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The Long-Run Relationship Between Public Debt and Economic Growth In Advanced Economies

By

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Master's Programme in Economics

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Abstract

This study revives a heavily debated relationship between government debt accumulation and economic growth. Although indebtedness levels stay high, current economic growth forecasts seem optimistic not only in the euro area but in other advanced countries, as well. 24 advanced OECD economies are investigated to understand if financial outcomes are detrimental to the real economic variables in the long-run. In effect, three related hypotheses are tested. *H1*: existence of the long-run negative public debt effect; *H2*: possibility of the impact through crowding out stock of capital; *H3*: existence of thresholds beyond which the effect changes. Methodologically, this study builds on the recent panel techniques and the work of Pesaran et al. (2013) to employ a dynamic model which accounts for a cross-sectional dependence of interconnected countries. The results show a negative relationship between debt/GDP growth and real GDP per capita growth in the long-run for the OECD economies. The impact is larger for the euro area sub-sample. According to the threshold study for *H3*, countries with 90% ratio or higher tend to grow slower in the long-run, but this result is not robust. Importantly, despite the growth theory suggesting that capital accumulation transmits negative debt effects in the long-run, *H2* is rejected. A seeming absence of crowding out effect in long horizons motivates the search for alternative explanation of causality and to explore impacts of real interest rates or Total Factor Productivity channels in the future research.

Keywords: Public debt, long-run, growth, OECD, common factors, dynamic models

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1.Introduction

The relationship between countries‘ debt and economic growth rate has been a long-standing empirical question for developing economies. For example, Krugman (1989) points to a possibility of debt overhang. Emerging countries collect an *external* debt which later becomes unsustainable and the necessity to reorientate resouces towards obligation payments harms future growth potential (Krugman, 1989). Another study hinging on simillar theoretical arguments is that of Patillo et al. (2004) who investigate 61 developing countries over 30 years and find that debt affects growth through reduced productivity. However, given a recent severe financial crisis, the empirical focus started shifting – research on advanced interconnected economies has lately attracted much attention. As governments of developed countries started running large deficits to counteract the crisis, impacts of *public* debt (with both dosmestic and external elements owed by governments) overhang became and important question to explore.

The investigation momentum for public finance of the developed countries emerged in the aftermath of the debt crisis which evolved from the financial crisis. It was at the centre of an economic turmoil in the euro area a few years ago and still remains a highly political issue. Contrary to emerging economies, the euro area and other advanced countries maintain highly developed financial markets. They allow government debt to accumulate not only externally as often the case of developing countries is, but also internally via active bond trading (Panizza and Presbitero, 2013). Paradoxically, financial development and interconnectedness gave a strong base for instability and a fiscal crisis (Lane, 2012; Gourinchas & Obstfeld, 2012). Starting with the financial crisis of 2007 – 2008 which affected banking sector through defaulting loans and a slump in asset prices, the sovereign debt issue became very serious because public recapitalization of banking system was needed (Lane, 2012). As of 2009, countries started reporting unusually high debt to GDP ratios. For example, Spain and Ireland were heavily dependent on the construction sector and its weakening resulted in a tax revenues deteriorating faster than GDP.

Currently, fiscal climate and economic prospects seem more optimistic, although debt/GDP ratios stay above 100% in Greece, Italy, the U.S. or Japan. According to the most recent projections, economic growth is accelerating and advanced economies are likely to grow annually by 1.9 percent in 2017 and 2.0 percent in 2018 (World Bank, 2017). These prospects are higher than the projections in 2016 for the same year. According to the World Bank (2017), the main reasons for optimism include a boost in manufacturing output and diminishing drag from inventories. The positive signs of growth but persistent indebtedness raise a question if a heavy accumulation of debt by governments of advanced countries have any negative impacts on economic growth in the future. Again, this puzzle is highly empirical and it naturally spurred multiple studies. They generally vary in terms of sample, length of time series and estimation techniques. Apart from these nuances, a crucial categorization is whether studies investigate the *short*- or *long-run*. Given the current optimistic forecasts and the past crisis that still cause political backlash, it is natural to pay attention to resilience of growth process in long horizons.

The present study has three aims that work as a contribution to the analysis of debt-growth relationship. First, the recent developments in panel data techniques are used to identify parameters which can be problematic in the presence of high cross-sectional dependence among countries. This arises because of strong and weak common factors. The former affect all countries simultaneously while the latter work among some pairs or clusters of countries. Secondly, the new methodology is applied to the sample of advanced OECD economies, which are heavily interrelated, hence making the identification problem very relevant. Previous studies applied similar methodology to more general broad samples of both developed and emerging economies (e.g. Pesaran et al. (2013) or Eberhardt and Presbitero (2015)). Lastly, the study draws motivation for causal logic and estimation approach from the growth theory. This helps to identify the interim variables that transmit effect of debt accumulation to economic growth. In effect, the causal channels are explored not only technically (e.g. endogeneity problem) but theoretically, as well. Estimation-wise, it motivates measuring the long-run impacts in the steady state, when variables reach their time equilibrium. Hence time series properties, such as persistence and unit roots need to be accounted for.

2. Public Debt in the Advanced Countries: A Brief Exploratory View

Focusing specifically on public debt domestically and externally accumulated by governments works as a test of public finance management and brings policy dimension into analysis. Issuing bonds to finance public investment, services or government consumption in the short-run seems like a conventional strategy of fiscal policy. Elmendorf and Mankiw (1999) describe a standard view on public debt within a Keynesian framework: public spending financed by public debt stimulates aggregate demand in the short-run and helps to utilize resources which are underutilized. On the other hand, debt accumulation may occur due to major shocks, such as wars or crises hindering collection of tax revenues. The latter case defines the recent debt dynamics in advanced economies (Reinhart and Rogoff, 2010b). Irrespective of reasons, Greiner and Fincke (2009) argue that keeping public debt measure (debt/GDP ratio, for instance) sustainable and mean-reverting over time, primary government surpluses need to react positively to increasing debt in the future periods. However, the lack of such positive reaction and persistent upward trends appear evident in advanced economies given a historical glance in Figure 1.

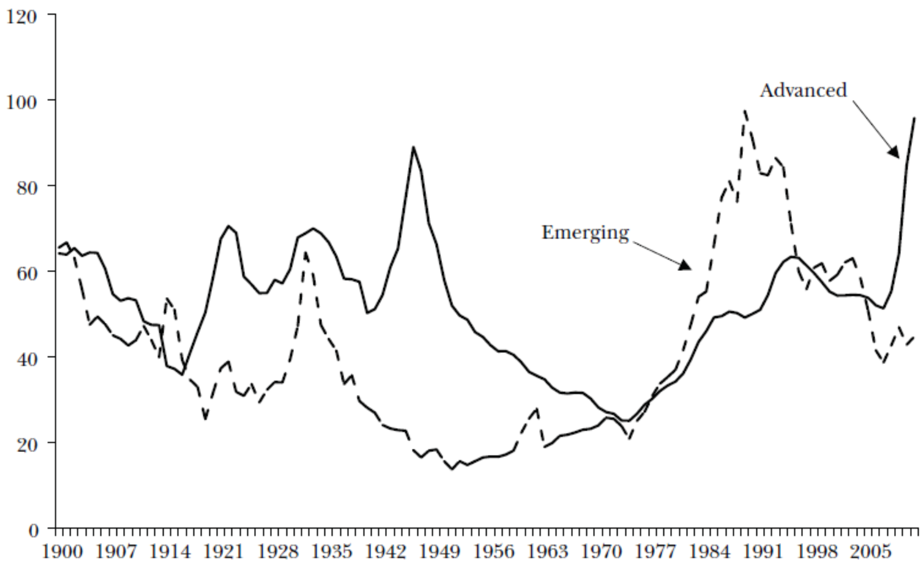

Figure 1. Central government debt/GDP in advanced and emerging countries. Source: Reinhart and Rogoff (2010b).

As can be seen from the historical data of Reinhart from Rogoff (2010b), in the course of 110 years, advanced economies were accumulating higher levels of public debt than their developing counterparts most of the time. The solid line, which tracks an unweighted cross-sectional average of 22 advanced OECD countries, starts to climb persistently at 1970's and reaches 100% debt to GDP ratio in 2010. The last decade is marked with a systematic crisis, in particular. The need of fiscal stabilizers, surging unemployment benefits and falling taxation revenues boosted debt levels dramatically both in the euro area and a wider OECD circle (Elmeskov & Sutherland, 2012). Looking back, piles of debt, followed by a gradual consolidation, emerged in the time of World War II but fairly high debt ratios were evident around the World War I era and the Great Depression, as well. More specifically, around the sequence of these major events public debts reached peaks in the U.S., Italy, France, the United Kingdom (Reinhart & Rogoff, 2010b; Balassone et al., 2011).

Importantly, the debt history of advanced countries suggests that surges of debts occur as an *answer* to deteriorating economic conditions and hindered growth: lowered taxation revenues, falling asset prices (Lane, 2012) or detrimental war periods. However, Reinhart and Rogoff (2010b) point at a lack of consolidation and persistence of large public debts. They find episodes of slower growth systematically following persistent episodes of large debt/GDP ratios. Thus, a tendency for large debt/GDP ratios to grow for years in advanced economies motivates the exploration of a one-sided effect from debt to economic growth and its consequences in a long perspective.

3. Theoretical Framework: Debt Economics and Econometrics

Understanding the long-run relationship between public debt accumulation and economic outocomes, particularly in the developed countries, is a complex task. There are two problems: one that is theoretical and one that is empirical. While the theoretical foundations of the impact of debt on economic growth lie in economic models encompassing micro decisions and their generalizations to macroeconomic effects, the econometric task is rather subtle. This is because public debt becomes one of the many regressors that can explain economic growth. Importantly, they are often interrelated which causes a threat to validity of partial effects estimation (Pesaran and Smith, 2014). Hence, it is important to consider models that are general enough and do not suffer from interdependencies of explanatory variables. Moreover, an extra step is to explore the connection between debt and important growth drivers. Therefore, it is necessary to deduct the research hypotheses based on core growth theory concepts. In particular, the long-run causal relationship between public debt and capital formation helps to formulate the hypotheses and motivates the methodology.

3.1. Public Debt and Long-Term Capital Acumulation

Early theoretical arguments on long-term effect of public debt accumulation rest on intergenerational links and inter-temporal redistribution of production. According to works of Buchanan (as quoted in Tempelman, 2007), incidence of public debt goes to the future generations which results in utility loss hindering productivity. Effectively, shifting financing of public expenditure from taxation to deficit (i.e. increasing debt accumulation via bonds emission) induces a boost in expected taxation of future periods since government has to obey an inter-temporal budget constrain (Modigliani, 1961). The translation of debt-induced tax burden to real economic variables is reflected in formal models of overlapping generations (Diamond, 1965; Saint-Paul, 1992). Particularly, Diamond (1965) and Saint-Paul (1992) consider a setting where two generations live at every time period t and younger workers invest their savings into capital and reap the return at $t + 1$ already belonging to elder generation. As Obstfeld (2012) illustrates, long-run equilibrium in this class of models results in a *steady state*

[1] with a lower capital stock if the elder generation acquires bonds as an asset. This is due to working generation facing taxes that are a deterministic function of debt accumulation. As a result, a portion of their savings is wasted on unproductive debt[2]. Therefore, public debt accumulation hinders productivity in the steady state.

More general ideas which abandon temporal links between generations but still target capital investments are provided by Gale and Orzag (2003) and Reinhart and Rogoff (2010b). Shifting financing sources from public taxation to issuing debt reduces the pool of national savings, causing pressure on real interest rate (Gale and Orzag, 2003). In effect, private agents compete for lower funds for investment. Reinhart and Rogoff (2010b) further point at the risk factor when the indebtedness is high. This argument applies for both externally and internally accumulated debt. Boosting the risk premium on government bonds spills to long-term real interest rates that influence long-term investments in capital and other durables (Sorensen and Whitta-Jacobsen, 2005). Thus, rising risk premiums have a negative effect on economic growth. This transmission channel is actively put under empirical scrutiny and gives mixed evidence (e.g. Laubach, 2009; Baum et al., 2011). Lastly, Greiner and Fincke (2009) bring the capital crowding out argument to the micro-decision level. They consider two cases: 1) the government runs permanent deficits but obeys budget constraints in the sense that primary budget surpluses increase enough as a reaction to debt that is *growing* over time; 2) the budget is balanced and debt/GDP ratio becomes zero in the long-run. They consider situation where steady state growth under 1) is always lower than under 2). This happens because bonds are included in household wealth and ever growing debt creates incentives to consume more out of wealth. This leaves less savings for productive capital formation (Greiner and Fincke, 2009).

3.2. Role of Capital in Exogenous and Endogenous Growth Theory

As the discussed models clearly suggest, capital accumulation is pivotal for long-run effect of public debt. The subtle point is to what extent capital is important for the long-run economic growth. According to the exogenous growth theory, capital accumulation has only *transitory*[3] effects and brings zero growth in the steady state due to diminishing returns to scale (Barro and Sala-i-Martin, 2004). By constrast, endogenous growth ideas emphasize non-diminishing returns to scale of capital due to spill-over effects – Total Factor Productivity (TFP) is a

[1] Or a balanced growth path. This is a state where various macroeconomic variables grow at constant rates in the long-run, i.e. reach a long-run equilibrium (Barro & Sala-i-Martin, 2004).
[2] This argument implicitly assumes absence of Ricardian effects. Thus, people, are myopic enough and do not rationally compensate for expected increase in future taxation by saving more.
[3] This is just a temporary state after a shock while variables converge to a new steady state (King & Rebelo, 1989).

function of capital accumulation. This is the key idea of the *AK* growth models (Rogers, 2003). In the words of Rossi (2012), increasing capital stock increases the ability of economies to use new technologies which results in a higher growth. This growth channel is particularly relevant for advanced economies that are capital rich. This link can be illustrated using the reduced form Cobb-Douglas production function ($Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}; A_t = K_t^{\phi}$) that can be tested empirically (also adding additional explanatory variables if necessary) in the spirit of Rao (2010):

$$\gamma_t = \delta g_{k_t} \rightarrow \gamma^* = \delta g_k^* \; as \; t \rightarrow \infty \quad (1)$$

Equation (1) is obtained by considering variables per worker in the Cobb-Douglas production function and taking first differences of natural logarithms which gives the growth rates γ and g_k. Here, the steady state GDP per worker growth rate is driven by the capital per worker growth rate. The interpretation of capital as a TFP source suggests a possibility of growth in a steady state, which is emphasized by Kobayashi (2015) when comparing the discused overlapping generation models of Diamond (1965) and Saint-Paul (1992). The former model hinges on exogenous growth theory, implying that debt accumulation might not cause lower growth because crowding out affects already unproductive capital. In the latter model, capital has constant returns to scale, so debt brings a steady state with a decreasing GDP growth.

