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Topological model and analysis of the P2P BitTorrent protocol

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Abstract: In this paper, we propose a directed and weighted graph model for describing P2P networks using the BitTorrent protocol. Then, we analyse the topological properties of the model based on the complex networks theory. We find that the node strength follows a power-law distribution and that there exists a positive correlation between flow betweenness and out-strength. We also include some other findings about clustering and shortest path length of these networks in our work.

Keywords: betweenness; BitTorrent; clustering; complex networks; P2P; simulation.

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1 Introduction

A Peer-to-Peer (P2P) overlay network is composed of participants that make a portion of their resources directly available to others in the same network, without the need for central coordination instances (Schollmeier, 2001). Due to the increasing number of internet users globally and the fast development of digital multimedia technologies, P2P has become dominating in dealing with file-sharing and content delivery such as online TV broadcasting, Video-on-Demand (VOD), etc., for which the traditional client-server model showed its limitation and weakness with respect to bandwidth, storage space and computing power. Even some of the internet telephony applications such as Skype are favoured in using P2P technique (Baset and Schulzrinne, 2006).

Among the various popular P2P applications, BitTorrent is one of the most widely used protocols. Originally designed for file-sharing, BitTorrent is a robust P2P protocol that takes advantage of peers' bandwidth to efficiently replicate and transfer content without adding too much load to the servers. It has been shown to be very efficient with featuring a game theoretical incentive mechanism, which is used to ensure the fair distribution of content and prevent selfish peer behaviour. In BitTorrent networks, each peer has to maximise its uploading capacity while owning a wish to maximise the downloading speed, which essentially contributes to the connectivity and continuity of the networks. Also, as multimedia contents' popularity increases on the internet, BitTorrent Assisted Streaming Systems (BASSs) (Dana et al., 2005), have been deployed to provide real-time content delivery, like online video and TV broadcasting. Therefore, studying the BitTorrent network is important to investigate how this protocol works and how it can be improved. The network topology has significant impact on how peers interact and cooperate with each other, which can be a very essential issue in terms of the performance, functionality, efficiency, and resource cost of P2P networks. It makes sense to study the overlay topology of P2P networks for the purpose of uncovering the inner characteristics of certain networks that concern their sophisticated behaviours.

In recent years, the complex network theory has been widely applied to networks in many different research fields from sociology to cell biology, microelectronics to computer science. There are currently three kinds of basic topology for complex networks: random graph, small-world and scale-free networks. Random graphs are constituted with nodes and links which depend on certain probabilities between two of them. Small-world networks have small average shortest path lengths, but a clustering coefficient significantly higher than expected in random graphs. And many real networks like those of science collaboration, movie actors on IMDb, and even the internet have been found to follow a power-law scale-free degree distribution. The clustering coefficient is a measure of degree to which nodes in a graph tend to cluster together. Evidence suggests that in most real-world networks, nodes tend to create tightly knit groups characterised by a relatively high density of ties (Watts and Strogatz, 1998). The shortest path is a path between two vertices such that the sum of the weights of its constituent edges is minimised. The betweenness measure helps determine the relative importance of a vertex within the graph. Performance, robustness and stability in real networks have been studied and proved to be related to their specific structures in other works (Beygelmizer et al., 2004; Wang et al., 2009; Qi et al., 2009; Chism et al., 2009).

In previous works based on the topology of P2P systems, it has been found that communication efficiency is improved due to the topological model used in P2P networks (Theotokis and Spinellis, 2004). Network topology has also been proved to be relevant to the accessibility of resources on P2P networks (Kedar, 2004). A simple scheme for participants to build P2P networks with topological properties such as low diameter has been proposed (Pandurangan, 2001). Some specific protocols such as Small World Overlay Protocol (Hui et al., 2006) have also been proposed, which were shown to be helpful for improving object lookup performance and dealing with flash crowds efficiently in P2P networks. Further, the way the performance and network resilience of a P2P communication network depend on the topology in a two-coupled network model has also been investigated (Wang et al., 2009). In another work, the performance of a P2P video streaming system has been studied under several basic topology models such as random, small-world and scale-free graphs, which indicates that a more connected graph does not necessarily imply a higher streaming rate (Chism et al., 2009). And for another P2P protocol named Gnutella, a complex network model of a P2P network structure has been constituted with some basic analysis (Qi et al., 2009). There is another paper (Mohamad Dikshie Fauzie, 2010) which has proposed a way for studying the overlay topologies of BitTorrent networks by doing experiments on actual networks.

