resulting in a nuclear evolved red giant branch star. Alternatively, the main-sequence star evolves into a red giant (phase 5), filling its Roche lobe, leading to the onset of a second common-envelope phase (6). The red giant envelope then engulfs the helium white dwarf and the degenerate helium core. If coalescence occurs in the common envelope, a degenerate helium core forms surrounded by the remainder of the common envelope. If it is sufficiently massive to ignite helium ($M_c > 0.48$ $M_\odot$) it may appear as a horizontal branch star (phase 7a) otherwise it will appear as a red giant. Alternatively, after the second common-envelope phase a tight double degenerate helium white-dwarf system is formed (phase 7b).

The envelope binding energy is given by

$$E_{\rm bind} = -\frac{G M_{\rm c1} M_{\rm env1}}{\lambda R_1}, \qquad (1)$$

where $M_{\rm c1}$ and $M_{\rm env1}$ are the core and envelope mass of the red giant respectively, R_1 is its radius and λ is a constant. A typical value for λ is 0.5, although values greater than 1 are possible (Dewi & Tauris

2000; Tauris & Dewi 2001). The change in orbital binding energy is given by

$$\delta E_{\rm orb} = -\frac{1}{2}\left(\frac{GM_{\rm c1}M_2}{a_{\rm f}} - \frac{GM_{\rm c1}M_2}{a_{\rm i}}\right), \tag{2}$$

where $a_{\rm i}$ and $a_{\rm f}$ are the initial and final separations, respectively, and M_2 is the mass of the secondary. The energy balance is then given by

$$E_{\rm bind} = \alpha_{\rm CE}\delta E_{\rm orb}, \tag{3}$$

where $\alpha_{\rm CE}$ is the common-envelope efficiency. The ratio of initial to final separation only depends on the product $\alpha_{\rm CE}\lambda$ and not on their separate values. We take the value of $\alpha_{\rm CE}\lambda = 1$ in our calculations. We have run with different values for $\alpha_{\rm CE}\lambda$ but found that changing $\alpha_{\rm CE}\lambda$ did not alter the size of parameter space where coalescences occur (see Section 2.2 for further details).

2.1 Constraints on initial masses

In order for the observed red giant population depletion to occur, without depleting the horizontal branch, the secondaries in the binaries require masses near the turn-off mass for the cluster. Fig. 2 shows the age of a star at the end of the red giant branch as a function of initial mass and for metallicities $z = 0.0001$, 0.001 and 0.002. From Fig. 2 it is clear that a large range of cluster ages is represented if we consider a mass range for the secondary of 0.6–1.0 $\mathrm{M}_\odot$. This represents the range of masses of secondaries which are observed as red giants in globular clusters today.

2.2 Constraints on initial separations

The initial separation needs to be such that both binary components overflow their Roche lobes as red giants. If the separation is too small, then the secondary will overflow its Roche lobe before it reaches the red giant branch. This will deplete both the red giant and horizontal branch. Alternatively, if the separation is too large then the binary will not come into contact for a second time. In this case, no depletion of either the red giant or the horizontal branch occurs.

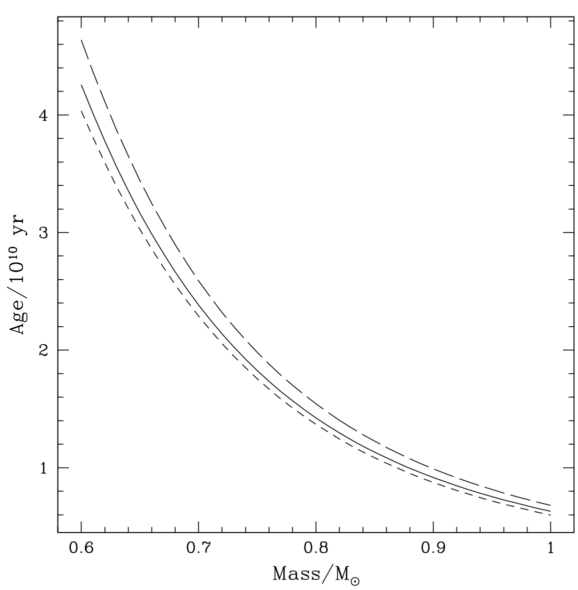

Figure 2. A figure showing the time to reach the end of the red giant branch as a function of the initial age for different metallicities. The solid line, short-dashed line and long dashed line are for metallicities of 0.001, 0.0001 and 0.002, respectively.