3.3. Possibility of Threshold Effects

The *crowding in* alongside the argument of crowding out can be introduced, as well. For example, Elmendorf and Mankiw (1999) describe public debt as standard tool to stimulate economy in the short-run, marely exchanging source of government financing from taxes to issuing debt. Hence, from the Keynesian point of view, public debt can bring positive growth, irrespective if it collected externally or internally. As a result, detrimental growth impacts arise after some threshold of indebtedness is crossed. Arai et al. (2014), on the other hand, discuss crowding in in terms of internally produced debt. They analyse an economy under financial constraints along the lines of Woodford (1990) who describes public debt as means to increase liquidity in constrained economies. In the model of Arai et al. (2014) agents with different productivity levels coexist and they all face borrowing constraints. When government issues more debt, a shrinkage of financial resources occur (Gale and Orszag, 2003) and interest rates rise which results in investment crowding out. Despite that, resources become more intensively used by more productive economic agents since the less productive ones are excluded and face even more constraints. This redistribution of resources along with an opportunity to save in form of government bonds allow productive agents to raise their investments. Arai et al. (2014) demonstrate that this crowding in effect is stronger when debt to GDP ratio is low.

3.4. Research Hypotheses

Given implications of the growth theory and the channels through which debt accumulation can affect long-term growth rate, three following research hypotheses can be deducted:

H1: There is a negative long-run relationship between public debt and economic growth

H2: Accumulation of debt has negative long-run impact on capital accumulation

H3: There exist levels of debt to GDP ratio beyond which negative impact on growth becomes stronger

All three hypotheses are closely related. As the discussion of the economic theory above implies, accumulation of public debt negatively impacts economic growth (*H1*) via reduced capital accumulation (*H2*) as a steady state with less capital emerges in the long-run. Hence, the hypotheses should be tested under equilibrium conditions, i.e. when a permanent shock happens to public debt variable and a new steady state comes after the transitory period. This requires an adequate econometric model and investigation of time series properties. Moreover, *H1-H3* do not assume a *uniquely* non-linear effect of public debt. For example, the model of Arai et al. (2014) suggests that the effect is only non-linear, but the theory related to a long perspective assumes a monotonic relationship in the long-run. Consequently, *H3* is a prediction which suggests measuring the additional effect of an average level of indebtedness (i.e. the threshold) in a steady state.

3.5 Theory of Long-Run Estimation

To reflect the discussed implications of economic theory in an econometric setting and estimate particularly the long-run effects of public debt on growth, it is necessary to take *two* steps. The first is to model economy in the equilibrium, or a steady state. Secondly, it is important to estimate more general and flexible models than (1) which includes both debt and capital variables. As was shown in Greiner and Fincke (2009) and Saint-Paul (1992), causality from debt accumulation to growth runs indirectly through capital formation. As a result, the very construction of *H1* and *H2* imposes incorrect coefficients. Because a regression coefficient is interpreted as a partial effect *holding other variables constant*, Pesaran and Smith (2014) show that historical interrelations between regressors do not allow to keep one variable constant when taking the partial effect of the other (a sketch of the proof for the debt-growth case is provided in Appendix A).

To model an economy in the equilibrium, a common way in the growth literature is to use data averages over time periods. This method is used in the earlier prominent debt and growth studies of Kumar and Woo (2010) and Baum et al. (2012). Both studies average data over 5 years. However, as Rao (2010) or Eberhardt and Teal (2011) notice, to reflect an economy converging to a steady state, it is advisable to average observations over 10 or 20 years. However, this procedure drastically reduces the number of observations, which means that resulting estimates become less precise had the full sample been used. It is possible to use overlapping averages, but this necessarily induces a serial correlation in the errors (Panizza & Presbitero, 2014).

Naturally, the data on public debt and growth cannot tell if an economy is in a steady state because it is never observed in practice irrespective of the series being averaged or not (Rao, 2010). However, an alternative route can be taken by creating a counterfactual and relying on time series properties of the variables. The literature distinguishes two cases: when series are $I(1)$, or unit root, and $I(0)$, or stationary. Gonzalo et al. (2001) suggest modelling long-run equilibrium between two $I(1)$ variables and calculating long-run coefficient via cointegration. Essentially, this would detect equilibrium relationship between two series, where shock to one or the other variable induces a temporary detour from the attracting equilibrium. However, based on the long-term growth and steady state ideas (Romer, 1990; Barro, 1990), it is not hard to see that persistence and unit roots are inconsistent with an equilibrium state that is reached by the variable *itself* over time and not *between* drifting series.

An alternative that is more appealing to economic growth is given by Pesaran et al. (2013) in a setting of Autoregressive Distributed Lag Model (ARDL) with $I(0)$ series. Here, an equivalent of a steady state in deterministic growth models is a constant expectation assumed by the stochastic series over a long time. In this case, the long-run parameter can be calculated as a non-linear function of other parameters in the regression equation that gives a marginal effect of a *permanent change* in public debt variable. This is a cumulative change in expectation of the series. The equations (2) and (3) adapted from Pesaran et al. (2013) present the idea.

$$\gamma_{t+j} = \alpha + \varphi_1 \gamma_{t-1+j} + \beta_0 Debt_{t+j} + \beta_1 Debt_{t-1+j} + \varepsilon_{t+j} \quad (2)$$

$$\gamma^* = \alpha + \varphi_1 \gamma^* + \beta_0 Debt^* + \beta_1 Debt^* \quad (3)$$

Equation (3) is derived by taking the limit of a conditional expectation of each variable in (2): $\lim_{j\to\infty} E[.|\mathcal{M}]$. Here, $\mathcal{M} = \{\boldsymbol{\mathcal{F}}_t,\ e_{Debt,t+s} = \sigma, s = 0,1,2 \ldots\}$, where $\boldsymbol{\mathcal{F}}_t$ is a sigma-algebra generated until t and $e_{Debt,t+s}$ is a permanent shock to public debt series. Equation (3) emulates relationship between GDP growth and debt accumulation in a long-run equilibrium.

As shock of the size σ happens to the debt variable and persists to the future[4], in the long horizon $I(0)$ series comes back to the equilibrium with a shifted expected value. The long-run equilibrium parameter is then solved for as a non-linear function of parameters:

$$\theta = \frac{(\beta_0 + \beta_1)}{1 - \varphi_1} \tag{4}$$

Note that solution would not be possible if the variables were $I(1)$ as $\lim_{j \to \infty} E[.|\mathcal{M}] = f(t)$. ARDL in (2) can be augmented with p and q lags for the growth and debt series, respectively. Therefore, the long-run parameter[5] for ARDL (p, q) is received according to the same logic:

$$\theta = \frac{\sum_{j=0}^{q} \beta_j}{1 - \sum_{l=1}^{p} \varphi_l} \tag{5}$$

It is important to differentiate between ARDL and a simple Distributed Lag Model without the lagged dependent variable. This is illustrated in the Monte Carlo simulations of Pesaran et al. (2013) and Chudik and Pesaran (2015) who show that inclusion of the lagged dependent variable accounts for feedbacks and reduces bias if lags of the dependent variable are modelled adequatelly and series are long, i.e. $T > 50$ in their experiment. As will be seen, feedbacks and absence of strong exogeneity is important to debt and growth relationship.

3.6. Strands of the Previous Empirical Research

The issue of public debt and growth relationship has been examined with a wide variety of empirical tools. The empirics focus on hypotheses that are similar to *H1*, *H2* and *H3*: negative long horizon effects, threshold effects and a direction of causality.

Pesaran et al. (2013) are the first to adopt the flexible ARDL framework to calculate permanent cumulative effects. They investigate a large sample of developing and advanced countries from 1965 to 2010 and focus on $I(0)$ public debt growth rate. They find that, on average, 1 percentage point change in debt growth rate reduces GDP growth rate by 0.055 or 0.075 percentage points, depending on the specification. The results do not differ dramatically if the DL model is employed: on average, a 1 percentage point increase in debt growth rate reduces economic growth by 0.068 or 0.089 percentage points, depending on a number of lags included.

[4] Permanence can be clearly observed from cumulative effect to the future: $\frac{\partial \gamma_t}{\partial Debt_t} + \frac{\partial \gamma_t}{\partial Debt_{t-1}} + \frac{\partial \gamma_t}{\partial Debt_{t-2}} + \cdots = \beta_0 + (\varphi_1\beta_0 + \beta_1) + \varphi_1(\varphi_1\beta_0 + \beta_1) + {\varphi_1}^2(\varphi_1\beta_0 + \beta_1) + \ldots = \beta_0(1 + \varphi_1 + {\varphi_1}^2 \ldots) + \beta_1(1 + \varphi_1 + {\varphi_1}^2 \ldots) = \frac{\beta_0+\beta_1}{1-\varphi_1}$, which is received using the formula for infinite sums, which applies if the series are $I(0)$.

[5] Standard error of this parameter as a function of other estimated parameters is calculated by Delta method. See Greene (2003) for a detailed discussion of the approach.

Nevertheless, they explore only the direct link between debt and growth and do not tackle the interim relationships predicted by the theory. Eberhardt and Presbitero (2015) research a similarly large sample of 118 emerging and advanced economies in the time span of 1961 – 2012 but base their long-run estimation on cointegration and do not model growth rates. They consider debt to GDP ratio, real GDP levels and capital stock as one of the core variables determining the long-term growth. Depending on the estimator used, their long-run cointegration debt coefficients range from -0.031 to 0.050, which are lower in absolute value than the ones of Pesaran et al. (2013). Balassone et al. (2011) also use cointegration techniques but for a single country. They focus on long time series of Italy counting back to 19th century. As in Eberhardt and Presbitero (2015), the study is done in levels and not in growth rates. The authors find a negative long-run relationship between public debt and GDP. According to them, a 1 percentage point change in debt/GDP ratio reduces real GDP by -0.027 percentage points. Yet, the fallacy occurs in a claim that debt necessarily Granger-causes reduction in GDP. Existence of cointegration implies Granger causality from X to Y, vice versa or even both directions, which is a straightforward implication from VEC models reflecting the responsiveness of the variables when it comes to error correction (Gonzalo et al., 2001).

Another class of research detects long-run dynamics by impulse responses in a VAR setting and establishes Granger causality between debt and economic outocomes. For example, Ferreira (2016) uses panel causality tests between growth and public debt for the European Union between 2007 and 2012. He finds a positive impact of debt on growth, which he takes as evidence of Keynesian effects of a short-term stimulation. For the full panel including advanced and emerging countries outside the EU, a strong negative reverse Granger causality from growth to debt accumulation is noticed. The null hypothesis of no Granger causality from debt to growth is not rejected in the study of Ajovin and Navarro (2014) who focus on OECD countries from 1980 to 2009. They estimate seemingly unrelated regressions (SUR) by controlling for country heterogeneity and cross-sectional correlation which are issues to be seriously addressed in panels (Pesaran and Tosseti, 2011). By bootstrapping Wald statistics for every country in the sample they reject the null at the 5% only for Austria and the Netherlands. Additionally, more links of causality are found going from growth to debt.

A similar OECD sample is used by Lof and Malinen (2013) who consider a panel VAR model. Similarly to the previously discussed causal studies, signs of the reverse causality are evident: a positive shock to GDP has negative effect of debt to GDP which lasts around three years. For the economies whose data from period 1905 – 2008 are available, the results do not change.

This route of research is developed further by Ogawa et al. (2016) who move from a bivariate to a trivariate VAR analysis and incorporate long-term interest rates (on 10-year-maturity government bonds) as a transmission variable to account for a deeper causal link. In line with Laubach (2009), they hypothesize that accumulation of debt boost interest rates, hence growth rates should decline due to reduced incentives for private investment. No Granger causality is found from debt to growth neither for moderately nor highly indebted countries. Oppositely, debt to GDP ratio is reduced by 3.24 percentage points in four years after one standard deviation shock in GDP in heavily indebted economies.

Some studies on threshold effects take a long-run perspective by employing the already discussed averaging over time. For instance, Kumar and Woo (2010) investigate a sample of 38 advanced and emerging economies in the time span of 1970 – 2007 by creating non-overlapping 5-year averages. They employ various estimators for robustness checks to investigate both linear and non-linear links between government debt and growth: ordinary least squares, between- and within- estimation and system GMM. The results obtained from the latter technique show that an increase in debt to GDP ratio by 10% reduces growth by 2.9%. Additionally, a dummy variable approach to non-linearity signals an extra negative impact when debt to GDP surpasses 90%. System GMM estimator is also applied by Checherita and Rother (2010) whose sample consists of 12 Eurozone countries. Again, they rely on 5-year non-overlapping averages. By exploiting flexibility of the estimator to select exogenous and endogenous variables and a variety of instruments to avoid the issue of reverse causality, they capture a significant inverted U-shape relationship between public debt and long-run growth rate. The turning point emerges from 90% to 100% of debt burden in all specifications when squared term of the debt variable is included (Checherita and Rother, 2010). A standard averaging over 5-year intervals to emulate steady state is used by Kourtellos et al. (2013), as well. Yet, they take a different perspective with respect to the threshold variable and create a selection equation resembling selection criteria in limited dependent variable models. In a sample of 82 advanced and developing economies, they indicate that debt to GDP itself is not a suitable candidate for sample splitting and select low and high quality democratic rule as regimes mediating relationship between public debt and growth rate (Kourtellos et al., 2013).

