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Garpinger, Olof

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LUND UNIVERSITY

PO Box 117  
221 00 Lund  
+46 46-222 00 00

# Optimal PI and PID Parameters for a Batch of Benchmark Process Models Representative for the Process Industry

Olof Garpinger

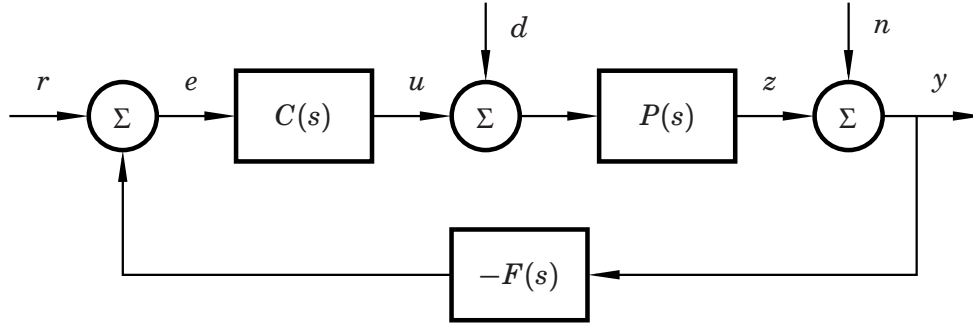
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## Abstract

This report presents a large number of optimal PI and PID parameters for a batch of 134 benchmark process models representative for the process industry. This information can e.g. be used as a reference for other controller design methods. The optimal parameters have been derived using the MATLAB<sup>®</sup>-based software called SWORD, which determines the PI or PID controller that gives the best load disturbance performance given some closed-loop robustness. The performance measure used here is the integrated absolute error during a step disturbance on the process input, and the robustness constraints are given by upper limits on maximum sensitivity and complementary sensitivity. It has previously been shown that SWORD can be used to find near-optimal controllers also with respect to an additional noise sensitivity constraint. A controller design method based on this finding gives a set of PI and PID controllers all with the same robustness, but instead with a trade-off between performance and noise sensitivity. Five such sets of controllers are presented here for process models with varying dynamics. There are also a large number of optimal, unfiltered, SWORD PI and PID parameters presented, and related to four different robustness values. All data has been collected in tables which are given at the end of this report. The same tables have been collected in MATLAB<sup>®</sup> data files, Excel spreadsheets, as well as in a tex-file.

## 1. Introduction

The purpose of this document is to provide a large number of IAE optimal PI and PID controllers for a batch of benchmark process models, which can be used for comparison with other design methods. Both for the unfiltered case (with regards to four different robustness levels) and for the filtered, noise sensitivity constrained, case. The first few sections are used to define the optimization problem used to determine the controllers, and to present the batch of benchmark models. 77 tables filled with optimal PID parameters and related data are given in Sections 8 to 10. The tables have also



**Figure 1** A load disturbance,  $d$ , measurement noise,  $n$ , and set-point,  $r$ , act on the closed-loop system with process  $P(s)$ , controller  $C(s)$  and measurement filter  $F(s)$ .

been collected in MATLAB<sup>®</sup> mat-files, Excel spreadsheets (different sheets for different process types), as well as in a tex-file. These can be downloaded from:

<http://www.control.lth.se/Research/ProcessControl/PIDControl/>

## 2. The closed-loop system

The single-input single-output feedback loop in Fig. 1 will be used here to set up a constrained optimization problem for the design of PI and PID controllers. The process,  $P(s)$ , is manipulated by a controller,  $C(s)$ , such that the controlled variable,  $z$ , is kept as close as possible to a set-point,  $r$ , in order to minimize the control error,  $e$ . The process is affected by a load disturbance,  $d$ , at the process input. The measurements of the controlled variable,  $y$ , typically contain noise,  $n$ , and are fed through a low-pass filter,  $F(s)$ , to keep the noise level of the control signal,  $u$ , low.

Assuming regulatory control around a constant set-point,  $r = 0$ , the closed-loop system can be described by three equations

$$Z(s) = \frac{P(s)}{1 + P(s)C(s)F(s)}D(s) - \frac{P(s)C(s)F(s)}{1 + P(s)C(s)F(s)}N(s), \quad (1)$$

$$Y(s) = \frac{P(s)}{1 + P(s)C(s)F(s)}D(s) + \frac{1}{1 + P(s)C(s)F(s)}N(s), \quad (2)$$

$$U(s) = -\frac{P(s)C(s)F(s)}{1 + P(s)C(s)F(s)}D(s) - \frac{C(s)F(s)}{1 + P(s)C(s)F(s)}N(s), \quad (3)$$

with frequency-domain signals in capital letters.

It is well-known that the four transfer functions

$$S(s) = \frac{1}{1 + P(s)C(s)F(s)}, \quad (4)$$

$$T(s) = \frac{P(s)C(s)F(s)}{1 + P(s)C(s)F(s)}, \quad (5)$$

$$S_p(s) = \frac{P(s)}{1 + P(s)C(s)F(s)}, \quad (6)$$

$$S_c(s) = \frac{C(s)F(s)}{1 + P(s)C(s)F(s)}, \quad (7)$$

are sufficient to describe the closed-loop system in Fig. 1.  $S(s)$  is called the sensitivity function and  $T(s)$  the complementary sensitivity function.

### 3. Controllers and noise filters

In this document, optimal PI and PID parameters will be presented, sometimes together with a corresponding noise filter time constant. There are several PID and filter forms commonly used, and it is therefore important to define the ones used here. The PI controller will be given by the transfer function

$$C_{PI}(s) = K \left( 1 + \frac{1}{sT_i} \right), \quad (8)$$

where  $K$  is the proportional gain and  $T_i$  is the integral time. For PID control the parallel form has been chosen since it is more general than the series form. It has the transfer function

$$C_{PID}(s) = K \left( 1 + \frac{1}{sT_i} + sT_d \right). \quad (9)$$

The derivative part comes with the derivative time parameter,  $T_d$ . Low-pass filters are used here to decrease closed-loop noise throughput. This is especially important for PID control, but also useful for PI control. A first-order low-pass filter has been used for PI control,

$$F_{PI}(s) = \frac{1}{sT_f + 1}, \quad (10)$$

which gives the controller high-frequency roll-off. A second order filter

$$F_{PID}(s) = \frac{1}{(sT_f)^2/2 + sT_f + 1}, \quad (11)$$

is used for PID control, also for the sake of high-frequency roll-off. The filter time constant  $T_f$  is the only parameter that needs to be set in both  $F_{PI}$  and  $F_{PID}$ .

$$\begin{aligned}
P_1(s) &= \frac{e^{-s}}{1+sT}, \\
T &= 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, \\
&\quad 1.3, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500, 1000 \\
P_2(s) &= \frac{e^{-s}}{(1+sT)^2}, \\
T &= 0.01, 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 1, \\
&\quad 1.3, 1.5, 2, 4, 6, 8, 10, 20, 50, 100, 200, 500 \\
P_3(s) &= \frac{1}{(s+1)(1+sT)^2}, \\
T &= 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 2, 5, 10 \\
P_4(s) &= \frac{1}{(s+1)^n}, \\
n &= 3, 4, 5, 6, 7, 8 \\
P_5(s) &= \frac{1}{(1+s)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)}, \\
\alpha &= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 \\
P_6(s) &= \frac{1}{s(1+sT_1)} e^{-sL_1}, \\
L_1 &= 0.01, 0.02, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.9, 1.0, \quad T_1 + L_1 = 1 \\
P_7(s) &= \frac{1}{(1+sT)(1+sT_1)} e^{-sL_1}, \quad T_1 + L_1 = 1, \\
T &= 1, 2, 5, 10 \quad L_1 = 0.01, 0.02, 0.05, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0 \\
P_8(s) &= \frac{1-\alpha s}{(s+1)^3}, \\
\alpha &= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1 \\
P_9(s) &= \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}, \\
T &= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0
\end{aligned}$$

**Figure 2** Test batch of 134 processes representative for the process industry.

## 4. A batch of benchmark models

A batch of 134 benchmark process models, representative for the process industry, was introduced in [1] and can be found in Fig. 2, on the next page. Sections 8 to 10 holds optimal PI and PID parameters for this batch.

In addition to these models, the second order time-delayed model

$$P_{2,mod}(s) = \frac{e^{-0.1s}}{(20s+1)^2}, \quad (12)$$

will also be used to derive a set of optimal controller parameters (with

corresponding filters).

The benchmark models have been approximated with first-order time-delayed (FOTD) models

$$P_m(s) = \frac{K_p}{sT + 1} e^{-sL}, \quad (13)$$

in order to classify them based on the normalized time delay,

$$\tau = \frac{L}{L + T}. \quad (14)$$

## 5. Criteria for PID controller design

A rational way to start the design of a controller is to first derive a process model and a collection of requirements. Constrained optimization can then be applied to make a trade between often conflicting requirements. Tuning of PID controllers for the process industry is seldom done this way since the effort that can be devoted to a single loop is severely limited.

Requirements typically include specifications on load disturbance attenuation, robustness to process uncertainty, noise sensitivity and set-point tracking. Load disturbance attenuation is a primary concern in process control where steady-state regulation is a key issue, while set-point tracking is a major concern in motion control. Set-point tracking can, however, be treated separately by using a control architecture having two degrees of freedom. Therefore, the controllers listed here will be optimal with respect to load disturbance attenuation.

### 5.1 Performance

Closed-loop performance, i.e. load disturbance attenuation will here be characterized by the integrated absolute error (IAE)

$$\text{IAE} = \int_0^{\infty} |e(t)| dt, \quad (15)$$

for a unit step load disturbance on the process input, see Fig. 1.

### 5.2 Robustness

It is very important to ensure that all plant processes are safe and stable around their set-points. For this reason, the primary goal of many PID design methods is to secure the robustness of the closed-loop system, i.e. to keep good margins to the point of instability. Robustness can be captured by the sensitivity function,  $S(s)$ , and the complementary sensitivity function,  $T(s)$ , see Eqs. (4–5). The maximum values of these functions

$$|S(i\omega)| \leq M_s, \quad |T(i\omega)| \leq M_t, \quad \forall \omega \in \mathbb{R}^+, \quad (16)$$

i.e. the  $H_\infty$ -norms, will be used here as robustness constraints on the closed-loop system. Large values of  $M_s$  and  $M_t$  will lead to poor robustness, while values close to 1 give very high robustness. Reasonable robustness is typically given with  $M_s$  and  $M_t$  between 1.2 and 2.0. For simplicity,  $M_s$  and  $M_t$  will have the same values in each new optimization here ( $M_s = M_t = 1.2, 1.4, 1.6, \text{ and } 1.8$ ). Note that  $\omega$  is the frequency in [rad/s] and  $\mathbb{R}^+$  denotes the set of all non-negative real numbers.

### 5.3 Noise sensitivity

Large control signal activity generated by measurement noise could cause undesirable actuator wear and tear. The impact of the transfer function from measurement noise to control action,  $S_c(s)$ , depends on many factors, with the controller parameters and low-pass filter being particularly important. In this document, the  $H_2$ -norm of  $S_c(s)$  (see Eq. 7)

$$\|S_c(s)\|_2 \leq \kappa_u, \quad (17)$$

will be used to constrain the noise sensitivity. Assuming continuous-time white Gaussian measurement noise with unit spectral density,  $\|S_c(s)\|_2$  can be derived using the integral formula

$$\|S_c(s)\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |S_c(i\omega)|^2 d\omega}. \quad (18)$$

## 6. Optimal PID design using SWORD

The MATLAB<sup>®</sup> software tool called SWORD (Software-based optimal robust design), was first presented in [2]. This tool is used to derive optimal PI and PID controllers with respect to the optimization problem

$$\begin{aligned} & \underset{K, T_i, T_d \in \mathbb{R}^+}{\text{minimize}} && \int_0^{\infty} |e(t)| dt, \\ & \text{subject to} && |S(i\omega)| \leq M_s, \\ & && |T(i\omega)| \leq M_t, \quad \forall \omega \in \mathbb{R}^+. \end{aligned} \quad (19)$$

The low-pass filter time constant,  $T_f$ , is fixed in each optimization. To simplify the problem solving, the software finds the controller that makes at least one of the robustness constraints active. This will give the optimal solution also to (19), unless  $M_s$  and  $M_t$  are quite large (i.e. when SWORD starts giving highly oscillating responses). This typically happens for  $M_s$  and  $M_t$  greater than 2.0.

Sections 8 and 9 hold tables with unfiltered SWORD PI and PID parameters for the benchmark models in Fig. 2, with respect to  $M_s = M_t = 1.2, 1.4, 1.6, \text{ and } 1.8$ .

The SWORD software tool is freely available at:

<http://www.control.lth.se/Research/ProcessControl/PIDControl/>

An older version of the SWORD software is also available on this webpage. It has been tested more thoroughly and can e.g. be used if the most recent SWORD version would not work.

## 7. Using SWORD to determine the noise filter

In [3], it is shown that SWORD can provide controllers that are close to optimal also with respect to the extended optimization problem

$$\begin{aligned} & \underset{K, T_i, T_d, T_f \in \mathbb{R}^+}{\text{minimize}} && \int_0^{\infty} |e(t)| dt, \\ & \text{subject to} && |S(i\omega)| \leq M_s, \\ & && |T(i\omega)| \leq M_t, \quad \forall \omega \in \mathbb{R}^+, \\ & && \|S_c(s)\|_2 \leq \kappa_u. \end{aligned} \tag{20}$$

This is accomplished by varying the low-pass filter time constant,  $T_f$ , as well as the controller type (PID, PI or I). The result is a set of controllers and low-pass filters. The user can then use this set to choose the best possible controller giving an acceptable level of the control signal activity. This level can be determined by visual feedback of the control signal.

Section 10 presents tables with controller sets for five different models (with  $M_s = M_t = 1.4$ ), containing: lag-dominant, balanced, delay-dominant, integrating, and dominant second-order processes.



