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Workshop - Systems Design Meets Equation-based Languages

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LCCC Workshop: Systems Design meets Equation-based Languages

19-21 September 2012

Old Bishop's Palace at Biskopsgatan 1 in Lund

Scientific Committee

Johan Åkesson, Lund University, Sweden (Chair)

Moritz Diehl, KU Leuven, Belgium

Hilding Elmquist, Dassault Systèmes, Sweden

Claus Führer, Lund University, Sweden

Clas Jacobson, United Technologies Research Center, USA

Eric Van Wyk, University of Minnesota, USA

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1. Introduction

LCCC workshops are organized in a 3-day format. About 20-25 speakers from academia and industry are invited for the workshop, selected for excellence and for an optimal coverage of the theme. The speakers are also encouraged to extend their stay beyond the workshop for further interaction with the local research environment. For each workshop, the research theme is chosen strategically to support the vision of a LCCC, usually with a cross-disciplinary perspective. An international scientific committee is responsible for the program.

1.1 WORKSHOP THEME

Equation-based object-oriented languages (EOOL), such as Modelica and VHDL-AMS, have become widely used in academia and industry during recent years. While these languages are mainly oriented towards dynamic simulation, they are well suited as a basis for solving a wider range of engineering design problems, making use of existing and new algorithms. Examples include sensitivity analysis, state and parameter estimation, optimal control and MPC, robust design, and model reduction.

1.2 SCOPE

The workshop focused on how EOOLs can be extended to support this wider range of problems in systems design. The following aspects are of primary interest:

1. *Extension examples:* What kind of engineering design problems could benefit from support through EOOLs, or extensions to an EOOL language? What existing or new algorithms could be used for such extensions? An existing example for such an extension is Optimica which adds optimization capabilities to Modelica.
2. *Language extension design:* How can such

extensions be formulated as language extensions? What different techniques, e.g., annotations, syntactic extensions, semantic extensions, or embedded DSLs are appropriate for different extensions? How can model execution standards, e.g., the Functional Mock-up Interface (FMI) be explored to link language extensions to algorithms?

3. *Language extension implementation:* How can these extensions be implemented in supporting tools like compilers? How can modularity with respect to core languages be maintained? How can interactive tools like IDEs be extended to support the language extensions? Examples of new metacompilation frameworks supporting language extensions include JastAdd, Silver, and Kiama.
4. *Applications:* What interesting industrial cases can be found that could benefit from such new developments?

Supporting such extensions to EOOL's would answer the strong industrial need for integrating existing EOOL models with systems design algorithms and on-line control systems.

The problems are cross disciplinary, and the aim of the workshop was therefore to bring together researchers and industrial practitioners in several fields, including engineering design (modeling, simulation, optimization, etc.), computer science (languages and tools), numerical analysis (algorithms for solving design problems), and applications.

The workshop supported the LCCC theme Modeling for design and verification. During the last few years, a local community has emerged, consisting of researchers at the departments of Mathematics, Computer Science and Automatic Control, and companies, notably Modelon, Lund, and ABB, Malmö. The local community is oriented towards the two open source projects

JModelica.org (an open-source implementation of Modelica) and JastAdd (a meta-compilation tool supporting language extension). The theme of the workshop stemmed from this environment cross-disciplinary interactions between researchers at Lund university, local companies, and students are frequent. Such interactions include joint master's thesis projects, joint scientific publications, joint PhD student advising, all inspired by industrial applications.

1.3 ORGANIZATION AND VENUE

The workshop was initiated by Claus Führer (Center for Mathematical Sciences), Görel Hedin (Department of Computer Science) and Johan Åkesson (Department of Automatic Control).

The scientific committee consisted of Johan Åkesson (chair), Moritz Diehl, Hilding Elmquist, Claus Führer, Clas Jacobson and Eric van Wyk.

The local organization and interactions with workshop speakers and participants was handled by Eva Westin.

The workshop was held at the Pufendorf Institute at Lund University 19-21 September 2012.

2. Panel discussion

Participants: **Albert Benveniste, Hilding Elmqvist, Carl D. Laird, Edward E. Lee, Clas Jacobson**

Moderator: **Karl Johan Åström**

The panel discussion circled around three main themes; modeling and systems engineering in education, employing equation-based languages in systems design, and formalization of model representations.

2.1 MODELING AND SYSTEMS ENGINEERING IN EDUCATION

C. Jacobson put forward the observation that systems engineering is no longer taught by academic institutions. As a consequence, graduated engineers lack experience with systems design tools, which are widely used in industry. In cases where systems design courses are offered by universities, they are often taught by industrial practitioners that are brought in for the occasion.

E. Lee suggested to introduce a new topic into program curricula: Model Engineering. While this topic would build on established disciplines, it would emphasize that the concept of modeling as a key element in systems engineering. What is currently offered by universities in this area is generally weak. E. Lee made an analogy to software engineering, which has a long-time tradition within academia, and which contains a number of structured concepts that are taught systematically. Concepts suggested to be integrated into the topic model engineering include object-orientation, represented by languages such as Modelica, and refactoring of model code, which is a standard technique in software engineering.

A. Benveniste noted that mathematics is and must remain a fundamental element of systems engineering – mathematics is every-

where! It was also noted that French software industry emphasizes systems engineering for this particular reason.

2.2 EMPLOYING EQUATION-BASED LANGUAGES IN SYSTEMS DESIGN

In his opening note, H. Elmqvist talked about recent directions in the development of the Modelica language. The latest version of Modelica supports synchronous constructs. State-machines have been added in order to promote modeling of clock and sequential control systems. H. Elmqvist stressed the need to continue to expand the scope of Modelica to cover areas such as requirements management, integration with 3D modeling tools, Monte Carlo analysis, embedded optimization in physical models and systems design in general. H. Elmqvist also took the opportunity to invite everybody to interact and to contribute to the further development of Modelica.

C. Jacobson commented that equation-based languages are currently not used to their full potential. Given the languages and tools available today, we can move from experimentation based on simulation to computations in systems design. C. Jacobson mentioned Six Sigma and Monte Carlo techniques as targets for integration with computational frameworks based on physical models, and he highlighted rich opportunities for research in the area, for example in propagation of uncertainty.

C. Laird talked about the interplay between algorithm design and modeling, specifically in the context of dynamic optimization of large-scale non-linear systems. In effect, the way models are constructed is affected by the capabilities of such algorithms. In addition, the need for exploitation of structure in models was stressed.

2.3 FORMALIZATION OF MODELS

A. Benveniste used the fighter aircraft Rafale to exemplify the need for integrated and formal methods in requirements management and verification. Approximately 250.000 requirements were considered in the design. The process was characterized by informal handling of the requirements, multiple engaged sub-contractors, and often, requirements verification without models. In other activities in the project, however, models were developed and used extensively, including system dimensioning, control design and Product Lifecycle Management (PLM). Typically, very different modeling tools were used for these purposes. Based on the example, A. Benveniste put forward questions to be addressed in research and in industrial practice. How to fuse the model-based tools in order for models to become widely available in different processes? How does the V-model for product development come into play in this context? What is needed in terms of Modelica extensions in order to accommodate the needs exemplified in the Rafale project?

In his remarks, E. Lee reasoned about what properties of models we should value. Three aspects were brought forward. Firstly, fidelity of models is a key property, that is to what degree the models mimic a given system. Secondly, understandability of a model, something we are often eager to sacrifice, should be valued. E. Lee called for a cultural change in this respect – we should be proud of small models! Thirdly, analyzability of a model is important in order to perform model-based analyses such as model checking and verification. E. Lee stressed in this context the need for formal model description formats.

3. Summary and outlook

3.1 IMPORTANT OBSERVATIONS

- Different approaches to modeling of hybrid systems were discussed during the workshop. This seems to be one of the core challenges in the area, i.e., to develop a rigorous mathematical formalism to describe the semantics of models encoded in languages such as Modelica, Ptolemy and VHDL-AMS, and in model exchange standards such as FMI.
- The interest in model exchange formats which are neutral with respect to physical domain, modeling language, and software tool is increasing. The Functional Mock-up Interface is rapidly being adopted in research and in industry, which was evident from several presentations. In addition, the CIF format which resulted from the MULTIFORMS project was presented.
- The interest in Modelica is broadening, and the scope of the language is expanding from primarily modeling of physical systems to control systems and systems design. Specifically, synchronous extensions to Modelica and optimization based on Modelica models were discussed. Also, the potential of Modelica in systems design was high-lighted during the panel discussion.
- The need for formal verification of requirements, and approaches to solving such problems was a strong theme during the workshop. This topic was high-lighted during the panel discussion in the context of aircraft control systems and in several presentations.
- Some speakers bore witness to difficulties in applying software for non-convex dynamic optimization to industrial problems. The level of maturity of existing algorithms for such problems seems to be significantly less than

for simulation tools targeting the same class of systems.

- Extensible languages and compilers is becoming feasible through research efforts in the computer science community. Two different approaches to compiler extensibility was discussed in the workshop presentations.
- Python holds a strong position in the scientific computing field, which was underlined in a number of presentations.

3.2 OPEN PROBLEMS

- **Modeling formalisms for hybrid systems.** Several speakers touched upon modeling formalisms for hybrid systems. While there are different frameworks available for description of hybrid systems, consensus is yet to be reached upon the semantic behaviour and a unified mathematical theory.
- **Robustness of numerical optimization algorithms for large-scale non-linear dynamic systems.** The academic community has produced a large body of algorithms for optimization of large-scale non-linear dynamic systems. Still, industrial practitioners experiences significant challenges in applying such algorithms to problems relevant for their applications.
- **Physical modeling languages for convex optimization.** Current modeling languages such as VHDL-AMS and Modelica target construction of non-linear and hybrid physical system models, which are not immediately useful as a basis for the large body of available optimization algorithms for convex optimization. Still, many physical systems can be modeled in order to fulfill the requirements of convex optimization. Accordingly, challenges remain in combining concepts from EoOL and convex optimization.

3.3 ACTIONS

From the discussions during the workshop, it is clear that there are rich opportunities for cross fertilization between different fields represented by speakers and participants. Based on these discussions, the following actions are recommended.

- **More efforts are needed in terms of language support for optimization.**

Several presentations touched upon this topic and several interesting directions were mentioned, including convex optimization formulations based on physical modeling languages, challenges in application of state-of-the-art optimization algorithms to large-scale physical models, and industrial applications.

- **Increased interaction is needed between communities working with modeling formalisms for hybrid systems.**

It is clear that there are several research groups developing modeling formalisms for hybrid systems, as well as industrial initiatives such as FMI and Modelica. Interactions between these groups would be beneficial in order to develop a unified framework for modeling of hybrid systems. An initiative in this direction was taken by the Modelica community, represented by H. Elmquist, who visited E. Lee's group in the weeks following the workshop.

- **Establishment of a repository of dynamic benchmark models of industrial grade to support research in systems design.**

Development of relevant industrial grade models requires a high level of expertise, that this not always available in research projects targeting systems design. Such projects benefit from freely available dynamic models.

Appendix A

PROGRAM

Wednesday, September 19, 2012

08.30-09.00	Registration
09.00-09.10	Opening session
09:10-10:10	<i>Non-standard semantics of hybrid systems modelers</i> Albert Benveniste , IRISA/INRIA <i>Equations, Synchrony, Time, and Modes</i> Edward A. Lee , EECS, UC Berkeley
10:10-10:40	Coffee
10:40-12:10	<i>Formal Modeling and Analysis of Software Systems with Lustre</i> Mike Whalen , University of Minnesota <i>Systems Engineering: Status of Industrial Use, Opportunities and Needs</i> Clas Jacobson , United Technologies Systems & Controls Engineering <i>The OpenModelica Environment including Static and Dynamic Debugging of Modelica Models and Systems Engineering / Design verification</i> Peter Fritzson , Linköping University, PELAB
12:10-13:30	Lunch
13:30-15:00	<i>The Dark Side of Object-Oriented Modelling: Numerical Problems, Existing Solutions, Future Perspectives</i> Francesco Casella , Politecnico di Milano <i>Bridging between different modeling formalisms – results from the MULTIFORM project</i> Sebastian Engell , TU Dortmund <i>Equation-based Modeling and Control of Industrial Processes</i> Johan Sjöberg , ABB AB, Corporate Research and Linköping university
15:00-15:30	Coffee
15:30-16:30	<i>FMI: Functional Mockup Interface for Model Exchange and Co-Simulation</i> Torsten Blochwitz , ITI GmbH Dresden <i>Vertical Integration in Tool Chains for Modeling, Simulation and Optimization of Large-Scale Systems</i> Johan Åkesson , Modelon AB and Lund University

Thursday, September 20, 2012

09:00-10:00	<i>System Design – From Requirements to Implementation</i> Alberto Ferrari , ALES S.r.l. <i>Synchronous Control and State Machines in Modelica</i> Hilding Elmquist , Dassault Systèmes AB
10:00-10:30	Coffee

10:30-12:00	<i>Extensible Programming and Modeling Languages</i> Eric Van Wyk , University of Minnesota <i>Extensible compiler architecture – examples from JModelica.org</i>
	Görel Hedin , Lund University <i>Constraint satisfaction methods in embedded system design</i>
	Krzysztof Kuchcinski , Lund University
12:00-13:30	Lunch
13:30-15:00	Discussion
15:00-15:30	Coffee
15:30-16:30	<i>Dynamical models for industrial controls: use cases and challenges</i> Fernando D'Amato , GE Global Research Center <i>Origins of Equation-Based Modeling Languages</i> Karl Johan Åström , Lund University
18:20	Gathering at Bangatan 14 (next to Ica Kvantum Malmöborgs)
19:00	Workshop dinner at Häckeberga castle

Friday, 21 September, 2012

09:15-10:00	Panel discussion
10:00-10:30	Coffee
10:30-12:00	<i>Pyomo: Optimization Modeling in Python</i> Carl Laird , Texas A&M University <i>Efficient symbolical and numerical algorithms for nonlinear model predictive control with OpenModelica</i>
	Bernhard Bachmann , Fachhochschule Bielefeld University of Applied Sciences <i>Algorithmic differentiation: Sensitivity analysis and the computation of adjoints</i>
	Andrea Walther , Universität Paderborn
12:00-13:00	Lunch
13:00-14:30	<i>CasADi: A Tool for Automatic Differentiation and Simulation-Based Nonlinear Programming</i> Moritz Diehl , OPTEC KU Leuven <i>Modeling Seen as Programming</i>
	Klaus Havelund , Jet Propulsion Laboratory, California Institute of Technology <i>Verification of Stiff Hybrid Systems by Modeling the Approximations of Computational Semantics</i>
	Pieter J. Mosterman , MathWorks
14:30-15:00	Coffee
15:00-16:00	<i>Assimulo – a Python package for solving differential equation with interface to equation based languages</i> Claus Führer , Lund University <i>Functional Development with Modelica</i>
	Stefan-Alexander Schneider , Schneider System Consulting
16:00-16:05	Closing

Appendix B

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Appendix C

PRESENTATIONS

NON-STANDARD SEMANTICS OF HYBRID SYSTEMS

MODELERS

Albert Benveniste, IRISA/INRIA

Hybrid system modelers have become a corner stone of complex embedded system development. Embedded systems include not only control components and software, but also physical devices. In this area, Simulink is a de facto standard design framework, and Modelica a new player. However, such tools raise several issues related to the lack of reproducibility of simulations (sensitivity to simulation parameters and to the choice of a simulation engine). In this paper we propose using techniques from non-standard analysis to define a semantic domain for hybrid systems. Non-standard analysis is an extension of classical analysis in which infinitesimal (the ϵ and η in the celebrated generic sentence $\forall\epsilon\exists\eta\dots$ of college maths) can be manipulated as first class citizens. This approach allows us to define both a denotational semantics, a constructive semantics, and a Kahn Process Network semantics for hybrid systems, thus establishing simulation engines on a sound but flexible mathematical foundation. These semantics offer a clear distinction between the concerns of the numerical analyst (solving differential equations) and those of the computer scientist (generating execution schemes). We also discuss a number of practical and fundamental issues in hybrid system modelers that give rise to non-reproducibility of results, non-determinism, and undesirable side effects. Of particular importance are cascaded mode changes (also called “zero-crossings” in the context of hybrid systems modelers).



Non-Standard Semantics of Hybrid Systems Modelers

Albert Benveniste Timothy Bourke
Benoit Caillaud Marc Pouzet

INRIA Rennes and ENS Ulm, France

September 14, 2012

Difficulties in Hybrid Systems Modelers

Some examples

Non-Standard Hybrid Systems (for the math-aware)

Non-Standard Analysis and Standardisation (for the fan)

Non-Standard Hybrid Systems and their Standardisation

The SIMPLEHYBRID mini-language

Conclusion

Difficulties in Hybrid Systems Modelers

- ▶ Cascaded zero-crossings and start'n'kills of ODE/DAE
 - ▶ ZC can traverse, tangent, be thick... how to define them?
 - ▶ cascades: finite? bounded?
 - ▶ solver can stop in zero time if initialized on a zero-crossing
 - ▶ is this the duty of Continuous or Discrete?

Difficulties in Hybrid Systems Modelers

- ▶ Cascaded zero-crossings and start'n'kills of ODE/DAE
 - ▶ ZC can traverse, tangent, be thick... how to define them?
 - ▶ cascades: finite? bounded?
 - ▶ solver can stop in zero time if initialized on a zero-crossing
 - ▶ is this the duty of Continuous or Discrete?
- ▶ Use of a global solver
 - ▶ non-interacting subsystems interact!
 - ▶ time scales propagate everywhere
 - ▶ HotCold restart of solvers

Difficulties in Hybrid Systems Modelers

- ▶ Cascaded zero-crossings and start'n-kills of ODE/DAE
 - ▶ ZC can traverse, tangent, be thick... how to define them?
 - ▶ cascades: finite? bounded?
 - ▶ solver can stop in zero time if initialized on a zero-crossing
 - ▶ is this the duty of Continuous or Discrete?

Use of a global solver

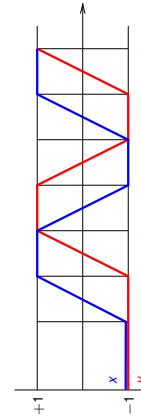
- ▶ non-interacting subsystems interact!
- ▶ time scales propagate everywhere
- ▶ Hot/Cold restart of solvers
- ▶ Slicing Discrete/Continuous is essential
 - ▶ strange hybrid D-C Simulink/Stateflow diagrams can be specified
 - ▶ they get strange returns from the tool
 - ▶ the Modelica consortium has made this a central effort
- ▶

- ▶
- ▶
- ▶
- ▶

Some examples 1: infinite cascade

$$\begin{cases} \dot{y} = 0 & \text{init } -1 \\ \dot{x} = 0 & \text{init } -1 \\ \dot{z} = 1 & \text{init } -1 \end{cases} \quad \begin{array}{l} \text{reset } [1, -1] \text{ every up}[x, -x] \\ \text{reset } [-1, 1, 1] \text{ every up}[y, -y, z] \end{array}$$

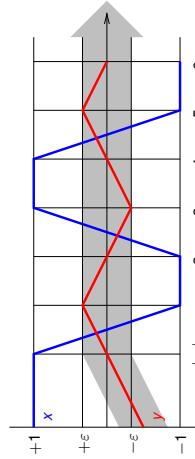
Note that z is just a physical clock. So, such an example can arise with "discrete" systems following the discrete/hybrid classification in force in the community of hybrid systems modelers.



here and subsequently, ε is infinitesimal

Some examples 2: sliding mode

$$\begin{cases} \dot{x} = 0 & \text{init } -\text{sgn}(y_0) \\ \dot{y} = x & \text{init } y_0 \end{cases}$$



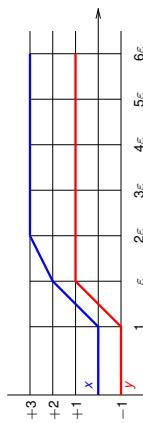
This is a simple form for an ABS system. Corresponding "averaged" system is:

$$\dot{y} = \begin{cases} -\text{sgn}(y_0), & \text{for the interval } [0, |y_0|], \\ 0 & \text{for } [|y_0|, \infty), \end{cases}$$

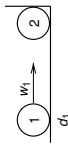
Some examples 3: finite cascade

Some examples 4: balls on wall

$$\begin{cases} \dot{x} = 0 \text{ init } 0 \text{ reset } [\text{last}(x) + 1, \text{last}(x) + 2] \text{ every up}[y, z] \\ \dot{z} = 1 \text{ init } -1 \\ \dot{y} = 0 \text{ init } -1 \text{ reset } [1] \text{ every up}[z] \end{cases}$$



Here the question is: how should the reset on x and y be performed? Here we have adopted a micro-step interpretation reflecting causality between the two resets. A different interpretation is often proposed by existing modelers.



$$\begin{cases} \dot{x}_1 = v_1 \text{ init } d_1 \\ \dot{x}_2 = v_2 \text{ init } d_2 \\ \dot{v}_1 = 0 \text{ init } w_1 \text{ reset last } (v_2) \text{ every up}[x_1 - x_2] \\ \dot{v}_2 = 0 \text{ init } w_2 \text{ reset } [\text{last } (v_1), -\text{last } (v_2)] \text{ every up}[x_1 - x_2; x_2] \end{cases}$$

Here the difficulty is the cascade involving

1. ball 1 hitting ball 2, resulting in ball 2 moving to the right (reset)
2. which causes ball 2 to hit the wall immediately (ODE activated for zero time)
3. resulting in ball 2 moving backward (reset)
4. followed by the symmetric scheme.

Questions

- ▶ Can we propose a semantic domain for these (and all) examples?
- ▶ Can we use it
 - ▶ to identify example (1) as pathological, but not example (2)?
 - ▶ to decide on the semantics of example (3)?
 - ▶ to give a semantics to example (4)?
- ▶ More generally, can we develop a semantic domain to serve as a mathematical basis for the management of (possibly cascaded) zero-crossings?

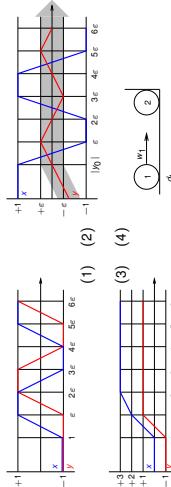
The great idea: non-standard analysis

Suppose for a while that we can give a formal meaning to the following:

$$\dot{y} = x \quad \text{means, by definition: } \frac{y_{t+\partial} - y_t}{\partial} = x_t$$

where ∂ is infinitesimal

Let's make a trial use of non-standard analysis.
The ε of our examples will be identified with the above ∂ .
By doing so, our drawings become the semantics of cascades and ODEs' semantics is written as transition relations involving ∂ .



Non-Standard Time Base

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Fix an infinitesimal base step ∂

$$\text{time base} : \mathbb{T} = \{t_0 = n\partial \mid n \in {}^*\mathbb{Z}\}$$

$$\begin{aligned} \text{define } \forall t \in \mathbb{T} : \bullet t &= \max\{s \mid s \in \mathbb{T}, s < t\} \\ \bullet t &= \min\{s \mid s \in \mathbb{T}, s > t\} \end{aligned}$$

\mathbb{T} offers "the butter and the money of the butter" (popular french idiom):

- (i) \mathbb{T} is totally ordered
- (ii) every subset of \mathbb{T} that is bounded from above by a finite (non-standard) number has a unique maximal element

- (iii) \mathbb{T} is dense in \mathbb{R}

By (i) and (ii) \mathbb{T} looks "discrete"

By (iii), \mathbb{T} looks "continuous"

Non-Standard Time Base

Back to the examples

Can we propose a semantic domain for these (and all) examples?
The drawings show the non-standard semantics with $\partial := \varepsilon$

Can we use it
yes we can

to identify example (1) as pathological?
easy

to identify example (2) as non-pathological?
less easy

to decide on the semantics of example (3)?
easy

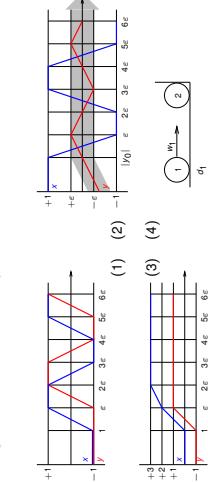
to give a semantics to example (4)?
subtle

$$\forall t \in \mathbb{T} : \bullet t = \begin{cases} \{t_0 = n\partial \mid n \in {}^*\mathbb{Z}\} \\ \max\{s \mid s \in \mathbb{T}, s < t\} \\ \min\{s \mid s \in \mathbb{T}, s > t\} \end{cases}$$

$$\dot{x} = f(x, u) \quad \longleftrightarrow \quad x_t = x_0 + \partial \times \int f(x_s, u_s) \quad (\text{always well defined})$$

Streams of events generated by the zero-crossings of x :
 $\zeta_x =_{\text{def}} \{t \in \mathbb{T} \mid x_t < 0 \wedge x_t \geq 0\}$ (always well defined)
 $\approx \{s \in \mathbb{R} \mid x_s < 0 \wedge x_s \geq 0\}$ (possibly not well defined)

Cascades following t :



No standard counterpart using $\mathbb{R}, \mathbb{R} \times \mathbb{N}$ sufficient for finite cascades ("super-dense" time). Some cascades are worse (example 1) and cannot find their semantics in super-dense time

Back to the examples

Can we propose a semantic domain for these (and all) examples?

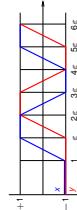
The drawings show the non-standard semantics with $\partial := \varepsilon$

Can we use it

- ▶ to identify example (1) as pathological?

easy

The figure shows the non-standard semantics. The system oscillates for the whole \mathbb{T} ('for ever'), for a non-standard number of times. Note that the sequence of instants $n\varepsilon$ tends to infinity because n can itself be an infinite non-standard integer. This trajectory possesses no standardisation.



Back to the examples

Can we propose a semantic domain for these (and all) examples?

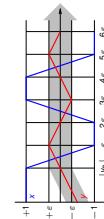
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Can we use it

- ▶ to identify example (2) as non-pathological?

less easy

The figure shows the non-standard semantics. The system oscillates for the whole \mathbb{T} ('for ever'), for a non-standard number of times. However, while the blue trajectory oscillates between -1 and $+1$, the red one oscillates between $-\varepsilon$ and $+\varepsilon$, and it can be proved that the standard part of this trajectory is indeed the thick grey polyline in which ε is interpreted as zero.



Back to the examples

Can we propose a semantic domain for these (and all) examples?

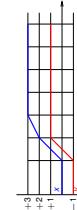
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Can we use it

- ▶ to decide on the semantics of example (3)?

easy

The figure shows the non-standard semantics. The system has a first zero-crossing at $t = 1$, which causes a second one to occur on the blue trajectory at $t = 1 + \varepsilon$. This yields a classical super-dense time semantics.



Back to the examples

Can we propose a semantic domain for these (and all) examples?

The drawings show the non-standard semantics with $\partial := \varepsilon$

Can we use it

- ▶ to give a semantics to example (4)?

non-standard semantics of the colliding balls example:

1. $t = \partial$, $x_1 = \partial \cdot w_1 > 0 \Rightarrow z \cdot c$ (zero-crossing) on $x_1 - x_2$.
2. \Rightarrow at $t = 2\partial$ balls exchange velocities: $w_1 = 0$ and $v_2 = w_1$.
3. $t = 3\partial$, $x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow$ ODE has immediate $z \cdot c$ on x_2
4. $t = 4\partial$, $x_1 = x_2 = 2\partial \cdot w_1$, $v_1 = 0$ and $v_2 = -w_1$.
5. $t = 5\partial$, $x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow z \cdot c$ $x_1 - x_2$
6. \Rightarrow at $t = 6\partial$, $x_1 = 2\partial \cdot w_1$, $x_2 = 0$, $v_1 = -w_1$ and $v_2 = 0$.

Then ball 1 moves toward $-\infty$ according to the ODEs and no further zero-crossings occur.



What is needed to establish the above on firm bases?

Two things are needed:

1. To establish on firm bases the juggling we play with ε and ∂ without care for both continuous and discrete dynamics
2. To relate it to "normal life semantics" where discrete dynamics, continuous dynamics and hybrid dynamics may or may not be well defined (existence/uniqueness/nonzeroeness of solutions), not to speak about composition thereof

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Answers to the above is provided by:

1. Non-Standard analysis seriously (don't be afraid...)
2. Standardisation of non-standard entities

What is needed to establish the above on firm bases?

► What Non-Standard semantics yields:

1. NS semantics is always defined; it involves "quasi-discrete" dynamical systems indexed by $\mathbb{T} = \partial \times \mathbb{N}$ (NS semantics is thus ∂ -dependent)
 - hybrid system program \rightarrow_{∂} NS semantics
2. Systems always compose

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1. NS semantics is always defined; it involves "quasi-discrete" dynamical systems indexed by $\mathbb{T} = \partial \times \mathbb{N}$ (NS semantics is thus ∂ -dependent)
 - hybrid system program \rightarrow_{∂} NS semantics
2. Systems always compose

- Standardisation principle: There exists a **standardisation map**
- hybrid system program \rightarrow_{∂} NS semantics \rightarrow_{∂} S semantics

such that

1. it is a partial map (sometimes NS systems have no S counterpart)
2. when standardisation exists, then the above end-to-end map does not depend on ∂ : NS semantics is **intrinsic**
3. when system composition is well defined in the S domain, then we get commutative diagrams

Non-Standard Analysis

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- A bit of history
 - ▶ Born in 1961 from Abraham Robinson, then developed by a small community of mathematicians.
 - ▶ Proposed as a conservative enhancement of Zermelo-Fraenkel set theory; some fancy axioms and principles; nice for the adicts
 - ▶ Subject of controversies: what does it do for you that you cannot do using our brave analysis with $\forall \exists \forall \dots ?$
 - ▶ 1988: a nice presentation of the topic by T. Lindstrom, kind of "non-standard analysis for the axiom-averse"
 - ▶ 2006: used in Simon Blauze PhD where he proposes the counterpart of a "Turing machine" for hybrid systems (supervised by D. Krob)

Non-Standard Analysis

A bit of history

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Why is non-standard analysis interesting for the computer scientist?

- ▶ It offers a step-based view of continuous and hybrid systems
- ▶ It is non-effective, still, it is amenable to symbolic executions and can thus be used for symbolic analyses at compile (and even run) time

Non-Standard Analysis

- The aim
 - ▶ to augment $\mathbb{R} \cup \{\pm\infty\}$ with elements that are infinitely close to x for each $x \in \mathbb{R}$, call " \mathbb{R}^* " the result;
 - ▶ \mathbb{R}^* should obey the same algebra as \mathbb{R} : total order, $+$, \times , ...
 - ▶ any $f : \mathbb{R} \rightarrow \mathbb{R}$ extends to $f^* : \mathbb{R}^* \rightarrow \mathbb{R}$, etc
- Idea:
 - ▶ mimic the construction of \mathbb{Q} from \mathbb{Z} as Cauchy sequences; candidates for infinitesimals include:

$$\begin{array}{lll} \text{close to } 0 & : & \left\{ \frac{1}{\sqrt{n}} \right\} > \left\{ \frac{1}{n} \right\} > \left\{ \frac{1}{r^2} \right\} > 0 \\ \text{close to } +\infty & : & \left\{ \sqrt{n} \right\} < \left\{ n \right\} < \left\{ r^2 \right\} \end{array}$$

Non-Standard Analysis

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infinitely close to x for each $x \in \mathbb{R}$, call $*\mathbb{R}$ the result;
- ▶ $*\mathbb{R}$ should obey the same algebra as \mathbb{R} : total order, $+$, \times , \dots
- any $f: \mathbb{R} \rightarrow \mathbb{R}$ extends to $*f: *R \rightarrow *\mathbb{R}$, etc
- Are we done? Not quite so:
- ▶ Sequences of reals $\{x_n\}$ generally do not converge
- ▶ Two sequences $\{x_n\}$ and $\{y_n\}$ converging to 0 may be s.t. $\{n \mid x_n > y_n\}$, $\{n \mid x_n < y_n\}$, and $\{n \mid x_n = y_n\}$ are all infinite sets

Are we done?

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- ▶ $P \cap Q = \emptyset \Rightarrow \mu(P \cup Q) = \mu(P) + \mu(Q)$; $\mu(\mathbb{N}) = 1$
- ▶ P finite implies $\mu(P) = 0$: P is **negligible**
- ▶ either P or $\mathbb{N} - P$ belongs to \mathcal{F}

Existence of \mathcal{F} follows from Zorn's lemma (\Leftrightarrow axiom of choice)

Non-Standard Analysis

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- Lindström: partition subsets of \mathbb{N} into **negligible**/**non-negligible** ones, so that:
 - ▶ finite or empty subsets are all negligible
 - ▶ negligible sets are stable under finite unions
 - ▶ for any subset P , either P or its complement is non-negligible
- Having such a decision mechanism relies on Zorn Lemma (\approx axiom of choice) and is formalized as explained next.

Non-Standard Analysis: the idea of Lindström

Pick \mathcal{F} a **free ultrafilter** of \mathbb{N} :

- ▶ $\emptyset \notin \mathcal{F}$, \mathcal{F} stable by intersection
- ▶ $P \in \mathcal{F}$ and $P \subseteq Q$ implies $Q \in \mathcal{F}$
- ▶ P finite implies $P \notin \mathcal{F}$
- ▶ either P or $\mathbb{N} - P$ belongs to \mathcal{F}

Existence of \mathcal{F} follows from Zorn's lemma (\Leftrightarrow axiom of choice)

Define:

$$\mu(P) = \begin{cases} 1 & \text{if } P \in \mathcal{F} \\ 0 & \text{else} \end{cases}$$

- ▶ $P \cap Q = \emptyset \Rightarrow \mu(P \cup Q) = \mu(P) + \mu(Q)$; $\mu(\mathbb{N}) = 1$
- ▶ P finite implies $\mu(P) = 0$: P is **negligible**

Non-Standard Analysis: the idea of Lindström

Non-Standard Analysis: the Idea of Lindstrom

$(x_n), (x'_n) \in \mathbb{R}^{\mathbb{N}}$, define $(x_n) \approx (x'_n)$ iff set $\{n \mid x_n \neq x'_n\}$ is negligible

$${}^*\mathbb{R} = \mathbb{R}^{\mathbb{N}} / \approx ; \text{ elements of } {}^*\mathbb{R} \text{ are written } [x_n]$$

- ▶ For any two $(x_n), (y_n)$ exactly one among the sets $\{n \mid x_n > y_n\}, \{n \mid x_n < y_n\}, \{n \mid x_n = y_n\}$, is non-negligible
 \Rightarrow

any two sequences can always be compared modulo \approx

- ▶ By pointwise extension, a 1st-order formula is true over ${}^*\mathbb{R}$ iff it is true over \mathbb{R} : this is known as the **transfer principle**
- ▶ Say that $x = st([x_n])$ if $x_n \rightarrow x$ modulo negligible sets

Non-Standard Analysis: the idea of Lindstrom

Theorem: [standardisation] Any non-standard real $[x_n]$ possesses a unique standard part

Proof:

1. Pick $x = \sup\{u \in \mathbb{R} \mid [u] \leq [x_n]\}$
 where $[u]$ denotes the constant sequence equal to u .
 2. Since $[x_n]$ is finite, x exists; remains to show that $[x_n] - x$ is infinitesimal.
 3. If this is not true,
 - ▶ then there exists $y \in \mathbb{R}, y > 0$ such that $y < |x - [x_n]|$.
 - ▶ that is, either $x < [x_n] - [y]$ or $x > [x_n] + [y]$,
 - ▶ which both contradict the definition of x .
 4. The uniqueness of x is clear, thus we can define $st([x_n]) = x$.
- Infinite non-standard reals have no standard part in \mathbb{R} .

Integrals, ODE, and the Standardisation Principle

- ▶ internal functions and sets by pointwise extension:
- ▶ $\forall n, g_n : \mathbb{R} \mapsto \mathbb{R}$ yields $[g_n] : {}^*\mathbb{R} \mapsto {}^*\mathbb{R}$ by $[g_n]([x_n]) = [g_n(x_n)]$
- ▶ Pick ∂ infinitesimal and $N \in {}^*\mathbb{N}$ such that $(N-1)\partial < 1 \leq N\partial$, and consider the set $T = \{0, \partial, 2\partial, \dots, (N-1)\partial, 1\}$
- ▶ By definition, if $\partial = [d_n]$, then $N = [N_n]$ with $N_n = \frac{1}{d_n}$ and $T = [T_n]$ with $T_n = \{0, \partial, 2\partial, \dots, (N_n-1)\partial, 1\}$
- ▶ For $f : [0, 1] \mapsto \mathbb{R}$ a continuous function and $\gamma = [f, f, \dots]$ its non-standard version

$$\left[\sum_{t \in T_n} \frac{1}{N_n} f(t_n) \right] = \sum_{t \in T} \frac{1}{N} f(t)$$

we claim this

$$st\left(\left[\sum_{t \in T_n} \frac{1}{N_n} f(t_n) \right]\right) = st\left(\sum_{t \in T} \frac{1}{N} f(t)\right) = \int_0^1 f(t) dt$$

Integrals, ODE, and the Standardisation Principle

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Integrals, ODE, and the Standardisation Principle

Theorem: [standardisation] if $f : [0, 1] \rightarrow \mathbb{R}$ is continuous, then

$$\text{st} \left(\left(\sum_{t \in T_n} \frac{1}{N_n} f(t_n) \right) \right) = \text{st} \left(\sum_{t \in T} \frac{1}{N} * f(t) \right) = \int_0^1 f(t) dt$$

Proof: If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a standard function, we always have

$$\sum_{t \in T} \frac{1}{N} * f(t) = \left[\sum_{t \in T_n} \frac{1}{N_n} f(t_n) \right] \quad (1)$$

Now, f continuous implies $\sum_{t \in T_n} \frac{1}{N_n} f(t_n) \rightarrow \int_0^1 f(t) dt$, so, by definition of non-standard reals,

$$\int_0^1 f(t) dt = \text{st} \left(\sum_{t \in T} \frac{1}{N} * f(t) \right) \quad (2)$$

Integrals, ODE, and the Standardisation Principle

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- Thus, if f is smooth so that its Riemann integral is well defined, then any non-standard formulation of the integral of f has $\int_0^1 f(t) dt$ as its standard part
- The same philosophy applies to ODEs and Hybrid Systems

Integrals, ODE, and the Standardisation Principle

For every $0 < t \leq 1$:

$$\int_0^t f(u) du = \text{st} \left(\sum_{u \in T, u \leq t} \frac{1}{N} * f(t) \right) \quad (\text{Non-standard Riemann integral})$$

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For every $0 < t \leq 1$:

$$\int_0^t f(u) du = st \left(\sum_{u \in T: u < t} \frac{1}{N} f(u) \right) \quad (\text{Non-standard Riemann integral})$$

Set $\partial = \frac{1}{N}$ and consider the ODE $\dot{x} = f(x, t)$, x_0 , in integral form

$$\begin{aligned} x(t) &= x_0 + \int_0^t f(x(u), u) du \quad (\text{with the needed smoothness}) \\ \textcolor{red}{x(t)} &= st \left(x_0 + \sum_{u \in T: u \leq t} \frac{1}{N} f(x(u), ku) \right) \end{aligned}$$

For every $0 < t \leq 1$:

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$$\begin{aligned} x(t) &= x_0 + \int_0^t f(x(u), u) du \\ \textcolor{red}{x(t)} &= st \left(x_0 + \sum_{u \in T: u \leq t} \frac{1}{N} f(x(u), ku) \right) \\ &= st(\star(x(s_i))) \quad , \text{ for } s_i = \max\{k \cdot t_k = k\partial \leq t\} \end{aligned} \quad (3)$$

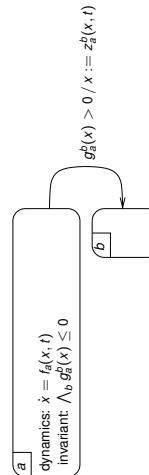
where $\star x$ is the non-standard semantics of the above ODE with time basis ∂ :

$$\begin{cases} \star x(t_k) &= \star x(t_{k-1}) + \partial \times (\star x(t_{k-1}), t_{k-1}) \\ \star x(t_0) &= x_0 \end{cases} \quad (4)$$

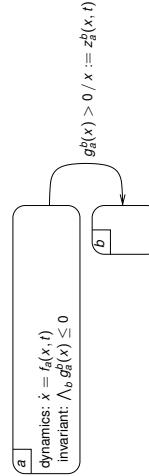
Theorem: [standardisation]

- (4) is always defined as a non-standard dynamical system
- (3) only holds if the ODE has a solution

Non-Standard Hybrid Systems, Standardisation Principle



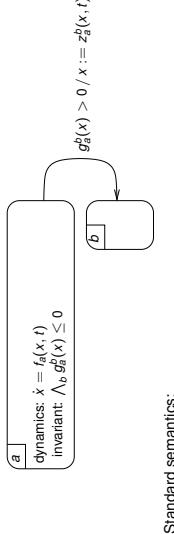
Non-Standard Hybrid Systems, Standardisation Principle



Standard semantics:

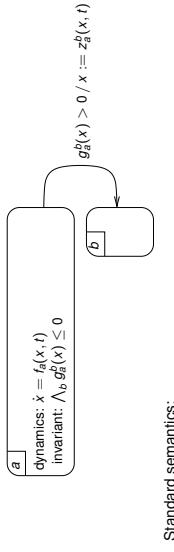
- ▶ spending standard > 0 duration within modes: ODE
- ▶ finite cascades of mode changes: super-dense time $(t, n) \in \mathbb{R} \times \mathbb{N}$

Non-Standard Hybrid Systems, Standardisation Principle



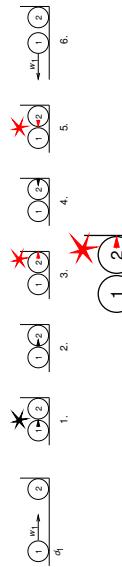
- In this example, we successively have, within an infinitesimal period of time:
1. a first cascade of z-c (a hit causing changes in velocities)
 2. **the launching of an ODE with an immediate z-c**
 3. another cascade of z-c followed by the symmetric scheme.
- Provided that such a cascade of [z-c + ODE micro-steps] remains finite, a super-dense time semantics can be given. Execution by executing the symbolic non-standard semantics: **Extended Standardisation Principle**.

Non-Standard Hybrid Systems, Standardisation Principle



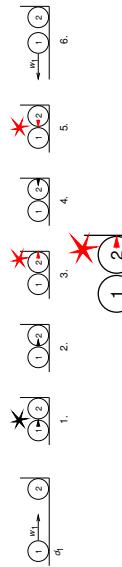
Theorem: [standardisation] If the S semantics is well-defined, then it is the standardisation of the NS (β -dependent) semantics, for any choice of β

Non-Standard Hybrid Systems, Standardisation Principle (extended)



In this example, we successively have, within an infinitesimal period of time:

1. $t = \partial, x_1 = \partial \cdot w_1 > 0 \Rightarrow z\text{-c}$ (zero-crossing) on $x_1 - x_2$.
2. \Rightarrow at $t = 2\partial$ balls exchange velocities: $v_1 = 0$ and $v_2 = w_1$.
3. $t = 3\partial, x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow$ ODE has immediate z-c on x_2 .
4. $t = 4\partial, x_1 = x_2 = w_2 = 2\partial \cdot w_1, v_1 = 0$ and $v_2 = -w_1$.
5. $t = 5\partial, x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow z\text{-c}$ $x_1 - x_2$.
6. \Rightarrow at $t = 6\partial, x_1 = 2\partial \cdot w_1, x_2 = 0, v_1 = -w_1$ and $v_2 = 0$.



Non-standard symbolic simulation of the colliding balls example:

1. $t = \partial, x_1 = \partial \cdot w_1 > 0 \Rightarrow z\text{-c}$ (zero-crossing) on $x_1 - x_2$.
2. \Rightarrow at $t = 2\partial$ balls exchange velocities: $v_1 = 0$ and $v_2 = w_1$.
3. $t = 3\partial, x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow$ ODE has immediate z-c on x_2 .
4. $t = 4\partial, x_1 = x_2 = w_2 = 2\partial \cdot w_1, v_1 = 0$ and $v_2 = -w_1$.
5. $t = 5\partial, x_1 = 2\partial \cdot w_1$ and $x_2 = \partial \cdot w_1 \Rightarrow z\text{-c}$ $x_1 - x_2$.
6. \Rightarrow at $t = 6\partial, x_1 = 2\partial \cdot w_1, x_2 = 0, v_1 = -w_1$ and $v_2 = 0$.

The SIMPLEHYBRID mini-language and its semantics

Difficulties in Hybrid Systems Modelers
 $\bullet_{x_i} =_{\text{def}} \{n\partial\}_{n \in \mathbb{N}}$ $\bullet^{(n\partial)} =_{\text{def}} (n-1)\partial$

Some examples

Non-Standard Hybrid Systems (for the math-averse)

Non-Standard Analysis and Standardisation (for the fan)

Non-Standard Hybrid Systems and their Standardisation

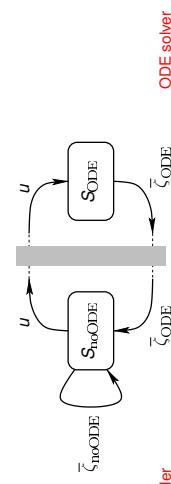
The SIMPLEHYBRID mini-language

Conclusion

statement	transition relation
$y = f(x)$	$y = f(x)$
$y = \text{last}(x) \text{ init } y_0$	$y = \bullet_x \text{ init } y_0$
$\zeta = \text{up}(x)$	$\zeta^* = \begin{cases} (\bullet_x < 0) \wedge [x \geq 0] \\ \vee ([x \leq 0] \wedge [x > 0]) \end{cases}$
$\dot{y} = x \text{ init } y_0 \text{ reset } z$	$\text{on } \tau \setminus \tau_z : y = \bullet_y + \partial \times \bullet_x$ $\text{on } \tau_z : y = z$
$y = \text{x every } \zeta \text{ init } y_0$	$\text{before } \zeta : y = y_0$ $\text{on } \zeta : y = x$
$y = \text{pre}(x) \text{ init } y_0$	$\tau_y = \tau_x$ $\text{before min}(\tau_y) : y = y_0$ $\text{on } \tau_y : y = \bullet_x$
$S_1 \parallel S_2$	conjunction

The SIMPLEHYBRID mini-language and its semantics

Slicing



discrete compiler

ODE solver

statement	transition relation
$y = f(x)$	$\{n\partial\}_{n \in \mathbb{N}}$
$y = \text{last}(x) \text{ init } y_0$	$\bullet_{x_i} =_{\text{def}} \{n\partial\}_{n \in \mathbb{N}}$
$\zeta = \text{up}(x)$	$\zeta^* = \begin{cases} (\bullet_x < 0) \wedge [x \geq 0] \\ \vee ([x \leq 0] \wedge [x > 0]) \end{cases}$
$\dot{y} = x \text{ init } y_0 \text{ reset } z$	$\text{on } \tau \setminus \tau_z : y = \bullet_y + \partial \times \bullet_x$ $\text{on } \tau_z : y = z$
$y = \text{x every } \zeta \text{ init } y_0$	$\text{before } \zeta : y = y_0$ $\text{on } \zeta : y = x$
$y = \text{pre}(x) \text{ init } y_0$	$\tau_y = \tau_x$ $\text{before min}(\tau_y) : y = y_0$ $\text{on } \tau_y : y = \bullet_x$
$S_1 \parallel S_2$	conjunction

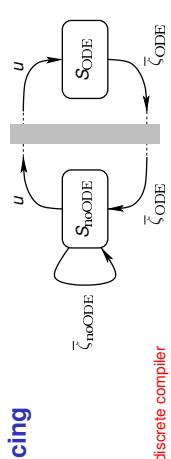
all ZC + aborting ODE in S ; $\bar{\zeta}_S$

aborting ODE

three types
of zero-crossing
no need for
left/right limit

Slicing

Further use of Non-Standard Semantics



statement of S	Assigned to S_{noODE}	Assigned to S_{ODE}
$y = f([x])$	on $\bar{\zeta}_S : y = f([x])$	outside $\bar{\zeta}_S : y = f([x])$
$y = \text{last}(x)$	on $\bar{\zeta}_S : y = \text{last}(x)$	outside $\bar{\zeta}_S : y = \text{last}(x)$
$\zeta = \text{up}(x)$		$\zeta = \text{up}(x)$
$\dot{y} = x \text{ init } y_0$	on $\bar{\zeta}_S \setminus \zeta : \dot{y} = x \text{ init } y_0$	outside $\bar{\zeta}_S : \dot{y} = x \text{ init } y_0$
	reset z	reset z
$y = [x] \text{ every } [\zeta]$	$y = [x] \text{ every } [\zeta]$	
	init y_0	
$y = \text{pre}(x)$	$y = \text{pre}(x)$	
	init y_0	

- Causality Analysis and Constructive Semantics
 - △ compilation and code generation
 - △ clock-aware compilation
 - △ **new application:** DAE and index analysis
- Kahn Network semantics (KPN arguments extend to \mathbb{N})
 - △ distributed simulation & multiple solvers
 - △ to avoid unwanted coupling due to adaptive step size

Conclusion

- Non-standard semantics is not just for the fun of Albert Benveniste
 - it gives a semantics to all syntactically well-formed programs
 - △ no hand waving, no need for obscure continuity/zero assumption
 - △ compositional
 - this is what the language designer needs
 - △ provides semantic support for clock-aware causality analysis
 - △ clock-aware co-simulation (getting rid of global solver(s))
 - △ future: extend to DAE
 - △ provides semantic support for Discrete/Continuous slicing
 - △ NS symbolic simulation of aborting ODEs
 - △ future: singular perturbations and multiple time-scales
 - △ Prevents the designer from the need for manual smoothing (non compositional because bandwidth-dependent)

You hybrid guys, go learning it!