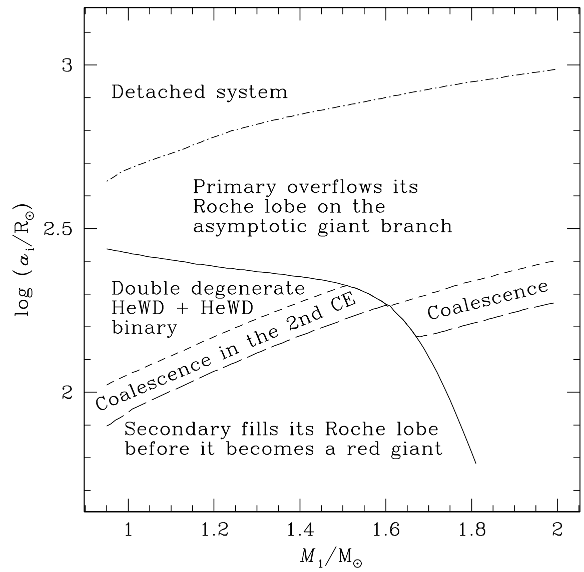

Figure 3. Plot of initial separation $a_{\rm i}$, for $M_2 = 0.8\ \mathrm{M}_\odot$, as a function of primary mass M_1 showing constraints for the formation of a binary as described in Section 2.

Fig. 3 is a plot of initial primary masses and separations and shows the constraints for the formation of a horizontal branch star via the coalescence of two degenerate helium cores during the second common-envelope phase. We take a metallicity of 0.001 and a secondary mass of 0.8 $\mathrm{M}_\odot$, both appropriate for a globular cluster. The long-dashed line shows the separation above which coalescence occurs in the second common envelope. Below this separation, the secondary fills its Roche lobe before it has reached the red giant branch. The short-dashed line represents the separation at which a double degenerate white-dwarf system is formed as opposed to coalescence occurring during the common envelope. The solid line represents the separation at which the primary does not fill its Roche lobe until the asymptotic giant branch. The dot-dashed line represents the separation above which the system remains detached.

Fig. 4 is a plot of initial secondary masses and separations and shows the constraints for the formation of a horizontal branch star via the coalescence of two degenerate helium cores during the second common-envelope phase. For the evolutionary calculation, we take a metallicity of 0.001 and a primary mass of 1 $\mathrm{M}_\odot$. The lines are the same as in Fig. 3. Fig. 4 shows that the constraints for formation of the merged object are only weakly dependent on the secondary mass.

In all of our calculations in which two degenerate helium cores merge during the second common envelope, we find that the combined core mass is sufficient to ignite helium. In all the cases which form a horizontal branch star, the primary overflows its Roche lobe near the end of the giant branch. The core mass of the primary in this case is around 0.4 $\mathrm{M}_\odot$. For the second common-envelope phase, we require the secondary to be a red giant and consequently its core mass is at least 0.2 $\mathrm{M}_\odot$. Consequently, the combined core masses are always large enough to ignite helium (which requires $M_{\rm c} > 0.48\ \mathrm{M}_\odot$).

