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# An Approach to Constraint-Based and Mass-Customizable Product Design

*In traditional product development, several iterations are usually necessary to obtain a successful compromise between constraints emanating from engineering, manufacturing, and aesthetics. Moreover, this approach to product development is not well suited for true mass-customization, as the manufacturing company remains in control of all aspects of the shape of the product-to-be. In this article, we propose an alternative approach that would (1) allow for an improved integration of industrial design into the product development process and (2) enhance the creative repertoire of industrial designers, which (3) would result in significantly improved prospects for mass-customization. The industrial design process may benefit from using advanced and aesthetically interesting morphologies emanating from the areas of mathematics and nature. Such complex morphologies can only be manipulated (analyzed and represented) by means of specific algorithms. On one hand, this requires a shift from established industrial design practice, where the industrial designer is in total control of the product form; on the other hand, it makes it fully possible to compute form so that it complies with engineering and manufacturing constraints. In this setup, the industrial designer still has control of the final result, in that she or he can choose from a set of valid forms. This approach would greatly reduce the number of iterations in the product development process between industrial design, engineering, and production. Naturally, such an approach also allows for advanced mass-customization by allowing consumers to use these tools. Within this approach, a table generation system has been developed: A system that generates tables whose support structure is based on a Voronoi diagram that fulfills structural and manufacturing constraints while being aesthetically appealing. [DOI: 10.1115/1.3569828]*

## 1 Introduction

In the traditional product development process, several iterations are usually needed to obtain a compromise between engineering and manufacturing constraints on the one hand and aesthetics on the other. The process typically begins with a design brief to which the industrial designer (hereafter, “designer”) develops initial concepts, which are reviewed and decided on in close collaboration with marketing, engineering design, and production departments. The chosen concept is refined until it satisfies the engineering and manufacturing requirements and can be produced. This traditional approach limits severely any subsequent option for mass-customization related to the design of the product: An option to which customers are becoming more attuned. Finally, while the new digital means of creation and fabrication empower designers with new levels of freedom to even become market actors in their own right, they are rarely educated to take full advantage of the new methodologies of creation, especially regarding the competence to exploit the extraordinary reservoir of morphologies from nature and mathematics, as has been the case within the area of architecture for quite a while now. In this article, we want to propose an alternative approach that would (1) allow for an increased integration of industrial design (hereafter, “design”) into the product development process and (2) enhance the creative repertoire of the designer, which would eventually (3) result in significantly improved prospects for mass-customization.

Today, many natural and mathematical structures can be computed, analyzed, and graphically represented in a manageable

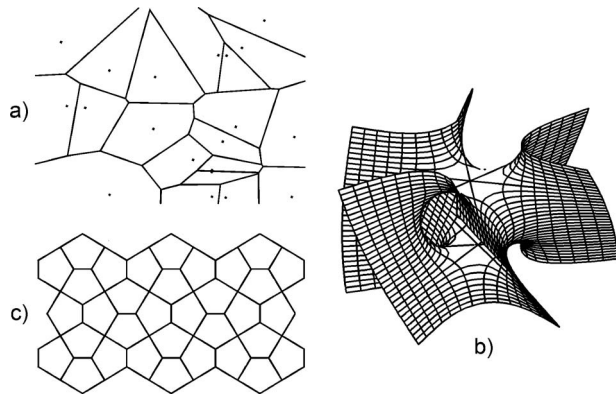
time. It is believed that designers could use specific tools in order to create concepts and designs that they could never have conceived without the support of such tools. The new morphologies can then be optimized with reference to customers’ specifications and engineering and manufacturing constraints while submitted to the critical judgment of the designer. This approach has the potential to allow active customer participation, which might even result in the actual origination of the design in the spirit of true mass-customization.

The main focus of this article is to elaborate on the feasibility of such an approach. Can the customers’ specifications as well as the engineering and manufacturing constraints be taken into account in a semi-automated design process without compromising aesthetics? To test this approach, a table generation system has been developed. The designer or end customer (hereafter, user) has some degrees of freedom concerning the form of tables. A 2D Voronoi diagram, see Fig. 1(a), was chosen as a novel tessellation for the supporting parts of tables and was submitted to a set of structural and manufacturing constraints. The developed system allows for the search of tables that fulfill those constraints while being aesthetically appealing. The first section of this article develops this approach in some depth and compares it to related works. Then, the heuristic used and the application developed are presented.

## 2 Background

Design is by and largely considered an integral part of most product development processes. Design activities are performed either by external designers (design agencies or freelance designers) or by an internal design department. In the first case, design is somewhat in the margin of product development: External designers are hired for one project, they often work relatively isolated and their proposed concepts will inevitably result in further dis-

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**Fig. 1** Examples of (a) a Voronoi diagram, (b) a minimal surface (twisted Scherk surface), and (c) a D1-tessellation

ussions in terms of engineering and manufacturing constraints. In the second case, even when there is extensive collaboration between the different functions of the company, iterations are still necessary. For the majority of companies, the development time is of utmost importance and a reduction of the number of iterations is a priority.

The traditional product development process also puts restrictions on the possibilities for mass-customization. Customers may occasionally decide between different colors or materials but it is almost impossible to allow them to generate a form of their own choice, apart from the possibility to select predefined product variants and models.

The designer is rarely educated to make conceptual use of complex forms such as those derived from Voronoi diagrams or minimal surfaces (Fig. 1). This is partly due to the fact that those forms generally require advanced knowledge in mathematics but more so due to the idea that the designer must be an intuitive *author*; hence, often disregarding morphologies that would require the expertise of others. This dilemma was discussed in length in Ref. [1].

### 3 Related Work

The association between design and related topics, such as new morphologies, mass-customization, and integration of engineering and manufacturing constraints, has already been considered in literature but mostly as discrete issues.