## 8. Optimal PI parameters for the test batch

The following tables hold process data, optimal unfiltered SWORD PI parameters, and corresponding IAE-values, for the benchmark models. Section 8.1 presents tables for a closed-loop robustness given by  $M_s = M_t = 1.2$ , Section 8.2 for  $M_s = M_t = 1.4$ , Section 8.3 for  $M_s = M_t = 1.6$ , and Section 8.4 for  $M_s = M_t = 1.8$ . The tables have also been collected in MATLAB<sup>®</sup> mat-files, Excel spreadsheets (different sheets for different process types), as well as in a tex-file. These can be downloaded from:

<http://www.control.lth.se/Research/ProcessControl/PIDControl/>

### 8.1 $M_s = M_t = 1.2$

**Table 1**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.020	0.98	0.097	0.37	0.097	0.26	3.77
0.050	0.95	0.098	0.38	0.098	0.26	3.87
0.10	0.91	0.10	0.40	0.10	0.25	4.02
0.20	0.83	0.11	0.46	0.11	0.23	4.26
0.30	0.77	0.12	0.53	0.12	0.23	4.43
0.50	0.67	0.14	0.67	0.14	0.21	4.66
0.70	0.59	0.17	0.82	0.17	0.21	4.78
1.00	0.50	0.22	1.05	0.22	0.21	4.85
1.30	0.43	0.26	1.28	0.26	0.21	4.87
1.50	0.40	0.29	1.43	0.29	0.21	4.85
2.00	0.33	0.37	1.79	0.37	0.21	4.78
4.00	0.20	0.72	3.12	0.72	0.23	4.41
6.00	0.14	1.07	4.19	1.07	0.25	4.05
8.00	0.11	1.41	5.04	1.41	0.28	3.71
10.0	0.091	1.76	5.73	1.76	0.31	3.42
20.0	0.048	3.48	7.85	3.48	0.44	2.40
50.0	0.020	9.12	11.7	9.12	0.78	1.31
100	0.0099	18.6	14.7	18.6	1.27	0.79
200	0.0050	37.6	16.7	37.6	2.25	0.45
500	0.0020	94.3	18.2	94.3	5.18	0.19
1000	0.0010	189	18.8	189	10.1	0.099

**Table 2**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.01	0.98	0.097	0.37	0.097	0.26	3.78
0.02	0.96	0.098	0.38	0.098	0.26	3.85
0.05	0.92	0.099	0.40	0.099	0.25	4.05
0.10	0.85	0.10	0.45	0.10	0.23	4.35
0.20	0.74	0.11	0.55	0.11	0.21	4.87
0.30	0.66	0.13	0.67	0.13	0.19	5.31
0.50	0.55	0.15	0.94	0.15	0.17	6.06
0.70	0.48	0.18	1.22	0.18	0.15	6.70
1.00	0.41	0.22	1.65	0.22	0.13	7.56
1.30	0.36	0.25	2.11	0.25	0.12	8.36
1.50	0.34	0.27	2.42	0.27	0.11	8.88
2.00	0.30	0.32	3.20	0.32	0.099	10.1
4.00	0.22	0.44	6.48	0.44	0.068	14.7
6.00	0.19	0.52	9.87	0.52	0.052	19.1
8.00	0.18	0.57	13.3	0.57	0.043	23.5
10.0	0.17	0.60	16.8	0.60	0.036	27.8
20.0	0.15	0.70	34.3	0.70	0.020	49.3
50.0	0.14	0.77	87.3	0.77	0.0088	113
100	0.14	0.80	176	0.80	0.0046	220
200	0.13	0.82	353	0.82	0.0023	433
500	0.13	0.83	888	0.83	0.0009	1070

**Table 3**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.0050	0.0094	20.7	0.14	20.7	149	0.0067
0.010	0.018	10.1	0.23	10.1	44.5	0.023
0.020	0.034	4.96	0.36	4.96	13.9	0.074
0.050	0.073	2.09	0.61	2.09	3.40	0.30
0.10	0.12	1.11	0.81	1.11	1.38	0.73
0.20	0.18	0.63	0.98	0.63	0.64	1.56
0.50	0.23	0.40	1.40	0.40	0.28	3.54
2.00	0.23	0.39	3.53	0.39	0.11	9.08
5.00	0.19	0.51	8.35	0.51	0.061	16.5
10.0	0.17	0.61	16.9	0.61	0.036	27.6

**Table 4**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PI,  $M_s = M_t = 1.2$

n	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
3.00	0.25	0.35	2.07	0.35	0.17	5.83
4.00	0.33	0.25	2.41	0.25	0.10	9.55
5.00	0.39	0.21	2.75	0.21	0.075	13.3
6.00	0.43	0.18	3.10	0.18	0.059	17.0
7.00	0.47	0.17	3.45	0.17	0.048	20.7
8.00	0.50	0.16	3.81	0.16	0.041	24.4

**Table 5**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PI,  $M_s = M_t = 1.2$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.067	2.20	0.66	2.20	3.32	0.30
0.20	0.12	1.03	0.89	1.03	1.17	0.86
0.30	0.17	0.65	1.02	0.65	0.64	1.56
0.40	0.22	0.48	1.14	0.48	0.42	2.39
0.50	0.25	0.38	1.26	0.38	0.30	3.34
0.60	0.28	0.32	1.41	0.32	0.23	4.41
0.70	0.30	0.28	1.59	0.28	0.18	5.57
0.80	0.32	0.26	1.80	0.26	0.15	6.82
0.90	0.33	0.25	2.07	0.25	0.12	8.15

**Table 6**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.2$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.99	0.01	0.00	0.25	15.8	0.25	0.016	63.1
0.98	0.02	0.00	0.25	15.9	0.25	0.016	63.7
0.95	0.05	0.00	0.24	16.1	0.24	0.015	65.6
0.90	0.10	0.00	0.24	16.4	0.24	0.015	68.8
0.80	0.20	0.00	0.23	17.0	0.23	0.013	74.8
0.70	0.30	0.00	0.22	17.5	0.22	0.012	80.5
0.50	0.50	0.00	0.20	18.4	0.20	0.011	90.5
0.30	0.70	0.00	0.19	19.0	0.19	0.010	98.0
0.10	0.90	0.00	0.19	19.4	0.19	0.0098	102
0.00	1.00	0.00	0.19	19.4	0.19	0.0098	102

**Table 7**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PI,  $M_s = M_t = 1.2$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
1.00	0.99	0.010	0.14	0.80	1.75	0.80	0.46	2.19
1.00	0.98	0.020	0.14	0.77	1.73	0.77	0.45	2.24
1.00	0.95	0.050	0.15	0.69	1.67	0.69	0.42	2.41
1.00	0.90	0.10	0.17	0.60	1.59	0.60	0.37	2.67
1.00	0.70	0.30	0.25	0.38	1.36	0.38	0.28	3.54
1.00	0.50	0.50	0.33	0.29	1.21	0.29	0.24	4.19
1.00	0.30	0.70	0.41	0.24	1.11	0.24	0.22	4.62
1.00	0.10	0.90	0.48	0.22	1.06	0.22	0.21	4.83
1.00	0.00	1.00	0.50	0.22	1.05	0.22	0.21	4.85
2.00	0.99	0.010	0.12	0.91	2.55	0.91	0.36	2.79
2.00	0.98	0.020	0.13	0.89	2.53	0.89	0.35	2.83
2.00	0.95	0.050	0.13	0.83	2.46	0.83	0.34	2.96
2.00	0.90	0.10	0.15	0.75	2.37	0.75	0.32	3.17
2.00	0.70	0.30	0.20	0.55	2.11	0.55	0.26	3.85
2.00	0.50	0.50	0.24	0.45	1.94	0.45	0.23	4.33
2.00	0.30	0.70	0.29	0.40	1.84	0.40	0.22	4.63
2.00	0.10	0.90	0.32	0.38	1.79	0.38	0.21	4.77
2.00	0.00	1.00	0.33	0.37	1.79	0.37	0.21	4.78
5.00	0.99	0.010	0.089	1.49	4.33	1.49	0.34	2.90
5.00	0.98	0.020	0.090	1.47	4.31	1.47	0.34	2.93
5.00	0.95	0.050	0.093	1.41	4.25	1.41	0.33	3.02
5.00	0.90	0.10	0.098	1.34	4.21	1.34	0.32	3.15
5.00	0.70	0.30	0.12	1.12	4.03	1.12	0.28	3.61
5.00	0.50	0.50	0.14	1.00	3.87	1.00	0.26	3.93
5.00	0.30	0.70	0.15	0.93	3.76	0.93	0.25	4.13
5.00	0.10	0.90	0.16	0.90	3.69	0.90	0.24	4.21
5.00	0.00	1.00	0.17	0.89	3.69	0.89	0.24	4.22
10.0	0.99	0.010	0.059	2.60	6.35	2.60	0.41	2.45
10.0	0.98	0.020	0.059	2.57	6.34	2.57	0.41	2.47
10.0	0.95	0.050	0.060	2.51	6.33	2.51	0.40	2.53
10.0	0.90	0.10	0.063	2.41	6.30	2.41	0.38	2.63
10.0	0.70	0.30	0.071	2.11	6.13	2.11	0.34	2.96
10.0	0.50	0.50	0.079	1.92	5.95	1.92	0.32	3.20
10.0	0.30	0.70	0.085	1.81	5.82	1.81	0.31	3.35
10.0	0.10	0.90	0.090	1.76	5.74	1.76	0.31	3.41
10.0	0.00	1.00	0.091	1.76	5.73	1.76	0.31	3.42

**Table 8**  $P_8(s) = \frac{1 - s\alpha}{(s + 1)^3}$ , Unfiltered PI,  $M_s = M_t = 1.2$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.27	0.32	2.05	0.32	0.16	6.38
0.20	0.29	0.29	2.04	0.29	0.14	6.95
0.30	0.32	0.27	2.02	0.27	0.13	7.52
0.40	0.34	0.25	2.01	0.25	0.12	8.10
0.50	0.36	0.23	2.00	0.23	0.12	8.69
0.60	0.38	0.22	1.99	0.22	0.11	9.30
0.70	0.40	0.20	1.98	0.20	0.10	9.91
0.80	0.42	0.19	1.97	0.19	0.096	10.5
0.90	0.44	0.18	1.96	0.18	0.091	11.2
1.00	0.45	0.17	1.95	0.17	0.086	11.8
1.10	0.47	0.16	1.95	0.16	0.082	12.5

**Table 9**  $P_9(s) = \frac{1}{(s + 1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.12	1.19	0.66	1.19	1.82	0.57
0.20	0.20	0.61	0.80	0.61	0.76	1.32
0.30	0.24	0.42	0.87	0.42	0.48	2.08
0.40	0.27	0.34	0.95	0.34	0.35	2.83
0.50	0.28	0.29	1.02	0.29	0.28	3.54
0.60	0.29	0.26	1.10	0.26	0.24	4.22
0.70	0.30	0.24	1.18	0.24	0.21	4.88
0.80	0.30	0.23	1.27	0.23	0.18	5.50
0.90	0.31	0.22	1.35	0.22	0.16	6.10
1.00	0.31	0.22	1.45	0.22	0.15	6.68

## 8.2 $M_s = M_t = 1.4$

**Table 10**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.4$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.02	0.98	0.16	0.34	0.16	0.46	2.16
0.05	0.95	0.16	0.35	0.16	0.45	2.22
0.10	0.91	0.16	0.38	0.16	0.44	2.30
0.20	0.83	0.18	0.43	0.18	0.41	2.43
0.30	0.77	0.19	0.49	0.19	0.40	2.52
0.50	0.67	0.24	0.62	0.24	0.38	2.63
0.70	0.59	0.28	0.76	0.28	0.37	2.67
1.00	0.50	0.36	0.98	0.36	0.37	2.68
1.30	0.43	0.46	1.22	0.46	0.38	2.66
1.50	0.40	0.52	1.38	0.52	0.38	2.65
2.00	0.33	0.68	1.74	0.68	0.39	2.58
4.00	0.20	1.30	2.84	1.30	0.46	2.26
6.00	0.14	1.90	3.59	1.90	0.53	1.96
8.00	0.11	2.51	4.13	2.51	0.61	1.71
10.0	0.091	3.11	4.53	3.11	0.69	1.52
20.0	0.048	6.12	5.61	6.12	1.09	0.96
50.0	0.02	15.1	6.54	15.1	2.31	0.46
100	0.0099	30.1	6.94	30.1	4.34	0.24
200	0.0050	60.1	7.15	60.1	8.41	0.13
500	0.0020	150	7.27	150	20.6	0.051
1000	0.0010	300	7.32	300	41.0	0.026

**Table 11**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.4$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.01	0.98	0.16	0.34	0.16	0.46	2.16
0.02	0.96	0.16	0.35	0.16	0.45	2.20
0.05	0.92	0.16	0.37	0.16	0.43	2.31
0.10	0.85	0.17	0.42	0.17	0.40	2.49
0.20	0.74	0.19	0.52	0.19	0.36	2.77
0.30	0.66	0.21	0.63	0.21	0.33	3.01
0.50	0.55	0.26	0.87	0.26	0.29	3.41
0.70	0.48	0.30	1.13	0.30	0.27	3.74
1.00	0.41	0.37	1.54	0.37	0.24	4.17
1.30	0.36	0.43	1.96	0.43	0.22	4.56
1.50	0.34	0.47	2.24	0.47	0.21	4.81
2.00	0.30	0.55	2.97	0.55	0.19	5.40
4.00	0.22	0.80	6.01	0.80	0.13	7.55
6.00	0.19	0.95	9.14	0.95	0.10	9.57
8.00	0.18	1.07	12.3	1.07	0.087	11.5
10.0	0.17	1.15	15.6	1.15	0.074	13.5
20.0	0.15	1.39	32.1	1.39	0.043	23.1
50.0	0.14	1.60	82.2	1.60	0.019	51.6
100	0.14	1.68	166	1.68	0.01	98.9
200	0.13	1.73	334	1.73	0.0052	193
500	0.13	1.76	838	1.76	0.0021	477



**Table 12**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.4$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.0050	0.0094	35.1	0.071	35.1	493	0.0020
0.01	0.018	17.8	0.13	17.8	133	0.0076
0.02	0.034	9.11	0.24	9.11	37.7	0.027
0.05	0.073	3.84	0.47	3.84	8.20	0.12
0.10	0.12	2.06	0.68	2.06	3.02	0.33
0.20	0.18	1.16	0.89	1.16	1.31	0.77
0.50	0.23	0.71	1.31	0.71	0.54	1.84
2.00	0.23	0.70	3.30	0.70	0.21	4.74
5.00	0.19	0.94	7.78	0.94	0.12	8.28
10.0	0.17	1.18	15.7	1.18	0.075	13.4

**Table 13**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PI,  $M_s = M_t = 1.4$

n	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
3	0.25	0.63	1.95	0.63	0.33	3.07
4	0.33	0.43	2.25	0.43	0.19	5.21
5	0.39	0.35	2.57	0.35	0.14	7.34
6	0.43	0.31	2.90	0.31	0.11	9.47
7	0.47	0.28	3.23	0.28	0.086	11.6
8	0.50	0.26	3.56	0.26	0.073	13.7

**Table 14**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PI,  $M_s = M_t = 1.4$

$\alpha$	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.10	0.067	4.20	0.49	4.20	8.51	0.12
0.20	0.12	1.93	0.75	1.93	2.58	0.39
0.30	0.17	1.19	0.91	1.19	1.31	0.76
0.40	0.22	0.85	1.04	0.85	0.82	1.22
0.50	0.25	0.67	1.17	0.67	0.57	1.75
0.60	0.28	0.56	1.31	0.56	0.43	2.35
0.70	0.30	0.49	1.48	0.49	0.33	3.01
0.80	0.32	0.46	1.69	0.46	0.27	3.71
0.90	0.33	0.44	1.95	0.44	0.23	4.44