However, most of these works assume that the P2P network is an un-weighted or un-directed graph. One defect of these models is that links without weight or direction do not clearly describe relations between corresponding peers. It can be assumed that there is a data transfer if there exists a link between a pair of peers. However, information on the amount of data transferred, and on the source peer and the destination peer between whom the data was transferred is omitted; this includes significant information in communication networks such as P2P.

In this paper, our objective is to model and analyse an overlay P2P network using a BitTorrent protocol. To characterise the BitTorrent networks, we use the complex networks theory (Albert and Barabási, 2002) which shows potential to be useful in analysing the topology of large networks, since the structure affects the function (Strogatz, 2001). We propose a directed and weighted graph model to describe the peers' behaviour in P2P networks derived from the BitTorrent protocol. Not only the amount of data transferred, but also the direction of the transfer has been taken into consideration in the analysis of evaluating parameters such as clustering coefficient and betweenness in our model.

Therefore, our results reflect the behaviour of BitTorrent networks more precisely. As far as we know, this should be the first work that uses both directed and weighted graphs to model the topology of P2P networks and give statistical analyses. These analysis techniques such as betweenness may provide a way for P2P protocol designers to solve some open issues such as selection of best peers (Meddour, 2006) in the networks. We believe that our methodology of modelling and analysis could also be a good choice for other communication networks.

P2P is such a hot subject that it arouses lots of research activities, which vary from search algorithms (de Mello et al., 2007) to simulation methods (LaFortune et al., 2009). Generally, it can be said that testing on real and active P2P networks would not be an easy task due to its large in peers amount, and high distributing and ad-hoc characteristics, as well as the fact that the risk of affecting the network when doing experiments on it is too high (Vogeleer et al., 2008). According to BitTorrent Specifications, although it is possible for one peer to know which peers it has connections with, it is still very difficult to obtain the data amount transferred between all pairs of peers, which is needed for constructing both weighted and directed graph models. Till now, we haven't found a good way to collect data from actual BT networks for analysis. Therefore, we use the popular P2P simulator Peersim (Jelasity and etc., 2009, Frioli and Pedrolli, 2008) for simulations of the BitTorrent protocol on a variety of networks which are arranged from small ones with dozens of nodes to large ones with thousands of nodes. In real P2P systems, communication quality, which is usually decided by parameters such as bandwidth and latency, affects the data transfer between peers directly. The bandwidth is usually decided by the Internet Service Provider (ISP), while the latency is seriously affected by the physical connection between machines or computers on which the P2P application runs. Since the physical connections of the internet are well reflected by the sum of latency between routers, the internet topology generator at the router level (Quoitin et al., 2009) is used in the P2P simulation of our model. Several topological parameters such as node strength distribution, clustering coefficient (Opsahl and Panzarasa, 2009), shortest path length and betweenness (Borgatti, 2005) will be calculated to get the network anatomy. Finally, we include some findings like the scale-free topology and positive correlation between flow betweenness and out-strength of BitTorrent networks.

2 BitTorrent

For better understanding of our work, a brief introduction to how the BitTorrent protocol works is given as follows. More details about how this protocol works can be found in the BitTorrent Specifications.

The network swarm constructed by the BitTorrent protocol is constituted with three different types of components: peers, seeders, and trackers. Peers are the majority of terminals which are joined in P2P networks to download files or other resources while contributing to other participants by uploading the parts they already obtained. Peers become seeders as soon as they have finished downloading and continue uploading to others. The resource file which is being shared in the network is divided into many small pieces which have a fixed size, such as 512KB or 256KB. Each piece can then be divided into smaller blocks of a certain size, such as 16KB, which can be transferred separately and assembled after downloading all of them.

In traditional file-sharing systems, all peers get the list of seeders and other participants with their file status from one (or several) tracker server(s) whose URL or IP address(es) could be found in a file called the torrent file. This torrent file also contains information on the file or resource being shared in the network. Therefore, it is necessary for BitTorrent users to obtain the torrent file before they participate in the network. Because the torrent file is small in size (usually dozens of KB) and does not contain sensitive information, it can be easily put on websites by resource publishers for downloading. Similar techniques could be found in other BitTorrent-like systems. Clearly, there must be at least one available seeder in the network for making resource sharing possible. This first seeder is often provided by the publisher when the BitTorrent network is created. Peers that join in afterwards will find the seeder from the tracker server and send it a request for transferring. They will become seeders after successfully downloading all the resources. As new peers appear and seeders quit, the network stays robust and stable by its game mechanism, which encourages uploading and refuses selfish participants.