We have run the evolution code with different values for the common-envelope efficiency and neglecting the mass loss due to

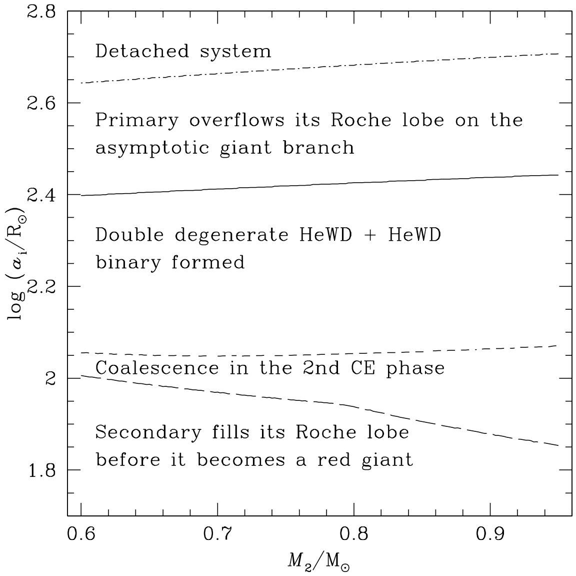

Figure 4. Plot of initial separation $a_{\rm i}$, for $M_1 = 1\ {\rm M}_\odot$, as a function of secondary mass M_2 showing constraints for the formation of a binary as described in Section 2.

the wind. Neglecting the wind did not significantly change the evolutionary outcome of the systems and the range of parameter space was found to be similar for each model. The common-envelope efficiency, however, determines the separations at which coalescence during the second common-envelope phase will occur. The solid line above which the primary overflows its Roche lobe on the asymptotic giant branch is unaffected but the separation after the first common-envelope phase is. This in turn affects whether a merger will occur in the second common-envelope phase, as this will depend on both the common-envelope efficiency and the initial separation. It was found that decreasing the efficiency by a factor of 2 shifted the parameter space in which coalescence occurs upwards by a factor of 0.2 dex but without significantly changing its area. The common-envelope efficiency consequently mainly affects the proportion of double degenerates which are formed relative to the horizontal branch stars. This demonstrates that the evolutionary scenario is a common outcome and not a consequence of the selection of certain parameters.

3 ROLE OF THE STELLAR ENCOUNTERS IN A PCC CLUSTER

In this section we evolve a population of stars in which the binaries have undergone varying numbers of encounters. Below we describe the assumptions which we use in these calculations.

If a star encounters a binary, the lowest mass star is usually ejected and the binary becomes harder (Davies 1995). The number of encounters between a binary and passing single stars in the core of a globular cluster depends on the encounter time-scale ($\tau_{\rm enc}$) which for binaries is given by

$$\tau_{\rm enc} = 7 \times 10^4 \frac{n}{10^5/{\rm pc}^3} \left(\frac{{\rm M}_\odot}{M_1 + M_2}\right) \frac{{\rm R}_\odot}{b_{\rm min}} \frac{v_\infty}{10\ {\rm km\ s}^{-1}}\ {\rm Myr}, \qquad (4)$$

where n is the number density of stars, $b_{\rm min}$ is the distance of closest approach and v_∞ is the stellar velocity. For typical parameters in the core of a PCC cluster, $\tau_{\rm enc}$ is approximately 400 Myr. This implies that during the main-sequence lifetime of a 0.8-${\rm M}_\odot$ star we can expect 20–40 encounters. However, as encounters make the binaries harder (see Section 3.1) the encounter time-scale will increase with each encounter. Consequently we will only consider up to 20 encounters per binary. To determine how encounters change the population of binary components we calculate the effect of a number of stellar encounters on a binary population. We assume an initial population of stars with an initial mass function given by (Eggleton, Fitchett & Tout 1989)

$$\frac{M}{{\rm M}_\odot} = \frac{0.19x}{(1-x)^{0.75} + 0.032(1-x)^{0.25}} \qquad (5)$$

where x is a random number between 0 and 1. This appears as a power law for masses greater than 1 ${\rm M}_\odot$ and with a turnover at low masses (with a peak at $M = 0.18\ {\rm M}_\odot$). Of our initial population of stars, we assume 10 per cent of them were in binaries. Both components of a binary were chosen at random from the initial population given by equation (5). To mimic encounters we assume that a binary has encounters with a star randomly drawn from the single star population. The effects of hardening and modification of the binary mass distribution are described below.