**Table 15**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.4$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.99	0.010	0	0.45	7.31	0.45	0.062	16.1
0.98	0.020	0	0.45	7.28	0.45	0.061	16.3
0.95	0.050	0	0.43	7.24	0.43	0.059	16.9
0.90	0.10	0	0.40	7.16	0.40	0.056	17.8
0.80	0.20	0	0.37	7.28	0.37	0.051	19.5
0.70	0.30	0	0.36	7.41	0.36	0.048	21.0
0.50	0.50	0	0.33	7.48	0.33	0.044	23.5
0.30	0.70	0	0.31	7.43	0.31	0.042	25.0
0.10	0.90	0	0.30	7.38	0.30	0.041	25.8
0.00	1.00	0	0.30	7.37	0.30	0.041	25.8

**Table 16**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PI,  $M_s = M_t = 1.4$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
1.00	0.99	0.010	0.14	1.69	1.66	1.69	1.02	0.98
1.00	0.98	0.020	0.14	1.58	1.62	1.58	0.98	1.02
1.00	0.95	0.050	0.15	1.39	1.57	1.39	0.88	1.13
1.00	0.90	0.10	0.17	1.13	1.47	1.13	0.77	1.30
1.00	0.70	0.30	0.25	0.68	1.26	0.68	0.54	1.85
1.00	0.50	0.50	0.33	0.49	1.11	0.49	0.44	2.26
1.00	0.30	0.70	0.41	0.40	1.02	0.40	0.39	2.54
1.00	0.10	0.90	0.48	0.37	0.98	0.37	0.38	2.67
1.00	0.00	1.00	0.50	0.36	0.98	0.36	0.37	2.68
2.00	0.99	0.010	0.12	1.93	2.36	1.93	0.82	1.22
2.00	0.98	0.020	0.13	1.85	2.33	1.85	0.80	1.25
2.00	0.95	0.050	0.13	1.68	2.26	1.68	0.74	1.34
2.00	0.90	0.10	0.15	1.45	2.15	1.45	0.67	1.48
2.00	0.70	0.30	0.20	0.99	1.91	0.99	0.52	1.93
2.00	0.50	0.50	0.24	0.83	1.86	0.83	0.44	2.25
2.00	0.30	0.70	0.29	0.73	1.79	0.73	0.41	2.47
2.00	0.10	0.90	0.32	0.68	1.75	0.68	0.39	2.57
2.00	0.00	1.00	0.33	0.68	1.74	0.68	0.39	2.58
5.00	0.99	0.010	0.089	3.17	3.69	3.17	0.86	1.16
5.00	0.98	0.020	0.090	3.08	3.66	3.08	0.84	1.19
5.00	0.95	0.050	0.093	2.87	3.58	2.87	0.80	1.25
5.00	0.90	0.10	0.098	2.59	3.47	2.59	0.75	1.34
5.00	0.70	0.30	0.12	2.08	3.39	2.08	0.61	1.63
5.00	0.50	0.50	0.14	1.83	3.38	1.83	0.54	1.87
5.00	0.30	0.70	0.15	1.68	3.31	1.68	0.51	2.02
5.00	0.10	0.90	0.16	1.61	3.26	1.61	0.49	2.09
5.00	0.00	1.00	0.17	1.60	3.25	1.60	0.49	2.10
10.0	0.99	0.010	0.059	5.39	4.80	5.39	1.12	0.89
10.0	0.98	0.020	0.059	5.28	4.78	5.28	1.10	0.91
10.0	0.95	0.050	0.060	4.98	4.70	4.98	1.06	0.94
10.0	0.90	0.10	0.063	4.58	4.60	4.58	1.00	1.00
10.0	0.70	0.30	0.071	3.88	4.66	3.88	0.83	1.20
10.0	0.50	0.50	0.079	3.48	4.66	3.48	0.75	1.36
10.0	0.30	0.70	0.085	3.23	4.59	3.23	0.70	1.47
10.0	0.10	0.90	0.090	3.12	4.54	3.12	0.69	1.52
10.0	0.00	1.00	0.091	3.11	4.53	3.11	0.69	1.52

**Table 17**  $P_8(s) = \frac{1 - s\alpha}{(s + 1)^3}$ , Unfiltered PI,  $M_s = M_t = 1.4$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.27	0.57	1.92	0.57	0.29	3.40
0.20	0.29	0.51	1.90	0.51	0.27	3.74
0.30	0.32	0.46	1.89	0.46	0.25	4.08
0.40	0.34	0.43	1.88	0.43	0.23	4.43
0.50	0.36	0.39	1.86	0.39	0.21	4.78
0.60	0.38	0.36	1.85	0.36	0.20	5.15
0.70	0.40	0.34	1.84	0.34	0.18	5.52
0.80	0.42	0.32	1.84	0.32	0.17	5.91
0.90	0.44	0.30	1.82	0.30	0.16	6.30
1.00	0.45	0.28	1.82	0.28	0.15	6.69
1.10	0.47	0.27	1.81	0.27	0.15	7.10

**Table 18**  $P_9(s) = \frac{1}{(s + 1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.4$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.12	2.14	0.56	2.14	3.81	0.27
0.20	0.20	1.12	0.76	1.12	1.47	0.69
0.30	0.24	0.78	0.86	0.78	0.91	1.11
0.40	0.27	0.60	0.91	0.60	0.66	1.52
0.50	0.28	0.50	0.96	0.50	0.52	1.92
0.60	0.29	0.45	1.04	0.45	0.43	2.30
0.70	0.30	0.42	1.12	0.42	0.37	2.67
0.80	0.30	0.40	1.21	0.40	0.33	3.02
0.90	0.31	0.39	1.30	0.39	0.30	3.35
1.00	0.31	0.38	1.39	0.38	0.27	3.68

### 8.3 $M_s = M_t = 1.6$

**Table 19**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.6$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.020	0.98	0.20	0.33	0.20	0.62	1.62
0.050	0.95	0.20	0.34	0.20	0.60	1.66
0.10	0.91	0.21	0.36	0.21	0.58	1.73
0.20	0.83	0.23	0.41	0.23	0.55	1.82
0.30	0.77	0.25	0.47	0.25	0.53	1.89
0.50	0.67	0.33	0.64	0.33	0.51	1.96
0.70	0.59	0.40	0.81	0.40	0.50	1.99
1.00	0.50	0.53	1.05	0.53	0.50	2.00
1.30	0.43	0.65	1.28	0.65	0.51	1.97
1.50	0.40	0.73	1.43	0.73	0.51	1.95
2.00	0.33	0.94	1.76	0.94	0.54	1.88
4.00	0.20	1.76	2.69	1.76	0.65	1.56
6.00	0.14	2.56	3.27	2.56	0.78	1.30
8.00	0.11	3.37	3.67	3.37	0.92	1.12
10.0	0.091	4.17	3.95	4.17	1.06	0.97
20.0	0.048	8.18	4.67	8.18	1.75	0.59
50.0	0.020	20.2	5.26	20.2	3.84	0.27
100	0.0099	40.2	5.48	40.2	7.34	0.14
200	0.0050	80.3	5.60	80.3	14.3	0.072
500	0.0020	200	5.68	200	35.3	0.029
1000	0.0010	401	5.71	401	70.2	0.015

**Table 20**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.6$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.010	0.98	0.20	0.33	0.20	0.62	1.62
0.020	0.96	0.20	0.33	0.20	0.61	1.65
0.050	0.92	0.20	0.35	0.20	0.58	1.74
0.10	0.85	0.21	0.40	0.21	0.54	1.87
0.20	0.74	0.24	0.49	0.24	0.48	2.08
0.30	0.66	0.28	0.62	0.28	0.44	2.25
0.50	0.55	0.36	0.91	0.36	0.39	2.54
0.70	0.48	0.43	1.21	0.43	0.36	2.78
1.00	0.41	0.54	1.66	0.54	0.32	3.08
1.30	0.36	0.64	2.13	0.64	0.30	3.35
1.50	0.34	0.69	2.44	0.69	0.28	3.52
2.00	0.30	0.82	3.21	0.82	0.26	3.91
4.00	0.22	1.19	6.27	1.19	0.19	5.26
6.00	0.19	1.43	9.28	1.43	0.15	6.51
8.00	0.18	1.59	12.2	1.59	0.13	7.72
10.0	0.17	1.70	15.2	1.70	0.11	8.91
20.0	0.15	2.08	30.7	2.08	0.068	14.8
50.0	0.14	2.47	79.1	2.47	0.031	32.0
100	0.14	2.64	160	2.64	0.017	60.6
200	0.13	2.75	323	2.75	0.0085	118
500	0.13	2.82	813	2.82	0.0035	289

**Table 21**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.6$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.0050	0.0094	47.0	0.053	47.0	889	0.0011
0.010	0.018	23.9	0.10	23.9	236	0.0042
0.020	0.034	12.2	0.19	12.2	65.4	0.015
0.050	0.073	5.18	0.39	5.18	13.5	0.074
0.10	0.12	2.85	0.61	2.85	4.70	0.21
0.20	0.18	1.70	0.88	1.70	1.93	0.52
0.50	0.23	1.08	1.40	1.08	0.77	1.30
2.00	0.23	1.06	3.55	1.06	0.30	3.35
5.00	0.19	1.41	7.95	1.41	0.18	5.65
10.0	0.17	1.74	15.3	1.74	0.11	8.79

**Table 22**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PI,  $M_s = M_t = 1.6$

n	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
3.00	0.25	0.97	2.12	0.97	0.46	2.19
4.00	0.33	0.65	2.50	0.65	0.26	3.83
5.00	0.39	0.52	2.83	0.52	0.18	5.45
6.00	0.43	0.45	3.14	0.45	0.14	7.05
7.00	0.47	0.40	3.44	0.40	0.12	8.64
8.00	0.50	0.37	3.74	0.37	0.098	10.2

**Table 23**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PI,  $M_s = M_t = 1.6$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.067	6.24	0.42	6.24	14.7	0.068
0.20	0.12	2.78	0.68	2.78	4.07	0.25
0.30	0.17	1.77	0.90	1.77	1.96	0.51
0.40	0.22	1.28	1.09	1.28	1.18	0.85
0.50	0.25	1.00	1.26	1.00	0.80	1.25
0.60	0.28	0.84	1.43	0.84	0.59	1.70
0.70	0.30	0.75	1.63	0.75	0.46	2.19
0.80	0.32	0.69	1.87	0.69	0.37	2.72
0.90	0.33	0.66	2.16	0.66	0.31	3.27

**Table 24**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.6$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.99	0.010	0.00	0.72	5.68	0.72	0.13	7.87
0.98	0.020	0.00	0.70	5.65	0.70	0.12	8.03
0.95	0.050	0.00	0.66	5.58	0.66	0.12	8.50
0.90	0.10	0.00	0.60	5.49	0.60	0.11	9.20
0.80	0.20	0.00	0.52	5.38	0.52	0.096	10.4
0.70	0.30	0.00	0.48	5.40	0.48	0.088	11.3
0.50	0.50	0.00	0.44	5.70	0.44	0.077	13.0
0.30	0.70	0.00	0.41	5.75	0.41	0.072	14.2
0.10	0.90	0.00	0.40	5.73	0.40	0.070	14.7
0.00	1.00	0.00	0.40	5.73	0.40	0.070	14.8



**Table 25**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PI,  $M_s = M_t = 1.6$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
1.00	0.99	0.010	0.14	2.65	1.60	2.65	1.66	0.60
1.00	0.98	0.020	0.14	2.46	1.56	2.46	1.58	0.63
1.00	0.95	0.050	0.15	2.06	1.49	2.06	1.38	0.72
1.00	0.90	0.10	0.17	1.68	1.44	1.68	1.16	0.86
1.00	0.70	0.30	0.25	1.02	1.34	1.02	0.76	1.32
1.00	0.50	0.50	0.33	0.73	1.21	0.73	0.60	1.66
1.00	0.30	0.70	0.41	0.59	1.11	0.59	0.53	1.88
1.00	0.10	0.90	0.48	0.53	1.06	0.53	0.50	1.98
1.00	0.00	1.00	0.50	0.53	1.05	0.53	0.50	2.00
2.00	0.99	0.010	0.12	3.06	2.25	3.06	1.36	0.74
2.00	0.98	0.020	0.13	2.90	2.21	2.90	1.31	0.76
2.00	0.95	0.050	0.13	2.53	2.13	2.53	1.19	0.84
2.00	0.90	0.10	0.15	2.12	2.02	2.12	1.05	0.95
2.00	0.70	0.30	0.20	1.46	1.94	1.46	0.75	1.33
2.00	0.50	0.50	0.24	1.16	1.85	1.16	0.63	1.60
2.00	0.30	0.70	0.29	1.01	1.79	1.01	0.56	1.78
2.00	0.10	0.90	0.32	0.95	1.76	0.95	0.54	1.87
2.00	0.00	1.00	0.33	0.94	1.76	0.94	0.54	1.88
5.00	0.99	0.010	0.089	5.02	3.34	5.02	1.50	0.66
5.00	0.98	0.020	0.090	4.84	3.31	4.84	1.47	0.68
5.00	0.95	0.050	0.093	4.39	3.22	4.39	1.36	0.73
5.00	0.90	0.10	0.098	3.84	3.10	3.84	1.24	0.81
5.00	0.70	0.30	0.12	2.87	3.01	2.87	0.95	1.05
5.00	0.50	0.50	0.14	2.48	3.04	2.48	0.82	1.23
5.00	0.30	0.70	0.15	2.27	3.05	2.27	0.75	1.35
5.00	0.10	0.90	0.16	2.17	3.02	2.17	0.72	1.41
5.00	0.00	1.00	0.17	2.16	3.01	2.16	0.72	1.42
10.0	0.99	0.010	0.059	8.56	4.15	8.56	2.07	0.48
10.0	0.98	0.020	0.059	8.31	4.12	8.31	2.02	0.50
10.0	0.95	0.050	0.060	7.63	4.03	7.63	1.89	0.53
10.0	0.90	0.10	0.063	6.80	3.92	6.80	1.73	0.58
10.0	0.70	0.30	0.071	5.24	3.82	5.24	1.37	0.73
10.0	0.50	0.50	0.079	4.70	3.95	4.70	1.19	0.84
10.0	0.30	0.70	0.085	4.35	3.98	4.35	1.09	0.93
10.0	0.10	0.90	0.090	4.19	3.96	4.19	1.06	0.97
10.0	0.00	1.00	0.091	4.17	3.95	4.17	1.06	0.97

**Table 26**  $P_8(s) = \frac{1 - s\alpha}{(s + 1)^3}$ , Unfiltered PI,  $M_s = M_t = 1.6$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.27	0.86	2.11	0.86	0.41	2.45
0.20	0.29	0.77	2.09	0.77	0.37	2.72
0.30	0.32	0.70	2.07	0.70	0.34	2.98
0.40	0.34	0.63	2.04	0.63	0.31	3.25
0.50	0.36	0.58	2.02	0.58	0.29	3.53
0.60	0.38	0.53	1.99	0.53	0.27	3.81
0.70	0.40	0.49	1.96	0.49	0.25	4.11
0.80	0.42	0.45	1.93	0.45	0.23	4.40
0.90	0.44	0.42	1.90	0.42	0.22	4.71
1.00	0.45	0.39	1.87	0.39	0.21	5.02
1.10	0.47	0.36	1.84	0.36	0.20	5.34