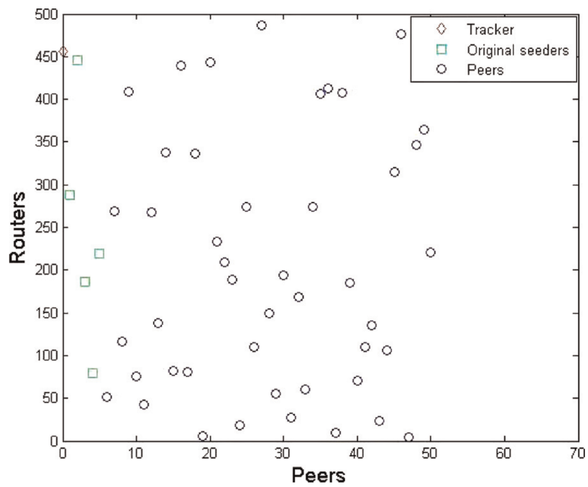
3 Simulation model

This objective of this paper is to construct a directed and weighted graph model to analyse the BitTorrent protocol using the complex network theory. Although network protocols are usually designed to be independent of the network topology, topology in many cases has a major impact on the performance of network protocols (Tangmunarunkit et al., 2002). Therefore, we will evaluate the topological characteristics of overlay BitTorrent networks and make comparisons by adjusting parameters such as network size, original seeder proportion, resource file size, swarm size, etc.

Any P2P system relies on a network of peers within which requests and messages must be routed with efficiency and fault tolerance, and through which peers and content can be efficiently located. The total time for downloading each block of the sharing resource file is decided by the bandwidth and latency of the corresponding pair of peers according to the router topology. To describe peers' capacity more accurately, we constructed a router network by using a router-level internet topology in which every node represents a router located in a real geographic place on the earth with longitude and latitude parameters. Each link's weight represents the distance between the two nodes, which can be calculated using geographic parameters. Using the internet topology generator Igen (Quoitin et al., 2009), some heuristics in a real router network, such as the geographic distribution, identification of points of presence which are the access points to the internet, topology of backbone routers and link capacity were considered in constructing the router-graph. The total number of routers in the network was denoted Num_r.

Each peer should be assigned randomly to one router in the router network. Suppose that the size of the BitTorrent network is Num_p. Then Num_p peers will be assigned to Num_r routers. Two or more peers are allowed to associate with the same router. An example of an assignment with 50 peers to routers can be seen in Figure 1. The latency between two peers is usually caused by the number of hops on the router trace, which is decided by certain interior/Border Gateway Protocols. For simplicity as well as accuracy, the sum of latency is calculated in the shortest path of the router graph for any pair of routers. A local latency is set for the case in which the source and the destination router are the same. We set the allowed transferring bandwidth value to be 1Mb/s, 2Mb/s, 4Mb/s and 10Mb/s. Each peer is randomly assigned to one of the values above.

Figure 1 An example assignment of 50 peers to routers. The assignments are performed randomly for each simulation. A local latency of 50 ms was set for the case in which source and destination router is the same (see online version for colours)



Some other parameters have to be adjusted before performing simulations and making comparisons of BitTorrent protocol under different circumstances. These parameters include original seeder proportion, resource file size, swarm size, etc. With communication capacity allocated to each peer, we can then simulate the BitTorrent protocol using the Peersim simulator (Jelasity et al., 2009; Frioli and Pedrolli, 2008). The simulation should record the amount of data transferred between all pairs of peers from the first downloading action till the accomplishment of the whole network.

After the simulation has been completed, a directed and weighted graph can finally be constructed to analyse the BitTorrent protocol. One peer P_i in the network is represented by a node N_i in the graph. There exists an arc A_{ij} from N_i to N_j if there is data transferred between the corresponding peers of P_i and P_j. The data amount transferred between P_i and P_j is denoted L_{ij}. We assign arc A_{ij} a weight W_{ij} which is proportional to L_{ij}. L_{ij} is divided by the sharing file size *Res_Size* for comparing networks with different resource sizes. Therefore, arc A_{ij} is evaluated with a weight of W_{ij} that is equal to $L_{ij}/\text{Res\_Size}$.