4. Econometric Analysis

4.1. Estimation Issues for Debt-Growth Relationship

The majority of the reviewed empirical studies investigating the relationship between public debt accumulation and growth in a long perspective in principle rely on estimating augmented growth regressions similar to (1). This implies that econometric problems arising from specifying growth equations can hinder quality of results when debt variables are incorporated among other explanatory variables as shown by Pesaran and Smith (2014). Additional drawbacks arise when time series properties are not accounted for.

4.1.1. Panel unit roots and unbalanced equations. The question of an *unbalanced* regression equation becomes relevant if it contains integrated and stationary series on the left- and right-hand sides. Then usual *t* and *F* statistics asymptotically diverge and do not have a distribution (Phillips, 1986). This is likely to happen when the dependent variable is growth rate of GDP and not GDP in levels. Specifically, debt to GDP ratio is usually a highly persistent variable since debt accumulates for relatively long periods before fiscal consolidations take place. For example, Baldacci and Kumar (2010) find evidence of this variable being $I(1)$. Additionally, Paesani et al. (2006) theorize debt to GDP ratio as a sum of $I(1)$ and transitory $I(0)$ processes in their VAR model for debt effects on the long-run interest rates. On the other hand, growth rates do not appear to be very persistent, but are likely to have some limited memory (Keele and De Bouf, 2004; Pesaran et al., 2013). Additionally, using averages of data to approximate steady state or diminish effects of business cycles do not remove unit roots. If sequence $\{Debt_t\}$ is non-stationary, so is $\{Debt_{t-1}\}$ and their linear combination (an average) still contains a stochastic trend (Eberhardt and Teal, 2011).

4.1.2. Cross-sectional dependence and parameter identification. Although not specific to growth estimation, this problem is particularly relevant to advanced economies which are financially interrelated. As demonstrated by Chudik and Pesaran (2013), the common factors can drive both the dependent variable (through the unobservable) and the regressors along the cross-section resulting in an identification problem. In the context of government debt and economic growth, an example of a strong common factor is a financial crisis. It affects both growth rates and accumulation of public debt through the need to support a weakening banking sector when asset prices fall and bad loans accumulate (Lane, 2012). Based on Eberhardt et al. (2013), the identification problem can be summarized by the equations (5) – (7):

$$\gamma_{it} = \alpha + \beta Debt_{it} + u_{it} \tag{5}$$

$$u_{it} = \lambda f_t + \varepsilon_{it} \tag{6}$$

$$Debt_{it} = \delta f_t + \eta_{it} \tag{7}$$

Here f_t describes the unobserved common factor and λ together with δ stand for the factor loadings which show how strongly growth rate and debt variables are affected by a common variation in the factor. As can be seen from (5) – (7), $Debt$ is still endogenous unless the common factors are accounted for. Additionally, u_{it} cannot be i.i.d. in presence of the common factors which makes hypothesis testing invalid.

4.1.3. Endogeneity and feedback effects. As could be seen from reviewed VAR and Granger causality studies, the direction of causality remains a serious issue. Causality can run from debt to growth as suggested by the theory, yet accelerating economic growth can also reduce debts because a country 'outgrows' debt if interest payment rate is lower than growth rate (Greiner and Fincke, 2009). Nevertheless, according to Panizza and Presbitero (2013), instrumental variable approach is unlikely to work since the usual exclusion restriction is extremely hard to satisfy: public debt is likely to be instrumented with other macro series whose independence from error term in the growth equation is hard to justify. Despite this, endogeneity might be alleviated in two ways: 1) it can be partially accounted for by controlling for the common factors that might accelerate economic growth rate and reduce debt at the same time as a mediating variable (of course, endogeneity which comes from idiosyncratic errors is still present); 2) lags of the dependent variable can be included in the regression. It is likely that a feedback from growth at $t-1$ to debt series at t exists, i.e. public debt is only weakly exogenous at best. This possibility has a support from empirical VAR studies implementing Granger-causality tests in samples of advanced and developing economies (Lof and Malinen (2013); Ogawa et al. (2016)). As simulations in Pesaran et al. (2013) and Chudik et al. (2015) show, when common factors are controlled for and an adequate number of lags of the dependent variable are added, the estimation bias deteriorates.

4.1.4. Heterogeneous parameters. Temple (1999) notes that cross-sectional growth regressions typically fail to account for multiple long-run equilibria that exist among heterogeneous countries. Additionally, traditional estimators of panel models such as fixed- or random-effects can estimate only the *pooled* parameter for countries in the sample. However, extra unit-specific information can be gained if slope coefficients and their variances are allowed to differ along the cross section. This estimation can be carried out consistently if individual parameters vary around the mean randomly and they are orthogonal to the explanatory variables included in regression (Eberhardt and Teal, 2011). In effect, pooled estimates can result in a loss of important information which drives parameter heterogeneity.

4.2. Parameter Identification Strategy

4.2.1. Model for H1 and H2. Given the issues of growth modelling, the empirical strategy has three main aims. First, to employ the theoretically motivated flexible ARDL model to estimate long-run parameters and control for feedback effects from growth to public debt. Secondly, to augment the regression equation with the common factors in order to control for co-movements over the cross-section resulting in identification problems. Lastly, to ensure that the fitted model is balanced. Equations (8) – (10) present ARDL (p, q) model with common factors structure for economic growth and capital.

$$\gamma_{it} = \alpha_i + \sum_{l=1}^{p} \varphi_{il}\gamma_{it-l} + \sum_{m=0}^{q} \beta_{im}\, Debt_{it-m} + u_{it} \tag{8}$$

$$k_{it} = \tilde{\alpha}_i + \sum_{l=1}^{p} \tilde{\varphi}_{il} k_{it-l} + \sum_{m=0}^{q} \tilde{\beta}_{im}\, Debt_{it-m} + \tilde{u}_{it} \qquad (9)^{6}$$

$$u_{it} = \boldsymbol{\lambda}_i' \boldsymbol{f}_t + \varepsilon_{it} \tag{10}$$

$$\tilde{u}_{it} = \tilde{\boldsymbol{\lambda}}_i' \tilde{\boldsymbol{f}}_t + \tilde{\varepsilon}_{it}$$

Here, $\boldsymbol{\lambda}_i$ $(\tilde{\boldsymbol{\lambda}}_i)$ and $\boldsymbol{f}_t$ $(\tilde{\boldsymbol{f}}_t)$ are vectors that represent heterogenous factor loadings and multiple common factors, respectively. In line with Chudik et al. (2011), u_{it} can be divided into m_1 strong[7] common factors and m_2 weak factors, where $m_2 \gg m_1$. Strong factors represent underlying variables that are common to all members of cross-section. For example, business cycles or more severe financial crises. As in Lane (2012), falling prices of financial assets bring losses to banking sector which cannot help in financing investments, while governments incur larger debts to alleviate banking crisis. Weak factors, on the other hand, represent smaller spill-overs among the neighbouring countries that can drive idiosyncratic business cycles (Eberhardt and Teal, 2011). As shown by Pesaran (2006), the strong factors can be approximated by inclusion of cross-sectional averages of the dependent variable and the regressors in the regression equation[8]. This leads to the Common Correlated Effects (CCE) estimator. Additionally, Chudik et al. (2011) model situations with large number (possibly $m_2 \to \infty$) of weak factors by performing Monte Carlo simulations. They demonstrate that the same approximation by cross-sectional averages performs well in terms of parameter estimation and tests for cross-sectional dependence because vector of slope parameters continues to be asymptotically normal and consistent (Chudik et al., 2011).

[6] Here ~ emphasizes that models are different, hence the coefficients and the errors are not the same.

[7] Weak cross-sectional dependence means that at every time t the (weighted) cross-sectional average of the process converges to its mean (conditional on information set $\boldsymbol{\mathcal{F}}_{t-1}$) and the contrary applies to strong dependence. Weak factors have absolutely summable factor loadings ($\sum_i^{\infty} |\lambda_i^w| < \infty$), while the strong ones do not. For formal definitions, see Chudik and Pesaran (2015).

[8] Intuitively, cross-sectional averages capture a common underlying variation of all series along the cross-section in the sample, hence they approximate a number of common factors driving the variation.

It is useful to reparameterize the ARDL (p, q) model in (8) – (10) into the Error Correction Model, hence ECM, as they are equivalent. This allows an easier procedure to estimate the long-run parameter θ_i in (5) (Keele and De Bouf, 2004), and the ECM distinguishes between immediate short-run and long-run effects. Therefore, the simplest ARDL (p, q) becomes:

$$\triangle \gamma_{it} = \alpha_i + \pi_i[\gamma_{it-1} - \theta_i Debt_{it} - \boldsymbol{\Phi}_i' \boldsymbol{X}_{it}] + \sum_{m=1}^{p-1} \psi_{im}^{\gamma} \triangle \gamma_{it-m} + \sum_{n=0}^{q-1} \psi_{in}^{D} \triangle Debt_{it-n} + \sum_{k=0}^{q-1} \boldsymbol{\psi}_{Xik}' \triangle \boldsymbol{X}_{it} + \boldsymbol{\lambda}_i' \boldsymbol{f}_t + \varepsilon_{it} \tag{11}$$

$$\triangle k_{it} = \tilde{\alpha}_i + \tilde{\pi}_i\left[k_{it-1} - \tilde{\theta}_i Debt_{it} - \widetilde{\boldsymbol{\Phi}_i'} \boldsymbol{X}_{it}\right] + \sum_{m=1}^{p-1} \widetilde{\psi_{im}^{k}} \triangle k_{it-m} + \sum_{n=0}^{q-1} \widetilde{\psi_{in}^{D}} \triangle Debt_{it-n} + \sum_{k=0}^{q-1} \widetilde{\boldsymbol{\psi}}_{Xik}' \triangle \boldsymbol{X}_{it} + \widetilde{\boldsymbol{\lambda}}_i' \widetilde{\boldsymbol{f}}_t + \tilde{\varepsilon}_{it} \tag{12}$$

Here, (11) and (12) represent ECM form as in Kripfganz and Schneider (2016)[9]. $\boldsymbol{X}_{it}$ contains any other possible variables. Pesaran and Chudik (2013) discuss the estimation of this type of models, which requires a dynamic CCE. Augmentation with the *current* cross-sectional averages does not give consistency since the lagged dependent variable is present. Consistency is gained only when lagged cross-sectional averages are allowed to increase with T [10]. In fact, the rate of consistency in this case is $\sqrt{N}$ and not $\sqrt{NT}$. This is due to factors and the mean group estimation which gives full parameter heterogeneity: $\hat{\beta} = \frac{1}{N}\sum_1^N \hat{\beta}_i$.

Lastly, it is important to note, that (11) or (12) nest models that have important qualitative and quantitative differences. If all variables are $I(1)$, the model can explore cointegration in a classical ECM setting as in Gonzalo et al. (2001) and π_i measures the speed of adjustment to equilibrium between series that drift stochastically. In case of $I(0)$ series, the concept of equilibrium is different because two series do not have an error-correcting relationship as in case of cointegration. In the stationary case, the parameters are obtained as if variables *themselves* are in equilibrium. This coincides with the derivation of equation (3). As a result, the interpretation of π_i changes: it shows how fast the impact of $Debt_{it}$ on γ_{it} diminishes over

[9] Particularly this reparameterization is carried out in Stata panel data programs, e.g. 'xtpmg' by Blackburne III and Frank (2007) or 'xtdcce' by Ditzen (2016). There is a different version where the long-run parameter belongs to lagged debt variable but it involves more parameters, while the former is simpler computationally.
[10] See Pesaran and Chudik (2013) for derivations and proofs.

time (Enns et al., 2014) and *it is not related to corrections*. Therefore, this gives the ECM with stationary variables as in Keele and De Bouf (2004)[11]. The term π_i becomes *pseudo speed-of-adjustment parameter*.

4.2.2. Updated model for H3. To introduce thresholds into the ECM, a dummy variable approach is chosen, in line with Pesaran et al. (2013) and Kumar and Woo (2010). Consequently, it provides an additional effect ($\omega_{i\tau}$) of certain indebtedness level along with the impact given by the long-run parameter θ_i.

$$\triangle \gamma_{it} = \alpha_{i\tau} + \omega_{i\tau} D_{it}(\tau) + \pi_{i\tau}\left[\gamma_{it-1} - \theta_{i\tau} Debt_{it}\right] + \sum_{m=1}^{p-1} \psi_{im\tau}^{\gamma} \triangle \gamma_{it-m} + \sum_{n=0}^{q-1} \psi_{in\tau}^{D} \triangle Debt_{it-m} + \boldsymbol{\lambda}_i' \boldsymbol{f}_t + \varepsilon_{it} \qquad (13)$$

Here, τ is an exogenously picked threshold level. Naturally, as indicated by subscripts, the rest of the parameters become functions of the threshold under consideration because everytime sample is modified: in every estimation for the given τ, countries which had at least one period out of T with debt/GDP larger or equal to τ need to be included in the sub-sample. Although this approach is rather simplistic, it has a clear interpretation, provided that long-run parameter is already a non-linear function of ARDL parameters[12].

Given this identification strategy for *H1*, *H2* and *H3*, empirical research is divided into following steps: 1) conducting a preliminary analysis by testing feedbacks and running traditional growth regressions for comparative purposes 2) testing *H1*, *H2* and *H3* with factor-augmented ECM model 3) implementing robustness checks.

4.3. Data and Variables

The sample consists of 24 advanced economies belonging to the OECD, including: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain, Australia, Canada, Denmark, Japan, Iceland, New Zealand, Norway, Chile, Sweden, Switzerland, Turkey, the United Kingdom and the United States. The time period is 1965 – 2014. The starting date is chosen in line with Pesaran et al. (2013) in order to obtain a balanced

[11] They exploit ECM in the field of political science where variables are usually not persistent. Their simulations demonstrate performance of ECM with stationary series. See Keele and De Bouf (2004).

