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1998

Document Version:

Publisher's PDF, also known as Version of record

[Link to publication](#)

Citation for published version (APA):

Bernhardsson, B., Möllerstedt, E., & Mattsson, S. E. (1998). *A New Approach to Steady-State Analysis of Power Distribution Networks*. Paper presented at Reglermöte 1998, Lund Institute of Technology, Sweden.

Total number of authors:

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A New Approach to Steady State Analysis of Power Distribution Networks

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Keywords: electrical distribution networks, modeling, harmonics, steady state analysis, harmonic balance

ABSTRACT

Today's complex distribution networks need special models. They have many and non-linear components, which means that it is necessary to work with aggregated models. This paper presents a modularized approach to modeling of harmonics at steady state. It exploits that loads are connected in parallel and that frequency and amplitude of the voltage across a load are close to known nominal values. It means that linearization of the nonlinear relations is tractable. The method of harmonic balancing is used to model the harmonics. The model for a component is given by a linear relation between the Fourier coefficients for spectra of deviations of the voltage and the current from nominal values. This linear relationship between the voltage and the current is conveniently described by an admittance matrix. It is then straightforward linear algebra to build models also for nonlinear networks. Applications show that we get accurate results. The advantage of this approach is that it results in fast calculations since the basic operation is solving linear equation systems.

INTRODUCTION

Modern distribution networks are very complex. They are widespread and contain numerous non-linear and switching devices. Distribution networks constitute the link between high voltage transmission lines and the power consumers. As a definition, the voltage level of Swedish distribution networks range between 230 V and 40 kV.

The increased use of switched power supplies and power electronics for motor control creates disturbances in the wave form of the supplied electricity. These disturbances result in increased losses and also in failure of sensitive equipment. Today, problems have been encountered in, for instance, hospitals and industries, with sensitive equipment and high power loads, but the situation is expected to become severe even in domestic areas. The power companies want to be able to guarantee the power quality. It has also been proposed that there will be a fee for polluting the power. Consequently, there is a desire for methods to analyze these complex systems.

Models for static and dynamic simulation of electricity distribution networks need to be non-linear and accurate up to frequencies of a few kHz. Typical network configurations are a shopping center, an office building or a local district with fifty houses, two factories, and two transformer stations. The complex and non-linear nature of the systems requires

methods for aggregation and modularization. The big question is then how to make the aggregation. It would be very useful to have tools that can take a component-based model of a part of a network and make a simplified aggregated model. One difficulty is that a network includes non-linear components. For linear dynamic models there are well-established model reduction techniques.

We are developing a method to describe non-linear elements in the frequency domain. Using these models, aggregation of loads and solving the network is done by direct calculation using linear algebra, i.e., no iterative solution is necessary. This means that convergence problems, and large computational efforts are avoided. The models are valid for steady-state analysis.

HARMONIC BALANCE

For many electrical components, the voltage and current are related through a differential-algebraic equation system

$$f(v, \frac{dv}{dt}, i, \frac{di}{dt}, t) = 0. \quad (1)$$

For static analysis, we assume the signals to be periodic, with a period of 20 ms (50 Hz), and represent voltages and currents by truncated Fourier series

$$\begin{aligned} v(t) &= \sum_{k=1}^N a_k \cos k\omega t + b_k \sin k\omega t & \Rightarrow & \quad V = [a_1 \quad \dots \quad a_N \quad b_1 \quad \dots \quad b_N]^T \\ i(t) &= \sum_{k=1}^N A_k \cos k\omega t + B_k \sin k\omega t & \Rightarrow & \quad I = [A_1 \quad \dots \quad A_N \quad B_1 \quad \dots \quad B_N]^T. \end{aligned}$$

With this, Equation(1) transforms to an algebraic equation system

$$F(V, I) = 0. \quad (2)$$

The equation system can be solved iteratively, e.g., by Newton's method. This way of solving non-linear systems is referred to as the harmonic balance method, [2, 3]. There are however some problems with using this method on distribution systems. The complexity of the networks make the size of the equation system very large, which result in time consuming Jacobian calculations, and problems with convergence of the iterations. With many non-linear elements the problems become severe. It would also be attractive to be able to reuse parts of the result if the network is modified, e.g., a component is substituted. This bring up the desire for some form of modularization.