In academia, expanding the creative repertoire by using morphologies from nature and mathematics has been the concern of design research for some time now, see, e.g., Ref. [2], although such ideas have been deployed mainly within architecture [3,4] or graphic design. This concern has also been elaborated within engineering design [5,6] and in the development of materials [7,8]. More recently, McCormack et al. [9] presented a model of what they call generative design and proposed an appropriate agenda for design research, although the examples were not taken from the area of industrial design. The research of generative design in industrial design concerns primarily the (1) generation of artifacts based on a particular *style* [10,11] (2) and *branding* related issues [12–14]. Shea and Cagan [15] used a combination of shape grammar and simulated annealing for both functional and aesthetic purposes and applied it to truss structures. In Ref. [15], the aesthetic constraints were formalized by using the properties of the golden number. Shea and Cagan's [15] approach is the most similar to ours. In our case, though, it is necessary to let users decide on their own preferences. This is achieved by using an interactive method.

Some applications can be found in industry, mainly developed by design agencies and leading designers. For example, Trubridge's ceiling lamp is based on classic polyhedral geometry [16]. In general, for products with few structural constraints, such

as lamps, mathematical algorithms have been used to develop new forms: This permits to focus on the aesthetic characteristics while the remaining constraints can be neglected. These structures have also been used together with rapid prototyping, permitting the production of virtually every form but is an expensive and time-consuming production system: Wertel and Oberfell from Platform Studio (Leuven, Belgium) developed a mineral-based table, Fractal-T [17], manufactured with rapid prototyping. Moreover, many designs remain concepts: They are presented at design fairs and showrooms, with the purpose of showing the aesthetic and functional potential of complex structures and inspire other designers rather than making a commercial exploitation of them. For example, van der Veer developed a paper table, demonstrating the strength of paper in combination with mathematical models [18], and Mayor's burnout bench is based on a sculptural wave described in 3D by a computer [19]. In the automotive industry, Mercedes has developed a concept car based on the properties of the boxfish, "respecting at once physics, design, and aerodynamics" [20]. In comparison to industrial design, there are numerous examples of algorithmically generated and optimized architecture: London Town Hall, London Swiss Re Building, etc. Nevertheless, the constraints are different. Since a building is most often a one-off product, once a proposal has been accepted at the conceptual level, the architect is assured to receive financing for "manufacturing." The risks associated with the project are thus more or less eliminated.

Focus on form has usually been decoupled from the other aspects commended in this project: mass-customization and integration of production constraints in the design activity. Concerning the latter, many methodologies and tools have been developed for the integration of engineering and manufacture [21] but industrial design has been neglected. In the domains of product platform and product family design, the algorithms developed to automate the generation of product variants and models, e.g., Refs. [22,23], take engineering and production constraints into account but not aesthetics. DASSAULT SYSTÈMES CATIA® and PTC's (Needham, MA) PROENGINEER® each have implemented a module that permits freeform design in a format compatible with their computer-aided design (CAD) system: Imagine and Shape and Pro/Concept, respectively. Such plug-ins can accelerate the design process and represent a step toward increased integration but still do not implement any extended morphological repertoire, and the engineering and manufacturing constraints are treated after the first concepts have been produced.

Concerning advanced mass-customization, where the user has a direct influence, graphic design has experienced some success stories, where customers are completely free to design whatever motive they want, as shown by the Harvard Business School case reported in Ref. [24]. These are among the examples described by von Hippel [25,26], who initiated the user innovation paradigm. In electronics and software industry, mass-customization is also highly present. The iPhone® from Apple is no longer primarily considered a mobile phone with a user-friendly interface but as a platform in which everyone can build on. Once again, however, the production constraints are not identical [27]. The novelty of this project is thus to bring together these three aspects, new morphologies, mass-customization and integration of design, and engineering and production, which should, in a long term perspective if successful, reduce time-to-market and costs, and give a competitive advantage to those companies implementing it.

### 4 Approach and Application

In this paper, we propose an approach that can be appropriate when aesthetic aspects and mass-customization are prioritized. Many morphologies from nature (which has long been a source of inspiration in design) have been described in a computational form. In geometry, many shapes have been developed that have structural properties (for example, minimal surfaces, Fig. 1(b)) and are aesthetically remarkable. By coupling such computer-

based morphologies to engineering and manufacturing constraints, it is possible to generate forms that are both structurally sound and visually appealing. Advances in optimization methods and artificial intelligence make this integration possible. At the same time, this approach facilitates the development of variants that can be tailored to each customer (individuals or groups). It also allows for the designer or the customer (the user) to intervene during the optimization process (interaction method). Letting the customer to *co-create* the final product makes a truly mass-customized production system a reality.

This approach is detailed in Sec. 4.1. The approach is illustrated by an application presented in the next sections.

**4.1 The Approach.** The class of problems concerned by our approach has the following characteristics: (1) a complex morphology is integrated in a product, (2) the user can partially control the features of the morphology in order to make the product unique, (3) the morphology must comply with engineering and manufacturing constraints, and (4) the complex morphology affects the product's functionality and aesthetics. The third characteristic implies that the morphology is not just decorative but has some structural function; it implies also that the approach takes into account manufacturing systems other than just rapid prototyping.

In this general case, then, some functional and aesthetic engineering and manufacturing objectives  $\mathbf{F}(\mathbf{x})$  need to be minimized or maximized (for example, costs and weight) while other functional and aesthetic engineering and manufacturing conditions are constraints  $\mathbf{G}(\mathbf{x}) \leq \mathbf{0}$  and  $\mathbf{H}(\mathbf{x}) = \mathbf{0}$ . This is a problem of multi-objective optimization

$$\begin{aligned} & \text{minimize } \mathbf{F}(\mathbf{x}) \\ & \text{subject to } \mathbf{G}(\mathbf{x}) \leq \mathbf{0} \quad \text{and} \quad \mathbf{H}(\mathbf{x}) = \mathbf{0} \end{aligned} \quad (1)$$

Some aesthetic qualities of the solutions can be very difficult to formalize and most consumer preferences are plainly subjective. It is therefore necessary to let them express their preferences during the optimization process. This approach, called interactive multi-objective optimization, directs the optimization process toward solutions that satisfy the user. Typically, the user must be able to choose from a set of optimized solutions and possibly to relaunch the optimization system if not satisfied. An interactive multi-objective optimization presents several advantages: The user can "learn about the interdependencies in the problem as well as about one's own preferences" (p. 3 in Ref. [28]), this can save some computational costs as the user's indication directs efficiently the search, and it avoids the need to compare many Pareto optimal solutions simultaneously (see p. 28 in Ref. [29]).