3.1 Binary hardening due to stellar encounters

During stellar encounters the orbital separation is hardened as the third body interacts with the system (Davies 1995). Initially binaries will have a range of orbital separations but the wide binaries will become tighter, pushing more binaries into the parameter space where they are likely to undergo two common-envelope phases and merge to form a horizontal branch star. Our assumptions for the encounters are described below.

The separation of a binary changes with an encounter according to

$$a_{\rm new} = \frac{a_{\rm old}}{1+\Delta} \frac{(M_1 M_2)_{\rm new}}{(M_1 M_2)_{\rm old}}, \qquad (6)$$

where Δ is the fractional change in binding energy of the binary and the probability distribution for Δ is given by (Heggie & Hut 2003, p. 221)

$$\frac{{\rm d}p(\Delta)}{{\rm d}\Delta} \propto \Delta^{-\alpha} (1+\Delta)^{(-4.5+\alpha)}, \qquad (7)$$

for a fly-by (no exchange) where α is 0.54 and $\Delta > 0.01$ (Heggie & Hut 1993). In the case of an exchange Hut (1984) has shown that

$$\frac{{\rm d}p(\Delta)}{{\rm d}\Delta} \propto (1+\Delta)^{-4.5} \qquad (8)$$

provides a good fit to numerical simulations. The eccentricity of a binary after an encounter is given by a thermal distribution (Heggie 1975)

$$\frac{{\rm d}p(e)}{{\rm d}e} = 2e. \qquad (9)$$

As encounters occur, the mean separation of the binaries becomes smaller. The more compact the binary, the less likely it is to encounter a star, as the collision cross-section is smaller. In addition, the circularization time-scales depend sensitively on the separation (Tassoul 1995). Consequently, the tighter the binary, the more likely it is to have circularized and thus have a smaller collision cross-section (an effect which we do not explicitly consider but which is included in the binary evolution code). In our calculations, any binary whose periastron distance is less than 5 ${\rm R}_\odot$ will be assumed to have been circularized and have too small a collision cross-section to encounter any more stars. This simplistic method stops the binaries becoming arbitrarily (and unrealistically) small while avoiding the complications of performing a full N-body calculation which