[12] It is possible to obtain long-run parameters in the ECM and estimate endogenous thresholds by inclusion of square term only if original series are $I(1)$. This is achieved by looking for co-summability, which is a generalization of cointegration for non-linear functions of variables. See Gonzalo and Beranguer-Rico (2014).

panel and to maintain a long series which is required for estimation of the long-run relationships and application of estimators which produce estimates of country-specific parameters (Temple, 1999). The so-far-generic $Debt$ is *the central government debt relative to GDP* mainly obtained from the historical database of Reinhart and Rogoff (2010a). Their series reach 2010 but the sample is extended until 2014 to cover the latest debt crisis in Europe which most severely touched Greece, Ireland and Spain. Data on the central government debt for extension is retrieved from the IMF Historical Public Debt database (2010), Eurostat and Trading Economics database. In total, 1197 observations are collected. As also notified by Eberhardt and Presbitero (2015), central government debt does not reflect the full indebtedness of a public sector since it does not include accounts of municipal governments, for example. However, sufficiently long series of general government debt are not available even for the advanced economies (Reinhart and Rogoff, 2010a). The dependent variable is *an annual growth of real GDP per capita* calculated as the first difference of natural logarithms of real GDP per capita.

Other macroeconomic variables used in the study come from the latest (9^{th} edition) Penn World Table (Feenstra et al., 2015). Control variables which are important for growth are chosen in line with the long-term growth literature, particularly the work of Sala-i-Martin et al. (2004), who provide the list[13] of variables that are most likely to be included in growth regressions. Table 1 summarizes information of all the variables of this study.

Table 1. Review of the variables used

Code	Variable	Description
γ	GDP Growth	First difference in natural logarithms of real GDP per capita
$Debt$ (g_D for growth rate)	Public Debt	Central government debt to GDP
k (per capita) (g_k for growth rate)	Capital stock	Total capital stock in levels in the country at given period (constant dollars of 2011).
l (per capita) (g_l for growth rate)	Number of persons engaged	Total number of people from age of 15 that performed work at least for 1 hour during reference week (Feenstra et al. 2015). This is a proxy for labour force.

[13] Out of 67 candidate variables derived from various theories, they select 18. Obviously, to avoid overfitting, only the subset which appeared in similar empirical works is selected for this study.

HC	Human capital index	Index constructed from averaging years of schooling and estimated return from education.
$Open$	Openness	Sum of exports and imports relative to GDP
G (g_G for growth rate)	Government consumption	Day-to-day government expenditures (not investments with long-run value) to reflect general public sector proneness to spend.

4.4. Descriptive Statistics and Tests

Table 2 gives brief summary statistics of the two main variables: public debt and GDP growth. Over the course of 50 years, the maximum growth was experienced by Iceland in 1971 and Chile suffered the severest slump in 1970. Japan had the most significant public debt burden of 196.64% in 2014, whereas the smallest burden of 3.67% was experienced by Finland in 1971. Additionally, low p-values of cross-sectional dependence test (CD P-Value)[14] in the last column signal strong tendencies of debt to GDP ratio and economic growth to co-vary across the OECD cross-section, which already suggests taking common factors $\boldsymbol{f}_t$ into account.

Table 2. Descriptive statistics

Variable	Mean	St. Dev.	Min.	Max.	CD P-Value
γ	2.52%	3.67%	-24.12%	18.9%	0.000
$Debt$	46.5%	32.09%	3.27%	196.64%	0.000

In the Figure 2, eight countries were selected randomly. Plus, the cross-sectional average for all 24 countries is plotted to reflect tendency of the whole sample. The top part compares debt to GDP ratio and annual economic growth. As can be seen, public debt series, although normalized by GDP, are drifting upwards for Japan and Greece, for example. This hints that they can be at best trend-stationary. Series for Ireland or Chile, on the other hand, resemble a stochastic drift. This is consistent with the theoretical predictions that debt to GDP ratio can be $I(1)$. At the same time, growth rates for the same 8 countries appear to hover around the time-constant mean with evident negative spikes in the end of 1970's and in the beginning of 1980's. Also, slumps in growth are notable for Greece in 2010 and 2011 which coincide with the financial crisis aftermaths and start of the debt crisis. The overall view together with the cross-sectional average is consistent with the fact that an annual GDP growth rate tends to be $I(0)$.

[14] The test statistic is constructed from pairwise correlation coefficients between regression residuals along the cross-section. The null is absence of correlation. See Ditzen (2016) for an instructive explanation.

The bottom left part on the left exhibits the annual debt to GDP ratio growth, where the series are less persistent. Both debt and economic growth rates appear to mirror each other at least to some degree. For example, positive spikes in debt growth and negative spikes in economic growth are evident around 1980. The same situation reoccurs in 2008 – 2010 for the majority of selected countries. The bottom right part illustrates the sub-sample dynamics of the cross-sectional averages. The central takeaway from Figure 2 is a potential for growth equations with the variables of an unbalanced order of integration and a suggestion to model relationship between growth rates of the two central variables as in Pesaran et al. (2013).

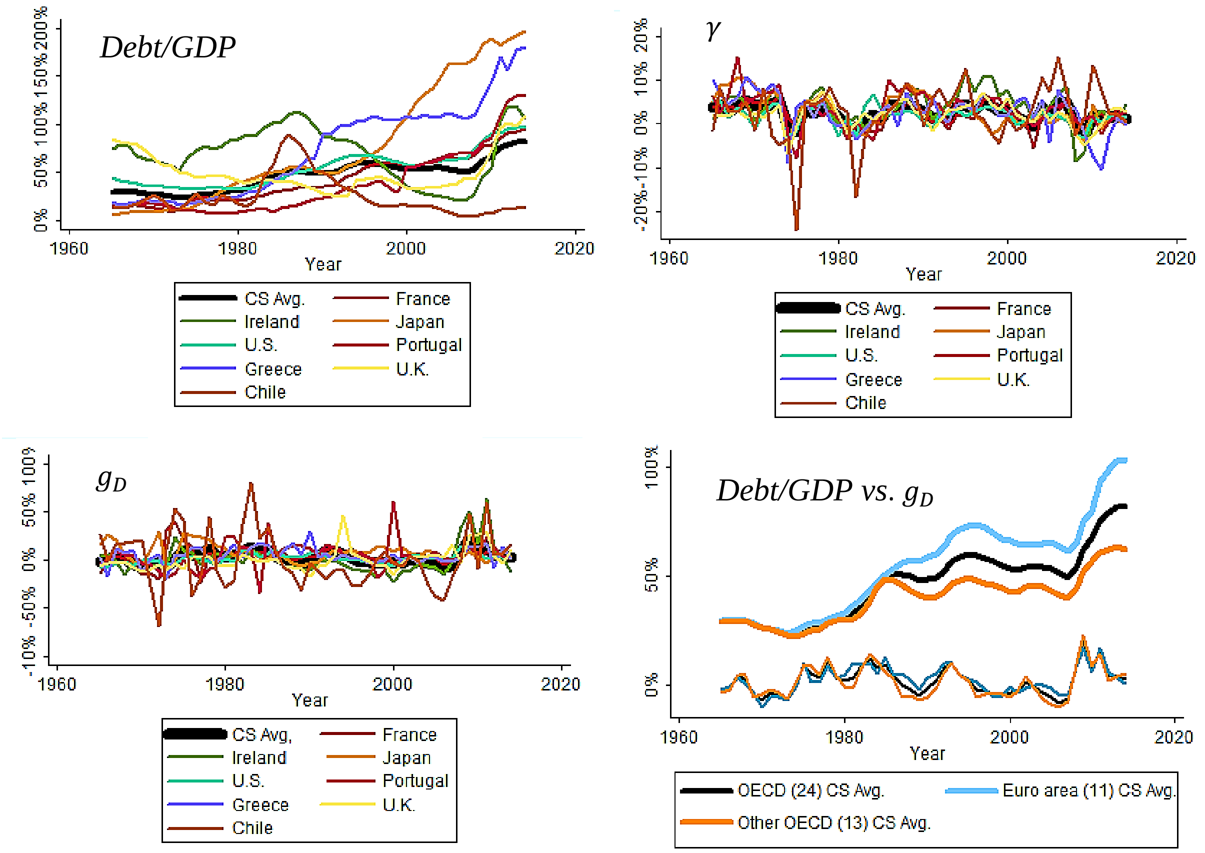

Figure 2. Comparison of the series in levels and growth rates (the full sample and the sub-samples)

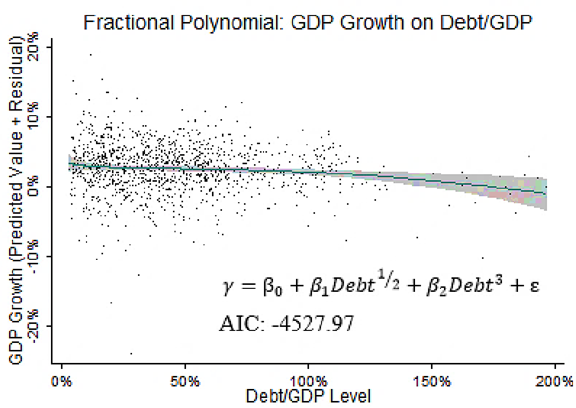

Figure 3. Fractional polynomial for the full sample with 95% confidence bands

Figure 3 above plots debt to GDP ratio together with growth. The full sample is employed. A fractional polynomial regression is fit that searches for the smoothest univariate function linking two series for exploratory purposes. Here, a polynomial of degree (0.5, 3) suggestively signals a slight negative relationship between public debt to GDP ratio and economic growth.

As the last step, a panel unit root test proposed by Pesaran (2003) is performed [15] and, as expected from other studies, the null of non-stationarity cannot be rejected for debt to GDP ratio. On contrary, annual growth rate is $I(0)$. Therefore, debt to GDP variable is transformed to growth rates (g_D) in order avoid estimation of unbalanced equations. Also, growth rates (difference of natural logarithms) give a clear interpretation in terms of *percentage point changes*. The same transformation to growth rates (g_G) is performed for government consumption. Table 3 summarizes the results. Additionally, standard tests for a conditional heteroskedasticity and a serial correlation are performed, leading to the usage of robust standard errors (results of the tests are provided in Appendix B).

Table 3. Results of Pesaran (2003) panel unit root test

Variable	P-Value (1 Lag)	P-Value (2 Lags)	P-Value (3 Lags)
γ	0.000	0.000	0.000
$Debt$	0.994	0.967	0.995
g_D	0.000	0.000	0.000
k	0.916	0.990	0.386
l	0.194	0.532	0.633
HC	0.009	0.037	0.108
G	0.609	0.773	0.887
$Open$	0.037	0.186	0.147
g_k	0.000	0.000	0.000
g_l	0.000	0.000	0.000
g_G	0.000	0.000	0.000

4.5. Preliminary Results

4.5.1. Evidence of weak exogeneity. As a starting point, a panel VAR study is carried out to learn about the direction of Granger causality between debt/GDP growth rate and economic growth, which are both $I(0)$. This guides how many lags of the dependent and independent variable include in ARDL before rewriting it as the ECM.

[15] Essentially, it is an Augmented Dickey-Fuller test taking into account a cross-sectional dependence. The test is based on an average value of the DF statistic for each cross-section. Under the null of non-stationarity, the average statistic follows a standard normal distribution (Lewandowski, 2007). A constant is included as a deterministic component because existence of deterministic elements can dominate a random walk component leading to seemingly trend-stationary processes as in Figure 2. Also, to correct for serial correlation, up to 3 lags are included in the DF equation.

$$\boldsymbol{g}_{it} = \sum_{s=1}^{p} \boldsymbol{\phi}_s \boldsymbol{g}_{it-s} + \boldsymbol{\Gamma}_i \boldsymbol{F}_t + \boldsymbol{\varepsilon}_{it} \tag{18}$$

In equation 18, $\boldsymbol{g}_{it} = (g_{D_{it}}, \gamma_{it})'$, $\boldsymbol{\Gamma}_i$ and $\boldsymbol{F}_t$ represent matrix of loadings and vector of factors for the system, respectively. To establish Granger causality, it is sufficient to employ reduced form VAR and not structural VAR imposing causal ordering of the series. It is important to note that this panel VAR setting cannot control for the common factors in such a flexible manner as strong and weak factor structure. Inclusion of cross-sectional averages in both regressions as exogenous variables does not suffice as averages are linear combinations of variables that are endogenous by the very construction of VAR. On the other hand, the problem can be alleviated by subtracting cross-sectional averages to account for exactly common time effects and subtracting time averages to remove individual specific affects[16]. Since this exercise works as a guidance for lag inclusion in ARDL model, the tested p is chosen to be 3: this looks at the deeper lags to be safe about the delayed effects and does not allow the regression equation to become large given limited *T*. As a result, Granger causality from growth to debt is detected (irrespective if cross-sectional averages (CS) are subtracted or not and the number or lags used). At the same time, for debt accumulation rate, null for no Granger causality is failed to reject for the case of a unit lag. A delayed causal effect is detected if deeper lags are accounted for. Table 4 describes the results[17].

Table 4. Granger causality test for debt and growth relationship

Equations	χ^2 Statistic (L1)	χ^2 Statistic (L2)	χ^2 Statistic (L3)
$\gamma \rightarrow g_D$	17.288***[18]	16.585***	18.408***
$g_D \rightarrow \gamma$	0.242	9.044**	15.766***
Equations (CS)	**χ^2 Statistic (L1)**	**χ^2 Statistic (L2)**	**χ^2 Statistic (L3)**
$\gamma \rightarrow g_D$	7.300***	7.117**	8.275**
$g_D \rightarrow \gamma$	0.077	2.907	7.894**

The crucial observation is the robust feedback from economic growth to debt growth appearing in all specifications. This implies that it is necessary to control for the absence of strong exogeneity of debt/GDP growth. This is achieved by following the result of Monte Carlo simulation of Pesaran et al. (2013) and Chudik et al. (2011). Inclusion of the lagged dependent variable reduces the bias stemming from feedback effects if $T > 50$. In this study, the length of

[16] Particularly, this procedure is carried out by forward orthogonal deviations (see Abrigo and Love (2015)) to avoid loss of observations through first-differencing. The panel VAR is estimated by GMM.
[17] All eigenvalues of the companion matrices are inside the unit circle. Appendix D provides similar Granger causality tests fot *H2* and modules (as some roots are complex) of eigenvalues for every VAR specification.
[18] * – 10%, ** – 5%, *** – 1% significance levels throughout the study.

series for each economy is precisely 50 years. Moreover, accounting for the common factors should alleviate simultaneity bias, therefore debt effect can be isolated. Granger causality from debt to growth appearing at higher-lag specifications suggests controlling for lagged debt effects in upcoming models, as well.