Difficulties in Hybrid Systems Modelers

Some examples

Non-Standard Hybrid Systems (for the math-averse)

Non-Standard Analysis and Standardisation (for the fan)

Non-Standard Hybrid Systems and their Standardisation

The SIMPLEHYBRID miniLanguage

Conclusion



EQUATIONS, SYNCHRONY, TIME, AND MODES

Edward A. Lee, EECS, UC Berkeley

The key principle behind equation-based languages is that components in a system interact with one another not by reacting to inputs to produce outputs, but rather by asserting relationships between the values of variables that they share. This principle is closely related to key principle behind synchronous-reactive (SR) languages, where the meaning of a composition of components is a fixed-point solution to a system of equations. In both cases, interactions between components is a dialog, with give and take, rather than a monolog. SR languages have been used to model discrete behaviors primarily, whereas equation-based languages, particularly Modelica, have been used to model continuous dynamics primarily. In this talk, I will show how to bridge the two.

Synchronous programs execute a sequence of (conceptually) simultaneous and instantaneous computations. Each step in the sequence is called a "tick" of a conceptual clock that governs the execution. Distinctly lacking, however, is any notion of metric or measurable time in this clock, so there is no foundation in these languages for modeling continuous dynamics. The ticks form a sequence, not a time line. In fact, a correct execution of a synchronous program (conformant with the semantics) can take as much time as it likes between ticks. The intervals need not even be constant or defined.

In this talk, I will review the principles of synchronous semantics and show how they can be extended to provide a rigorous foundation for timed systems that do have a metric notion of time. In particular, I will show how discrete-event (DE) and continuous-time models can be built on top of synchronous semantics. I will also introduce a hierarchical multiform time that allows time progress at different rates in different parts of the system, and I will show how the underlying synchronous semantics ensures determinacy and preserves causality. This multiform model of time provides a foundation for modal behaviors and hybrid systems.

Equations, Synchrony, Time, and Modes



Collaborative with:

- Adam Cataldo
 - Patricia Dener
 - John Ellison
 - Xiaojun Liu
 - Eleftherios Matsikoudis
 - Haixiang Zheng
- Edward A. Lee**
Robert S. Pepper Distinguished Professor
UC Berkeley

Invited Talk at Workshop:
System Design meets Equation-based
Languages: Workshop Program
Lund, Sweden,
Sept. 18-21



What is the momentum of the middle ball as a function of time?

$$\mathbf{p}(t) = m\mathbf{v}(t)$$

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What is the momentum of the middle ball as a function of time?

$$\mathbf{p}(t) = m\mathbf{v}(t)$$

It might seem:

$$\mathbf{v}(t) = 0 \quad \Rightarrow \quad \mathbf{p}(t) = 0$$

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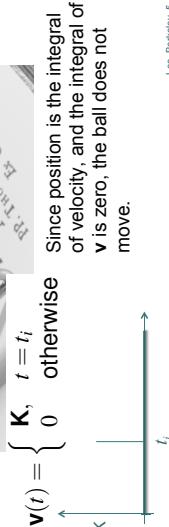
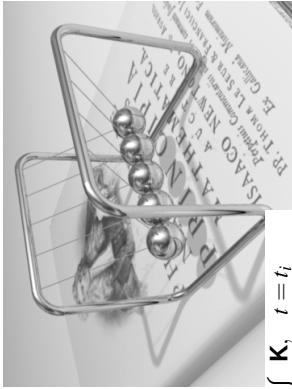


But no, it is:

$$\mathbf{v}(t) = \begin{cases} \mathbf{K}, & t = t_i \\ 0 & \text{otherwise} \end{cases}$$

where t_i is the time of collision

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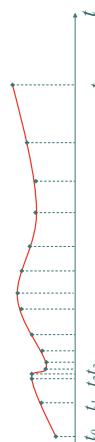


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Samples yield discrete signals

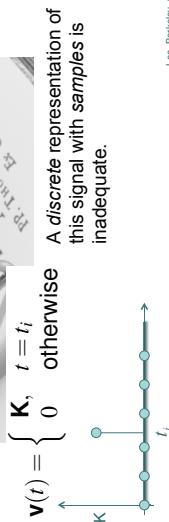
A signal $s: T \rightarrow D$ is sampled at tags

$$\pi(s) = \{t_0, t_1, \dots\} \subset T$$

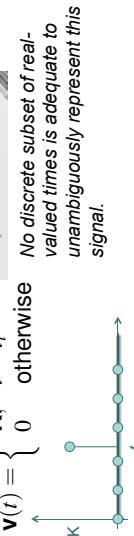


A signal s is **discrete** if there is an *order embedding* from its tag set $\pi(s)$ (the tags for which it is defined and not absent) to the natural numbers (under their usual order). Note: Benveniste et al. use a different (and less useful?) notion of "discrete."

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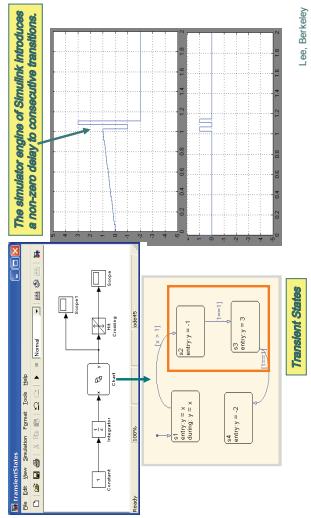
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Lee, Berkeley 8

Simulink/Stateflow cannot accurately model such events.

In Simulink, a signal can only have one value at a given time. Hence Stateflow introduces solver-dependent behavior.



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Ptolemy II uses Superdense Time

[Maier, Manna, Pnueli, '92]
for Continuous-Time Signals

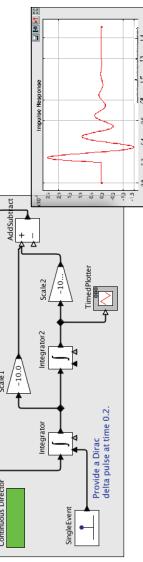
$$\mathbf{v}: (\mathbb{R} \times \mathbb{N}) \rightarrow \mathbb{R}^3$$

Initial value: $\mathbf{v}(t_i, 0) = 0$

Intermediate value: $\mathbf{v}(t_i, 1) = \mathbf{K}$

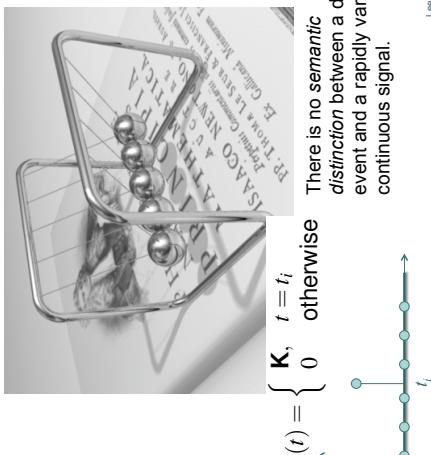
Final value: $\mathbf{v}(t_i, n) = 0, \quad n \geq 2$

o Infinitessimals (even Dirac delta functions):



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At each tag, the signal has exactly one value. At each time point, the signal has a sequence of values. Signals are piecewise continuous, in a well-defined technical sense, a property that makes ODE solvers work well.



$$\mathbf{v}(t) = \begin{cases} \mathbf{K}, & t = t_i \\ 0 & \text{otherwise} \end{cases}$$

There is no semantic distinction between a discrete event and a rapidly varying continuous signal.

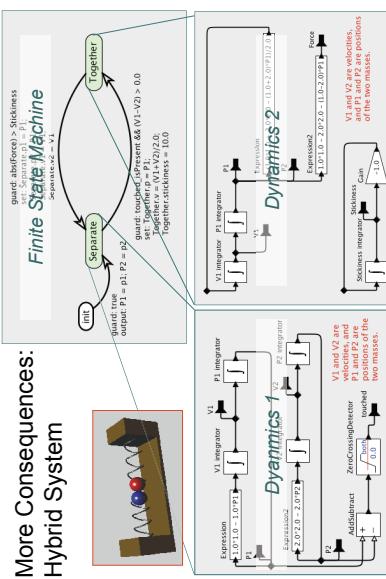
Consequences of using Superdense Time

- o Transient states are well represented:

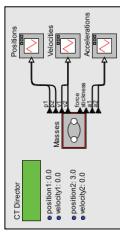
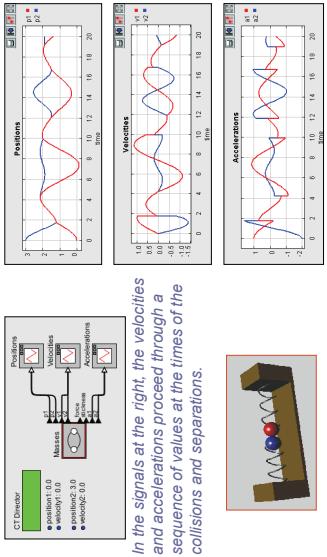


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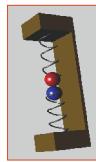
More Consequences: Hybrid System



Transitions between modes are instantaneous

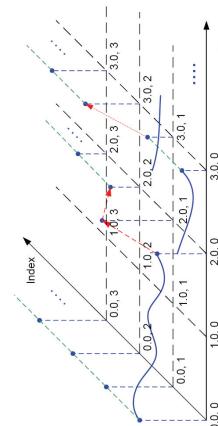
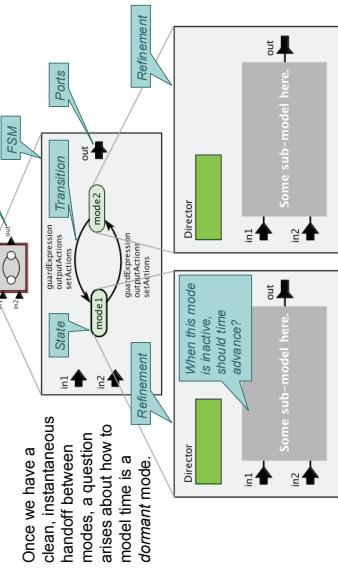


In the signals at the right, the velocities and accelerations proceed through a sequence of values at the times of the collisions and separations.



Superdense Time

Modal Models and Multiform Time



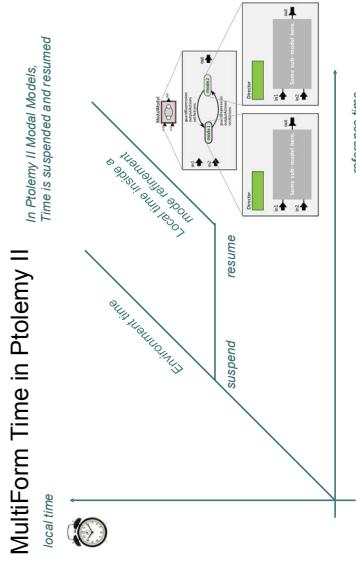
The red arrows indicate value changes between tags, which correspond to discontinuities. Signals are continuous from the left and continuous from the right at points of discontinuity.

The Modal Model Muddle

It's about time

After trying several variants on the semantics of modal time, we settled on this:

A mode refinement has a *local* notion of time. When the mode refinement is inactive, local time does not advance. Local time has a monotonically increasing gap relative to environment time.



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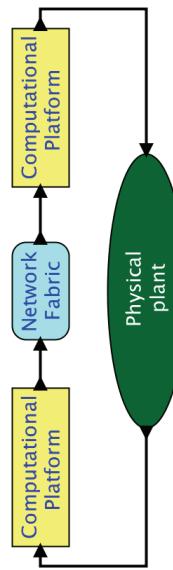
Variants for the Semantics of Modal Time that we Tried or Considered, but that Failed

- Mode refinement executes while “inactive” but inputs are not provided and outputs are not observed.
- Time advances while mode is inactive, and mode refinement is responsible for “catching up.”
- Mode refinement is “notified” when it has requested time increments that are not met because it is inactive.
- When a mode refinement is re-activated, it resumes from its first missed event.

All of these led to some very strange models...

Final solution: Local time does not advance while a mode is inactive. Monotonically growing gap between local time and environment time.

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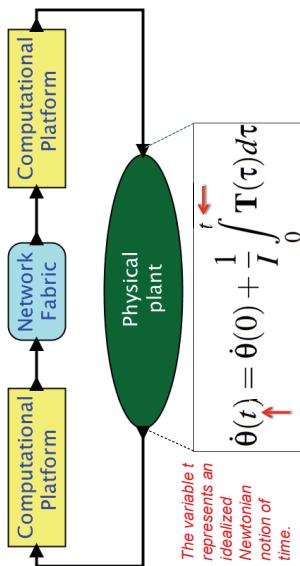


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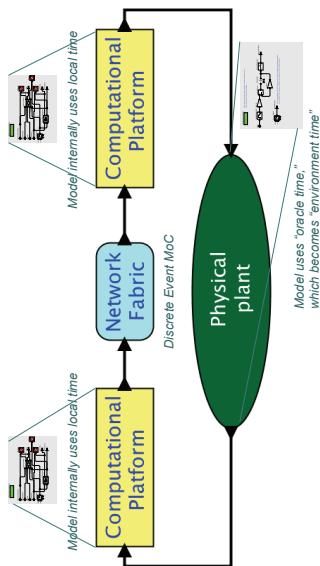
Once we have multiform time, we can build accurate models of cyber-physical systems

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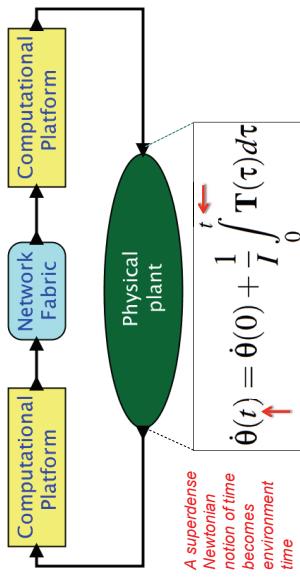
Engineers model physical dynamics using differential-algebraic equations.



Local time within a hierarchy can advance at different rates.



But computational platforms have no access to t . Instead, local measurements of time are used.

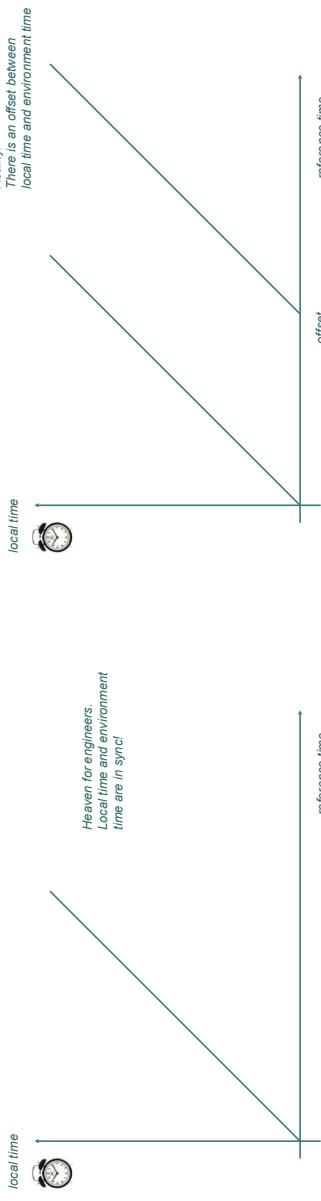


Clocks drift

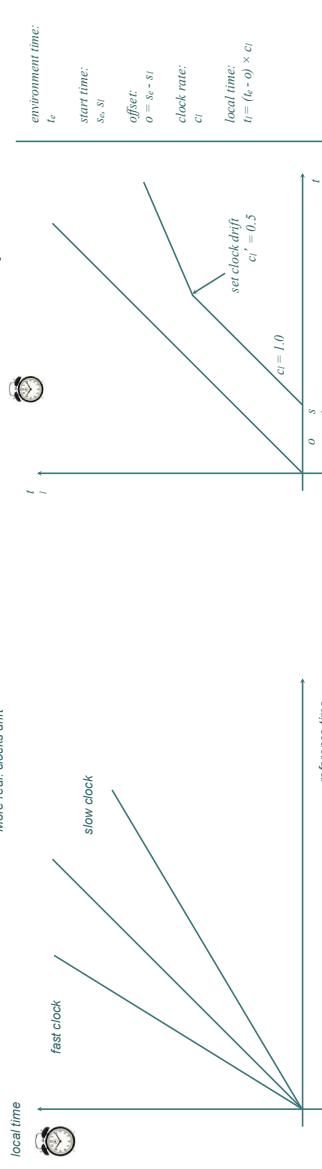


- Fabrication tolerance
- Aging
- Temperature
- Humidity
- Vibrations
- Quality of the quartz.
- Clock drifts measured in "parts per million" or ppm
- 1 ppm corresponds to a deviation of 1μs every second

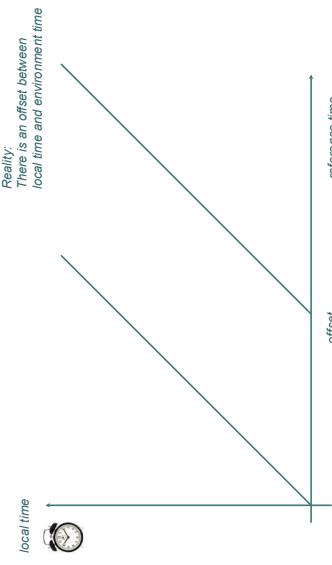
Multiform Time in Ptolemy



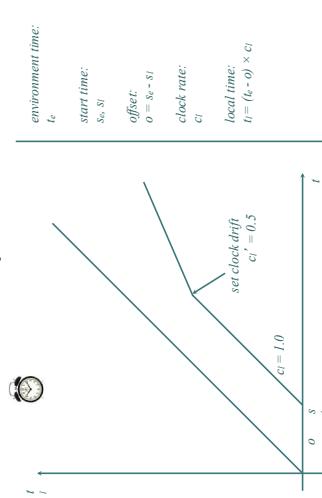
Multiform Time in Ptolemy



Multiform Time in the Real World



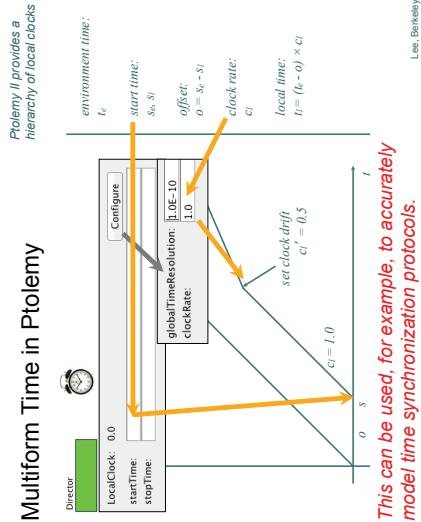
Multiform Time in Ptolemy



Lee, Berkeley 27.2

Lee, Berkeley 28.2

Multiform Time in Ptolemy



Other Questions about Time:

1. **Precision**
 - In floating-point formats, precision degrades as magnitude increases
 2. **Clear Semantics of Simultaneity**
 - Requires precise addition and subtraction, e.g. $(a + b) + c = a + (b + c)$. Floating-point numbers don't have this property.
- Floating point numbers are a poor choice for modeling time!

Conclusions

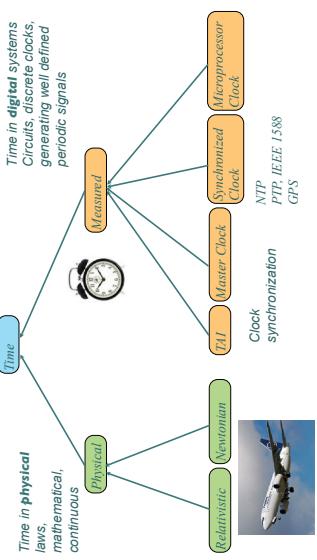
- Modeling time as a simple continuum is not adequate.
 - Superdense time** offers clean semantics for instantaneous events.
- Homogeneous time advancing uniformly is not adequate.
 - Hierarchical multiform time** enables accurate and practical models of heterogeneous distributed systems.
- Floating point numbers for time are not adequate.
 - A model with invariant precision and precise addition and subtraction is.



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Source: Patricia Denter and John Edison

Multiform Time is Intrinsic!



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**FORMAL MODELING AND ANALYSIS OF SOFTWARE
SYSTEMS WITH LUSTRE**
Mike Whalen, University of Minnesota

Rockwell Collins and the University of Minnesota have used the synchronous dataflow language Lustre as a basis for a variety of analyses of industrial critical systems both for component level models written in Simulink and system architectural models written in AADL. This talk describes the approach, several examples of analyzed models as well as several challenges to extend the scale and variety of systems that can be practically analyzed.



Compositional Analysis of System Architectures (using Lustre)



Acknowledgements

- Rockwell Collins (**Darren Cofer, Andrew Gacek**, Steven Miller; Lucas Wagner)
- UPenn: (Insup Lee, Oleg Sokolsky)
- UMN (Mats P.E. Heimdahl)
- CMU SEI (Peter Feiler)

Acknowledgements

- Rockwell Collins (**Darren Cofer, Andrew Gacek**, Steven Miller; Lucas Wagner)
- UPenn: (Insup Lee, Oleg Sokolsky)
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- CMU SEI (Peter Feiler)

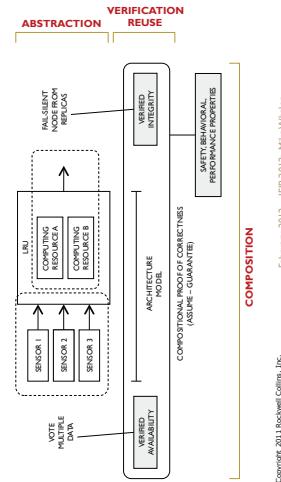
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September, 2012 | LCCC 2012: Mike Whalen

Component Level Formal Analysis Efforts



System design & verification through pattern application and compositional reasoning

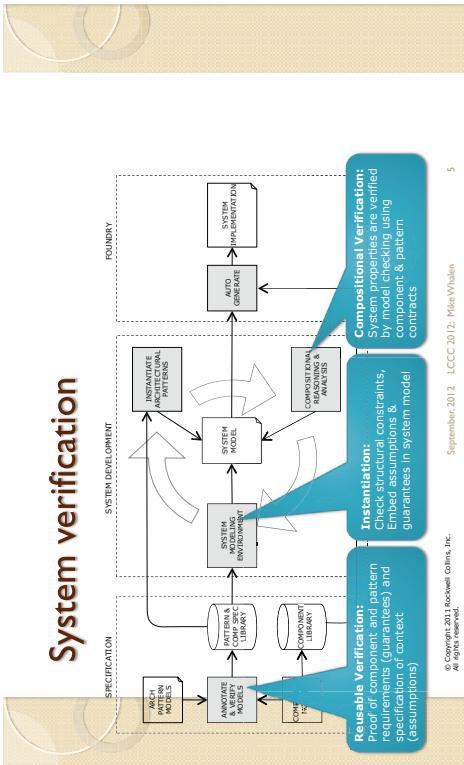


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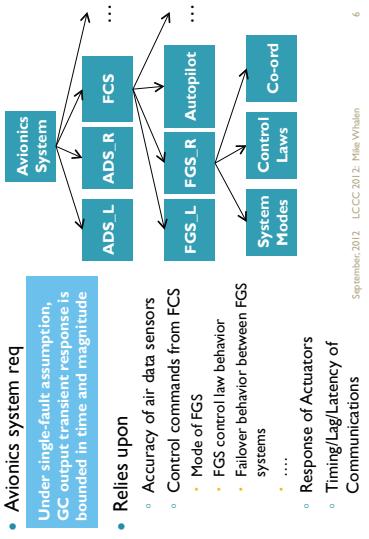
February, 2012 | FIP 2012: Mike Whalen

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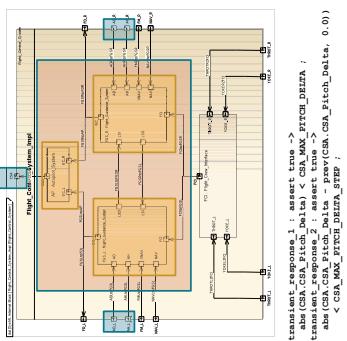
System verification



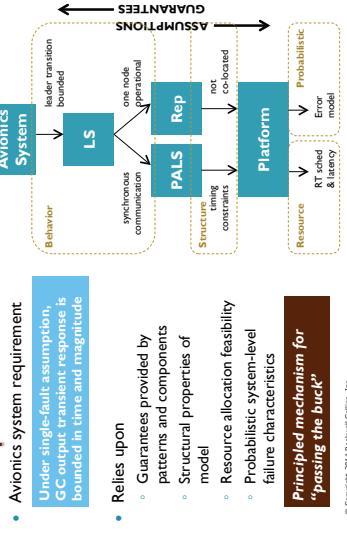
Hierarchical reasoning about systems



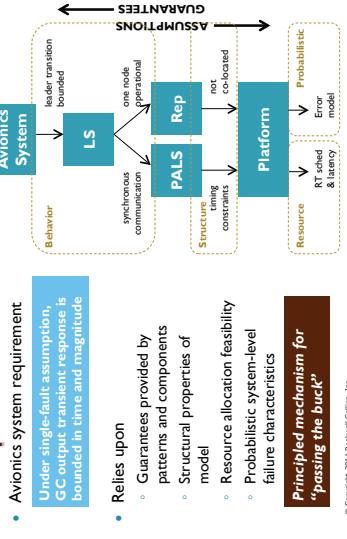
Compositional Reasoning for Active Standby



Contracts between patterns and components



Contracts between patterns and components



5

6

7

8

Contracts

- Derived from Lustre and Property Specification Language (PSL) formalism
- IEEE standard
- In wide use for hardware verification
- Assume / Guarantee style specification
- Assumptions: "Under these conditions"
- Promises (Guarantees): "... the system will do X"
- Local definitions can be created to simplify properties

```
Contract;
fun abs(x : real) : real = if (x > 0) then x else -x ;
const ABS_MAX_PITCH_DELTA : real := 3.0 ;
const PC_MAX_PITCH_SIDE_DELTA : real := 2.0 ;
...
property AD_L_Pitch_Stop_Delta_Valid =
true => ABS(AD_L_Pitch_Stop_Delta) <= prev(AD_L_Pitch_Stop_Delta);
abs(AD_L_Pitch_Stop_Delta) <=
AD_L_Pitch_Stop_Delta ;
...
active_assumption : assume some_actuve ;
assumption_Act_L_Pitch_Stop_Delta_Valid_and_Pitch_1c_OK ;
...
create_en_response_1 : AD_L_Pitch_Stop_Delta_Valid_and_Pitch_1c_OK ;
assert_en_response_1 : AD_L_Pitch_Stop_Delta_Valid_and_Pitch_1c_OK ;
assert_en_response_2 : AD_L_Pitch_Stop_Delta_Valid_and_Pitch_1c_OK ;
assumption_pc_max_pitch_delta =
abs(AD_L_Pitch_Stop_Delta) <= prev(AD_L_Pitch_Stop_Delta, 0.0) ;
AD_MAX_PITCH_STOP ;
```

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Reasoning about contracts

- Notionally: It is always the case that if the component assumption is true, then the component will ensure that the guarantee is true.



$G(A \Rightarrow P);$

- An assumption violation in the past may prevent component from satisfying current guarantee, so we need to assert that the assumptions are true up to the current step:

$G(H(A) \Rightarrow P);$

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Reasoning about Contracts

- Given the set of component contracts:
 $\Gamma = \{ G(H(A_c) \Rightarrow P_c) \mid c \in C \}$
- Architecture adds a set of obligations that tie the system assumption to the component assumptions

$$Q = \{ H(A_s) \Rightarrow P_s \} \cup \\ \{ H(A_s) \Rightarrow A_c \mid c \in C \}$$
- This process can be repeated for any number of abstraction levels

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Composition Formulation

- Suppose we have
 - Sets of formulas Γ and Q
 - A well-founded order \prec on Q
 - Sets $\Theta_q \subseteq \Delta_q \subseteq Q$, such that $r \in \Theta_q$ implies $r \prec q$
- Then if for all $q \in Q$
 - $\Gamma \Rightarrow G((Z(H(\Theta_q)) \wedge \Delta_q) \Rightarrow q)$
 - Then:
 - $G(q)$ for all $q \in Q$
 - [Adapted from McMillan]

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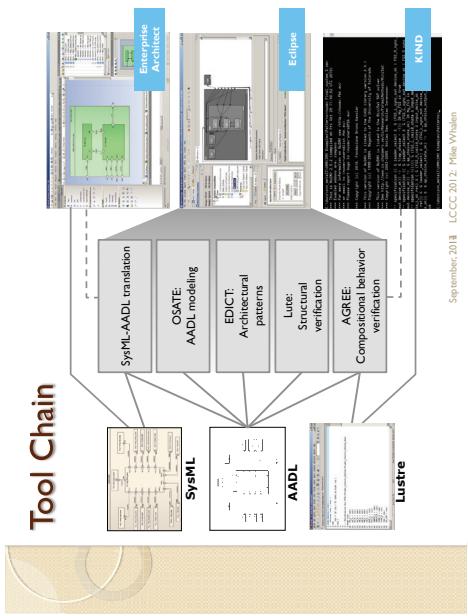
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A concrete example

- Order of data flow through system components is computed by reasoning engine
 - $\{FGS_L|FGS_R\} \rightarrow \{System\ inputs\}$
 - $\{FGS_L|FGS_R\} \rightarrow \{AP\}$
 - $\{AP\} \rightarrow \{System\ outputs\}$
- Based on flow, we establish four proof obligations
 - System assumptions \rightarrow FGS_L assumptions
 - System assumptions \rightarrow FGS_R assumptions
 - System assumptions + FGS_L guarantees + FGS_R guarantees \rightarrow AP assumptions
 - System assumptions + $\{FGS_L|FGS_R|AP\}$ guarantees \rightarrow System guarantees
- System can handle circular flows, but user has to choose where to break cycle

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Tool Chain



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Research Challenges



Proving

- Current back-end analysis performed using SMT-based k-induction model checking technique [Hagen and Tinelli: FMCAD 2008]
 - Very scalable if properties can be inductively proven
 - Unfortunately, inductive proofs often fail because properties are too weak
 - Lots of work on lemma/invariant discovery to strengthen properties
- Biesse and Claessen: SMT-based verification without State Space Traversal
 - Bradley: SMT-based Model Checking without Unrolling
 - Timelic: Instantiation-Based Invariant Discovery, [NFM 2011]
- These strengthening methods are not targeted towards our problem
 - Only supports analysis of linear models

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Scaling

- What do you do when systems and subcomponents have hundreds of requirements?
- FGS mode logic: 280 requirements
- DWM: >600 requirements
- Need to create automated slicing techniques for predicates rather than code.
- Perhaps this will be in the form of counterexample-guided refinement



Assigning blame

- Counterexamples are often hard to understand for big models
- It is much worse (in my experience) for property-based models
- Given a counterexample, can you automatically assign blame to one or more subcomponents?
- Given a ‘blamed’ component, can you automatically open the black box to strengthen the component guarantee?

	Signal	Step:
AD_Undefined	TRUE	2.2
AD_0n_called	TRUE	2.35
AD_0n_called	FALSE	2.39
AD_patch_cold	TRUE	2.83
AD_patch_cold	FALSE	3.74
ARCSA_patched	TRUE	0.09
ARCSA_patched	FALSE	0.26
ARCSA_patched	TRUE	0.35
ARCSA_patched	FALSE	0.35
ARCSA_patched	TRUE	0.57
ARCSA_patched	FALSE	0.57
ARCSA_patched	TRUE	0.74
ARCSA_patched	FALSE	0.74
ARCSA_patched	TRUE	0.81
ARCSA_patched	FALSE	0.81
ARCSA_patched	TRUE	0.88
ARCSA_patched	FALSE	0.88
ARCSA_patched	TRUE	0.91
ARCSA_patched	FALSE	0.91
ASSumption_AP	TRUE	0.43
ASSumption_AP	FALSE	0.43
ASSumption_for_FC_L	TRUE	0.43
ASSumption_for_FC_R	TRUE	0.43
FGS_LC_cards_active	TRUE	0.45
FGS_LC_cards_active	FALSE	0.45
FGS_LLC_master	TRUE	0.45
FGS_LLC_master	FALSE	0.45
FGS_RFC_cards_active	TRUE	0.43
FGS_RFC_cards_active	FALSE	0.43
FGS_RFC_cards_active	TRUE	0.48
FGS_RFC_cards_active	FALSE	0.48
FGS_RFC_cards_active	TRUE	0.51
FGS_RFC_cards_active	FALSE	0.51
FGS_RFC_cards_active	TRUE	0.54
FGS_RFC_cards_active	FALSE	0.54
FGS_RFC_cards_active	TRUE	0.55
FGS_RFC_cards_active	FALSE	0.55
FGS_RFC_cards_active	TRUE	0.58
FGS_RFC_cards_active	FALSE	0.58
FGS_RFC_cards_active	TRUE	0.61
FGS_RFC_cards_active	FALSE	0.61
FGS_RFC_cards_active	TRUE	0.65
FGS_RFC_cards_active	FALSE	0.65
FGS_RFC_cards_active	TRUE	0.68
FGS_RFC_cards_active	FALSE	0.68
FGS_RFC_cards_active	TRUE	0.71
FGS_RFC_cards_active	FALSE	0.71
FGS_RFC_cards_active	TRUE	0.74
FGS_RFC_cards_active	FALSE	0.74
FGS_RFC_cards_active	TRUE	0.77
FGS_RFC_cards_active	FALSE	0.77
FGS_RFC_cards_active	TRUE	0.80
FGS_RFC_cards_active	FALSE	0.80
FGS_RFC_cards_active	TRUE	0.83
FGS_RFC_cards_active	FALSE	0.83
FGS_RFC_cards_active	TRUE	0.86
FGS_RFC_cards_active	FALSE	0.86
FGS_RFC_cards_active	TRUE	0.89
FGS_RFC_cards_active	FALSE	0.89
FGS_RFC_cards_active	TRUE	0.92
FGS_RFC_cards_active	FALSE	0.92
FGS_RFC_cards_active	TRUE	0.95
FGS_RFC_cards_active	FALSE	0.95
FGS_RFC_cards_active	TRUE	0.98
FGS_RFC_cards_active	FALSE	0.98
FGS_RFC_cards_active	TRUE	1.00
FGS_RFC_cards_active	FALSE	1.00

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“Argument Engineering”

- Disparate kinds of evidence throughout the system
 - Probabilistic
 - Resource
 - Structural properties of model
 - Behavioral properties of model
- How do we tie these things together?
 - Evidence graph, similar to proof graph in PVS
 - Shows evidential obligations that have not been discharged

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Dealing with Time

- Current analysis is synchronous
 - It assumes all subcomponents run at the same rate
 - It assumes single-step delay between subcomponents
- This is not how the world works!
 - ...unless you use Time-Triggered Architectures or PALS
 - Adding more realistic support for time is crucial to accurate analyses
 - Time intervals tend to diverge in hierarchical verification
 - E.g. synchronization.

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Provocations

We do not yet have a clear idea of how to effectively partition system analyses to perform effective compositional reasoning across domains

We need research to combine analyses to make overall system analysis more effective.

The Collins/JMN META tools are a first step towards this goal.

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Conclusions

- Still early work...
 - Many AADL constructs left to be mapped
 - Many timing issues need to be resolved
 - Better support for proof engineering needs to be found
- **But**
 - Already can do some interesting analysis with tools
 - Sits in a nice intersection between requirements engineering and formal methods
 - Lots of work yet on how best to specify requirements

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Thank you!



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**SYSTEMS ENGINEERING: STATUS OF INDUSTRIAL USE,
OPPORTUNITIES AND NEEDS**

**Clas Jacobson, United Technologies Systems & Controls
Engineering**

This talk will survey aspects of systems engineering and the relevance of equation based modeling in industrial design flows. The particular areas of requirements, architectures, model based design and design flow processes will be addressed. The current landscape in certain industrial areas will be presented along with opportunities and also research needs.



Systems Engineering *Status of Industrial Use, Opportunities and Needs*

Clas A. Jacobson
 Chief Scientist
 Systems & Controls Engineering
 LCCC
 Lund

September 19, 2012

AGENDA

System Design

- Systems engineering:
- (1) requirements,
- (2) architecture,
- (3) model based design,
- (4) (design/development) process

Platform Based Design – design flows (orthogonalize concerns, hierarchy)

Opportunities & progress

- System level modeling – positive on reusability, speed...
- Architecture exploration – not fully exploited - but enabled
- Requirements – potential to move between formal languages (in progress for embedded systems)
- Model based development – positive on controls - MPC (and optimization), uncertainty (and use for robust design not there yet)
- Process – progress on integration of tool chains; level of abstraction change (slightly) with domain (but separate into main product development cycles)

TEAM

Alberto Sangiovanni Vincentelli, Alberto Ferrari, Mark Myers, John Cassidy, Richard Murray, Andzej Banaszuk, Sean Meyn, Johan Akesson, Hubertus Tummesscheit, Karl Astrom, Manfred Morari, Elco Schole, Rich Poisson, Satish Narayanan, Kevin Otto, John Burns, Igor Mezic, Marco Di Natale, Scott Bortoff...

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DRIVERS

System interactions ("emergent behavior")

Requirements & acceptance testing (verification)

Safety (critical) (software intensive) systems

Reusable architectures (modularity)

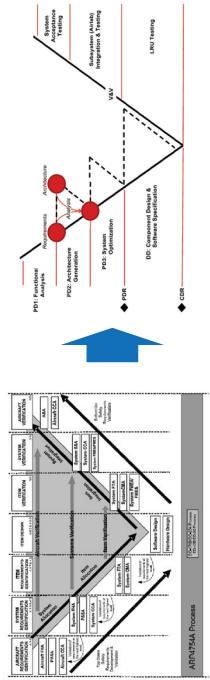
Robustness (risk, lifting)

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SYSTEMS ENGINEERING (DESIGN)

Process



From process to analysis (model based development)

Bring forward in time the verification testing (SIL => HIL => acceptance)

Orthogonalize requirements (requested behavior) and architecture (delivering services)

SYSTEMS ENGINEERING (DESIGN)

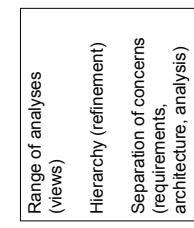
Definition

Systems engineering is a methodology for product system level design, optimization and verification that:

1. Provides guarantees of performance and reliability against customer **requirements** while achieving business cost and time-to-market objectives;
2. Produces modular, extensible architectures for products incorporating mechanical components, embedded systems and application software;
3. Exploits **model-based analytical tools and techniques** to determine design choices and ensure robust system performance despite variations caused by product manufacturing, integration with other products and customer operation; and
4. achieves these objectives through the coordinated execution of a prescriptive, repeatable and measurable **process**.

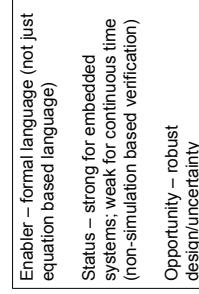
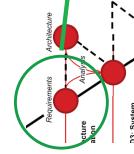
DESIGN PROCESS

Status & Opportunities



REQUIREMENTS

Status & Opportunities



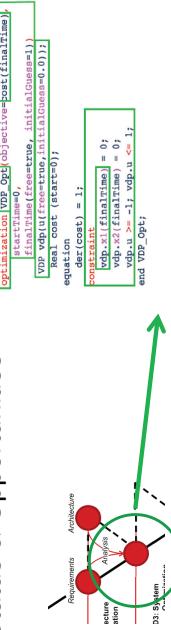
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MODEL BASED DEVELOPMENT

Status & Opportunities



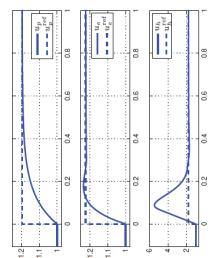
Opportunity – robust design/uncertainty

Utilize interconnection structure to tear system into strong & weak connections; propagate uncertainty (Meyn-Matthew (and others) DARPA RUM 2008)

ROBUST DESIGN

Optimization [Optimization.vdp Opt|Objective=Part((finaltime)]

```
Set finaltime;
W0 = cost -(startcost);
initialtime = initialtime+0.01;
equation
    startcost;
    bdp.x(t,finaltime) = 0;
    vdp.u(>>> -1; vdp.u(>>> 1;
end vdp Opt;
```

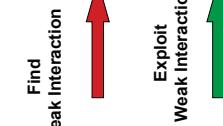


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ROBUST DESIGN & UNCERTAINTY

Status & Opportunities: Exploit Structure

Probability Distribution of input parameters



SUMMARY

System Design

- Systems engineering :
- (1) requirements,
- (2) architecture,
- (3) mode based design,
- (4) process

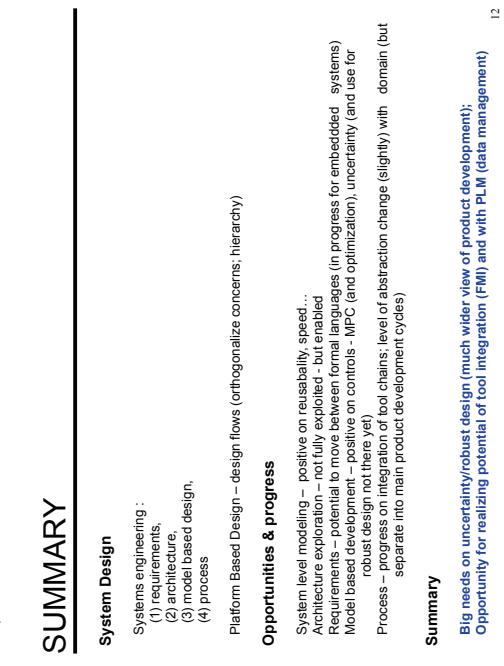
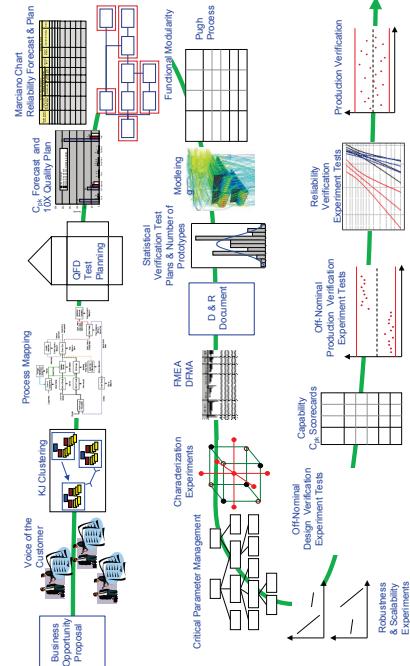
Platform Based Design – design flows (orthogonalize concerns; hierarchy)

Opportunities & progress

System level modeling – positive on reusability, speed...
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Model based development – positive on controls - MPC (and optimization), uncertainty (and use for robust design not there yet)
Process – progress on integration of tool chains; level of abstraction change (slightly) with domain (but separate into main product development cycles)

Summary

Big needs on uncertainty/robust design (much wider view of product development);
Opportunity for realizing potential of tool integration (FMI) and with PLM (data management)



KEY POINTS

System Design

- Systems engineering :
- (1) requirements,
- (2) architecture,
- (3) model based design,
- (4) process

Platform Based Design – design flows (orthogonalize concerns; hierarchy)

Opportunities & progress

- System level modelling – positive on reusability, speed ...
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Summary

- Big needs on uncertainty/robust design;
- Opportunity for realizing potential of integration (FMI) with tool chain and PLM

THE OPENMODELICA ENVIRONMENT INCLUDING STATIC AND DYNAMIC DEBUGGING OF MODELICA MODELS AND SYSTEMS ENGINEERING/DESIGN VERIFICATION

Peter Fritzson, Linköping University, Department of Computer and Information Science, PELAB – Programming Environment Laboratory



This talk gives an overview of the OpenModelica environment, especially highlighting two aspects:

- 1) model debugging and 2) systems engineering including design verification against requirements.

The high abstraction level of equation-based object-oriented languages (EOO) such as Modelica has the drawback that programming and modeling errors are often hard to find. In this paper we present static and dynamic debugging methods for Modelica models and a debugger prototype that addresses several of those problems. The goal is an integrated debugging framework that combines classical debugging techniques with special techniques for equation-based languages partly based on graph visualization and interaction.

The static transformational debugging functionality addresses the problem that model compilers are optimized so heavily that it is hard to tell the origin of an equation during runtime. This work proposes and implements a prototype of a method that is efficient with less than one percent overhead, yet manages to keep track of all the transformations/operations that the compiler performs on the model. Modelica models often contain functions and algorithm sections with algorithmic code. The fraction of algorithmic code is increasing since Modelica, in addition to equation-based modeling, is also used for embedded system control code as well as symbolic model transformations in applications using the MetaModelica language extension.

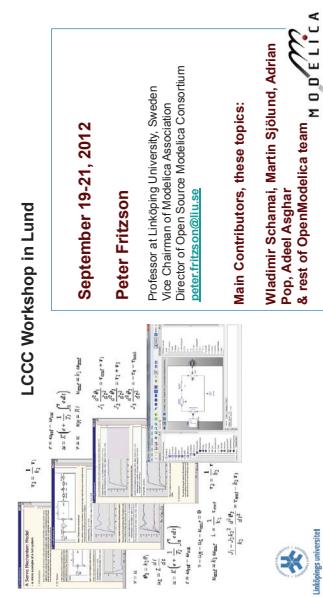
Our earlier work in debuggers for the algorithmic subset of Modelica used instrumentation-based techniques which are portable but turned out to have too much overhead for large applications. The new debugger is the first Modelica debugger that can operate without run-time information from instrumented code. Instead it communicates with a low-level C-language symbolic debugger to directly extract information from a running executable, set and remove break-points, etc. This is made possible by the new bootstrapped OpenModelica compiler which keeps track of a detailed mapping from the high level Modelica code down to the generated C code compiled to machine code. The debugger is operational, supports both standard Modelica data structures and tree/list data structures, and operates efficiently

on large applications such as the OpenModelica compiler with more than 100 000 lines of code. Moreover, an integrated debugging approach is proposed that combines static and dynamic debugging. To our knowledge, this is the first Modelica debugger that supports transformational debugging and algorithmic code debugging.