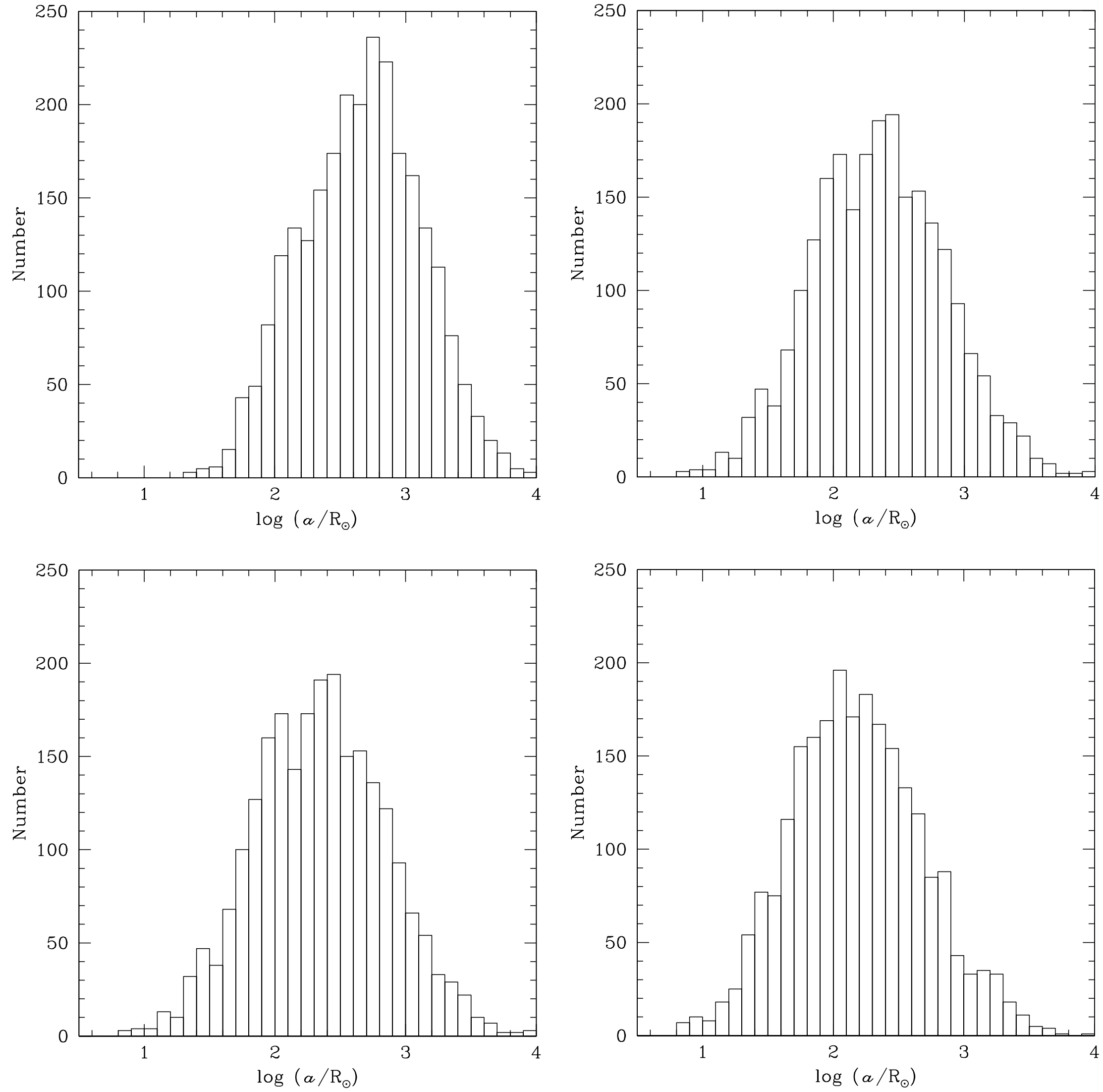

Figure 5. A figure showing how the separation of a population of binaries changes with encounters for 5, 10, 15 and 20 encounters, respectively. The initial population was taken to be flat in log a between 2.75 ± 0.75 R$_\odot$.

simultaneously calculates the binary evolution and the stellar dynamics. It is necessary to decouple the population synthesis from the stellar dynamics in this way, as a full *N*-body simulation of a cluster including the effects of stellar evolution is outside the current capabilities.

During encounters there is a tendency to equipartition energy between the total kinetic energy of both the binary barycentre and the third star and the internal energy of the binary (Heggie & Hut 2003). If the internal energy of the binary is much larger than the total kinetic energy, then the internal energy will decrease (which typically makes the binary tighter) and the total kinetic energy will increase. The separation at which the binary internal energy and kinetic energy of the system are equal is known as the hard/soft boundary. Hard binaries become tighter and soft binaries will eventually become unbound. In our calculations, the hard/soft boundary is included by calculating for each encounter whether the encountered star has sufficient energy to unbind the binary. If the binary is unbound, its components are assumed not to encounter any other stars and are evolved separately as single stars.

We calculate encounters for a population of 10^5 stars in which initially 10 per cent are in binaries. Fig. 5 shows a histogram of the binary separations after 5, 10, 15 and 20 encounters. The population was initially assumed to be flat in log a between 2.75 ± 0.75 R$_\odot$. Fig. 5 clearly shows that as the binaries undergo more encounters their mean separation becomes smaller. We also note that, as very tight binaries have large encounter time-scales, only a few binaries have short separations around 1 R$_\odot$.