**Table 27**  $P_9(s) = \frac{1}{(s + 1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.6$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.12	2.89	0.51	2.89	5.67	0.18
0.20	0.20	1.53	0.73	1.53	2.10	0.48
0.30	0.24	1.08	0.85	1.08	1.27	0.79
0.40	0.27	0.87	0.96	0.87	0.91	1.10
0.50	0.28	0.76	1.07	0.76	0.71	1.41
0.60	0.29	0.69	1.17	0.69	0.59	1.70
0.70	0.30	0.65	1.28	0.65	0.51	1.97
0.80	0.30	0.62	1.39	0.62	0.45	2.23
0.90	0.31	0.61	1.50	0.61	0.40	2.48
1.00	0.31	0.59	1.61	0.59	0.37	2.72

#### 8.4 $M_s = M_t = 1.8$

**Table 28**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.8$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.020	0.98	0.30	0.42	0.30	0.72	1.40
0.050	0.95	0.30	0.42	0.30	0.70	1.45
0.10	0.91	0.31	0.46	0.31	0.67	1.50
0.20	0.83	0.34	0.53	0.34	0.63	1.59
0.30	0.77	0.37	0.61	0.37	0.61	1.65
0.50	0.67	0.45	0.77	0.45	0.59	1.71
0.70	0.59	0.54	0.93	0.54	0.58	1.73
1.00	0.50	0.68	1.15	0.68	0.59	1.71
1.30	0.43	0.82	1.36	0.82	0.60	1.67
1.50	0.40	0.91	1.48	0.91	0.62	1.63
2.00	0.33	1.15	1.77	1.15	0.65	1.54
4.00	0.20	2.12	2.57	2.12	0.82	1.22
6.00	0.14	3.09	3.06	3.09	1.01	1.00
8.00	0.11	4.06	3.38	4.06	1.20	0.84
10.0	0.091	5.02	3.61	5.02	1.39	0.72
20.0	0.048	9.84	4.16	9.84	2.37	0.43
50.0	0.020	24.3	4.58	24.3	5.30	0.19
100	0.0099	48.4	4.74	48.4	10.2	0.099
200	0.0050	96.4	4.82	96.4	20.0	0.051
500	0.0020	241	4.88	241	49.4	0.021
1000	0.0010	481	4.89	481	98.4	0.010

**Table 29**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.8$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.01	0.98	0.29	0.41	0.29	0.72	1.40
0.02	0.96	0.30	0.43	0.30	0.70	1.43
0.05	0.92	0.30	0.45	0.30	0.67	1.51
0.10	0.85	0.32	0.51	0.32	0.62	1.63
0.20	0.74	0.36	0.64	0.36	0.56	1.82
0.30	0.66	0.40	0.79	0.40	0.51	1.98
0.50	0.55	0.50	1.10	0.50	0.45	2.23
0.70	0.48	0.59	1.42	0.59	0.42	2.42
1.00	0.41	0.73	1.91	0.73	0.38	2.66
1.30	0.36	0.85	2.40	0.85	0.35	2.86
1.50	0.34	0.92	2.73	0.92	0.34	2.99
2.00	0.30	1.09	3.55	1.09	0.31	3.27
4.00	0.22	1.60	6.74	1.60	0.24	4.23
6.00	0.19	1.94	9.85	1.94	0.20	5.10
8.00	0.18	2.18	12.9	2.18	0.17	5.94
10.0	0.17	2.36	15.9	2.36	0.15	6.76
20.0	0.15	2.86	30.8	2.86	0.093	10.8
50.0	0.14	3.39	76.8	3.39	0.044	22.7
100	0.14	3.70	157	3.70	0.024	42.4
200	0.13	3.87	316	3.87	0.012	81.7
500	0.13	3.99	795	3.99	0.0050	199

**Table 30**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PI,  $M_s = M_t = 1.8$

T	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.0050	0.0094	58.3	0.045	58.3	1292	0.0008
0.010	0.018	29.7	0.087	29.7	341	0.0029
0.020	0.034	15.3	0.16	15.3	93.1	0.011
0.050	0.073	6.58	0.35	6.58	18.6	0.054
0.10	0.12	3.68	0.59	3.68	6.25	0.16
0.20	0.18	2.25	0.91	2.25	2.46	0.41
0.50	0.23	1.46	1.55	1.46	0.95	1.06
2.00	0.23	1.44	3.93	1.44	0.37	2.74
5.00	0.19	1.92	8.52	1.92	0.23	4.43
10.00	0.17	2.42	16.1	2.42	0.15	6.64

**Table 31**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PI,  $M_s = M_t = 1.8$

n	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
3.00	0.25	1.32	2.38	1.32	0.56	1.81
4.00	0.33	0.89	2.90	0.89	0.31	3.30
5.00	0.39	0.71	3.35	0.71	0.21	4.76
6.00	0.43	0.62	3.78	0.62	0.16	6.21
7.00	0.47	0.56	4.20	0.56	0.13	7.64
8.00	0.50	0.52	4.62	0.52	0.11	9.06

**Table 32**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PI,  $M_s = M_t = 1.8$

$\alpha$	$\tau$	K	$T_i$	$k_p$	$k_i$	IAE
0.10	0.067	8.21	0.38	8.21	21.5	0.047
0.20	0.12	3.72	0.67	3.72	5.52	0.18
0.30	0.17	2.34	0.93	2.34	2.52	0.40
0.40	0.22	1.69	1.16	1.69	1.47	0.68
0.50	0.25	1.34	1.37	1.34	0.97	1.03
0.60	0.28	1.13	1.60	1.13	0.71	1.43
0.70	0.30	1.01	1.86	1.01	0.54	1.86
0.80	0.32	0.94	2.15	0.94	0.44	2.33
0.90	0.33	0.90	2.49	0.90	0.36	2.81

**Table 33**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.8$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.99	0.01	0.00	1.01	4.86	1.01	0.21	4.80
0.98	0.02	0.00	0.98	4.82	0.98	0.20	4.94
0.95	0.05	0.00	0.89	4.76	0.89	0.19	5.34
0.90	0.10	0.00	0.79	4.66	0.79	0.17	5.93
0.80	0.20	0.00	0.66	4.54	0.66	0.14	6.92
0.70	0.30	0.00	0.59	4.55	0.59	0.13	7.71
0.50	0.50	0.00	0.53	4.73	0.53	0.11	8.96
0.30	0.70	0.00	0.50	4.88	0.50	0.10	9.84
0.10	0.90	0.00	0.48	4.91	0.48	0.098	10.3
0.00	1.00	0.00	0.48	4.91	0.48	0.098	10.4

**Table 34**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PI,  $M_s = M_t = 1.8$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
1.00	0.99	0.010	0.14	3.67	1.55	3.67	2.37	0.42
1.00	0.98	0.020	0.14	3.38	1.52	3.38	2.22	0.45
1.00	0.95	0.050	0.15	2.84	1.50	2.84	1.89	0.53
1.00	0.90	0.10	0.17	2.32	1.52	2.32	1.53	0.65
1.00	0.70	0.30	0.25	1.35	1.45	1.35	0.93	1.08
1.00	0.50	0.50	0.33	0.96	1.33	0.96	0.72	1.40
1.00	0.30	0.70	0.41	0.77	1.23	0.77	0.63	1.60
1.00	0.10	0.90	0.48	0.69	1.16	0.69	0.59	1.70
1.00	0.00	1.00	0.50	0.68	1.15	0.68	0.59	1.71
2.00	0.99	0.010	0.12	4.30	2.18	4.30	1.97	0.51
2.00	0.98	0.020	0.13	4.01	2.14	4.01	1.88	0.53
2.00	0.95	0.050	0.13	3.41	2.05	3.41	1.67	0.60
2.00	0.90	0.10	0.15	2.93	2.06	2.93	1.42	0.70
2.00	0.70	0.30	0.20	1.92	2.01	1.92	0.95	1.05
2.00	0.50	0.50	0.24	1.47	1.91	1.47	0.77	1.30
2.00	0.30	0.70	0.29	1.25	1.82	1.25	0.69	1.46
2.00	0.10	0.90	0.32	1.16	1.77	1.16	0.66	1.53
2.00	0.00	1.00	0.33	1.15	1.77	1.15	0.65	1.54
5.00	0.99	0.010	0.089	7.04	3.11	7.04	2.26	0.44
5.00	0.98	0.020	0.090	6.72	3.08	6.72	2.18	0.46
5.00	0.95	0.050	0.093	5.96	3.00	5.96	1.99	0.50
5.00	0.90	0.10	0.098	5.01	2.86	5.01	1.75	0.57
5.00	0.70	0.30	0.12	3.68	2.90	3.68	1.27	0.79
5.00	0.50	0.50	0.14	3.05	2.87	3.05	1.06	0.94
5.00	0.30	0.70	0.15	2.74	2.85	2.74	0.96	1.04
5.00	0.10	0.90	0.16	2.62	2.84	2.62	0.92	1.09
5.00	0.00	1.00	0.17	2.60	2.84	2.60	0.92	1.10
10.0	0.99	0.010	0.059	12.0	3.76	12.0	3.20	0.31
10.0	0.98	0.020	0.059	11.5	3.72	11.5	3.10	0.32
10.0	0.95	0.050	0.060	10.3	3.63	10.3	2.84	0.35
10.0	0.90	0.10	0.063	8.91	3.51	8.91	2.53	0.39
10.0	0.70	0.30	0.071	6.63	3.50	6.63	1.89	0.53
10.0	0.50	0.50	0.079	5.69	3.54	5.69	1.61	0.62
10.0	0.30	0.70	0.085	5.23	3.58	5.23	1.46	0.69
10.0	0.10	0.90	0.090	5.04	3.61	5.04	1.40	0.72
10.0	0.00	1.00	0.091	5.02	3.61	5.02	1.39	0.72

**Table 35**  $P_8(s) = \frac{1 - s\alpha}{(s + 1)^3}$ , Unfiltered PI,  $M_s = M_t = 1.8$

$\alpha$	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.27	1.17	2.38	1.17	0.49	2.05
0.20	0.29	1.04	2.37	1.04	0.44	2.29
0.30	0.32	0.94	2.36	0.94	0.40	2.54
0.40	0.34	0.85	2.34	0.85	0.37	2.78
0.50	0.36	0.78	2.32	0.78	0.34	3.03
0.60	0.38	0.72	2.30	0.72	0.31	3.29
0.70	0.40	0.66	2.27	0.66	0.29	3.55
0.80	0.42	0.62	2.25	0.62	0.27	3.81
0.90	0.44	0.57	2.22	0.57	0.26	4.08
1.00	0.45	0.54	2.20	0.54	0.24	4.35
1.10	0.47	0.50	2.17	0.50	0.23	4.63

**Table 36**  $P_9(s) = \frac{1}{(s + 1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PI,  $M_s = M_t = 1.8$

T	$\tau$	$K$	$T_i$	$k_p$	$k_i$	IAE
0.10	0.12	3.47	0.47	3.47	7.40	0.14
0.20	0.20	1.90	0.72	1.90	2.63	0.38
0.30	0.24	1.40	0.91	1.40	1.54	0.66
0.40	0.27	1.17	1.08	1.17	1.08	0.94
0.50	0.28	1.04	1.23	1.04	0.84	1.20
0.60	0.29	0.96	1.38	0.96	0.70	1.46
0.70	0.30	0.91	1.52	0.91	0.60	1.71
0.80	0.30	0.87	1.66	0.87	0.53	1.94
0.90	0.31	0.85	1.81	0.85	0.47	2.16
1.00	0.31	0.84	1.95	0.84	0.43	2.37



## 9. Optimal PID parameters for the test batch

The following tables hold process data, optimal unfiltered SWORD PID parameters, and corresponding IAE-values, for the benchmark models. Section 9.1 presents tables for a closed-loop robustness given by  $M_s = M_t = 1.2$ , Section 9.2 for  $M_s = M_t = 1.4$ , Section 9.3 for  $M_s = M_t = 1.6$ , and Section 9.4 for  $M_s = M_t = 1.8$ . The tables have also been collected in MATLAB<sup>®</sup> mat-files, Excel spreadsheets (different sheets for different process types), as well as in a tex-file. These can be downloaded from:

<http://www.control.lth.se/Research/ProcessControl/PIDControl/>

### 9.1 $M_s = M_t = 1.2$

**Table 37**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.020	0.98	0.10	0.38	0.032	0.10	0.27	0.0033	3.65
0.050	0.95	0.11	0.40	0.075	0.11	0.28	0.0083	3.58
0.10	0.91	0.13	0.44	0.13	0.13	0.29	0.017	3.48
0.20	0.83	0.16	0.52	0.21	0.16	0.30	0.033	3.31
0.30	0.77	0.19	0.60	0.26	0.19	0.31	0.050	3.18
0.50	0.67	0.25	0.75	0.33	0.25	0.33	0.083	2.99
0.70	0.59	0.30	0.86	0.39	0.30	0.35	0.12	2.85
1.00	0.50	0.37	1.01	0.44	0.37	0.36	0.16	2.80
1.30	0.43	0.45	1.20	0.44	0.45	0.37	0.20	2.78
1.50	0.40	0.50	1.32	0.44	0.50	0.38	0.22	2.77
2.00	0.33	0.63	1.60	0.44	0.63	0.40	0.28	2.72
4.00	0.20	1.17	2.48	0.44	1.17	0.47	0.52	2.42
6.00	0.14	1.70	3.10	0.44	1.70	0.55	0.76	2.13
8.00	0.11	2.24	3.56	0.44	2.24	0.63	0.99	1.88
10.0	0.091	2.80	4.11	0.41	2.80	0.68	1.14	1.70
20.0	0.048	5.50	6.39	0.40	5.50	0.86	2.19	1.24
50.0	0.020	13.6	9.47	0.40	13.6	1.44	5.38	0.71
100	0.0099	27.1	11.3	0.40	27.1	2.41	10.7	0.42
200	0.0050	54.1	12.4	0.40	54.1	4.35	21.4	0.23
500	0.0020	135	13.2	0.40	135	10.2	53.5	0.098
1000	0.0010	270	13.5	0.40	270	19.9	107	0.050