4 Results

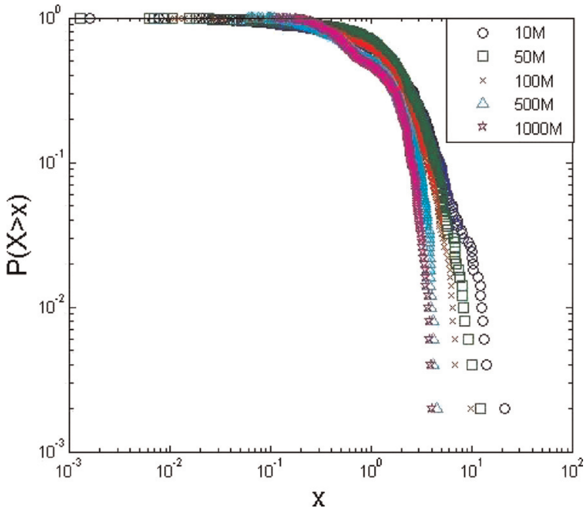
In this section, we present the results from our simulations and complex network analysis of BitTorrent networks.

4.1 Node strength distribution

One node's strength is defined as the sum of weights of all the arcs that are either from or to this node. The in-strength of a node is the sum of weights of all the arcs that point to it, and the out-strength of a node is the sum of weight of all the arcs that point to other nodes from this node. Since every peer's total downloaded data amount (in-strength) should be the same for any one BitTorrent network, we can calculate the out-strength D_i of node N_i as the sum of each peer's uploaded data:

$$D_i = \sum_{j=1}^{n} L_{ij}/\text{Res\_Size}. \tag{1}$$

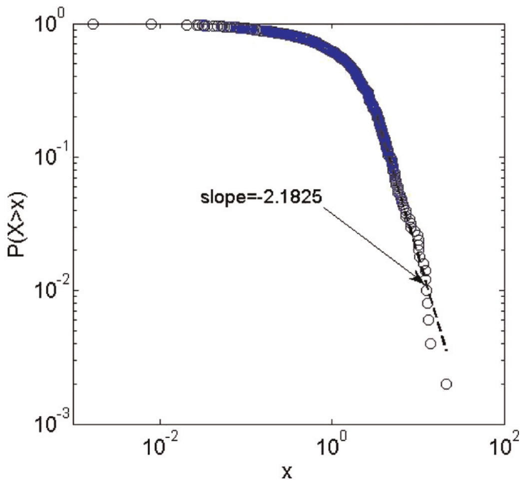
Figure 2 Cumulative out-strength distribution of 1000-peer BitTorrent networks with different resource file sizes (see online version for colours)



We performed simulations on a BitTorrent network of 1000 peers with varying resource file size from 10 MBytes to 1000 Mbytes. Five original seeders were provided at the beginning of each experiment. Other peers were simultaneously added afterwards. Results were analysed when every peer had completed downloading. The resulting cumulative out-strength distribution is shown in Figure 2.

Figure 3 shows the same result when the resource file size is 10 Mbytes. As can be seen, when $x > 3$, the cumulative out-strength distribution obeys a power-law function of $P(X > x) = x^{-\alpha}$ for $\alpha = -2.1825$. Then the out-strength's distribution should be $P(x) \sim x^{-(\alpha+1)} = x^{-3.1825}$ when $x > 3$.

Figure 3 Cumulative out-strength distribution when resource file size is 10 M. As can be seen from the figure, it obeys a power-law distribution when approximately $x > 3$ (see online version for colours)



By comparing results from the BitTorrent networks with different *Res_Size* values, we find that a higher value of α is obtained for a larger *Res_Size*, which can be explained by the fact that peers have a higher possibility to get a small out-strength and a lower possibility to get a large out-strength when downloading larger resource files.

4.2 Clustering

The clustering coefficient is a popular measure to describe the degree to which nodes tend to cluster together. In non-weighted and for undirected networks, the global clustering coefficient is defined as the ratio between the number of closed triplets and the number of all triplets. A triplet is composed of three nodes that have at least two links connecting them. A closed triplet is one with three links, which form a triangle centred in a specific node.

For BitTorrent networks which are both weighted and directed, we use a method called generalised clustering coefficient proposed in Opsahl and Panzarasa (2009). It is based on the classical global clustering coefficient with several modifications. First, it uses the definition of transitivity (Karlberg, 1997) for directed graphs. A brief introduction is given below.