THE PROPOSED METHOD

Harmonic balance applied to electrical distribution systems creates some interesting possibilities to support modularity. The idea is that for distribution systems, the line voltage is known in advance (e.g. 230 V, 50 Hz) and the maximum level of distortion, e.g., energy contents in harmonics, is regulated by norms and standards not to exceed certain values. The operating conditions for loads, which are connected in parallel, are hence approximately known in advance. This makes linearization of the nonlinear algebraic relations tractable.

Let V , and, I , be defined as in the previous section. If only small deviations from the nominal conditions, V_0 , I_0 , are considered, the algebraic equation system can be linearized

$$I = I_0 + Y(V - V_0). \quad (3)$$

The Jacobian matrix, Y , is a matrix that describes the linear relationship between the voltage and the current.

$$Y = \frac{\partial I}{\partial V} = \begin{bmatrix} \frac{\partial A}{\partial a} & \frac{\partial A}{\partial b} \\ \frac{\partial B}{\partial a} & \frac{\partial B}{\partial b} \end{bmatrix}, \quad \text{where} \quad \frac{\partial A}{\partial a} = \begin{bmatrix} \frac{\partial A_1}{\partial a_1} & \cdots & \frac{\partial A_1}{\partial a_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial A_N}{\partial a_1} & \cdots & \frac{\partial A_N}{\partial a_N} \end{bmatrix}, \text{ etc...} \quad (4)$$

A non-linear device is then represented by a nominal current spectrum, I_0 , and a Jacobian matrix, Y . The Jacobian is sometimes referred to as the harmonic admittance matrix, but could also be called the linearized describing function.

The method proposed is a version of Newton's method of Harmonic Balancing. It can be seen as one iteration of that method where the nonlinear elements have been linearized around their nominal working conditions, a network with pure sinusoidal voltages. The main point is that one iteration can be achieved using precomputed, modularized information. This gives a very fast method that is interesting also for analytical investigations since only linear algebra is used. The model, (3,4), can be obtained either through measurements on real systems, time domain simulation of simple circuits, or analytical calculations. Moreover, a global admittance matrix can be computed by interconnecting local models. The main problem to investigate is how accurate solutions we get, using only one iteration. Our first experiments show promising results. The idea is presented using two examples.

EXAMPLES

Analytical calculation of a model for a dimmer

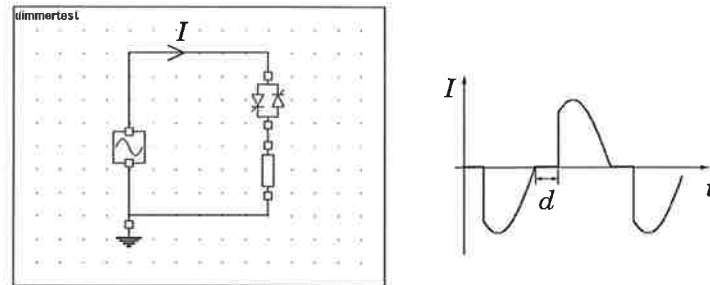


Figure 1 A dimmer in series with a resistive load. The current through the dimmer is turned off for a time, d , every half period.

To illustrate the idea, we consider a simple dimmer, consisting of two thyristors in parallel. When the current through the dimmer becomes zero, the dimmer is turned off. The current through the dimmer remains zero for a time, d , after which the dimmer is turned on again. The dimmer is connected in series with a linear resistor, see Figure 1. When connected to a stiff voltage source, the current will be distorted. Since the current is symmetric, there will only be odd harmonics, i.e., we have

$$u_{\text{nom}}(t) = a_1^0 \cos \omega t, \quad i_{\text{nom}}(t) = \sum_{k \text{ odd}} A_k^0 \cos k\omega t + B_k^0 \sin k\omega t.$$

If, however, the voltage source is a bit distorted, we approximately have that the deviation from the nominal current is linearly dependent on the distortion of the voltage

$$u(t) = a_1^0 \cos \omega t + \sum_{k \text{ odd}} \hat{a}_k \cos k\omega t + \hat{b}_k \sin k\omega t,$$

$$i(t) = \sum_{k \text{ odd}} (A_k^0 + \hat{A}_k) \cos k\omega t + (B_k^0 + \hat{B}_k) \sin k\omega t,$$

where we have put $A_k = A_k^0 + \hat{A}_k$ etc. and

$$[\hat{A}_1 \quad \dots \quad \hat{B}_1 \quad \dots]^T = Y [\hat{a}_1 \quad \dots \quad \hat{b}_1 \quad \dots]^T.$$