Since the investigated structures derived from mathematics or nature are highly nonlinear and often discrete, as can be the constraints and objectives, a stochastic solver instead of a classical (e.g., gradient-based) optimization approach is to be favored. A type of algorithm suitable for these types of problems is the genetic algorithm (GA) [30]. A GA tries to artificially simulate the process of evolution [31], by which the structures in nature were first created. For a review of applications using GA for multi-objective optimization, see Ref. [32]. The interactive genetic algorithm (IGA) approach is used when the user intervenes during the optimization process. The purpose of the user interaction is slightly different from that of classical IGA approaches, see, e.g., Ref. [29]. The user does not help the system to minimize  $\mathbf{F}(\mathbf{x})$  by bringing in expertise (the results presented to the user are already optimized) but chooses the alternatives that are thought to fit the preferences and restarts the optimization if not satisfied.

Because some constraints can require long computational times (e.g., finite element analyses (FEAs)), it is preferable to handle the constraints sequentially, as this does not require that all constraints be evaluated at each iteration. This is implemented by scoring each individual according to which constraints it has fulfilled (see

Sec. 4.4.2). The scoring system ensures that an individual that passes the  $m$  first constraints will always have a higher score than an individual that passes the  $m-k$  first constraints ( $k=1, \dots, m-1$ ). Gradually, the number of individuals fulfilling the constraints will increase. Individuals that fulfill all constraints are scored based on how well they minimize  $\mathbf{F}(\mathbf{x})$ ; this score is obtained by computing a weighted sum of the different objectives. The individuals are then ranked according to their scores. This ranking is used as basis for the selection of individuals with which to create the next generation of new individuals. The scoring and selection algorithm for the application is developed in Sec. 4.4. The ranking system for our application is presented in Sec. 4.4.2. This way of handling constraints sequentially, or lexicographically, will be denominated, hereafter, lexicographic constraint-handling technique (Lexcoht).

In summary, in order to solve the class of problems defined above, our approach is to model them as interactive multi-objective optimization problems, using stochastic solvers such as GA. In line with this approach, the following process is proposed.

The first step is, naturally, to let the designer or consumer customize parts of the object. Some of these will be required for the initialization of the optimization process. This can be material choice, contour, and choice of morphology. In the application presented below, the user first specifies the tabletop outline and the legs; the Voronoi structure was imposed. The optimization system is then launched and follows the same steps as a classical evolutionary optimization system. Once a first optimization run has been performed, a set of optimized products fulfilling all constraints is proposed to the user. If the user finds one that is deemed preferential, the process stops. Otherwise, the user is asked to choose a set of alternatives that are nearest the preferences, and the optimization process is rerun with the chosen individuals as "parents" for the initial population. The user can perform several iterations until completely satisfied, or give up.

## 4.2 Application

**4.2.1 The Type of Structure.** Among the infinite number of possible mathematical structures, cf. Ref. [1], the choice went to utilize a simple 2D tessellation, namely, the Voronoi diagram, which is easy to generate programmatically. Phenomena as diverse as the wing of a dragonfly, the structure of bone marrow, and a honeycomb can be described with Voronoi diagrams [33]. Such structures are often found in lightweight and strong structures in nature [2,34].

The Voronoi diagram is created from a number of Voronoi sites, or points. Each Voronoi site  $s$  is contained in a Voronoi cell, which contains all points closer to  $s$  than to any other Voronoi site. In the case of two sites  $s_1$  and  $s_2$ , the space is divided in two by a straight line, which is the bisecting line of the segment  $[s_1s_2]$ . If all sites are coplanar, the *Voronoi* diagram consists of polygons, see Fig. 1(a). Formally, a Voronoi diagram is described as follows. Let  $S$  be a set of  $n$  sites in Euclidean space of dimension  $d$ . For each site  $p$  of  $S$ , the Voronoi cell  $V(p)$  of  $p$  is the set of points that are closer to  $p$  than to other sites of  $S$ . The Voronoi diagram  $V(S)$  is the space partition induced by Voronoi cells (Ref. [35], Chap. 7).

**4.2.2 The Product.** Furniture has always been a suitable canvas for designers on which to paint future technologies and aesthetics [36]. In this sense, furniture provides a suitable test bed for innovation because any person understands what furniture represents. The furniture industry is also an important part of Swedish industry, representing 25 billion SEK (3.1 billion USD as of September 2009) [37]. Furniture is important also in terms of image in relation to what is widely known as Scandinavian design.

Moreover, tables pose high demand on low weight, stiffness, and visual appeal. This makes them suitable as illustration objects. Three tables with different sizes and loads have been chosen for this test: a coffee table, a side table, and a dining table.



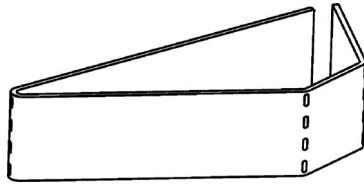


Fig. 2 Representation of a laser cut and bent Voronoi cell

The Voronoi diagram is used as the supporting structure of the table. The tabletop itself is made of glass so that the Voronoi structure is visible.