The second aspect, systems engineering including design verification against requirements, is supported by the OpenModelica ModelicaML profile. This profile implements the vVDR (Virtual Verification of Designs against Requirements) method that enables a model-based design verification against requirements. In the vVDR method there are different kinds of models that are created independently and that will become dependent and need to be related to each other in some concrete verification usage. The aim is to reduce modeling errors and modeling effort by automatically generating composite verification models from their constituting sub-models based on data dependencies that are defined using so-called mediators, which allow expressing data dependencies between models without affecting, i.e., changing, the models themselves.

The OpenModelica Environment including Static and Dynamic Debugging of Modelica Models and Systems Engineering / Design Verification

Overview

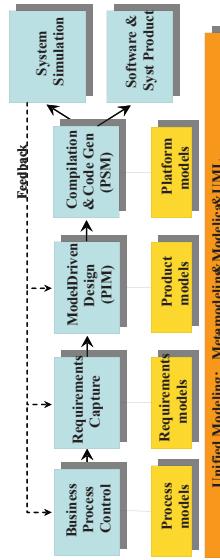


- Background

- Debugging models
- Dynamic verification of requirements

MOFETIA

Vision of Integrated Model-Based Development



Vision of unified modelling framework for model-driven product development from platform independent models (PIM) to platform specific models (PSM)

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MOFETIA

Formal Specification of Modelica Static Semantics

- First Structured Operational Semantics (SOS) Modelica subset formal specification
 - First version 1998, main parts of Modelica static semantics
 - Primarily Big step semantics /Natural Semantics
 - Generating first version of the OpenModelica compiler
- Generating efficient compiler using RML tool
 - 2005 converting rule-based syntax into MetaModelica syntax
 - 2011 full integration with standard Modelica
 - Bootstrapping of the OpenModelica compiler

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MOFETIA

Main Language Extensions

- **MetaModelica 2005**
 - Recursive data structures, lists
 - Pattern matching
 - Failure/exception handling, backtracking
- **ParModelica 2011**
 - Dataparallel language constructs, multi-core, e.g. mapping to OpenCL
 - Memory hierarchy for data allocation
- **Optimization extension 2012**
 - Follow same syntax as Optimica in jmodelica.org
- **ModelicAML extension from 2007**
 - Integrate UML/SysML graphical language and requirement handling
 - Separate tool, not yet integrated in Modelica and the OpenModelica compiler

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OpenModelica – An Open Source Environment Open Source Modelica Consortium, 43 org members Aug 2012

Founded Dec 4, 2007

Open-source community services

- Website and Support Forum
- Version-controlled source base
- Bug database
- Development courses
- www.openmodelica.org

Interactive Modelica compiler (OMC)

- Compile the Modelica Language
- Modelica and Python scripting
- Environment for creating models
- OMShell - scripting commands
- OMNotebook - interactive notebook
- MOT - Eclipse plugin
- OMEdit - graphic Editor
- OMOptim optimization tool
- ModelicAML UML Profile

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Problems

- Large Gap in Abstraction Level
from Equations to Executable Code
- Example error message (hard to understand)


```
Error solving nonlinear system 132
time = 0.002
residual[0] = 0.288956
x[0] = 1.105149
residual[1] = 17.000400
x[1] = 1.248448
...

```

Debugging Equation-Based Languages and Background

Modelica

Modelica

Static vs Dynamic Debugging

- Static Debugging
 - Analyze the model/program at compile-time
 - Explain inconsistencies and errors, trace error dependencies
 - Example: Underconstrained/overconstrained systems of equations
 - Example: errors in symbolic transformations of models
- Dynamic Debugging
 - Find sources of errors at run-time, for a particular execution
 - **Declarative dynamic debugging** – compare the execution with a specification and semi-automatically find the location of the error
 - **Traditional dynamic debugging** – interactively step through the program, set breakpoints, display and modify data structures, trace, stack inspection
- Goal: Integrated Static and Dynamic Debugging

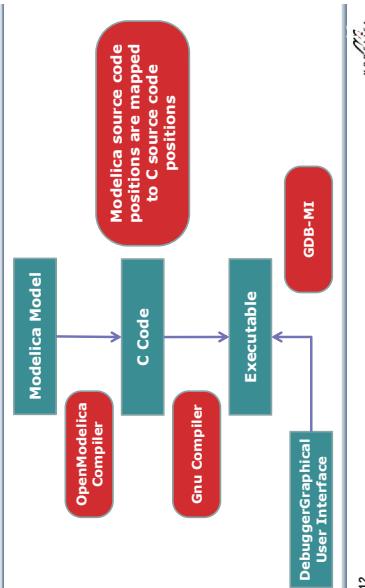
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Previous PhD Theses on Dynamic/Static Debugging in Our Group

- *Dynamic*. Nahid Shahmeri(1991). Generalized Algorithmic Debugging
- *Dynamic*. Mariam Kamkar(1993). Interprocedural Dynamic Slicing with Applications to Debugging and Testing
- *Dynamic*. Henrik Nilsson(1998). Declarative Debugging for Lazy Functional Languages
- *Static/Dynamic*. Peter Bunus (June 2004). Debugging Techniques for Equation-Based Languages.
- *Dynamic*. Adrian Pop (June 5, 2008). Integrated Model-Driven Development Environments for Equation-Based Object-Oriented Languages

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Tool Architecture and Communication



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Modelica

Modelica

Dynamic Debugging Large Modelica Algorithmic Code Models

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Modelica

Example Mapping Modelica Positions to C Code

- Convert Modelica code to C source code by adding Modelica line number references.

```

HelloWorld.mo
1 <function HelloWorld>
2   input Real x;
3   output Real y;
4   algorithm
5     y = sin(x);
6 end HelloWorld;

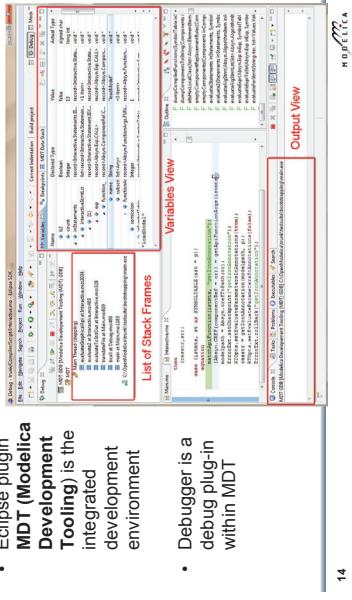
HelloWorld.c
57 #line 29 "HelloWorld.c"
58 /* FunctionBodyRegularFunction: var init */
59 #line 30 "HelloWorld.c"
60 /* FunctionBodyRegularFunction: body */
61 #line 5 "/C/WorkSpace/HelloWorld>HelloWorld.mo"
62 temp2 = sin(x);
63 #line 5 "/C/WorkSpace/HelloWorld>HelloWorld.mo"
64 y = temp2;
65 #line 35 "HelloWorld.c"

```

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Debugger Integrated in Eclipse OpenModelica MDT Environment

- Eclipse plugin MDT (Modelica Development Tooling) is the integrated development environment
- Debugger is a debug plugin within MDT



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Debugging Equation Systems

- Modelica Compiler Backend
- Complex mathematical transformations
 - Hidden to users
 - Users want to access this information
 - Not intuitive, because
 - No explicit control flow
 - Numerical solvers
 - Linear/Non-linear blocks
 - Optimization
 - Events

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Static Debugging Transformational Debugging of Equation-Based Models

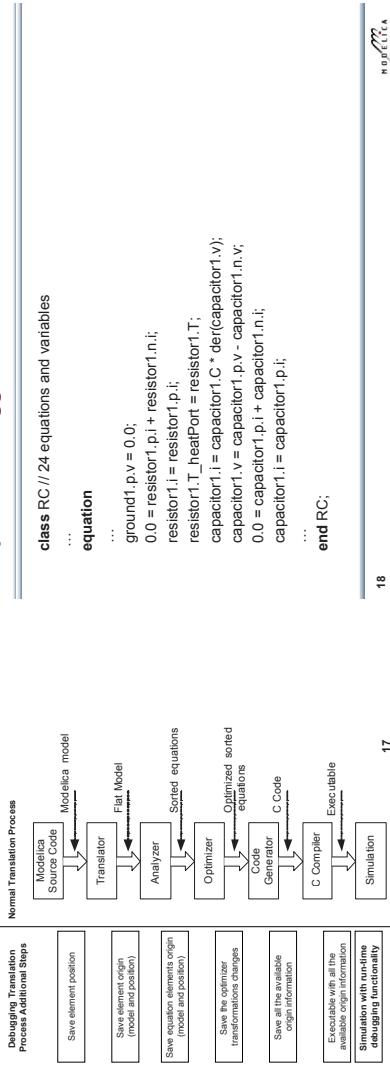
Modelica

Modelica

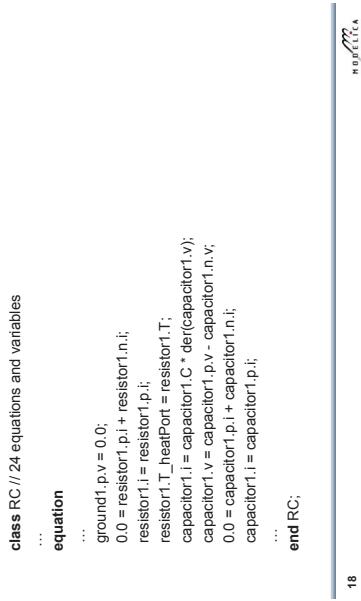
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Translation Phases with Model Debugging

- Include debugging support within the translation process



Input to Debugger: Modelica Model



Output from Compiler Frontend: Sorted ODE or DAE (Differential Algebraic Equations)

```
class RC // 24 equations and variables
...
equation
...
ground1.p.v = 0.0;
0.0 = resistor1.p.i + resistor1.n.i;
resistor1.i = resistor1.p.i;
resistor1.T_heatPort = resistor1.T;
capacitor1.i = capacitor1.C * der(capacitor1.v);
capacitor1.v = capacitor1.p.v - capacitor1.n.v;
0.0 = capacitor1.p.i + capacitor1.n.i;
capacitor1.i = capacitor1.p.i;
...
end RC;
```

Symthic Transformations

- From source code to flat equations
 - Most of the structure remains
 - Few symbolic manipulations (mostly simplification/evaluation)
- Equation System Optimization
 - Changes structure
 - Strong connected components
 - Variable replacements
 - ... and more



Tracing Symbolic Transformations

- Simple Idea
 - Store transformations as equation metadata
- Works best for operations on single equations
 - Alias Elimination ($a = b$)
 - Equation solving $f_1(a,b) = f_2(a,b)$, solve for a
- Multiple equations require special handling
 - Gaussian Elimination (linear systems, several equations)
 - ...

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Modelica

Tracing Overhead?

- OpenModelica compiler implementation is so fast that tracing is enabled by default
 - 1 extra comparison and/or cons operation per optimization
 - Not noticeable during normal compilation
 - Less than 1% time overhead for tracing
- No real overhead unless you output the trace

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Modelica

Substitution Example, Storing the Trace

- | | |
|-------------------------------|--|
| $a = b$ | The alias relation $a=b$
stored in variable a |
| $c = a + b$ | |
| $d = a \cdot b$ | |
|
 | |
| $c = a + b$ (subst $a=b$) => | The equations are e.g.
stored as
(lhs,rhs,list<ops>) |
| $c = b + b$ (simplify) => | |
| $c = 2 * b$ | |
|
 | |
| $d = a - b$ (subst $a=b$) => | |
| $d = b - b$ (simplify) => | |
| $d = 0.0$ | |

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Modelica

Debugging Using the Transformation Trace

- Text output
 - Initial implementation
 - Verify performance and correctness of the trace
- Structured output based on database storage
 - Graphical debugging
 - Cross-referencing equations (dependents/parents)
 - Ability to see why a variable is solved in a particular way
- Requires a schema
- Future work/work in progress

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Modelica

Trace Example (1)

```

0 = y + der(x * time * z);      z = 1.0;

(1) substitution:
    y + der(x * (time * z))      (3) expand derivative
    =>                                (symbolic diff):
        y + der(x * time)          y + (x + der(x) * time)
        =>                                y + (x + der(x) * time)
    (2) simplify:
        y + der(x * (time * 1.0))  (4) solve:
        =>                                0.0 = y + (x + der(x) * time)
        y + der(x * (time * 1.0))  =>                                0.0 * (der(x) * x + der(y) * y)
        =>                                2.0 * (der(x) * x + der(y) * y)
        y + der(x * time)          =>                                2.0 * (u * x + v * y)
        der(x) = ((y) - x) / time

```

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Modelica

Trace Example (2)

differentiation:	Substitution:
d/dtime L ^ 2.0	2.0 * (der(x) * x + der(y) * y)
=>	=>
0.0	2.0 * (\$DER.x * x + \$DER.y * y)
differentiation:	differentiation:
d/dtime x ^ 2.0 + y ^ 2.0	d/dtime x ^ 2.0 + y ^ 2.0
=>	=>
2.0 * (der(x) * x + der(y) * y)	2.0 * (u * x + v * y)
=>	=>
2.0 * (u * x + v * y)	2.0 * (u * x + v * y)
	=>
	2.0 * (u * xloc[1] + v * xloc[0])

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Modelica

Readability of Transformation Trace

- Most equations have **very few** transformations on them
- Most of the interesting equations have a few
 - Still rather readable
- Some extra care to handle Modellica variable aliasing

MSL 3.1 MultiBody DoublePendulum

- Structural debug information queries based on a database
- Graphical debugger
- Simulation runtime uses database
- More operations recorded
 - Dead code elimination
 - Control flow and events
 - Forgotten optimization modules

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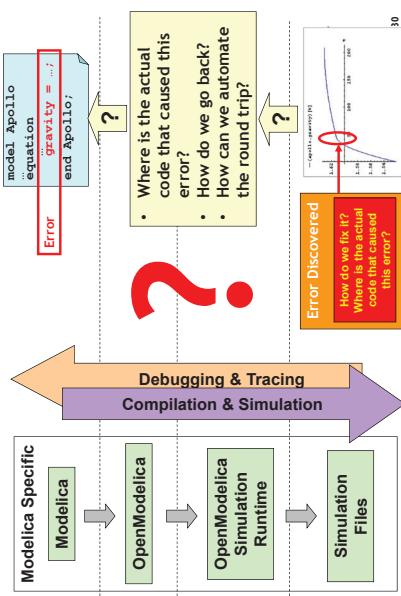
Modelica

Future Work on Transformational Debugging

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Modelica

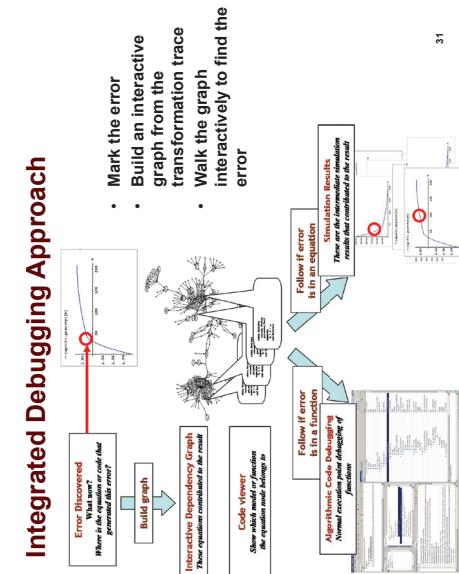
Need to Combine Approaches to Help the User



Integrated Debugging



Integrated Debugging Approach



Debugging Based on User Interaction

- The interactive dependency graph contains two types of edges:
 - Displaying simulation results through selection of the variables
 - Classifying a variable as having wrong values
 - Building a new dependency graph based on the new set of variables with wrong values (classified variables) or by modifying the equations or parameter values nodes.
 - Displaying model code by following origin edges
 - Involving the algorithmic code debugging subsystem

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n^o32%
n^o31%

Debugging Summary

- Debugging equation-based models present new challenges
- Equation systems are **transformed** symbolically to a form hard for the user to recognize
- Static transformational debugging **explains** the transformations and maintains a mapping between the low level and the high level model
- **Dynamic debugging** helps to **walk** through a model/program and inspect data for an execution
- **Goal:** integrated static/dynamic debugging approach

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Modelica

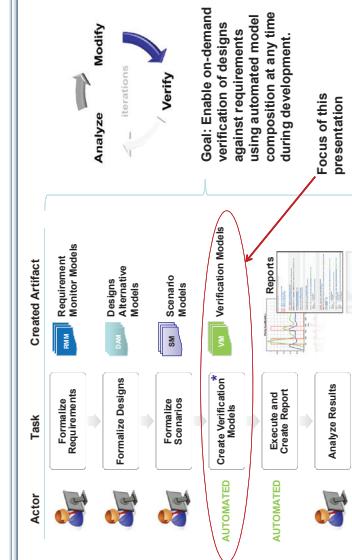
Introduction: ModelicaML Background

- **ModelicaML Eclipse plug-in**
ModelicaUML profile integrates a subset of the UML and the Modelica language in order to leverage standardized graphical notations of UML for system modeling and the simulation power of Modelica
 - ModelicaML enables engineers to describe
 - System requirements
 - System design (structure and behavior)
 - Usage-, test scenarios
 - VDR (Virtual Verification of Designs against Requirements) is a method that enables a model-based design verification against requirements
 - VDR is supported in ModelicaML

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Modelica

Introduction: vVDR Method



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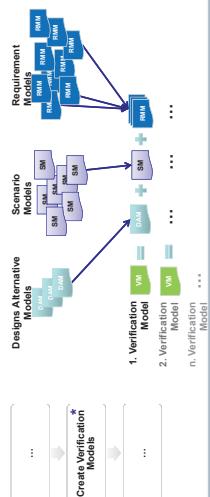
Modelica

Challenge

- We want to verify different design alternatives against sets of requirements using different scenarios, issues:

1) How to find valid combinations of design alternatives, scenarios and requirements in order to enable an automated composition of verification models?

2) Having found a valid combination: How to bind all components correctly?



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Solution Proposal: Value Bindings

- Value Binding enables the automation of verification model composition

Value Bindings include the definition of:

- Client (component that requires data from other components)
- Provider (component that provides data for other components)
- Mediator (mediates between clients and providers)
- Requirement Models
- RMM
- SM
- Scenario Models
- Other Required Models

- Depending on which mediators and providers are in place we can:
 - Determine which clients can be satisfied
 - Find value combinations and generate verification models
 - Generate binding code for client components in verification models

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Page 38

- Value Bindings include the definition of verification model composition
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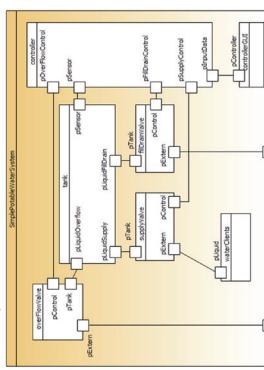
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Page 38

Example: Design Alternative Model "The time to fill an empty tank shall be 300 sec. max."

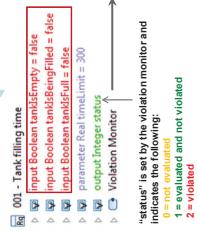
Simplified Aircraft Potable Water System

- Overhead tank system that can be filled using a liquid source from bottom with the aircraft on ground.
- Controller monitors the level of liquid and controls the valves according to its mode (e.g. "fill", "drain", "pre-selected value fill" mode).



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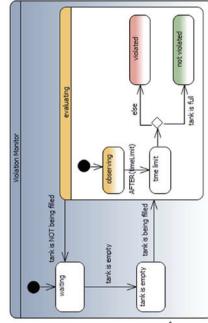
Clients to get input values
from design model providers



"status" is set by the violation monitor and indicates the following:
 0 = not evaluated
 1 = evaluated and not violated
 2 = violated

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Example: Requirement Monitor Model "The time to fill an empty tank shall be 300 sec. max."



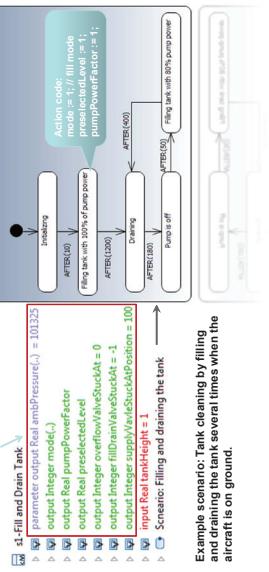
Model 2

Model 1

Model 3

Example: Scenario Model “Filling and draining the tank”

Providers for design model clients



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Simulation and Report Generation in ModelicaML

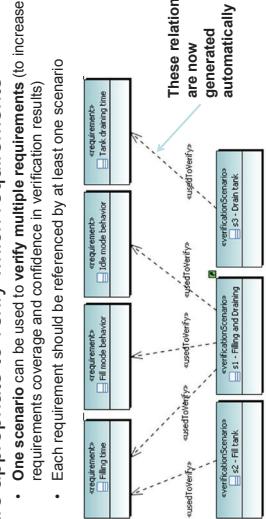
- Verification models are simulated.
- The generated Verification Report is a prepared summary of:
 - Configuration, bindings
 - Violations of requirements
 - etc.



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Example: Mapping Scenarios to Requirements

- Automatic generation/selection of which scenarios are appropriate to verify which requirements



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Conclusion

- The ModelicaML Value Bindings approach enables automated model composition, which is used in ModelicaML for **automatic generate verification models**
- Bindings do not modify client or provider models (important when libraries are used)
- Using binding definitions we can find **valid combinations and automatically generate verification models**
- The generated **verification models** become artifacts that are **created automatically on-demand** and do not need to be maintained

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Overall Summary

- Goal of integrated model-based development

This talk covers two aspects

• Integrated static/dynamic debugging of models

- Dynamic debugging of large algorithmic models fully functional
 - Static Equation debugging prototype need to be integrated and scaled up for large models

• Requirements traceability and verification

- Automated dynamic verification and generation of verification models
 - Need to be integrated in Modelica standard

**THE DARK SIDE OF OBJECT-ORIENTED MODELLING:
NUMERICAL PROBLEMS, EXISTING SOLUTIONS, FUTURE
PERSPECTIVES**

**Francesco Casella, Politecnico di Milano, Dipartimento di
Elettronica e Informazione**

Object-Oriented Modelling languages allow to tackle the system-level modelling of engineering system in a modular and convenient way, ideally focusing on clarity and generality of the code rather than on numerical robustness and computational efficiency. Symbolic and numerical methods are now available to generate efficient simulation code from this models; however, there are still some problems (most notably initialization) that are far from being solved in a completely satisfactory way from the end-user perspective. O-O models are now also starting to be used for optimization, which has very different numerical requirements than simulation; numerical and symbolic methods similar in purpose to those already existing for simulation are badly needed in order to automatically turn high-level O-O models into low-level equations that can be optimized reliably and efficiently, thus avoiding the manual tuning of the code for this purpose. The talk will present the current state of the art and point out the needs that haven't been satisfied yet, making the case for new cross-disciplinary research in the field.



Introduction

The Dark Side of Object-Oriented Modelling: Numerical Problems, Existing Solutions, Future Perspectives

Francesco Casella
 (francesco.casella@polimi.it)

Dipartimento di Elettronica e Informazione
 Politecnico di Milano



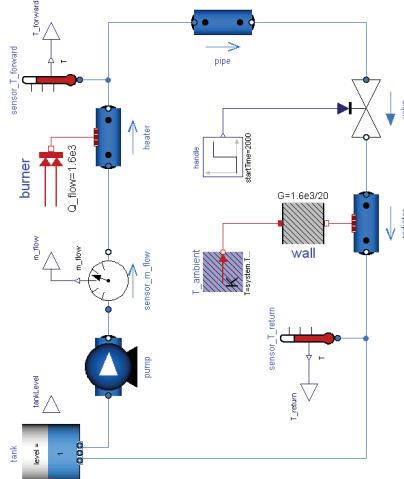
- Equation-Based Object-Oriented Modelling (OOM) approach well established in industry and academia

- Modelica Language emerging

- A-causal approach (write equations, not how they are solved) makes fully modular modelling of multi-domain physical system possible
- Hierarchical modelling, handling of complex systems
- Replaceable models, handling of reconfigurable systems
- Fancy GUIs, model management tools, ...

2

A (not so nice) anecdote



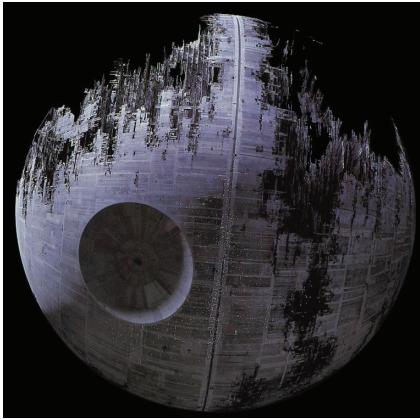
but...

3

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The dark side of O-O modelling

Numerical Errors in OOM: A Taboo Topic?



Numerical Errors in OOM: A Taboo Topic?

- All users of OOM encounter this kind of problems
- High-level expertise in simulation techniques required for troubleshooting
- Domain experts use the model!
- These errors are a blocker (– no result obtained at all until solved)
- They can scare off people from OOM technology



- Little progress in solving these problems in a sound and systematic way
- Error handling and debugging techniques much less developed than simulation techniques

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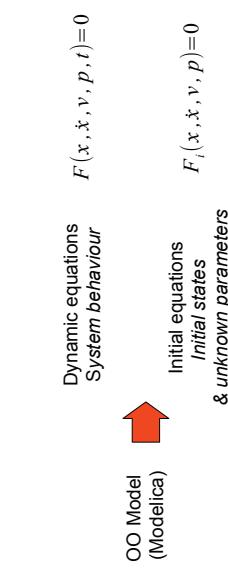
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Numerical Problems in OOM Simulation

8

Numerical Problems in OOM Simulation

Simulation Problem



Simulation Problem

- At each time step,
solve $F(x, \dot{x}, v, p, t) = 0$ given x, p, t
- Behaviour equations
 - well-tested model library
 - physically meaningful connection equations
- Known states problem broken into many small subproblems
- Advanced numerical/symbolic techniques used for efficient solution
- Modelling errors wrong behaviour wrong trajectories



*Errors can be investigated
by domain-expert modeller*

Initialization problem

- At time $t = 0$, solve
 - $F(x, \dot{x}, v, p, 0) = 0$
 - $F_i(x, \dot{x}, v, p) = 0$
- Typically large systems of highly non-linear equations
 - In case of solver errors: no trajectories available for inspection

Initialization problem

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 $F_i(x, \dot{x}, v, p) = 0$
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Error scenarios

- (1) Well-posed problem, solver fails
- (2) Underconstrained/overconstrained problem
- (3) System-level singular problem, solver fails
- (4) Structurally well-posed problem, wrong parameters (no solution)

(1) Well-Posed Problem, Simulation Fails

- Nonlinear solver needs initial guess for all iteration variables
- Providing close enough values often unfeasible or very tedious
- Can't tell whether the guess values are wrong, or the problem has no solution at all

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Homotopy-based initialization

- New Modelica operator `homotopy` (actual, simplified) introduced in 2011



Homotopy-based initialization



High-level debugging

- Beware of singularities!

- Define a simplified problem which is easier to solve

$$F_s(x) = 0$$

- Continuously transform into the actual problem

$$(1 - \lambda) F_s(x) + \lambda F_a(x) = 0$$

- Track divergence to ∞

- Regular path with turning points

- Isola

- Bifurcation

- Solution of function x

- Homotopy parameter ,

- 1

- 0

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Homotopy-Based Initialization

- Define a simplified problem which is easier to solve

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- 1

- 0

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Homotopy-Based initialization – First Results

Modellica Conference 2011
(Casella, Sielmann et al.)

- Multibody systems with multiple configurations
- Analog electronic circuits
- Hydraulic networks
- Calibration of air-conditioning systems
- Large power plants
- Probability one homotopy

Modellica Conference 2011
(Sielmann)

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Modellica Conference 2012
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High-Level Debugging: the Open Modelica Debugger

- Idea #1: keep track of all model transformations in the generated code
- Idea #2: explore this information graphically, by traversing graphs

- The solver fails when trying to solve an equation in the BLT
 - or
- A result is found, clearly wrong from a physical perspective

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High-Level Debugging: the Open Modelica Debugger

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 - A result is found, clearly wrong from a physical perspective



Code Dependency Graph

- where does the equation come from in the original model?
- which transformations have been applied?
- from which other variables it depends?
- what values they have?

see paper by Pop et al., Modelica Conference 2012

(2) Underconstrained/Overconstrained problem

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- Initial conditions are usually set at the single component level
- Initialization problem is a system-level problem (exp. with control systems and steady-state conditions!)
- Additional initial equations added determine unknown parameters (trimming problem)
- Initial equations made redundant in case of index reduction



(3) System-level singular problem, solver fails

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- The initialization problem can be square but singular for system-level structural reasons
- Examples:
 - Closed thermohydraulic systems with steady-state initial equations
 - Closed hydraulic systems with incompressible fluid
 - Electrical circuits without ground connection
- The Jacobian of the initialization problem equations is singular, no unique solution, solver fails

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- The Jacobian of the initialization problem equations is singular, no unique solution, solver fails

No satisfactory solution available yet

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(3) System-level singular problem, solver fails

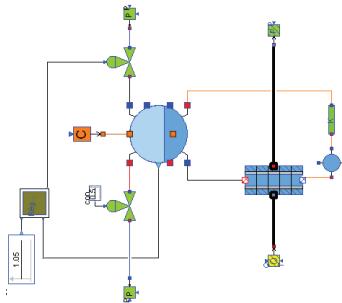
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 - Closed hydraulic systems with incompressible fluid
 - Electrical circuits without ground connection
 - The Jacobian of the initialization problem equations is singular, no unique solution, solver fails
- ➡
- Analysis of the nullspace of the Jacobian can reveal the subset of equations causing the singularity
 - Meaningful annotations on key equations which are always part of the singular system can be used for high-level diagnostics to the end user
 - Proposed by Casella, Modelica Conference 2012, not yet implemented

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Numerical Problems in OOM Optimization

(4) Structurally well-posed problem, wrong parameters

- The strange case of the steam generator
- Steam valve Cv too small
- No steady state solution exists!
- Difficult to determine "the" wrong parameter
 - steam valve too small?
 - too much flue gas flow?
 - flue gas too hot?
- Traditional troubleshooting strategy:
divide & conquer
- New ideas?



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Dynamic optimization of OO Models

- OO models can be used for dynamic optimization (optimal control) of physical systems
- The OO modelling language is used to write the model DAEs which are the dynamic constraint of the optimization problem
- Extension of Modelica: Optimica (J. Åkesson PhD work)

```
optimization DMINTime (
  objects;
    var finalTime;
    startTIme;
    finalTime (fromTime, initialGuess=1);
    Real (Cstart=0, fixed=true);
    Real (Cstart=0, fixed=true);
    input Real u;
    equation
      der (x)=u;
      der (v)=u;
    constraint
      x (finalTime)=1;
      v (finalTime)=0;
      v=0.5;
      u=-1;
      u=+1;
  end DMINTime;
```

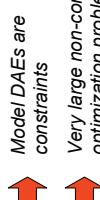
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Dynamic optimization of OO Models

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- Extension of Modelica: Optimica (J. Åkesson PhD work)

```
optimization DIMinTime (
    objective=finalTime,
    startTime=0;
    finalTime(free=true, initialGuess=1))
Real x(start=0, fixed=true);
Real v(start=0, fixed=true);
input Real u;
equation
der(x)=v;
der(v)=u;
constraint
x(finalTime)=1;
v(finalTime)=0;
u=0.5;
v=0;
u=-1;
v=-1;
end DIMinTime;
```



Convergence of solver is a major issue
(more than in simulation problems!)



Symbolic Manipulation of Equations

- | <i>for Simulation</i> | <i>for Optimization</i> |
|--|---|
| <ul style="list-style-type: none"> • DAEs solved for state derivatives, then time integration • alias elimination • BLT ordering • index reduction / dummy derivative symbolic solution of implicit equations • tearing of implicit systems • common subexpression elimination | <ul style="list-style-type: none"> • DAEs used as constraints • alias elimination • scaling of variables (via nominal attribute) • bounds on variables (via min/max attributes) |
| ... | ? |

Example 1

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for Optimization

```
optimization Example1A(
    objective = a1/b1*x1 - x10)^2 +
    a2/b2*x2 - x20)^2 +
    a3/b3*(x3 - x30)^2,
    ...
end Example1A;
```

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for Simulation

```
optimization Example1B(
    objective = f1 + f2 + f3,
    ...
    equation
    f1 = a1/b1*(x1 - x10)^2;
    f2 = a2/b2*(x2 - x20)^2;
    f3 = a3/b3*(x3 - x30)^2;
    ...
end Example1B;
```

Consequences on Modelling

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- | <i>for Simulation</i> | <i>for Optimization</i> |
|--|--|
| <ul style="list-style-type: none"> • The modeller can focus on clarity, readability, generality • The modeller can (almost) ignore how equations will be solved • Efficient and robust simulation code automatically generated regardless how the equations are written | <ul style="list-style-type: none"> • The modeller must focus on solvability and convergence issues when writing the equations • Mathematically equivalent models completely different optimization problems • completely different convergence properties |

~~Declarative OOM~~

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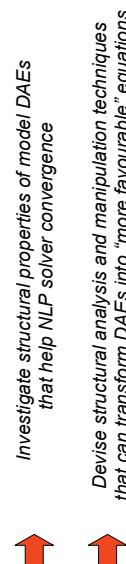
Example 2

Research Goals

```

optimization Example2A{
    parameter Real PR = 10;
    ...
    equation
        p_in*p_out = PR "Turbine pressure ratio";
    ...
    end Example1A;
}

```



optimization Example2A{

 parameter Real PR = 10;

 ...

 equation

 p_in = PR*p_out "Turbine pressure ratio";

 ...

 end Example1A;

*Bring the field of OOM Optimization from adolescence
(or childhood?) into adulthood*

Conclusions

- Object-Oriented Modelling concepts are getting more and more widely used in system engineering and control
- OOM has a huge potential to help closing the gap between theory and practice in optimal control
- Use of OOM for simulation is now fairly mature, but still has some dark areas related to initialization problems that need to be addressed for wider acceptance and use
- Use of OOM for optimization still quite young, comparably little effort spent in automatic (or assisted) streamlining of model equations
- Some steps in the right direction have already been made in recent years
- More research is needed to
 - improve the numerical robustness and mitigate convergence problems in tools
 - develop effective and easy-to-use debugging tools for domain experts
 - possibly by embedding expert a-priori knowledge in the modelling code and exploiting it cleverly

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Thank you for your kind attention!

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BRIDGING BETWEEN DIFFERENT MODELING FORMALISMS**- RESULTS FROM THE MULTIFORM PROJECT****Sebastian Engell, Process Dynamics and Operations
Group, Department of Biochemical and Chemical
Engineering, TU Dortmund**

The European project MULTIFORM that ended in May, 2012, had the goal to provide interoperability between different modeling, simulation, optimization and analysis tools for hybrid dynamic systems (systems with continuous dynamics modeled by DAE and switching logic). For this purpose, the Compositional Interchange Format (CIF) that was developed by a group at TU Eindhoven was employed. The CIF is a formally defined, automata-based modeling language for hybrid systems. Transformations between the CIF and Modelica, gPROMS, Uppaal (a tool for the analysis of timed-automata models), SpaceEx (a tool for the reachability analysis of hybrid automata) and other tools for logic controller design and synthesis and control software verification have been developed. Tool chains for the development and simulation of logic controllers interacting with continuous dynamic systems were demonstrated. The model transformations and results handling are supported by the MULTIFORM Design Framework in which supports (partly) model-based design processes. We illustrate the idea and the results of MULTIFORM by the design of a miniature pipeless plant for education and demonstration purposes and discuss the experience gained.



Outline

m u l t i f o r m
Integrated Multi-formalism Tool Support for the Design of Networked Embedded Control Systems

- Bridging between different modeling formalisms
- results from the **MULTIFORM** project
- The MULTIFORM project
 - Design flow example
 - Tool developments
 - Model exchange and model transformations
 - Lessons learned

Martin Hüfner, Christian Sonntag, Sebastian Engel

Process Dynamics and Operations Group
Department of Biochemical and Chemical Engineering
TU Dortmund
Germany

System Design meets Equation-based Languages, Sept. 19, 2012, Lund

MULTIFORM: EU ICT STREP 9/2008 – 5/2012



- **TUDO** (Coordinator)
 - TU Dortmund, Germany
Sebastian Engel
 - TU Eindhoven, Netherlands
Koos Rooda, Bert van Beek, Jos Baeten
 - Université Joseph Fourier, Grenoble, France
Coran Frehse, Oded Maler
 - RWTH Aachen, Germany
Stefan Kowalewski
 - aalborg universitet,
Denmark
Kim Larsen, Brian Nielsen
 - Stichting Embedded Systems Institute
Ed Brinksma, Boulewin Havkerk
- **AAU**
 -
- **KVCA**
 - "Danish Cooling Cluster"
 - Jens Andersen
 - Closely working with DANFOSS
- **VEMAC**
 - Aachen, Germany
Michael Reke

Project acronym MULTIFORM and logo are registered trademarks of the European Commission.

Example: Design of a Pipeless Plant



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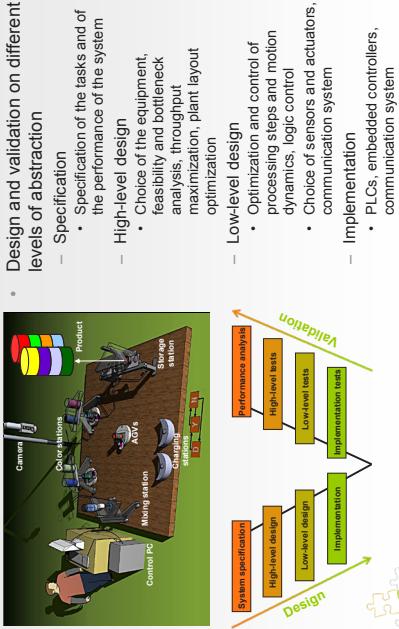
m u l t i f o r m
Integrated Multi-formalism Tool Support for the Design of Networked Embedded Control Systems

2

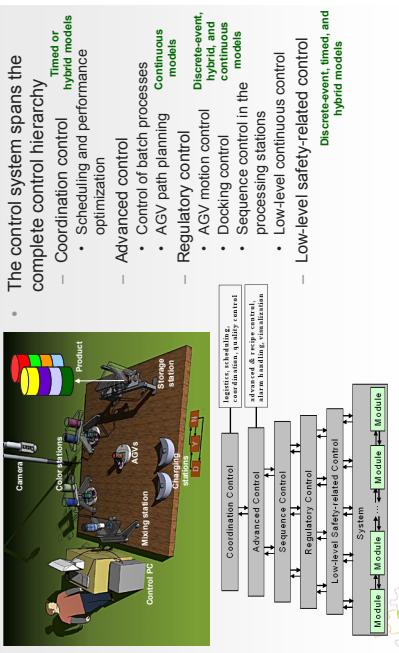
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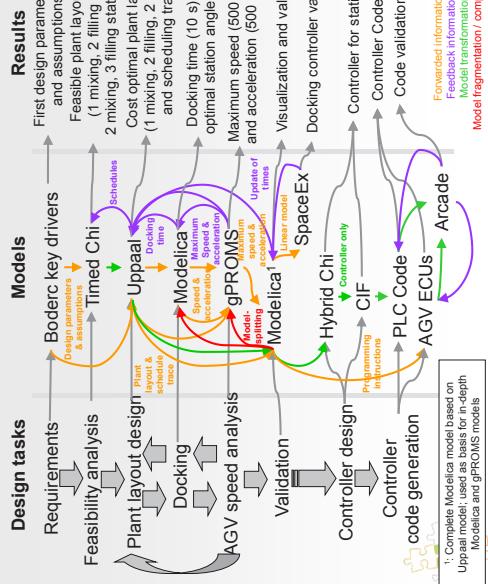
Challenges for Model-based Design (1)



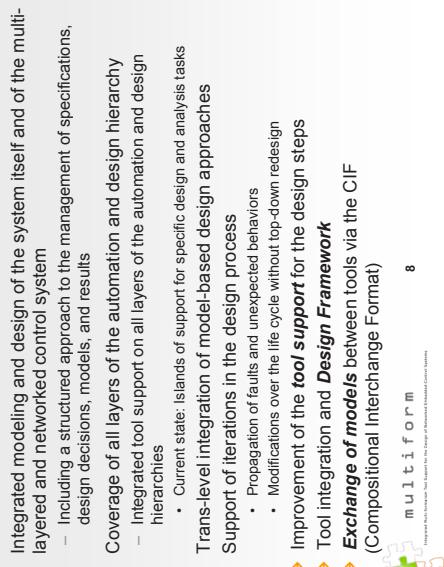
Challenges for Model-based Design (2)



Design tasks



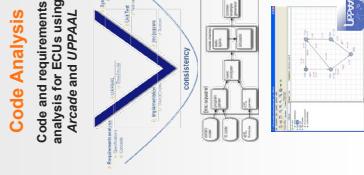
Integrated Model-based Design



MULTIFORM Tools and Tool Chains

Logic Controller Design

- Integrated controller design and analysis
- Informal and vague systematic analysis
- Consistency checking methods using UPPAAL
- Step-wise refinement based on H/C/F



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The MULTIFORM Design Framework [ESI]

Code Analysis

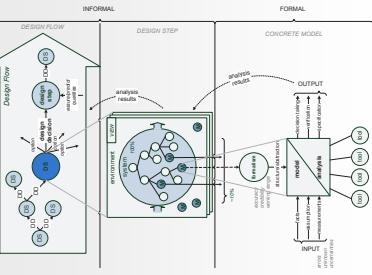
- Code and requirements analysis for ECUs using Arcade and UPPAAL



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Design Flow

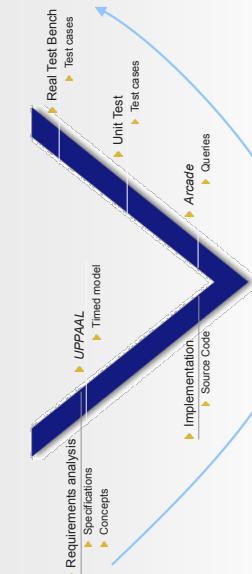


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VEMAC Development Process

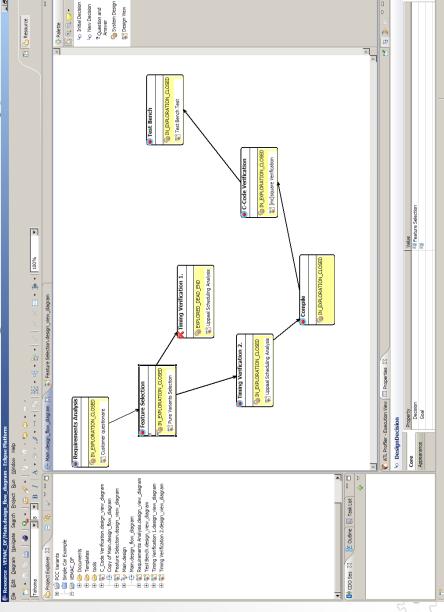
V-model



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Customized Design Framework Prototype



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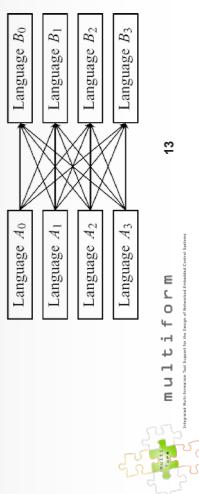
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Model Exchange Using the Compositional Interchange Format (CIF)

- Incompatibility of tools is one of the major obstacles for a broader acceptance of model-based design in industry
- Achieve inter-operability by (algorithmic) model transformations
- One possibility: Bi-lateral transformations
 - Problems
 - Many transformations may be needed
 - The developer of a transformation must be familiar with many different formalisms



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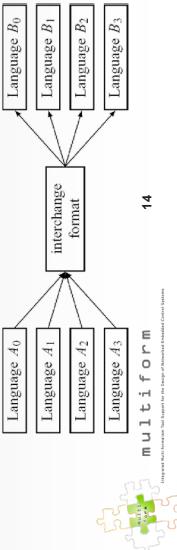
The Compositional Interchange Format (CIF) [Bert van Beek et al., TUE]

- Purposes
 - Establish inter-operability of a wide range of tools
 - Provide a generic formalism for general hybrid systems
- Major features
 - Formal and compositional semantics
 - Independence of implementation aspects
 - Mathematical correctness proofs of translations
 - Property-preserving model transformations possible
 - Fully implicit DAE dynamics (possibly discontinuous)
 - Hierarchy and re-usability
 - Parallel composition with different communication concepts
 - Model component interaction
 - Point to point communication, multi-component synchronization, broadcast communication, shared variables
- Different urgency concepts
 - <http://develse.wtb.tue.nl/trac/cif/>

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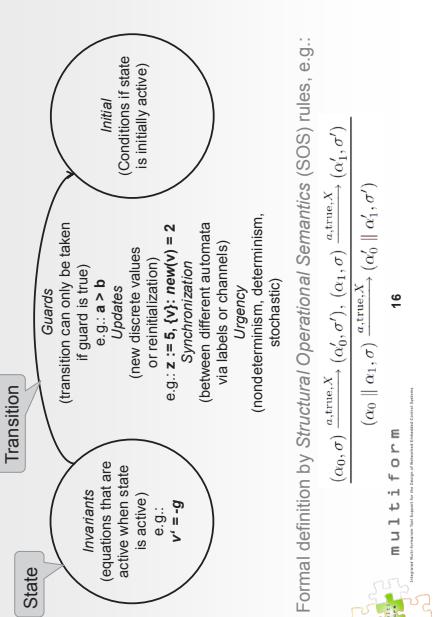
Model Exchange with the Compositional Interchange Format (CIF)

- Incompatibility of tools is one of the major obstacles for a broader acceptance of model-based design in industry
- Achieve inter-operability by (algorithmic) model transformations
- One possibility: Bi-lateral transformations
 - Interchange format
 - Generic and sufficiently rich modelling formalism
 - Only transformations from/to the interchange format are necessary
 - Reduction of the implementation effort



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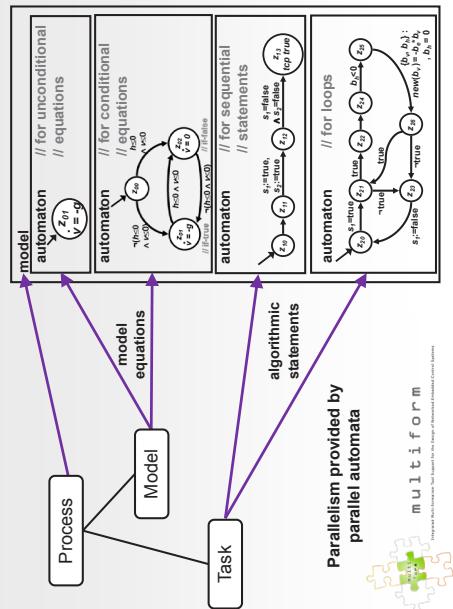
The Compositional Interchange Format (CIF) [Bert van Beek et al., TUE]



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Transformations – gPROMS \Rightarrow CIF (Excerpt)



Simple Example: tank.cif

```
model TankController()=
  || cont control nat V = 10.0
  || var real Qi, Qo
  || disc control nat n = 0
  :: Tank : || mode physics = initial
  inv V' = Qi - Qo
  , Qi = n * 5.0
  , Qo = sqrt(V)

  Controller : ( mode closed = initial
  (when V <= 2 now do n := 1) goto opened
  , opened = (when V >= 10 now do n := 0) goto closed
  )
  )||
```



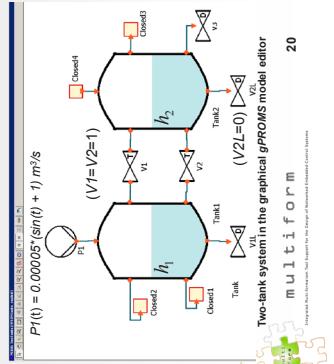
Flattened Example: tank_flat.cif

```
model TankController()=
  || var real V = 10.0 ; var real Qi ; var real Qo ; var nat n = 0
  :: Tank_Controller:
  {
    var string Controller_LP ; var string Tank_LP_V,n
    ; controlset Controller_LP ; Tank_LP_V,n
    ; dynparam disc Controller_LP, disc Tank_LP ; disc n; cont V;
    mode X = initial (((Tank_LP) = ("physics")) and (true))
    and (((Controller_LP) = ("closed")) and (true))
    inv (((Tank_LP) = ("physics")) => (((V') = ((Qi) - (Qo)))
    and (((Qi) = ((n) * (5.0))) and (((Qo) = (sqrt(V))))))
    top (((Controller_LP) = ("opened")) => (not((V) <= (2)))
    (when (V) <= (2), (Controller_LP) = ('closed')) => (not((V) >= (10)))
    (Controller_LP, n) : (new(n)) = (1), (new(Controller_LP)) = ("opened"))
    (when (V) >= (10), (Controller_LP) = ('opened')) do
    {Controller_LP, n} : (new(n)) = (0), (new(Controller_LP)) = ("closed")) goto
    X
  }
```

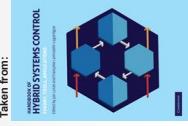
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A Two-tank System under Discrete Control

- Hybrid non-linear model of a two-tank system, modeled in gPROMS
- Designed to contain many constructs of the gPROMS language



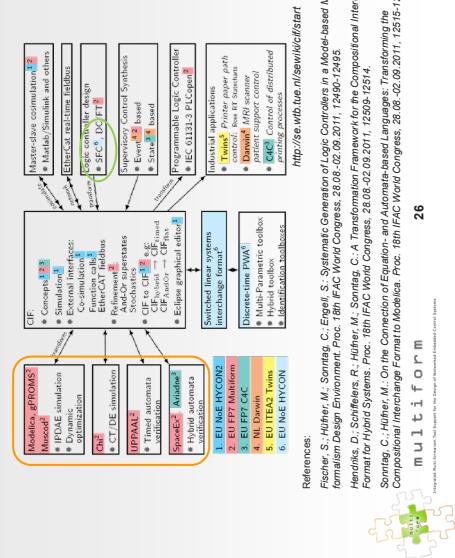
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HYBRID SYSTEMS CONTROL

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Compositional Interchange Format (CIF)

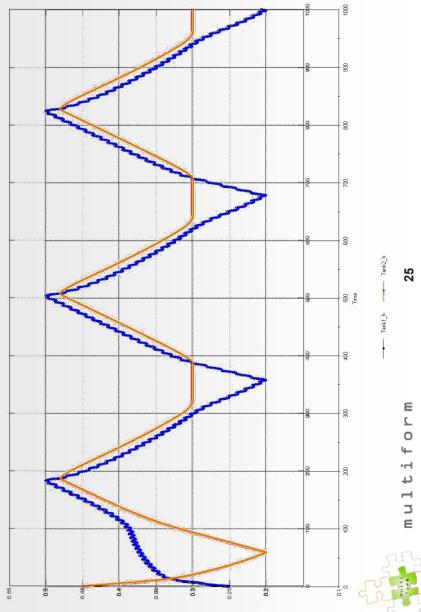


Summary

- There is a need for efficient model-based support of the design of complex automated systems with trans-level propagation and iteration, and re-use of models
- An all-encompassing mega-tool for the design of complex automated systems is not realistic, so several tools and modeling formalisms must be used in the design process.
- Three different routes to tool and model integration and design support were pursued in MULTIFORM:
 - Model exchange and tool chains via the CIF
 - Direct coupling of tools for testing of specifications
 - Propagation of parameters via the Design Framework



Output Identical



Equation-based vs. Automaton-based Formalisms

- Simulation/Solver/Tool options encoded in model code (e.g. EcosimPro, gPROMS)
 - Tool specific options cannot be transformed
 - Other tools might not find a solution for a difficult initialization problem
- Formal semantics not available → Transformation not provably correct
- Equation-based models can be more restrictive than automata models
 - E.g. Modelica enforces globally and locally balanced models
 - Automata models need to be preprocessed
 - Either by flattening of the model
 - Or by rebuilding the automata structure in an equation-based formalism



Lessons and Challenges from MULTIFORM

- The C/F and its tool set are stable and relatively mature
- Available under open source licence
- The effort for developing model transformations is high
- Transformations from the C/F in most cases can only be performed for subsets of the models which can be represented in the formalism.
 - A formal specification of the supported C/F subset of a tool is needed
 - It should be possible to trace elements of a model after the transformation
- Model blow-up is not as bad as could be expected



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Lessons and Challenges from MULTIFORM (2)

- The C/F is very expressive and well suited for model exchange between automata-based formalisms, but conceptually different from equation-oriented languages (e.g. Modelica, gPRO/M, EcosimPro)
 - Possible solution: Use a Modelica subset as an exchange formalism for equation-oriented languages, bridge equation- and automata-oriented formalisms via the C/F \leftrightarrow Modelica transformation
- Often only some elements of a system are modeled precisely, and these models are formulated in different formalisms (*fragmented modeling*)
 - How can the interdependencies between model fragments be formally described and exploited?
- **Model ontology needed**
 - Specification of model formalism expressivity using a common formal vocabulary
 - Equipping model artifacts with meta data on their origin(s) \rightarrow traceability
 - Description of relations of partial models to an overall model



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EQUATION-BASED MODELING AND CONTROL OF INDUSTRIAL PROCESSES

Johan Sjöberg, ABB AB, Corporate Research and Linköping university

Control systems are used in a vast number of different applications. However, despite the differences, the objective is similar. That is, to control the application in the most efficient manner possible. In order to find out what is most efficient, models are an important tool. The models can be derived using more or less physical relations, but there is a trend towards using more and more physical insights since it has the advantage that the model can be used in an extrapolating way. That is, it is possible to draw conclusions based on the model without having measurements from the considered operating region. This is a strength when optimizing the process and the best possible operating condition might very well be significantly different compared to usual operating conditions for which data is available.