To compute Y , we assume that $u(t)$ has the form above. If \hat{a}_k and \hat{b}_k are sufficiently small there will still be two zero-crossings per period located close to the distortion free case. Denote the new first positive zero-crossing with $t_1 = t_1^0 + \hat{t}_1$, where $t_1^0 = \pi/2\omega$ and \hat{t}_1 is small if \hat{a}_k and \hat{b}_k are small. Linearization gives

$$u(t_1) = a_1^0 \cos \omega t_1 + \sum_{k \text{ odd}} \hat{a}_k \cos k\omega t_1 + \hat{b}_k \sin k\omega t_1$$

$$= a_1^0(-\omega \hat{t}_1) + \sum_{k \text{ odd}} \left(\hat{b}_k (-1)^{\frac{k-1}{2}} + O(a_k^2 + b_k^2) \right) = 0,$$

which shows that up to first order terms we have $t_1 = \frac{\pi}{2\omega} + \frac{1}{\omega a_1^0} \sum_{k \text{ odd}} \hat{b}_k (-1)^{\frac{k-1}{2}}$. The Fourier coefficients are given by $RA_k = a_k - \frac{4}{T} \int_{t_1}^{t_1+d} u(t) \cos k\omega t dt$, and if we insert the Taylor expansion of t_1 direct calculation shows that the coefficient before \hat{a}_l in the expression for \hat{A}_k is given by

$$R \frac{\partial \hat{A}_k}{\partial \hat{a}_l} = \delta_{kl} - \frac{4}{T} \int_{t_1^0}^{t_1^0+d} \cos l\omega t \cos k\omega t dt.$$

The coefficient before \hat{b}_l in the expression for \hat{A}_k is given by

$$R \frac{\partial \hat{A}_k}{\partial \hat{b}_l} = -\frac{4}{T} \int_{t_1^0}^{t_1^0+d} \sin l\omega t \cos k\omega t dt + (-1)^{\frac{k+1}{2}} \sin k\omega d.$$

Similarly we obtain

$$R \frac{\partial \hat{B}_k}{\partial \hat{a}_l} = -\frac{4}{T} \int_{t_1^0}^{t_1^0+d} \cos l\omega t \sin k\omega t dt,$$

$$R \frac{\partial \hat{B}_k}{\partial \hat{b}_l} = \delta_{kl} - \frac{4}{T} \int_{t_1^0}^{t_1^0+d} \sin l\omega t \sin k\omega t dt - \frac{2}{\pi} (-1)^{\frac{k+l}{2}} \sin \omega d \cos k\omega d.$$

In Figure 2, the resulting coefficients, calculated from this linearized model, are compared with the true values obtained from a simulation of the network. The results show that the linearized model is a good approximation if the deviations in voltage are limited to 10%. Note that the current is severely distorted due to the nonlinearity, as shown in Figure 1. The accuracy should be enough for investigations of common electrical networks.

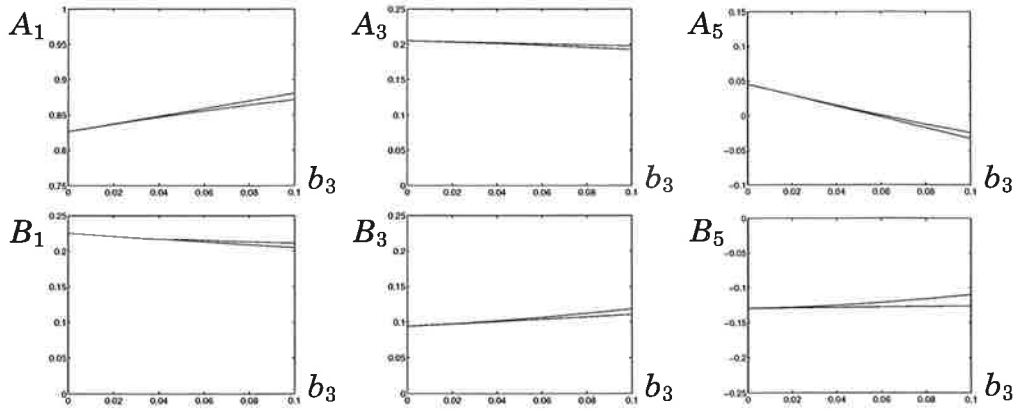


Figure 2 Plots showing how the linearized and simulated Fourier coefficients for the current in the circuit in Figure 1 depend on b_3 , when $u(t) = \cos t + b_3 \sin 3t$.