**Table 38**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.2$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.010	0.98	0.10	0.37	0.031	0.10	0.27	0.0032	3.66
0.020	0.96	0.11	0.39	0.060	0.11	0.28	0.0064	3.62
0.050	0.92	0.12	0.43	0.13	0.12	0.29	0.016	3.51
0.10	0.85	0.15	0.50	0.22	0.15	0.30	0.033	3.39
0.20	0.74	0.20	0.61	0.39	0.20	0.32	0.077	3.12
0.30	0.66	0.22	0.69	0.50	0.22	0.31	0.11	3.22
0.50	0.55	0.28	0.93	0.62	0.28	0.30	0.17	3.36
0.70	0.48	0.35	1.18	0.70	0.35	0.29	0.24	3.47
1.00	0.41	0.45	1.56	0.83	0.45	0.29	0.38	3.61
1.30	0.36	0.56	1.92	0.96	0.56	0.29	0.54	3.71
1.50	0.34	0.64	2.15	1.04	0.64	0.30	0.66	3.76
2.00	0.30	0.83	2.72	1.25	0.83	0.30	1.03	3.83
4.00	0.22	1.66	4.67	2.02	1.66	0.36	3.35	3.67
6.00	0.19	2.60	6.66	2.66	2.60	0.39	6.93	3.30
8.00	0.18	3.66	8.95	3.23	3.66	0.41	11.8	2.93
10.0	0.17	4.83	11.2	3.75	4.83	0.43	18.1	2.62
20.0	0.15	11.8	20.3	6.28	11.8	0.58	74.2	1.74
50.0	0.14	40.5	32.9	11.9	40.5	1.23	480	0.83
100	0.14	117	41.4	16.6	117	2.82	1940	0.37
200	0.13	380	48.0	20.5	380	7.91	7770	0.13
500	0.13	1980	49.9	24.7	1980	39.5	48800	0.027

**Table 39**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.2$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.0050	0.0094	88.9	0.045	0.0095	88.9	1980	0.84	0.0005
0.010	0.018	46.2	0.082	0.019	46.2	561	0.87	0.0019
0.020	0.034	23.9	0.15	0.037	23.9	163	0.88	0.0067
0.050	0.073	10.2	0.28	0.090	10.2	36.3	0.91	0.032
0.10	0.12	5.56	0.42	0.17	5.56	13.3	0.95	0.096
0.20	0.18	3.27	0.57	0.32	3.27	5.68	1.06	0.25
0.50	0.23	2.00	0.87	0.70	2.00	2.30	1.40	0.70
2.00	0.23	2.01	2.19	1.78	2.01	0.92	3.59	1.75
5.00	0.19	3.53	4.94	3.05	3.53	0.72	10.8	2.01
10.0	0.17	6.95	9.16	4.57	6.95	0.76	31.8	1.67

**Table 40**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PID,  $M_s = M_t = 1.2$

n	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
3.00	0.25	1.76	1.32	1.15	1.76	1.34	2.03	1.22
4.00	0.33	0.73	1.91	1.47	0.73	0.38	1.07	3.43
5.00	0.39	0.50	2.44	1.80	0.50	0.20	0.90	5.56
6.00	0.43	0.40	2.89	2.15	0.40	0.14	0.86	7.73
7.00	0.47	0.34	3.30	2.51	0.34	0.10	0.86	9.96
8.00	0.50	0.31	3.70	2.86	0.31	0.083	0.88	12.3

**Table 41**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PID,  $M_s, M_t = 1.2$

$\alpha$	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.067	35.0	0.20	0.068	35.0	174	2.39	0.0062
0.20	0.12	7.26	0.47	0.16	7.26	15.6	1.19	0.075
0.30	0.17	3.15	0.63	0.27	3.15	5.02	0.87	0.26
0.40	0.22	1.85	0.73	0.40	1.85	2.53	0.74	0.57
0.50	0.25	1.29	0.85	0.54	1.29	1.52	0.70	0.97
0.60	0.28	1.01	1.02	0.70	1.01	0.99	0.71	1.42
0.70	0.30	0.86	1.21	0.87	0.86	0.71	0.75	1.90
0.80	0.32	0.78	1.41	1.05	0.78	0.55	0.82	2.40
0.90	0.33	0.74	1.65	1.25	0.74	0.45	0.93	2.91

**Table 42**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.2$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.99	0.010	0.00	93.8	0.51	0.20	93.8	184	19.0	0.0056
0.98	0.020	0.00	28.0	0.84	0.34	28.0	33.2	9.47	0.032
0.95	0.050	0.00	7.03	1.72	0.52	7.03	4.09	3.65	0.25
0.90	0.10	0.00	2.73	2.80	0.63	2.73	0.98	1.71	1.02
0.80	0.20	0.00	1.14	4.67	0.67	1.14	0.24	0.76	4.10
0.70	0.30	0.00	0.70	6.44	0.66	0.70	0.11	0.46	9.15
0.50	0.50	0.00	0.41	9.69	0.57	0.41	0.042	0.24	23.6
0.30	0.70	0.00	0.31	12.2	0.47	0.31	0.025	0.15	39.4
0.10	0.90	0.00	0.27	13.7	0.41	0.27	0.020	0.11	49.9
0.00	1.00	0.00	0.27	13.9	0.40	0.27	0.019	0.11	51.3

**Table 43**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PID,  $M_s = M_t = 1.2$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
1.00	0.99	0.010	0.14	120	0.47	0.16	120	254	18.9	0.0040
1.00	0.98	0.020	0.14	39.3	0.65	0.24	39.3	60.2	9.39	0.017
1.00	0.95	0.050	0.15	11.5	1.01	0.31	11.5	11.4	3.50	0.089
1.00	0.90	0.10	0.17	4.52	1.06	0.36	4.52	4.25	1.64	0.27
1.00	0.70	0.30	0.25	1.16	1.07	0.46	1.16	1.08	0.54	1.14
1.00	0.50	0.50	0.33	0.65	1.07	0.48	0.65	0.60	0.31	1.85
1.00	0.30	0.70	0.41	0.45	1.04	0.48	0.45	0.44	0.22	2.39
1.00	0.10	0.90	0.48	0.38	1.01	0.45	0.38	0.37	0.17	2.74
1.00	0.00	1.00	0.50	0.37	1.01	0.44	0.37	0.36	0.16	2.80
2.00	0.99	0.010	0.12	211	0.47	0.18	211	451	37.9	0.0023
2.00	0.98	0.020	0.13	66.7	0.71	0.28	66.7	93.6	18.9	0.011
2.00	0.95	0.050	0.13	18.7	1.23	0.38	18.7	15.2	7.15	0.066
2.00	0.90	0.10	0.15	7.44	1.49	0.44	7.44	5.01	3.24	0.21
2.00	0.70	0.30	0.20	1.85	1.45	0.54	1.85	1.28	1.01	1.01
2.00	0.50	0.50	0.24	1.05	1.55	0.54	1.05	0.67	0.56	1.76
2.00	0.30	0.70	0.29	0.76	1.58	0.50	0.76	0.48	0.37	2.33
2.00	0.10	0.90	0.32	0.65	1.60	0.45	0.65	0.40	0.29	2.67
2.00	0.00	1.00	0.33	0.63	1.60	0.44	0.63	0.40	0.28	2.72
5.00	0.99	0.010	0.089	467	0.45	0.20	467	1050	95.5	0.0010
5.00	0.98	0.020	0.090	158	0.88	0.30	158	180	47.0	0.0056
5.00	0.95	0.050	0.093	39.8	1.45	0.46	39.8	27.4	18.1	0.037
5.00	0.90	0.10	0.098	15.9	2.01	0.53	15.9	7.89	8.36	0.13
5.00	0.70	0.30	0.12	3.97	2.48	0.59	3.97	1.60	2.34	0.72
5.00	0.50	0.50	0.14	2.28	2.47	0.57	2.28	0.92	1.29	1.34
5.00	0.30	0.70	0.15	1.68	2.67	0.51	1.68	0.63	0.86	1.90
5.00	0.10	0.90	0.16	1.46	2.80	0.45	1.46	0.52	0.66	2.23
5.00	0.00	1.00	0.17	1.44	2.82	0.44	1.44	0.51	0.64	2.27
10.0	0.99	0.010	0.059	931	0.47	0.20	931	1990	191	0.0005
10.0	0.98	0.020	0.059	276	0.80	0.35	276	347	95.2	0.0031
10.0	0.95	0.050	0.060	75.0	1.57	0.48	75.0	47.7	36.4	0.021
10.0	0.90	0.10	0.063	29.6	2.32	0.57	29.6	12.8	16.9	0.079
10.0	0.70	0.30	0.071	7.50	3.52	0.61	7.50	2.13	4.60	0.50
10.0	0.50	0.50	0.079	4.34	3.92	0.55	4.34	1.11	2.41	1.00
10.0	0.30	0.70	0.085	3.25	4.06	0.47	3.25	0.80	1.54	1.43
10.0	0.10	0.90	0.090	2.84	4.11	0.42	2.84	0.69	1.18	1.67
10.0	0.00	1.00	0.091	2.80	4.11	0.41	2.80	0.68	1.14	1.70

**Table 44**  $P_8(s) = \frac{1-s\alpha}{(s+1)^3}$ , Unfiltered PID,  $M_s = M_t = 1.2$

$\alpha$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.27	1.21	1.43	1.08	1.21	0.85	1.31	1.77
0.20	0.29	0.93	1.55	1.06	0.93	0.60	0.98	2.25
0.30	0.32	0.76	1.66	1.05	0.76	0.46	0.80	2.72
0.40	0.34	0.64	1.75	1.05	0.64	0.37	0.67	3.19
0.50	0.36	0.56	1.81	1.05	0.56	0.31	0.58	3.66
0.60	0.38	0.49	1.86	1.06	0.49	0.26	0.52	4.14
0.70	0.40	0.44	1.90	1.07	0.44	0.23	0.47	4.64
0.80	0.42	0.39	1.92	1.08	0.39	0.20	0.43	5.17
0.90	0.44	0.36	1.94	1.10	0.36	0.18	0.39	5.72
1.00	0.45	0.33	1.96	1.11	0.33	0.17	0.36	6.30
1.10	0.47	0.30	1.97	1.13	0.30	0.15	0.34	6.91

**Table 45**  $P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.2$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.12	3.22	0.39	0.15	3.22	8.18	0.49	0.16
0.20	0.20	1.86	0.49	0.31	1.86	3.82	0.57	0.39
0.30	0.24	1.43	0.55	0.46	1.43	2.60	0.66	0.61
0.40	0.27	1.22	0.60	0.62	1.22	2.03	0.75	0.82
0.50	0.28	1.12	0.70	0.75	1.12	1.60	0.84	1.02
0.60	0.29	1.05	0.79	0.87	1.05	1.34	0.92	1.19
0.70	0.30	1.02	0.88	0.99	1.02	1.16	1.01	1.35
0.80	0.30	1.00	0.97	1.10	1.00	1.03	1.11	1.50
0.90	0.31	1.00	1.07	1.21	1.00	0.94	1.20	1.63
1.00	0.31	1.00	1.16	1.31	1.00	0.86	1.31	1.75

## 9.2 $M_s = M_t = 1.4$

**Table 46**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.4$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.02	0.98	0.17	0.35	0.034	0.17	0.48	0.0057	2.08
0.05	0.95	0.18	0.38	0.078	0.18	0.49	0.014	2.04
0.1	0.91	0.21	0.40	0.14	0.21	0.51	0.029	1.97
0.2	0.83	0.26	0.49	0.22	0.26	0.54	0.057	1.86
0.3	0.77	0.33	0.58	0.26	0.33	0.56	0.086	1.80
0.5	0.67	0.43	0.71	0.33	0.43	0.61	0.14	1.74
0.7	0.59	0.52	0.80	0.38	0.52	0.66	0.20	1.74
1.0	0.50	0.66	0.96	0.40	0.66	0.68	0.26	1.73
1.3	0.43	0.79	1.11	0.41	0.79	0.71	0.32	1.70
1.5	0.40	0.88	1.21	0.41	0.88	0.73	0.36	1.67
2.0	0.33	1.12	1.43	0.41	1.12	0.78	0.46	1.58
4.0	0.20	2.05	1.99	0.43	2.05	1.03	0.88	1.25
6.0	0.14	2.99	2.34	0.43	2.99	1.28	1.29	1.02
8.0	0.11	3.93	2.57	0.43	3.93	1.53	1.71	0.85
10	0.091	4.88	2.84	0.42	4.88	1.72	2.04	0.74
20	0.048	9.61	3.61	0.41	9.61	2.66	3.94	0.45
50	0.020	23.8	4.32	0.41	23.8	5.50	9.67	0.21
100	0.010	47.4	4.63	0.41	47.4	10.2	19.2	0.11
200	0.0050	94.6	4.80	0.41	94.6	19.7	38.4	0.056
500	0.0020	236	4.91	0.41	236	48.2	95.7	0.023
1000	0.0012	472	4.94	0.40	472	95.6	191	0.011

**Table 47**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.4$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.01	0.98	0.16	0.34	0.027	0.16	0.48	0.0043	2.10
0.020	0.96	0.17	0.36	0.064	0.17	0.49	0.011	2.06
0.050	0.92	0.20	0.40	0.14	0.20	0.50	0.028	1.99
0.10	0.85	0.25	0.47	0.23	0.25	0.53	0.057	1.90
0.20	0.74	0.35	0.59	0.37	0.35	0.58	0.13	1.83
0.30	0.66	0.39	0.70	0.43	0.39	0.56	0.17	1.95
0.50	0.55	0.50	0.92	0.53	0.50	0.55	0.27	2.12
0.70	0.48	0.62	1.15	0.61	0.62	0.54	0.38	2.23
1.00	0.41	0.81	1.51	0.74	0.81	0.53	0.60	2.32
1.30	0.36	1.00	1.88	0.86	1.00	0.53	0.86	2.35
1.50	0.34	1.14	2.12	0.94	1.14	0.54	1.07	2.35
2.00	0.30	1.49	2.68	1.13	1.49	0.56	1.68	2.29
4.00	0.22	3.12	4.48	1.80	3.12	0.70	5.61	1.86
6.00	0.19	5.08	5.77	2.34	5.08	0.88	11.9	1.47
8.00	0.18	7.38	7.02	2.77	7.38	1.05	20.4	1.18
10.0	0.17	10.0	8.34	3.14	10.0	1.20	31.4	0.97
20.0	0.15	27.5	12.3	4.62	27.5	2.25	127	0.48
50.0	0.14	120	15.8	6.74	120	7.60	812	0.14
100	0.14	411	17.5	7.95	411	23.5	3270	0.046
200	0.13	1500	18.4	8.74	1500	81.4	13100	0.013
500	0.13	8830	19.0	9.30	8830	465	82100	0.0023



**Table 48**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.4$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.005	0.0094	196	0.016	0.0089	196	12600	1.74	0.0001
0.010	0.018	103	0.029	0.018	103	3490	1.82	0.0004
0.020	0.034	53.5	0.056	0.035	53.5	963	1.89	0.0016
0.050	0.073	22.8	0.12	0.087	22.8	184	1.98	0.0090
0.10	0.12	12.4	0.22	0.17	12.4	57.5	2.09	0.030
0.20	0.18	7.24	0.36	0.32	7.24	20.2	2.31	0.090
0.50	0.23	4.38	0.71	0.68	4.38	6.17	2.98	0.29
2.00	0.23	4.43	1.80	1.72	4.43	2.46	7.62	0.72
5.00	0.19	8.21	3.11	2.85	8.21	2.64	23.4	0.68
10.0	0.17	17.4	4.74	3.96	17.4	3.68	69.0	0.45