The model building is a very large share of the work and it is therefore imperative to be able to reuse models developed for earlier applications. This makes equation-based modeling very interesting for companies such as ABB that develop control strategies for many different applications. ABB has therefore developed tools for equation-based modeling and control for a long time. These tools have also been successfully applied to many different processes, such as pulp and paper mills, power plants, cement industries, etc.

In this presentation, we focus on two different applications where equation-based modeling has been key in the process optimization. The first application is hot rolling mills, while the second application is harbor cranes.

Hot rolling mills are complicated industries where the overall model has to incorporate physical relations from many different domains, such as mechanics, thermodynamics and metallurgy. For hot rolling mills, ABB has developed a model that describes the material throughout the whole rolling including reheating, rolling and cooling. This model has been used to compute operating conditions that minimizes the energy consumption while maintaining the material properties.

For harbor cranes, ABB has developed a tool to improve the process of picking up coal from a ship and put it in a hopper at shore. In this context, process improvement normally means minimization of the cycle time, but today it is also important to limit the energy consumption.

A Global Leader in Power and Automation Technologies Leading Market Positions in Main Businesses

- 135,000 employees in about 100 countries
- \$38 billion in revenue (2011)
- Formed in 1988 merger of Swiss and Swedish engineering companies
- Predecessors founded in 1883 and 1891
- Publicly owned company with head office in Switzerland



Johan Sjöberg, ABB Corporate Research, Västerås, Sweden

Equation-based Modeling and Control of Industrial Processes

How ABB is organized
Five global divisions

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Power and productivity =
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Power and automation are all around us
You will find ABB technology ...

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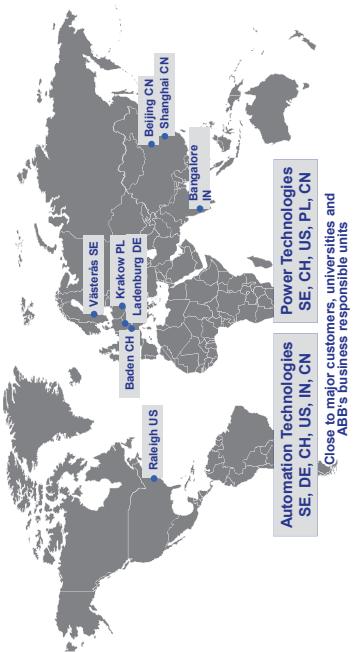
ABB's portfolio covers:

- Motors and drives
- Intelligent building systems
- Robots and robot systems
- Services to improve customer's productivity and reliability
- Electrical automation, controls and instrumentation for power generation and industrial processes
- Power transmission
- Distribution solutions
- Low-voltage products

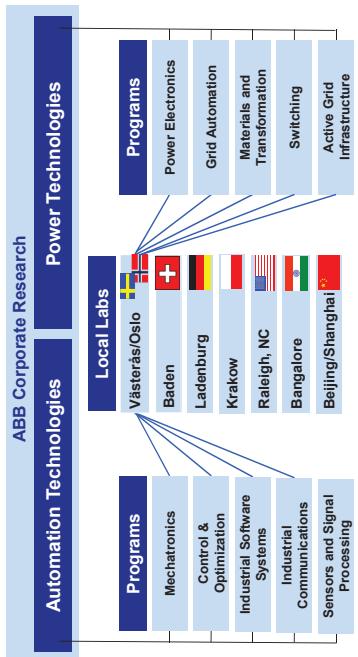
Power Products	Power Systems	Discrete Automation and Motion	Low Voltage Products	Process Automation
\$10.3 billion (2011 revenues, consolidated)	\$7.7 billion 19,500 employees	\$8.4 billion 27,500 employees	\$5.0 billion 21,000 employees	\$7.7 billion 28,800 employees

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Global Labs... ... and Local Lab Locations



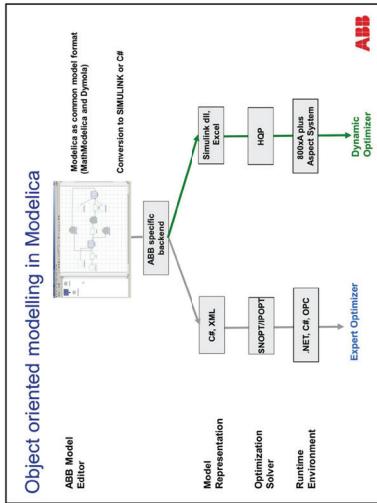
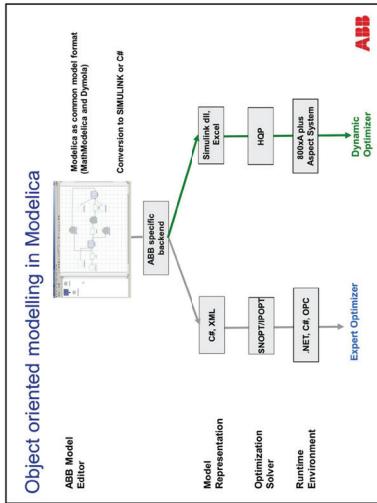
Global Labs, Corporate Programs and Locations
700 Researchers World-Wide – 250 in Västerås/Oslo



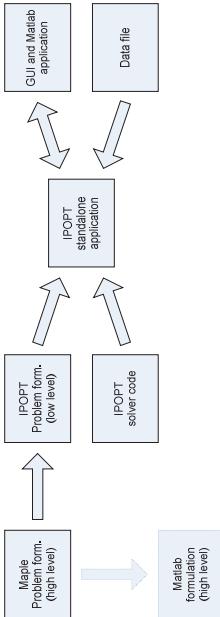
Examples of Industrial Systems modeled using an Equation-based Approach at ABB



Tools for Equation-based Simulation and Control Modelica



Tools for Equation-based Simulation and Control Maple to IPOPT



• Maple (high level description)

```

<<<
acts := [];
acts := [exp(acts), Height:=Width:=Area:=];
# Height, cut
acts := [exp(acts), HeightCut:=AreaCover:=MaxCoverWidth:=Gap];

```

• IPOPT (low level description)

```

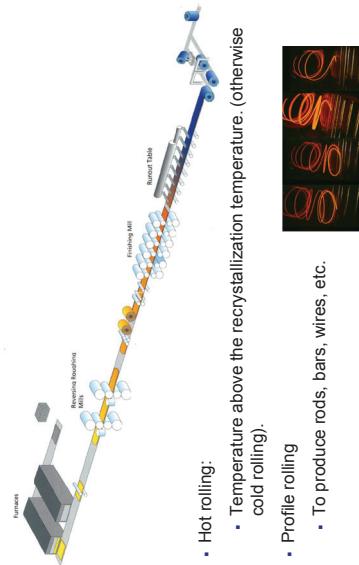
g1[i+1] = x2 + 170 * x1 - x125 / 170 * x1;
g2[i+1] = x200 + 170 * x1 - x16 + 170 * x1 / x15 - 170 * x1 - x1170 * x1;

```

(Energy) Optimization in Hot Rolling Mills Huge Potential

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Introduction to Hot Rolling



- Hot rolling:
- Temperature above the recrystallization temperature. (otherwise cold rolling).
- Profile rolling
- To produce rods, bars, wires, etc.

Optimization of Hot Rolling



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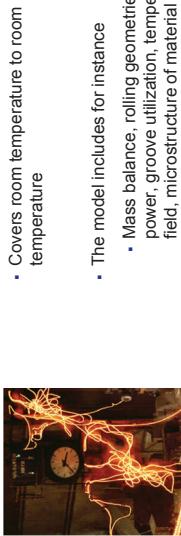
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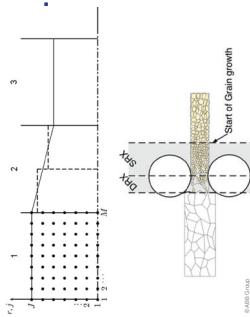
- Profile mills:
 - 7% reduction => 1.2 GWh/yr (>1000 profile mills globally)
- Flat mills:
 - 0.5% reduction => 1.6 GWh/yr (~400 flat mills globally)

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Process Model



- Covers room temperature to room temperature
- The model includes for instance
 - Mass balance, rolling geometries, power, groove utilization, temperature field, microstructure of material



- Model properties:
 - Large but sparse
 - Discontinuities due to switching behavior
 - Bad numerical scaling of certain equations

Optimization Formulation

$$\begin{aligned} & \min f(v, g, htc, T_0, \mathcal{X}) \\ \text{s.t. } & c_i(v, g, htc, T_0, \mathcal{X}) = 0, i \in \mathcal{E} \\ & c_i(v, g, htc, T_0, \mathcal{X}) \geq 0, i \in \mathcal{I} \end{aligned} \approx 700 + 45$$

$$A(htc, \mathcal{X})T^{m+1} = b(T^m, htc, \mathcal{X}) \approx 10000 - 100000$$

v = roll speed

g = gap

T_0 = furnace temperatures

htc = heat transfer coefficients

\mathcal{X} = intermediate variables and parameters such as T_j , σ, \dots

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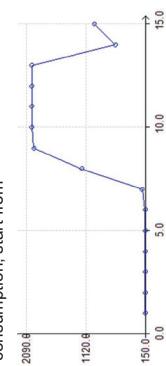
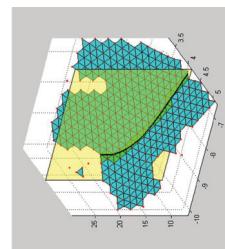
Multiojective Optimization

- Real life: Compromise between different objectives:

- Austenite grain size (related to strength of material)
- Total power
- Production speed
- Pareto front analysis yields many insights, for instance, for the cooling.
- Grain size reduction requires cooling. For low energy consumption, start from behind.

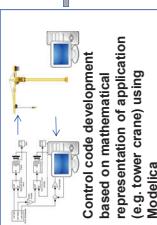
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Optimization & Optimal control are important but... Hardware-in-the-loop



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Optimization & Optimal control are important but... System Identification - Grey-box identification

- In an optimization project, modelling is by far the most time-consuming part
- Physical model should be gradually extended while testing statistical significance against experimental data
- Estimation of parameters using horizon estimation (HE) gives biased parameters without the right regularization in many cases.



Book by Torsten Bohlin
Springer, 2006

$$\begin{aligned} & \min_{x_k, \theta} \frac{1}{2} \sum_{k=0}^N x_k^T Q^{-1} x_k + v_k^T R^{-1} v_k + \log(\det(S_k(\theta))) \\ \text{s.t. } & x_{k+1} = f_k(x_k, u_k) + w_k \\ & y_k = h_k(x_k, u_k) + o_k \\ & x(0) = x_0 \end{aligned}$$

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Outcome 2A 30.11.2011 10

Equation-based Modeling and Control for Industry Conclusions, cont'd.

- Identification
 - Parameter identification is not supported enough yet.
 - Virtualization
- Hardware-In-the-Loop / training simulators gain popularity
- Generally
 - Ease of use (incl. look-and-feel)
 - Model management
 - Integration (process, data, tool etc.)



Equation-based Modeling and Control for Industry Conclusions

- Modeling
 - More and more use of first principle modeling
 - Requires considerable knowledge to succeed – process as well as theoretical
- Optimization
 - Optimization and decision support are slowly gaining ground
 - Increased competition will force more and more optimization solutions
 - More plant-wide & wider scope (production scheduling etc)
 - Optimal solution often used for comparison, not for the actual control



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Proposed grey-box scheme for nonlinear systems

- Model process in Modelica
- Discretize symbolically and export equations

- Linearize model symbolically

- Import and prepare data

- Carefully introduce noise variables at equations motivated by physical insight

- Solve by nonlinear programming (e.g. using IPOPT)

$$\min_{x_k, \beta} V = \frac{1}{2} \sum_{k=0}^N (w_k^T Q^{-1} w_k + v_k^T R^{-1} v_k + \log(\det(S_k(\theta)))$$

subject to

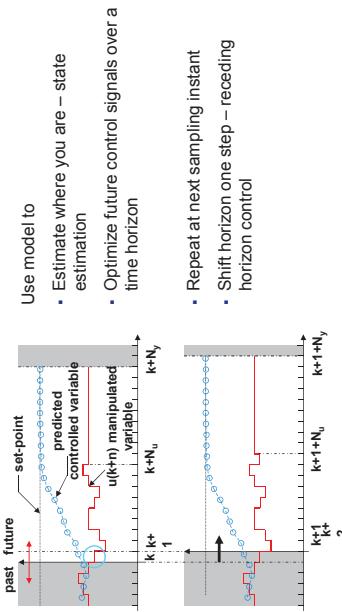
$$x_{k+1} = g(x_k, u_k) + w_k$$

$$y_k = h(x_k, u_k) + v_k$$

- For every evaluation of V calculate the (time varying) linearized system along the trajectory to compute S_k

- Test which parameters to make free (including noise parameters) by hypothesis testing using the chi-squared risk calculation
- Repeat 5-8 until no further improvement

Model Predictive Control (MPC) Algorithm



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FMI: FUNCTIONAL MOCKUP INTERFACE FOR MODEL**EXCHANGE AND CO-SIMULATION****Torsten Blochwitz, ITI GmbH Dresden, Germany**

The Functional Mockup Interface (FMI) is a tool independent standard for the exchange of dynamic models and for co-simulation. The FMI was developed in a close collaboration between simulation tool vendors, research institutes and industrial users within the European joint research project MODELISAR. It is continued as Modelica Association Project since 2012. More than 30 tools support FMI, and it is heavily used in industrial and scientific projects, not only in the automotive sector. The presentation explains the technical concepts of FMI and demonstrates some industrial applications. Additionally an overview about version 2.0 of FMI is given that combines the formerly separated interfaces for Model Exchange and Co-Simulation in one standard.



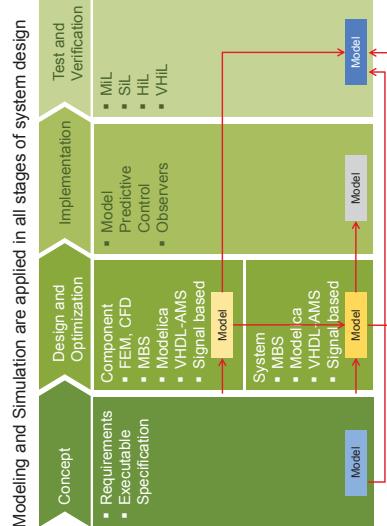
Functional Mockup Interface for Model Exchange and Co-Simulation

Contents

- Motivation
 - Main Design Idea
 - FMI for Model Exchange and Co-Simulation
 - New Features of FMI 2.0
 - Unification
 - Classification of Interface Variables
 - Save and Restore FMU State
 - Dependency Information
 - Partial Derivatives, Jacobian Matrices
 - Tools supporting FMI
 - FMU Modelica Association Project
 - Conclusion
 - Outlook
- T. Blochwitz
ITI, Dresden
M. Otter
DLR, Oberpfaffenhofen
M. Åkesson
Modelon, Lund,
University of Halle
C. Clauss
Fraunhofer IIS EAS, Dresden
H. Elmquist, H. Olsson
Dassault Systèmes, Lund
M. Friedrich,
Simpack AG, Gilching
A. Jungmann, J. Mäuss
QTronic, Berlin
D. Neumerkel
Daimler AG, Stuttgart
A. Viel
LMS Imagine, Roanne

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Motivation



Modeling and Simulation are applied in all stages of system design

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Motivation



Functional mockup interface for model exchange and tool coupling

Challenges for Functional Mockup:

- Different tools and languages are involved
- No standards for model interface and co-simulation available
- Protection of model IP and know-how of supplier

Modellisar project:

- **Functional Mockup Interface for Model Exchange and Co-Simulation**

2

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4

5

Functional Mockup Interface

FMI – Main Design Idea

- EU project Modelisar (2008 – 2011, 26 Mill. €, 178 partners)
- Initiated by Daimler AG, 28 European partners

- Tool vendors

- Users

- Research organizations

- Proof of concept in industrial use cases

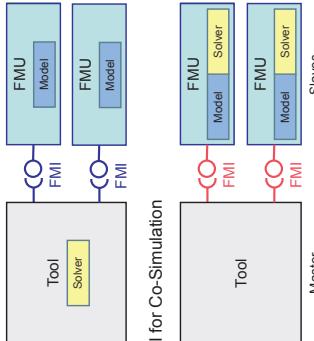
After 2011

- Continuation as Modelica Association Project
- Modelica Association changes its bylaws to become an umbrella organization for projects related to model based system design



FMI – Main Design Idea

- FMI for Model Exchange

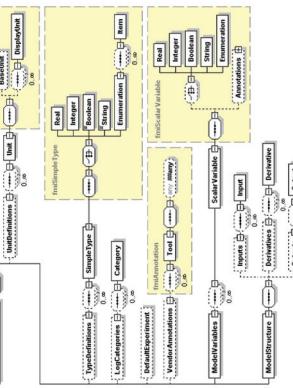


FMI Model Description

Interface definition is stored in one xml-file:

- A component which implements the interface is called a Functional Mockup Unit (FMU)
- Separation of:
 - Description of interface data (XML file)
 - Functionality (API in C)
- An FMU is a zipped file (*.fmu) containing:
 - modelDescription.xml
 - Implementation in source and/or binary form
 - Additional data and functionality
 - One FMU can contain implementations of both interfaces

- Implementation and capability flags
- Definition of units



- Variables and their attributes
- Dependency information

Example

Example

```
<?xml version="1.0" encoding="UTF-8"?>
<fmModel Identifier="www.w3.org/2001/XMLSchema"
XsiNamespace="http://www.w3.org/2001/XMLSchema-instance"
Location="file:///C:/Program Files/Modelica/Modelica/Modelica2.0-Coupling/DriveTrain/TorqueAtEnd"
modelName="FMU_Coupling.DriveTrain.TorqueAtEnd"
guid="1a4976b5-c09f-432a-94d1-e80baa3acd60">
...
<ModeExchange>
  modeIdentifier="FMU_Occupling" ...
    providesPartialDerivativesOnDerivativeFunction="true" ...
    providesDirectDerivatives="true" />
<CoSimulationIdentifier>
  modelIdentifier="FMU_Occupling" ...
    canHandleVariableCommunicationStepsize="true"
    canInterpolateInputs="true" ...
  </CoSimulationIdentifier>
  <UnitDefinitions>
    <Unit name="N" type="Angle" m="1" n="2" /> <Unit>
      <UnitDefinition>
        <TypeDefinition>
          <SimpleType name="Modelica.Slunits.Torque" unit="Nm" />
          <SimpleType name="Modelica.Slunits.Angle" unit="Radian" />
        </TypeDefinition>
        <TypeDefinition startTime="0.0" stopTime="1.0" tolerance="0.001" />
      </UnitDefinition>
    </Unit>
  </UnitDefinitions>
  <TypeDefinitions>
    <TypeDefinition name="fmiStatus" fmiType="Enum" />
  </TypeDefinitions>
</CoSimulationIdentifier>
<InputDependence>
  <InputDependence name="phi" index="1" value="1" type="Fixed" />
  <InputDependence name="w" index="2" type="Fixed" />
</InputDependence>
<OutputDependence>
  <OutputDependence name="torque" index="1" value="1" type="Fixed" />
  <OutputDependence name="phi" index="2" value="2" type="Fixed" />
</OutputDependence>
<ModelStructure>
  <ModelStructure description="FMU description" />
</ModelStructure>
</CoSimulationIdentifier>
</fmModel>
```

C-Interface

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FMI for Model Exchange Features

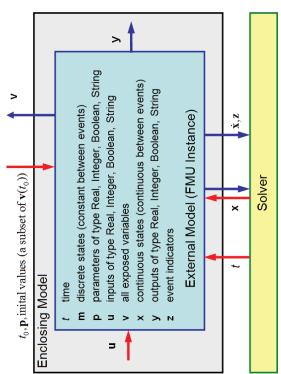
- Instantiation:
 - `fmiComponent fmiInstantiateModel(fmString instanceName, ...)`
 - `fmiComponent fmiInstantiateSlave(fmString instanceName, ...)`
 - Returns an instance of the FMU. Returned fmicomponent is an argument of the other interface functions.
- Functions for initialization, termination, destruction
- Support of real, integer, boolean, and string inputs, outputs, parameters
- Set and Get functions for each type:


```
fmiStatus fmiSetReal(fmiComponent C,
const fmValueReference vr[], size_t nvr,
```

```
const fmReal value[])
```
- Identification by value reference, defined in the XML description file for each variable

- Functionality of state of the art modelling methods can be expressed
- Support of continuous-time and discrete-time systems
- Model is described by differential, algebraic, discrete equations
- Interface for solution of Ordinary Differential Equations (ODE)
 - Handling of time, state and step events, event iteration
- Discarding of invalid inputs, state variables
- No explicit function call for computation of model algorithm
- FMU decides which part is to be computed, when a `fmiGetXXXX` function is called
- Allows for efficient caching algorithms

FMI for Model Exchange Signals



FMI for Co-Simulation Features

- State-of-the-Art Co-Simulation:
 - Fixed communication step size
 - To improve accuracy and robustness:
 - Optional variable communication step size
 - Optional higher order approximation of inputs and outputs
 - Optional repetition of communication steps
 - Capabilities of the slave are contained in the XML-file, for example:
 - canHandleVariableCommunicationStepSize
 - canInterpolateInputs
 - canGetAndSetFMUstate
 - Master can decide which coupling algorithm is applicable
 - Asynchronous execution (allows parallel execution)

Co-Simulation

Definition:

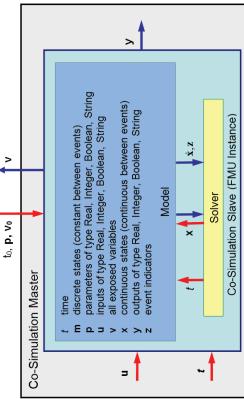
- Coupling of several simulation tools
- Each tool treats one part of a modular coupled problem
- Data exchange is restricted to discrete communication points
- Subsystems are solved independently between communication points

Motivation

- Simulation of heterogeneous systems
- Partitioning and parallelization of large systems
- Multirate integration
- Hardware-in-the-loop simulation

- Simulation of heterogeneous systems
- Partitioning and parallelization of large systems
- Multirate integration
- Hardware-in-the-loop simulation

FMI for Co-Simulation Signals



Additional:

- Status information
- Derivatives of inputs, outputs w.r.t. time for support of higher order approximation

FMI for Co-Simulation

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FMI for Model Exchange and Co-Simulation Sample Code

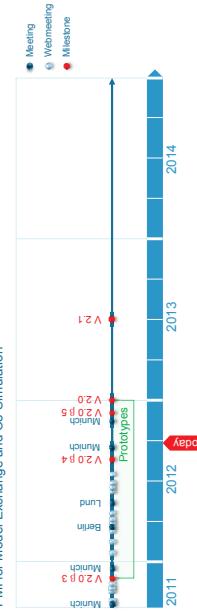
- Model Exchange:
(One model evaluation)
- Co-Simulation:
(One communication step)

```
/* Set inputs */
fmiSetReal(m, id_u1, ul, null);
fmiSetContinuous(m, tC);
fmiSetContinuousStates(m, x, rx);
/* Get results */
fmiGetDerivatives(m, derx, rx);
fmiGetEventIndicators(m, z, nz);
fmiGetReal(m, id_u1, ul, null);
```



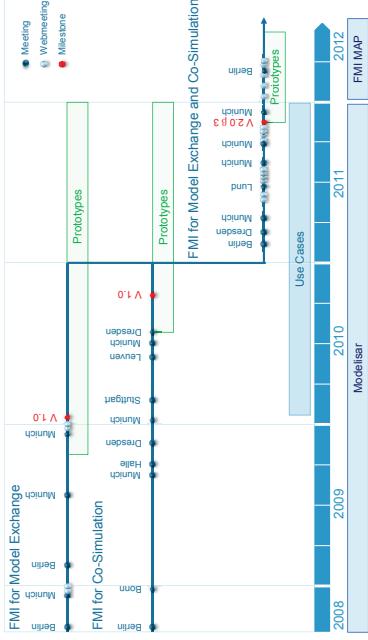
Development Process

FMI for Model Exchange and Co-Simulation



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Development Process



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FMI 2.0 New Features

- Motivation for FMI 2.0
 - Clarification of specification document
 - Ease usability
 - Increase performance for large models
- Unification of Model Exchange and Co-Simulation Standard
 - FMU can contain implementations of both interfaces
 - Distributed and tool based use cases now also for Model Exchange
- Many minor changes
 - Definition of log categories
 - Removal of alias and anti alias variables to ease usage
 - Continuous state variables are named and ordered
 - Improved unit handling

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Development Process



FMI 2.0 Specification:

- Release December 2012
- Valid for several years
- Backwards-compatible enhancements in minor releases

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Current status of FMI 2.0

FMI 2.0 Classification of interface variables

Classification of specification:	
▪ Instantiation	
▪ Classification of variables	
▪ Calling sequence	
Features:	
▪ Tunable parameters	
▪ Improved unit handling	
▪ Save and restore FMU state	
▪ Detailed dependency information (inputs, outputs, derivatives)	Contained in public Beta 4
▪ Efficient interface to partial derivatives	Under Discussion
▪ Improved handling of time events	

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FMI 2.0 Save and Restore FMU State

- FMI 1.0: implicit save and restore depending on arguments of fmiDoStep
- FMI 2.0: explicit function calls
 - fmiSetFMUstate(fmiComponent c, fmiFMUstate * FMUstate)
 - fmiGetFMUstate(fmiComponent c, fmiFMUstate FMUstate)
- Iterative co-simulation algorithms
 - Repeat more than one communication step
- Model Predictive Control
 - Simulate some steps starting from the same state with different sets of input values
 - Use the optimal set as control value for the real system
- FMU state can be serialized into a byte vector
 - Usage: start a training simulator from a certain scenario

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FMI 2.0 Dependency Information

- FMI 1.0:
 - Only dependencies of outputs on inputs can be indicated
- FMI 2.0:
 - Dependencies of outputs on continuous states
 - Dependencies of derivatives on continuous states and inputs
- Usage:
 - Detection of algebraic loops
 - Definition of sparsity pattern of Jacobian matrices

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FMI 2.0 Dependency Information

- Kind of dependency is also defined:
 - nonLinear: Jacobian entry is not constant
 - fixed: Jacobian entry is constant
 - discrete: Jacobian entry may change after events
- Allows optimizations:
 - Generate linear systems of equations for solution of algebraic loops if possible
 - Reduce number of Jacobian computations

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FMI 2.0 Time Event Handling (under Development)

Requirements:

- Guarantee synchronicity of time events
- Support a subset of the synchronous extensions from Modelica 3.3 (time triggered clocks with constant and variable period)
- Allow backward compatible extensions
- Usable for tools without synchronous features

Main design idea:

- FMU exposes base rates and clocks in the XML model description
- Clock ticking is signaled by `fmiSetClock(..)` before `fmiEventUpdate(..)`
- Discrete variables can be associated with clocks (optional) in XML model description

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FMI 2.0 Directional Derivatives (Jacobian Matrices)

- Jacobians are needed for:
 - Implicit integration methods
 - Solution of systems of equations resulting from algebraic loops
 - Linearization of FMU
 - Extended Kalman filters
- Numerical computation is expensive for large models
- Optional function for providing directional derivatives
 - `fmiStatus fmiGetDirectionalDerivative fmiComponent C,..)`
 - Arguments define which derivative(s) w.r.t. which variable(s) are to be retrieved

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FMI 2.1 Hierarchical Data, Buses, Physical Connectors (planned)

Requirements:

- Group variables to hierarchical structures, connectors
- Signal based tools must not be excluded
- Keep type information of connectors
 - (e.g. Modelica.Electrical.Analog.Interfaces.Pin)
- Add connector type definition for reconstruction of connector type or mapping to existing types

Main design idea:

- Additional "layer" in XML model description
- Mark input/output variables as flow or across quantities
- Causality (input, output) is fixed

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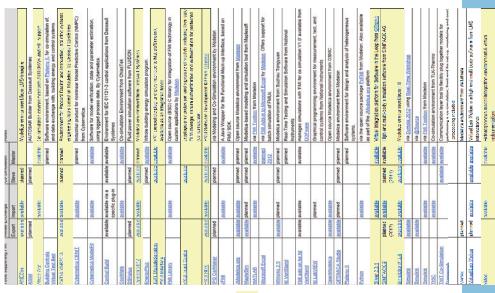
Roadmap

- 2012:
 - Finalize time event handling
 - October: FMI Meeting
 - November: Release of public beta 5
 - December: Release of FMI 2.0
 - Coordinated prototype implementations by tool vendors

- 2013:
 - Backwards-compatible extensions
 - Support of arrays and hierarchical data
 - Bus and physical connectors
 - Graphical appearance
 - ...

FMI Support in Tools fmi-standard.org/tools

- Tool support started immediately after release of FMI 1.0
- 32 tools support FMI, 9 intend to
 - Within Modelisar project: 15

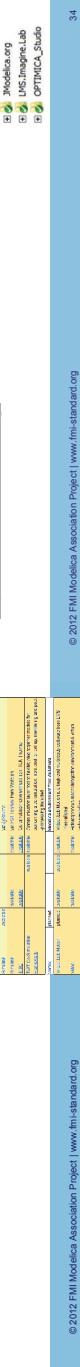


Quality of FMI Implementations

- 31 © 2012 FMI Modelling Association Project | www.fmi-standard.org
- Authoring Tools: 12
- Integration Tools: 20 (Co-Simulation master, HIL, optimization, control, analysis)
- Software Development Kits: 3 (C, Python, Java)
- FMI Compliance Checker
 - Open source implementation under contract of MA
 - Checks XML model description
 - Simulates single FMUs for Model Exchange and Co-Simulation
 - https://svn.fmi-standard.org/fmi/branches/public/Test_FMU_1.0/Compliance-Checker/
- Repository of FMUs, generated by different tools
 - https://svn.fmi-standard.org/fmi/branches/public/Test_FMU_1.0/FMUs/

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- <https://trac.fmi-standard.org/>
- https://fmi-standard.org/doc/PDFs/FMI_1_0.pdf



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Applications outside of Automotive

FMI Modelica Association Project (MAP)

Power plant simulation and control

- Siemens, ABB, EDF
- EU Project MODRIO (19 Mill. €, 150 man-years, 2012 – 2015)

Building simulation

- Situation is similar to automotive industry:
 - Heterogeneous systems (building, heating, air conditioning, ...)
 - Components of different nature and from several suppliers

Research

- Co-Simulation master algorithms
- Model based control

General conditions

- FMI project members need not to be Modelica Association (MA) members
- Project results are owned by the MA
- Project results are freely usable under copyleft license
- Meetings are open to the public

Situation is similar to automotive industry:

- Heterogeneous systems (building, heating, air conditioning, ...)
- Components of different nature and from several suppliers

Research

- Co-Simulation master algorithms
- Model based control

FMI Project Rules How to participate

Steering Committee

- Prove active FMI support by participation at 2 meetings in the last 24 months
- Support FMI or part of it in a commercial or open source tool, and/or active FMI usage in industrial projects
- Be accepted by Steering Committee with qualified majority

Advisory Board

- Prove active FMI support by participation at 2 meetings in the last 24 months
- Send e-mail to contact@fmi-standard.org for registration in mailing list

Guests

- Altair Engineering, Berkeley University, Bosch, ETAS, Equa Simulation, IBM Research

FMI MAP Members

Steering Committee

- Atego, Daimler, Dassault Systèmes, IFP EN, ITI, LMS, Modelon, QTronic, Siemens, SIMPACK

Advisory Board

- Armines, DLR, Fraunhofer (IIS/EAS, First, SCAI), Open Modelica Consortium, TWT, University of Halle

Guests

- Altair Engineering, Berkeley University, Bosch, ETAS, Equa Simulation, IBM Research

Conclusions

Outlook

FMI for Model Exchange and Co-Simulation is an established standard

- 32 tools currently support FMI 1.0, 9 intend to
- Is used in industrial and research applications
- Is maintained as Modelica Association Project

FMI project is open for non Modelica tool vendors and organizations

FMI 2.0 improves:

- Compatibility of implementations (clarified specification)
- Usability (tunable parameters, unit handling)
- Efficiency and robustness for large models (dependency information, directional derivatives)

FMI 2.0 Release planned for December 2012

Current tasks:

- Precise handling of time events for periodic and aperiodic sampled data systems

Ideas for FMI 2.1

- Arrays, hierarchical data, buses, physical ports
- Graphical appearance, connector placement



VERTICAL INTEGRATION IN TOOL CHAINS FOR MODELING, SIMULATION AND OPTIMIZATION OF LARGE- SCALE SYSTEMS

**Johan Åkesson, Modelon AB and Lund University, Lund,
Sweden**

In recent years, languages such as Modelica and VHDL-AMS have emerged as intuitive user and application-oriented high-level description formats suitable for modeling of physical systems. This trend has been further strengthened by the availability of software tools for modeling, simulation and optimization, which enable engineers to rapidly develop detailed models of complex systems composed from sub-systems from different physical domains. While the capabilities of such tools in terms of performance match the requirements in challenging industrial applications, tool interoperability has traditionally received little attention. Rather, tools have been designed as monolithic software environments, with dedicated interfaces to numerical algorithms. As a result, flexible creation of tool chains where several computational tools are assembled into a workflow tailored to a particular design process is often difficult. Driven by the observation that one single tool will not be able to provide the solution to the computational needs of the future, new challenges have emerged.

In order to meet these challenges, three aspects of computational tool chains need considered. Firstly, modeling languages are critical to provide comprehensive environments for engineering practitioners, as well as for enabling formal analysis of model properties. In addition, language extensibility, both in terms of the language itself and in terms of tools supporting it, requires attention in order to enable flexible tailoring of existing languages to specific needs arising when formulating different systems design problems. Secondly, open interfaces plays a key role in achieving tool interoperability. A recent example is the Functional Mock-up Interface (FMI), which has received considerable attention in the simulation tool community. Finally, symbolic and numerical algorithms designed to solve systems design problems need to be employed. In this presentation, challenges arising when integrating different tools into complete tools chains will be discussed. Particular attention will be given to the three mentioned aspects: languages, open interfaces and algorithms. Examples will be drawn from experiences from developing, integrating and using open source tools, notably CasADi and Jmodelica.org, in industrial projects where large-scale optimization techniques has been applied to Modelica models derived from first principles.

In 2006...

Vertical Integration in Tool Chains for Modeling Simulation and Optimization of Large-Scale Systems

Johan Åkesson, Modelon AB/Lund University

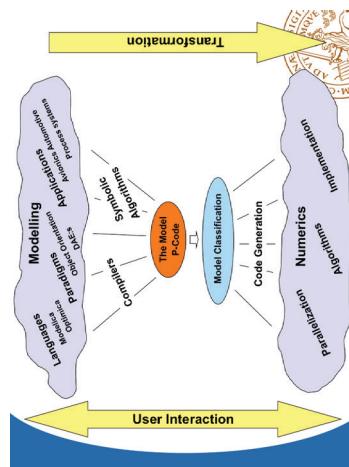
Thanks to
 Joel Andersson, Niklas Andersson, Magnus Gäfvert, Staffan Hägglund,
 Görel Hedin, Per-Ola Larsson, Alexandra Lind, Kilian Link,
 Fredrik Magnusson, Elin Saalberg, Stéphanie Veut



The Landscape

Outline

- Modelica
- Application examples
- Extension example
- Interface example
- Towards a vertically integrated tool chain
- Challenges



What is Modelica?

- A language for modeling of complex heterogeneous physical systems
 - Open language
 - Modelica Association (www.modelica.org)
 - Several tools supporting Modelica
 - Dymola
 - OpenModelica (free)
 - Mosilab
 - Scilab/Scicos (free)
 - Extensive (free) standard library
 - Mechanical, electrical, thermal etc.

Key Features of Modelica

- Declarative equation-based modeling
 - Text book style equations
- Multi-domain modeling
 - Heterogeneous modeling
- Object oriented modeling
 - Inheritance and generics
- Software component model
 - Instances and (acausal) connections
- Graphical and textual modeling

A Simple Modelica model

Differential equation

$$\dot{x}(t) = ax(t) + bu(t)$$

Variable declaration

```
model FirstOrder
  input Real u;
  parameter Real b = 1;
  parameter Real a = -1;
  variable Real x(start=1);
```

Initialization

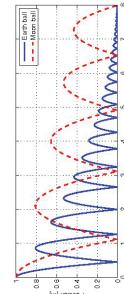
```
der(x) = a*x + b*u;
end FirstOrder;
```

Equation

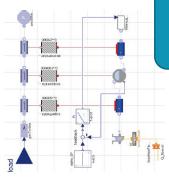
Hybrid modeling

```
class BouncingBall //A model of a bouncing ball
parameter Real g = 9.81; //Acceleration due to gravity
parameter Real e = 0.9; //Elasticity coefficient
Real pos(start=1); //Position of the ball
Real vol(start=0); //Velocity of the ball
equation
  der(pos) = vel; // Newtons second law
  der(vel) = -g;
  when pos <=0 then
    reinit(vel, -e*pre(vel));
  end when;
end BouncingBall;
```

```
class BBex
  BouncingBall eBall;
  BouncingBall mBall(g=1.62);
end BBex;
```



Graphical Modeling



```

model MotorControl1
  Modelica.Mechanics.Rotational.Inertia inertia;
  Modelica.Mechanics.Rotational.Sensors.SpeedSensor speedSensor;
  Modelica.Electrical.Machines.BasicMachines.DCMotor DCM;
  Modelica.Electrical.Analog.BasicGround ground;
  Modelica.Electrical.Analog.Sources.SignalVoltage signalVoltage;
  Modelica.Blocks.Math.Feedback feedback;
  Modelica.Blocks.Sources.Ramp ramp{height=100, startTime=0};
  Modelica.Blocks.Continuous.PI PI{k=-2};

equation
  connect(inertia.flange_b, speedSensor.flange_a);
  connect(DCM.flange_a, inertia.flange_a);
  connect(speedSensor.w, feedback.u1);
  connect(ramp.y, feedback.u1);
  connect(signalVoltage.n, DCPM.pin_ap);
  connect(signalVoltage.p, ground.p);
  connect(ground.p, DCPM.pin_an);
  connect(feedback.y, PI.u);
  connect(PI.y, signalVoltage.v);
end MotorControl1;

```

Industrial Application I Power Plant Start-up Optimization

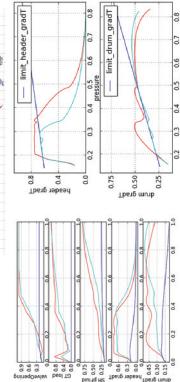
- Start-up optimization of combined cycle power plants
- Reduce start-up time
- Model-based optimization
- Siemens AG LU, Modelon collaboration

Continuous time states: 39

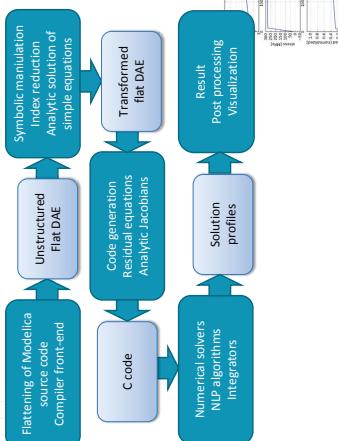
Scalar equations: 569

Algebraic variables: 530

NLP equations: 26824

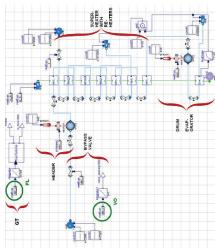


A Modelica-based Tool Chain



Industrial Application I Power Plant Start-up Optimization

- Large effort to develop models suitable for optimization
- Scaling of problem significantly more challenging than in simulation
- Convergence and robustness of numerical algorithms



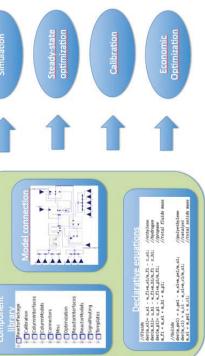
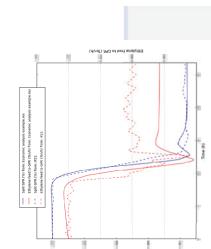
Industrial Application I Power Plant Start-up Optimization

- Design-patterns from Modelica media model libraries applied to optimization-friendly modes
- Intuitive high-level descriptions of dynamic optimization problem appreciated by users – a vehicle for communicating ideas



Industrial Application II Grade Changes in Polyethylene Production

- Optimization of economics of polyethylene grade changes
- Model calibration to data
- Modeling with Modelica and Optimica
- Development of end-user GUI
- PIC-LU – Lund University and Borealis



Extension Example – Optimica

- High-level description of optimization problems
 - Steady-state
 - Dynamic
- Extension to Modelica
- Optimization of physical models

```

optimization vdp_opt(objective=cost{finalTime},
                      startTime=0,
                      finalTime=true,
                      initialGuess=1)
vdp.vdpOpt{free=true,initialGuess=0,0};
Real cost (start=0);
equation
  def(cost) = 1;
  vdp..x1{finalTime} = 0;
  vdp..x2{finalTime} = 0;
  vdp..u >= -1; vdp..u <= 1;
end vdp_opt;

```

```

min Y(z,p)
subject to the dynamic system
F'(x(t),x(t),u(t),p)<=0, t in [t_0,t_f]
and the constraints
c_u(x(t),y(t),u(t),p)<=0, t in [t_0,t_f]
c_y(x(t),y(t),u(t),p)<=0, t in [t_0,t_f]
c_w(z,p)=0
c_w(z,p)=0
while
z=[x(t_1),...,x(t_N),y(t_1),...,y(t_N),u(t_1),...,u(t_N)], t in [t_0,t_f]

```

Industrial Application II Grade Changes in Polyethylene Production

- Model reuse across different computations
 - High-level model and optimization problem formulation enabled promotion focus on problem formulation
 - Custom GUI in Python appreciated by end-users



Extension Example – Optimica

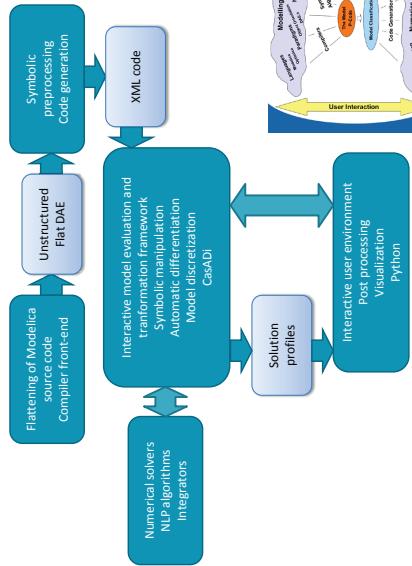
- High-level problem descriptions promote focus on formulation rather than encoding
 - New users without optimization experience quickly gets up to speed
 - Model reuse for different usages
 - Automatic model transformation reduce user effort

```

optimization vdp_opt(objective=cost{finalTime},
                      startTime=0,
                      finalTime=true,
                      initialGuess=1)
vdp.vdpOpt{free=true,initialGuess=0,0};
Real cost (start=0);
constraint
  vdp..x1{finalTime} = 0;
  vdp..x2{finalTime} = 0;
  vdp..u >= -1; vdp..u <= 1;
end vdp_opt;

Lessons learnt
• High-level descriptions make optimization technology available to non-experts
• Automatic model transformation reduces design cycle times
• Modern compiler construction technology is accessible to non-experts (e.g. JastAdd)
```

Towards a vertically integrated toolchain



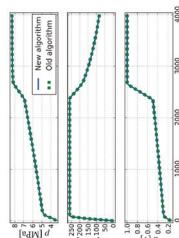
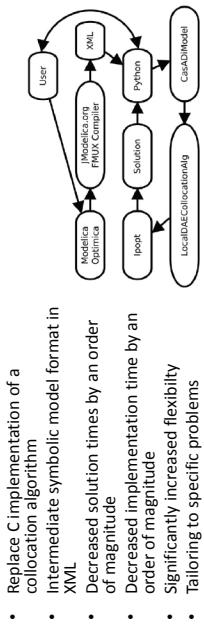
Interfacing Example – Modelica, XML Models and CasADI



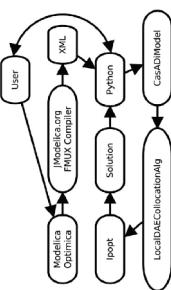
Challenges

- How do we make advanced algorithms in systems design in general and in optimization in particular PhD-free?
- How do we combine declarative modeling languages with ideas from interactive model transformation/evaluation frameworks?
- How do we propagate consistent error/diagnostics through the tool chain?
- Open interfaces and interoperability, FMI and extensions
- Classify models applicable to different solution algorithms

Interfacing Example – Modelica, XML Models and CasADI



	Off-line	On-line	Total	Iterations
New alg.	4.9	3.0	7.9	75
Old alg.	13.2	23.9	37.2	75



Conclusions

- In users' perception, current optimization algorithms for large-scale non-linear dynamic systems require high level of expertise
- Very different cultures and best practices in simulation and optimization communities – expectation management
- Users sometimes need to/desire to interact with both mathematical model and solution algorithm implementation
- Challenges in usability and robustness of numerical algorithms
- Challenges in vertically integrated tool chains – languages and open interfaces and tool decoupling

Thank you!