Triplet r is evaluated to be ω_r by one of the following four methods: Arithmetic mean, Geometric mean, Maximum, Minimum. In the methods, the arcs' weight is taken into consideration in different ways. Since the arc's weight in our model stands for the transferred amount of data, it should include a bottleneck effect. So we use the Minimum method in which ω_r is equal to the smaller arc's weight out of both arcs which are related to the central node. For a closed triplet, the weight of the third arc will not be taken into account for the value of this triplet, since the aim of the clustering coefficient is to assess the possibility of this arc's appearance instead of its weight. Then we get the generalised clustering coefficient C_ω by calculating the sum of all triangles and triplets' value as the numerator and denominator respectively:

$$C_\omega = \frac{\sum_{r,\Delta} \omega_r}{\sum_r \omega_r} \tag{2}$$

where $\sum_r \omega_r$ is the sum of ω_r for which triplet r is considered to be nontrivial and $\sum_{r,\Delta} \omega_r$ is the sum of ω_r for which triplet r is a triangle included in the denominator.

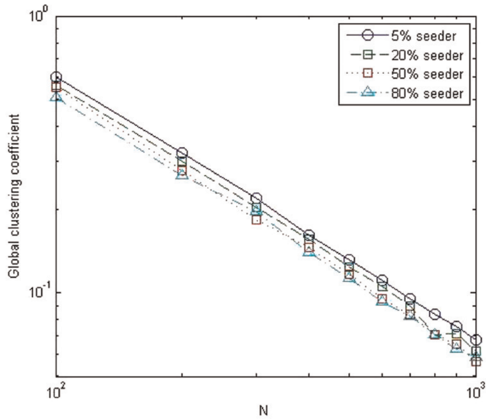
In our model, the clustering coefficient (or transitivity) helps describe the capacity of the transferred data from some peers (like seeders that do not download when uploading) to others (like leechers that do not upload at all when downloading), and then to the rest of the others until all peers have obtained the data. Therefore, a network with a higher clustering coefficient has a better capacity to propagate data from the original seeders to all other peers.

We start the simulation of BitTorrent networks with two different parameters, network size and proportion of original seeders in all peers. The first parameter is for analysing how the number of participants in the BitTorrent network will affect the clustering. The objective of the second parameter is to investigate if the popularity of certain BitTorrent networks will have an effect on the structure, since more seeders exist in more popular networks. The dynamics of adding or deleting peers is not considered in this step. The generalised clustering coefficient C_ω for varieties of BitTorrent networks is shown in Figure 4.

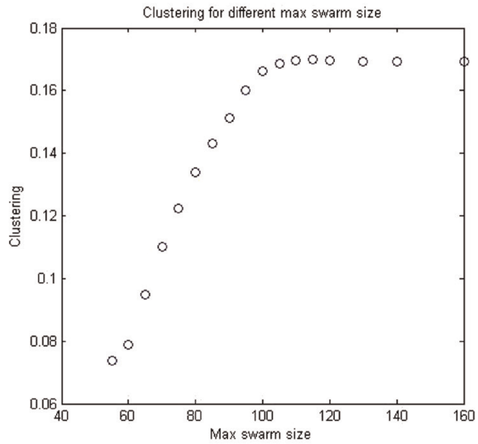
First, from Figure 4(a), we can see that the clustering coefficient of the BitTorrent network which has 5% of its node members being original seeders gets smaller from approximately 0.6 to 0.06 when the size of the network increases from 100 to 1000. All other networks with different original seeder proportion have similar results in our experiments. This means that BitTorrent networks with fewer peers are more clustered together than those with more peers. This is similar to many other networks such as random graphs, for it tends to be 'easier' for smaller networks to cluster together than larger ones. Second, we find that the clustering coefficient decreases gently when the proportion of original seeders increases, except for a very few cases. We believe that this is due to the decreasing number of arcs when more peers are seeders originally that do not need to create a connection to download data from each other.

Then Figure 4(b) shows the clustering of BitTorrent networks with different maximum swarm sizes. Here, maximum swarm size refers to the upper limit of the number of the neighbour peers that one peer downloads data from in the BitTorrent network. This is a parameter which could be set by BitTorrent users, since a larger maximum swarm size may brings faster downloading speed with more peers sending data simultaneously, while satisfying more system resources for dealing with the communication which gets denser. From the results of our experiments, it can be seen that clustering increases from 0.07 to 0.17 as maximum swarm size grows from 50 to 100. But the clustering stays stable as the maximum swarm size continues growing. It can be interpreted that, when a higher value of maximum swarm size is set, the networks tend to cluster more than if maximum swarm size is comparably

Figure 4 (a) Global clustering coefficient for BitTorrent networks with different size and proportion of original seeders; (b) Global clustering coefficient for max swarm size varying from 50 to 160 in a 500-peer network. All other parameters are set to be unchanged (see online version for colours)



(a)



(b)

small. When maximum swarm size grows to some extent, like 100 in our 500-peer network, the clustering stops increasing and stays stable. We can explain this in this way: larger swarm sizes may require more resources such as calculation and bandwidth, which may have their own limits and therefore, peers could not have too large a number of neighbours simultaneously, even if the maximum swarm size is large. We can know from the result that one cannot always get a better connected network (with higher clustering coefficient) by setting a larger swarm size.