**Table 49**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PID,  $M_s = M_t = 1.4$

n	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
3	0.25	3.84	1.14	1.12	3.84	3.38	4.29	0.52
4	0.33	1.33	2.11	1.34	1.33	0.63	1.79	2.14
5	0.39	0.89	2.61	1.59	0.89	0.34	1.42	3.71
6	0.43	0.71	3.00	1.86	0.71	0.24	1.32	5.22
7	0.47	0.61	3.36	2.14	0.61	0.18	1.31	6.69
8	0.50	0.55	3.71	2.43	0.55	0.15	1.34	8.14

**Table 50**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PID,  $M_s, M_t = 1.4$

$\alpha$	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.067	89.3	0.084	0.052	89.3	1070	4.67	0.0013
0.20	0.12	15.9	0.26	0.14	15.9	60.5	2.31	0.024
0.30	0.17	6.41	0.44	0.26	6.41	14.4	1.65	0.11
0.40	0.22	3.61	0.66	0.38	3.61	5.43	1.36	0.27
0.50	0.25	2.45	0.88	0.50	2.45	2.79	1.24	0.51
0.60	0.28	1.89	1.09	0.65	1.89	1.73	1.22	0.81
0.70	0.30	1.59	1.31	0.80	1.59	1.21	1.27	1.13
0.80	0.32	1.43	1.55	0.97	1.43	0.92	1.38	1.47
0.90	0.33	1.35	1.81	1.15	1.35	0.75	1.55	1.81

**Table 51**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.4$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.99	0.010	0	314	0.15	0.11	314	2150	33.3	0.0006
0.98	0.020	0	97.2	0.33	0.17	97.2	294	16.1	0.0039
0.95	0.050	0	18.1	0.64	0.35	18.1	28.1	6.40	0.0450
0.90	0.10	0	6.48	1.32	0.44	6.48	4.92	2.88	0.21
0.80	0.20	0	2.32	2.06	0.56	2.32	1.13	1.29	0.96
0.70	0.30	0	1.35	2.64	0.59	1.35	0.51	0.80	2.16
0.50	0.50	0	0.75	3.66	0.55	0.75	0.20	0.41	5.46
0.30	0.70	0	0.55	4.46	0.47	0.55	0.12	0.26	8.96
0.10	0.90	0	0.48	4.92	0.41	0.48	0.0970	0.20	11.3
0.00	1.00	0	0.47	4.98	0.41	0.47	0.0950	0.19	11.6

**Table 52**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PID,  $M_s = M_t = 1.4$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
1.00	0.99	0.010	0.14	378	0.17	0.086	378	2280	32.6	0.0005
1.00	0.98	0.020	0.14	115	0.32	0.14	115	360	16.0	0.0031
1.00	0.95	0.050	0.15	25.6	0.56	0.24	25.6	45.5	6.20	0.025
1.00	0.90	0.10	0.17	9.30	0.80	0.31	9.30	11.7	2.85	0.10
1.00	0.70	0.30	0.25	2.12	1.04	0.42	2.12	2.05	0.89	0.63
1.00	0.50	0.50	0.33	1.15	1.02	0.44	1.15	1.12	0.50	1.14
1.00	0.30	0.70	0.41	0.80	0.97	0.43	0.80	0.83	0.34	1.50
1.00	0.10	0.90	0.48	0.67	0.96	0.40	0.67	0.70	0.27	1.70
1.00	0.00	1.00	0.50	0.66	0.96	0.40	0.66	0.68	0.26	1.73
2.00	0.99	0.010	0.12	773	0.19	0.082	773	4100	63.8	0.0003
2.00	0.98	0.020	0.13	219	0.40	0.14	219	550	31.6	0.0018
2.00	0.95	0.050	0.13	46.4	0.68	0.26	46.4	68.3	12.2	0.016
2.00	0.90	0.10	0.15	15.9	1.00	0.35	15.9	15.9	5.63	0.069
2.00	0.70	0.30	0.20	3.45	1.30	0.49	3.45	2.65	1.69	0.50
2.00	0.50	0.50	0.24	1.88	1.36	0.50	1.88	1.38	0.93	0.98
2.00	0.30	0.70	0.29	1.34	1.37	0.46	1.34	0.97	0.62	1.35
2.00	0.10	0.90	0.32	1.14	1.41	0.42	1.14	0.81	0.48	1.55
2.00	0.00	1.00	0.33	1.12	1.43	0.41	1.12	0.78	0.46	1.58
5.00	0.99	0.010	0.089	1650	0.17	0.10	1650	9630	165	0.0001
5.00	0.98	0.020	0.090	538	0.43	0.15	538	1250	78.2	0.0008
5.00	0.95	0.050	0.093	108	0.80	0.28	108	135	30.2	0.0075
5.00	0.90	0.10	0.098	35.5	1.17	0.40	35.5	30.4	14.2	0.035
5.00	0.70	0.30	0.12	7.48	1.73	0.55	7.48	4.33	4.08	0.30
5.00	0.50	0.50	0.14	4.10	1.91	0.54	4.10	2.15	2.20	0.63
5.00	0.30	0.70	0.15	2.98	2.04	0.49	2.98	1.46	1.45	0.93
5.00	0.10	0.90	0.16	2.57	2.16	0.44	2.57	1.19	1.13	1.10
5.00	0.00	1.00	0.17	2.52	2.18	0.43	2.52	1.16	1.08	1.12
10.0	0.99	0.010	0.059	3640	0.19	0.087	3641	19300	318	0.0001
10.0	0.98	0.020	0.059	1040	0.38	0.15	1040	2740	158	0.0004
10.0	0.95	0.050	0.060	207	0.79	0.29	207	263	61.0	0.0039
10.0	0.90	0.10	0.060	68.0	1.24	0.42	68.0	54.8	28.6	0.019
10.0	0.70	0.30	0.071	14.2	2.08	0.56	14.2	6.84	8.03	0.17
10.0	0.50	0.50	0.079	7.82	2.49	0.54	7.82	3.15	4.25	0.40
10.0	0.30	0.70	0.085	5.72	2.72	0.48	5.72	2.11	2.74	0.60
10.0	0.10	0.90	0.090	4.97	2.82	0.43	4.97	1.76	2.11	0.72
10.0	0.00	1.00	0.091	4.88	2.84	0.42	4.88	1.72	2.04	0.74

**Table 53**  $P_8(s) = \frac{1-s\alpha}{(s+1)^3}$ , Unfiltered PID,  $M_s = M_t = 1.4$

$\alpha$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.27	2.36	1.50	1.01	2.36	1.58	2.38	0.93
0.20	0.29	1.74	1.67	0.97	1.74	1.04	1.69	1.31
0.30	0.32	1.39	1.76	0.95	1.39	0.79	1.32	1.67
0.40	0.34	1.16	1.81	0.94	1.16	0.64	1.09	2.02
0.50	0.36	1.00	1.85	0.93	1.00	0.54	0.94	2.36
0.60	0.38	0.88	1.86	0.94	0.88	0.47	0.82	2.69
0.70	0.40	0.78	1.87	0.94	0.78	0.42	0.74	3.02
0.80	0.42	0.71	1.86	0.95	0.71	0.38	0.67	3.35
0.90	0.44	0.65	1.90	0.95	0.65	0.34	0.61	3.68
1.00	0.45	0.59	1.94	0.95	0.59	0.31	0.56	4.02
1.10	0.47	0.55	1.97	0.96	0.55	0.28	0.52	4.36

**Table 54**  $P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.4$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.12	6.53	0.22	0.16	6.53	29.4	1.06	0.06
0.20	0.20	3.79	0.35	0.33	3.79	10.8	1.23	0.17
0.30	0.24	2.92	0.48	0.47	2.92	6.13	1.39	0.29
0.40	0.27	2.52	0.59	0.61	2.52	4.29	1.55	0.41
0.50	0.28	2.31	0.69	0.74	2.31	3.35	1.72	0.52
0.60	0.29	2.20	0.79	0.86	2.20	2.79	1.90	0.61
0.70	0.30	2.13	0.88	0.98	2.13	2.43	2.09	0.70
0.80	0.30	2.11	0.97	1.09	2.11	2.18	2.29	0.77
0.90	0.31	2.10	1.05	1.19	2.10	1.99	2.50	0.84
1.00	0.31	2.12	1.14	1.28	2.12	1.86	2.72	0.90

### 9.3 $M_s = M_t = 1.6$

**Table 55**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.6$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.020	0.98	0.22	0.35	0.034	0.22	0.64	0.0075	1.57
0.050	0.95	0.25	0.38	0.075	0.25	0.65	0.019	1.53
0.10	0.91	0.30	0.44	0.13	0.30	0.67	0.037	1.49
0.20	0.83	0.38	0.55	0.20	0.38	0.70	0.075	1.45
0.30	0.77	0.45	0.63	0.25	0.45	0.72	0.11	1.44
0.50	0.67	0.57	0.73	0.33	0.57	0.78	0.19	1.44
0.70	0.59	0.69	0.86	0.35	0.69	0.80	0.24	1.44
1.00	0.50	0.87	1.03	0.36	0.87	0.84	0.31	1.41
1.30	0.43	1.05	1.17	0.37	1.05	0.89	0.39	1.35
1.50	0.40	1.17	1.27	0.38	1.17	0.92	0.44	1.31
2.00	0.33	1.48	1.42	0.39	1.48	1.04	0.58	1.21
4.00	0.20	2.73	1.82	0.41	2.73	1.50	1.13	0.90
6.00	0.14	3.98	2.02	0.42	3.98	1.97	1.68	0.70
8.00	0.11	5.24	2.15	0.42	5.24	2.44	2.22	0.58
10.0	0.091	6.49	2.25	0.43	6.49	2.89	2.78	0.49
20.0	0.048	12.8	2.62	0.42	12.8	4.87	5.39	0.28
50.0	0.020	31.6	2.94	0.42	31.6	10.8	13.3	0.12
100	0.0099	63.1	3.07	0.42	63.1	20.6	26.3	0.062
200	0.0050	126	3.13	0.42	126	40.2	52.7	0.031
500	0.0020	315	3.17	0.42	315	99.2	131	0.013
1000	0.0010	629	3.18	0.42	629	197	263	0.0064

**Table 56**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.6$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.010	0.98	0.21	0.34	0.027	0.21	0.63	0.0057	1.58
0.020	0.96	0.24	0.37	0.06	0.24	0.65	0.014	1.55
0.050	0.92	0.29	0.44	0.13	0.29	0.67	0.037	1.51
0.10	0.85	0.36	0.53	0.20	0.36	0.68	0.072	1.49
0.20	0.74	0.45	0.62	0.34	0.45	0.73	0.16	1.55
0.30	0.66	0.52	0.76	0.38	0.52	0.68	0.20	1.65
0.50	0.55	0.66	1.04	0.47	0.66	0.64	0.31	1.77
0.70	0.48	0.82	1.32	0.55	0.82	0.62	0.45	1.83
1.00	0.41	1.08	1.71	0.68	1.08	0.63	0.73	1.85
1.30	0.36	1.34	2.08	0.80	1.34	0.64	1.07	1.82
1.50	0.34	1.53	2.31	0.87	1.53	0.66	1.34	1.79
2.00	0.30	2.02	2.84	1.06	2.02	0.71	2.14	1.69
4.00	0.22	4.37	4.38	1.67	4.37	1.00	7.30	1.23
6.00	0.19	7.29	5.35	2.13	7.29	1.36	15.6	0.91
8.00	0.18	10.8	6.01	2.49	10.8	1.80	26.9	0.70
10.0	0.17	14.9	6.46	2.78	14.9	2.30	41.4	0.55
20.0	0.15	43.8	8.75	3.74	43.8	5.01	164	0.23
50.0	0.14	212	10.4	4.89	212	20.4	1040	0.055
100	0.14	764	11.1	5.47	764	69.1	4180	0.016
200	0.13	2890	11.4	5.80	2890	253	16700	0.0045
500	0.13	17300	11.5	6.06	17300	1510	105000	0.0008

**Table 57**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.6$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.0050	0.0094	315	0.0094	0.0085	315	33500	2.69	0.0001
0.010	0.018	168	0.017	0.017	168	9790	2.92	0.0002
0.020	0.034	87.7	0.034	0.034	87.7	2600	3.00	0.0008
0.050	0.073	37.5	0.082	0.084	37.5	459	3.16	0.0043
0.10	0.12	20.4	0.15	0.17	20.4	136	3.39	0.015
0.20	0.18	11.9	0.27	0.31	11.9	43.6	3.72	0.046
0.50	0.23	7.13	0.56	0.67	7.13	12.8	4.79	0.15
2.00	0.23	7.24	1.41	1.69	7.24	5.12	12.2	0.38
5.00	0.19	13.9	2.29	2.72	13.9	6.07	37.7	0.33
10.0	0.17	30.9	2.98	3.60	30.9	10.4	111	0.20

**Table 58**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PID,  $M_s = M_t = 1.6$

n	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
3.00	0.25	6.23	0.91	1.10	6.23	6.86	6.88	0.28
4.00	0.33	1.85	2.21	1.30	1.85	0.83	2.39	1.56
5.00	0.39	1.20	2.93	1.49	1.20	0.41	1.80	2.90
6.00	0.43	0.95	3.47	1.71	0.95	0.27	1.62	4.21
7.00	0.47	0.81	3.94	1.93	0.81	0.21	1.57	5.48
8.00	0.50	0.73	4.38	2.16	0.73	0.17	1.57	6.74

**Table 59**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PID,  $M_s, M_t = 1.6$

$\alpha$	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.067	154	0.051	0.043	154	3040	6.65	0.0006
0.20	0.12	25.0	0.19	0.13	25.0	135	3.36	0.012
0.30	0.17	9.57	0.38	0.24	9.57	25.0	2.32	0.062
0.40	0.22	5.23	0.61	0.36	5.23	8.52	1.87	0.17
0.50	0.25	3.49	0.85	0.49	3.49	4.12	1.69	0.34
0.60	0.28	2.65	1.09	0.62	2.65	2.43	1.65	0.56
0.70	0.30	2.21	1.34	0.77	2.21	1.66	1.71	0.81
0.80	0.32	1.98	1.61	0.93	1.98	1.23	1.85	1.07
0.90	0.33	1.88	1.90	1.10	1.88	0.99	2.07	1.32

**Table 60**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.6$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.99	0.01	0.00	693	0.12	0.060	693	5640	41.3	0.0002
0.98	0.02	0.00	182	0.21	0.11	182	872	20.7	0.0014
0.95	0.05	0.00	33.4	0.63	0.24	33.4	52.9	7.99	0.019
0.90	0.10	0.00	10.3	0.90	0.36	10.3	11.4	3.70	0.099
0.80	0.20	0.00	3.41	1.40	0.50	3.41	2.43	1.70	0.50
0.70	0.30	0.00	1.91	1.78	0.55	1.91	1.08	1.06	1.19
0.50	0.50	0.00	1.02	2.40	0.55	1.02	0.42	0.56	3.06
0.30	0.70	0.00	0.74	2.89	0.48	0.74	0.26	0.35	4.98
0.10	0.90	0.00	0.64	3.16	0.43	0.64	0.20	0.27	6.21
0.00	1.00	0.00	0.63	3.20	0.42	0.63	0.20	0.26	6.37



