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# Control Using Local Distance Measurements Cannot Prevent Incoherence in Platoons

Richard Pates, Carolina Lidström, and Anders Rantzer.

**Abstract**—When local control strategies are used to arrange a platoon into a string like formation, there is mounting evidence that a poorly regulated accordion like motion will emerge. In this paper we prove that this is an inevitable consequence of using local distance measurements to design the control. More specifically we demonstrate that no controller, irrespective of its dynamical complexity, sparsity, or linearity, can prevent the appearance of macroscopic behaviours in the platoon if only noisy measurements of the distances between neighbouring vehicles are available.

## I. INTRODUCTION

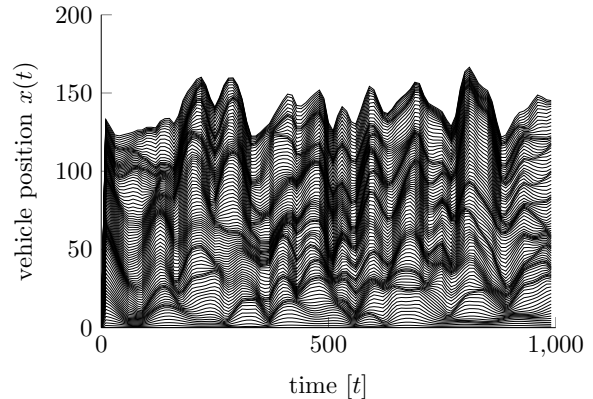
The formation control problem for a set of vehicles is arguably the simplest of the difficult network control problems. The problem is by now very well studied [1], [2], [3], [4], [5], and the basic objective is to arrange the vehicles into a line with specified constant inter-vehicle spacings. Despite this simple objective, when a range of local control strategies are used, undesirable macroscopic behaviours such as string instability [4] or oscillations on a large spacial scale [6], start to emerge. There is even growing evidence that the presence of these phenomena is an *inevitable consequence* of the type of information available for control, and the controller architecture used [6], [7]. In this paper we investigate the role of local distance measurements, that is control based on the distances between neighbouring vehicles, in the appearance of these apparently fundamental behaviours. In particular, inspired by the analysis in [6], we show that macroscopic accordion like behaviours such as those in Figure 1(a) *cannot be prevented* if only local distance measurements are available for control.

The use of automatic control has the potential to make road use significantly more efficient. This has been demonstrated in practice in, for example, the platooning of convoys of trucks [8]. Naturally similar questions have been asked about platooning vehicles to increase throughput on highways. However undesirable macroscopic behaviours have the potential to jeopardise what can be achieved here, especially when these behaviours occur on a similar scale to the length of the roads being used. The topological simplicity of the platooning problem also makes it an ideal test bed when trying to understand similar large scale behaviours in other application areas, such as inter-area oscillations in electrical

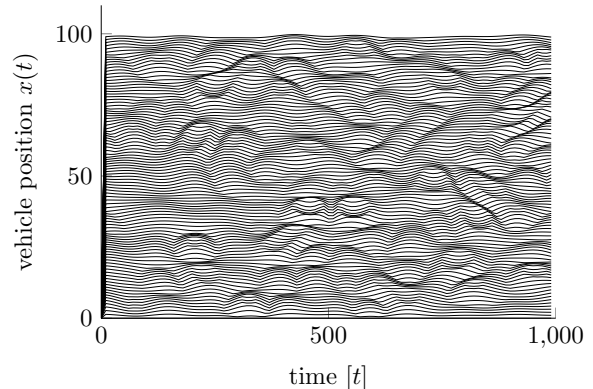
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(a) Local measurements available for control.



(b) Distances to leader available for control.

Fig. 1. Illustration of the behaviour of a platoon of 100 vehicles under two different measurement strategies. Each figure consists of 100 lines, each of which shows how the position of a single vehicle evolves over time. In (a), only measurements of the distances between neighbouring vehicles are available to the controller, and in (b) only distances relative to the leader are available. Both these measurements are subject to noise, and the figures have been generated when the measurements are subject to the *same* noise vector. Just as observed in Figure 1 of [6], in (a) a macroscopic accordion like behaviour emerges. This is not present in (b), where the designed control successfully regulates the large scale behaviour of the platoon.