4.3 Shortest path length

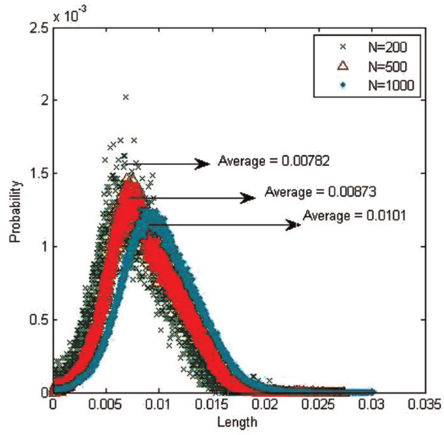
The shortest path has long played an essential role in the research of complex networks. It helps describe the relation between different vertices which are not connected directly by calculating the nearest distance from one to the other. In telecommunication networks such as BitTorrent, the length of the shortest paths between these pairs of peers reflects the dependence between them. The shorter the length is, the more dependent the two nodes are, and vice versa. In our model, we find shortest paths in the following steps

- Invert the weight of all the arcs (dividing them with 1). We do so because in weighted networks, the transaction between two nodes might be quicker along arcs with higher weight, which could be interpreted as a stronger or closer connection between them, than with lower weight. This is due to the fact that the weight in our model is proportional to the total amount of data transferred after all peers finish downloading. Paths are composed of arcs in the same direction, one by one. The length of the path describes the total amount of data transferred from one peer to another along the 'path'.
- Find all pairs' shortest paths and calculate their lengths by adding the inverted weight of each arc along the paths. The Floyd-Warshall (Floyd, 1962) algorithm is used to get the finest time complexity, which is $O(n^3)$. After that, the average shortest path length of the network can be calculated.

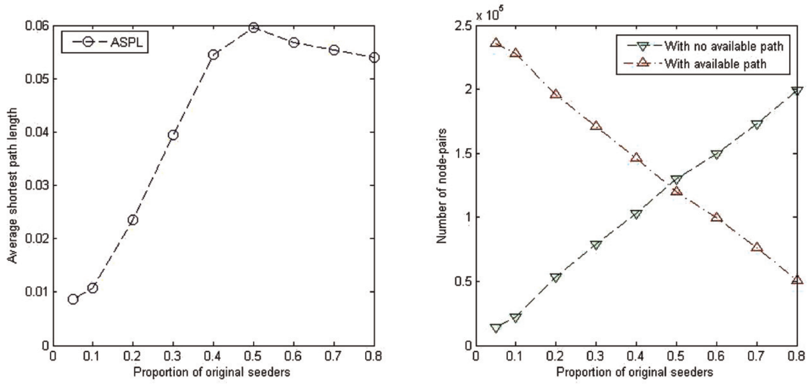
In Figure 5(a), the frequency distribution of the shortest path lengths of BitTorrent networks with different sizes is shown. It is not difficult to see that when the network size becomes larger, from 200 to 500 and then 1000, the average length of the shortest paths grows from 0.00782 to 0.00873 and then 0.0101, respectively. Therefore, the length of the shortest paths between peers tends to have a higher possibility of increasing as well. This is the same with other networks, since larger ones may bring longer shortest paths. But the peak's value decreases from 2‰ to 1.5‰, and then to 1.3‰ approximately, as the network size grows from 200 to 500, and then to 1000 respectively. We believe that this is because larger networks own a wider range of lengths, for example, 0–0.03 for a 1000-peer network compared to 0–0.027 for a 200-peer one. It makes the 200-peer curve look steeper than the 1000 peer one.