3.2 Modification of binary masses due to stellar encounters

When a star encounters a binary, the lowest mass star is usually ejected (Davies 1995). This could be either the third star or one of the binary components, i.e. the remaining binary always consists of the two most massive stars. The mass range, which would have evolved on to the red giant branch in the recent past, is 0.8–0.81 M$_\odot$. Fig. 6 shows the fraction of stars with this mass range, which are the primary and secondary components of binary systems as a function of the number of encounters the binary population has experienced. The results did not significantly change if a different range of masses

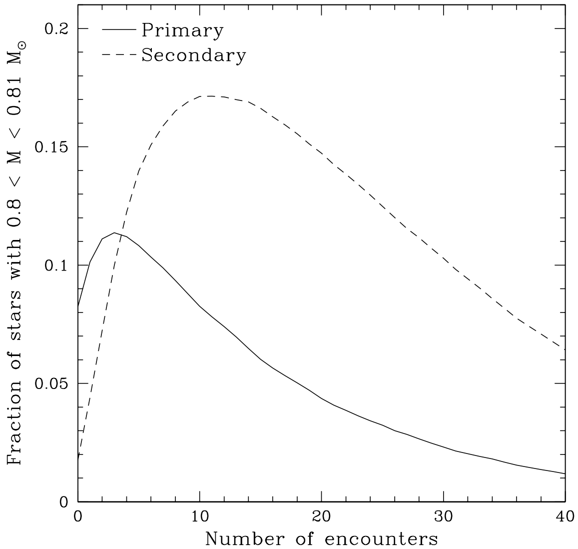

Figure 6. A plot of the proportion of stars with masses between 0.8 and 0.81 $M_{\odot}$ in the different binary components as a function of the number of stellar encounters a binary population has experienced. The solid line is the primary population and the dashed line is the secondary.

Table 1. A table showing the proportion of stars, in the mass range 0.8–0.81 $M_{\odot}$, which are single or the components of a binary after 0, 5, 10, 15 and 20 encounters per binary system. During the lifetime of a 0.8-$M_{\odot}$ star, around 20 encounters per binary are expected to occur in the core of a PCC cluster.

$N_{\rm enc}$	Proportion of stars which are single (per cent)	primary (per cent)	secondary (per cent)
0	90.0	7.9	2.1
5	76.7	11.2	13.1
10	73.8	8.8	17.4
15	76.1	6.6	17.3
20	79.5	4.8	15.0

was chosen. Table 1 shows the proportions of primary and secondary components after 0, 5, 10, 15 and 20 encounters per binary. From Table 1 and Fig. 6 it is clear that in a non-PCC cluster the population of stars would be the same as the field with roughly 8 per cent of the stars in the mass range of interest as the primaries of binaries and almost none as the secondaries. Consequently, the red giant population will not be significantly depleted through binaries undergoing two common-envelope phases in a non-PCC cluster. After 10 encounters, however, the situation is reversed. The secondary population has increased up to 15 per cent while the primary component has dropped to 5 per cent. Consequently, more systems can merge in the second common-envelope phase to form horizontal branch stars in a cluster whose members have had a number of stellar encounters.

3.3 Results of simulations

We evolved a population of stars, using the above assumptions, whose binaries had experienced 0, 5, 10, 15 and 20 encounters respectively. We evolved them for a time, which corresponded to a main-sequence turn-off mass of 0.8 $M_{\odot}$ in a cluster of metallicity 0.001. If coalescence occurred during the second common-envelope phase, we assumed that a core helium burning star was formed, which has a mass equal to the sum of the masses of the two degenerate helium cores of the binary components. Table 2 shows the proportion of stars which are red giant branch stars and blue objects at the end of the simulation compared to the number which would have formed if all the stars had evolved isolated. The numbers represent an average over 10 simulations. We considered as a blue object any star which was hotter and more luminous than the main-sequence turn-off. Table 2 also shows the number of binaries in which both components have evolved through the red giant branch and the number of binaries which have coalesced during the second common-envelope phase. From Table 2 it is clear that, as the number of encounters increases, far more binaries have components in which both evolve off the main sequence. In addition, more binaries have components which have coalesced during the second common-envelope phase. As these coalescences occur throughout the simulations, the majority of them will not be visible as blue objects today as they will have evolved through this phase of their evolution unless they were formed recently.