power systems [9]. Questions as to the fundamental nature of such macroscopic phenomena are then of paramount importance when considering the control architectures that should be employed in these applications. For example it has been observed that if measurements of the absolute positions of the vehicles are available, controllers that eliminate undesirable macroscopic behaviours can be designed [6]. However this clearly comes at a cost, both financially and in the complexity of the controller. The question is then, is

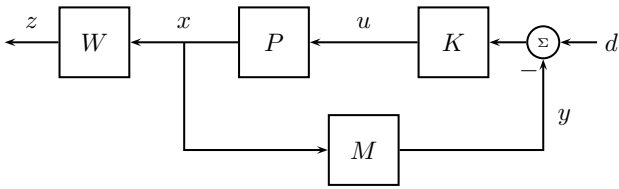


Fig. 2. Block diagram used to describe platooning problems.

this really necessary?

The main contribution of this paper is to show that if only noisy local distance measurements are available to design the control, then the accordion like behaviour shown in Figure 1(a) cannot be prevented. In particular, these guarantees come independently of the locality of the implemented controller. That is, even if every vehicle had a noisy measurement of the distance between *every* neighbouring vehicle in the platoon, there exists no controller, linear or nonlinear, that can eliminate an accordion like motion in the platoon. This is in contrast to existing explanations of this behaviour, which typically attribute this to a combination of factors, including the use of local control, and relative measurements.

The intuitive explanation for our observation is that these local measurements contain insufficient information about the large scale behaviour of the platoon to allow it to be regulated. We illustrate this point by additionally demonstrating that if instead only the distances to the leader are known, this behaviour can be eliminated, as shown in Figure 1(b). Note that these are still *relative* measurements, and this is not the same as knowing the global position of the lead vehicle. The explanation in this case is that this time we have information at a range of length scales, and it is this fact that allows the large scale behaviours to be regulated.

## II. RESULTS

In this section we present a performance limit that applies to any system that can be modelled by the block diagram in Figure 2. In particular we show that the structure of  $M$  imposes a lower bound on the gain between the external disturbance  $d$  and performance output  $z$ . The focus of this section is purely on the mathematical derivation of this result. We will give it an extensive interpretation from the perspective of platooning problems in Section III.

We use an operator theoretic notation throughout.  $\mathcal{L}_2^n$  denotes the space of  $n$  vectors of signals defined for positive time with bounded energy

$$\|w\|^2 := \int_0^\infty w^T(t) w(t) dt.$$

This is a subspace of  $\mathcal{L}_{2e}^n$ , whose members need only be square integrable on finite intervals. Operators are mappings between  $\mathcal{L}_{2e}$  spaces, and we use the shorthand  $Aw$  to denote the signal obtained by mapping the signal  $w$  through the operator  $A$ . The gain of an operator  $A$  is given by

$$\|A\| := \sup_{w \in \mathcal{L}_2^n: w \neq 0} \frac{\|Aw\|}{\|w\|}. \quad (1)$$

The block diagram in Figure 2 encodes the following feedback configuration:

$$\begin{aligned} z &= Wx, \\ x &= Pu, \\ y &= Mx, \\ u &= K(d - y). \end{aligned} \quad (2)$$

Here  $z, x, u \in \mathcal{L}_{2e}^n$ ,  $y, d \in \mathcal{L}_{2e}^{n-1}$ , and  $W, P, M, K$  are causal operators between appropriately dimensioned  $\mathcal{L}_{2e}$  spaces. We use the shorthand  $T_{zd}$  to denote the operator from  $d$  to  $z$  as defined by eq. (2), and use a similar notation for the maps from  $d$  to the other signals. We assume throughout that these maps are well posed.

All the results in this paper are contingent on the following additional assumptions.

(A1) For all  $v \in \mathbb{R}^{n-1}$ ,

$$\lim_{t \rightarrow \infty} T_{yd} v H(t) = v,$$

where  $H \in \mathcal{L}_{2e}$  is the unit step function ( $T_{yd} v H$  is the signal obtained by mapping  $vH$  through  $T_{yd}$ ).

(A2)

$$W := \frac{1}{n} \left( I_n - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^T \right),$$

where  $I_n$  is the  $n \times n$  identity matrix and  $\mathbf{1}_n$  the  $n$  vector of ones.

(A1) essentially implies the presence of integral action in the feedback loop, since it states that  $d(t) - y(t)$  tends to zero in response to step disturbances. (A2) simply specifies how  $z$  depends on  $x$ . With these assumptions in place, we are now ready to state the theoretical contribution of the paper. The following theorem gives a lower bound on the gain of  $T_{zd}$  that depends only on  $M$ . Observe in particular that there is a fundamental difference in how the bound scales with the problem size  $n$ , which will later correspond to the number of vehicles. In the first case the lower bound it tends to a positive number for large  $n$ , and in the second it tends to zero.