The left graph of Figure 5(b) gives the Average Shortest Path Length (ASPL) of nine 500-peer BitTorrent networks with different original seeder proportions, which vary from 5% to 80%. We can see that the ASPL increases when the seeder proportion varies from 5% to 50%. And then the ASPL decreases gently after the seeder proportion gets larger than 50%. It could be explained this way: when the seeder proportion increases, one peer tends to receive data from a larger number of seeders rather than from a very few of them. Therefore, the amount of data which comes from one seeder tends to decrease for common peers as the result of the decreasing of weight along one path. Since the weight is inverted in our model, the ASPL increases correspondingly as seeder proportion increases. However, when there is enough number of seeders, the number of arcs in the path decreases, while the number of seeders continues growing. Consider the extreme case when there is only 1 peer with all partners being seeders, in which all paths consist of only 1 seeder-to-peer arc. This contributes to decrease in ASPL when the seeder proportion is larger than 50%. We can also see from the right graph of Figure 5(b) that the number of node-pairs decreases and increases almost linearly for those that

Figure 5 (a) Frequency distribution of shortest paths length of BitTorrent networks with different sizes; (b) Average shortest path length with different proportion of original seeders is given in the left figure. All other conditions are set to be unchanged. It is showed in the right figure that the number of node-pairs decreases for those with available paths connecting them and increases for those with no available paths, respectively (see online version for colours)



(a)



(b)

have available paths existing between them and those that do not, respectively. This is due to the fact that seeders may not be in the middle or end of a path, since no data may possibly be transferred to them. As a result, fewer paths may exist between node-pairs when seeder proportion grows.

4.4 Betweenness

Betweenness (Borgatti, 2005) is one of the structural indicators which relates to the centrality of different nodes. In a network, certain nodes occupy advantageous positions, whereas some other nodes may rely on them to connect to nodes further out. The extent to which a node contributes itself to the communications of others can be studied using the betweenness measure.

In our model, we propose to use flow betweenness (Rousseau and Zhang, 2008) to see how peers' performance affects the data flow of the whole BitTorrent networks. First we define the capacity *Cap*(*i, j*) of the network connection between nodes *i* and *j* as

$$\mathrm{Cap}(i,j) = \max_{\mathrm{Path}(i,j)\in\mathrm{Paths}(i,j)} \min_{A_{uv}\in\mathrm{Path}(i,j)} W_{uv} \tag{3}$$

which gives the maximum flow between nodes *i* and *j*. The capacity between nodes *i* and *j* in a network assuming that node *k* has been wiped off from the network is defined as

$$\mathrm{Cap}(i-k-j) = \max_{\substack{\mathrm{Path}(i,j)\in\mathrm{Paths}(i,j)\\ k\notin\mathrm{Path}(i,j)}} \min_{A_{\mathrm{uv}}\in\mathrm{Path}(i,j)} W_{uv} \tag{4}$$

where A_{uv} is the arc from node *u* to *v* and W_{uv} is the weight of A_{uv}. Paths(*i*, *j*) is the set of all the possible simple paths between nodes *i* and *j*. Path(*i*, *j*) *is* denoted as one of them. For directed networks, Cap(*i*, *j*) is distinct from *Cap*(*j*, *i*). If there is no connection between nodes *i* and *j* then Cap(*i, j*) is set equal to 0. The same is valid when *i* equals *j*. When there is only one possible path between two nodes, this definition could be interpreted as showing how strong a chain is: that always depends on its weakest part.

The flow betweenness of node *k*, $b_w(k)$, is given as:

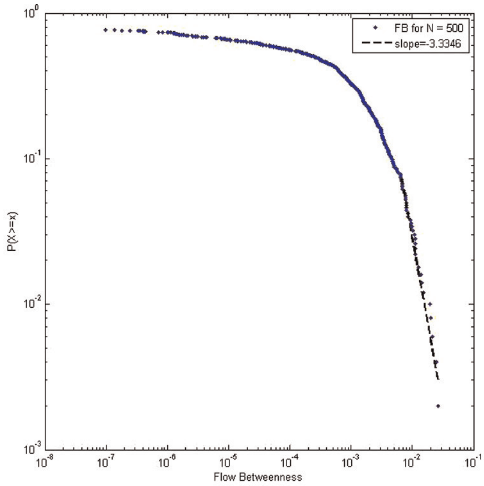
$$b_w(k) = \frac{\sum_{j\neq k,\ k\neq i,\ j\neq i}(\mathrm{Cap}(i,j) - \mathrm{Cap}(i-k-j))}{\sum_{j\neq k,\ k\neq i,\ j\neq i}\mathrm{Cap}(i,j)} \tag{5}$$

which can be interpreted as the ratio of the sum of capacity increment with the contribution of node *k* to the sum of the capacity of all. The more the capacity increases with the participation of node *k*, the higher $b_w(k)$ is, the more important role node *k* plays in the communication of the whole network. It is not difficult to see that flow betweenness is always a value between 0 and 1. For nodes which only act as the first or last node of the path, we consider them to be trivial. For example, the flow betweenness of all the seeder and leecher nodes in BitTorrent networks are set equal to 0.