Questions, comments?

SYSTEM DESIGN – FROM REQUIREMENTS TO IMPLEMENTATION

Alberto Ferrari, ALES S.r.l.

The design of cyber-physical systems by successive refinements starts from a set of requirements and incrementally adds design decisions till implementation is built. Equation based languages are essential to support with rigorousness these decisions and enable a formal exploration of the solution space.

For real industrial cases, none of the current equation based language is capable of covering the entire design flow and different languages must be used. In this talk some of the current gaps and challenges will be described and partially addressed.

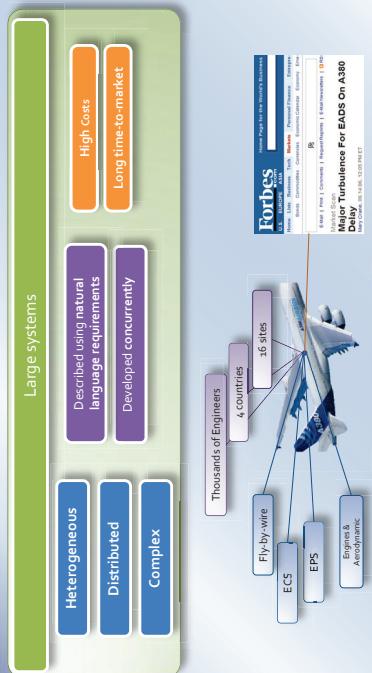


System Design: From Requirements to Implementation

A.Ferrari
O.Ferrante, L.Mangeruca



System Engineering Challenges

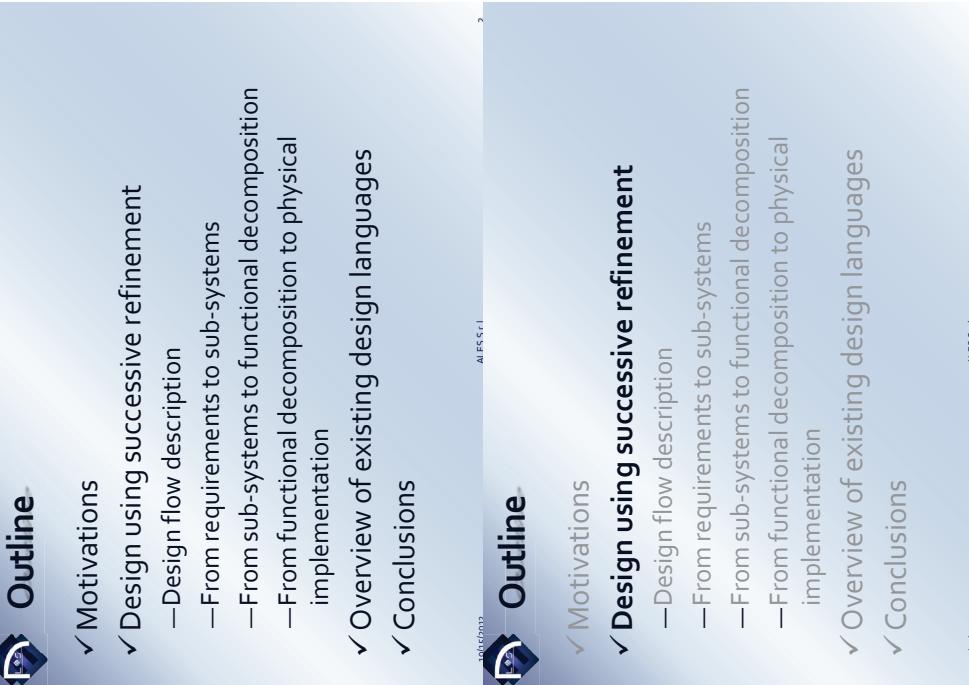


Outline

- ✓ Motivations
- ✓ Design using successive refinement
 - Design flow description
 - From requirements to sub-systems
 - From sub-systems to functional decomposition
 - From functional decomposition to physical implementation
- ✓ Overview of existing design languages
- ✓ Conclusions

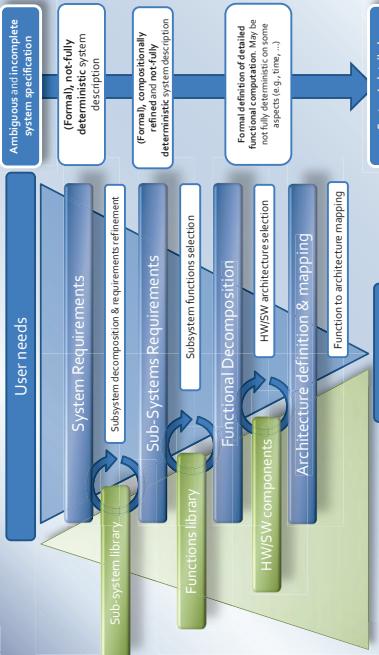
Outline

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Design using successive refinement

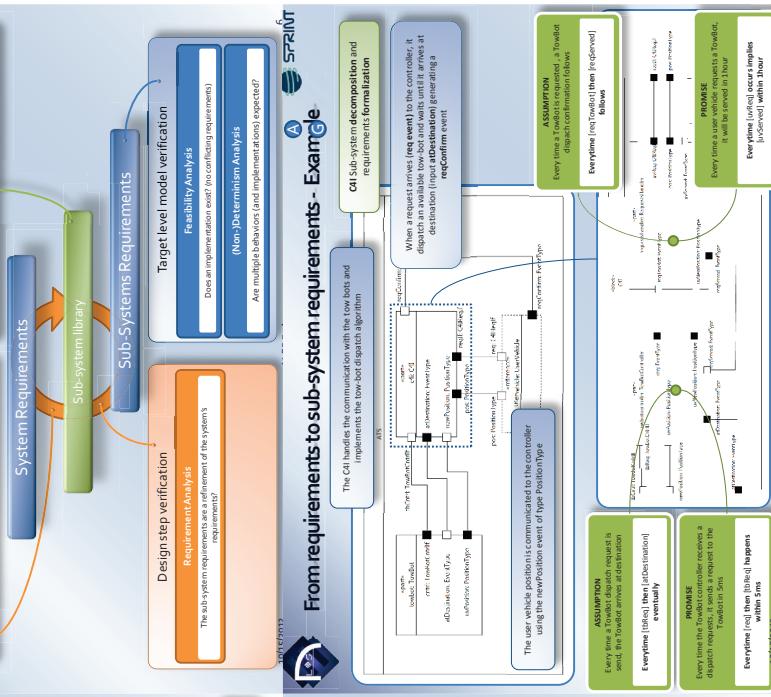
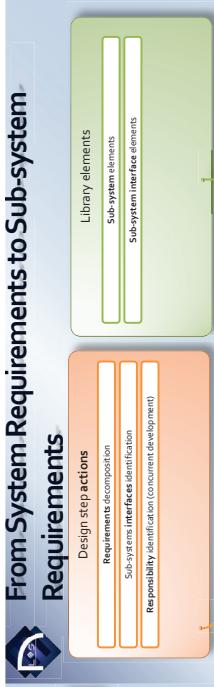
From System Requirements to Sub-system Requirements



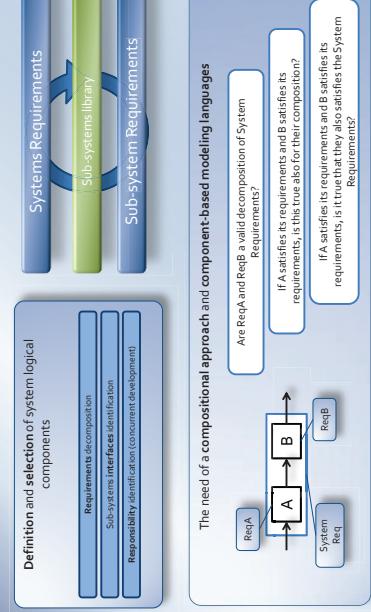
SPRINT ATS use case



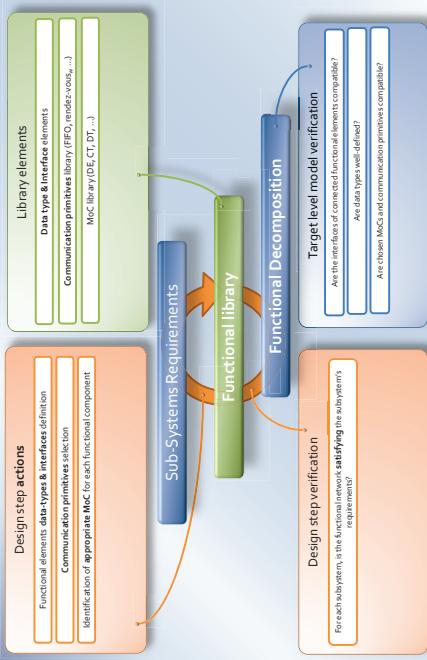
*Command Control Communications Computer, and Intelligence



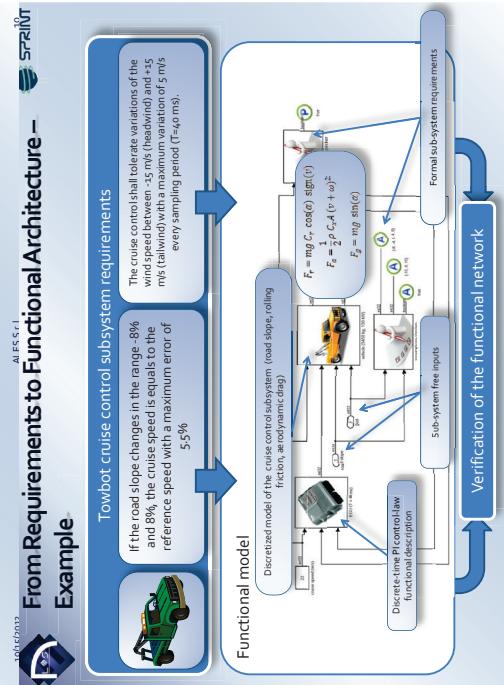
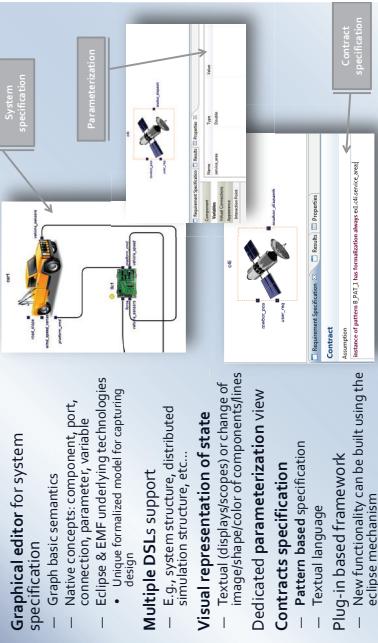
Design using successive refinement



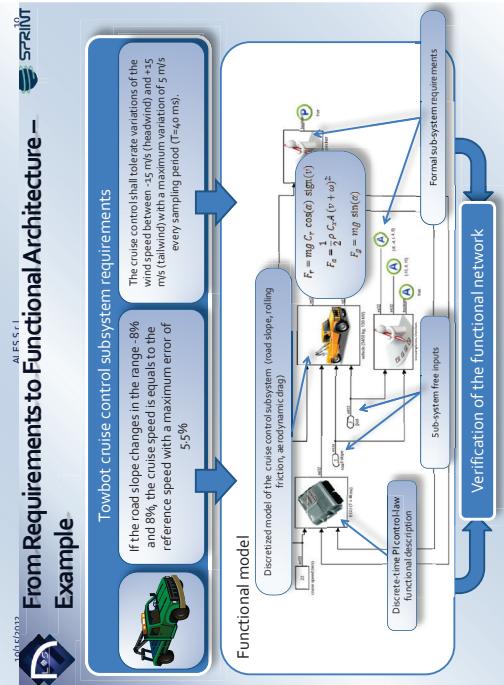
From Requirements to Functional Architecture



ALES Experience – Requirements formalization using the Contract Editor tool

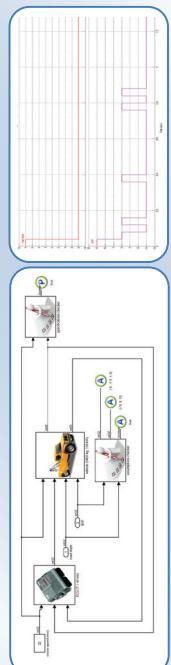


ALES Experience – Requirements formalization using the Contract Editor tool

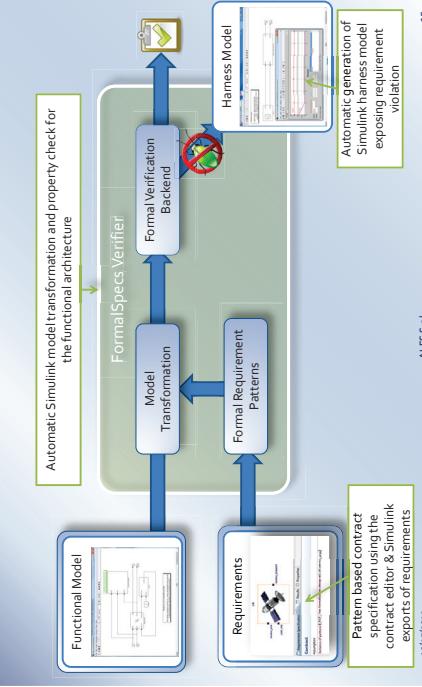


Cruise control contracts

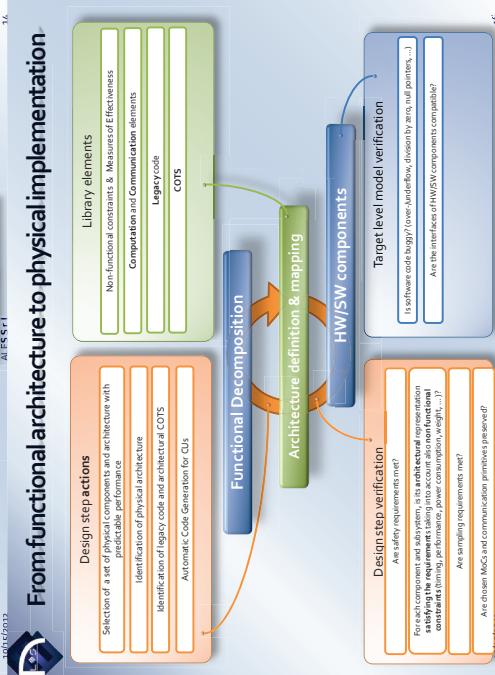
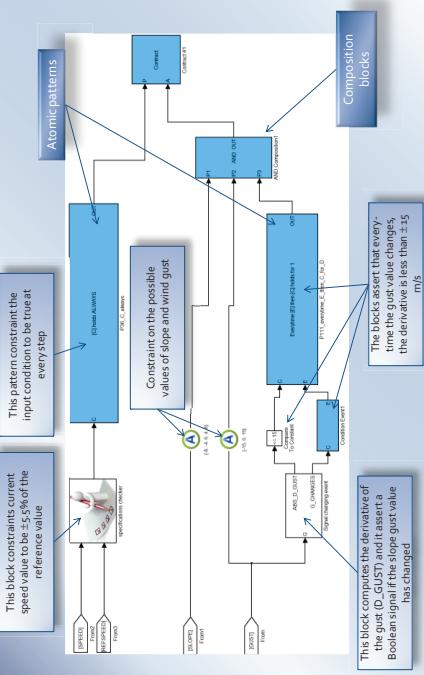
- ✓ Contract specification
 - **Assumption** is the conjunction of three assertions
 - The slope value (slope percentage) is in $\{-8, -4, 0, 4, 8\}$
 - The wind gust value is in $\{-15, 0, -15\}$ m/s
 - The wind gust, every 40 ms, can change of a maximum absolute value of 15 m/s
 - **Promise:**
 - the actual speed value is $\pm 5.5\%$ of the reference speed value



ALES Experience – Requirement & Functional architecture description & formal verification



From requirements to functional architecture – Example

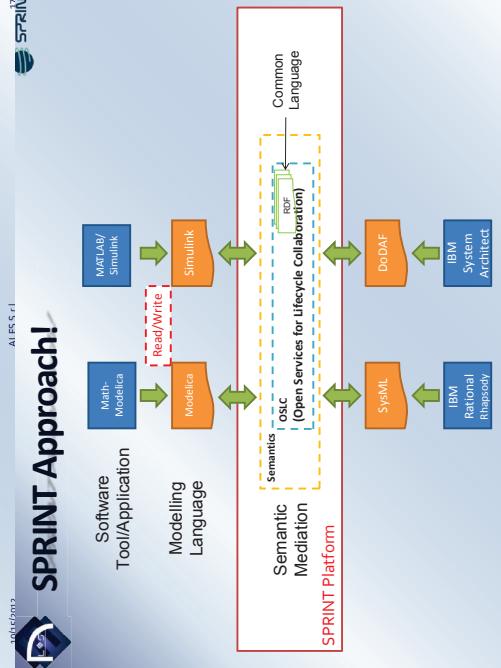


Outline

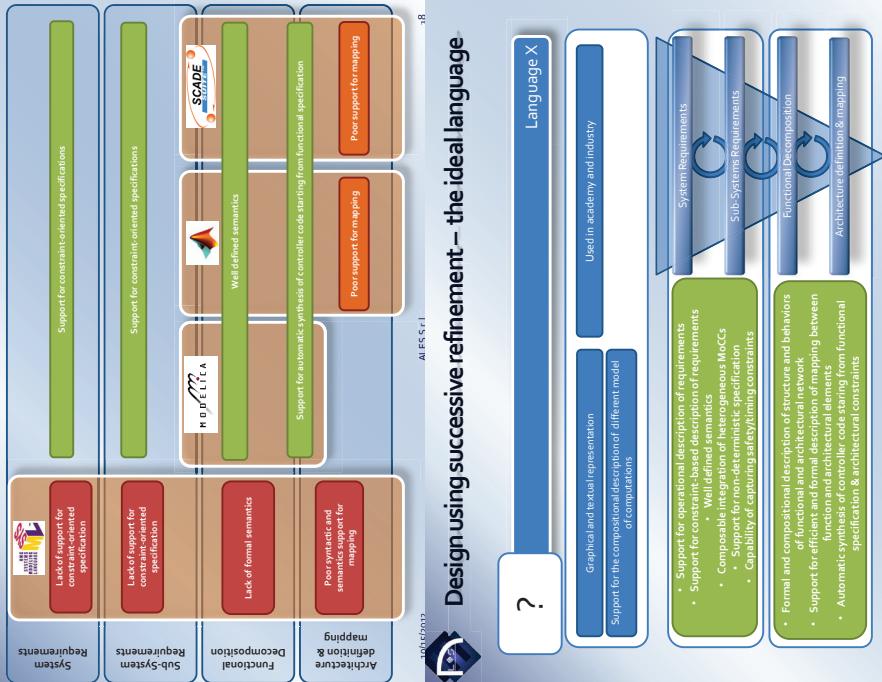
- ✓ Motivations
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 - From functional decomposition to physical implementation
- ✓ Conclusions

✓ Overview of existing design languages

SPRINT Approach!



Where Languages Map?



10/16/2012

20

ALFES



Conclusion

- ✓ Summary
 - Design flow using successive refinement
 - From requirements to sub-system
 - From sub-system to functional architecture
 - From functional architecture to physical implementation
 - Equation-based language
 - Overview
 - Limitations



SYNCHRONOUS CONTROL AND STATE MACHINES IN MODELICA

Hilding Elmqvist, Dassault Systèmes AB

The scope of Modelica has been extended from a language primarily intended for physical systems modeling to modeling of complete systems by allowing the modeling of control systems and by enabling automatic code generation for embedded systems. Much focus has been given to safe constructs and intuitive and well-defined semantics.

The presentation will describe the fundamental synchronous language primitives introduced for increased correctness of control systems implementation. The approach is based on associating clocks to the variable types. Special operators are needed when accessing variables of another clock. This enables clock inference and increased correctness of the code since many more checks can be done during translation. Furthermore, the sampling period of a clocked partition needs to be defined only at one place (either in absolute time or relatively to other clocked partitions). The principles of partitioning a system model into different clocks (continuous, periodic, non-periodic, multi-rate) will be explained.

The new language elements follow the synchronous approach. They are based on the clock calculus and inference system of Lucid Synchrone. However, the Modelica approach also uses multi-rate periodic clocks based on rational arithmetic and also non-periodic and event based clocks are supported.

Parallel and hierarchical state machines will be introduced including submodels within states. The supporting Modelica library will also be introduced.

Content

Synchronous Control and State Machines in Modelica

- Introduction
- Synchronous Features of Modelica
 - Synchronous Operators
 - Base-clock and Sub-clock Partitioning
 - Modelica_Synchronous library
 - State Machines
 - Conclusions

Hilding Elmquist

Dassault Systèmes

Sven Erik Mattsson, Fabien Gaucher, Francois Dupont

Dassault Systèmes

Martin Otter, Bernhard Thiele

DLR

Introduction

- Why synchronous features in Modelica 3.3?

```
model Asynchronous_Modelica32
  Real xstart=0,fixed=true,
  ystart=0,fixed=true,z;
  equation
    when sample(0,0.33) then
      x = previous(x)+1;
    end when;
    when sample(0,1/3) then
      y = previous(y)+1;
    end when;
    z = x*y;
  end Asynchronous_Modelica32;
```

Rational number 1/3

x and y must have

the same clock

Modelica33;

A subclock pattern includes clocks that cannot be deduced to be equal.

Clock(1/3)

appears in the partition

when Clock(1/3)

end when;

when Clock(1/3)

x = previous(x)+1;

end when;

z = x*y;

end when;

z = y*x;

end when;

z = y*x;

- Error Diagnostics for safer systems!

Introduction

- Scope of Modelica extended
 - Covers complete system descriptions including controllers
- Clocked semantics
 - Clock associated with variable type and inferred
 - For increased correctness
 - Based on ideas from Lucid Synchrone and other synchronous languages
 - Extended with multi-rate periodic clocks, varying interval clocks and Boolean clocks

Slide 2

Slide 4

Slide 2

Slide 4

Slide 4

Synchronous Features of Modelica

- Plant and Controller Partitioning
 - Boundaries between continuous-time and discrete-time equations defined by operators.
 - `sample()`: samples a continuous-time variable and returns a clocked discrete-time expression
 - `hold()`: converts from clocked discrete-time to continuous-time by holding the value between clock ticks
 - sample operator may take a Clock argument to define when sampling should occur

Mass with Spring Damper

- Consider a continuous-time model
- ```

partial model MassWithSpringDamper
 parameter Modelica.SIunits.Mass m=1;
 parameter Modelica.SIunits.TranslationalSpringConstant k=1;
 parameter Modelica.SIunits.TranslationalDampingConstant d=0.1;
 Modelica.SIunits.Position x(start=1,fixed=true) "Position";
 Modelica.SIunits.Velocity v(start=0,fixed=true) "Velocity";
 Modelica.SIunits.Force f "Force";
equation
 der(x) = v;
 m*der(v) = f - k*x - d*v;
end MassWithSpringDamper;
```

Kontroll

## Synchronous Controller

- Discrete-time controller

```

model SpeedControl
 extends MassWithSpringDamper;
 parameter Real K = 20 "Gain of speed P controller";
 parameter Modelica.SIunits.Velocity vref = 100 "Speed ref.";
 discrete Real vd;
 discrete Real u(start=0);
equation
 vd = sample(v,Clock(0.01));
 // P controller for speed
 u = K*(vref-vd);
```

The clock of the equation  
is inferred to be the same as for the variable  
`vd` which is the result of `sample()`

```

// force actuator
f = hold(u);
end SpeedControl;
```

Kontroll

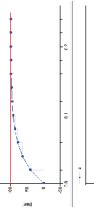
Kontroll

## Discrete-time State Variables

- Operator `previous()` is used to access the value at the previous clock tick (cf `pre()` in Modelica 3.2)
- Introduces discrete state variable
- Initial value needed
- `interval()` is used to inquire the actual interval of a clock

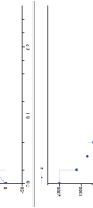
Slide 6

Kontroll



Slide 5

Kontroll



Slide 6

Kontroll

Slide 7

Kontroll

Slide 8

Kontroll

## Base-clocks and Sub-clocks

- A Modelica model will typically have several controllers for different parts of the plant.
- Such controllers might not need synchronization and can have different **base clocks**.
- Equations belonging to different base clocks can be implemented by asynchronous tasks of the used operating system.
- It is also possible to introduce **sub-clocks** that tick a certain factor slower than the base clock.
- Such sub-clocks are perfectly **synchronized** with the base clock, i.e. the definitions and uses of a variable are **sorted** in such way that when sub-clocks are activated at the same clock tick, then the definition is evaluated before all the uses.
- New base type, **Clock**:  
`Clock_Control = Clock(0.1);`  
`Clock_Center = subsample(ccControl, 5);`

## Exact Periodic Clocks

- Clocks defined by Real number period are not synchronized:

```
Clock c1 = Clock(0.1);
```

```
Clock c2 = superSample(c1,3);
```

```
Clock c3 = Clock(0.1/3); // Not synchronized with c2
```

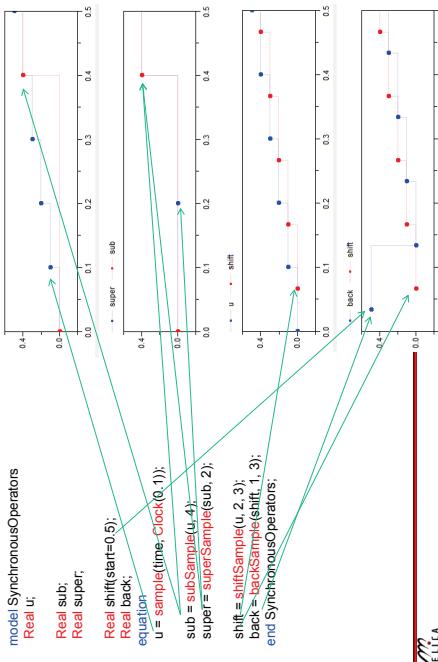
- Clocks defined by rational number period are synchronized:

```
Clock c1 = Clock(1/10); // period = 1/10
```

```
Clock c2 = superSample(c1,3); // period = 1/30
```

```
Clock c3 = Clock(1/30); // period = 1/30
```

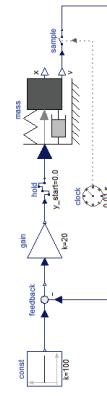
## Sub and super sampling and phase



Slide 9

## Modelica\_Synchronous library

- Synchronous language elements of Modelica 3.3 are "low level":  
`v_d = sample(v, Clock(0.01));`  
`// P controller for speed`  
`u = K*(vref-vd);`  
`// force actuator`  
`f = hold(u);`
- Modelica\_Synchronous library developed to access language elements in a convenient way graphically.



Slide 9

Modelica

Modelica

Modelica

Slide 12

## Blocks that generate clock signals

Generates a periodic clock with a Real period

```
Parameter Modelica.SIunits.Time period;
equation
y = Clock(period);

```



## Sample and Hold

Generates a periodic clock as an integer multiple of a resolution (defined by an enumeration).

```
Code for 20 ms period:
y = superSample(Clock(20), 1000);

```

Clock with period 20 s      super-sample clock with 1000 period =  $20 / 1000 = 20 \text{ ms}$

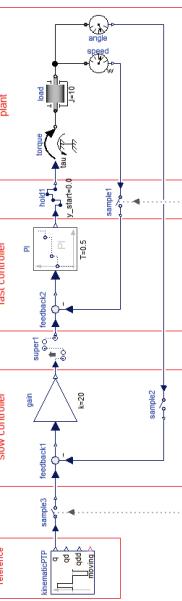
Generates an event clock: The clock ticks whenever the continuous-time Boolean input changes from false to true.

```
y = Clock(t);
```

## Sub- and Super-Sampling

Defines that the output signal is an integer factor faster than the input signal, using a "hold" semantics for the signal. By default, this factor is inferred. It can also be defined explicitly.

```
y = superSample(t);
```



Defines that the output signal is an integer factor slower than the input signal, picking every n-th value of the input.

```
y = subSample(t, factor);
```

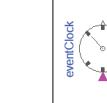


## Slide 13

Generates a periodic event clock

```
Parameter Modelica.SIunits.Time period;
equation
y = eventClock();

```

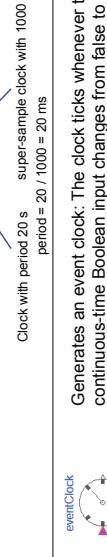


## Slide 14

Generates a periodic event clock with a Real period

```
Parameter Modelica.SIunits.Time period;
equation
y = eventClock(period);

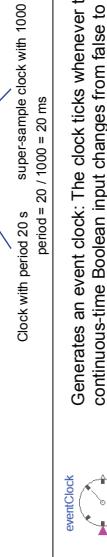
```



## Slide 15

Purely algebraic block from Modelica.Blocks.Math

```
y = sample(t, clock);
```

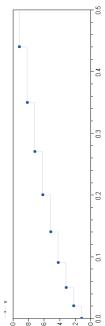


## Slide 16

## Varying Interval Clocks

- The first argument of `Clock(ticks, resolution)` may be time dependent
- Resolution must not be time dependent
- Allowing varying interval clocks
- Can be sub and super sampled and phased

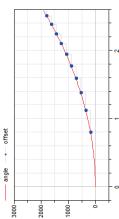
```
model VaryingClock
 Integer nextInterval;
 Clock c = Clock(nextInterval, 100);
 Real visitor=0.2;
 equation
 when c then
 nextInterval = previous(nextInterval) + 1;
 v = previous(v)+1;
 end when;
 end VaryingClock;
```



## Boolean Clocks

- Possible to define clocks that tick when a Boolean expression changes from false to true.
- Assume that a clock shall tick whenever the shaft of a drive train passes 180°.

```
model BooleanClock
 Modelica.Slunits.Angle angle(start=0,fixed=true);
 Modelica.Slunits.Angle(velocity) wstart=0,fixed=true);
 Modelica.Slunits.Torque tau=10;
 parameter Modelica.Slunits.Inertia J=1;
 Modelica.Slunits.Angle offset;
 equation
 w = der(angle);
 J*der(w)=tau;
 when Clock(angle >= hold(offset)-Modelica.Constants.pi) then
 offset = sample(angle);
 end when;
 end BooleanClock;
```



## Discretized Continuous Time

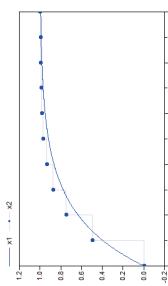
- Possible to convert continuous-time partitions to discrete-time
- A powerful feature since in many cases it is no longer necessary to manually implement discrete-time components
- Build-up a inverse plant model or controller with continuous-time components and then sample the input signals and hold the output signals.
- And associate a solverMethod with the Clock.



Slide 17

### Discretized

```
model Discretized
 Real x1(start=0,fixed=true);
 Real x2(start=0,fixed=true);
 equation
 der(x1) = x1 + 1;
 der(x2) = -x2 + sample(1,Clock(Clock(0.5),solverMethod=d=ExplicitEuler));
```



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## State Machines

- Modelica extended to allow modeling of control systems
- Any block without continuous-time equations or algorithms can be a state of a state machine.
- Transitions between such blocks are represented by a new kind of connections associated with transition conditions.
- The complete semantics is described using only 13 Modelica equations.
- A cluster of block instances at the same hierarchical level which are coupled by transition equations constitutes a state machine.
- All parts of a state machine must have the same clock. (*We will work on removing this restriction, allowing mixing clocks and allowing continuous equations, in future Modelica versions.*)
- One and only one instance in each state machine must be marked as initial by appearing in an `initialState` equation.



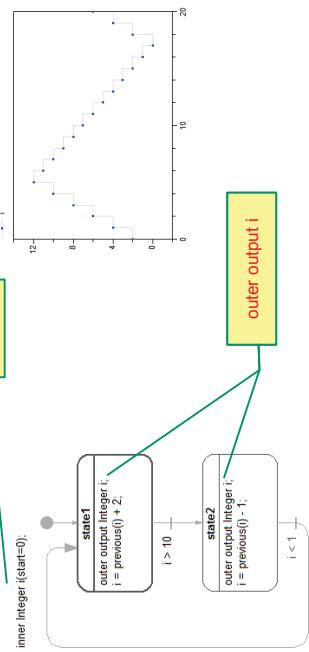
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Slide 20



## A Simple State Machine

## A Simple State Machine – Modelica Text Representation

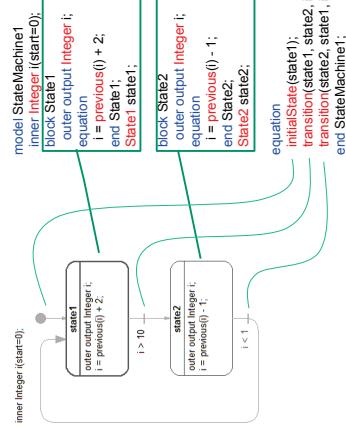


## Merging Variable Definitions

- An **outer output** declaration means that the equations have access to the corresponding variable declared **inner**.
- Needed to maintain the **single assignment** rule.
- Multiple definitions** of such outer variables in different mutually exclusive states of one state machine need to be **merged**.
  - In each state, the outer output variables ( $y$ ) are solved for ( $\text{expr}$ ) and, for each such variable, a single definition is automatically formed:
  - $v := \text{if activeState(state}_1\text{)} \text{ then } \text{expr}_1 \text{ elseif activeState(state}_2\text{)} \text{ then } \text{expr}_2 \text{ else last}(y)$
- last()** is a special internal semantic operator returning its input. It is just used to mark for the sorting that the incidence of its argument should be ignored.
- A start value must be given to the variable if not assigned in the initial state.
- Such a newly created assignment equation might be merged on higher levels in nested state machines.

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K n o w l e d g e



## Defining a State machine

**transition**(*from*, *to*, *condition*, *immediate*, *reset*, *synchronize*, *priority*)

- This operator defines a transition from instance "from" to instance "to". The "from" and "to" instances become states of a state machine.
- The transition fires when condition = true if immediate = true (this is called an "immediate transition") or previous(condition) when immediate = false (this is called a "delayed transition").
- If reset = true, the states of the target state are reinitialized, i.e. state machines are restarted in initial state and state variables are reset to their start values.
- If synchronize = true, the transition is disabled until all state machines within the from-state have reached the final states, i.e. states without outgoing transitions.
- "from" and "to" are block instances and "condition" is a Boolean expression.
- "immediate", "reset", and "synchronize" (optional) are of type Boolean, have parametric variability and a default of true, true, false respectively.
- "priority" (optional) is of type Integer, has parametric variability and a default of 1 (highest priority). Defines the priority of firing when several transitions could fire.

## InitialState(state)

- The argument "state" is the block instance that is defined to be the initial state of a state machine.

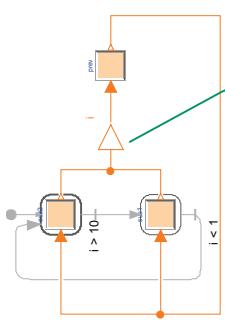
Slide 22

K n o w l e d g e

Slide 24

## Conditional Data Flows

- Alternative to using **outer output** variables is to use conditional data flows.



```

block Increment
 extends Modelica.Blocks.Interfaces.PartialIntegerSISO;
 parameter Integer increment;
 equation
 y = u + increment;
 end Increment;

block Prev
 extends Modelica.Blocks.Interfaces.PartialIntegerSISO;
 equation
 y = previous(u);
 end Prev;

```

protected connector (node) i

## Merge of Conditional Data Flows

- It is possible to **connect several outputs to inputs** if all the outputs come from states of the same state machine.

$$u_1 = u_2 = \dots = y_1 = y_2 = \dots$$

with  $u_i$  inputs and  $y$  outputs.

- Let variable  $v$  represent the signal flow and rewrite the equation above as a set of equations for  $u_i$  and a set of assignment equations for  $v$ :

```

v := if activeState(state1) then y1 else last(v);
 ...
u1 = v
u2 = v
...

```

- The **merge** of the definitions of  $v$  is then made as described previously:

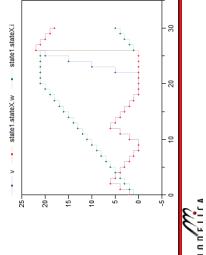
```

v = if activeState(state1) then y1
 elseif activeState(state2) then y2
 elseif ... else last(v)
 ...

```

## Hierarchical State Machine Example

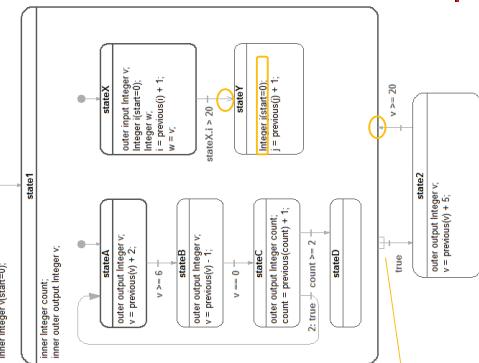
- stateA declares  $v$  as '**outer output**'.
- state 1 is on an intermediate level and declares  $v$  as '**inner outer output**', i.e. matches lower level outer  $v$  by being inner and also matches higher level inner  $v$  by being outer.
- The top level declares  $v$  as **inner** and gives the start value.



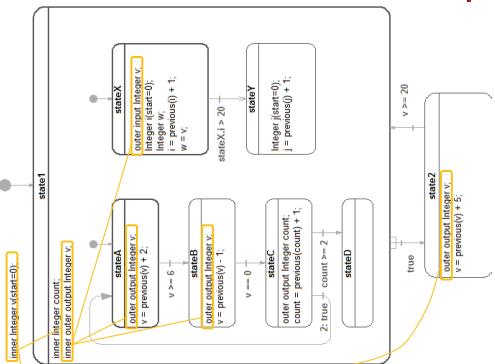
## Reset and Synchronize

- count is defined with a start value in state 1. It is **reset** when a transition ( $v >= 20$ ) is made to state 1.
- stateY declares a local counter, j.
- It is **reset** at start and as a consequence of the reset transition ( $v >= 20$ ) from state2 to state 1.
- The **reset** of j is **deferred until** stateY is entered by transition (stateY,  $i > 20$ ) although this transition is not a reset transition. **Synchronizing** the exit from the two parallel state machines of state1 is done by using a synchronized transition.

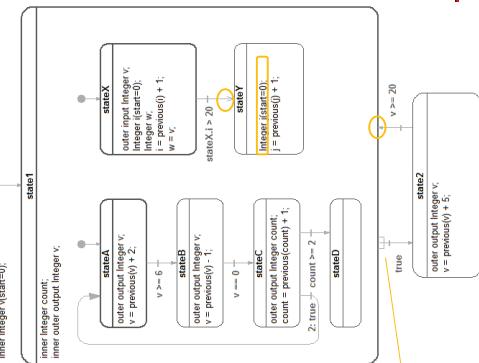
## Slide 26



## Slide 25

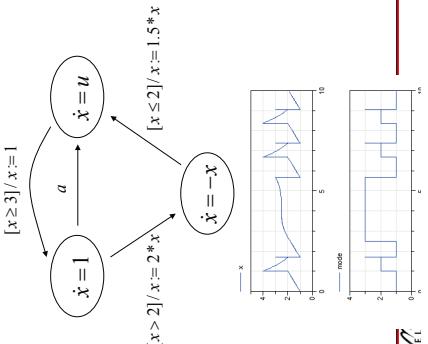


## Slide 26



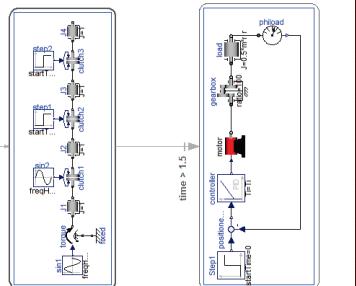
## Summary

## Hybrid Automata (Modelica 3.2-, 2006)

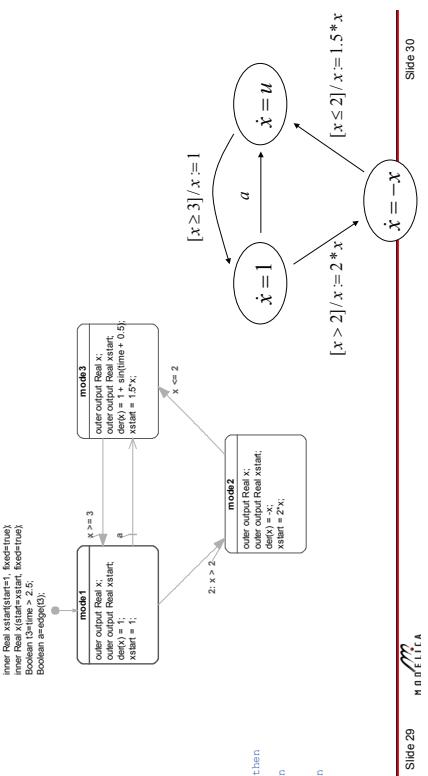


## Acausal Models in States – Modelica 3.3+

- The equations of each state is guarded by the activity condition
- Should time variable be stopped when not active?
- Should time be reset locally in state by a reset transition?
- Special Boolean operator exception() to detect a problem in one model and transition to another model

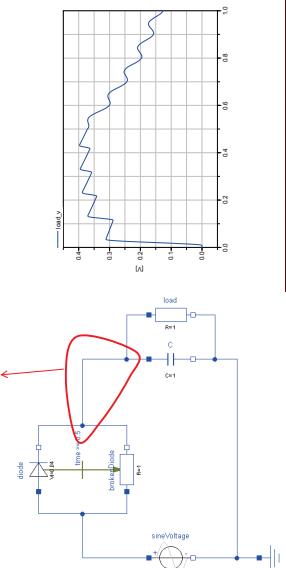


## Hybrid Automata with Modelica 3.3+ (prototype)



## Multiple Acausal Connections

- `// C_p_l+brokenDiode_n_i+diode_n_i+load_p_j = 0;`
- Replaced by:
- `C_p_l+  
(if activateState(brokenDiode)  
then brokenDiode_n_i else 0)+  
diode_n_i +  
load_p_j = 0;`



## Side 30

## Side 31

## Side 32

## Conclusions

- We have introduced synchronous features in Modelica 3.3.
  - For a discrete-time variable, its clock is associated with the variable type and inferencing is supported.
  - Special operators have to be used to convert between clocks.
  - This gives an additional safety since correct synchronization is guaranteed by the compiler.
- We have described how state machines can be modeled in Modelica 3.3.
  - Instances of blocks connected by transitions with one such block marked as an initial state constitute a state machine.
  - Hierarchical state machines can be defined with reset or resume semantics,
  - when re-entering a previously executed state.
  - Parallel sub-state machines can be synchronized when they reached their final states.
  - Special merge semantics have been defined for multiple outer output definitions in mutually exclusive states as well as conditional data flows.



**EXTENSIBLE PROGRAMMING AND MODELING  
LANGUAGES**  
**Eric Van Wyk, University of Minnesota**

Extensible programming and modeling languages allow their users to import new features into their language. These may be new syntax (notations), new semantics (e.g. analysis for additional error checking), new optimizations, and new translations packaged as language extensions. Ideally, programmers and engineers with no knowledge of language design or implementation can direct tools to compose a "host" language with their chosen set of language extensions resulting in a custom translator or compiler for their extended language. To achieve this goal, languages and extensions are specified declaratively using context free grammars and attribute grammars. We describe a set of tools for generating translators and compilers from these specifications and a set of analyses that language extensions designers can use to verify that the composition of their extension and other similarly verified, independently developed, extensions will work as desired with the host language. These analyses ensure that the generated LR parser will be deterministic with no conflicts and that the attribute grammar will be complete, that is, has equations defining all needed attributes. Thus, the user is assured that their chosen language extensions will all work together. Example extensions to Java, C, Lustre, and Modelica will be discussed.

## Extensible Programming and Modeling Languages

Ted Kaminski, Yogesh Malai, August Schwendfeger  
and Eric Van Wyk

University of Minnesota

September 20, 2012, Lund, Sweden

### Extensible Language Frameworks — ableP

- ▶ add features to a "host" language — Promela
- ▶ new language constructs - their syntax and semantics
  - ▶ `select (altitude: 1000 .. 10000);`
  - ▶ `select (altitude: 1000 .. 10000 step 100);`
  - ▶ `select (altQuality: High, Med, Low);`
  - ▶ DTSPin constructs: `timer t; t = 1; expire(t);`
- ▶ new semantics of existing constructs
  - ▶ semantic analysis, translations to new target languages, ...
    - ▶ type checking
    - ▶ advanced ETCH-style type inference and checking
- ▶ Languages are not monolithic.
- ▶ But most language tools primarily support monolithic design and implementation.