4.2.3 *The Manufacturing Method.* The following numbers of manufacturing methods were considered:

1. laser cutting strips of sheet metal and robot-welding them together
2. laser cutting strips of sheet metal and computer numerical control (CNC)-bending them into individual Voronoi cells and assembling them by welding, screwing, or gluing
3. same as method 2 but with laser cut perforations along the edges of the cells
4. sheet metal corrugated along the cell walls of the Voronoi diagram

Method 2 could allow the table to be assembled by the customer and the cells are easily individualized with different materials and colors. This is not possible with method 4 and very difficult with method 1 (because of welding issues). Method 3 allows for the same freedom as method 2 and furthermore makes it possible for the table to be bent by the customers (see Fig. 2). This reduces manufacturing and transportation costs (as the cells can be flat-packed) while the customers have the option to purchase a cheaper table (at the expense of added assembly time). In front of those arguments, both manufacturing methods 2 and 3 were chosen in order to let each customer decide of the assembly type she or he prefers.

4.2.4 *Possibilities for Customization.* For this product, the user/customer is able to fully define the contour and dimensions of the tabletop (see Fig. 3(a)). The user also chooses the table height, and the number and position of the legs. For each user-defined leg position, the Voronoi cell closest to it is set as a leg. The legs themselves are formed by the walls of cells, and have the same height as the table. They are fixed in all degrees of freedom during the finite element evaluation.

At the end of the first optimization run, the customer chooses the table that maximizes her or his preferences. The tables subsequently displayed have all fulfilled the constraints and have different costs (that is the cost minimization function presenting different values). Typically, four or five out of a population of 50 are presented to the user. The initial population for the next optimization run is then created by mutating and crossing the individuals selected by the user. The algorithm goes on until the user is satisfied (in this example, only two runs were performed).

4.3 **Specifications.** Beyond the table properties specified by the user/customer (Sec. 4.2.4), the inherent constraints necessary for a table to perform its function, the manufacturing requirements and cost (the manufacturing cost of the table must be minimized), were taken into account.

4.3.1 *Functional and Aesthetic Constraints.* The tables need to be able to handle the weight put on them without buckling, exceeding the yield stress of the material or deforming noticeably. A load of 500 N was used for the coffee table and the side table, and a load of 1000 N was used for the dining table to model the typical vertical loads. It is also important that the deformation of the table structure is not noticeable to someone sitting at the table,

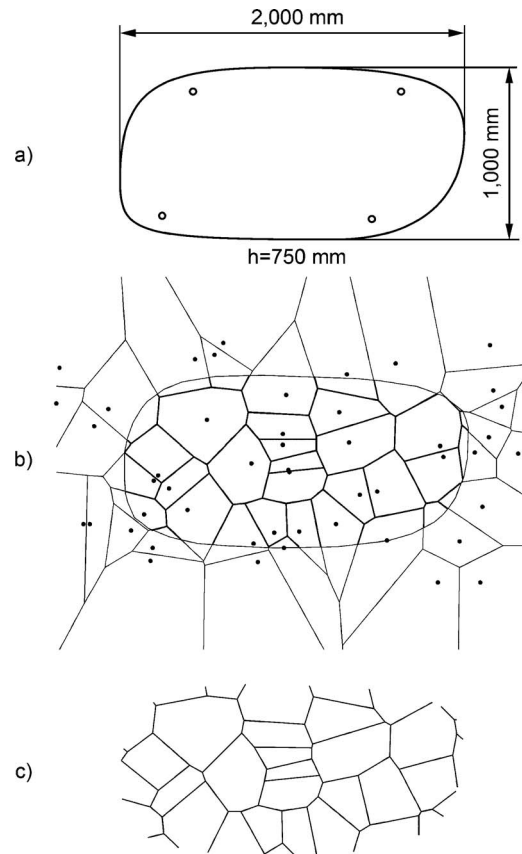


Fig. 3 (a) User-defined outline and leg positions of the table (dining table). (b) Example of how the Voronoi structure is created. The Voronoi diagram is generated from the Voronoi sites and cut off at the table boundary. (c) Final appearance of the structure after the cells have been cut off.

as the tabletop is in transparent glass. It was decided arbitrarily that a deformation under  $a_{\text{allowed}}=2.5$  mm would not be perceptible. This constraint is expressed as

$$\max(a) \leq a_{\text{allowed}} \quad (2)$$

where  $\max(a)$  is the largest displacement measured in an individual.

4.3.2 *Manufacturing Requirements.* All tables were optimized with respect to the requirements for bending with a CNC-bending machine (manufacturing method 2).

As the CNC-bending machine will intersect with itself if the distance between two bends is too short, the cells cannot have walls less than 30 mm ( $l_{\text{allowed}}$ ), or the bends have sharper angles than 33 deg ( $\alpha_{\text{allowed}}$ ) [38]. This is expressed by the following constraints:

$$\min(l) \geq l_{\text{allowed}} \quad (3)$$

where  $\min(l)$  is the shortest cell wall found in an individual, and

$$\min(\alpha) \geq \alpha_{\text{allowed}} \quad (4)$$

where  $\min(\alpha)$  is the smallest angle measured in an individual.

4.3.3 *Cost Objective.* The number of cells to be produced mostly governs the manufacturing cost, rather than the amount of material used. Also, the time of assembly is mostly dependent on the number of cells [39]. This means that to minimize the cost of the product, the optimization should focus on the number of cells,

rather than on the amount of material used. The cost is the only objective to optimize for this product. The goal is thus to *minimize*  $n$ , where  $n$  is the number of Voronoi cells in the table structure.