These simulations naturally explain why the depletion of red giants is only observed in PCC clusters. In non-PCC clusters there is not a significant number of binaries which have secondaries of the cluster turn-off mass. After a number of encounters, however, the situation is reversed and a number of turn-off mass stars are the secondaries in binaries. These binaries may undergo two common-envelope phases with coalescence in the second common envelope. These mergers form horizontal branch stars and deplete the red giant population.

4 DISCUSSION

We have shown that, if a binary system undergoes two common-envelope phases, a merger of the two cores is possible. We have

Table 2. A table showing the proportion of stars which are red giant branch stars and blue objects after the evolution of a population of stars which has undergone 0, 5, 10, 15 and 20 encounters per binary system. Also shown is the number of binaries in which both components evolve through the red giant branch and the number of binaries which coalesce during the second common-envelope phase.

$N_{\rm enc}$	Proportion which form red giants (per cent)	blue objects (per cent)	Mean number of binaries in which both stars evolve through the RGB	coalescence occurs during the second CE phase
0	99.1	96.2	102	12
5	89.7	91.0	562	44
10	86.8	89.5	651	78
15	85.3	90.7	523	78
20	86.1	92.3	329	49

assumed that this results in a horizontal branch star. The masses of the cores will add up to greater than the mass required for helium ignition on the red giant branch. If all the helium ignited simultaneously then the nuclear energy generated would be greater than the binding energy of the star. However, as in the case of degenerate helium ignition at the end of the red giant branch, the envelope will remain bound if it can survive the initial burst of luminosity reaching of order 10^7 L$_\odot$. It is generally assumed that stars do survive this.

If the cores merge near the end of the common-envelope phase, then the final star will have ejected a large portion of the hydrogen envelope and so the merged object may resemble an sdB star. Han et al. (2002) have investigated the formation channels for sdB stars and, although one of the routes considered is dynamical mass transfer followed by a common-envelope phase, two common-envelope phases are not discussed (although the merger of two white dwarfs is one of the formation channels considered). Whether or not this is a main evolution channel is unknown as it may depend strongly on the initial separation.

The merged objects in our calculations have a higher core mass and a lower envelope mass than normal horizontal branch stars. The higher core mass results in a larger luminosity and so the SHBs observed by Bailyn (1994) and Shara et al. (1998) (or a subset of them) may be the merged objects which form after coalescence during the second common-envelope phase. It is reasonable to expect that a horizontal branch star formed through a coalescence would (at least initially) have a faster rotation than those which have evolved off the red giant branch. Recio-Blanco et al. (2002) have found a population of fast rotating ($\sim$30 km s^{-1}) horizontal branch stars in galactic globular clusters. These (or a subset of them) may be the result of a coalescence of the type proposed in this paper.

We have shown that in the cores of PCC clusters the conditions are such that depletion of the red giant population is possible. We have not, however, performed detailed binary population synthesis of the cluster as a whole. This is complicated due to the necessity of including stellar encounters, stellar evolution and binary interactions. The solution to this may be the use of *N*-body modelling codes. However, all we are showing here is that this evolutionary route is both possible and likely in PCC cluster cores. In this paper we do not attempt the determination of the depletion due to common-envelope mergers including the effects of encounters, exchanges and binary evolution simultaneously.

In addition to the horizontal branch stars formed through mergers, our calculations predict the existence of a substantial double degenerate population. The double degenerate population we find in our calculations has two main components. The first is where both are helium white dwarfs. In this case the primary will have overflowed its Roche lobe near the end of the red giant branch while the secondary will typically overflow near the start of the red giant branch. This is a consequence of the inspiral during the common-envelope phase resulting in the radii of the two stars at Roche lobe overflow being significantly different. The mass ratio in this case is typically between 1.5 and 2 and the final orbital period is a few hours.