4.5.2. H1: Conventional growth regression approach. Traditional growth regression in the spirit of Rao (2010) is estimated. Although a possibility of bias is motivated by the construction of *H1* and *H2*, it gives the baseline for comparison with more general ARDL/stationary ECM models. Equation 15 presents the model derived from production function in (1) after augmenting with extra variables. Here, g_k and g_l are capital and labour growth rates.

$$\gamma_{it} = \delta_{1i}g_{k_{it}} + \delta_{2i}g_{l_{it}} + \beta_i g_{D_{it}} + \boldsymbol{\varphi}_i'\boldsymbol{X}_{it} + \boldsymbol{\lambda}_i^{'}\boldsymbol{f}_t + \varepsilon_{it} \tag{15}$$

The column (iii) in Table 5 presents the result from a conventional long-run estimation when non-overlapping averages of the series are taken to reflect the economy that converges to a steady state. In line with study of Kuman and Woo (2010), 5-year averages are employed, thus reducing total number of observations to 240 and effectively leaving T=10 per country.

Table 5. Baseline short- and long-run estimation

Variables	CCE (i)	OLS (ii)	OLS[19] (iii)	CCE (iv)	CCE (v)
g_k	0.222***	0.240***	0.263***	0.240***	0.296***
	(0.0391)	(0.0231)	(0.0363)	(0.0394)	(0.0508)
g_l	0.592***	0.651***	0.689***	0.625***	0.660***
	(0.119)	(0.0711)	(0.104)	(0.122)	(0.132)
g_D	-0.0328**	-0.0379***	-0.0389***		
	(0.0146)	(0.00875)	(0.0129)		
$L1.g_D$				0.0157	
				(0.0107)	
D					0.00112
					(0.00613)
HC	-0.0442	-0.00853***	-0.00989***	-0.0240	-0.0433
	(0.0548)	(0.00217)	(0.00224)	(0.0829)	(0.0662)
g_G	-0.227***	-0.147***	-0.355***	-0.225***	-0.239***
	(0.0459)	(0.0342)	(0.0443)	(0.0439)	(0.0443)
Open	-0.00755	-0.00149	-0.00195	0.000477	0.0161
	(0.0335)	(0.00173)	(0.00208)	(0.0329)	((0.0346)
Constant	0.0665	0.0422***	0.0473***	0.102	0.0562
	(0.0674)	(0.00700)	(0.00714)	(0.0832)	(0.0578))
Observations	1,197	1,197	240	1,197	1,197
R-squared		0.363	0.544		
Number of Countries	24	24	24	24	24
CD Statistic	-0.357	30.818***	13.150***	-0.730	-0.219
CD P-Value	0.721	0.000	0.000	0.465	0.827

[19] CCE for averaged data is skipped as it requires a large T.

As can be seen from column (i) that includes the factors, a 1 percentage point increase in growth rate of debt accumulation, reduces (short-term) GDP growth by approximately 0.032 percentage points. Also, accounting for the factor structure takes care of the cross-sectional dependence in the error terms as can be noted from the cross-sectional dependence test statistic. This suggests that the CCE result is more reliable, although coefficients which are significant do not lose significance when applying OLS (except for human capital index which gives a counterintuitive result in every specification). Importantly, according to the results reported in collumn (iii), there are no fundamental changes when non-overlapping averages of the data are employed and effectively 10 episodes of growth are analysed. All coefficients are systematically slightly higher in absolute value but the effect of 1 percentage point change appears to be almost identical to the one from a simple OLS regression. In column iv, current debt/GDP growth rate is replaced with the same variable lagged by one year. The effect is insignificant referring to absence of Granger causality from debt growth rate to economic growth rate when a unit lag is included. Lastly, (v) includes logarithm of debt/GDP *level* (D). Intuitively, no systematic relationship between trending debt and stationary GDP growth is detected.

4.6. Long-Run Estimation with the Stationary ECM[20]

4.6.1. H1: Impact on economic growth. Moving from the baseline to the theory-implied estimation methods with stationary series, specification in equations 11 or 12 represent ECM with $I(0)$ data as in Keele and De Bouf (2004). Therefore, the parameter of interest θ_i is equivalent to a long-run cumulative effect parameter from the original ARDL model in (6). The parameter π_i stands for the speed with which effect of debt on economic growth vanishes and *not* equilibrium error correction. The regression equation in growth rates is outlined in (16):

$$\vartriangle \gamma_{it} = \alpha_i + \pi_i[\gamma_{it-1} - \theta_i g_{D\,it}] + \sum_{m=1}^{p-1} \psi_{im}^{\gamma} \vartriangle \gamma_{it-m} + \sum_{n=0}^{q-1} \psi_{in}^{D} \vartriangle g_{D\,it-n} + \boldsymbol{\lambda}_i^{'} \boldsymbol{f}_t + \varepsilon_{it} \tag{16}$$

Table 6 shows the result from ECM specification with and without accounting for the common factors. Since it is important to control for feedbacks from growth to debt accumulation, models up to 3 lags are considered as growth is not a long memory process (Pesaran et al., 2013; Keele and De Bouf, 2004). In line with Granger Causality test, up to 3 lags of debt growth are included. Also, since the focus is on long horizons, results on average $\hat{\theta}_i$ and $\hat{\pi}_i$ are reported

[20] As a natural addition, Appendix E presents the results from an alternative version of stationary DL model introduced by Pesaran et al. (2013). Additionally, a proof based on the Beveridge-Nelson decomposition is provided. Although the long-run parameters are similar, a cross-sectional dependence is not eliminated.

here. See Appendix C for the short-run immediate effects in this and the upcoming models. ECM (p, q) stands for number of lags of each variable before ARDL is written as the ECM; CS highlights inclusion of cross-sectional averages[21]. For every specification, $p = q$, in line with Pesaran et al. (2013) because there is no theoretical guidance how the lags should be specifically combined.

Table 6. Long-run results: debt and growth

Variables	ECM (1,1)	ECM (2,2)	ECM (3,3)	CS-ECM (1,1)	CS-ECM (2,2)	CS-ECM (3,3)
Pseudo-EC term $(\hat{\pi})$	-0.739***	-0.805***	-0.800***	-0.734***	-0.686***	-0.612***
	(0.0408)	(0.0574)	(0.0564)	(0.0448)	(0.0820)	(0.0781)
g_D $(\hat{\theta})$	-0.0553*	-0.0575**	-0.0499*	-0.0703**	-0.387	0.0588
	(0.0297)	(0.0268)	(0.0270)	(0.0316)	(0.322)	(0.0850)
Observations	1.173	1.149	1.125	1.173	1.149	1.125
R-squared Adjusted	0.498	0.519	0.534	0.503	0.567	0.580
Number of Countries	24	24	24	24	24	24
CD Statistic	29.32***	28.01***	27.35***	1.34	1.40	0.53
CD P-Value	0.000	0.000	0.000	0.18	0.16	0.59

As can be seen, when the common factors are not accounted for, a *permanent* 1 percentage point increase in public debt growth rate leads to a reduction in GDP growth rate in the long-term from 0.05 to 0.058 percentage points, depending on the number of lags. The strongest negative effect of -0.058 is obtained when using 2 lags in the dependent and independent variables. In all specifications, long-term parameter is statistically significant at the 10 or 5 percent level, albeit not at 1 percent level. Importantly, the size of coefficients is systematically larger than in the baseline estimation reported in the Table 5. The difference can be intuitively explained by the ARDL construction of the parameter where changes accumulate into the future periods in the long-run. Regarding the pseudo EC term, it is negative and lower than 1 in absolute value across all specifications, which is in line with the theory. On average, the magnitude of the term is large and it signals a fast reduction of the debt accumulation effect into the future periods.[22]

[21] At minimum, an integer part of $\sqrt{T}$ lags (3, in this case) of the cross-sectional averages must be included (Ditzen, 2016). In this study, 3 – 5 lags are employed, depending if it helps to reduce the cross-sectional dependence and does not alter the results strongly.

[22] The 'inflation' of EC term when the process is not persistent can be explained with the logic of Enns et al. (2014). Rewriting growth process in the spirit of Dickey-Fuller test: $\gamma_{it} = \rho\gamma_{it-1} + \varepsilon_{it} \rightarrow \Delta\, \gamma_{it} = (\rho - 1)\gamma_{it-1} + \varepsilon_{it}$. Then, estimation of error-correction term lumps two parameters: $\pi_i + (\rho - 1)$. As π_i is usually negative, the less persistent the series are (the lower ρ is), the higher absolute value with negative sign of error correction term is received.

Pesaran (2015) cross-sectional dependence test rejects the null of a weak cross-sectional dependence in the errors. This calls for taking the common factors into consideration. Here, long-run result demonstrates robustness and significance at the 5 percent level for CS-ECM (1,1) specification. The cross-sectional dependence in errors is reduced but the significance of long-term impact vanishes for larger lags. On average, a 1 percentage point increase in debt growth rate, leads to a reduction of economic growth by 0.07 percentage points. Inclusion of cross-sectional averages increase an absolute value of the coefficient which is the sign of a corrected bias. These results for the advanced OECD economies are consistent with the result of Pesaran et al. (2013) obtained from large sample of developed and developing countries. In their research, the coefficients range from -0.055 to -0.075. However, their effects stay significant irrespective of 1 or 3 lags of the dependent and independent variables included.

4.6.2. H2: Impact on capital accumulation rate. The main causal mechanism coming from growth and debt theory is a hindered capital accumulation (Saint-Paul, 1992). Before testing *H2*, the long-run effect of capital accumulation on economic growth is tested. The model is reflected in equation (17).

$$\triangle \gamma_{it} = \alpha_i + \pi_i[\gamma_{it-1} - \theta_i g_{k_{it}}] + \sum_{m=1}^{p-1} \psi_{im}^{\gamma} \triangle \gamma_{it-m} + \sum_{n=0}^{q-1} \psi_{in}^{k} \triangle g_{k_{it-n}} + \boldsymbol{\lambda}_i^{'} \boldsymbol{f}_t + \varepsilon_{it} \tag{17}$$

Table 7 provides the long-term result. Additionally, lags of dependent variable are very important because, according to panel Granger causality analysis[23], economic growth tends to feedback into capital accumulation growth.

Table 7. Long-run results: capital and growth

Variables	ECM (1,1)	ECM (2,2)	ECM (3,3)	CS-ECM (1,1)	CS-ECM (2,2)	CS-ECM (3,3)
Pseudo-EC term ($\hat{\pi}$)	-0.729***	-0.759***	-0.765***	-0.696***	-0.610***	-0.569***
	(0.0455)	(0.049)	(0.0555)	(0.0519)	(0.0614)	(0.0729)
g_k ($\hat{\theta}$)	0.216***	0.209***	0.163***	0.196***	0.143**	0.182*
	(0.0412)	(0.0463)	(0.0479)	(0.0728)	(0.0660)	(0.0948)
Observations	1.173	1.149	1.125	1.173	1.149	1.125
R-squared Adjusted	0.49	0.51	0.52	0.519	0.532	0.567
Number of Countries	24	24	24	24	24	24
CD Statistic	35.42***	35.11***	34.18***	1.30	1.87*	0.73
CD P-Value	0.000	0.000	0.000	0.19	0.06	0.47

[23] See Appendix D for the result. Again, as in the case for debt-GDP growth VAR, time averages and cross-sectional averages are subtracted to control for country- and time-specific effects

Depending on the lag specification, if capital accumulation increases by 1 percentage point permanently, economic growth rises from 0.163 to 0.216 percentage points and from 0.143 to 0.196 percentage points when the factors are controlled for. In comparison to the coefficients from the baseline estimation in (17), absolute value of the coefficients is smaller across the board but does not differ much in specifications of (1,1) and (3,3). Note that specification (2,2) might still retain cross-sectional dependence in the error term.

Focusing on *H2* and impacts of public debt accumulation, Figure 4 illustrates a negative relationship between public debt and capital accumulation rates for the whole sample of 24 OECD economies with an emerging slight positive effect. Contrary to debt/GDP level and growth relationship in Figure 2, this fractional polynomial has an element of natural logarithm.

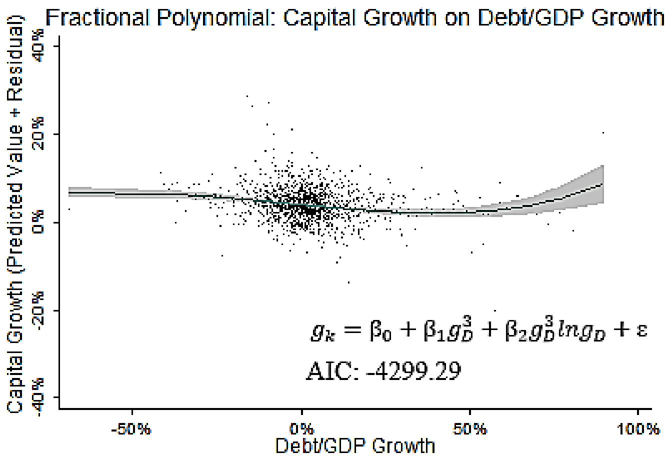

Figure 4. Fractional polynomial for debt/GDP and capital growth rates

Following the illustrative result from Figure 4, equation (18) gives the stationary ECM model for relationship between public debt and capital accumulation growth rates.

$$\Delta g_{k_{it}} = \alpha_i + \pi_i[g_{k_{it-1}} - \theta_i g_{D_{it}}] + \sum_{m=1}^{p-1} \psi_{im}^k \Delta g_{k_{it-m}} + \sum_{n=0}^{q-1} \psi_{in}^D \Delta g_{D_{it-n}} + \boldsymbol{\lambda}_i^{'} \boldsymbol{f}_t + \varepsilon_{it} \quad (18)$$

Table 8 gives the long-run results with and without accounting for common factors. Again, to stay consistent with the previous specifications, up to three lags are considered to account for possible capital growth feedbacks into debt accumulation rate. The causality analysis signals a statistically weak (10% significance) negative effect and Granger causality from debt growth to capital growth. Appendix D provides the Granger causality study.