*Theorem 1:* Define all signals and operators as in eq. (2), and assume (A1–2). If

$$M = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix},$$

then

$$\|T_{zd}\| \geq \frac{1}{2n \sin\left(\frac{\pi}{2n}\right)} \geq \frac{1}{\pi}.$$

If

$$M = [1_{n-1} \quad -I_{n-1}],$$

then

$$\|T_{zd}\| \geq \frac{1}{n}.$$

*Proof:* First note that since both  $M$ 's are incidence matrices for connected graphs, it follows that  $W = \frac{1}{n} M^\dagger M$ ,

where ‘ $\dagger$ ’ denotes the Moore-Penrose pseudoinverse. This implies that

$$\begin{aligned} T_{zd} &= WT_{xd} \\ &= \frac{1}{n} M^\dagger M T_{xd} \\ &= \frac{1}{n} M^\dagger T_{yd}. \end{aligned} \quad (3)$$

Next, define the ‘unit step on interval  $T$ ’ as

$$H_T(t) := \begin{cases} 1 & \text{if } 0 \leq t \leq T, \\ 0 & \text{otherwise.} \end{cases}$$

Recalling the definition of the gain of an operator in eq. (1), by simply restricting the class of disturbance signals  $d$  we get the following lower bound, which is valid for any  $v \in \mathbb{R}^{n-1}$  and  $T > 0$ :

$$\|T_{zd}\| \geq \frac{\|T_{zd}vH_T\|}{\|vH_T\|}.$$

By instead using the  $\mathcal{L}_2[0,T]$  norm, which we denote  $\|\cdot\|_{\mathcal{L}_2^T}$ , we get a further lower bound, also valid for any  $v$  and  $T$ :

$$\|T_{zd}\| \geq \frac{\|T_{zd}vH_T\|_{\mathcal{L}_2^T}}{\sqrt{T}\|v\|}.$$

Substituting in for  $T_{zd}$  from eq. (3) and applying the reverse triangle inequality gives

$$\begin{aligned} \|T_{zd}\| &\geq \frac{\|\frac{1}{n}M^\dagger T_{yd}vH_T\|_{\mathcal{L}_2^T}}{\sqrt{T}\|v\|}, \\ &\geq \left| \frac{\|M^\dagger vH_T\|_{\mathcal{L}_2^T}}{n\sqrt{T}\|v\|} - \frac{\|M^\dagger e\|_{\mathcal{L}_2^T}}{n\sqrt{T}\|v\|} \right|, \end{aligned} \quad (4)$$

where  $e = (vH_T - T_{yd}vH_T)$ . Now examine the second term on the right hand side of the above. Clearly

$$\begin{aligned} \frac{\|M^\dagger e\|_{\mathcal{L}_2^T}}{n\sqrt{T}\|v\|} &\leq \frac{\|M^\dagger\| \|e\|_{\mathcal{L}_2^T}}{n\sqrt{T}\|v\|} \\ &= \frac{\|M^\dagger\|}{n\|v\|} \sqrt{\frac{1}{T} \int_0^T e(t)^T e(t) dt}. \end{aligned}$$

By L’Hôpital’s rule

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T e(t)^T e(t) dt = \lim_{T \rightarrow \infty} e(T)^T e(T),$$

which equals zero by (A1). It therefore follows from the above and eq. (4) that

$$\begin{aligned} \|T_{zd}\| &\geq \lim_{T \rightarrow \infty} \frac{\|M^\dagger vH_T\|_{\mathcal{L}_2^T}}{n\sqrt{T}\|v\|} \\ &= \frac{\|M^\dagger v\|}{n\|v\|}. \end{aligned}$$

Since the above holds for any  $v$ , we arrive at the lower bound  $\|T_{zd}\| \geq \frac{1}{n} \|M^\dagger\|$ . The result follows by computing  $\|M^\dagger\|$ .