**Table 61**  $P_7(s) = \frac{e^{-sL_1}}{(sT + 1)(1 + sT_1)}$ , Unfiltered PID,  $M_s = M_t = 1.6$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
1.00	0.99	0.01	0.14	625	0.091	0.071	625	6850	44.1	0.0002
1.00	0.98	0.02	0.14	170	0.21	0.13	170	817	22.2	0.0016
1.00	0.95	0.05	0.15	37.3	0.65	0.23	37.3	57.2	8.51	0.018
1.00	0.90	0.10	0.17	13.8	0.64	0.27	13.8	21.6	3.73	0.058
1.00	0.70	0.30	0.25	2.92	1.06	0.39	2.92	2.76	1.15	0.44
1.00	0.50	0.50	0.33	1.54	1.12	0.41	1.54	1.38	0.63	0.87
1.00	0.30	0.70	0.41	1.07	1.08	0.39	1.07	0.99	0.42	1.20
1.00	0.10	0.90	0.48	0.89	1.04	0.37	0.89	0.85	0.33	1.38
1.00	0.00	1.00	0.50	0.87	1.03	0.36	0.87	0.84	0.31	1.41
2.00	0.99	0.01	0.12	1340	0.13	0.063	1340	10500	84.9	0.0001
2.00	0.98	0.02	0.13	375	0.20	0.11	375	1840	41.9	0.0007
2.00	0.95	0.05	0.13	76.2	0.49	0.20	76.2	157	15.6	0.0070
2.00	0.90	0.10	0.15	24.0	0.73	0.31	24.0	32.8	7.40	0.037
2.00	0.70	0.30	0.20	4.79	1.28	0.46	4.79	3.76	2.20	0.33
2.00	0.50	0.50	0.24	2.53	1.43	0.47	2.53	1.77	1.18	0.71
2.00	0.30	0.70	0.29	1.78	1.44	0.44	1.78	1.24	0.78	1.02
2.00	0.10	0.90	0.32	1.51	1.43	0.40	1.51	1.06	0.60	1.19
2.00	0.00	1.00	0.33	1.48	1.42	0.39	1.48	1.04	0.58	1.21
5.00	0.99	0.01	0.089	3420	0.13	0.061	3420	25700	209	0.00
5.00	0.98	0.02	0.090	717	0.17	0.16	717	4240	113	0.0004
5.00	0.95	0.05	0.093	170	0.50	0.24	170	338	41.0	0.0034
5.00	0.90	0.10	0.098	54.9	0.83	0.34	54.9	66.3	18.4	0.017
5.00	0.70	0.30	0.12	10.5	1.49	0.51	10.5	7.04	5.37	0.18
5.00	0.50	0.50	0.14	5.57	1.78	0.51	5.57	3.12	2.86	0.43
5.00	0.30	0.70	0.15	3.98	1.89	0.47	3.98	2.10	1.87	0.64
5.00	0.10	0.90	0.16	3.41	1.94	0.42	3.41	1.76	1.45	0.77
5.00	0.00	1.00	0.17	3.35	1.93	0.42	3.35	1.73	1.40	0.79
10.0	0.99	0.01	0.059	7070	0.12	0.058	7070	566	409	0.00
10.0	0.98	0.02	0.059	1820	0.21	0.11	1820	8540	208	0.0001
10.0	0.95	0.05	0.060	329	0.42	0.25	329	790	82.9	0.0017
10.0	0.90	0.10	0.063	106	0.86	0.35	106	123	36.9	0.0093
10.0	0.70	0.30	0.071	20.0	1.59	0.53	20.0	12.6	10.7	0.10
10.0	0.50	0.50	0.079	10.6	1.96	0.53	10.6	5.44	5.68	0.25
10.0	0.30	0.70	0.085	7.66	2.13	0.48	7.66	3.60	3.71	0.39
10.0	0.10	0.90	0.090	6.61	2.22	0.44	6.61	2.98	2.88	0.48
10.0	0.00	1.00	0.0910	6.49	2.25	0.43	6.49	2.89	2.78	0.49

**Table 62**  $P_8(s) = \frac{1-s\alpha}{(s+1)^3}$ , Unfiltered PID,  $M_s = M_t = 1.6$

$\alpha$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.27	3.43	1.42	0.97	3.43	2.42	3.33	0.61
0.20	0.29	2.45	1.68	0.92	2.45	1.45	2.26	0.92
0.30	0.32	1.92	1.85	0.90	1.92	1.04	1.73	1.22
0.40	0.34	1.59	1.95	0.89	1.59	0.81	1.40	1.52
0.50	0.36	1.35	2.02	0.88	1.35	0.67	1.19	1.82
0.60	0.38	1.18	2.07	0.87	1.18	0.57	1.03	2.12
0.70	0.40	1.05	2.10	0.87	1.05	0.50	0.91	2.42
0.80	0.42	0.94	2.12	0.87	0.94	0.45	0.82	2.72
0.90	0.44	0.86	2.13	0.87	0.86	0.40	0.75	3.03
1.00	0.45	0.79	2.14	0.87	0.79	0.37	0.68	3.33
1.10	0.47	0.73	2.14	0.87	0.73	0.34	0.63	3.64

**Table 63**  $P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.6$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.12	10.0	0.17	0.17	10.0	60.3	1.73	0.034
0.20	0.20	5.86	0.30	0.33	5.86	19.7	1.96	0.10
0.30	0.24	4.54	0.41	0.48	4.54	11.1	2.20	0.17
0.40	0.27	3.94	0.51	0.63	3.94	7.74	2.47	0.25
0.50	0.28	3.62	0.60	0.76	3.62	6.05	2.74	0.31
0.60	0.29	3.45	0.68	0.88	3.45	5.05	3.03	0.37
0.70	0.30	3.37	0.76	0.99	3.37	4.44	3.35	0.42
0.80	0.30	3.34	0.84	1.10	3.34	4.00	3.67	0.47
0.90	0.31	3.35	0.91	1.20	3.35	3.69	4.01	0.51
1.00	0.31	3.38	0.97	1.30	3.38	3.48	4.38	0.54

#### 9.4 $M_s = M_t = 1.8$

**Table 64**  $P_1(s) = \frac{e^{-s}}{(sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.8$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.020	0.98	0.31	0.42	0.026	0.31	0.74	0.0080	1.36
0.050	0.95	0.34	0.45	0.065	0.34	0.76	0.022	1.36
0.10	0.91	0.39	0.51	0.11	0.39	0.77	0.044	1.33
0.20	0.83	0.47	0.59	0.19	0.47	0.80	0.089	1.29
0.30	0.77	0.55	0.68	0.24	0.55	0.81	0.13	1.26
0.50	0.67	0.68	0.81	0.30	0.68	0.84	0.20	1.27
0.70	0.59	0.82	0.95	0.32	0.82	0.86	0.26	1.26
1.00	0.50	1.03	1.09	0.35	1.03	0.94	0.36	1.21
1.30	0.43	1.25	1.25	0.35	1.25	0.99	0.44	1.15
1.50	0.40	1.39	1.34	0.36	1.39	1.04	0.50	1.11
2.00	0.33	1.76	1.50	0.37	1.76	1.18	0.66	1.01
4.00	0.20	3.26	1.87	0.40	3.26	1.74	1.29	0.72
6.00	0.14	4.76	2.05	0.41	4.76	2.32	1.93	0.56
8.00	0.11	6.26	2.16	0.41	6.26	2.90	2.57	0.45
10.0	0.091	7.77	2.23	0.41	7.77	3.48	3.21	0.38
20.0	0.048	15.3	2.38	0.42	15.3	6.42	6.41	0.21
50.0	0.020	37.9	2.48	0.42	37.9	15.3	16.0	0.089
100	0.0099	75.5	2.52	0.42	75.5	30.0	32.1	0.046
200	0.0050	151	2.54	0.43	151	59.5	64.2	0.023
500	0.0020	377	2.54	0.43	377	148	161	0.0093
1000	0.0010	753	2.54	0.43	753	296	321	0.0047

**Table 65**  $P_2(s) = \frac{e^{-s}}{(sT + 1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.8$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.01	0.98	0.31	0.43	0.024	0.31	0.74	0.0074	1.37
0.02	0.96	0.33	0.45	0.048	0.33	0.74	0.016	1.37
0.05	0.92	0.38	0.50	0.11	0.38	0.76	0.042	1.35
0.10	0.85	0.45	0.58	0.19	0.45	0.77	0.084	1.33
0.20	0.74	0.54	0.70	0.31	0.54	0.77	0.17	1.38
0.30	0.66	0.62	0.86	0.35	0.62	0.72	0.21	1.46
0.50	0.55	0.79	1.15	0.43	0.79	0.68	0.34	1.55
0.70	0.48	0.98	1.44	0.52	0.98	0.68	0.51	1.58
1.00	0.41	1.29	1.84	0.64	1.29	0.70	0.82	1.57
1.30	0.36	1.62	2.20	0.76	1.62	0.73	1.22	1.52
1.50	0.34	1.85	2.42	0.83	1.85	0.76	1.54	1.48
2.00	0.30	2.46	2.91	1.01	2.46	0.84	2.49	1.35
4.00	0.22	5.43	4.26	1.58	5.43	1.27	8.58	0.93
6.00	0.19	9.22	5.03	2.00	9.22	1.83	18.4	0.66
8.00	0.18	13.8	5.54	2.31	13.8	2.50	31.9	0.49
10.0	0.17	19.3	5.88	2.55	19.3	3.28	49.2	0.37
20.0	0.15	58.8	6.88	3.26	58.8	8.54	192	0.14
50.0	0.14	299	7.98	4.03	29	37.5	1200	0.0310
100	0.14	1100	8.32	4.38	1100	132	4830	0.0089
200	0.13	4220	8.55	4.58	4220	494	19300	0.0024
500	0.13	25600	8.58	4.73	25600	2990	121000	0.0004

**Table 66**  $P_3(s) = \frac{1}{(s+1)(sT+1)^2}$ , Unfiltered PID,  $M_s = M_t = 1.8$

T	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.0050	0.0094	444	0.0068	0.0085	444	65000	3.78	0.00
0.010	0.018	240	0.014	0.017	240	17700	4.09	0.0001
0.020	0.034	126	0.026	0.033	126	4840	4.22	0.0004
0.050	0.073	54.2	0.060	0.083	54.2	899	4.50	0.0025
0.10	0.12	29.5	0.12	0.16	29.5	254	4.78	0.0087
0.20	0.18	17.2	0.21	0.31	17.2	80.3	5.28	0.027
0.50	0.23	10.3	0.45	0.66	10.3	22.9	6.81	0.094
2.00	0.23	10.4	1.12	1.68	10.4	9.32	17.5	0.23
5.00	0.19	20.5	1.76	2.62	20.5	11.6	53.7	0.19
10.00	0.17	46.9	2.21	3.38	46.9	21.2	158	0.11

**Table 67**  $P_4(s) = \frac{1}{(s+1)^n}$ , Unfiltered PID,  $M_s = M_t = 1.8$

n	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
3.00	0.25	8.95	0.73	1.10	8.95	12.3	9.83	0.18
4.00	0.33	2.29	2.26	1.27	2.29	1.01	2.90	1.25
5.00	0.39	1.46	3.16	1.44	1.46	0.46	2.10	2.43
6.00	0.43	1.14	3.83	1.62	1.14	0.30	1.84	3.59
7.00	0.47	0.97	4.39	1.81	0.97	0.22	1.76	4.73
8.00	0.50	0.87	4.86	2.04	0.87	0.18	1.78	5.86

**Table 68**  $P_5(s) = \frac{1}{(s+1)(s\alpha+1)(s\alpha^2+1)(s\alpha^3+1)}$ , Unfiltered PID,  $M_s, M_t = 1.8$

$\alpha$	$\tau$	K	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.067	224	0.040	0.039	224	5600	8.82	0.0003
0.20	0.12	34.1	0.16	0.13	34.1	218	4.30	0.0077
0.30	0.17	12.6	0.34	0.23	12.6	36.5	2.91	0.042
0.40	0.22	6.70	0.57	0.35	6.70	11.8	2.33	0.12
0.50	0.25	4.40	0.81	0.47	4.40	5.44	2.08	0.26
0.60	0.28	3.32	1.07	0.61	3.32	3.10	2.02	0.43
0.70	0.30	2.76	1.34	0.75	2.76	2.06	2.08	0.63
0.80	0.32	2.46	1.62	0.91	2.46	1.52	2.25	0.84
0.90	0.33	2.32	1.93	1.08	2.32	1.20	2.52	1.05

**Table 69**  $P_6(s) = \frac{e^{-sL_1}}{s(sT_1 + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.8$

$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.99	0.01	0.00	966.00	0.11	0.0510	966.00	8523.00	49.50	0.0001
0.98	0.02	0.00	262.00	0.16	0.0930	262.00	1654.00	24.30	0.0008
0.95	0.05	0.00	47.50	0.42	0.19	47.50	114.00	9.09	0.0099
0.90	0.10	0.00	13.70	0.69	0.32	13.70	19.70	4.33	0.0620
0.80	0.20	0.00	4.36	1.18	0.46	4.36	3.70	2.01	0.34
0.70	0.30	0.00	2.39	1.58	0.52	2.39	1.52	1.25	0.84
0.50	0.50	0.00	1.24	2.09	0.53	1.24	0.59	0.66	2.22
0.30	0.70	0.00	0.89	2.39	0.48	0.89	0.37	0.43	3.66
0.10	0.90	0.00	0.77	2.54	0.43	0.77	0.30	0.33	4.56
0.00	1.00	0.00	0.75	2.56	0.43	0.75	0.29	0.32	4.68