In summary, the table generation system has to

$$\begin{aligned} & \text{minimize } n \\ & \text{subject to } \max(a) \leq a_{\text{allowed}} \\ & \min(l) \geq l_{\text{allowed}} \\ & \min(\alpha) \geq \alpha_{\text{allowed}} \end{aligned} \quad (5)$$

#### 4.4 Implementation

**4.4.1 Representation of the Voronoi Diagram.** To give the optimization algorithm full control of the Voronoi structure, the structures were represented by the location of the Voronoi sites. Each table was represented by the coordinates of  $n_V=70$  Voronoi sites. The number of Voronoi sites was determined by experiment ranging from 50 to 100. With this range, the GA had enough points to construct a valid table structure while not having too many points to optimize. The number of sites could not be varied during the search. Nevertheless, the sites can move across the table boundary (Fig. 3(b)) to reduce or increase the number of cells in the table structure.

$$\text{genome} = \begin{pmatrix} x_{1,1} & x_{2,1} \\ x_{1,2} & x_{2,2} \\ \vdots & \vdots \\ x_{1,70} & x_{2,70} \end{pmatrix} \quad (6)$$

To mutate the structures, the Voronoi sites were randomly moved by a quantity  $\delta_{i,j}$ , varying between 0% and 10% around  $x_{i,j}$  at the beginning and decreasing linearly until the maximum number of generations is reached. To crossover two individual solutions, their vector of Voronoi site coordinates was exchanged at a random mutation point  $p$ .

**4.4.2 Fitness Function and Selection.** The constraints were handled, following the Lexcoht method described in Sec. 4.1. At each generation and for each individual, the structural requirements were tested first, then the cell wall length, and, finally, the cell angle. The individuals that possessed a larger displacement than allowed ( $a_{\text{allowed}}=2.5$  mm) were scored the lowest. The score is based on Eq. (7a) with  $\max(a)$  being the largest displacement  $a$  measured in the individual. The individuals passing the structural requirement but not the shortest wall requirement ( $l_{\text{allowed}}=30$  mm) were scored the second lowest. The score is computed using Eq. (7b), where  $\min(l)$  is the shortest cell wall found in an individual. Individuals passing the first two requirements but containing cells with bending angles sharper than  $\alpha_{\text{allowed}}=33$  deg got a score according to Eq. (7c), where  $\min(\alpha)$  is the smallest angle measured in an individual. Finally, the individuals that passed all the constraints were given a score  $p_4$  inversely proportional to their number of cells, Eq. (7d), where  $n$  is the number of Voronoi cells in the structure.

$$f_1 = 0 + \begin{cases} \frac{a_{\text{allowed}}}{\max(a)} & \text{if } \max(a) \geq a_{\text{allowed}} \\ 1 & \text{else(constraint 1 fulfilled)} \end{cases} \quad (7a)$$

$$f_2 = 1 + \begin{cases} \frac{\min(l)}{l_{\text{allowed}}} & \text{if } \min(l) \leq l_{\text{allowed}} \\ 1 & \text{else(constraint 2 fulfilled)} \end{cases} \quad (7b)$$

$$f_3 = 2 + \begin{cases} \frac{\min(\alpha)}{\alpha_{\text{allowed}}} & \text{if } \min(\alpha) \leq \alpha_{\text{allowed}} \\ 1 & \text{else(constraint 3 fulfilled)} \end{cases} \quad (7c)$$

$$f_4 = 3 + \frac{n_V - n}{n_V - 1} \quad (7d)$$

An individual that does not fulfill any constraint gets the score  $f_1$  and the other scores are discarded. Likewise, an individual that passes the first constraint but not the second gets the score  $f_2$  and an individual that passes the first constraint but not the second gets the score  $f_3$ . Finally, an individual that passes all constraints gets the score  $f_4$ . The scores  $f_1, f_2, f_3$ , and  $f_4$  have been normalized and offset such that

$$0 < f_1 \leq 1 < f_2 \leq 2 < f_3 \leq 3 < f_4 \leq 4 \quad (8)$$

With Eq. (8), an individual fulfilling  $m$  constraints is certain to get a higher score  $f_m$  than an individual fulfilling  $m-k$  constraints (and gets a score  $f_{m-k}$ ,  $k=1, \dots, m-1$ ). The advantage of this scoring system over a weighted sum is that if some constraints are not feasible, this can be discovered quickly.

After the scoring is done, a number of individuals are selected to populate the next generation. The individuals are ranked, following their score. The probability for an individual solution to be selected is related to its ranking.

The scoring and selection algorithm is represented in Fig. 4.

**4.4.3 Termination.** To ensure the convergence of the optimization, while still keeping the feedback time to the user reasonable, the termination of the optimization was set to occur after 600 generations, which is a moderate number of generations for truss optimization problems according to Giger and Ermanni [40].

**4.5 The Table Generation System Software.** The code was developed in MATLAB<sup>®</sup>. An interface has been developed for the user to draw the table boundary and indicate the leg positions. The implementation of GA is based on MATLAB's<sup>®</sup> genetic algorithm solver in the global optimization toolbox. To evaluate the structural stability, a finite element package developed at Lund University (Lund, Sweden) denoted CALFEM<sup>®</sup> [41] was used. This package makes it possible to calculate displacements and stresses in a structure by defining the structure's degrees of freedom, their coordinates, how they are connected to each other, and the boundary conditions for each degree of freedom. Each cell wall was represented by a beam element. The load was applied evenly across the nodes of the frame to simulate an even pressure from the glass top. The nodes that were meant to be leg nodes and lead down to the floor were set as fixed in all six degrees of freedom. Using the functions provided by CALFEM<sup>®</sup>, it was then possible to analyze displacements and stresses in the structure. MATLAB<sup>®</sup> has its own function to generate a Voronoi diagram [42].

**4.6 Results.** Three different boundaries were used to test the application: a dinner table of dimensions 2000 mm(L)  $\times$  1000 mm(B)  $\times$  750 mm(H), see Fig. 3(a), a coffee table (1000  $\times$  1000  $\times$  250 mm<sup>3</sup>), and a side table (500  $\times$  500  $\times$  500 mm<sup>3</sup>). Two runs were performed before the final tables were chosen. The first search for suitable individuals used a population of 50 individuals and 600 generations (a moderate number of generations for truss problems, as pointed out in Ref. [40]). The search took approximately 1.5 h of CPU time on a single core 3.0 GHz processor. After the first search was done, the user was presented with the different possible solutions. The selected individuals were then further optimized for another 600 generations in separate searches; the resulting best individuals from the different populations were then presented to the user for a final choice. An example of the result for the coffee table is presented in Fig. 5.