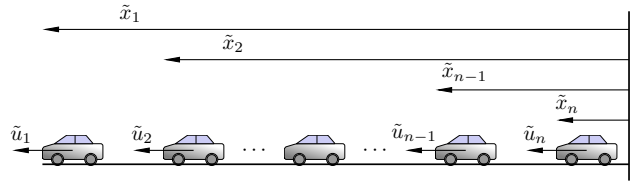


Fig. 3. Vehicle platoon with  $n$  cars. The position of car  $i$  is specified by  $\tilde{x}_i$  and measured relative a global reference frame. Each car has a control input  $\tilde{u}_i$  available, which can be used to adjust its position.

*Case 1:* Since  $M^\dagger = M^T (MM^T)^{-1}$ ,  $\|M^\dagger\|^2$  is equal 1 over the smallest eigenvalue of  $MM^T$ . Now

$$MM^T = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & & -1 & 2 \end{bmatrix}.$$

This matrix is very closely related to the Laplacian for the path graph, and is diagonalised by

$$Q_{ik} = \sin\left(\frac{ik\pi}{n}\right).$$

The corresponding eigenvalues are  $(2 \sin(\frac{i\pi}{2n}))^2$ . Therefore

$$\|M^\dagger\| = \frac{1}{2 \sin(\frac{\pi}{2n})}.$$

*Case 2:* In this case

$$MM^T = I_{n-1} + 1_{n-1} 1_{n-1}^T.$$

By proceeding as before, the result follows easily.  $\blacksquare$

### III. DISCUSSION

*A. Is Theorem 1 applicable to platooning problems?*

We consider the problem of how to adjust the inputs to individual cars in a platoon so that they can maintain a string like formation in the presence of disturbances. The basic setup is shown in Figure 3. In this figure the signal  $\tilde{x} \in \mathcal{L}_{2e}^n$  gives the positions of the vehicles in a fixed reference frame, and  $\tilde{u} \in \mathcal{L}_{2e}^n$  the control inputs available to manipulate the vehicles. We make the following assumption about the platoon.

(A3) The desired inter-vehicle spacings are all equal to  $L$ . Furthermore, there exists a nominal input  $\tilde{u}_0$  such that if  $\tilde{u} \equiv \tilde{u}_0$ , then the resulting vehicle positions  $\tilde{x} \equiv \tilde{x}_0$  satisfy

$$\tilde{x}_{i-1}(t) - \tilde{x}_i(t) = L, \quad \forall t > 0.$$

(A3) means that, in the absence of disturbances, it is possible to arrange the platoon into a string like formation. In this section we will show how to use eq. (2) (the block diagram in Figure 2) to model a wide range of control strategies for maintaining the desired formation  $\tilde{x}_0$  in the presence of disturbances.

In this context, the signals  $x$  and  $u$  are

$$\begin{aligned} x &= \tilde{x} - \tilde{x}_0, \\ u &= \tilde{u} - \tilde{u}_0. \end{aligned}$$

That is they are the positions and inputs of the vehicles relative to those in the nominal platoon behaviour. The operators  $P, M, K$  specify the vehicle dynamics, the information about  $x$  that is available to the controller (the signal  $y$ ), and the controller dynamics respectively. The disturbance  $d$  adds noise to the measurement  $y$ . In the following we will go into each of these in more detail, explaining why  $T_{yd}$  satisfies (A1) if typical vehicle dynamics are considered, and also why so many different ‘standard’ controller architectures for solving this platooning problem are covered.

1) *Vehicle dynamics*: The only restriction imposed on the vehicle dynamics is that they are captured by a causal operator mapping input to position (relative to the nominal platoon behaviour). This is satisfied by every simple vehicle model in the literature. For example, if each car is modelled as a point mass that can be manipulated by a force, then

$$m_i \frac{d^2}{dt^2} x_i(t) = u_i(t),$$

which clearly has such an operator representation. It can also cover more sophisticated car models, which may also be performing *local feedback*. For example, the model

$$\begin{aligned} m_i \frac{d^2}{dt^2} x_i(t) &= v_i(t), \\ a_i \frac{d}{dt} v_i(t) + v_i(t) &= u_i(t) - k_i \frac{d}{dt} x_i(t), \end{aligned}$$

which includes simple throttle dynamics and proportional feedback on velocity, also has an operator representation. Observe also that both these models guarantee that (A1) is satisfied (provided  $T_{yd}$  is stable). This is because in both cases there is at least one integrator in the mapping from  $u_i$  to  $x_i$ . As a result of this integral action, if  $d \equiv vH$  (where  $H$  is the unit step), then for any  $v \in \mathbb{R}^{n-1}$

$$\lim_{t \rightarrow \infty} y(t) = v.$$

This is precisely (A1). Therefore in order to satisfy the assumptions required by Theorem 1, we need only restrict the vehicle dynamics to have an operator representation from input to position, that contains at least one pure integrator.