**Table 70**  $P_7(s) = \frac{e^{-sL_1}}{(sT+1)(1+sT_1)}$ , Unfiltered PID,  $M_s = M_t = 1.8$

$T$	$T_1$	$L_1$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
1.00	0.99	0.010	0.14	974	0.12	0.052	974	8000	50.5	0.0001
1.00	0.98	0.020	0.14	284	0.14	0.086	284	1990	24.4	0.0007
1.00	0.95	0.050	0.15	41.8	0.30	0.26	41.8	140	11.0	0.012
1.00	0.90	0.10	0.17	17.7	0.62	0.24	17.7	28.7	4.31	0.040
1.00	0.70	0.30	0.25	3.58	1.06	0.37	3.58	3.37	1.34	0.35
1.00	0.50	0.50	0.33	1.86	1.17	0.39	1.86	1.58	0.72	0.72
1.00	0.30	0.70	0.41	1.27	1.16	0.37	1.27	1.10	0.47	1.02
1.00	0.10	0.90	0.48	1.05	1.12	0.34	1.05	0.94	0.36	1.19
1.00	0.00	1.00	0.50	1.03	1.09	0.35	1.03	0.94	0.36	1.21
2.00	0.99	0.010	0.12	1560	0.070	0.069	1560	22300	108	0.0001
2.00	0.98	0.020	0.13	539	0.19	0.091	539	2860	48.9	0.0004
2.00	0.95	0.050	0.13	103	0.34	0.18	103	306	18.7	0.0043
2.00	0.90	0.10	0.15	31.4	0.62	0.28	31.4	50.8	8.74	0.0240
2.00	0.70	0.30	0.20	5.93	1.24	0.44	5.93	4.77	2.58	0.25
2.00	0.50	0.50	0.24	3.06	1.45	0.45	3.06	2.11	1.37	0.56
2.00	0.30	0.70	0.29	2.14	1.50	0.42	2.14	1.43	0.89	0.84
2.00	0.10	0.90	0.32	1.80	1.50	0.38	1.80	1.20	0.68	0.99
2.00	0.00	1.00	0.33	1.76	1.50	0.37	1.76	1.18	0.66	1.01
5.00	0.99	0.010	0.089	4990	0.080	0.049	4990	62600	246	0.0000
5.00	0.98	0.020	0.090	1260	0.13	0.10	1260	9690	127	0.0002
5.00	0.95	0.050	0.093	241	0.49	0.19	241	493	47.0	0.0021
5.00	0.90	0.10	0.098	72.5	0.66	0.30	72.5	110	21.7	0.011
5.00	0.70	0.30	0.12	13.1	1.41	0.49	13.1	9.27	6.34	0.13
5.00	0.50	0.50	0.14	6.77	1.76	0.49	6.77	3.85	3.35	0.33
5.00	0.30	0.70	0.15	4.78	1.91	0.45	4.78	2.50	2.16	0.51
5.00	0.10	0.90	0.16	4.08	1.96	0.41	4.08	2.08	1.67	0.61
5.00	0.00	1.00	0.17	4.01	1.98	0.40	4.01	2.03	1.61	0.63
10.0	0.99	0.010	0.059	9960	0.088	0.049	9960	113000	490	0.0000
10.0	0.98	0.020	0.059	2390	0.24	0.11	2390	9860	255	0.0001
10.0	0.95	0.050	0.060	483	0.47	0.19	483	1030	90.8	0.0010
10.0	0.90	0.10	0.063	141	0.67	0.31	141	211	43.4	0.0059
10.0	0.70	0.30	0.071	25.0	1.48	0.50	25.0	16.9	12.6	0.074
10.0	0.50	0.50	0.079	13.0	1.90	0.51	13.0	6.82	6.66	0.19
10.0	0.30	0.70	0.085	9.22	2.12	0.46	9.22	4.34	4.28	0.30
10.0	0.10	0.90	0.090	7.91	2.21	0.42	7.91	3.57	3.33	0.37
10.0	0.00	1.00	0.091	7.77	2.23	0.41	7.77	3.48	3.21	0.38

**Table 71**  $P_8(s) = \frac{1-s\alpha}{(s+1)^3}$ , Unfiltered PID,  $M_s = M_t = 1.8$

$\alpha$	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.27	4.43	1.33	0.95	4.43	3.33	4.19	0.45
0.20	0.29	3.06	1.66	0.90	3.06	1.85	2.75	0.71
0.30	0.32	2.37	1.87	0.87	2.37	1.27	2.06	0.97
0.40	0.34	1.94	2.01	0.85	1.94	0.96	1.65	1.24
0.50	0.36	1.64	2.11	0.84	1.64	0.78	1.38	1.51
0.60	0.38	1.43	2.18	0.83	1.43	0.66	1.19	1.78
0.70	0.40	1.26	2.22	0.83	1.26	0.57	1.04	2.05
0.80	0.42	1.13	2.26	0.82	1.13	0.50	0.93	2.33
0.90	0.44	1.03	2.28	0.82	1.03	0.45	0.84	2.62
1.00	0.45	0.94	2.30	0.82	0.94	0.41	0.77	2.91
1.10	0.47	0.87	2.31	0.82	0.87	0.38	0.71	3.20

**Table 72**  $P_9(s) = \frac{1}{(s+1)((sT)^2 + 1.4sT + 1)}$ , Unfiltered PID,  $M_s = M_t = 1.8$

T	$\tau$	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	IAE
0.10	0.12	13.8	0.14	0.18	13.8	99.9	2.44	0.022
0.20	0.20	8.09	0.25	0.34	8.09	32.4	2.78	0.066
0.30	0.24	6.30	0.34	0.50	6.30	18.4	3.14	0.12
0.40	0.27	5.49	0.43	0.64	5.49	12.8	3.52	0.16
0.50	0.28	5.07	0.51	0.77	5.07	10.0	3.91	0.21
0.60	0.29	4.85	0.58	0.89	4.85	8.38	4.32	0.25
0.70	0.30	4.74	0.64	1.01	4.74	7.40	4.78	0.28
0.80	0.30	4.71	0.70	1.11	4.71	6.69	5.24	0.31
0.90	0.31	4.73	0.76	1.21	4.73	6.22	5.73	0.33
1.00	0.31	4.79	0.82	1.30	4.79	5.87	6.24	0.35



## 10. Controller sets for five process models

The following tables hold process data, sets of optimal SWORD PI and PID parameters, corresponding IAE-, and  $\|S_c\|_2$ -values, for five process models with varying dynamics. The controller sets are related to a PID design method presented in [3]. The tables have also been collected in MATLAB<sup>®</sup> mat-files, Excel spreadsheets (different sheets for different process types), as well as in a tex-file. These can be downloaded from:

<http://www.control.lth.se/Research/ProcessControl/PIDControl/>

**Table 73**  $P_{2,mod}(s) = \frac{e^{-0.1s}}{(20s+1)^2}$ ,  $M_s = M_t = 1.4$

Controller	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	$T_f$	IAE	$\ S_c\ _2$
PID	280	4.25	1.88	280	65.8	526	0.14	0.016	10000
PID	170	5.41	2.33	170	31.5	396	0.22	0.034	4000
PID	114	6.52	2.74	114	17.5	314	0.30	0.061	2000
PID	76.1	7.87	3.22	76.1	9.67	245	0.41	0.11	1000
PID	44.8	9.99	3.91	44.8	4.48	175	0.61	0.24	400
PID	30.1	11.8	4.46	30.1	2.55	134	0.81	0.42	200
PID	20.4	13.7	5.03	20.4	1.49	103	1.08	0.72	100
PID	12.2	15.9	5.87	12.2	0.77	71.7	1.57	1.47	40.0
PID	8.29	17.0	6.66	8.29	0.49	55.3	2.10	2.51	20.0
PID	5.67	18.2	7.61	5.67	0.31	43.1	2.84	4.22	10.0
PID	3.49	21.1	9.00	3.49	0.17	31.4	4.27	8.03	4.00
PID	2.45	23.1	10.3	2.45	0.11	25.4	5.93	12.6	2.00
PID	1.74	24.8	12.2	1.74	0.070	21.3	8.48	19.2	1.00
PID	1.50	25.6	13.4	1.50	0.059	20.2	10.1	22.9	0.74
PID	1.30	26.6	15.0	1.30	0.049	19.5	12.2	27.5	0.55
PI	0.95	31.0	-	0.95	0.031	-	3.65	32.4	0.40
PI	0.83	31.2	-	0.83	0.027	-	5.42	37.4	0.30
PI	0.69	33.0	-	0.69	0.021	-	9.94	47.6	0.20

**Table 74**  $P_{2,T=0.05}(s) = \frac{e^{-s}}{(0.05s + 1)^2}$ ,  $M_s = M_t = 1.4$

Controller	$K$	$T_i$	$k_p$	$k_i$	$T_f$	IAE	$\ S_c\ _2$
PI	0.16	0.37	0.16	0.43	0.0001	2.31	8.00
PI	0.16	0.37	0.16	0.43	0.0008	2.32	3.88
PI	0.16	0.37	0.16	0.43	0.0034	2.32	2.00
PI	0.16	0.37	0.16	0.43	0.0075	2.33	1.41
PI	0.16	0.38	0.16	0.43	0.019	2.35	0.99
PI	0.16	0.38	0.16	0.42	0.038	2.39	0.80
PI	0.16	0.39	0.16	0.41	0.065	2.44	0.70
PI	0.17	0.43	0.17	0.39	0.15	2.58	0.60
PI	0.18	0.48	0.18	0.37	0.25	2.69	0.56
PI	0.20	0.57	0.20	0.36	0.41	2.82	0.53

**Table 75**  $P_{4,n=4}(s) = \frac{1}{(s + 1)^4}$ ,  $M_s = M_t = 1.4$

Controller	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	$T_f$	IAE	$\ S_c\ _2$
PID	1.33	2.11	1.34	1.33	0.63	1.78	0.0031	2.15	10000
PID	1.32	2.11	1.34	1.32	0.63	1.78	0.0058	2.15	4000
PID	1.32	2.12	1.34	1.32	0.62	1.76	0.0092	2.16	2000
PID	1.31	2.13	1.34	1.31	0.62	1.75	0.015	2.18	1000
PID	1.29	2.13	1.34	1.29	0.60	1.72	0.026	2.21	400
PID	1.26	2.14	1.34	1.26	0.59	1.69	0.041	2.26	200
PID	1.23	2.16	1.33	1.23	0.57	1.64	0.065	2.33	100
PID	1.16	2.19	1.33	1.16	0.53	1.54	0.11	2.47	40.0
PID	1.09	2.22	1.33	1.09	0.49	1.45	0.17	2.63	20.0
PID	1.01	2.25	1.33	1.01	0.45	1.34	0.26	2.87	10.0
PID	0.87	2.31	1.35	0.87	0.38	1.18	0.45	3.34	4.01
PID	0.76	2.36	1.40	0.76	0.32	1.07	0.68	3.87	2.01
PID	0.71	2.40	1.45	0.71	0.30	1.03	0.82	4.18	1.50
PID	0.64	2.46	1.56	0.64	0.26	1.00	1.09	4.72	1.00
PID	0.60	2.55	1.70	0.60	0.23	1.02	1.40	5.24	0.75
PID	0.57	2.70	1.90	0.57	0.21	1.08	1.78	5.80	0.60
PID	0.55	2.93	2.18	0.55	0.19	1.21	2.28	6.40	0.50
PI	0.35	2.53	-	0.35	0.14	-	0.95	7.27	0.40
PI	0.36	2.89	-	0.36	0.13	-	1.51	7.97	0.35
PI	0.59	5.47	-	0.59	0.11	-	5.01	9.27	0.29

**Table 76**  $P_{5,\alpha=0.1}(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)}$ ,  $M_s = M_t = 1.4$

Controller	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	$T_f$	IAE	$\ S_c\ _2$
PID	46.3	0.13	0.057	46.3	345	2.66	0.0041	0.0035	10000
PID	35.6	0.16	0.060	35.6	228	2.14	0.0066	0.0052	4000
PID	28.2	0.18	0.063	28.2	159	1.77	0.0093	0.0073	2000
PID	21.8	0.20	0.066	21.8	109	1.44	0.013	0.011	1000
PID	15.0	0.23	0.073	15.0	64.7	1.10	0.020	0.019	400
PID	11.2	0.26	0.080	11.2	42.9	0.89	0.029	0.029	200
PID	8.20	0.29	0.087	8.20	28.0	0.71	0.040	0.046	98.2
PID	5.40	0.33	0.10	5.40	16.1	0.56	0.064	0.085	40.0
PID	3.86	0.37	0.13	3.86	10.4	0.49	0.096	0.14	19.9
PI	2.62	0.58	-	2.62	4.49	-	0.041	0.22	10.0
PI	2.35	0.64	-	2.35	3.66	-	0.061	0.27	7.48
PI	2.03	0.68	-	2.03	2.99	-	0.088	0.34	5.50
PI	1.73	0.72	-	1.73	2.40	-	0.12	0.42	4.01
PI	1.53	0.80	-	1.53	1.91	-	0.18	0.52	3.01
PI	1.25	0.90	-	1.25	1.40	-	0.29	0.71	2.00
PI	1.14	1.05	-	1.14	1.08	-	0.43	0.92	1.50
PI	1.14	1.50	-	1.14	0.76	-	0.89	1.32	1.02

**Table 77**  $P_{6,L_1=0.01,T_1=0.99}(s) = \frac{e^{-0.01s}}{s(0.99s + 1)}$ ,  $M_s = M_t = 1.4$

Controller	$K$	$T_i$	$T_d$	$k_p$	$k_i$	$k_d$	$T_f$	IAE	$\ S_c\ _2$
PID	78.6	0.40	0.18	78.6	197	14.0	0.013	0.0055	10000
PID	48.6	0.52	0.22	48.6	94.3	10.6	0.020	0.011	4000
PID	32.7	0.62	0.26	32.7	52.6	8.45	0.027	0.020	2000
PID	21.8	0.76	0.30	21.8	28.6	6.63	0.037	0.037	1000
PID	12.8	0.99	0.37	12.8	12.9	4.76	0.055	0.082	400
PID	8.52	1.20	0.43	8.52	7.09	3.67	0.074	0.15	200
PID	5.74	1.46	0.49	5.74	3.93	2.82	0.098	0.27	100
PID	3.44	1.87	0.58	3.44	1.84	1.98	0.14	0.58	40.1
PID	2.35	2.25	0.64	2.35	1.04	1.51	0.19	1.04	20.0
PID	1.61	2.70	0.72	1.61	0.60	1.17	0.26	1.85	10.0
PID	0.99	3.54	0.84	0.99	0.28	0.84	0.38	4.03	4.00
PID	0.69	4.43	0.96	0.69	0.16	0.67	0.53	7.35	2.00
PID	0.48	5.67	1.12	0.48	0.085	0.54	0.75	13.6	1.00
PID	0.37	6.93	1.30	0.37	0.053	0.48	1.00	21.9	0.60
PID	0.30	8.21	1.50	0.30	0.036	0.45	1.26	32.4	0.40
PI	0.25	10.5	-	0.25	0.024	-	0.42	41.5	0.30
PI	0.21	12.5	-	0.21	0.017	-	0.68	59.5	0.20
PI	0.18	14.8	-	0.18	0.012	-	0.99	84.3	0.14
PI	0.16	16.1	-	0.16	0.010	-	1.15	99.0	0.12
PI	0.15	17.8	-	0.15	0.0084	-	1.38	120	0.10

## 11. References

- [1] T. Hägglund, K. J. Åström, Revisiting the Ziegler-Nichols step response method for PID control, *Journal of Process Control* 14 (6) (2004) 635–650.
- [2] O. Garpinger, T. Hägglund, A Software Tool for Robust PID Design, in: 17th IFAC World Congress, Seoul, South Korea, 2008.
- [3] O. Garpinger, T. Hägglund, Software-based optimal PID design with robustness and noise sensitivity constraints, *Journal of Process Control* 33 (9) (2015) 90–101.