Table 8. Long-run results: debt and capital

Variables	ECM (1,1)	ECM (2,2)	ECM (3,3)	CS-ECM (1,1)	CS-ECM (2,2)	CS-ECM (3,3)
Pseudo-EC term ($\hat{\pi}$)	-0.431***	-0.489***	-0.499***	-0.505***	-0.703***	-0.367***
	(0.0323)	(0.0499)	(0.0402)	(0.0479)	(0.0917)	(0.0585)
g_D ($\hat{\theta}$)	-0.293	0.0304	-0.860	0.00899	0.131	0.0192
	(0.1932)	(0.0903)	(0.7754)	(0.0529)	(0.138)	(0.0585)
Observations	1.173	1.149	1.125	1.173	1.149	1.125
R-squared Adjusted	0.31	0.37	0.41	0.383	0.651	0.531
Number of Countries	24	24	24	24	24	24
CD Statistic	36.15***	33.45***	31.35***	0.49	0.64	1.51
CD P-Value	0.000	0.000	0.000	0.63	0.52	0.13

The result goes contrary to the predictions of debt accumulation models (Diamond, 1965; Saint-Paul, 1992). No long-run negative effect of debt growth rate on economic growth is detected. Depending on the specification, size of the coefficient, along with the sign, differs strongly, however, they are insignificant irrespective if cross-sectional dependence is taken into account or not. The result generally implies that a one percentage point change in growth rate of public debt to GDP does not result in crowding out in a steady state, i.e. the rate of change of capital acummulation is not reduced in the long-run. Thus, there is some statistical evidence that negative debt effect on growth trickles through different channels. Only immediate short run effects of -0.03 percentage points exist (again, for short-term effects, refer to Appendix C). Lower absolute value of pseudo-EC, as in the case of GDP growth and capital equation, shows that capital stock growth is a lower memory process than real GDP per capita growth.

4.6.3. H3: Debt thresholds and debt growth trajectories. Given earlier studies that focused on existence of the threshold effects (e.g. Chechrita and Rother (2010), Reinhart and Rogoff (2010b)) of public debt *level*, a simillar account is taken in this study. The exogenous threshold τ in (13) reflects debt/GDP levels found in previous studies: 50%, 60%, 70%, 80%, 90% as empirically detected by Baum et al., 2011, Egert, 2014, Chechrita and Rother, 2010 or Reinhart et al., 2012. As a result, an extra effect of debt/GDP level is distinguished from the long-run parameter θ_i that is based on growth rates of the variables. Consequently, the coefficient $\omega_{i\tau}$ reflects changes in the steady state given the average level of indebtedness. Moreover, since both series are transformed into growth rates, another justification for the level dummy variable approach can be given: a visual comparison in Figure 2 can hardly suggest a parabolic relationship between both growth rates.

As an additional step, $Debt_{it}(\tau)$ is interracted with a variable defined as $max[0, g_{D_{it}}]$. As $g_{D_{it}}$ is the $I(0)$ public debt growth rate used throughout the empirical study, the defined variable selects only positive growth and indicates *an impact of debt growth when specific level threshold is crossed*. Therefore, the final most general model to fit becomes:

$$\triangle\, y_{it} = \alpha_{i\tau} + \omega_{i\tau} Debt_{it}(\tau) + \upsilon_{i\tau}\left(Debt_{it}(\tau)\ x \max[0, g_{D_{it}}]\right) + \pi_{i\tau}\left[y_{it-1} - \theta_i g_{D_{it}}\right] + \sum_{m=1}^{p-1} \psi_{im\tau}^{y} \triangle\, y_{it-m} + \sum_{n=0}^{q-1} \psi_{in\tau}^{D} \triangle\, g_{D_{it-n}} + \boldsymbol{\lambda}_i' \boldsymbol{f}_t + \varepsilon_{it} \quad (19)$$

According to the significance of results above, CS-ECM (1,1) specification augmented with cross-sectional averages is employed. Results regarding dummies and interaction terms are represented in Table 9

Table 9. Debt thresholds and trajectories for GDP growth

Threshold	**Coefficient ω**	**Trajectory**	**Coefficient υ**
$Debt(50)$	0.00289	$Debt(50)\ x\ max[0, g_D]$	-0.0230
	(0.00348)		(0.0451)
$Debt(60)$	0.00272	$Debt(60)\ x\ max[0, g_D]$	-0.0874*
	(0.00399)		(0.0511)
$Debt(70)$	0.00200	$Debt(70)\ x\ max[0, g_D]$	-0.0818
	(0.00441)		(0.0509)
$Debt(80)$	-0.0769	$Debt(80)\ x\ max[0, g_D]$	3.704[24]
	(0.0722)		(3.697)
$Debt(90)$	-0.0108*	$Debt(90)\ x\ max[0, g_D]$	0.117*
	(0.00611)		(0.0685)

The first observation is that coefficient on the trajectory term marking growing debt when 60% debt to GDP ratio is crossed is weakly significant at 10%. This result for the OECD is consistent with Pesaran et al. (2013) who included the developing economies in their sample as well. A positive growth rate of public debt beyond 60% of public debt to GDP ratio additionally reduces economic growth by 0.08 percentage points. An interesting observation is a positive effect on economic growth which occurs when debt grows beyond 90% ratio to GDP. A positive debt growth beyond 90% debt to GDP raises real GDP per capita growth by approximately 0.12 percentage points. Moving to the level thresholds, only 90% level is significant at 10 percent and brings a reduction of approximately 0.011 percentage points in GDP growth rate. This result, although statistically weak, is consistent with (informal) findings of Reinhart and Rogoff (2010b) for advanced economies, who used a simple correlation.

An analogous model as in (19) is estimated for the growth of capital stock to see if there exist any threshold effects on capital accumulation. Naturally, if *H2* suggests that effect of public

[24] For 80% dummy, the covariance matrix becomes nearly singular, therefore results are imprecise.

debt accumulation runs through a reduced capital accumulation, similar threshold effects are anticipated.

Table 10. Debt thresholds and trajectories for capital stock growth

Threshold	Coefficient ω	Trajectory	Coefficient υ
$Debt(50)$	0.000988 (0.006931)	$Debt(50)\ x\ max[0, g_D]$	0.014818 (0.042723)
$Debt(60)$	-0.00211 (0.005696)	$Debt(60)\ x\ max[0, g_D]$	-0.009069 (0.071597)
$Debt(70)$	-0.007038 (0.00461)	$Debt(70)\ x\ max[0, g_D]$	0.038237 (0.046312)
$Debt(80)$	-0.034835 (0.03154)	$Debt(80)\ x\ max[0, g_D]$	1.56659 [25] (1.59607)
$Debt(90)$	-0.015014 (0.012724)	$Debt(90)\ x\ max[0, g_D]$	0.367257 (0.288926)

The results are consistent with the original model for debt-capital relationship in (18), which shows no significant effect of public debt accumulation rate and goes against the theoretical implications. No threshold effect is found neither in terms of public debt to GDP level, nor in terms of debt growth when a certain threshold is passed. Therefore, the rate of capital accumulation seems to be resilient to a rising debt accumulation rate in the long-run even when large stocks of public debt are accumulated.

4.7. Robustness Checks: Sub-sample, Alternative Lag Distribution, Additional Control and Half-Lives

Augmenting stationary ECM model with extra lags works as a partial robustness test which is not completely passed as seen from the Table 6 – significance of the results disappears when more lags are controled for. However, it is important to see if the results are not heavily dependent on the sample and if the results alter when ARDL lags are distributed differently before the reparameterization. Also, whether they still hold if more long-term parameters are estimated at the same time. Plus, a temporal interpretation is given to the effects in order to have a *qualitative* grasp of their importance.

4.7.1. H1, H2 and H3: Euro area sub-sample. The same flexible ARDL/ECM models are implemented only for 11 OECD countries belonging to the euro area. Since the dataset of Reinhart and Rogoff (2010a) is extended to include aftermaths of the debt crisis in Europe, a consideration of the sub-sample of countries some of which had to undergo severe austerity

[25] Again, for 80% dummy, covariance matrix becomes nearly singular.

measures is of an additional interest. Table 11 summarizes the results from the ECM models for *H1*, *H2* and *H3*.

Table 11. Long-run results and thresholds for euro area members

Variables	ECM (1,1)	ECM (2,2)	ECM (3,3)	CS-ECM (1,1)	CS-ECM (2,2)	CS-ECM (3,3)
H1: $g_D \rightarrow \gamma$	-0.095** (0.046)	-0.086 (0.050)	-0.08 (0.049)	-0.127** (0.035)	-0.128** (0.050)	-0.350 (0.555)
CD stat.	20.86***	20.57***	19.98***	1.57	0.31	-0.88
H2: $g_D \rightarrow g_k$	-0.580 (0.413)	0.092 (0.197)	0.062 (0.086)	0.0078 (0.120)	0.086 (0.096)	0.1.60 (0.113)
CD stat.	20.00***	19.91***	19.93***	0.76	-0.56	-2.36***

H3: **Threshold**	**Coefficient** ω	**Trajectory**	**Coefficient** υ
$Debt(50)$	-0.005348 (0.005115)	$Debt(50)\ x\ max[0, g_D]$	0.00601 (0.052827)
$Debt(60)$	-0.00016 (0.007244)	$Debt(60)\ x\ max[0, g_D]$	-0.114093 (0.073605)
$Debt(70)$	-0.003189 (0.005457)	$Debt(70)\ x\ max[0, g_D]$	-0.072284 (0.068225)
$Debt(80)$	-0.027817 (0.025408)	$Debt(80)\ x\ max[0, g_D]$	1.24073 (1.25449)
$Debt(90)$	0.00156 (0.012944)	$Debt(90)\ x\ max[0, g_D]$	-0.201759 (0.227386)

The results appear to be consistent with the full OECD sample estimations. 1 percentage point change in debt to GDP growth rate reduces economic growth by 0.095 percentage points in the long-run, according to the ECM (1,1) specification. Equations augmented with the cross-sectional averages show slightly higher negative impacts in absolute values. The fact that the impacts – which are significant at 5 percent level in 3 out of 6 specifications – are larger in absolute value than the ones for all 24 countries (where the impact is on average -0.07) can be explained as signs of the debt crisis when debt accumulation was accelerating fast. Regarding *H2*, the outcome is also consistent with the total sample. No statistically significant impact is detected across all specifications. Additionally, the CS-ECM (3,3) specification still shows a very significant cross-sectional dependence in the errors, hence appears to be unreliable. Lastly, test of *H3* does not show any threshold effects for the euro area sub-sample, contrary to the full OECD sample with 24 economies.

4.7.2. Alternative distribution of lags for H1 and H2. In every specification above, $p = q$, which allows augmenting models in a balanced way. An extra route of the analysis can be taken focusing on strength of feedbacks from economic growth or capital stock growth to public debt growth. Thus, for this exercise, q is fixed at 1, while p is allowed to vary from 2 to 3 because the cases for p = 1 have already been examined. Table 12 presents the result.

Table 12. Alternative lag distribution

Variables	ECM (2,1)	ECM (3,1)	CS-ECM (2,1)	CS-ECM (3,1)
H1: $g_D \rightarrow \gamma$	-0.057* (0.031)	-0.056* (0.031)	-0.096*** (0.030)	-0.186** (0.076)
CD stat.	28.87***	28.10***	1.73*	1.22
H2: $g_D \rightarrow g_k$	0.019 (0.413)	0.051 (0.111)	-0.015 (0.042)	-0.07 (0.065)
CD stat.	33.78***	34.40***	1.74*	1.05

As can be seen, when different distribution of lags is chosen, the results for *H1* and *H2* do not change significantly in terms of the evidence. Still, no effect coming through a reduced capital accumulation rate is detected. On the other hand, a negative long-term impact of a permanent change in debt/GDP growth shows a varying significance and ranges from -0.056 to -0.186 percentage points.

4.7.3. Control variables for H1. As the specification of CS-ECM (1,1) appears to be the most robust irrespective if the common factors are controlled for or not, equation (20) gives its generalized case.

$$\triangle \gamma_{it} = \alpha_i + \pi_i[\gamma_{it-1} - \theta_i g_{D\,it} - \boldsymbol{\Phi}_i'\boldsymbol{X}_{it}] + \psi_i^D \triangle g_{D\,it} + \boldsymbol{\psi}_{\boldsymbol{X}i}' \triangle \boldsymbol{X}_{\boldsymbol{it}} + \boldsymbol{\lambda}_i'\boldsymbol{f}_t + \varepsilon_{it} \quad (20)$$

Here, $\boldsymbol{X}_{it} = (g_{l_{it}}, g_{k_{it}})'$. The extra long-run determinants include growth of labour force in per capita terms and capital accumulation rate. The latter variable is ‘safely’ included because the model consistently predicts absence of the negative effect on capital accumulation, contrary to the theory. Table 13 summarizes the outcome.

Table 13. Robustness check with extra growth variables

Variables	ECM (1,1)	CS-ECM (1,1)
Pseudo-EC term (π)	-0.835***	-0.965***
	(0.0503)	(0.0465)
g_D (θ_D)	-0.0352	-0.0220
	(0.0216)	(0.0168)
g_l (θ_l)	0.388***	0.408***
	(0.107)	(0.117)
g_l (θ_k)	0.202***	0.255***
	(0.0472)	(0.0525)
Observations	1.125	1.125
R-squared	0.670	0.680
Number of Countries	24	24
CD Statistic	23.88***	-0.42
CD P-Value	0.000	0.671

Evidently, the results lose the statistical significance when more growth variables important in the long-run are included. The result does not seem to be robust when cross-sectional dependence in the errors is eliminated. This is consistent with observations from Tables 6 and 7. Statistical significance detected there is at most at 5 percent and it diminishes when more lags are included to control for the feedback effects and the equation is augmented with the cross-sectional averages to account for the common factors.