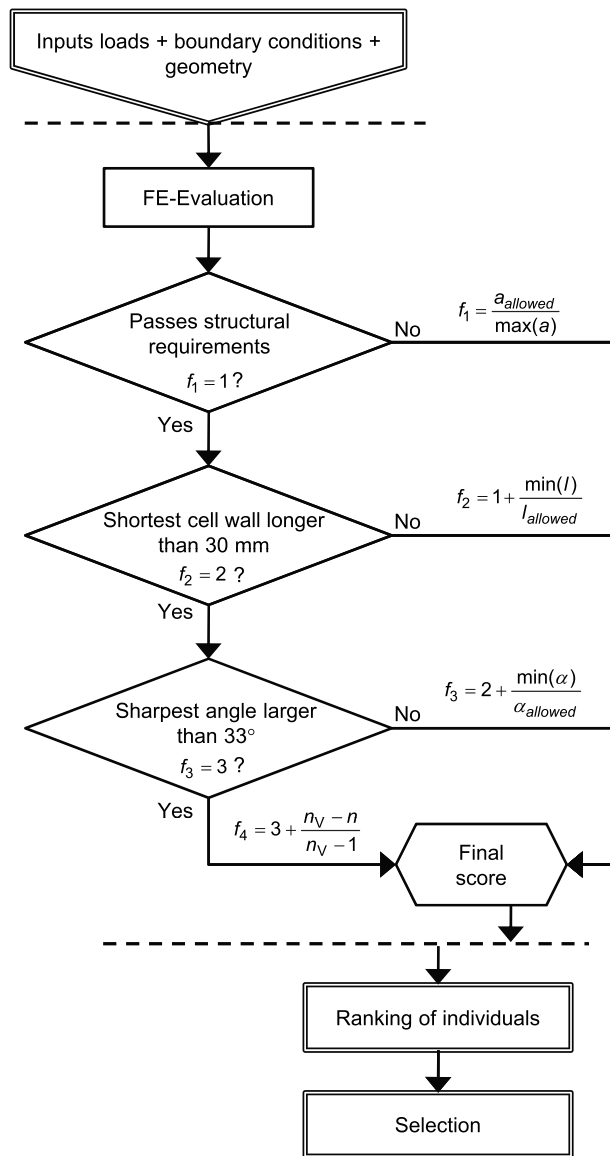


Fig. 4 Diagram of the evaluation function

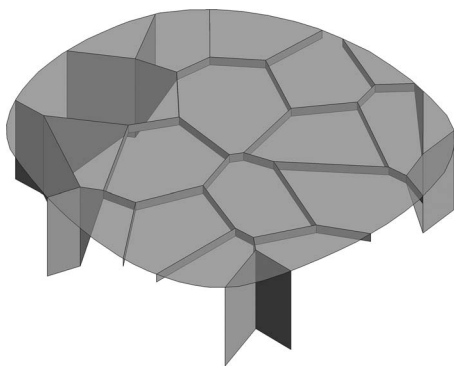


Fig. 5 The final optimized structure of the coffee table

The resulting table structures were studied in detail in ANSYS® that confirmed that the structural constraints were respected. Prototypes of all three tables have been built and were exhibited at international design fairs.

## 5 Discussion and Conclusion

This paper has proposed an approach that addresses three issues relevant to the industry: (1) the low integration of industrial design into the product development process, (2) the limited morphological repertoire currently available to designers, and (3) the limited possibilities of product form customization. These issues have been tackled separately in literature but have not been integrated. The proposed approach is to couple complex morphologies with an interactive optimization system. By using complex morphologies, designers can deal with forms they could scarcely imagine. Systems containing these formalized structures can then be optimized, taking into account aesthetical and functional engineering and manufacturing constraints. The user/customer can customize parts of the object, some of which will be required for the initialization of the optimization process (material, contours, and morphology), and then interact with the optimization system by selecting the resulting products according to her or his preferences. This approach allows for a true mass-customization without resorting to rapid prototyping. With an integration of morphologies with engineering and manufacturing constraints, the iterations between industrial designers and engineers are reduced.

The optimization problem contains a mixed (continuous and discrete) set of constraints, and the morphologies are not described by linear equations; therefore, stochastic search algorithms, such as GA, are recommended for finding solutions. Special emphasis must also be put on constraints: The solution space for engineering problems is often small and scarce, and some constraints are either hard or time-consuming to fulfill (requires extensive FEA). In our application, they have been prioritized accordingly.

In our approach, we propose that the multi-objective minimization be handled by a weighted sum. With the Lexcoht approach, the individuals fulfilling all the constraints can readily minimize the objective functions while the others are still evolving in other areas of the search space. However, the weighted sum is not always an efficient multi-objective optimization approach and the setting of the weights may be an arduous task (p. vi in Ref. [43]). Alternatively, one could separate the constraint-handling activity from the objective optimization (see, e.g., Ref. [44]), allowing the use of many more multi-objective optimization techniques. Making the population evolve until a certain percentage of the population fulfills all constraints could do this. These individuals would then be used for the optimization of the objective functions. This promising alternative requires further research.

We have now successfully tested our approach with another type of furniture (a bookshelf, see Ref. [45]). It is not difficult to imagine many other applications for the use of morphologies from nature and mathematics in design. For example, many 2.5D objects can be designed with regular or irregular tessellations (see Fig. 1(c)), furniture (as above), flooring, and wall elements are obvious examples, others are façade elements (window grates and balustrades), enclosure elements (wind deflectors and noise barriers), driveway elements (drainage gates and banisters), etc. (a product typology of such objects is available upon request). Our approach may also be used for parts of other products as long as the interface with the other parts is well defined. It could lead to new business strategies and models, building on augmented design automation and customer involvement instead of the traditional business model development-manufacturing-distribution-consumption.