### Various means for extending Promela

- ▶ `select (v: 1 .. 10) added in SPIN version 6.`
- ▶ DTSPIN features
  - ▶ as CPP macros — lightweight
  - ▶ or modifying the SPIN implementation — heavyweight
- ▶ ETCH, enhanced type checking
  - ▶ built their own scanner and parser using SableCC
  - ▶ ableP — middleweight approach

## An example

An altitude switch model that uses

- ▶ enhanced `select` statements
- ▶ DTSPIN-like constructs
- ▶ tabular Boolean expressions (à la RSM<sub>L</sub> and SCR)

An instance of ableP parses and analyzes the model, then generates its translation to pure Promela.

```
% java -jar ableP-aviation.jar AltSwitch.xml
% spin -a AltSwitch.pml
```

## Extending ableP with independently developed extensions

### Our approach:

- ▶ Users choose (independently developed) extensions.
- ▶ Tools compose the extensions and Promela host language.
- ▶ Distinguish
  - 1. extension user
    - ▶ has no knowledge of language design or implementations
  - 2. extension developer
    - ▶ must know about language design and implementation
- 1. Tools and formalisms support automatic composition.
- 2. Modular analyses ensure the composition results in a working translator.
- ▶ Value easy composition over expressivity, accept some restrictions
  - ▶ on syntax
  - ▶ new constructs are translated to "pure" Promela
  - ▶ ableP "instances" are smart pre-processors

### Developing language extensions

Two primary challenges:

1. composable syntax — enables building a scanner and parser
  - ▶ context-aware scanning [GPCE07]
  - ▶ modular determinism analysis [PLDI09]
  - ▶ Copper
2. composable semantics — analysis and translations
  - ▶ attribute grammars with forwarding, collections and higher-order attributes
  - ▶ set union of specification components
    - ▶ sets of productions, non-terminals, attributes
    - ▶ sets of attribute defining equations, on a production
    - ▶ sets of equations contributing values to a single attribute
  - ▶ modular well-definedness analysis [SLE12]
  - ▶ monolithic termination analysis [SLE12]
  - ▶ Silver

## Context aware scanning

## Allows parsing of embedded C code

- ▶ Scanner recognizes only tokens valid for current "context"
- ▶ keeps embedded sub-languages, in a sense, separate
- ▶ Consider:
  - ▶ `char in, out;`
  - ▶ `for i in a { a[i] = i*i ; }`
  - ▶ Two terminal symbols that match "`\n`".
    - ▶ terminal `\n`',
    - ▶ terminal `\t`',
    - ▶ terminal ID `/[a-zA-Z]/[a-zA-Z_0-9]*`/ submits to `{promela.kwd}`;
    - ▶ terminal `FOR` 'for' lexer classes `{promela.kwd}`;

## Semantics for host language assignment constructs

```
grammar edu:umn:cs:melt:ableP:host:core:abstractsyntax;

abstract production defaultAssign
s::Smt ::= Lhs::Expr rhs::Expr
{ s.pp = Lhs.pp ++ " ++ rhs.pp ++ " ;\n" ;
 Lhs.env = s.env; rhs.env = s.env;
 s.defs = emptyDefs();}

s.errors ::= Lhs.errors ++ rhs.errors ;
}
```

Adding extension constructs involves writing similar productions.

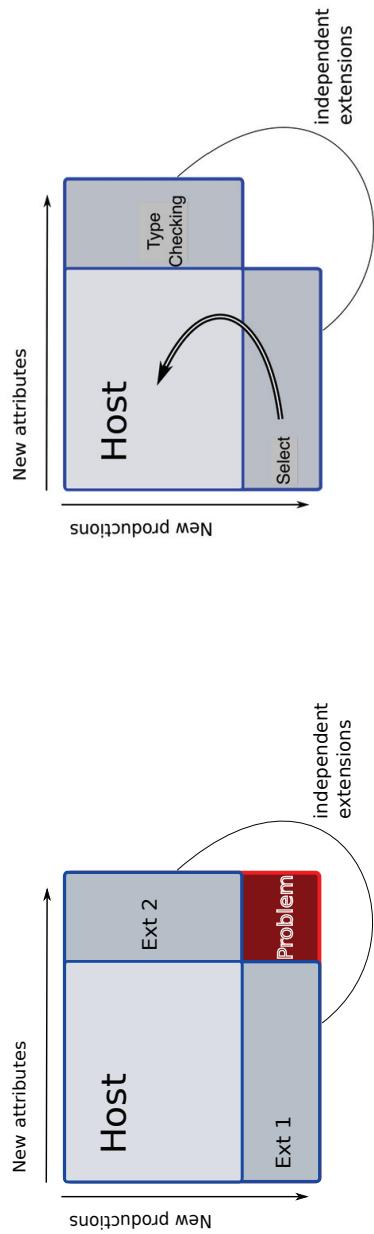
## Adding ETCH-like semantic analysis.

```
grammar edu:umn:cs:melt:ableP:extensions:typeChecking ;

synthesized attribute typeRep : TypeRep
occurs on Expr, Decl;
aspect production varRef
e::Expr ::= id::ID
{ e.typeRep = ... retrieve from declaration
 found in e.env ... ; }

aspect production defaultAssign
s::Smt ::= Lhs::Expr rhs::Expr
{ s.errors <- if isCompatible(Lhs.typeRep, rhs.typeRep)
 then [] else [mkError ("Incompatible types ..."];
```

Extensibility: safe composability



Extensibility: safe composability

Extensions get undefined semantics from host translation.

```
grammar edu:umm:cs:melt:ableP:extensions:enhancedSelect ;
abstract production selectFrom
s::Stmt ::= s1::'select' v:Expr es::Exprs
{
 s.pp = "select (" ++ v.pp ++ ":" ++ es.pp ++ "); \n" ;
 s.errors := v.errors ++ es.errors ++
 if ... check that all expressions in 'es' have
 same type as 'v', ...
 then [mkError ("Error: select statement " ++
 "requires same type ... ")]
 else [] ;
 forwards to ifStmt(mkOptions (v, es)) ;
}
```

Ensuring that the composition will be successful.

Modular analysis

## Context free grammars

## Attribute grammars

$$G_H \cup G_E^1 \cup G_E^2 \cup \dots \cup G_E^i$$

►  $\cup$  of sets of nonterminals, terminals, productions

► Composition of all is an context free grammar.

► Is it non-ambiguous, useful for deterministic (LR) parsing?

►  $conflictFree(G_H \cup G_E^1)$  holds

►  $conflictFree(G_H \cup G_E^2)$  holds

►  $conflictFree(G_H \cup G_E^1)$  holds

►  $conflictFree(G_H \cup G_E^1 \cup G_E^2 \cup \dots \cup G_E^i)$  may not hold

$$AG_H \cup AG_E^1 \cup AG_E^2 \cup \dots \cup AG_E^i$$

►  $\cup$  of sets of attributes, attribute equations, occurs-on declarations

► Composition of all is an attribute grammar.

► Completeness:  $\forall$  production,  $\forall$  attribute,  $\exists$  an equation

►  $complete(AG_H \cup AG_E^1)$  holds

►  $complete(AG_H \cup AG_E^2)$  holds

►  $complete(AG_H \cup AG_E^i)$  holds

►  $complete(AG_H \cup AG_E^1 \cup AG_E^2 \cup \dots \cup AG_E^i)$  may not hold

► similarly for non-circularity of the AG

## Detecting problems, ensuring composition

When can some analysis of the language specification be applied?

When ...

1. the host language is developed ?

2. a language extensions is developed ?

3. when the host and extensions are composed ?

4. when the resulting language tools are run ?

## Libraries, and modular type checking

► Libraries "just work"

► Type checking is done by the library writer, modularly.

► Language extensions should be like libraries, composition of "verified" extensions should "just work."

## Modular determinism analysis for grammars, 2009

## Modular completeness analysis for attribute grammars



- ▶  $\text{isComposable}(G_H, G_E^1) \wedge \text{conflictFree}(G_H \cup G_E^1)$  holds
- ▶  $\text{isComposable}(G_H, G_E^2) \wedge \text{conflictFree}(G_H \cup G_E^2)$  holds
- ▶  $\text{isComposable}(G_H, G_E^i) \wedge \text{conflictFree}(G_H \cup G_E^i)$  holds
- ▶  $\text{these imply } \text{conflictFree}(G_H \cup G_E^1 \cup G_E^2 \cup \dots)$  holds
- ▶  $(\forall i \in [1, n], \text{isComposable}(G_H, G_E^i) \wedge \text{conflictFree}(G_H \cup \{G_E^i\})) \implies \text{conflictFree}(G_H \cup \{G_E^1, \dots, G_E^n\})$
- ▶ Some restrictions to extension introduced syntax apply, of course.



- ▶  $\text{isComposable}(G_H, G_E^1) \wedge \text{conflictFree}(G_H \cup G_E^1)$  holds
- ▶  $\text{modComplete}(AG_H \cup AG_E^1)$  holds
- ▶  $\text{modComplete}(AG_H \cup AG_E^2)$  holds
- ▶  $\text{modComplete}(AG_H \cup AG_E^i)$  holds
- ▶  $\text{these imply complete}(AG_H \cup AG_E^1 \cup AG_E^2 \cup \dots)$  holds
- ▶  $(\forall i \in [1, n], \text{modComplete}(AG_H, AG_E^i)) \implies \text{complete}(AG_H \cup \{AG_E^1, \dots, AG_E^n\})$ .
- ▶ similarly for non-circularity of the AG
- ▶ Again, some restrictions on extensions.

So ...

- ▶ ableP supports the simple composition of language extensions
- ▶ This creates translators and analyzers for customized Promela-based languages.
- ▶ extensions can be verified to (syntactically) compose, with other verified extensions — done by extension developers
  - ▶ adding (independently developed) extensions that add new features and new analysis on host features is supported
- ▶ Challenge: SPIN verification still occurs on the generated pure Promela specification.
- ▶ Future work
  - ▶ More extensions: multi-dimensional array, unit/dimension analysis, ...
  - ▶ Improve type analysis
  - ▶ Semantic analysis of embedded C code?

Thanks for your attention.

Questions?

<http://melt.cs.umn.edu/~evv@cs.umn.edu>

**EXTENSIBLE COMPILER ARCHITECTURE – EXAMPLES****FROM JMODELICA.ORG****Görel Hedin, Dept of Computer Science, Lund University,  
Sweden**

The JModelica.org platform is built around an extensible compiler, implemented in reference attribute grammars (RAGs) using the JastAdd metacompiler. In this talk, I will give an overview of how extensible compiler architectures can be built using JastAdd and RAGs. Examples from the JModelica.org platform will be used for illustration



## Extensible Compiler Architecture Examples from JModelica.org

### Uses of the **JastAdd** systems

- JastAdd: an open source metacompiler for generating extensible compilers
- Object-orientation (Java as host language)
- Aspect-oriented programming / Open classes
- Attribute Grammars [Knuth 1968]
- Higher-order attributes [Vogt et al. 1989]
- Reference attributes [Hedin 2000]
- Context-dependent transformations [Ekmekci and Hedin 2004]
- ...

Görel Hedin  
Computer Science, Lund University



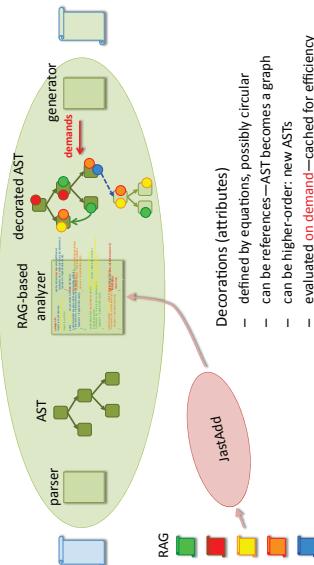
LCCC workshop, Lund, Sept 20, 2012

## Background

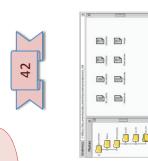
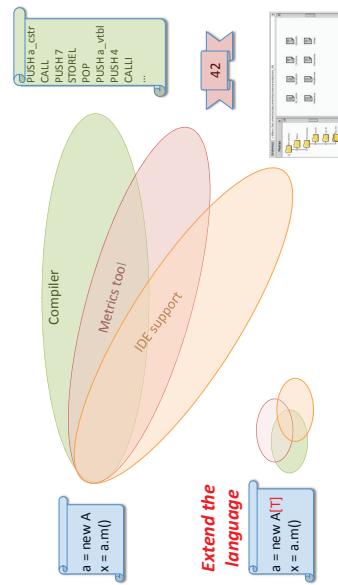


## Why extensible compilers?

### Modularizing the compiler using lastAdd

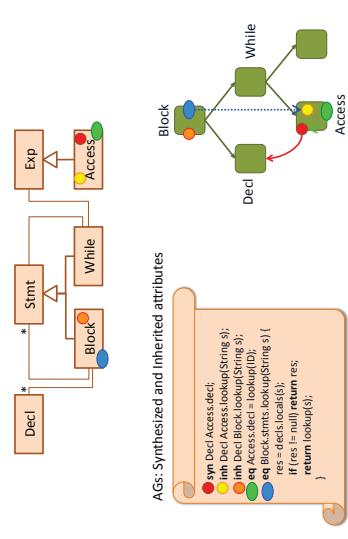
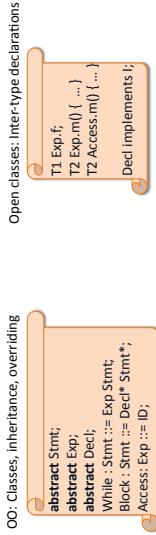


### Extend the language

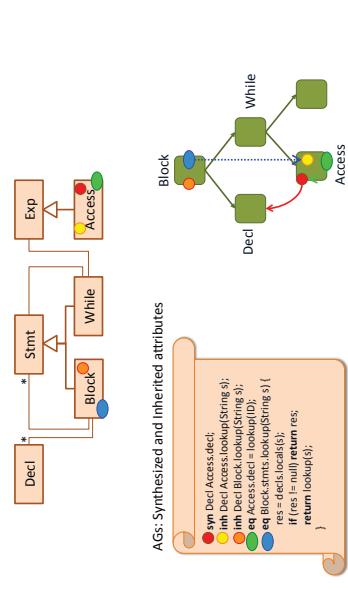


- Decorations (attributes)
  - defined by equations, possibly circular
  - can be references—AST becomes a graph
  - can be higher-order: new ASTs
  - evaluated on **demand**—cached for efficiency
  - arbitrary modularization of individual definitions

## JustAdd programming mechanisms

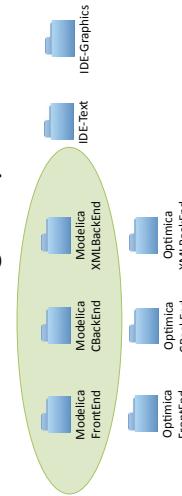


## JustAdd programming mechanisms

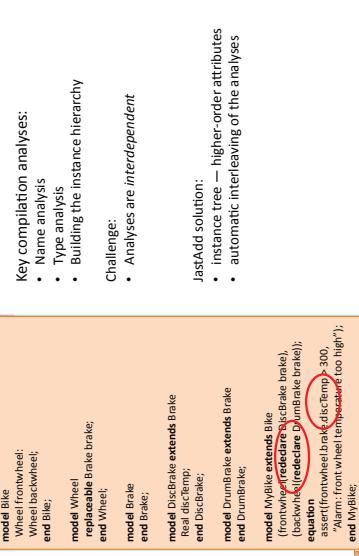


5

## JModelica.org components



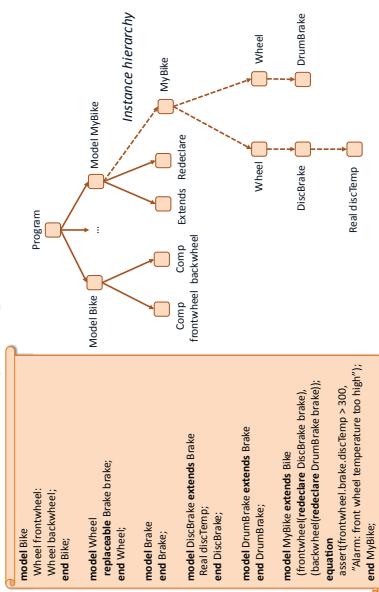
## Compiling Modelica



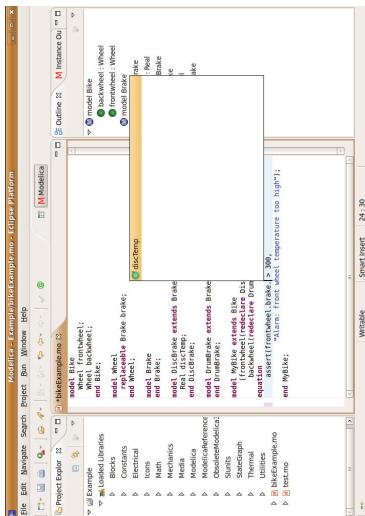
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## Compiling Modelica

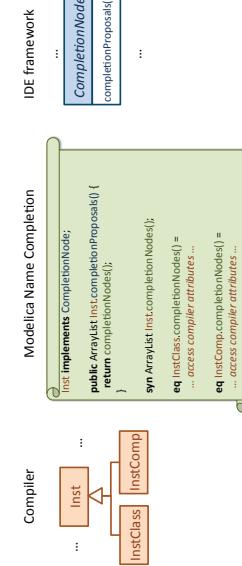


## IDE name completion

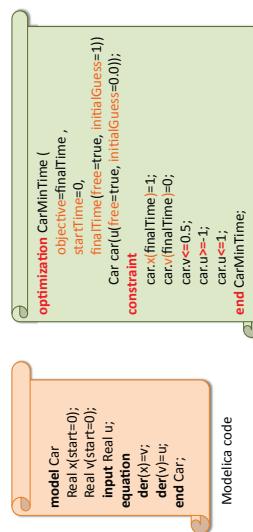


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## Extending the compiler with name completion



## Optimica: an extended language



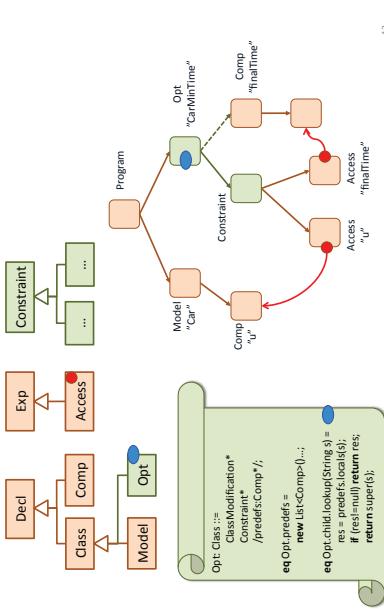
Optimica code  
- extends Modelica with new syntax  
- and changed semantics

11

12

## Extending Modelica to Optimica

### Ongoing and future work



13

- Incremental updating
- General IDE support
- Graphical editing
- Performance
- Higher-level specification

14

## Conclusions

JModelica.org, a great case for JustAdd!

For more information, see [jastadd.org](http://jastadd.org)

Thank you!

Questions?

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## CONSTRAINT SATISFACTION METHODS IN EMBEDDED SYSTEM DESIGN

**Krzysztof Kuchcinski, Dept. of Computer Science, Lund University**

Constraints can be used to define embedded systems parameters, requirements and specific design problem restrictions. They can be further formalized using Constraint Programming (CP) models. These models represent instances of Constraint Satisfaction Problem (CSP) and can be solved using CP solvers. CP is relatively young area that gains attention because of its flexibility to define different problems and possibility of using both complete and heuristic methods for their solving. Moreover, CP offers global constraint, such as scheduling constraints, that implement specific algorithms for efficient handling of a given class of problems. This provides an easy way to use several advanced algorithms in one problem that is difficult or time consuming in pure heuristic solutions. In this talk, we will concentrate on finite domain constraints and the related constraint programming framework. We will illustrate it with classical examples from embedded systems, such as scheduling, design mapping, register and memory allocation. Bandwidth auctions and their parallels to power.



## Outline

### Constraint satisfaction methods in embedded system design

Krzysztof Kuchcinski  
Dept. of Computer Science,  
Lund University, Sweden

- 1 Motivation an Example
- 2 CP Basics
- 3 Advanced Example- Sub-graph Isomorphism
- 4 Summary and Conclusions



## Outline

### Why constraints?

- Examples of combinatorial optimization problems in embedded systems
  - Scheduling, allocation and assignment,
  - Partitioning,
  - Memory and register assignment,
  - Instruction selection.
- Different constraints:
  - timing,
  - resource,
  - power consumption, etc.
- Constraint programming offers a *unified* approach to model and solve problems with *heterogeneous* constraints.

## Scheduling example



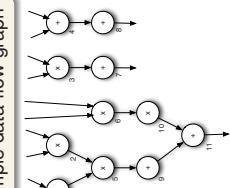
Simple data-flow graph



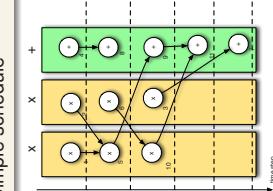
## Scheduling example



Simple data-flow graph



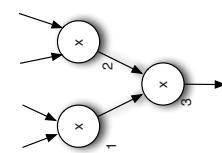
Simple schedule



Krystof Kuchcinski

LCCC workshop 2012

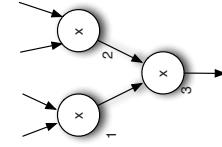
## Scheduling Constraints



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## Scheduling Constraints



### Variables

operation start  
 $t_1 :: \{0..10\}$ ,  $t_2 :: \{0..10\}$ ,  $t_3 :: \{0..10\}$

Assigned resource  
 $r_1 :: \{1..2\}$ ,  $r_2 :: \{1..2\}$ ,  $r_3 :: \{1..2\}$



## Scheduling Constraints

Variables  
 operation start  
 $t_1 :: \{0..10\}$ ,  $t_2 :: \{0..10\}$ ,  $t_3 :: \{0..10\}$   
 Assigned resource  
 $r_1 :: \{1..2\}$ ,  $r_2 :: \{1..2\}$ ,  $r_3 :: \{1..2\}$

Krystof Kuchcinski

6(21)





## Scheduling Constraints

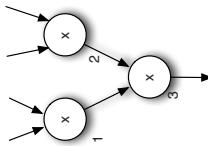
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### Global Constraints

#### Variables

Operation start  
 $t_1 :: \{0..10\}$ ,  $t_2 :: \{0..10\}$ ,  $t_3 :: \{0..10\}$

Assigned resource  
 $r_1 :: \{1..2\}$ ,  $r_2 :: \{1..2\}$ ,  $r_3 :: \{1..2\}$



#### Constraints

Precedence constraints

$$t_1 + d_1 \leq t_2 \wedge$$

$$t_2 + d_2 \leq t_3 \wedge$$

Resource constraints

$$(t_1 + d_1 \leq t_2 \vee t_2 + d_2 \leq t_1) \vee r_1 \neq r_2$$

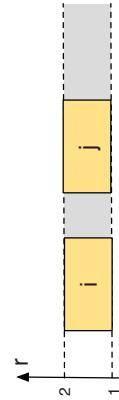
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7(2)

## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee r_i \neq r_j$



Diff2 constraint (non-overlapping rectangles)

Krzysztof Kucharski

7(2)



### Global Constraints

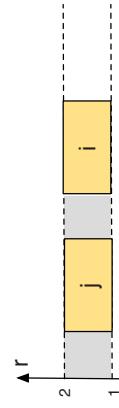
#### Variables

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee r_i \neq r_j$

7(2)

## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee r_i \neq r_j$



Diff2 constraint (non-overlapping rectangles)

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7(2)

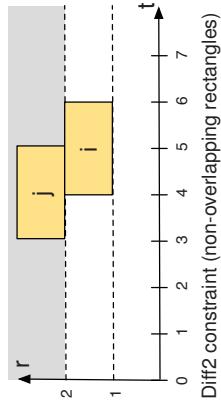
## Global Constraints

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## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee t_i \neq t_j$



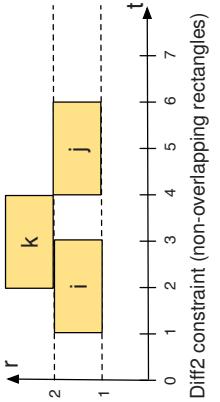
Diff2 constraint (non-overlapping rectangles)

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## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee t_i \neq t_j$



Diff2 constraint (non-overlapping rectangles)

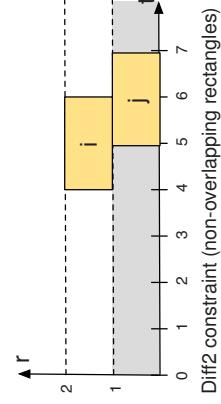
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## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee t_i \neq t_j$

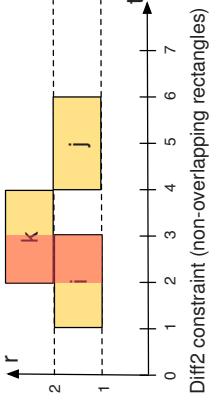


Diff2 constraint (non-overlapping rectangles)



## Global Constraints

$\forall i, j \text{ where } i < j : t_i + d_i \leq t_j \vee t_j + d_j \leq t_i \vee t_i \neq t_j$



Diff2 constraint (non-overlapping rectangles)



Krzysztof Kuchcinski

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## Final Model

## Model Advantages

```
array[1..n] of var 0..100 : t;
array[1..n] of var 1..2 : r;

% precedence constraints
constraint
 t[1] + 2 <= t[6] / \ t[2] + 2 =< t[6] / \ t[3] + 2 <= t[7] / \
 t[4] + 2 <= t[8] / \ t[5] + 1 =< t[9] / \ t[6] + 2 <= t[10] / \
 t[7] + 2 <= t[11] / \ t[10] + 1 =< t[11];

% resource constraints for adders
diff2([(t[5],r[5],1,1), (t[8],r[8],1,1)], [(t[9],r[9],1,1)],
 [(t[10],r[10],1,1), (t[11],r[11],1,1)]);
\

% resource constraints for multipliers
diff2([(t[1],r[1],2,1), (t[2],r[2],2,1), [t[3],r[3],2,1],
 [t[4],r[4],2,1], (t[6],r[6],2,1), [t[7],r[7],2,1]];
```



## Model Advantages

- Separation of a model and solving method
- Time-constrained and resource-constrained scheduling
- Easy to add new constraints
- Non-linear constraints
- Combination of consistency algorithms (e.g., diff2 and cumulative constraints)
- Standard and heuristic methods for solving the model

## Outline

### 1 Motivation an Example

### 2 CP Basics

### 3 Advanced Example- Sub-graph Isomorphism

### 4 Summary and Conclusions

## CP basics

- Finite domain variables, e.g.,  $t :: 0..10$
- Constraints; defined by their consistency methods (propagators)
- Primitive constraints
  - $a + b < c, x \cdot y = z, A \cup B = C$ , etc.
  - bounds and domain consistency
- Global constraints
  - diff2, alldifferent, etc.
  - can be decomposed to primitive constraints BJT
  - specialized algorithms from operation research, graph theory, computational geometry, etc. are more efficient

## Propagators

Propagator for  $x + y = z$  (bounds consistency)

```

x in {min(z) - max(y) .. max(z) - min(y)}
y in {min(z) - max(x) .. max(z) - min(x)}
z in {min(x) + min(y) .. max(x) + max(y)}

```

### Example

```

x :: {1..10}, y :: {1..10} and z :: {1..10}
yields
x :: {1..9}, y :: {1..9} and z :: {2..10}.

```

Krzysztof Kuchcinski

LCCC workshop 2012

12(21)

12(21)

## Global Constraints

- alldifferent, cumulative, table, etc.
- geometrical constraints: diff2, geost,
- combinatorial problems: binpacking, knapsack, network flow, etc.
- graph constraints: (sub-)graph isomorphism, clique, Hamiltonian path, simple path, connected components.

## Global Constraints

- alldifferent( $x_1 :: \{1..2\}, x_2 :: \{1..2\}, x_3 :: \{1..4\}$ )
- geometrical constraints: diff2,
- geometric problems:
- combinatorial problems:
- binpacking, knapsack, network flow, etc.
- graph constraints: (sub-)graph isomorphism, clique, Hamiltonian path, simple path, connected components.



12(21)

Krzysztof Kuchcinski

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Krzysztof Kuchcinski

13(21)

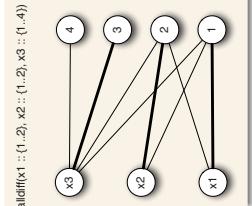
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# Global Constraints

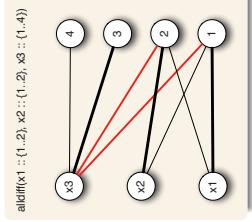
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## Global Constraints

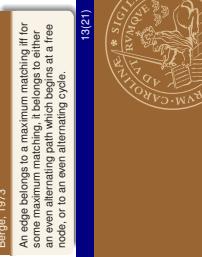
- alldifferent, cumulative, table, etc.
- geometrical constraints: diff2, geost,
- combinatorial problems: binpacking, knapsack, network flow, etc.
- graph constraints: (sub-)graph isomorphism, clique, Hamiltonian path, simple path, connected components.



alldiff(x1 :: {1..2}, x2 :: {1..2}, x3 :: {1..4})



alldiff(x1 :: {1..2}, x2 :: {1..2}, x3 :: {1..1})



Berge, 1973

An edge belongs to a maximum matching iff for some maximum matching, it belongs to either an even alternating path which begins at a free node, or to an even alternating cycle.

## Outline

13(21) Krzysztof Kucharski LCCC workshop 2012

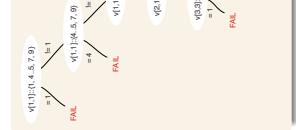


## Solving

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- Systematically assign values to variables and check if the problem is still consistent
- Implemented usually as depth-first-search
- Other methods can be used instead of assigning values, i.e., constraints on tasks ordering
- Heuristics can be incorporated



### 1 Motivation an Example

### 2 CP Basics

### 3 Advanced Example- Sub-graph Isomorphism

### 4 Summary and Conclusions

Krzysztof Kucharski

14(21)

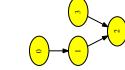
15(21)



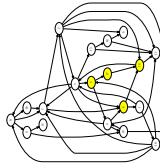
## Subgraph Isomorphism Constraint

### Definition (Subgraph isomorphism)

Target  $G_t = (N_t, E_t)$  and pattern  $G_p = (N_p, E_p)$  graphs are subgraph isomorphic iff there exist an injective function  $f : N_p \rightarrow N_t$  respecting  $(u, v) \in E_p \Leftrightarrow (f(u), f(v)) \in E_t$ .



pattern graph



target graph with matching

## Subgraph Isomorphism Constraint

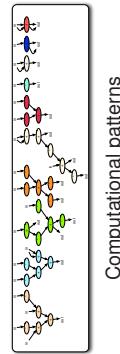
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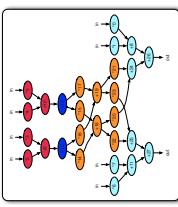
target graph with matching



## Instruction Identification and Selection



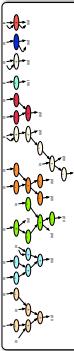
Computational patterns



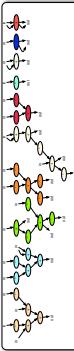
Covered data-flow graph

- Computational patterns - connected components of the graph

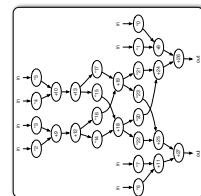
- Find sub-graph isomorphism that fulfills additional constraints (e.g., shortest schedule)



Computational patterns



Computational patterns



Data-flow graph

## Outline

### Our Solver



#### 1 Motivation an Example

#### 2 CP Basics

#### 3 Advanced Example- Sub-graph Isomorphism

#### 4 Summary and Conclusions



## Java Constraint Programming

- 1 constraint programming paradigm implemented in Java.
- 2 provides different type of constraints
  - 3 primitive constraints, such as arithmetical constraints ( $+$ ,  $*$ ,  $-$ ,  $/$ , mod, etc.), equality ( $=$ ) and inequalities ( $<$ ,  $>$ ,  $\leq$ ,  $\geq$ ,  $\neq$ ).
  - 4 logical, refined and conditional constraints
  - 5 global constraints.
- 6 set constraints, such as  $=$ ,  $\cup$ ,  $\cap$ .
  - 7 stochastic variables and constraints.
  - 8 High-level language, minimizing, interface
- 9 <http://www.jacop.eu>
- 10 <http://sourceforge.net/projects/jacop-solver/>

## Conclusions

- Easy way of modeling problems with heterogeneous constraints
- Easy to extend the problem with new constraints
- Can handle non-linear constraints
- Combination of different algorithms through global constraints
- Separation between modeling and solving
- Both complete and heuristic methods can be used for finding solutions





## DYNAMICAL MODELS FOR INDUSTRIAL CONTROLS: USE CASES AND CHALLENGES

**Fernando D'Amato, GE Global Research Center**

This presentation will introduce a few cases of industrial model-based controls in which model development has been critical for implementation success. Then, a brief description will be given of desired model properties for advanced controls, especially when dealing with formal optimization processes. Finally, a sample of the main challenges faced by modelling practices for industrial controls will be discussed.

# Dynamical models for industrial controls: use cases and challenges

**Fernando D'Amato**  
Principal Engineer, Controls, Electronics & Signal Processing  
General Electric Global Research

LCCC workshop: Systems Design Meets Equation-Based Languages



Lund, September 2012



## GE ... a heritage of innovation

- Founded in 1892
- 300,000 employees worldwide
- \$150 billion in annual revenues
- Only company in Dow Jones index originally listed in 1896



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## Outline

- Overview of controls at General Electric
- Train trip optimization example
- Power plant predictive control example
- From control system challenge to model challenge
- Conclusions



## GE today



## Aligned for growth



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## Expanding global presence in research

3000 technologists worldwide



## Controls at GE Research Labs



## Transportation: Optimal train control

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Optimize fuel utilization  
in every trip

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## Products with Controls



6

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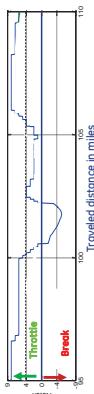
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# The Problem

Online calculation of optimal acceleration and braking for fuel efficiency



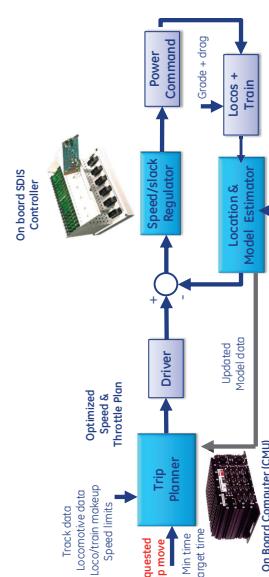
## Constraints

- Arrival timing
- Speed limits (mile per mile)
- Fuel reserves
- Maximum internal forces
- Train weight
- Track conditions
- Other trains operation



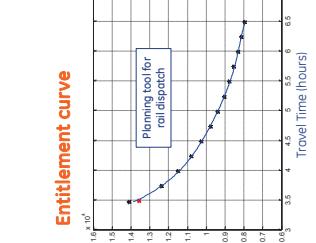
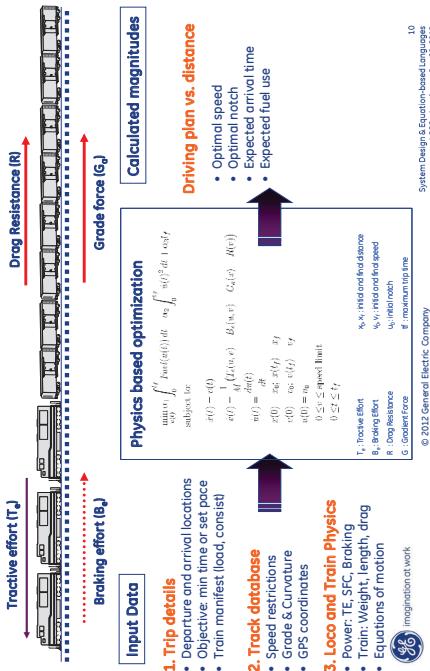
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## Implementation



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# Approach: Online optimal control

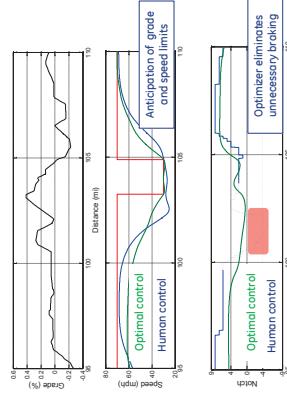


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## Results

### Improvements from optimal control



## Impact

- Runs on BNSF, CP, CSX, CN, Coal, grain & general merchandise
- 97 Subdivisions, 17000 Track Miles
- 10+ % system-wide average fuel savings, no velocity impact

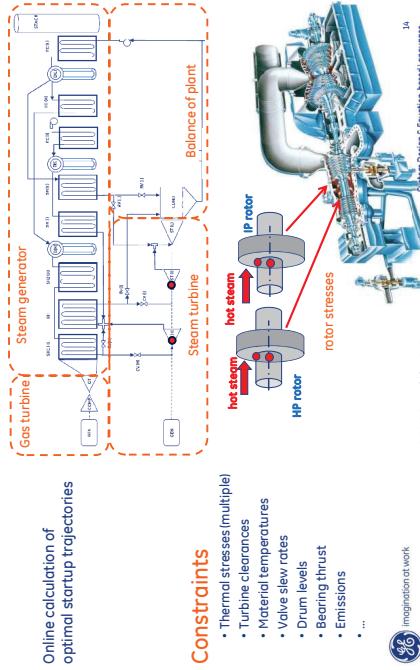
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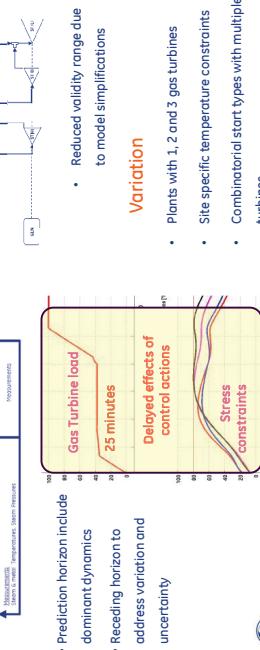
# Power Generation: Automated startup of combined cycle plants



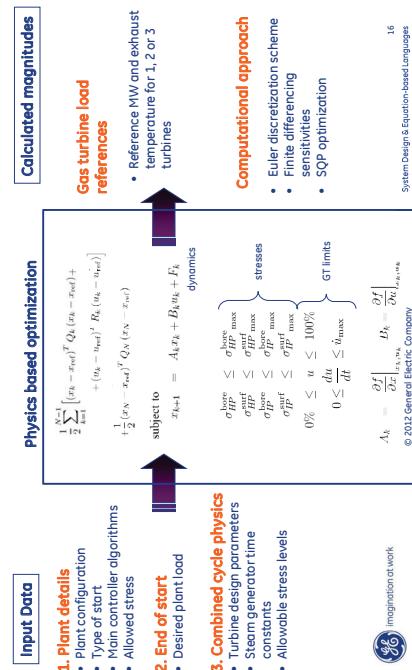
# The startup problem



# Approach: Model Predictive Control



# Approach: Optimization formulation



# Implementation

## Results

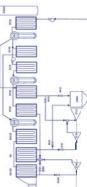


## Trends

### Advanced Model Based Controls, the answer?

- More detailed physical models
- Rely more on optimization

### Significant challenges ahead ...



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GE Power & Water



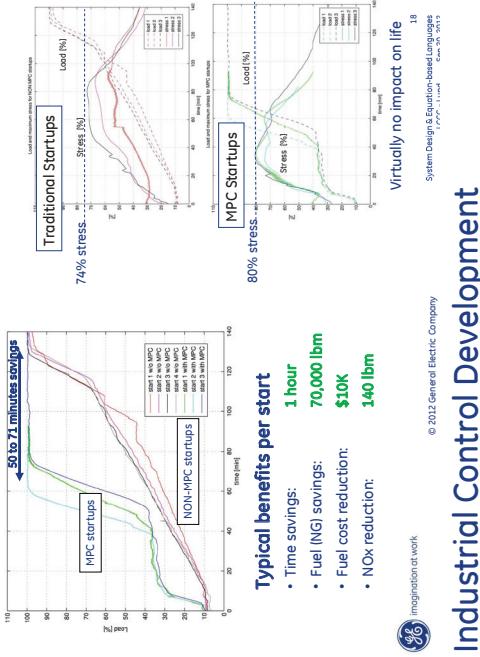
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PRESENTATIONS  
153



- Increasing performance demands
  - Competitiveness in market place
  - Increased operation flexibility
  - Transient efficiency
  - Environmental regulations

### Increasing performance demands

- Computing HW performance ↑
- Algorithms performance ↑
- Computing cost ↓



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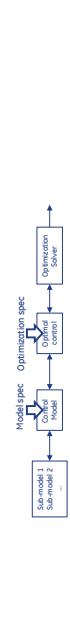
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PRESENTATIONS  
153

## How can modeling help? SV reliability

| Product                                                                                                           |                                                                                                                                |
|-------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <b>RTOS requirements</b>                                                                                          | <b>Modeling needs</b>                                                                                                          |
| WANT: Embed complex calculations<br>• Accurate models<br>• Online optimization process                            | Code requirements<br>Model requirements<br>Algorithm requirements                                                              |
| NEED: Aids to test embedded code quality<br>• SW infrastructure<br>• Rigorous coding practice<br>• Test as you go | Model Spec<br>Sub-module 1<br>Sub-module 2<br>Optimization Spec<br>Optimizer Solver<br>Optimizer Ref Model<br>Optimizer Corres |



### RTOS requirements

- Memory management
- Min math errors (i.e. MISRA compatible)
- Time consistency

| Product              |              |
|----------------------|--------------|
| Research             | Code         |
| Platform             | Model        |
| Hardware abstraction | Design data  |
| Model                | Parameter ID |



## How can modeling help? Product dev. need

| Product      |                                                |
|--------------|------------------------------------------------|
| Research     | Configuration tools based on requirements      |
| Code         | Integrated requisition tools with design dbase |
| Model        | Model tuning tools, i.e., parameter ID         |
| Parameter ID | Definition of system level test vectors        |

| Product                  |                                                |
|--------------------------|------------------------------------------------|
| Production requirements  | Modeling needs                                 |
| Ease for reconfiguration | Configuration tools based on requirements      |
| Fast requisition         | Integrated requisition tools with design dbase |
| Functional test          | Model tuning tools, i.e., parameter ID         |
|                          | Definition of system level test vectors        |
|                          | Testing plan, auto-testing tools               |



## How can modeling help? Function reliability & maintainability

| System                                        |                                                                                                                                                                                              |
|-----------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| WANT: Ensure physics is captured (physically) | • Trust model in pre-defined operating envelope                                                                                                                                              |
| NEED: Validation tools                        | <ul style="list-style-type: none"> <li>• Test every branch?</li> <li>• Model compatibility checks</li> <li>• Test vectors for every component &amp; system models</li> </ul>                 |
| Maintainability requirements                  | <p><b>Modeling needs</b></p> <ul style="list-style-type: none"> <li>Physical correctness</li> <li>Low complexity</li> <li>Error diagnostics and traceability</li> <li>Consistency</li> </ul> |
|                                               | <p>System Design &amp; Equation-based Languages<br/>© 2012 General Electric Company</p>                                                                                                      |

## Summary

| System                                              |                                                                                                                         |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| WANT: Ensure performance in industrial applications | <ul style="list-style-type: none"> <li>• Model Based Control to boost performance in industrial applications</li> </ul> |
| NEED: Tools for model reduction                     | <ul style="list-style-type: none"> <li>• MBC solutions are as good as models allow</li> </ul>                           |
| Maintainability needs                               | <ul style="list-style-type: none"> <li>• For MBC to be competitive, models need to</li> </ul>                           |
|                                                     | <ul style="list-style-type: none"> <li>• Reduce development cost &amp; time</li> </ul>                                  |

- Ensure maintainability
- Good modeling practices & tools are essential for viable products

|                                                                                  |
|----------------------------------------------------------------------------------|
| Need tools to accelerate transfer of academic solutions into industrial products |
|----------------------------------------------------------------------------------|



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**ORIGINS OF EQUATION-BASED MODELING LANGUAGES**  
**Karl Johan Åström, Department of Automatic Control,**  
**LTH, Lund University Lund, Sweden**

Modeling and simulation are indispensable tools for design and operation of complex engineered systems. Models are used in the design phase to select system architecture and configuration and for optimization of the design. Simulation is used to investigate dynamic behavior and to explore control architecture and control design. Simulation can be combined with real hardware in hardware-in-the-loop simulation for system testing. Models are also integral parts of feedback systems. Dynamic models are used during operation for control, dynamic optimization, supervision and fault diagnosis. They are also used in simulators for operator training. Models and simulation can also be used for decision support systems. Modeling is a rich field. It covers large parts of natural science and engineering. Statistics, design of experiments and parameter estimation are also essential ingredients. Numerical mathematics is important for simulation of a model and for optimization. Computer algebra is indispensable for safe transformation of models and for dealing with models of complex systems. Concepts and tools from computer science and software engineering are necessary to deal with large systems. Development of modeling and simulation worked hand in hand with emergence of computing starting with analog computers. The paper presents some of the ideas that lead to equation-based languages like Modelica.



## Modeling is Important

### Origins of Equation-Based Modeling

There will be growth in areas of simulation and modeling around the creation of new engineering “structures”. Computer-based design-build engineering ... will become the norm for most product designs, accelerating the creation of complex structures for which multiple subsystems combine to form a final product.

Karl Johan Åström  
Department of Automatic Control LTH  
Lund University



*NAE The Engineer of 2020*

Origins of Equation-based Modeling LCCC Sept 2012

## Vannevar Bush 1927

Engineering can progress no faster than the mathematical analysis on which it is based. Formal mathematics is frequently inadequate for numerous problems, a mechanical solution offers the most promise.



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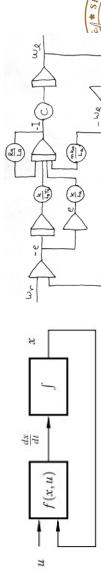
1. Introduction
2. Block diagram modeling
3. Equation-based modeling
4. Summary



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## Analog Computing

- Use a feedback loop to solve ODEs
- Integrators and function generation
- Linear systems integrators,  $+$ ,  $-$ ,  $*$
- Parallelism
- Algebraic loop (loop without integrator)
- Scaling and alarms for out of scale!!



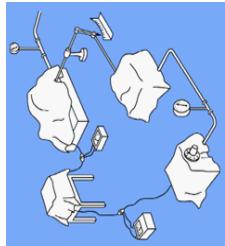
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Oppelt 1954

## Block Diagram Modeling

- Information hiding
- Very useful abstraction
- Essential for control
- Causal inputs-output models
- Blocks described by ODE
- Base for analog computing
- BUT not for serious physical modeling



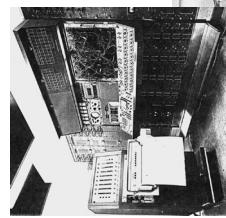
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Oppelt 1954

## Analog Simulation - HIL

- Ordinary differential equations  $\frac{dx}{dt} = f(x, p)$
- Scaling, patching
- Set initial conditions and parameters
- Direct manipulation of parameters
- Manifestation of algebraic loops
- Print results
- Hardware-in-the-loop simulation
- Simulation centers



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PC Matlab 1984, Simulink 1991

## Digital Emulators

- Precompilers to FORTRAN
- MIMIC Wright-Patterson 1965
- CSMP IBM 1962
- Babels tower > 30 emulators by 1965
- CSSL Simulation Council 1967
- ACSL Gauthier and Mitchell 1975
- SIMNON Elmquist 1975
- MATLAB Cleve Moler 1980
- System Build, MatrixX 1984
- LabView 1986
- PC Matlab 1984, Simulink 1991

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PC Matlab 1984, Simulink 1991

## LTH in the 70s

- New control department at LTH (1966) in new school (1961) close to an old university
- Research program in Control Department: Optimization, Computer Control, System Identification, Adaptive Control, Applications; **Computer Aided Control Engineering (CACE)**
- Embedded systems taught in the control department from 1970
- Interactive computing Wieslander: INTRAC, SYNpac, IDPAC, MODPAC. FORTRAN based widely distributed
- A nonlinear simulator was missing



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## Simmon Elmquist 1972

A block diagram language and an interactive simulator

**Formal syntax** in Bachus Naur format  
Six basic commands: SYSTEM, PAR, INIT, SIMU, PLOT, AXES  
Seven auxiliaries: STORE, SHOW, DISP, SPLIT, HCOPY, ALGOR, ERROR

```

DISCRETE SYSTEM reg
Input y
Output u
State I
New nl
Timestamp ts
t:=t+h
v:=t+h
u:=sat(u,0,1)
n:=h+k'h/el/Ti+u-v
k:1
h:0.1
END

CONTINUOUS SYSTEM proc
Input u
Output y
State x
Der dx
dx:=sat(u,0,1)
END

CONNECTING SYSTEM
y(reg)=1; y(reg)=y(proc)
u(proc)=u(proc)
END

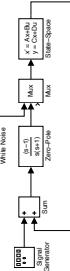
```

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## Simulink 1991 the Ultimate Block Diagram Tool

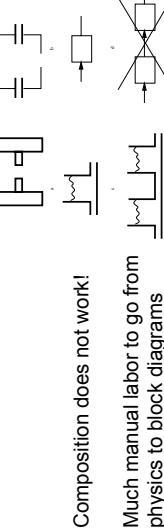
- Mimics the analog computer with more general blocks
- Each block a state model
- MATLAB, Stateflow
- Granularity and Structuring
- Graphical aggregation and disaggregation
- Much manual manipulation from physics to blocks
- Neither formal syntax nor formal semantics



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## But!!