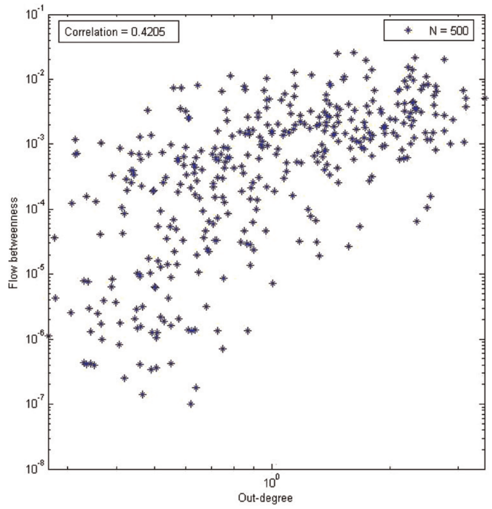
Figure 6(a) gives the distribution of flow betweenness (FB) in a 500-peer BitTorrent network. In our experiments, the FB value varies from a minimum of 0.0000000991842 to a maximum of 0.02599. Zero-value FB nodes are wiped off in our results. We can see that it obeys a power-law distribution with the slope being -3.3346 when FB is approximately >0.0064.

Figure 6(b) shows the correlation between node strength and flow betweenness in a 500-peer BitTorrent network. The Pearson correlation (Newman, 2002) value is 0.4205, which indicates a positive (increasing) relationship between the two variables. We can see from the figure that the FB value of nodes with small out-strength varies widely, which means that a small out-strength does not decide whether the FB value is low or high at all. However, when out-strength grows larger, the lower limit of the FB value grows as well. Since the upper limit of the FB value does not change much, it can be concluded that for nodes with large out-strength, their FB value is comparably high as well.

Figure 6 (a) The distribution of flow betweenness for a 500-peer BitTorrent network; (b) This figure shows the correlation between node strength and flow betweenness of each node in a 500-peer BitTorrent network. With Pearson correlation value being 0.4205, a positive (increasing) relationship between the two variables could be seen in the figure above (see online version for colums)



(a)



(b)

We can see that in our model flow betweenness is such a measure that it helps describing the contribution of certain peer to the communication of the whole network. We believe that peers with high value of flow betweenness play a more important role in the spreading of data from seeders to peers in the network than those with low value of flow betweenness. These peers with high flow betweenness value should then be encouraged, and become very good candidates for other peers to choose to download data from with high priority. Therefore, betweenness may be a good measure for selection of the best peers, which is one of the opening issues in the improvement of P2P protocols (Meddour, 2006). However, there are still some difficulties, such as computing complexity, which need to be resolved before betweenness can be applied into the improvement of the BitTorrent protocol. We hope more progress can be made in the future.

5 Conclusion and future work

In this work, we have proposed a plane graph model to exhibit the topology of P2P communication networks using the BitTorrent protocol. By valuing the connection between peers with the data amount transferred, evaluating parameters such as node strength distribution, clustering coefficient, shortest path, betweenness, etc., are calculated for revealing the topological characteristics of BitTorrent networks with different network sizes, seeder proportions, resource file sizes, swarm sizes, etc. The node strength of BitTorrent networks follows a power-law distribution. When the network size and seeder proportion grow, BitTorrent networks tend to be less clustered. However, a higher clustering value is obtained when the maximum swarm size is set to be larger, which allows peers to have more adjacent neighbours. The average shortest path length grows as network size expands, which is quite a common phenomenon in real networks. But the growing of seeders' proportion affects the ASPL (first increase, then decrease) as well, since the seeder can only act as the first peer in paths of which both the length and the number get smaller. Finally, a positive correlation between flow betweenness and node strength is found in our work.

In our future work, we will emphasise analysis of the dynamical evolution of BitTorrent networks, such as how peers' downloading and uploading speeds change at different time and what would happen under failure or attack on important peers. The methodology of modelling and analysing BitTorrent networks presented in this paper may be applied to other communication networks as well.

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