A Network with Two Dimmers

In radial networks, loads are connected in parallel. If the line losses are small, the load voltage is close to its nominal value, which means that our method is suitable. The steady-state solution for the system in Figure 3 is easily derived using our method. Applying Kirchhoff's laws, it is a matter of solving a linear equation system. Let the two dimmers, with resistive loads of 20Ω , be represented by admittance matrices, Y_1 and Y_2 . The line impedances, $z_{l1} = (0.75 + j\omega 0.0024) \Omega$ and $z_{l2} = (0.25 + j\omega 0.0008) \Omega$, are represented by matrices, Z_{l1} and Z_{l2} . The system is described by the following equation:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{I} & -\mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 & -Y_1 & 0 \\ 0 & 0 & \mathbf{I} & 0 & -Y_2 \\ Z_{l1} & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & Z_{l2} & -\mathbf{I} & \mathbf{I} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ \hat{U}_1 \\ \hat{U}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ I_{1\text{nom}} \\ I_{2\text{nom}} \\ 0 \\ 0 \end{bmatrix}.$$

Here I_0 , I_1 , and I_2 are vectors with the Fourier coefficients of the currents, and \hat{U}_1 and \hat{U}_2 contain the Fourier coefficients of the deviations from the nominal voltage, $u_{\text{nom}}(t) = (240\sqrt{2} \cdot \cos 2\pi 50t) \text{ V}$. The identity matrix of appropriate size is denoted \mathbf{I} . Instead of solving a non-linear algebraic equation system, we now have to solve a linear equation system of the same size.

The currents become more distorted the longer the dimmers are turned off. It is natural to assume that the method will be less accurate the larger d is. We therefore

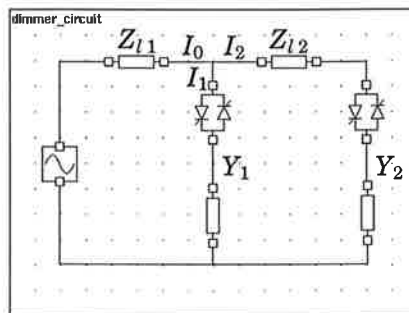


Figure 3 A circuit with two dimmers, Y_1 and Y_2 , and line losses, Z_{l1} and Z_{l2}

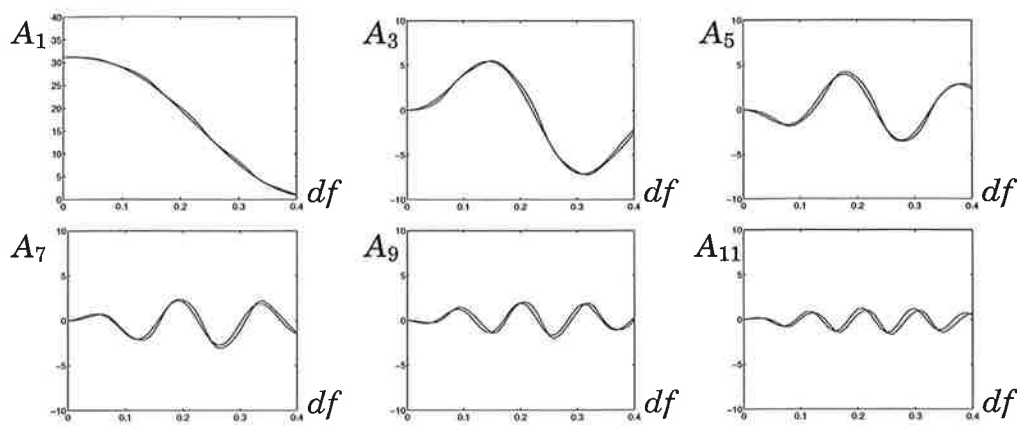


Figure 4 Plots showing how the linearized and simulated Fourier coefficients for the current in the circuit in Figure 3 depend on the turn on time, d , for the dimmers. When $df = 0.5$ the dimmer is completely turned off.

tested the accuracy of the method for different turn on delays, d . Figure 4 shows the first six cosine coefficients for the Fourier series of the current, when d is varied from zero to almost a half period. The plots show that the method works well for all d .

CONCLUSIONS

We have described a new method to model large electrical distribution networks that contain nonlinear components. A modularized approach is supported. The method is based on harmonic balancing in the frequency domain. Preliminary experiments show promising results. We now intend to use the method on larger systems and to measure the accuracy on real data. The obtained model is well suited for control design and resonance problems on compensated lines can probably be predicted.

ACKNOWLEDGEMENT

This work is the result from Elforsk project 3153 sponsored by Elforsk AB and Nutek.

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