States may disappear when systems are interconnected – warning algebraic loop!

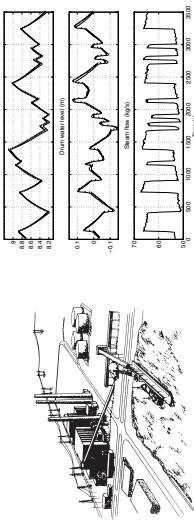


**Lesson 1:** Block diagrams not suitable for physical modeling  
**Lesson 2:** Don't stick to a paradigm based on old technology when new technology emerges!!



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## Boiler Control at LTH



1. Introduction
2. Block diagram modeling
3. Equation-based modeling
4. Summary



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## Inspiration

- Bond Graphs Henry Paynter MIT 1961  
Excellent if there is one dominating balance equation. Difficult to deal with many balances.
- Circuit theory  
Two ports systems: Kirchoffs current and voltage law  
Differential algebraic systems DAE Gear 1971 & Petzold  
Spice Peterson Berkeley 1973  
Good solution for circuits. Attempts at generalizations:  
System dynamics, through and across variables  
Multi-body systems: Adams, SolidWorks, ...  
➤ Chemical Engineering: Complex plants, no dynamics optimization

Origins of Equation-based Modeling LCCC Sept 2012

## Good Old Physical Modeling

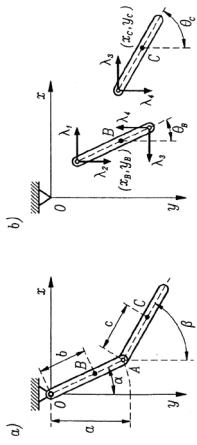
- Experiments, modeling, system identification
- Eklund Linear DrumBoiler-Turbine Models 1971
- Lindahl Design and Simulation of a Coordinated Drum Boiler-Turbine Controller Dec 1976

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## Mechanical Systems



- Split into subsystems (free body diagrams)
- Write equations of motion for each subsystem
- Add constraints to describe connections

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## Model Manipulations

- Eliminate redundant variables
- Use graph algorithms to reduce to lower block diagonal form LBD
- Solve linear blocks analytically
- Use tearing to generate iterative solution for nonlinear blocks
- Generate code for finding equilibria
- Generate code for DAE solvers
- Connect to optimizers
- Generate inverse models for feedforward control (reverse causality) e.g. computed torque
- Generate linear models for control design

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## Elmqvist's PhD Thesis

- Strong industrial interest in SIMNON, demands for extensions, matrices, hierarchies. Is this a good thesis topic? Transpiration/inspiration?
- More interesting to make a modeling language
- Modeling paradigm – balance equations
- Object orientation (Simula)
- Symbolic computations DAE
- Boiler model worked
- Great ideas but premature
- Demanding application useful

[www.control.ict.se/Publication/elm78dis.html](http://www.control.ict.se/Publication/elm78dis.html)  
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## Omola-Omsim

- Work on CACE stopped around 1980 because of FORTRAN and MATLAB
- New research project 1990 Object Oriented Modeling and Simulation: Sven Erik Mattsson, Mats Andersson, Bernt Nilsson, Dag Bruck, Jonas Eborn, Hubertus Tummesscheit, Johan Åkesson
- Experiments with OO in Lisp & KEE
- C++ for object orientation
- Language (Omola) and simulator (OmSim)
- Extensive symbolic manipulation (Mattsson)
- Jmodelica.org Optimica

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# Modelica

- Intensive interaction with Dynasim 1991
- ESPRIT Simulation in Europe, Lund Sept 1996
- COSY meeting Lund Sept 5-7, 1996
- European groups: 23 participants, 17 talks by groups from Dynasim Lund, ETH Zurich, INRIA Paris, DLR Munich, VTT Helsinki, Imperial College London, LTH Lund, RWTH Aachen and universities in Barcelona,, Groningen, Valencia, Wien
- Formation of the Modelica language group
- First Modelica language specification Sept 1997
- 7 Modelica compilers at 9th Modelica conf 2012

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# Original Language Team

- Hilding Elmqvist, Dynasim AB, Lund, Sweden
- Fabrice Boudaud, Gaz de France,
- Jan Broenink, University of Twente, Netherlands
- Dag Bruck, Dynasim AB, Lund, Sweden
- Thilo Ernst, GMD-FIRST, Berlin, Germany
- Peter Fritzson, Linköping University, Sweden
- Alexandre Jeandel, Gas de France
- Kai Justin, VTT, Finland
- Matthias Klose, Technical University of Berlin, Germany
- Sven Erik Mattsson, Lund University, Sweden
- Martin Otter, DLR, Oberpfaffenhofen, Germany
- Per Sahlén, BrisData, Stockholm, Sweden
- Hubertus Timmerscheidt, DLR Cologne, Germany
- Hans Vangheluwe, University of Gent, Belgium

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# Many Views on Modeling

- Engineering: Free body diagrams, circuit diagrams, block diagrams, P&I diagrams
- Behavioral systems Willems 1981 (CSM 2007)
- Physics: Mass, energy, momentum balances constitutive material equations
- Mathematics: ODE, DAE, PDE
- Computer Science: Languages, datastructures, programming, imperative, declarative
- **Block Diagram Modeling:** Causal modeling, imperative
- **Equation-Based Modeling:** Acausal, declarative

Origins of Equation-based Modeling LC/C/C Sept 2012



1. Introduction
2. Block diagram modeling
3. Equation-based modeling
4. Summary



## Equation-based Modeling

- Has come a long way
- Serious industrial use
- 9th Modelica conference, several commercial compilers
- Strong potential for education
- Lower the entrance barrier
- Many challenges
- Much work remains
- Step back and think!
- This workshop and ...



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## Challenges

- Is it time to sit it back and think about fundamentals?
- Make Modelica an international standard, compliance checking!
- Make it widely used!
- More than simulation
- Embedded systems
- Lower entrance barrier
- The tool chain



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## Modeling

- Solomon Golomb: Mathematical models – Uses and limitations. Aeronautical Journal 1968



Solomon Wolf Golomb (1932) a mathematician and engineer and a professor of electrical engineering at the University of Southern California. Best known to the general public and fans of mathematical games as the inventor of polyominoes, the inspiration for the computer game Tetris. He has specialized in problems of combinatorial analysis, number theory, coding theory and communications.

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## Golomb On Modeling

- Don't apply a model until you understand the simplifying assumptions on which it is based and can test their applicability. **Validity / ranges**
- Distinguish at all times between the model and the real world. **You will never strike oil by drilling through the map!**
- Don't expect that by having named a demon you have destroyed him
- The purpose of notation and terminology should be to enhance insight and facilitate computation – not to impress or confuse the uninitiated



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**ASSIMULO – A PYTHON PACKAGE FOR SOLVING  
DIFFERENTIAL EQUATION WITH INTERFACE TO EQUATION  
BASED LANGUAGES**

**Claus Führer, Centre of Mathematical Sciences,  
Lund University**

We present a Python package which gives access to state-of-the art industrial differential equation algorithms in C or FORTRAN and which is open for experimental methods in Python. The interesting feature of Assimulo is that it comes with a specially designed problem class to import models (=differential equations) from JMODELICA. Such an equation based modeling language can provide much more information to the solver than just the problem description itself. Equation coupling information, information about equation type, discontinuities and others can be used to improve and control efficiently the solution process. The talk includes even a wish-list for additional language constructions.

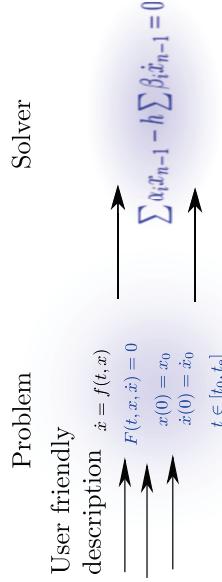


Let's move the focus ...

Assimulo - a Python package  
for solving differential equations  
with interface to equation based languages



LCCC workshop Lund  
September 2012



## ODE and DAE solvers in two disjoint worlds

### Industrial Simulation Tasks

- ▶ highly complex models
  - ▶ high robustness standards
  - ▶ high documentation standards
  - ▶ long life cycle
- one or two ODE/DAE packages meet these requirements.

### Academic Simulation Tasks

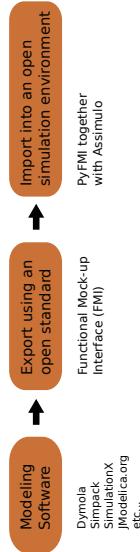
- ▶ a few, low scale test models
  - ▶ lab standard quality (validation of concept)
  - ▶ good analyzed algorithms, poor code documentation
  - ▶ short life cycle, often coupled on individual career steps.
- dozen of codes produced (and forgotten) this way.
- ... highly valid still today.

Hans Stetter in:  
Mathematics of computation, 1943-1993: a half-century of computational mathematics - Mathematics: 50th Anniversary Symposium, August 9-13, 1993, Vancouver, British Columbia

## Motivation

## Functional Mock-up Interface (FMI)

- ▶ Give the academic world access to complex models → FMI
- ▶ Give the industrial world access to a variety of ODE/DAE codes (even experimental ones): → ASSIMULO
- ▶ Give students in scientific computing an intuitive access to industrial standard solvers: → ASSIMULO



## Assimulo is written in Python, why?

### Benefits of using Python:

- ▶ Open-source language
- ▶ Interpreted
- ▶ Object-oriented
- ▶ Many freely available packages
  - ▶ NumPy
  - ▶ SciPy
  - ▶ Matplotlib
  - ▶ Cython
- ▶ Highly flexible for interfacing to C, FORTRAN ...
- ▶ Ideal in teaching.



FMI is an open interface for model exchange with the idea that tools may generate and exchange dynamic system models.

The FMI supports model defined as discontinuous ordinary differential equations.

Model interface The equations are evaluated and the model interaction is performed by standardized C functions.

Model description The variable information of the model is contained in an XML-file.

Additional data Model data, such as tables and maps may also exists.

⇒ Talk by [Torsten Blochwitz](#) on Wednesday

## ASSIMULO

Python workbench for simulation of ordinary differential equations.

The intention is to provide a common high-level interface for a variety of different solvers.

Supports

- ▶ problems formulated as first or second order ordinary differential equations
- ▶ problems formulated as implicit ordinary differential equations including overdetermined problems.

## ASSIMULO, problem formulations

- ▶ Explicit hybrid ODEs

$$\dot{y} = f(t, y, sw), \quad y(t_0) = y_0, \quad sw(t_0) = sw_0$$

- ▶ Implicit hybrid ODEs (also called DAEs)

$$F(t, y, \dot{y}, sw) = 0, \quad y(t_0) = y_0, \quad \dot{y}(t_0) = \dot{y}_0, \quad sw(t_0) = sw_0$$

- ▶ Mechanical systems in second order explicit ODE form

$$\ddot{p} = M(p)^{-1} f(t, p, \dot{p})$$

- ▶ Mechanical systems in (overdetermined) implicit ODE form

$$\dot{p} = v$$

$$M(p)v = f(t, p, v) - G^T(p)\lambda$$

$$0 = g_{\text{const}}(p)$$

$$0 = G(p)v$$

- ▶ Delay (retarded) differential equations

## ASSIMULO, overview

## ASSIMULO, solvers

- ▶ Currently, solvers written in Python, FORTRAN and C are available.

► IDA - Multistep method for DAEs

► CVode - Multistep methods for ODEs

► ODASSL - Multistep methods for overdetermined DAEs

► RADAU5 - Runge-Kutta method for DAEs

► GLIMDA - General linear methods methods for DAEs

► and we are working on a "solver museum" (oldest code in restoration 1983).

IDA and CVode are production quality solvers from the **SUNDIALS** suite.

## Simple example workflow

### Make a problem

```
def rhs(t,y):
 A = array([[0, 1], [-2, -1]])
 yd = N.dot(A, y)
 return yd
```

```
y0 = array([1.0, 1.0])
t0 = 0.0
linmodel = Explicit_Problem(rhs, y0, t0)
```

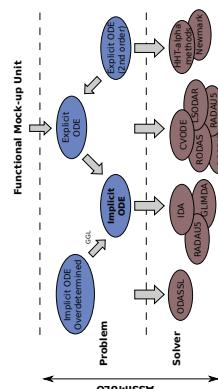
### Create a solver instance

```
sim = CVode(linmodel)
```

... and simulate

```
t, y = sim.simulate(tfinal)
```

**Figure :** Connection between the different problem formulations and the different solvers available in ASSIMULO. The connection of the Functional Mock-up Interface to ASSIMULO is also shown.



Assimulo can be quite verbose...

Final Run Statistics : Linear Test ODE

|                                          |       |
|------------------------------------------|-------|
| Number of Error Test Failures            | = 4   |
| Number of F-Eval During Jac-Eval         | = 0   |
| Number of Function Evaluations           | = 153 |
| Number of Jacobian Evaluations           | = 0   |
| Number of Nonlinear Convergence Failures | = 0   |
| Number of Nonlinear Iterations           | = 149 |
| Number of Root Evaluations               | = 0   |
| Number of Steps                          | = 84  |

Solver options:

|                         |              |
|-------------------------|--------------|
| Solver                  | : CVode      |
| Linear Multistep Method | : Adams      |
| Nonlinear Solver        | : FixedPoint |
| Maxord                  | : 12         |

## Discontinuities – a Continuous Challenge

```

class Extended_Problem(Explicit_Problem):
 #Sets the initial conditions directly into the problem
 y0 = [0.0, -1.0, 0.0]
 sw0 = [False, True, True]

 #The right-hand-side function (rhs)
 def rhs(self, t, y, sw):
 ...
 ...

 #The event function
 def state_events(self, t, y, sw):
 event_0 = y[1] - 1.0
 ...
 return array([event_0, event_1, event_2])

#Responsible for handling the events.
def handle_event(self, solver, event_info):
 event_info = event_info[0]
 while True: # Event Iteration
 self.event_switch(solver, event_info) #Turns the swi
 ...

```

Controlling the method

|                                     |
|-------------------------------------|
| sim.atol=M.array ([1.0, 0.1])*1.e-5 |
| sim.rtol=1.e-8                      |
| sim.maxord=3                        |
| sim.discr='BDF',                    |
| sim.iter='Newton',                  |

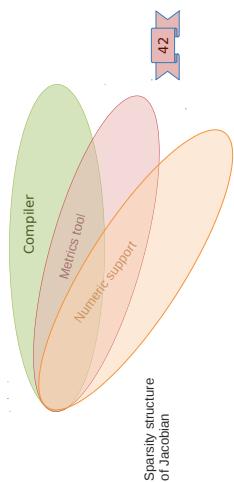
Languages have the potential to inform

- ▶ Are there discontinuities?
- ▶ State/Time events?
- ▶ Are there linear components?
- ▶ What are differential, what are algebraic variables? ("loop closure" conditions versus algebraic equations)
- ▶ Derivatives?

The compiler might know more

Plans/ideas/wishes for the future

## Why extensible compilers?



(Sorry Garel for changing your slide ...)

Thank you!

... and feel free to try it out!

► Assimulo [www.assimulo.org](http://www.assimulo.org)

► PyFMI [www.pyfmi.org](http://www.pyfmi.org)

- ▶ Would like to stimulate to open the **FMI** for a wider range of problem formulations - higher index DAES(?)
- ▶ Continue to expand the solvers available in ASSIMULO
  - ▶ Work on the museum.
  - ▶ Introduce problem formulation for delay differential equations
  - ▶ Generalize solvers for discontinuity handling
  - ▶ Potentials of language/compiler aided numerics.
- ▶ Automatic differentiation: a separate tool or an integrated part of the language-solver chain?

**CASADI: A TOOL FOR AUTOMATIC DIFFERENTIATION AND  
SIMULATION-BASED NONLINEAR PROGRAMMING**  
**Moritz Diehl, Electrical Engineering Department and  
Optimization in Engineering Center OPTEC KU Leuven**

We present CasADi, an open-source symbolic environment for simulation based nonlinear programming and automatic differentiation (AD). Casadi offers a level of abstraction that is higher than conventional AD tools and is in particular designed to enable calls to solvers of initial-value problems in differential-algebraic equations (DAE) within nonlinear programming formulations, with derivative information efficiently calculated through automatic formulation of the corresponding forward and adjoint sensitivity equations.

In this talk, we give an overview of the tool, with a focus on the AD approach and the symbolic environment. This environment allows users to formulate problems in a high-level language such as Python, but solve it with the speed of optimized C-code thanks to fast interpreters and just-in-time compilation. We also show how optimal control problems formulated in the physical modelling language Modelica can be imported into the symbolic environment. Joint work with Joel Andersson, Joris Gillis, and Johan Akesson.



## OPTEC - Optimization in Engineering Center

### CasADi: A Tool for Automatic Differentiation and Simulation-Based Nonlinear Programming

Moritz Diehl\*

with Joel Andersson\*, Joris Gillis\*, Johan Akesson\*\*

\*OPTEC, KU Leuven, Belgium

\*\*Lund University / Modelon

LCCC, Sept 20, 2012



*Center of Excellence of KU Leuven, since 2005  
70 people, working jointly on methods and applications of optimization,  
in 5 departments:*

- Electrical Engineering
- Mechanical Engineering
- Chemical Engineering
- Computer Science
- Civil Engineering



*Many real world applications at OPTEC...*



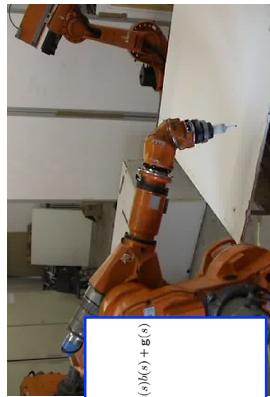
### OPTEC Research Example: Time Optimal Robot Motion

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 54, NO. 10, OCTOBER 2009  
DOI 10.1109/TAC.2009.2028432  
Time-Optimal Path Tracking for Robots:  
A Convex Optimization Approach  
Diego de Vries, Bram Dehandschutter, Jan Swevers, Joao De Schutter, and Moritz Diehl



Robot shall  
write as fast  
as possible.

Global solution found  
in 2 ms due to  
**convex reformulation**



$$\begin{aligned} & \min_{\mathbf{a}(s), \mathbf{c}(s), \tau(s)} \int_0^1 \frac{1}{\sqrt{\dot{s}(s)}} ds \\ & \text{subject to } \tau(s) = \mathbf{m}(s)\mathbf{a}(s) + \mathbf{c}(s)\dot{\mathbf{a}}(s) + \boldsymbol{\xi}(s) \\ & \dot{\mathbf{a}}(0) = \dot{\mathbf{a}}_0^2 \\ & \dot{\mathbf{a}}(1) = \dot{\mathbf{a}}_1^2 \\ & \dot{\mathbf{a}}(s) \geq 0 \\ & \mathcal{T}(s) \leq \tau(s) \leq \bar{\tau}(s) \\ & \text{for } s \in [0,1]. \end{aligned}$$

## Overview

- Optimization in Engineering Center OPTEC
- State of the Art in Optimal Control Algorithms (ACADO)
- CasADi: A Framework to WRITE Optimal Control Algorithms

## Optimal Control Problem in Continuous Time

$$\begin{aligned} & \text{minimize}_{x(\cdot), u(\cdot)} && \int_0^T L(x(t), u(t)) dt + E(x(T)) \\ & \text{subject to} && \\ & \dot{x}(t) - f(x(t), u(t)) = 0, && \text{(fixed initial value)} \\ & h(x(t), u(t)) \geq 0, && \text{(ODE model)} \\ & r(x(T)) = 0 && \text{(path constraints)} \\ & && \text{(terminal constraints).} \end{aligned}$$

How to solve these nonlinear problems reliably and fast?

## Sequential Approach (Single Shooting): Eliminate States

$$\begin{aligned} & \text{minimize}_{\bar{u}} && \sum_{i=0}^{N-1} L_i(\tilde{x}_i(u), \tilde{z}_i(u), u_i) + E(\tilde{x}_N(u)) \\ & \text{subject to} && \\ & h_i(\tilde{x}_i(u), \tilde{z}_i(u), u_i) \leq 0, && i = 0, \dots, N-1, \\ & r(\tilde{x}_N(u)) \leq 0. && \end{aligned}$$



Pros:

- Only control degrees of freedom (for NMPC)
- Can couple with "vanilla NLP" solver
- Sparsity of problem lost
- Unstable systems cannot be treated

Cons:

Historically first "direct" approach ("single shooting", Sargent&Sullivan 1978)

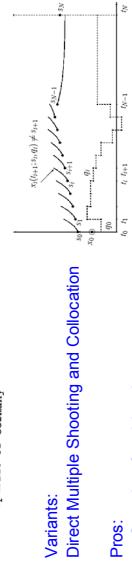
## Simultaneous Approach: Keep States in NLP



BUDAPEST, HUNGARY  
JULY 2-6 1994  
THE INTERNATIONAL  
SYNTHETIC DESIGN  
AND WORLD CONGRESS

A MULTIPLE SHOOTING ALGORITHM FOR DIRECT SOLUTION OF OPTIMAL CONTROL PROBLEMS\*

Hans Georg Bock and Karl J. Plitt  
Institut für Angewandte Mathematik, SFB 72, Universität Bonn, 5300 Bonn,  
Federal Republic of Germany



Variants:

Direct Multiple Shooting and Collocation

- Pros:
- Sparsity of problem kept
  - Unstable systems can be treated, nonlinearity reduced
  - Large scale problems
  - Need to develop (or use) structure exploiting NLP solver

## Nonlinear Program (NLP) in Multiple Shooting

$$\begin{aligned} & \text{minimize}_{x, z, u} && \sum_{i=0}^{N-1} L_i(x_i, z_i, u_i) + E(x_N) \\ & \text{subject to} && \\ & x_0 - \bar{x}_0 &=& 0, \\ & x_{i+1} - f(x_i, z_i, u_i) &=& 0, \quad i = 0, \dots, N-1, \\ & g_i(x_i, z_i, u_i) &=& 0, \quad i = 0, \dots, N-1, \\ & h_i(x_i, z_i, u_i) &\leq& 0, \quad i = 0, \dots, N-1, \\ & r(x_N) &\leq& 0. \end{aligned}$$

Structured parametric Nonlinear Program

- Initial Value  $\bar{x}_0$  is often not known beforehand ("online data" in NMPC)
- Discrete time dynamics from ODE simulation (we will need sensitivities)

## Sequential Convex Programming (SCP)

- Summarize problem as
 
$$\begin{array}{ll} \min_{x \in \mathbb{R}^n} & f(x) \\ \text{s.t.} & g(x) + M\xi = 0, \\ & x \in \Omega. \end{array}$$

Step 1: Linearize nonlinear constraints at  $x^k$  to obtain convex problem:

$$\begin{array}{ll} \min_{x \in \mathbb{R}^n} & f(x) \\ \text{s.t.} & g(x^k) + g'(x^k)(x - x^k) + M\xi = 0, \\ & x \in \Omega. \end{array}$$

Step 2: Solve convex problem to obtain next iterate.

Obtain new value of parameter  $\xi$  and go to step 1)

- Convergence to (and tracking of) local minima under mild assumptions [1]

[1] Tran Dinh, Savorgnan, Diehl: Adjoint-based predictor-corrector SCP for parametric nonlinear optimization. *SIAM Journal on Optimization* (in print)

## ACADO Toolkit [1]

- ACADO = Automatic Control and Dynamic Optimization
- Open source (LGPL/C++: www.acadotoolkit.org)
  - Implements direct multiple shooting [2] and real-time iterations [3]
  - User interface close to mathematical syntax
  - Automatic C-Code Export for Microsoft® Nonlinear MPC [4]
- Developed at OPTEC by B. Houska, H.J. Ferreau, M. Vukov, ...
- ~3000 downloads since first release in 2009

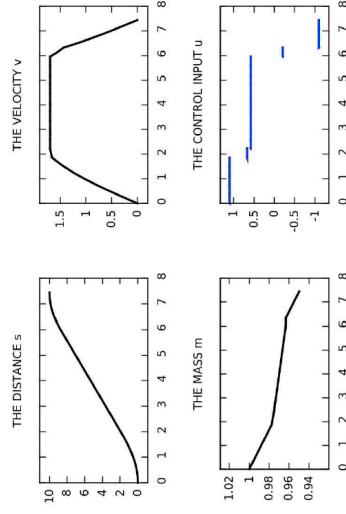


- [1] Houska, Ferreau, D., OCAM, 2011  
 [2] Bock, Pilt, IFAC WC, 1984  
 [3] D. Bock, Schöder, Nagy Algower, JPC, 2002  
 [4] Houska, Ferreau, D., Automatica, 2011

## Rocket Example in ACADO Language

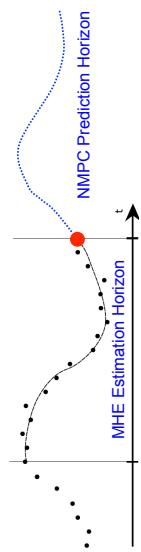
|                           |                                                                                                                                                                                                                                                            |
|---------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Mathematical Formulation: | $\begin{array}{ll} \text{minimize}_{s(t), v(t), m(t)} & T \\ \text{subject to} & \dot{s}(t) = v(t) \\ & \dot{v}(t) = \frac{u(t) - 0.2v(t)^2}{m(t)} \\ & \dot{m}(t) = -0.01u(t)^2 \\ s(0) = 0 & s(T) = 10 \\ v(0) = 0 & v(T) = 0 \\ m(0) = 1 & \end{array}$ |
| Control Parameters:       | $\begin{array}{ll} u(t) & \in [-1, 1] \\ T & \in [0, 15] \end{array}$                                                                                                                                                                                      |

## ACADO Results Plot (after few milliseconds)



## NMPC Practice: Estimation AND Optimization

- Moving Horizon Estimation (MHE): Get State by Least Squares Optimization
- Nonlinear Model Predictive Control (NMPC): Solve Optimal Control Problem



Gauss-Newton in ACADO:  
`ocp.minimizeMayerTerm() → ocp.minimizeLSQ();`

## MHE+NMPC Experiments (Aug 22, 2012)



## ACADO Code Generation for Tethered Airplanes



- 22 states, nonlinear, unstable
- 2 controls
- 1 s horizons in past / future



## Overview

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- CasADi: A Framework to WRITE Optimal Control Algorithms

## Optimal Control Problem (OCP) Solvers

### Two implementation approaches

- Write/use a general-purpose OCP solver
  - Examples: MUSCOD-II, ACADO Toolkit, DyOS, DIRCOL
    - + Easy to set up for the average user
    - + Can be very efficient for medium size problems
    - Many OCPs cannot be formulated
  - Write special-purpose OCP solvers
    - OCP→NLP using algebraic modelling language
      - + Full control of NLP formulation, easier to extend
      - So far only for collocation methods
    - Both approaches taken at OPT-EC using two in-house software tools
      - ACADO Toolkit: A general-purpose OCP solver for NMPC
      - CasADi: A framework for writing OCP solvers

## CasADi

### Main components of CasADi

- A symbolic framework with state-of-the-art algorithmic differentiation (all eight flavours of AD)
- Interfaces to other tools; NLP solvers, ODE/DAE integrators, ...
- In-house tools; NLP solvers, ODE/DAE integrators, ...
- Framework for import and symbolic reformulation of OCPs from Modelica

### Implementation

- Written in self-contained C++ code
- Full-featured front-ends to Python and Octave using SWIG

## Computer Algebra System for Algorithmic Differentiation

## CasADi

### What is CasADi?

A framework for C++, Python and Octave for quick, yet efficient, implementation of algorithms for numeric optimization

### In particular

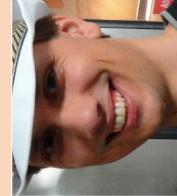
Facilitates OCP→NLP transcription for collocation methods and shooting methods (e.g. single-shooting method in 30 lines of code)

Permissive open-source license (GPL)

[www.casadi.org](http://www.casadi.org)

## CasADi

### Main developers



Joel Andersson



Joris Gillis

## CasADI

An illustrating example  
Drive a Van der Pol oscillator to the origin with minimal control effort.

$$\begin{aligned} \text{minimize}_{v,p,u}: \quad & \int_0^t u(t)^2 dt \\ \text{subject to:} \quad & \dot{x}(t) = \begin{bmatrix} \dot{v} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} (1 - p^2)v - p + u \\ v \end{bmatrix}, \quad t \in [0, t_f] \\ & v(0) = 0, \quad p(0) = 1, \\ & v(t_f) = 0, \quad p(t_f) = 0 \\ & -0.75 \leq u(t) \leq 1.0, \quad t \in [0, t_f] \end{aligned}$$

Solve with a direct-single shooting method.

## CasADI

### Step 1: Formulate symbolic expression ODE in CasADI

- The ODE:

$$\begin{bmatrix} \dot{v} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} (1 - p^2)v - p + u \\ v \end{bmatrix},$$

- Can be formulated in CasADI-Python:

```
Declare variables
u = sym('u')
v = sym('v')
p = sym('p')

ODE right hand side
vdot = (1 - p*p)*v - p + u
pdot = v
```

- Syntax  $\approx$  Matlab Symbolic Toolbox
- ODE can also be imported from Modelica

## CasADI

### Step 2: Create ODE function

- These expressions define the ODE rhs function  $f : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}^2$ :

```
f = SXFunction(\
 daeInC(x = vertcat([v, p]), p = u), \
 daeOutCode = vertcat([vdot, pdot]))
```

- Creating a function means topologically sorting the expression graph
- Function can be evaluated:

- In the CasADI interpreter: numerically or symbolically
- By generating and compiling C-code
- Through just-in-time compilation (using LLVM framework)
- Derivatives in CasADI are differentiable functions in CasADI and can be differentiated an arbitrary number of times
- Derivatives calculated through forward/adjoint sensitivity analysis

## CasADI

### Step 3: Formulate discrete time dynamics

- Assume a piecewise constant control with 20 intervals and let  $t_f$  be 10 s.

```
nk = 20 # Control discretization (uniform)
th = 10.0 # Length of the time horizon
f_d.init()
```

- Get the discrete time dynamics by allocating an ODE integrator instance, e.g. using CasADI's interface to Sundials:

```
f_d = CVodesIntegrator(f)
f_d.setParam("tf", th/nk) # Interval length
f_d.init()
```

- Integrators in CasADI are differentiable functions in CasADI and can be differentiated an arbitrary number of times
- Derivatives calculated through forward/adjoint sensitivity analysis

## CasADI

### Step 4: Formulate NLP

The integrator allows us to form an expression for the state at the final time:

```
U = msym('U',nk) # Controls for each interval
X0 = [0,1] # The initial state
Build a graph of integrator calls
X = X0
for k in range(nk):
 X,_,_ = I.call([X,U[k]])
this defines NLP objective functions and constraints:
Objective function: ||U||^2
F = MXFunction([U],[m1(U,T,U)])
Terminal constraints: x=[0,0]
G = MXFunction([U],[X])
```

## CasADI

### Step 5: Solve NLP

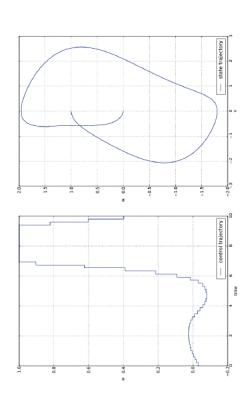
Solve NLP by using one of the interfaced NLP solvers, e.g. IPOPT:

```
import numpy # Standard linear algebra routines
Allocate an NLP solver
solver = IpoptSolver(F,G)
solver.init()
Set bounds and initial guess
solver.setInput(-0.75*numpy.ones(nk), 'NLP_LBX')
solver.setInput(1.0*numpy.ones(nk), 'NLP_UBX')
solver.setInput(numpy.zeros(nk),'NLP_X_INIT')
solver.setInput(numpy.zeros(2),'NLP_LBG')
solver.setInput(numpy.zeros(2),'NLP_UBG')
Solve the problem
solver.solve()
```

## CasADI

### Step 6: Visualize solution

Use standard Python packages visualizing the solution:



## CasADI Users

### Other OCP methods successfully implemented using CasADI

- Direct collocation (J. Andersson, J. Åkesson & F. Magnusson, M. Zanoni & S. Gross, J. Stenberg, J. Gillis ...)
- Direct multiple-shooting (J. Andersson, K. Gevlen, J. Frasch)
- Distributed multiple-shooting (A. Kozma & C. Savorgnan)
- Pseudospectral optimization (C. Andersson)

## Benchmarking CasADI vs AMPL Solver Library

| Problem  | Dimensions | Time ASL [s] | Time CasADI [s] | Diff. |
|----------|------------|--------------|-----------------|-------|
|          | #var       | Total AD     | Total AD        |       |
| gpp      | 250        | 498          | 0.492           | -3 %  |
| reading1 | 10001      | 5000         | 0.712           | -76 % |
| porous2  | 4900       | 4900         | 1.916           | -81 % |
| orthrgds | 10003      | 5000         | 0.949           | -71 % |
| cblbeam  | 1499       | 1000         | 0.776           | 0 %   |
| svanberg | 5000       | 5000         | 2.492           | -48 % |
| orthregd | 10003      | 5000         | 0.332           | -71 % |
| trainh   | 20000      | 10002        | 3.932           | -55 % |
| orthrgdm | 10003      | 5000         | 0.328           | -67 % |
| dtoc2    | 5994       | 3996         | 0.296           | -61 % |

### Benchmarking

- CasADI VM outperformed ASL VM by a factor 2 on average
- Most of the time spent in linear solver anyway
- Note:  $\approx 5\times$  faster still with C-codegen or just-in-time

## CasADI Usage in Leuven: Complex Plane Orbit

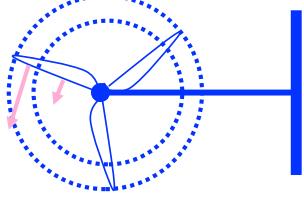
- Within ERC Project HIGHWIND, running from 2011-2016



**ERC HIGHWIND**  
SIMULATION, OPTIMIZATION & CONTROL OF  
HIGH-ALTITUDE WIND POWER GENERATORS

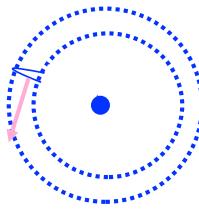
## What is the Optimal Wind Turbine ?

- Due to high speed, wing tips are **most efficient** part of wing
- Best winds are in high altitudes



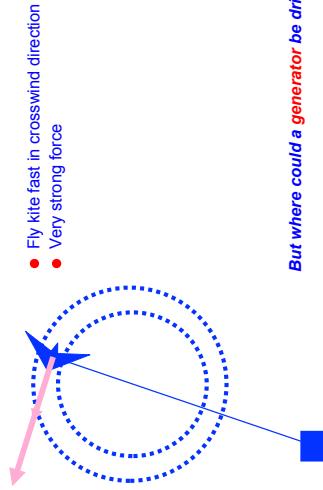
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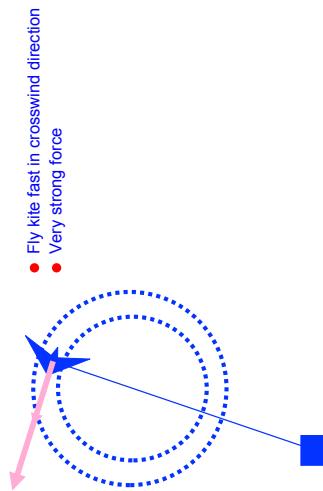


Could we construct a wind turbine  
with only wing tips and generator?

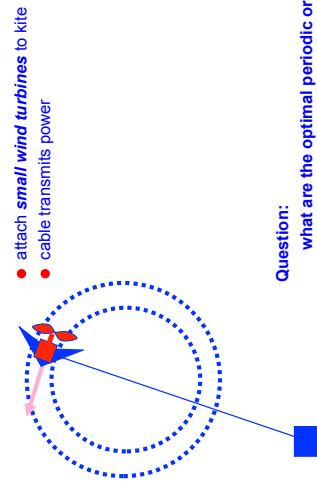
## Crosswind Kite Power



## Crosswind Kite Power



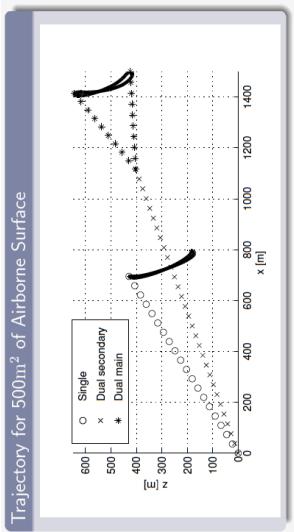
## One Variant: On-Board Generator



## CasADI Usage in Leuven: Complex Plane Orbits

- Complex aerodynamic models
- Periodic boundary conditions
- Connecting two tethers can increase the power output significantly...
  - ...but leads to even more complex models and optimal control problems

## Single vs. Dual Airfoils: Optimal Large System



Complex OCPs solved with CasADI, Collocation, IPOPT,  
from [Zanon et al., submitted]



## Visualization of Single vs. Dual Airfoils

## Summary

- Optimal Control Tools now 100000x faster than 1997, and ACADO Code Generation is currently tested in a variety of fast real world applications (cranes, airplanes, vehicles, induction motors, ...)
- But non-standard problems need non-standard solvers: CasADI allows the user to easily write competitive state-of-the-art optimal control algorithms specifically designed for one problem class
- CasADI distributed under permissive LGPL license and used by a growing number of people in and outside Leuven (e.g. Jmodelica)

## Appendix

**CasADI** [www.casadi.org](http://www.casadi.org)

## CasADI Performance

### Benchmarking Using CUTEr

- 10 NLPs from Bob Vanderbei's AMPL translation of CUTEr
- AMPL used to parse/pre-optimize AMPL models
- Solved using IPOPT 3.10 with MA27 as linear solver in two ways
  - Using AMPL Solver Library's (ASL) interface to IPOPT
  - Using CasADI's .nl import and interface to IPOPT
  - Only virtual machines (VM) for both tools, no codegen

## Complete CasADI Code for OCP Solution

```

from casadi import *
nK = 80 # Number of discretization
th = 10.0 # Final time
Build a graph of integrator calls
X = X0
for k in range(0,nK):
 X = callf([X],0)[0]
 X_i = X[0:nK-1]
 X_f = X[nK-1]
 # Integrate function: [0|1|1/2]
 F = EKFfunction([0],f,[0,T,i])
 # Terminal constraint: x=0, 0
 G = KFfunction([0],g,[0,T,i])
 # Set bounds and initial guess
 solver.setInput(-0.75*coss(i), MP_LBK)
 solver.setInput(0.75*coss(i), MP_RBK)
 solver.setInput(-0.75*sins(i), MP_X_LBK)
 solver.setInput(0.75*sins(i), MP_X_RBK)
 solver.setInput(zeros(2), MP_LBG)
 solver.setInput(zeros(2), MP_RBQ)
 # Solve the problem
 solver.evaluate()

```

## ACADO Code Generation for Benchmark CSTR

$$\begin{aligned}
 \dot{x}_A &= u_0(x_A - x_D) - k_A x_D x_C - k_B x_C x_D \\
 \dot{x}_B &= -u_0(x_A - x_D) + k_A x_D x_C - k_B x_C x_D \\
 \dot{x}_C &= -\frac{1}{k_A} (k_A x_D x_C + k_B x_C x_D) \\
 \dot{x}_D &= \frac{1}{k_A} (u_0 + k_A x_D x_C - k_B x_C x_D).
 \end{aligned}$$

CSTR Benchmark by [Klett, Engell, Kremling, Allgöwer 1995]

## ACADO Code Generation for Benchmark CSTR

$$\begin{aligned}
 \dot{x}(t) &= u(t)x(t) - \dot{x}(t) \\
 \dot{q}(t) &= -u(t)x(t) + \frac{1}{k_A} x_D(t) \\
 \dot{y}(t) &= u(t)x(t) + \frac{1}{k_A} x_D(t) + \frac{1}{k_A} x_C(t) \\
 \dot{z}(t) &= \frac{1}{k_A} x_D(t) - u(t)x(t) - k_B x_C(t).
 \end{aligned}$$

### CSTR Benchmark by [Klett, Engell, Kremling, Allgöwer 1995]

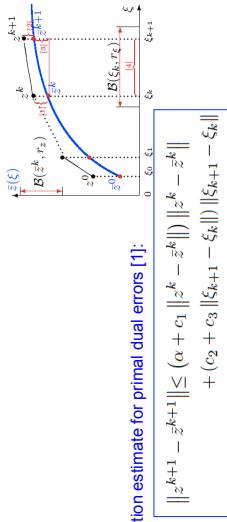
$$\begin{aligned}
 \dot{x}_A &= u_0(x_A - x_D) - k_A x_D x_C - k_B x_C x_D \\
 \dot{x}_B &= -u_0(x_A - x_D) + k_A x_D x_C - k_B x_C x_D \\
 \dot{x}_C &= -\frac{1}{k_A} (k_A x_D x_C + k_B x_C x_D) \\
 \dot{x}_D &= \frac{1}{k_A} (u_0 + k_A x_D x_C - k_B x_C x_D).
 \end{aligned}$$

## CPU Times for ACADO:

|                                         | CPU time (μs) | %   |
|-----------------------------------------|---------------|-----|
| Integration & sensitivities             | 121           | 30  |
| Condensing                              | 98            | 24  |
| QP solution (with qpOASES) <sup>3</sup> | 180           | 44  |
| Remaining operations                    | <5            | <2  |
| A complete real-time iteration          | 404           | 100 |

From [Houska, Ferreau, D., Automatica, 2011]  
**NMPC now 100 000x faster than 1997 (2000x by CPU, 500x by algorithms)**

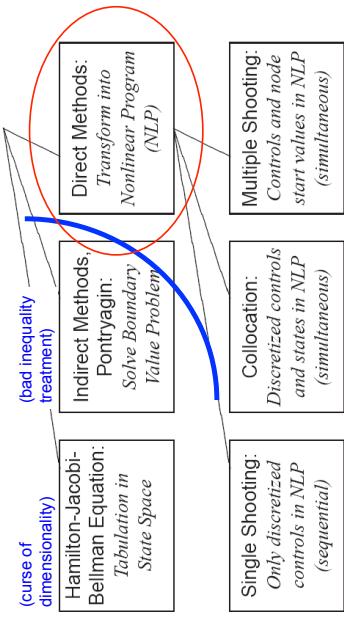
## (SCP) Real-Time Iteration Contraction Estimate



Depends only on nonlinearity of equalities, independent of active set changes!

[1] Tran Dinh, Savorgnan, Diehl: Adjoint-based predictor-corrector SCP for parametric nonlinear optimization. SIAM J. Opt. 2013 (in print)

## Optimal Control Family Tree





**PYOMO: OPTIMIZATION MODELING IN PYTHON**  
**Carl Laird, Artie McFerrin Department of Chemical**  
**Engineering, Texas A&M University**

Mathematical programming has proven to be an efficient tool for design, optimization, and online operation of complex engineered systems. Algebraic modeling languages provide a convenient mechanism for the user to formulate mathematical models and optimization formulations in a language that is similar to the mathematical description of the problem, including constructs for defining sets, expressions, constraints, and objectives. In addition these tools must provide reasonable interface functionality for solvers, for example, first (and possibly second) order derivative information.

Pyomo (Python Optimization Modeling Objects) is a new open-source algebraic optimization language. Pyomo is implemented in Python, and allows the user to make use of extensive scripting capabilities within a familiar, exhaustive, and well-documented programming environment. Pyomo provides general functionality to formulate and solve optimization problems with little or no programming knowledge, but also provides the flexibility to implement high-level language constructs. In this presentation, I will discuss the design and implementation of the Pyomo framework, and give examples of several language extensions including PySP, an extension that supports parallel programming for solution of difficult stochastic programming problems.

## LOCC Workshop: System Design Meets Equation-based Languages

Sep 20, 2012



William E. Hart  
Carl D. Laird  
Jean-Paul Watson  
David L. Woodruff  
  
Chemical Engineering, Texas A&M University  
  
William E. Hart, Jean-Paul Watson, John D. Sirota  
Sandia National Laboratories, Albuquerque, NM  
  
David L. Woodruff, Professor  
Business Management, University of California, Davis

- Algebraic equation-based modeling language for optimization
  - e.g AMPL, GAMS, AIMMS
  - acausal, equation-based modeling
  - currently no support for differential equations
  - initially driven by large-scale MILP
- Designed by Math Programmers
  - open-source, extensible alternative to existing tools
  - used to enable research and engineering solutions
- I work on algorithms and applications
  - I am a user of modeling languages, ... right?

## Pyomo - Python Optimization Modeling Objects

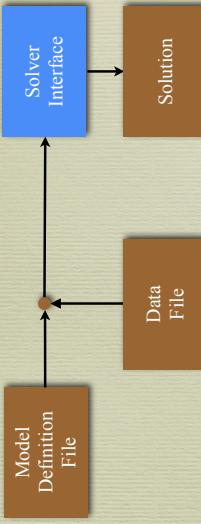


Arie McFerrin Department of  
**CHEMICAL ENGINEERING** | TEXAS A&M ENGINEERING

Chemical Engineering  
Thuesday, September 20, 12



Chemical Engineering  
Thuesday, September 20, 12



- Provide powerful, high-level problem specification
- Familiar math programming constructs (Sets, expressions)
- Very limited programming / scripting capability
  - model transformations? language extensions?
  - plotting? functions? numerical libraries?