2) *Signal available to the controller*: The two  $M$  matrices considered in Theorem 1 correspond to making the signals

$$y = \begin{bmatrix} x_1 - x_2 \\ \vdots \\ x_{n-1} - x_n \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} x_1 - x_2 \\ \vdots \\ x_1 - x_n \end{bmatrix}$$

available for control. Therefore the first case only allows the measurement of the distances between neighbouring vehicles. This is the setup considered in almost all prior work on vehicle platoons. The second case has largely been chosen to provide contrast, and corresponds to only allowing the distances to the lead vehicle to be measured.

3) *Controller dynamics*: Since the vehicle dynamics ensure the satisfaction of (A1), the only restriction that we impose on  $K$  is that can be described by a causal operator. In particular this means that virtually every existing controller that aims to set inter-vehicle displacements using local distance measurements is permitted, and is therefore subject to the first performance bound in Theorem 1. For example, in this case the control law

$$u_i = \begin{cases} c_i (d_i - y_i) & \text{if } i = 1 \\ k_{i-1} (d_{i-1} - y_{i-1}) & \text{if } i = n \\ k_{i-1} (d_{i-1} - y_{i-1}) + c_i (d_i - y_i) & \text{otherwise} \end{cases}$$

where  $c_i, k_i \in \mathbb{R}$ , corresponds to a simple heterogeneous bidirectional control strategy. Furthermore the conditions of the theorem clearly continue to hold even if  $u_i$  depends on a wider range of local distance measurements, the controller gains  $c_i, k_i$  are replaced with dynamic or even nonlinear relationships, or even if the controller is dense (every vehicle has every measurement in  $y$ ).

*B. What does Theorem 1 mean for platooning problems?*

In the previous section we established that the bounds in Theorem 1 must hold even when a very broad class of controllers for regulating the inter-vehicle displacements are used. But what does the lower bound in Theorem 1 mean in the context of the platooning problem? In the following we will show that Theorem 1 implies in the first case there exist disturbances  $d$  that cause behaviours on the scale of the platoon that cannot be attenuated, whereas no such conclusion can be drawn in the second case.

To measure the spread of vehicles in the platoon, we introduce the ‘average spread’ of a signal, which we define as follows

$$\bar{\sigma}(w) := \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \frac{1}{n} \sum_{i=1}^n (w_i(t) - \bar{w}(t))^2 dt}. \quad (5)$$

In the above  $\bar{w}(t)$  is the average of  $w(t)$ , that is

$$\bar{w}(t) := \frac{1}{n} \sum_{i=1}^n w_i(t).$$

Hence the term inside the integral in eq. (5) is the second moment of  $w(t)$ , which measures the spread of  $w(t)$  at time  $t$ . Therefore  $\bar{\sigma}(w)$  is the square root of the time average spread of  $w(t)$  across time.

We will now demonstrate that Theorem 1 proves that there exist disturbances  $d$  such that

$$\bar{\sigma} \left( \frac{\tilde{x} - \tilde{x}_0}{\bar{\sigma}(\tilde{x}_0)} \right) \propto \|T_{zd}\|. \quad (6)$$

The term on the left is the average spread of the vehicle positions as normalised by their desired positions  $\tilde{x}_0$ . This shows that in the local distance measurement case there exist disturbances which cause the vehicle positions to change in a manner that is comparable with the desired behaviour, that cannot be attenuated by any controller. For large  $n$ , no such

limit is imposed for the case of measurements relative to the leader.

The connection between eq. (6) and Theorem 1 hinges on the connection between  $\bar{\sigma}(x)$  and the signal power of  $z$ . First recall the definition of (the square root of) the power of a signal  $w$ :

$$\mathcal{P}(w) := \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \sum_{i=1}^n w_i(t)^2 dt}.$$

(A2) implies that

$$z(t) = \frac{1}{n} (x_i(t) - \bar{x}(t)),$$

from which it follows that

$$\mathcal{P}(z) = \frac{1}{\sqrt{n}} \bar{\sigma}(x). \quad (7)$$

As is well known, the gain of an operator, and the power gain of an operator, are the same, and so there exist disturbances  $d$  such that for arbitrarily small  $\epsilon > 0$ ,

$$\mathcal{P}(z) = (\|T_{zd}\| - \epsilon) \mathcal{P}(d).$$

In light of eq. (7), this means that there exist disturbances such that

$$\frac{\bar{\sigma}(x)}{\sqrt{n} \mathcal{P}(d)} = (\|T_{zd}\| - \epsilon). \quad (8)$$

Since the above holds for arbitrarily small  $\epsilon$ , the lower bounds in Theorem 1 impose fundamental limits on how well the average spread of the vehicles in the platoon can be regulated. Whether or not this results in macroscopic behaviour now depends on the size of the disturbances. We assume the following.