4.7.4. Estimation of half-lives. To give the long-run parameters a more physical interpretation, half-lives are estimated[26]. It gives a speed in annual terms with which 50% of the deviation from equlibrium between cointegrated series is reduced. For example, Eberhardt and Presbitero (2015) produce half-lives ranging from 1.05 to 0.90 (approximately, a year) when cross-sectional dependence is taken into consideration for broad sample of advanced and emerging economies and all series are $I(1)$. In the stationary case of this study, it indicates how fast half of the effect of debt growth on economic growth disappears. Taken average $\hat{\pi}$ from the CS-ECM (1,1) which is -0.734, the half-life for 24 OECD economies is 0.51 years or 6.4 months, approximatelly. This signals that debt accumulation rate effect on growth is not only small (from -0.05 to -0.07 percentage points), but it may be not long-lasting once the time perspective is introduced. Since the long-term parameter is constructed as an accumulated effect *to the*

[26] Its formula is $h_i = \frac{\ln(0.5)}{\ln(1+\hat{\pi}_i)}$, where $\hat{\pi}_i$ is an estimate of the pseudo speed of adjustment parameter. In addition, the full decay of effect is non-linear, hence the full 'survival' length is not simply a double of the half-life.

future given a permanent 1 percentage point change in debt accumulation rate, low half-lives imply a quick fade out of the effect. For the country-specific numbers, see Appendix F.

5. Discussion

5.1. Long-Run Parameters

The empirical results for 24 OECD countries provide some evidence in favor of *H1*: there is a small negative effect of government debt growth to real GDP growth rate in the long-run. Evaluating the test of *H1* from the estimation perspective only, the theoretically motivated transformed ARDL model gives the results different from the ones obtained from a conventional growth regression. Most importantly, the long-run coefficient is almost twice as large as the short-run coefficient, irrespective of specification and control for the common factors. This intuitively suggests a cumulative effect in the long horizon after a permanent shock in public debt growth happens. This result comes along the lines of the model by Greiner and Fincke (2009) which predicts that growth of public debt in a steady state brings a lower economic growth, contrary to the case of a balanced budget where debt/GDP ratio goes to zero. The coefficient size of -0.07 is similar and consistent with the work of Pesaran et al. (2013) who conducted study for a broad sample of emerging and advanced economies. Despite this, one cannot safely claim that growth of public debt reduces GDP growth in advanced economies by the same margin as in emerging ones. It is not clear which sub-sample dominates the results in the broad sample of Pesaran et al. (2013). On the other hand, the statistically significant coefficient (significant at 5 percent) in this study comes from CS-ECM (1,1) specification only. This suggests that the results are not very stable. Although negative and significant effects survive in the euro area sub-sample analysis, the robustness is lost when equation is augmented with extra core growth variables, such as capital stock or labor force (per capita) growth. Nonetheless, this has a technical explanation. One of the central aims of this study is to control for the common factors that violate independence of the errors. Inclusion of the cross-sectional averages that approximate the common factors brings the common variation in the model explicitly. Technically, this enlarges the regression equation by a great margin. Large statistics of cross-sectional dependence as seen in Tables 6 – 8, for example, signal that the dependence among advanced economies is statistically important. This can overwhelm the explanatory power of the other variables. Such explanation is supported by fixing q and increasing only p as a robustness check – when fewer variables and their cross sectional averages are included, the negative long-run impact stays statistically significant.

The extra estimation of half-lives gives some qualitative interpretation to *H1*. It hints that, on average, half of the shock to debt and its effect on GDP growth fade rather fast. 6.4 months is a period which can hardly be compatible with the long-run. Yet, half-life illustrates only half of necessary time for the effect to cease and it cannot be used to claim that debt/GDP growth is uninportant. As a result, a general policy implication is that slowing down the debt accumulation rates might help to avoid risks of negative long-run impacts on GDP development. Targetting the rates of fiscal deterioration is also intutive: constant or slowly moving debt levels do not signal instability in the economy as heavily as abrupt and unexpected changes in public deficits that require issuing more debt.

5.2. Thresholds

To begin with, evidence for threshold effects does not seem convincing for the interaction variable $Debt(\tau)\ x\ \max[0, g_D]$. Although the result is statistically significant for debt growth beyond 60% level, one would expect a continuing negative impact beyond 70%, 80% and so on. It is hard to find a rationale why growing debt has negative effects beyond 60% debt level, but they vanish beyond 70%. Additionally, the coefficient is significant at the 10% level only. Thus, it is implied that debt trajectory is not so important for advanced economies. In comparison, this result is opposite in the diverse sample of Pesaran et al. (2013) where the negative impact of growing debt persists for higher debt/GDP levels, also starting from 60%. The results remain dubious after testing *H3*, which looks at debt/GDP levels only. 90% debt/GDP level additionally slows down the growth by 0.011 percentage points. In effect, this replicates and gives more validity for the public debt overhang result by Reinhart and Rogoff (2010b) who observe a slower economic growth in countries with 90% or larger ratio using the same sample. Hence, the policy recommendation for fiscal discipline becomes only stronger. On the other hand, the robustness check for the sub-sample of 11 euro area countries shows no statistically important negative impact of any threshold found in debt-growth literature. This is a suggestion that 90% threshold, which already became a focal point in the empirical literature, may be sensitive to the outliers. For instance, the sub-sample excludes Japan or the United States – both of them surpassed 90% long time ago and stay beyond 100% with Japan approaching 200%.

5.3. Alternative Causal Channels

Despite some evidence in favour of *H1*, this study cannot empirically explain the interim causal relationship, proposed by *H2*. The growth of capital stock appears to have the long-run effect on economic growth and it is consistent with the evidence from Bond et al. (2010), who also employ ARDL model. Despite this, the effect of government debt growth trickles to economic growth without the impact on the long-run capital accumulation rate. The statistical absence of the negative effect is robust irrespective of specification: extra lags, control for the common factors or testing for the thresholds. Short-run negative hikes of -0.03 percentage points may suggest only temporary investors' confidence problems that disappear over time. This provides an important economic implication for advanced economies: capital growth seems to maintain resilience against growing debt or large debt/GDP levels in the long-run. At the same time, it calls for an alternative explanation of the negative debt impact which is compatible with shifts in the long-run equilibrium.

One possibility why capital growth seems to be resilient in the long-run is related to two conditions assumed by the theory (e.g. Diamond, 1965). First, Ricardian effects when rational individuals anticipate a future tax raise can be present. Hence, they increase their savings in advance, whereas the models of overlapping generations predict that portion of savings is wasted on unproductive public debt and resources for investments shrink. Secondly, a rising real interest rate is important for crowding out to occur (Gale and Orszag, 2003). Evidence against this transmission channel is found in Reinhart et al. (2012). With their historical analysis of 26 debt overhang episodes (e.g. when debt/GDP ratio is over 90%) in advanced countries, they indicate that in 11 cases growth *and* real interest rates appeared to be lower than in times when this ratio is below 90%. A similar absence of positive debt effect on interest rate of 10-year-maturity government bonds is found by Checherita and Rother (2010) for the euro area. Based on this evidence, the negative debt accumulation effect on growth can be partially explained in the spirit of Kobayashi (2015) who looks at the labour dynamics, instead. According to him, issuance of more government bonds gives more private liquidity and can bring higher growth in itself (as in Arai et al. (2014)). However, over the cycle in the long-run, stimulating fiscal policy and increased subsidies to workers make them reduce labour supply due to income effect. As a result, productivity declines through *labour* channel and, at the same time, real interest rate does not jump because lower production reduces borrowing needs of companies. Yet, this explanation heavily relies on debt being a liquid asset whose effect stays *within* the economy. Moreover, it is not entirely clear 1) how strong income effect is and 2) if a stimulated growth results in labour subsidies because it is conditional on a country's political spectrum.

Another explanation for the negative long-run debt effect can be found in the works by Ram (1986) or Barro (1990) who put emphasis on a productive *public* capital alongside the private capital. This shifts focus from the private agents and their investment decisions altered by public debt to the ability of governments to balance debt payback and creation of public goods. Capital stock series from Penn World Table 9 (Feenstra et al., 2015) lumps both public and private capital, therefore their dynamics are not considered separately and changes in public capital are not observed. Focusing on the public capital only, Ram (1986) and Barro (1990) argue that government long-run investments in infrastructure increase productivity and returns to the private capital because the latter can be used more efficiently. This implies that growing debt and interest payments force governments to forego some public investment. As a result, this opportunity cost reduces productivity and return to private capital in the long-run without crowding it out but a reduction in growth still occurs over time. Importantly, this channel is independent of the nature of debt holders and if debt is accumulated internally or from foreign funds. This explanation has some empirical evidence. For example, Krüger (2012) finds that public investment in infrastructure Granger-causes economic growth in the time scale from 2 to 4 years and from 8 to 16 years in Sweden.

Lastly, *H2* is built upon the *AK* growth model, which suggests that capital stock accumulation partially accounts for Total Factor Productivity and can preserve the long-run growth. However, a negative public debt effect can spill through reduced TFP investments which are less related to physical capital stock (Barro, 1990). For example, gross domestic expenditure on research and experimental development (R&D) financed by governments never surpassed 0.9% of GDP in the OECD in the period of 1981 – 2014, which is the longest available OECD public R&D series (OECD, 2017). Hence, for more than a half of this sample‘s length, research oriented public investments tended to be rather low and worked as only a small complement for a privately produced R&D. This implies that governments can find it easier to fulfill debt payment obligations by cutting funds for policies that are not of the main priority. This impact channel also has some empirical evidence, e.g. Checherita and Rother (2010) find a negative effect of debt/GDP level which is greater than 100% on a TFP index for 12 countries in the euro area.

5.4. Limitations

Despite this study using very recent panel data techniques to solve the identification problem, it is still subject to two main limitations.

- *Data and measurement.* As in the previous research, data availability and debt measurement remain the issues. First, to perform the mean group estimation and have reliable *country-specific* estimates of parameters is hard even though $T = 50$ in this study. The average reported parameter $\frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_i$, on the other hand, is much more reliable. Hence, individual half-lives in Appendix E are illustrative rather than precise. Moreover, series in the historical data-set by Reinhart and Rogoff (2010a) reach 18th century for some economies but then measurement becomes unclear due to historical reasons. For instance, early debt data for Italy are normalized with respect to exports and not GDP which would induce a break in a measurement standard if the longest series was employed. Secondly, debt series shows *gross* debt and not *net*. Calculation of net indebtedness requires exclusion of assets owned by governments, but their definitions and choice of reporting is a country-specific decision (Panizza and Presbitero, 2013).

- *Idiosyncratic endogeneity.* Controlling for the common factors accounts for endogeneity and helps identify parameters when both the variable and the unobservable are driven by the factors. This is a reasonable assumption for the interrelated OECD countries. Plus, the lagged dependent variable in ARDL/ECM setting can account for feedbacks coming from $t-1$ and further. Nonetheless, endogeneity can still occur at t due to the simulteneity and be driven by ε_{it} and not a vector of the factors $\boldsymbol{f}_t$.

5.5. Future Research

There are two central recommendations for the further research. The first is to test more minor and specific transmission channels by employing simillar approach to estimate the long-run parameters. For instance, long-term changes in real interest rate would revive the discussion of the crowding out argument. Also, depending on data availability, TFP, public capital or labor channel are an option. The second suggestion goes along the lines of Lane (2012) or Gourinchas and Obstfeld (2012) who consider the private debt. Particularly, the latter study finds empirical evidence that expansion of the private credit has a significant impact on a probability of crisis. Similarly, Ajovin and Navarro (2014) detect a negative effect and Granger causality from private debt to economic growth by SUR estimation. Hence, it is important to investigate the long-run impact of the private debt on growth potential using new panel techniques and controlling for the cross-sectional dependence. Additionally, a deeper research of the private debt is important not from a technical perspective only. It puts aside public finance and political implications and works as a test if financial development and credit accessibility can be detrimental in the long-term.

6. Conclusion

This study tested three hypotheses which relate public debt to economic growth in the long-term. A rapid accumulation of public debt externally and internally has been long observed in advanced economies. Thus, testing its connection to economic growth potential empirically is important for fiscal policy implications. Consequently, an issue of direction of causality is tackled on technical and theoretical grounds. This study builds on the recent panel techniques and applications pioneered by Pesaran et al. (2013) or Eberhardt and Presbitero (2015). At the same time, it follows causal links from public finance models and treats estimaton of growth according to growth theory implications. Drawing motivation from deteriorating fiscal positions by the advanced countries, the long-run focus is on 24 OECD economies during 50 years. Interdependency among countries and endogeneity is tackled by controlling for common factors. A weak negative long-term effect is suggested by ECM model with stationary series when debt/GDP growth is employed. On average, a 1 percentage point *permanen*t increase in debt accumulation rate results in 0.07 percentage points decline in economic growth. The coefficients are systematically larger for the euro area sub-sample which can be interpreted as a 'legacy' of an accelerated debt accumulation during debt crisis. Although the effects are similar to the ones received in Pesaran's et al. (2013) large and diverse sample, the results are not robust to inclusion of more long-run growth determinants to the same equation. This can be explained on technical grounds. Threshold analysis, based on heavily persistent debt/GDP levels, suggests that 90% and larger ratios reduce growth in the long run for the whole sample, yet no significant thresholds are detected for the euro area sub-sample. Contrary to the theoretical predictions, debt accumulation does not seem to hinder capital accumulation rates in the long-term. This is a positive sign for advanced economies where capital is an important growth determinant, according to the additional empirical evidence obtained in the study. Hence, the main policy takeaway is to control the rate at which public debt grows and avoid permanent upward shifts of that rate. However, transmission channels and their importance in the long-run for advanced economies still need to be empirically clarified.