The table generating system takes some engineering and manufacturing constraints into account but the potential issues linked with automating the production preparation (process planning and computer-aided manufacturing (CAM)) and detailed FEA need to be further investigated. The use of more advanced algorithms may enhance the heuristic proposed.

The table generating system in its current form does not allow for instant feedback. Even efficient, fully parallelized algorithms, while reducing the search time dramatically, will not solve this



general problem. Generative systems may simply take too long and, if the system is not designed carefully, the user may get to a point, where no design can be generated, given the constraints imposed. The impact of these issues on the user is under investigation, see Ref. [45]. It is necessary to consider usability features in such systems with special attention.

Another important point is the effort required to develop a dedicated application. Even if a high degree of freedom is conceded to the user, many form characteristics have to be frozen during the application development. In our example, the tables were optimized with constant Voronoi cell height. Nevertheless, the tables may be considered visually more pleasing and interesting if the cell heights in the final models are varied.

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## References

- [1] Hopf, A., 2009, "Renaissance 2.0—Expanding the Morphologic Repertoire in Design," 24th Cumulus Conference 24/09, pp. 78–85.
- [2] Pearce, P., 1978, *Structure in Nature is a Strategy for Design*, MIT, Cambridge, MA.
- [3] Oxman, R., 2006, "Theory and Design in the First Digital Age," *Des. Stud.*, **27**(3), pp. 229–265.
- [4] Shea, K., 2004, "Explorations in Using an Aperiodic Spatial Tiling as a Design Generator," First Design Computing and Cognition Conference DCC '04, pp. 137–156.
- [5] Vincent, J. F. V., and Mann, D. L., 2002, "Systematic Technology Transfer From Biology to Engineering," *Philos. Trans. R. Soc. London, Ser. A*, **360**(1791), pp. 159–173.
- [6] Chakrabarti, A., Sarkar, P., Leelavathamma, B., and Nataraju, B. S., 2005, "A Functional Representation for Aiding Biomimetic and Artificial Inspiration of New Ideas," *Artif. Intell. Eng. Des. Anal. Manuf.*, **19**(2), pp. 113–132.
- [7] Velcro, S. A., 1955, "Improvements in or Relating to a Method and a Device for Producing a Velvet Type Fabric," Swiss Patent No. 721338.
- [8] Hutchinson, H., 2008, "Beyond the Shark Skin Suit," *Mech. Eng. (Am. Soc. Mech. Eng.)*, **130**(4), pp. 42–44.
- [9] McCormack, J., Dorin, A., and Innocent, T., 2004, "Generative Design: A Paradigm for Design Research," *Futureground*.
- [10] Knight, T. W., 1980, "The Generation of Hepplewhite-Style Chair-Back Designs," *Environ. Plann. B*, **7**(2), pp. 227–238.
- [11] Orsborn, S., Cagan, J., Pawlicki, R., and Smith, R. C., 2006, "Creating Cross-Over Vehicles: Defining and Combining Vehicle Classes Using Shape Grammars," *Artif. Intell. Eng. Des. Anal. Manuf.*, **20**(3), pp. 217–246.
- [12] McCormack, J. P., Cagan, J., and Vogel, C. M., 2004, "Speaking the Buick Language: Capturing, Understanding, and Exploring Brand Identity With Shape Grammars," *Des. Stud.*, **25**(1), pp. 1–29.
- [13] Pugliese, M. J., and Cagan, J., 2002, "Capturing a Rebel: Modeling the Harley-Davidson Brand Through a Motorcycle Shape Grammar," *Res. Eng. Des.*, **13**(3), pp. 139–156.
- [14] Chau, H. H., Chen, X., McKay, A., and de Pennington, A., 2004, "Evaluation of a 3D Shape Grammar Implementation," First Design Computing and Cognition Conference DCC '04, pp. 357–376.
- [15] Shea, K., and Cagan, J., 1999, "Languages and Semantics of Grammatical Discrete Structures," *Artif. Intell. Eng. Des. Anal. Manuf.*, **13**(4), pp. 241–251.
- [16] Trubidge, D., 2004, "Coral [Lamp]," *Desire—The Shape of Things to Come*, R. Klanten, S. Ehmann, A. Kupetz, S. Moreno, and A. Mollard, eds., Die Gestalten Verlag, Berlin, p. 113.
- [17] Wertel, J., and Oberfell, G., 2007, "Fractal-T [Table]," *Desire—The Shape of Things to Come*, R. Klanten, S. Ehmann, A. Kupetz, S. Moreno, and A. Mollard, eds., Die Gestalten Verlag, Berlin, p. 134.
- [18] van der Veer, B., 2007, "Paper [Table]," *Desire—The Shape of Things to Come*, R. Klanten, S. Ehmann, A. Kupetz, S. Moreno, and A. Mollard, eds., Die Gestalten Verlag, Berlin, p. 99.
- [19] Mayor, J., 2007, "Burnout Bench [Bench]," *Desire—The Shape of Things to Come*, R. Klanten, S. Ehmann, A. Kupetz, S. Moreno, and A. Mollard, eds., Die Gestalten Verlag, Berlin, p. 117.
- [20] Sharfman, B., 2006, "Mercedes and the Boxfish," *The Scientist*, **20**(9), pp. 17–18.
- [21] Cheng, K., Harrison, D. K., and Jiang, Z., 2002, "An Integrated Concurrent Engineering Approach to the Design and Manufacture of Complex Components," *Int. J. Adv. Manuf. Technol.*, **20**(5), pp. 319–325.