The second component to the double degenerate population comes from carbon–oxygen and helium white-dwarf binaries. These are formed when the primary overflows its Roche lobe on the asymptotic giant branch resulting in both the carbon–oxygen white dwarf–helium white dwarf binary and a larger separation. The mass ratios in these cases are between 1 and 1.5 with larger periods of a couple of days upwards. Figs 3 and 4 show that there is clearly a large region of parameter space where such a double degenerate binary would be formed. An observed binary which could possibly be double degenerate with two white dwarfs of different compositions is HD 188112. This has a helium white dwarf which lies below the zero-age extreme horizontal branch in an HR diagram and a period of 0.61 d (Heber 2003). Its companion has a minimum mass of 0.72 M$_\odot$ which shows that it must be a compact object. The helium white-dwarf mass is larger than that expected for a binary millisecond pulsar companion (Rappaport et al. 1995) which suggests that the companion is a white dwarf and not a neutron star. The 0.61-d period can be explained if the primary was more massive than considered in our calculations ($>$2 M$_\odot$) and so had a larger envelope mass requiring a greater inspiral to eject it.

Compared with the observed population of Maxted, Marsh & Moran (2002) we find that the double degenerate helium white-dwarf binaries most closely resemble their population in which the mass ratios are around one and the periods are from a few hours to slightly greater than a day. There is a significant difference, however, in the ages determined from cooling models. In their models the white dwarfs are within a few hundred Myr of each other, while in our models the more massive primary overflows its Roche lobe typically a few Gyr before the secondary. A possible remedy to this scenario may be binaries which are closer in mass to each other or which initially were much more massive. In both these cases they would become giants in similar time-scales, allowing the white dwarfs to be formed closer in age to each other. The other populations of double degenerates, however, should still be present and it may be that there are selection effects that determine which binaries are observed or that the double degenerate population in the absence of a number of encounters is different.

Han (1998) has modelled the formation of double degenerates including two common-envelope phases. This evolutionary route was found to produce a significant portion of the total double degenerate population. Hurley et al. (2002) have investigated the region of parameter space from which double degenerates form. They find two main evolutionary paths. The first is Roche lobe overflow before the primary becomes a red giant followed by a common-envelope phase and the second is two common-envelope phases. The first path is preferred at close separations ($a < 10$ R$_\odot$) while the second is preferred at larger separations ($a > 200$ R$_\odot$). This is consistent with our work which finds two common-envelope phases the dominant formation route for double degenerates from binaries when $a > 100$ R$_\odot$. Nelemans et al. (2001) have also modelled the population of double degenerates although they use different assumptions concerning the inspiral during the first common-envelope phase and the mass loss due to a wind.

Of the double degenerates which do form, a number of them are close enough that they inspiral and mass transfer occurs. In the case of white dwarfs, the critical mass ratio at which mass transfer becomes unstable is 0.628 (see Hurley et al. 2002). The helium–helium white-dwarf binaries which form in our simulations occur when the primary fills its Roche lobe near the end of the red giant branch and the secondary fills its Roche lobe near the start of the red giant branch (this is a consequence of the inspiral during the first common-envelope phase). Consequently, the mass ratio is always less than the critical mass ratio for dynamic instability. In this case as mass is transferred the binary becomes wider again. The final binary will then have a very low mass secondary and a $\sim$0.6-M$_\odot$ helium white-dwarf primary, which might undergo ignition to form a carbon–oxygen white dwarf.

5 CONCLUSIONS

We have described an evolutionary scenario in which the red giants are depleted in the cores of PCC clusters. A binary undergoes two common-envelope phases, during the second of which coalescence of two degenerate helium cores occurs. The combined mass of these cores is sufficient for helium burning to commence, resulting in the formation of horizontal branch stars. The binary separations required for this to occur are around 100 $R_{\odot}$ for a primary of mass 1 $M_{\odot}$ and a range of secondary masses.

We have shown that this evolutionary scenario will be favoured in the cores of PCC due to the effects of stellar encounters. These encounters both harden the binary towards the required orbital separation and increase the population of secondaries which have the cluster turn-off mass.

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