(A4) The power of the disturbance equals

$$\mathcal{P}(d) = \eta \sqrt{n-1}.$$

Roughly speaking<sup>1</sup>, this corresponds to each individual measurement being subject to a disturbance of size  $\eta$ .

A routine calculation shows that

$$\bar{\sigma}(\tilde{x}_0) = L \sqrt{\frac{n^2 - 1}{12}}.$$

Therefore under (A1–4), it follows from eq. (8) that there exists disturbances such that

$$\bar{\sigma} \left( \frac{\tilde{x} - \tilde{x}_0}{\bar{\sigma}(\tilde{x}_0)} \right) = \frac{\eta}{L} \sqrt{\frac{12}{1 + \frac{1}{n}}} (\|T_{zd}\| - \epsilon).$$

Hence there exist disturbances such that the normalised average spread of the vehicle positions scales, to all intents and purposes, proportionally with  $\|T_{zd}\|$ . The impact of such a disturbance is particularly significant when the disturbance size  $\eta$  is high or the desired inter-vehicle spacing is small.

<sup>1</sup>This class of disturbances includes those that have their signal power uniformly distributed through the individual disturbances, but also those which focus the full power into an individual disturbance. A disturbance of this second type is not a realistic prospect, however we will see in the next section that the worst case disturbances are not of this type.

C. What does Theorem 1 have to do with accordion motions?

The short answer to this question is ‘very little’. Since Theorem 1 is simply a statement about the gain of  $T_{zd}$ , it provides no intuition about why the plots in Figure 1(a) look ‘accordion like’. To understand this requires further examination of the worst case disturbances. These can be clearly seen in the proof of Theorem 1. In both cases, they are step disturbances, though anything of the form

$$d(t) \equiv v f(t),$$

where  $v \in \mathbb{R}^{n-1}$  and  $f(t)$  is a slowly varying signal, is potentially troublesome. In particular, the proof shows that the worst case  $v$ 's correspond to the left singular vectors of  $M$  associated with the smallest singular values of  $M$ . For both the  $M$ 's considered, the singular values and vectors can be calculated analytically. It is not hard to show that in the first case the left singular vector

$$v_i = \sqrt{\frac{2}{n}} \sin\left(\frac{ik\pi}{2n}\right)$$

has singular value  $2 \sin\left(\frac{k\pi}{n}\right)$ . Therefore the worst case disturbances are given by signals that vary slowly in time with long spacial wavelengths. This is entirely consistent with an accordion like motion.

In the second case the vector  $v = 1_{n-1}$  has singular value  $\sqrt{n}$ , and all other others have singular values equal to 1. It is the fact that almost all the singular values are identical and far from zero that means there is no particularly bad disturbance predicted by Theorem 1 in this case. The intuitive explanation for this is that in this case the measurement  $y$  that contains information across the different length scales of the platoon.

It is precisely the above reasoning that allowed Figure 1 to be generated. There the simple car model

$$\frac{d}{dt} x_i(t) = u_i(t)$$

was used, and an optimal controller (with respect to  $\|T_{zd}\|$ ) computed for both cases. The disturbance used to generate the trajectories in this figure was of the form

$$d_i(t) \equiv \sum_{k=1}^{15} f_k(t) \sin\left(\frac{ik\pi}{2n}\right),$$

where  $f_k(t)$  were slowly varying signals. The disturbance was normalised to satisfy (A4), with  $\eta/L$  set to  $\sqrt{1/2}$ . Therefore the disturbance excites the slow time scale, large length scale, behaviour that Theorem 1 proves cannot be attenuated by any controller.

#### IV. CONCLUSION

It has been shown that large scale macroscopic behaviours cannot be prevented in platooning problems if only noisy local measurements are available for conducting control. This is not the case if measurements relative to the leader are available instead. This difference in behaviour occurs because in the first case the control has little information about the platoon across a range of length scales, whereas in the second it does.

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