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Appendix A

Let series $D(t)$ (debt) and $K(t)$ (capital variable) be continuous stochastic diffusion processes:

$$dD(t)^{27} = \mu_{1t}dt + \sigma_{1t}dW_1(t)$$

$$dK(t) = \mu_{2t}dt + \sigma_{2t}dW_2(t)$$

$$\boldsymbol{dX}(t) = \boldsymbol{\mu}_t dt + \boldsymbol{Q}_t^{1/2}\boldsymbol{dW}(t)$$

Here $W_i(t)$ is a Wienner process (or Brownian motion) and μ_{it} and σ_{it} represent drift and volatility terms, respectively. $\boldsymbol{X}(t)$ includes both series as a vector and the matrix $\boldsymbol{Q_t}$ emphasizes the existence of a covariance between both processes as off-diagonal elements. Assume, the relationship between economic growth rate $\gamma(t)$ and two latter variables can be modelled as at least twice differentiable function G:

$$\gamma(t) = G\big(D(t), K(t)\big) + u(t)$$

This relationship can be expanded by the bivariate Ito's lemma[28] and expectation (expectation is needed since every 'infinitesimal' here is a random draw) conditional on $dD(t)$ can be taken thus giving an expected change in growth rate given a change in a debt variable:

$$E[d\gamma(t)|dD(t)] = \frac{\partial G}{\partial K(t)}E[d\,K(t)|dD(t)] + \frac{\partial G}{\partial D(t)}dD(t)$$
$$+\frac{1}{2}\left[\frac{\partial^2 G}{\partial K_t^2}E[\big(dK(t)\big)^2|dD(t)] + 2\frac{\partial^2 G}{\partial K(t)\partial D(t)}E[dK(t)dD(t)|dD(t)] + \frac{\partial^2 G}{\partial D_t^2}(dD(t))^2\right]$$

Here, the expectation operator passes through the derivative term since $(D(t), K(t)) \in \boldsymbol{\mathcal{F}}(t)$ (measurable with respect to σ-algebra $\boldsymbol{\mathcal{F}}(t)$), also the error term is exogenous, thus zero in expectation (note that error term is *not* a martingale difference sequence). It can be seen that $E[dY(t)|dD(t)]$ is generally not equal to the derivative with respect to $D(t)$, because dependencies between the processes are evident and one cannot be held constant while looking at partial effect the other (Pesaran & Smith, 2014).

[27] This is a short-hand notation for a stochastic integral because infinitesimals do not exist in this setting – Brownian motion is almost surely non-differentiable.
[28] Ito's lemma can heuristically be interpreted as a generalization of the chain rule from a conventional calculus to stochastic calculus.

Appendix B

Table 14. Wooldridge test for serial correlation

Specification	F Statistic	P-Value
Baseline Growth Regression	4.367**	0.000
$g_D \rightarrow \gamma$		
ECM (1,1)	121.607***	0.000
ECM (2,2)	95.850***	0.000
ECM (3,3)	112.441***	0.000
$g_D \rightarrow g_k$		
ECM (1,1)	242.388***	0.000
ECM (2,2)	145.274***	0.000
ECM (3,3)	136.268***	0.000
$g_k \rightarrow \gamma$		
ECM (1,1)	216.790***	0.000
ECM (2,2)	146.736***	0.000
ECM (3,3)	156.420***	0.000

Table 15. Breusch-Pagan test for conditional heteroskedasticity

Specification	χ^2 Statistic	P-Value
Baseline Growth Regression	247.93***	0.000
$g_D \rightarrow \gamma$		
ECM (1,1)	40.76***	0.000
ECM (2,2)	67.87***	0.000
ECM (3,3)	60.92***	0.000
$g_D \rightarrow g_k$		
ECM (1,1)	86.40***	0.000
ECM (2,2)	620.66***	0.000
ECM (3,3)	575.95***	0.000
$g_k \rightarrow \gamma$		
ECM (1,1)	20.17***	0.000
ECM (2,2)	23.37***	0.000
ECM (3,3)	28.11***	0.000

Appendix C

Table 16. Short-run effects of debt and capital growth

Variables	ECM (1,1)	ECM (2,2)	ECM (3,3)	CS-ECM (1,1)	CS-ECM (2,2)	CS-ECM (3,3)
$g_D \rightarrow \gamma$						
$\Delta.\ g_D$	-0.0595**	-0.0638***	-0.0694***	-0.0272	-0.0473*	-0.0612**
	(0.0232)	(0.0226)	(0.0210)	(0.0209)	(0.0253)	(0.0244)
$L.\Delta.\ g_D$		-0.00991	-0.0128		-0.0286	-0.0342*
		(0.00966)	(0.0127)		(0.0203)	(0.0200)
$L2.\Delta.\ g_D$			-0.00675			-0.0120
			(0.00818)			(0.0118)
$g_D \rightarrow g_k$						
$\Delta.\ g_D$	-0.0150	-0.0329**	-0.0211	-0.00835	-0.0274	-0.0293
	(0.0118)	(0.0154)	(0.0178)	(0.0171)	(0.0328)	(0.0219)
$L.\Delta.\ g_D$		-0.0306***	-0.0113		-0.0229	-0.0289*
		(0.0104)	(0.0126)		(0.0197)	(0.0171)
$L2.\Delta.\ g_D$			0.0383***			0.00394
			(0.0108)			(0.0135)
$g_k \rightarrow \gamma$						
$\Delta\ g_k$	0.399***	0.396***	0.420***	0.429***	0.400***	0.452***
	(0.114)	(0.118)	(0.117)	(0.114)	(0.114)	(0.126)
$L.\Delta.\ g_k$		0.00499	0.0279		-0.00461	0.0550
		(0.0269)	(0.0292)		(0.0612)	(0.0805)
$L2.\Delta.\ g_k$			0.107***			0.111*
			(0.0365)			(0.0632)

Appendix D

Results from panel Granger causality tests for GDP growth-capital growth relationship and debt growth-capital growth relationship.

Table 17. Granger causality between GDP and capital growth

System I	χ^2 Statistic (L1)	χ^2 Statistic (L2)	χ^2 Statistic (L3)
$\gamma \rightarrow g_k$	6.640***	5.643*	6.222
$g_k \rightarrow \gamma$	4.360**	4.485	4.036
System I (CS)	**χ^2 Statistic (L1)**	**χ^2 Statistic (L2)**	**χ^2 Statistic (L3)**
$\gamma \rightarrow g_k$	8.447***	8.239**	12.766***
$g_k \rightarrow \gamma$	0.851	0.920	2.541

Table 18. Granger causality between debt and capital growth

System I	χ^2 Statistic (L1)	χ^2 Statistic (L2)	χ^2 Statistic (L3)
$g_k \rightarrow g_D$	5.293**	4.838*	7.876**
$g_D \rightarrow g_k$	1.549	4.380	6.112
System II (CS)	**χ^2 Statistic**	**χ^2 Statistic (L2)**	**χ^2 Statistic (L3)**
$g_k \rightarrow g_D$	1.788	1.967	3.014
$g_D \rightarrow g_k$	3.530*	4.786*	5.120

Table 19. Eigenvalues (modules) of panel VAR companion matrices

	Eigenvalues		
Systems	1 Lag	2 Lags	3 Lags
$\gamma \rightarrow g_D$	λ_1= 0.38	λ_1= 0.52	λ_1= 0.63
$g_D \rightarrow \gamma$	λ_2= 0.38	λ_2= 0.52	λ_2= 0.63
		λ_3= 0.27	λ_3= 0.31
		λ_4= 0.005	λ_4= 0.31
			λ_5= 0.19
			λ_6= 0.19
$\gamma \rightarrow g_k$	λ_1= 0.62	λ_1= 0.41	λ_1= 0.67
$g_k \rightarrow \gamma$	λ_2= 0.62	λ_2= 0.34	λ_2= 0.64
		λ_3= 0.34	λ_3= 0.43
		λ_4= 0.04	λ_4= 0.43
			λ_5= 0.33
			λ_6= 0.33
$g_k \rightarrow g_D$	λ_1= 0.64	λ_1= 0.47	λ_1= 0.68
$g_D \rightarrow g_k$	λ_2= 0.38	λ_2= 0.39	λ_2= 0.43
		λ_3= 0.39	λ_3= 0.43
		λ_4= 0.11	λ_4= 0.41
			λ_5= 0.41
			λ_6= 0.31

	Eigenvalues		
Systems (CS)	1 Lag	2 Lags	3 Lags
$\gamma \rightarrow g_D$ $g_D \rightarrow \gamma$	λ_1= 0.35 λ_2= 0.24	λ_1= 0.40 λ_2= 0.40 λ_3= 0.24 λ_4= 0.02	λ_1= 0.58 λ_2= 0.58 λ_3= 0.35 λ_4= 0.35 λ_5= 0.31 λ_6= 0.31
$\gamma \rightarrow g_k$ $g_k \rightarrow \gamma$	λ_1= 0.43 λ_2= 0.24	λ_1= 0.38 λ_2= 0.38 λ_3= 0.31 λ_4= 0.05	λ_1= 0.67 λ_2= 0.57 λ_3= 0.57 λ_4= 0.38 λ_5= 0.25 λ_6= 0.25
$g_k \rightarrow g_D$ $g_D \rightarrow g_k$	λ_1= 0.48 λ_2= 0.32	λ_1= 0.46 λ_2= 0.37 λ_3= 0.37 λ_4= 0.11	λ_1= 0.70 λ_2= 0.58 λ_3= 0.58 λ_4= 0.38 λ_5= 0.38 λ_6= 0.32

Appendix E

$$\gamma_{it} = \sum_{j=1}^{p} \phi_{ji} \gamma_{it-j} + \sum_{j=0}^{q} \beta_{ji} g_{D\,it-j} + u_{it} \quad \text{i}$$

$$u_{it} = \boldsymbol{\lambda}_i' \boldsymbol{f}_t + \varepsilon_{it} \quad \text{ii}$$

$$\gamma_{it} = \sum_{j=1}^{p} \phi_{ji} \gamma_{it-j} + \sum_{j=0}^{q} \beta_{ji} g_{D\,it-j} + u_{it} \rightarrow \gamma_{it} = \left[\sum_{j=1}^{p} \phi_{jt} L^j\right] \gamma_{it} + \left[\sum_{j=0}^{q} \beta_{ji} L^j\right] g_{D\,it} + u_{it} \quad \text{iii}$$

$$\gamma_{it} = \Phi_i(\mathrm{L})\gamma_{it} + \beta_i(L) g_{D\,it} + u_{it} \rightarrow \gamma_{it} = \frac{\beta_i(L)}{\Phi_i(L)} g_{Dit} + \Phi_i^{-1}(L) u_{it} \quad \text{iv}$$

$$\gamma_{it} = \theta_i g_{D\,it} + \theta_i(L) \vartriangle g_{D\,it} + \nu_{it} \quad \text{v}$$

Here, i, ii and iii represent a traditional common factor-augmented ARDL (p, q) model rewritten in terms of the lag operator. The operator is invertible as the series are stationary. Equation v is based on Beveridge-Nelson decomposition[29]. Hence,

$$\frac{\beta_i(L)}{\Phi_i(L)} = \delta_i(L) = \delta_i(1) - \widehat{\delta}_i(L)(1 - L)$$

Which can be explicitly written as:

$$\frac{\beta_i(L)}{\Phi_i(L)} = \frac{\sum_{j=0}^{q} \beta_{ji}}{1 - \sum_{j=1}^{p} \phi_{ji}} - \left[\sum_{j=0}^{\infty} \sum_{h=j+1}^{\infty} \delta_h L^j\right](1 - L) = \theta_i - \theta_i(L) \vartriangle$$

Table 20 gives the result from regression model in v, which is a distributed lag specification with directly computed long-run parameter θ_i. Of course, $\theta_i(L) \vartriangle g_{D\,it}$ gives an infinite number of lagged differences, thus the truncation is chosen in line with ARDL/ECM applications in section 4.6. Again, (p, q) indicate lags in ARDL before the reparameterization.

Table 20. Estimates from DL specifications

	DL (1,1)	DL (2,2)	DL (3,3)	CS-DL (1,1)	CS-DL (2,2)	CS-DL (3,3)
θ_i	-0.097***	-0.083***	-0.077***	-0.058***	-0.045**	0.035*
CD Statistic	30.952***	30.362***	29.876***	-3.286***	-3.244**	-3.037**

[29] Beveridge-Nelson decomposition allows transformation of any lag operator: $C(L) = C(1) - \hat{C}(L)(1 - L)$, where $C(1) = \sum_{j=1}^{\infty} c_j$, $\hat{C}(L) = \sum_{j=1}^{\infty} \hat{c}_j L^j$, $\hat{c}_j = \sum_{h=j+1}^{\infty} c_h$ and $(1 - L)$ is a difference operator.

Appendix F

Table 21. Country-specific half-lives

Country[30]	Long-run Parameter $\hat{\theta}_i$	$\hat{h}_i$	Approximation in Months
On Average	-0.07	0.53	6.4
Austria	-0.02	0.55	6.6
Belgium	-0.17	0.64	7.7
Finland	-0.05	0.43	5.2
France	0.003	1.49	17.9
Germany	-0.06	0.92	11.00
Greece	0.08	0.65	7.8
Ireland	-0.21	0.45	5.4
Italy	0.09	1.29	15.5
Netherlands	-0.27	0.32	3.8
Portugal	-0.018	0.41	4.9
Spain	-0.016	1.01	12.1
Canada	-0.07	0.49	5.9
Denmark	-0.003	0.73	8.76
Japan	0.089	1.80	21.6
New Zealand	-0.02	0.57	6.8
Norway	-0.14	0.46	5.5
Chile	-0.09	0.31	3.7
Sweden	0.027	0.32	3.8
Switzerland	-0.09	0.25	3.0
Turkey	-0.09	0.18	2.1
UK	-0.065	1.13	13.6
US	-0.075	0.26	3.1

[30] For Australia and Iceland pseudo EC term is greater than 1 in absolute value, hence calculating half-life has no meaning.