- [22] Simpson, T. W., Siddique, Z., and Jiao, J. R., 2005, *Product Platform and Product Family Design: Methods and Applications*, Springer, New York.
- [23] Huang, G. Q., Li, L., and Chen, X., 2007, "A Tandem Evolutionary Algorithm for Platform Product Customization," *ASME J. Comput. Inf. Sci. Eng.*, **7**(2), pp. 151–159.
- [24] Lakhani, K. R., and Kanji, Z., 2009, *Threadless: The Business of Community*, Harvard Business School Multimedia/Video Case 608–707, Harvard Business School, Cambridge, MA.
- [25] von Hippel, E., 1988, *The Sources of Innovation*, Oxford University Press, New York, NY.
- [26] von Hippel, E., 2005, *Democratizing Innovation*, MIT, Cambridge, MA.
- [27] Blecker, T., and Friedrich, G., 2006, *Mass Customization: Challenges and Solutions*, Springer Science+Business Media, Inc., Boston, MA.
- [28] Miettinen, K., 2008, "Introduction to Multiobjective Optimization: Noninteractive Approaches," *Multiobjective Optimization: Interactive and Evolutionary Approaches*, D. Hutchison, J. Branke, K. Deb, T. Kanade, J. Kittler, J. M. Kleinberg, F. Mattern, K. Miettinen, J. C. Mitchell, M. Naor, O. Nierstrasz, C. Pandu Rangan, R. Slowinski, B. Steffen, M. Sudan, D. Terzopoulos, D. Tygar, M. Y. Vardi, and G. Weikum, eds., Springer, Berlin, pp. 1–26.
- [29] Miettinen, K., Ruiz, F., and Wierzbicki, A. P., 2008, "Introduction to Multiobjective Optimization: Interactive Approaches," *Multiobjective Optimization: Interactive and Evolutionary Approaches*, D. Hutchison, J. Branke, K. Deb, T. Kanade, J. Kittler, J. M. Kleinberg, F. Mattern, K. Miettinen, J. C. Mitchell, M. Naor, O. Nierstrasz, C. Pandu Rangan, R. Slowinski, B. Steffen, M. Sudan, D. Terzopoulos, D. Tygar, M. Y. Vardi, and G. Weikum, eds., Springer, Berlin, pp. 27–57.
- [30] Koza, J. R., Keane, M. A., Streeter, M. J., Adams, T. P., and Jones, L. W., 2004, "Invention and Creativity in Automated Design by Means of Genetic Programming," *Artif. Intell. Eng. Des. Anal. Manuf.*, **18**(3), pp. 245–269.
- [31] Holland, J. H., 1975, *Adaptation in Natural and Artificial Systems: An Introductory Analysis With Applications to Biology, Control and Artificial Intelligence*, University of Michigan Press, Ann Arbor, MI.
- [32] Coello, C. A., 2000, "An Updated Survey of GA-Based Multiobjective Optimization Techniques," *ACM Comput. Surv.*, **32**(2), pp. 109–143.
- [33] Aurenhammer, F., 1991, "Voronoi Diagrams—A Survey of a Fundamental Geometric Data Structure," *ACM Comput. Surv.*, **23**(3), pp. 345–405.
- [34] Beukers, A., and van Hinte, E., 2005, *Lightness: The Inevitable Renaissance of Minimum Energy Structures*, 4th ed., 010 Publishers, Rotterdam.
- [35] de Berg, M., Cheong, O., van Kreveld, M., and Overmars, M., 2008, *Computational Geometry—Algorithms and Applications*, Springer-Verlag, Berlin.
- [36] Fayolle, C., 2006, "The Future is Less to Discover Than to Invent (in French. Original Title: L'avenir est moins à découvrir qu'à inventer)," Biennale Internationale Design Saint-Etienne, Cité du Design, Saint-Etienne, E. Francès, ed., Vol. 336, p. 126.
- [37] TMF/SCB, 2009, TMF in Figures—Statistics on the Swedish Tree and Furniture Industry (in Swedish. Original Title: TMF i siffror—Statistik om den svenska trä- och möbelindustrin), No. 1, MTF, Stockholm.
- [38] FERRUM Lasercut GmbH, personal communication.
- [39] Linde Metall AB, personal communication.
- [40] Giger, M., and Ermanni, P., 2006, "Evolutionary Truss Topology Optimization Using a Graph-Based Parameterization Concept," *Struct. Multidiscip. Optim.*, **32**(4), pp. 313–326.
- [41] Austrell, P. E., Dahlblom, O., Lindemann, J., Olsson, A., Olsson, K.-G., Persson, K., Petersson, H., Ristinmaa, M., Sandberg, G., and Wernberg, P.-A., 2004, *CALFEM: A Finite Element Toolbox—Version 3.4*, Structural Mechanics LTH, Lund University, Lund.
- [42] Mathworks, 2009, "Matlab Documentation—Voronoi Function," (<http://www.mathworks.com/access/helpdesk/help/techdoc/index.html?access/helpdesk/help/techdoc/ref/voronoi.html>), last accessed: Sep. 22.
- [43] Hutchison, D., Branke, J., Deb, K., Kanade, T., Kittler, J., Kleinberg, J. M., Mattern, F., Miettinen, K., Mitchell, J. C., Naor, M., Nierstrasz, O., Pandu Rangan, C., Slowinski, R., Steffen, B., Sudan, M., Terzopoulos, D., Tygar, D., Vardi, M. Y., and Weikum, G., 2008, *Multiobjective Optimization: Interactive and Evolutionary Approaches*, Springer, Berlin.
- [44] Schoenauer, M., and Xanthakis, S., 1993, "Constrained GA Optimization," Fifth International Conference on Genetic Algorithms—ICGA '93, pp. 573–580.
- [45] Nordin, A., Motte, D., Hopf, A., Björnemo, R., and Eckhardt, C.-C., 2010, "Complex Product Form Generation in Industrial Design: A Bookshelf Based on Voronoi Diagrams," Fourth Design Computing and Cognition Conference—DCC '10, pp. 701–720.