Chemical Engineering  
Thuesday, September 20, 12

Chemical Engineering  
Thuesday, September 20, 12

Chemical Engineering  
Thuesday, September 20, 12

## Seasonal Drivers in Infectious Disease Spread



## Seasonal Drivers in Infectious Disease Spread



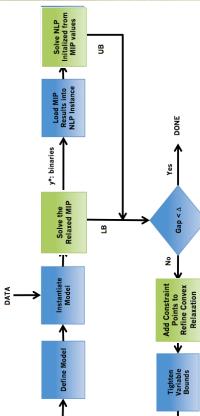
## Seasonal Drivers in Infectious Disease Spread



## Seasonal Drivers in Infectious Disease Spread



## Large Mixed Integer Non-Linear Programming Problem



## Large Mixed Integer Non-Linear Programming Problem

## Parallel Decomposition in Interior-Point Methods

$$\begin{array}{ll} \min_x & f(x) \\ \text{s.t.} & c(x) = 0 \\ & x \geq 0 \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{ll} \min_x & f(x) - \mu \cdot \sum_i \ln(x_i) \\ \text{s.t.} & c(x) = 0 \\ & x \geq 0 \end{array}$$

$\nabla f(x) + \nabla c(x)^T \cdot \lambda - z = 0$   
 $c(x) = 0$   
 $X \cdot z = \mu e$   
 $(x > 0, z > 0)$

$$\begin{bmatrix} W_k + \Sigma_k + \delta_w I & \nabla c(x_k)^T \\ \nabla c(x_k) & -\delta_c I \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta \lambda \end{pmatrix} = - \begin{bmatrix} \nabla \varphi_\mu(x_k) + \nabla c(x_k)^T \lambda_k \\ c(x_k) \end{bmatrix}$$

$(W_k = \nabla_{xx}^2 \mathcal{L} = \nabla_{xx}^2 f(x_k) + \nabla_{xx}^2 c(x_k) \lambda)$   
 $(\delta_w, \delta_c \geq 0) \quad (\Sigma_k = Z_k X_k^{-1})$

## Parallel Decomposition in Interior-Point Methods

$$\begin{array}{ll} \min_x & f(x) \\ \text{s.t.} & c(x) = 0 \\ & x \geq 0 \end{array} \quad \xrightarrow{\hspace{1cm}} \quad \begin{array}{ll} \min_x & f(x) - \mu \cdot \sum_i \ln(x_i) \\ \text{s.t.} & c(x) = 0 \\ & x \geq 0 \end{array}$$

$\nabla f(x) + \nabla c(x)^T \cdot \lambda - z = 0$   
 $c(x) = 0$   
 $X \cdot z = \mu e$   
 $(x > 0, z > 0)$

$$\begin{bmatrix} \nabla f(x) + \nabla c(x)^T \cdot \lambda - z & = 0 \\ c(x) & = 0 \\ X \cdot z & = \mu e \\ (x > 0, z > 0) \end{bmatrix} \quad \xrightarrow{\hspace{1cm}} \quad \begin{bmatrix} \nabla f(x) + \nabla c(x)^T \cdot \lambda - z & = 0 \\ c(x) & = 0 \\ X \cdot z & = \mu e \\ (x > 0, z > 0) \end{bmatrix}$$

$(W_k + \Sigma_k + \delta_w I - \nabla c(x_k)^T \cdot \lambda_k) \left( \begin{array}{c} \Delta x \\ \Delta \lambda \end{array} \right) = - \begin{bmatrix} \nabla \varphi_\mu(x_k) + \nabla c(x_k)^T \lambda_k \\ c(x_k) \end{bmatrix}$   
 $(W_k = \nabla_{xx}^2 \mathcal{L} = \nabla_{xx}^2 f(x_k) + \nabla_{xx}^2 c(x_k) \lambda)$   
 $(\delta_w, \delta_c \geq 0) \quad (\Sigma_k = Z_k X_k^{-1})$

## Parallel Decomposition in Interior-Point Methods

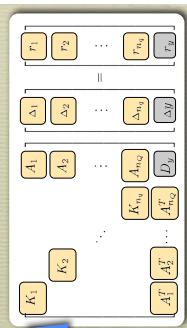
$$\begin{array}{ll} \min_{x,y} & \sum_q f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y = 0, \quad q \in Q \end{array}$$

- Nonlinear Stochastic Optimization
- Large-scale Parameter Estimation
- Design Under Uncertainty
- Spatially Decomposable Problems
- Very large-scale NLP Problems
  - Highly Structured

## Parallel Decomposition in Interior-Point Methods

$$\begin{array}{ll} \min_x & \sum_q f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y = 0, \quad q \in Q \end{array}$$

- Nonlinear Stochastic Optimization
- Large-scale Parameter Estimation
- Design Under Uncertainty
- Spatially Decomposable Problems
- Very large-scale NLP Problems
  - Highly Structured



6

## Parallel Decomposition in Interior-Point Methods

## Parallel Decomposition in Interior-Point Methods

$$\begin{array}{ll} \min_{x_q, y} & \sum_{q \in \mathcal{Q}} f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y y = 0, \end{array}$$

$$\begin{array}{ll} \min_{x_q, y} & \sum_{q \in \mathcal{Q}} f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y y = 0, \end{array}$$

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$$\begin{array}{ll} \min_{x_q, y} & \sum_{q \in \mathcal{Q}} f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y y = 0, \end{array}$$

$$\begin{array}{ll} \min_{x_q, y} & \sum_{q \in \mathcal{Q}} f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y y = 0, \end{array}$$

$$\begin{array}{ll} \min_{x_q, y} & \sum_{q \in \mathcal{Q}} f_q(x_q) \\ \text{s.t.} & c_q(x_q) = 0 \\ & x_q^L \leq x_q \leq x_q^U \\ & L_q^x x_q - L_q^y y = 0, \end{array}$$

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Thursday, September 20, 12

## Other Examples of Applications



Parallel Parameter Estimation for Spatial Transportation Affecting Disease Spread



Optimal Response to Water Contamination Events

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## Fragile tool chain

8  
Tuesday, September 20, 12

## Two Choices

## Two Choices

- 1. Design new language
  - modeling, scripting syntax
  - compiler tools
- 2. Use programming language
  - develop components in another language
  - import types/functionality

- Selected to develop in Python (Choice 2)
  - tired of writing parsers
  - not language experts
  - existing tools are not actively updated
  - not responsible for full language functionality and packages
  - want full-featured language and user-extensibility (for “free”)

## Requirements

- Powerful
  - full support for standard math programming constructs (LP, MILP, NLP, MINLP, ...)
  - full-featured programming environment (model interrogation, scripting, functions, classes, standard & numerical libraries)
  - extensive solver integration - ‘out-of-the-box’
- Open
  - like used under BSD (i.e. really open-source)
  - reduce barriers to adoption, ease of collaboration
  - transparency
- Flexible
  - extensible by users, contributors, not only by us
  - portable (Windows, Linux, OS X)
- Easy
  - language constructs familiar to math programmers - Abstract Models
  - scripting / programming capability well-defined
  - substantial documentation
- License
  - open-source
- Language Features
  - familiar lean syntax, rich set of existing data types, object-oriented, exceptions, dynamic loading, ...
- Support and stability
  - highly stable, well-supported
- Documentation
  - extensive online documentation, several books
- Libraries
  - significant external libraries, numerical & scientific packages
- Portability
  - widely available on many platforms

## Simple Modeling Example: Knapsack

$\mathcal{S}$ : set of items (set)  
 $v_i$ : value of item  $i$  (param)  
 $w_i$ : weight of item  $i$  (param)  
 $W_{max}$ : maximum weight (param)  
 $x_i$ : binary indicator (var)



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$\mathcal{S}$ : set of items (set)  
 $v_i$ : value of items (set)  
 $w_i$ : weight of items (set)  
 $W_m$ : maximum weight (set)  
 $x_i$ : binary indicator (var)

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

$$\text{s.t. } \sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_{max}$$

$$x_i \in \{0, 1\} \quad \forall i \in \mathcal{S}$$

## Knapsack Problem: Abstract Model

Thursday, September 20 | 12

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```

from copr.pyomo import *
model = AbstractModel()
model.ITEMS = SetO
model.v = Param(model.ITEMS, within=PositiveReals)
model.w = Param(model.ITEMS, within=PositiveReals)
model.x = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.v[i]*model.x[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.W_max

model.weight = Constraint()

```

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

$$\text{s.t. } \sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```

from copr.pyomo import *
model = AbstractModel()
model.ITEMS = SetO
model.v = Param(model.ITEMS, within=PositiveReals)
model.w = Param(model.ITEMS, within=PositiveReals)
model.x = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.v[i]*model.x[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.W_max

model.weight = Constraint()

```

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

$$\text{s.t. } \sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```

from copr.pyomo import *
model = AbstractModel()
model.ITEMS = SetO
model.v = Param(model.ITEMS, within=PositiveReals)
model.w = Param(model.ITEMS, within=PositiveReals)
model.x = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.v[i]*model.x[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.W_max

model.weight = Constraint()

```

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

$$\text{s.t. } \sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

## Knapsack Problem: Abstract Model

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## Knapsack Problem: Abstract Model

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## Knapsack Problem: Abstract Model

Thursday, September 20, 12

```

 \mathcal{S} : set of items
 v_i : value of items
 w_i : weight of items
 W_m : maximum weight
 x_i : binary indicator
 $\sum_{i \in \mathcal{S}} v_i \cdot x_i$
 $\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$
s.t.
 $x_i \in \{0, 1\}$

max
 $\sum_{i \in \mathcal{S}} v_i \cdot x_i$
model.value = objective()
model.weight = constraint()
def value_rule(model):
 return sum([model.v[i]*model.x[i] for i in model.ITEMS])
def weight_rule(model):
 return sum([model.w[i]*model.x[i] for i in model.ITEMS])
model.weight = constraint()

from copr.pyomo import *
model = AbstractModel()
model.ITEMS = SetO
model.v = Param(model.ITEMS, within=PositiveReals)
model.w = Param(model.ITEMS, within=PositiveReals)
model.w_max = Param(model.ITEMS, within=PositiveReals)
model.x = Var(model.ITEMS, within=Binary)

def val_ue_rule(model):
 return sum(model.v[i]*model.x[i] for i in model.ITEMS)
model.value = objective(sense=maximize)
def weight_rule(model):
 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.w_max
model.weight = constraint()

```

```

 \mathcal{S} : set of items
 v_i : value of items
 w_i : weight of items
 W_m : maximum weight
 x_i : binary indicator
 $\sum_{i \in \mathcal{S}} v_i \cdot x_i$
 $\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$
s.t.
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 return sum(model.v[i]*model.x[i] for i in model.ITEMS)
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 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.w_max
model.weight = constraint()

from copr.pyomo import *
model = AbstractModel()
model.ITEMS = SetO
model.v = Param(model.ITEMS, within=PositiveReals)
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model.w_max = Param(model.ITEMS, within=PositiveReals)
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def val_ue_rule(model):
 return sum(model.v[i]*model.x[i] for i in model.ITEMS)
model.value = objective(sense=maximize)
def weight_rule(model):
 return sum(model.w[i]*model.x[i] for i in model.ITEMS) \
 <= model.w_max
model.weight = constraint()

```

## Knapsack Problem: Abstract Model

Thursday, September 20, 12

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

s.t.

$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```
from coopr.pyomo import *
model = AbstractModel()
model.ITEMS = Set()
model.V = Param(model.ITEMS, within=PositiveReals)
model.W = Param(model.ITEMS, within=PositiveReals)
model.W_MAX = Param(within=PositiveReals)
model.X = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.V[i]*model.X[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

## Knapsack Problem: Abstract Model

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## Knapsack Problem: Abstract Model

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$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

s.t.

$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```
from coopr.pyomo import *
model = AbstractModel()
model.ITEMS = Set()
model.V = Objective(sense=maximize)
def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
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 $x_i$ : binary indicator

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$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

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```
from coopr.pyomo import *
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model.ITEMS = Set()
model.V = Param(model.ITEMS, within=PositiveReals)
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model.W_MAX = Param(within=PositiveReals)
model.X = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.V[i]*model.X[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

## Knapsack Problem: Abstract Model

Thursday, September 20, 12

Model is completely abstract - there is no data

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

s.t.

$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```
from coopr.pyomo import *
model = AbstractModel()
model.ITEMS = Set()
model.V = Objective(sense=maximize)
def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
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$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

s.t.

$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

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```
from coopr.pyomo import *
model = AbstractModel()
model.ITEMS = Set()
model.V = Param(model.ITEMS, within=PositiveReals)
model.W = Param(model.ITEMS, within=PositiveReals)
model.W_MAX = Param(within=PositiveReals)
model.X = Var(model.ITEMS, within=Binary)

def value_rule(model):
 return sum(model.V[i]*model.X[i] for i in model.ITEMS)

def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

## Knapsack Problem: Abstract Model

22

Model is completely abstract - there is no data

$\mathcal{S}$ : set of items  
 $v_i$ : value of items  
 $w_i$ : weight of items  
 $W_m$ : maximum weight  
 $x_i$ : binary indicator

$$\max \sum_{i \in \mathcal{S}} v_i \cdot x_i$$

s.t.

$$\sum_{i \in \mathcal{S}} w_i \cdot x_i \leq W_m$$

$$x_i \in \{0, 1\}$$

```
from coopr.pyomo import *
model = AbstractModel()
model.ITEMS = Set()
model.V = Objective(sense=maximize)
def weight_rule(model):
 return sum(model.W[i]*model.X[i] for i in model.ITEMS) \
 <= model.W_MAX
model.weight = constraint()
```

## Knapsack Problem: Abstract Model

Tuesday, September 20, 12

## Knapsack Problem: Abstract Model

23

Model is completely abstract - there is no data

```

S: set of items
v_i: value of items
w_i: weight of items
W_m: maximum weight
x_i: binary indicator
 $\sum_{i \in S} v_i \cdot x_i$
s.t.
 $\sum_{i \in S} w_i \cdot x_i \leq W_m$
 $x_i \in \{0, 1\}$

```

> pyomo --solver=glpk knapsack.py akesson\_art.dat

## Knapsack Problem: Abstract Model

Thursday, September 20, 12

```

from pyomo import *
model = ConcreteModel()
model.ITEMS = Set()
model.v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
model.w = {'hammer':5, 'wrench':7, 'screwdriver':4, 'towel':3}
model.W_max = 14
model.x = Var(model.ITEMS, within=Binary)
model.value = Objective(
 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
 sense = maximize)
model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= W_max)
model.sense = maximize)

```

from coopr.pyomo import \*

```

model = AbstractModel()
model.ITEMS = Set()
model.v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
model.w = {'hammer':5, 'wrench':7, 'screwdriver':4, 'towel':3}
model.W_max = 14
model.x = Var(model.ITEMS, within=Binary)
model.value = Objective(
 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
 sense = maximize)
model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= W_max)
model.sense = maximize)

```

## Knapsack Problem: Concrete Model

Thursday, September 20, 12

```

from coopr.pyomo import *
model = ConcreteModel()
model.ITEMS = Set(initialize=v.keys())
model.x = Var(model.ITEMS, within=Binary)
model.value = Objective(
 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
 sense = maximize)
model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= W_max)
model.sense = maximize)

```

from coopr.pyomo import \*

```

model = ConcreteModel()
model.ITEMS = Set(initialize=v.keys())
model.x = Var(model.ITEMS, within=Binary)
model.value = Objective(
 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
 sense = maximize)
model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= W_max)
model.sense = maximize)

```

## Knapsack Problem: Concrete Model

24

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Thursday, September 20, 12

```

from coopr.pyomo import *

v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
w = {'hammer':5, 'wrench':7, 'screwdriver':4, 'towel':3}
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 sense = maximize)

model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= w_max)

```

## Knapsack Problem: Concrete Model

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## Knapsack Problem: Concrete Model

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```

from coopr.pyomo import *

v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
w = {'hammer':5, 'wrench':7, 'screwdriver':4, 'towel':3}
w_max = 14

model = ConcreteModel()
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 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
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model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= w_max)

```

```

from coopr.pyomo import *

v = {'hammer':8, 'wrench':3, 'screwdriver':6, 'towel':11}
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w_max = 14

model = ConcreteModel()
model.ITEMS = Set(initialize=v.keys())
model.x = Var(model.ITEMS, within=Binary)

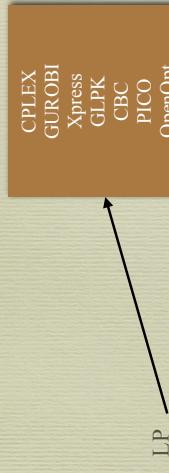
model.value = Objective(
 expr = sum(v[i]*model.x[i] for i in model.ITEMS),
 sense = maximize)

model.weight = Constraint(
 expr = sum(w[i]*model.x[i] for i in model.ITEMS) <= w_max)

```

## Solver Interfaces

Thursday, September 20, 12

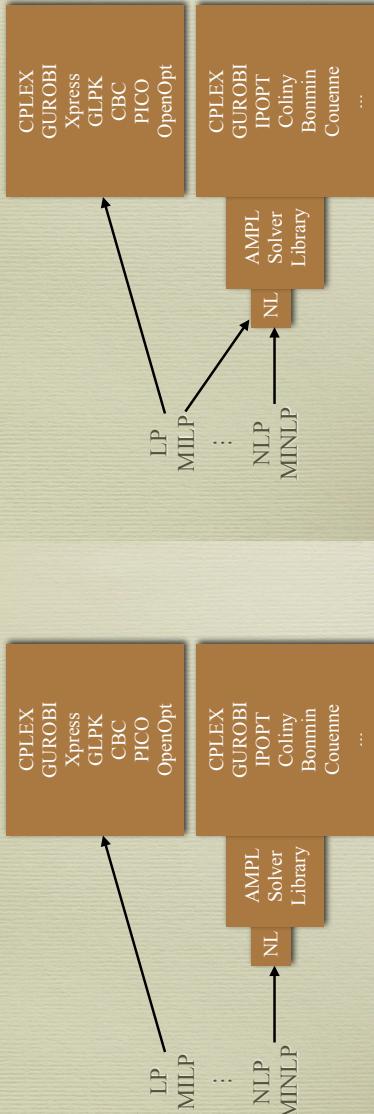


## Knapsack Problem: Concrete Model

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## Solver Interfaces



## Other Pyomo Features

- Advanced scripting capability
  - functions, OO, model interrogation & transformation
- Extensive set operations, tuples, multi-dimensional
- Load data from different sources
  - AMPL dat files, CSV files, Excel, databases
- Support for custom workflow with plugins
  - e.g. preprocess, create\_modeldata, save\_instance
- And more with extensions...

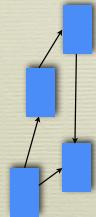
## Summary

- Pyomo is an equation-based, algebraic modeling language for optimization
- Pyomo is an object-oriented framework for building optimization-based applications
- Based on Python
  - simple syntax for modeling
  - full-featured language
- Significant solver integration
  - Open-source and Extensible
    - PySP: Stochastic Programming Framework
    - PH: Progressive Hedging Framework
    - Generalized Disjunctive Programming Capability
    - Blocks - Connectors
      - Piecewise-linear Constraints

## Some Closing Comments

- Performance?
  - Python is slow... but not that slow
  - Time dominated by solution, not construction
  - Compiled code for solver/AD
- Flat Model Specification
  - Abstract models
  - Computer scientists
- Object-Oriented Modeling
  - Concrete models
  - Programmatic creation
  - Engineers
- Karl Åström's Comment: Don't just do what you did before with new technology

$$\begin{aligned} & \max \sum_{i \in S} v_i \cdot x_i \\ \text{s.t. } & \sum_{i \in S} w_i \cdot x_i \leq W_m \\ & x_i \in \{0, 1\} \end{aligned}$$



## Acknowledgments (Development Community)

- Sandia National Laboratories
  - Bill Hart
  - Jean-Paul Watson
  - Ivan Smirnov
  - David Hart
  - Tom Brunsheim
  - University of California, Davis
  - Prof. David L. Woodruff
  - Prof. Roger Wets
  - Texas A&M University
  - Prof. Carl D. Laird
  - Daniel Word
  - James Young
  - Gabe Hackeborn
  - Texas Tech University
  - Zey Friedman
  - Rose Human Institute
  - Tim Ertl
  - William & Mary
  - Patrick Steege
  - North Carolina State
  - Kevin Hunter
- Plus our many users, including:
  - University of California, Davis
  - Texas A&M University
  - University of Texas
  - Rose-Hulman Institute of Technology
  - University of Southern California
  - George Mason University
  - Iowa State University
  - N.C. State University
  - University of Washington
  - Naval Postgraduate School
  - Universidad de Santiago de Chile
  - University of Pisa
  - Lawrence Livermore National Lab
  - Los Alamos National Lab

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## Learn More

- Project Homepage
  - <http://software.sandia.gov/coopr>

**Pyomo – Optimization Modeling in Python**

- Pyomo and PySP papers

Pyomo: Modelling and Solving Mathematical Programs in Python (Vol. 3, No. 3, 2011)  
PySP: Modelling and Solving Stochastic Programs in Python (Vol. 4, No. 2, 2012)



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Thursday, September 20 | 12

**EFFICIENT SYMBOLICAL AND NUMERICAL ALGORITHMS  
FOR NONLINEAR MODEL PREDICTIVE CONTROL WITH  
OPENMODELICA**

**Bernhard Bachmann, Fachhochschule Bielefeld University  
of Applied Sciences**

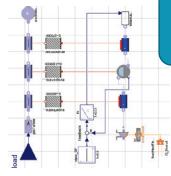
During the last decade nonlinear model predictive control (NMPC) has become increasingly important for today's control engineers. In order to apply NMPC a nonlinear optimal control problem (NOCP) must be solved, which needs a very high computational effort. Nowadays, corresponding modeling of the system dynamics and formulation of the optimization problem can be done in Modelica and Optimica, respectively.

State-of-the-art NOCP solution algorithms are based on multiple shooting and/or collocation algorithms. Only parallelizing these time-consuming algorithms can give reasonable performance appropriate for online-applications. In addition, efficient symbolical and numerical treatment of the underlying model formulation (e.g. matching, sorting, and tearing) are necessary, when solving NOCP involving complex system. Furthermore, for performance and stability reasons NOCP corresponding symbolically derived Jacobian and Hessian matrices and their efficient computation (e.g. identify and utilize the sparsity pattern of Jacobian matrices) are needed. This talk will discuss these mathematical aspects of NMPC as well as the current and future implementation of efficient, partly parallelized symbolical and numerical algorithms available in and with OpenModelica.



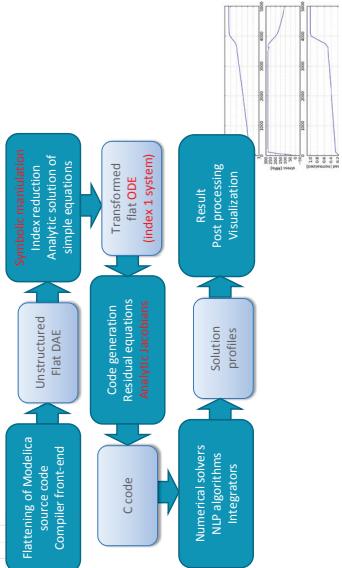
# Efficient Symbolical and Numerical Algorithms for nonlinear model predictive control with OpenModelica

Bernhard Bachmann  
Department of Engineering and Mathematics  
University of Applied Sciences Bielefeld



## A Modelica-based Tool Chain

(Johan Åkesson)



## Outline

1. Excerpt of OpenModelica's symbolic machinery
2. Symbolically derived Jacobians
  - i. Directional derivatives
  - ii. Sparsity pattern
  - iii. Coloring of the Jacobian
3. Nonlinear Optimal Control Problem
  - i. General Discretization Scheme
  - ii. Multiple Shooting/Collocation
  - iii. Total Collocation
  - iv. Applications
4. Lessons learned & Outlook

## Basic Transformation Steps

Transformation to explicit state-space representation:

$$\begin{aligned} \underline{0} &= \underline{f}(t, \underline{x}(t), \underline{x}(t), \underline{y}(t), \underline{u}(t), \underline{p}) \\ &\quad \downarrow \\ \underline{0} &= \underline{f}(t, \underline{x}(t), \underline{x}(t), \underline{y}(t), \underline{u}(t), \underline{p}), \quad \underline{x}(t) = \begin{pmatrix} \underline{\dot{x}}(t) \\ \underline{y}(t) \end{pmatrix} = \underline{g}(t, \underline{x}(t), \underline{u}(t), \underline{p}) \end{aligned}$$

$$\underline{0} = \underline{f}(t, \underline{x}(t), \underline{x}(t), \underline{y}(t), \underline{u}(t), \underline{p}), \quad \underline{x}(t) = \begin{pmatrix} \underline{\dot{x}}(t) \\ \underline{y}(t) \end{pmatrix} = \underline{g}(t, \underline{x}(t), \underline{u}(t), \underline{p})$$

Implicit function theorem:

Necessary condition for the existence of the transformation is that the following matrix is regular at the point of interest:

$$\det \left( \frac{\partial}{\partial \underline{z}} \underline{f}(t, \underline{z}(t), \underline{x}(t), \underline{u}(t), \underline{p}) \right) \neq 0$$

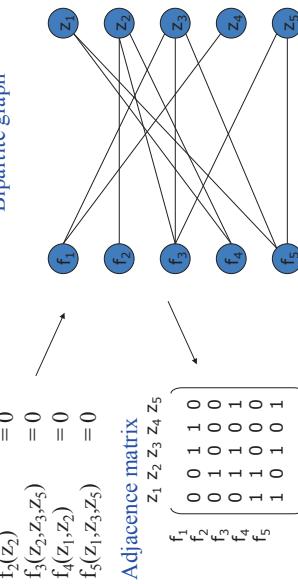
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Borrelli et al., 2014, p. 15

## DAEs and Bipartite Graph Representation

Example of a regular DAE:

$$\underline{0} = \underline{f}(t, \underline{z}(t), \underline{x}(t), \underline{y}(t), \underline{u}(t), \underline{p}), \quad \underline{z}(t) = \begin{pmatrix} \underline{\dot{x}}(t) \\ \underline{y}(t) \end{pmatrix}$$

Bipartite graph



Adjacency matrix

$$\begin{matrix} & z_1 & z_2 & z_3 & z_4 & z_5 \\ z_1 & 0 & 0 & 1 & 1 & 0 \\ f_1 & 0 & 0 & 1 & 0 & 0 \\ f_2 & 0 & 1 & 0 & 0 & 0 \\ f_3 & 0 & 1 & 1 & 0 & 1 \\ f_4 & 1 & 1 & 0 & 0 & 0 \\ f_5 & 1 & 0 & 1 & 0 & 1 \end{matrix}$$

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## Symbolic Transformation Algorithmic Steps

- DAEs and bipartite graph representation
  - Structural representation of the equation
- The matching problem
  - Assign to each variable exact one equation
  - Same number of equations and unknowns
- Construct a directed graph
  - Find sinks, sources and strong components
  - Sorting the equation system
- Adjacency Matrix and structural regularity
  - Block-lower triangular form (BLT-Transformation)

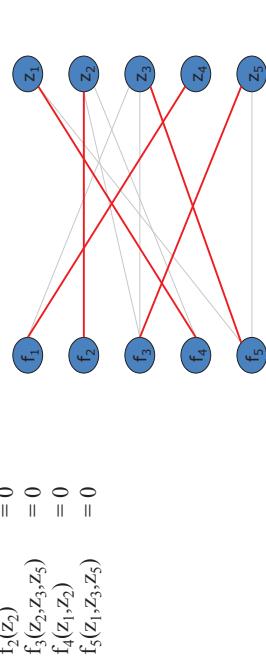
MODELICA

## Solve the Matching Problem

Example of a regular DAE:

$$\underline{0} = \underline{f}(t, \underline{z}(t), \underline{x}(t), \underline{y}(t), \underline{u}(t), \underline{p}), \quad \underline{z}(t) = \begin{pmatrix} \underline{\dot{x}}(t) \\ \underline{y}(t) \end{pmatrix}$$

Bipartite graph



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## Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica

### Bibliography

#### Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica

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### Bibliography

#### Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica

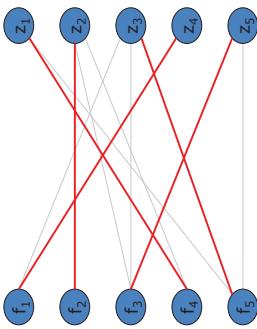
Borrelli et al., 2014

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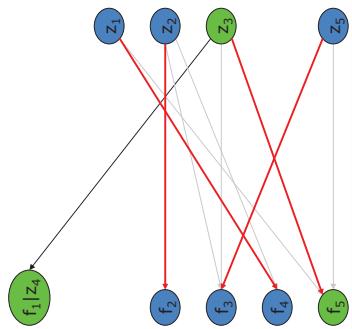
### Construct a Directed Graph

### Construct a Directed Graph



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### Construct a Directed Graph

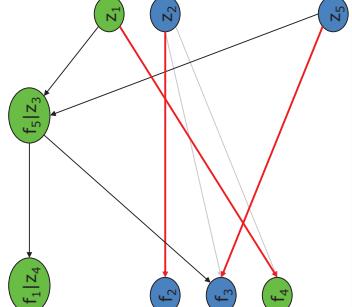


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### Construct a Directed Graph

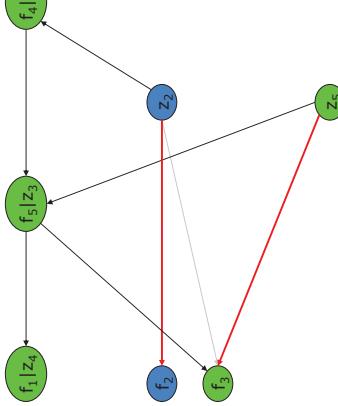


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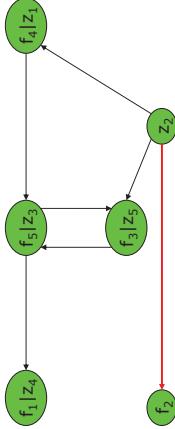
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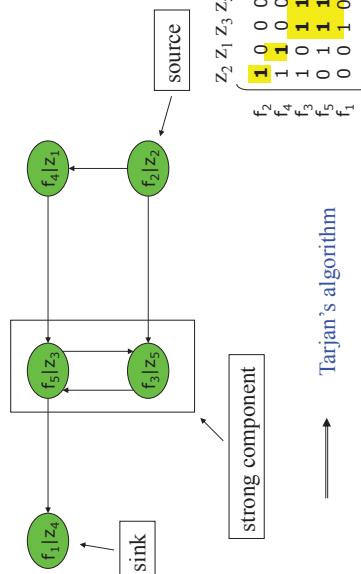
## Construct a Directed Graph



## Construct a Directed Graph



## Construct a Directed Graph



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## Further Efficiency Issues - Dummy-Derivative Method

- Matching algorithm fails
  - System is structurally singular
  - Find minimal subset of equations
    - more equations than unknown variables
  - Singularity is due to equations, constraining states
- Differentiate subset of equations
  - Static state selection during compile time
    - choose one state and corresponding derivative as purely algebraic variable
      - o so-called dummy state and dummy derivative
    - by differentiation introduced variables are algebraic
  - continue matching algorithm
  - check initial conditions
  - Dynamic state selection during simulation time
    - store information on constrained states
    - make selection dynamically based on stability criteria
    - new state selection triggers an event (re-initialize states)

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## Further Efficiency Issues – Algebraic Loops

- Solution of linear equation systems
  - Advanced solver packages (e.g. LAPACK) are used
  - Calculate LU-Decomposition for constant matrices
  - Small systems are inverted symbolically
- Solution of nonlinear systems
  - Advanced solver packages are used
    - Performance is depending on good starting values
    - Analytical Jacobian is provided symbolically
  - Tearing systems of equations
    - Reducing the iteration variables dramatically
  - Analytical Jacobians of the overall system
    - Minimize simulation/integration time needed

## Outline

- Excerpt of OpenModelica's symbolic machinery
- Symbolically derived Jacobians
  - Directional derivatives
  - Sparcity pattern
  - Coloring of the Jacobian
- Nonlinear Optimal Control Problem
  - General Discretization Scheme
  - Multiple Shooting/Collocation
  - Total Collocation
  - Applications
- Lessons learned & Outlook

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## Fast Simulation of Fluid Models with Colored Jacobians

**Willi Braun, Bernhard Bachmann**  
**Department of Engineering and Mathematics**  
**University of Applied Sciences Bielefeld**

**Stephanie Gallardo Yances, Kilian Link**  
**Siemens AG, Energy Section**  
**Erlangen**

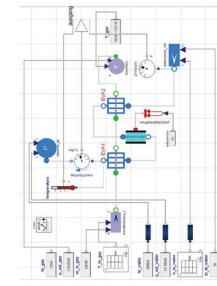
(see 9th International Modelica Conference)

## Symbolically Generation of Jacobians

How is simulation time effected by Jacobians?

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| Fluid Test Model  |      |
|-------------------|------|
| States            | 231  |
| Equations         | 942  |
| Simulation time   | 10.8 |
| J evaluations     | 111  |
| J evaluation time | 9.7  |

The evaluation of Jacobians effects  
the simulation time a lot!

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## Symbolically Generation of Jacobians

### Symbolically Generation of Jacobians

#### State-Space Equations

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} h(\underline{x}(t), \underline{u}(t), p; t) \\ k(\underline{x}(t), \underline{u}(t), p; t) \end{pmatrix}$$

#### Jacobian

$$J_A = \frac{\partial h}{\partial \underline{x}} = \begin{pmatrix} \frac{\partial h_1}{\partial x_1} & \dots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial x_1} & \dots & \frac{\partial h_n}{\partial x_n} \end{pmatrix}$$

#### Jacobian matrices

#### Simulation

- $A(t) = \frac{\partial h}{\partial \underline{x}}$

- $B(t) = \frac{\partial k}{\partial \underline{x}}$

- $C(t) = \frac{\partial k}{\partial \underline{u}}$

- $D(t) = \frac{\partial k}{\partial \underline{u}}$

## Symbolically Generation of Jacobians

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### Symbolically Generation of Jacobians

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#### Jacobian

$$J_A = \frac{\partial h}{\partial \underline{x}} \left( \frac{\partial \underline{x}}{\partial z} \right) = \begin{pmatrix} \frac{\partial h_1}{\partial z} & \dots & \frac{\partial h_1}{\partial z} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial z} & \dots & \frac{\partial h_n}{\partial z} \end{pmatrix}$$

#### Generic Directional Derivative

$$J_A = \frac{\partial h}{\partial \underline{z}} (\underline{\varepsilon}_k)$$

#### Jacobian

$$J_A = \frac{\partial h}{\partial \underline{z}} \left( \frac{\partial \underline{x}}{\partial z} \right) = \begin{pmatrix} \frac{\partial h_1}{\partial z} & \dots & \frac{\partial h_1}{\partial z} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_n}{\partial z} & \dots & \frac{\partial h_n}{\partial z} \end{pmatrix}$$

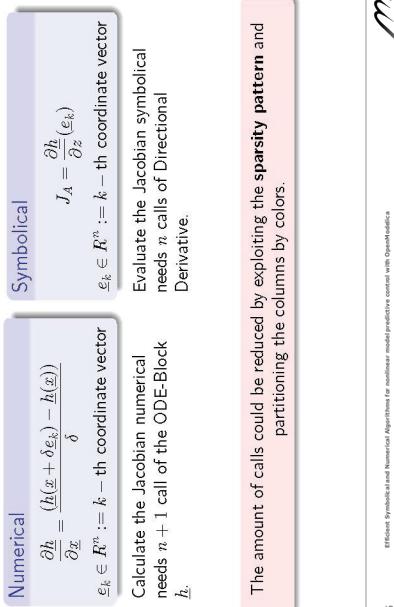
$$\begin{aligned} \frac{\partial d_{\text{err}}(k1)}{\partial z} &= \frac{\frac{\partial F1}{\partial z} * A2 - P1 * \frac{\partial A2}{\partial z}}{A2^2} - \frac{\frac{\partial F2}{\partial z} * A3 - P2 * \frac{\partial A3}{\partial z}}{A2^2} \\ \frac{\partial d_{\text{err}}(k2)}{\partial z} &= \frac{\frac{\partial F1}{\partial z} * A2 - P1 * \frac{\partial A2}{\partial z}}{A2^2} - \frac{\frac{\partial F2}{\partial z} * A3 - P2 * \frac{\partial A3}{\partial z}}{A2^2} \\ \frac{\partial d_{\text{err}}(k1)}{\partial z} &= R1 * \frac{\frac{\partial F1}{\partial z} * A2 - P1 * \frac{\partial A2}{\partial z}}{2 * \sqrt{A1 - k2}} \\ \frac{\partial d_{\text{err}}(k2)}{\partial z} &= R2 * \frac{\frac{\partial F2}{\partial z} * A3 - P2 * \frac{\partial A3}{\partial z}}{2 * \sqrt{A2 - k1}} \\ \frac{\partial d_{\text{err}}(k1)}{\partial z} &= \frac{\frac{\partial P}{\partial z} * A1 - P * \frac{\partial A1}{\partial z}}{A1^2} - \frac{\frac{\partial P}{\partial z} * A1 - P * \frac{\partial A1}{\partial z}}{A1^2} \end{aligned}$$

## Symbolically Generation of Jacobians

### Compute sparsity pattern of the Jacobians



The amount of calls could be reduced by exploiting the **sparsity pattern** and partitioning the columns by colors.



The amount of calls could be reduced by exploiting the **sparsity pattern** and partitioning the columns by colors.

## Compute sparsity pattern of the Jacobians

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### Utilize sparsity pattern of the Jacobians

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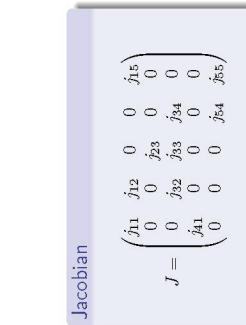
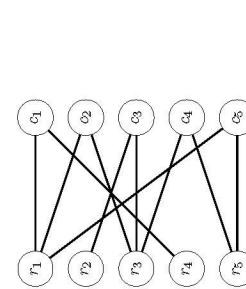
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$$J = \begin{pmatrix} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ * & 0 & 0 & * \\ 0 & * & 0 & 0 \\ 0 & 0 & * & * \end{pmatrix}$$

Jacobian

$$J = \begin{pmatrix} j_{11} & j_{12} & 0 & 0 & j_{15} \\ 0 & 0 & j_{23} & 0 & 0 \\ 0 & j_{32} & j_{33} & j_{34} & 0 \\ j_{41} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j_{54} & j_{55} \end{pmatrix}$$

$$J = \begin{pmatrix} 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \\ * & 0 & 0 & * \\ 0 & * & 0 & 0 \\ 0 & 0 & * & * \end{pmatrix}$$



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| z1 | z3 | z4 | z2 | z5 | x1 | x2 | x3 | x4 | x5 | Accumulation Lists |
|----|----|----|----|----|----|----|----|----|----|--------------------|
| f4 | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | f4: <5>            |
| f5 | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | f5: <5>            |
| f1 | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 1  | 0  | f1: <1,3,5>        |
| f2 | 0  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | f2: <2>            |
| f3 | 0  | 1  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | f3: <5,2>          |

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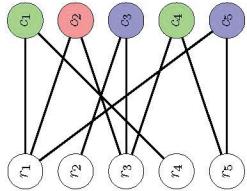
## Utilize sparsity pattern of the Jacobians

Jacobian

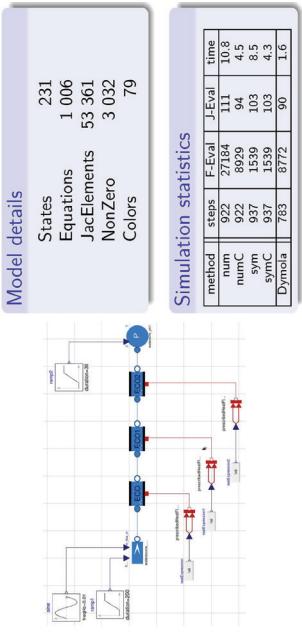
$$J = \begin{pmatrix} j_{11} & j_{12} & 0 & 0 & j_{15} \\ 0 & 0 & j_{23} & 0 & 0 \\ 0 & j_{32} & j_{33} & j_{34} & 0 \\ 0 & 0 & 0 & 0 & j_{45} \\ 0 & 0 & 0 & j_{54} & j_{55} \end{pmatrix}$$

$$J_R = \begin{pmatrix} j_{11} & j_{12} & j_{15} \\ 0 & 0 & j_{23} \\ j_{34} & 0 & 0 \\ j_{54} & 0 & j_{55} \end{pmatrix}$$

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## Performance gain of implementation



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## Parallel Multiple-Shooting and Collocation Optimization with OpenModelica

Bernhard Bachmann, Lennart Oehel, Vitalij Ruge  
Mathematics and Engineering  
University of Applied Sciences Bielefeld

Mahder Gebremedhin, Peter Fritzson,  
PELAB – Programming Environment Lab  
Vahheed Nezhadali, Lars Eriksson, Martin Sivertsson  
Vehicular Systems  
Linköping University

(see 9th International Modelica Conference)



## Outline

1. Excerpt of OpenModelica's symbolic machinery
2. Symbolically derived Jacobians
  - i. Directional derivatives
  - ii. Sparsity pattern
  - iii. Coloring of the Jacobian
3. Nonlinear Optimal Control Problem
  - i. General Discretization Scheme
  - ii. Multiple Shooting/Collocation
  - iii. Total Collocation
  - iv. Applications
4. Lessons learned & Outlook

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## Nonlinear Optimal Control Problem (NOCP)

Mathematical problem formulation

- objective function

$$\min_{u(t)} J(x(t), u(t), t) = E \left( x(t_f) \right) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt$$

- subject to

$$\begin{aligned} x(t_0) &= h_0 && \text{initial conditions} \\ \dot{x}(t) &= f(x(t), u(t), t) && \text{DAEs, Modelica} \\ g(x(t), u(t), t) &\geq 0 && \text{path constraints} \\ r(x(t_f), y(t_f)) &= 0 && \text{terminal constraints} \end{aligned}$$

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## Theoretical Background

Multiple Shooting/Collocation

- Solve sub-problem in each sub-interval

$$x_i(t_{i+1}) = h_i + \int_{t_i}^{t_{i+1}} f(x_i(t), u(t), t) dt \approx F(t_i, t_{i+1}, h_i, u_i)$$

$$x_i(t_i) = h_i$$

- Discretized Nonlinear Optimal Control Problem

$$\begin{aligned} \min_{u(t)} J(x(t), u(t), t) &= E(h_n) + \frac{\Delta t}{2} \sum_{i=0}^{n-1} L(h_i, u_i, t_i) + L(h_{i+1}, u_i, t_i) \\ &- \text{subject to} \end{aligned}$$

$$\begin{aligned} x(t_0) &= h_0 \\ F(t_i, t_{i+1}, h_i, u_i) &= h_{i+1} \\ g(h_i, u_i, t_i) &\geq 0 \\ g(h_{i+1}, u_i, t_{i+1}) &\geq 0 \end{aligned}$$

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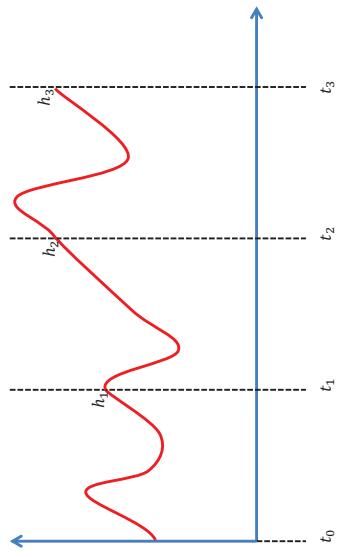
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## Theoretical Background

General discretization scheme

$$x_i(t_i) = h_i$$



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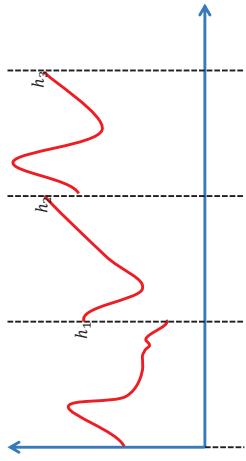
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- Multiple Shooting / Collocation Optimization

## Theoretical Background

Multiple Shooting/Collocation

- Discretized Nonlinear Optimal Control Problem
- objective function (integral approximation by trapezoidal rule)

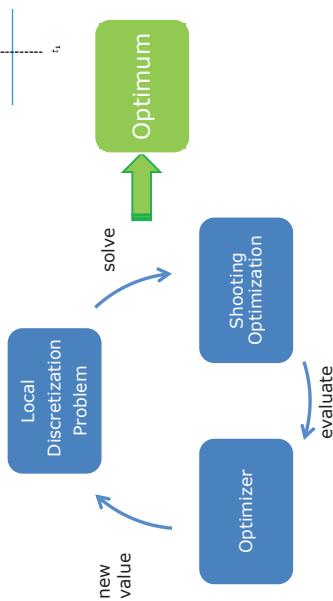


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## Theoretical Background

### Multiple Shooting / Collocation Optimization



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## Theoretical Background

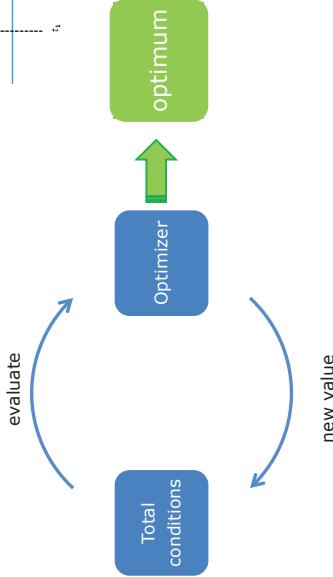
### Total Collocation Optimization

- Discretized Nonlinear Optimal Control Problem
  - objective function (integral approximation by Gauß quadrature)
 
$$\min_{u(t)} J(x(t), u(t), t) = E(h_n) + \Delta t \sum_{j=0}^m w_j \cdot \sum_{i=0}^{n-1} L(h_i^{(j)}, u_i, t_i + s_j)$$
  - subject to
 
$$\begin{aligned} x(t_0) &= h_0 \\ g(h_i, u_i, t_i) &\geq 0 \\ g(h_{i+1}, u_i, t_{i+1}) &\geq 0 \end{aligned}$$

$$\text{additional collocation conditions}$$

## Theoretical Background

### Total Collocation Optimization



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## Theoretical Background

### Collocation Condition – Approximation of States

- Assumption:  
States are locally polynomial
- Collocation Condition
 
$$x_i(t_i + \hat{s}_k \cdot \Delta t) = p_0(\hat{s}_k) \cdot h_i^{(m)} + \sum_{j=1}^m p_j(\hat{s}_k) \cdot h_i^{(j)}$$
- where  $x_i(t_i + \hat{s}_k \cdot \Delta t) = \delta_{k,0} \cdot h_i^{(m)} + \sum_{j=1}^m \delta_{k,j} \cdot h_i^{(j)} = h_i^{(k)}$
- $\hat{s}_k$  are the Radau points
- $p_j(\hat{s}_k)$  are the Lagrange Basis polynomial to the nodes  $\hat{s}_k$

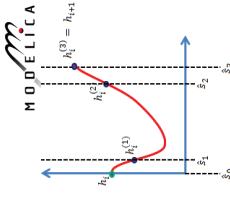
• Collocation conditions

$$\Delta t \cdot f(h_i^{(j)}, u_i, t_i + \hat{s}_k \cdot \Delta t) = p_0'(\hat{s}_k) \cdot h_{i-1}^{(m)} + \sum_{j=1}^m p_j'(\hat{s}_k) \cdot h_i^{(j)}$$

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## Theoretical Background

Collocation Condition – Approximation of State Derivatives

- Assumption:

State derivatives are locally polynomial

$$\Delta t \cdot f(x_i(t_i + \dot{s} \cdot \Delta t), u_i, t_i + \dot{s} \cdot \Delta t) = \sum_{j=0}^m p_j(\dot{s}) \cdot f_l^{(j)}$$

$$\text{where } \Delta t \cdot f(h_i^{(k)}, u_i, t_i + \dot{s}_k \cdot \Delta t) = \sum_{j=0}^m \delta_{k,j} \cdot f_l^{(j)} = f_i^{(k)}$$

$\dot{s}_j$  are the Lobatto points

$p_j(\dot{s})$  are the Lagrange Basis polynomial to the nodes  $\dot{s}_j$

- Collocation conditions

$$h_i^{(k)} = \sum_{l=0}^m p_j(\dot{s}_k) \cdot f_l^{(j)} + h_{i-1}^{(m)}$$



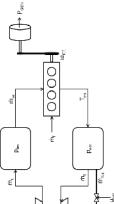
## Applications – Diesel Electric Powertrain

- Assumption:
- Find fuel optimal control and state trajectories from idling condition to a certain power level

Nonlinear mean value engine model

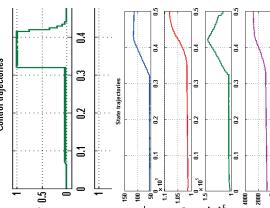
- Only diesel operating condition

- Mathematical problem formulation:
  - 2 inputs ( $u_f, u_w$ )
  - 4 states ( $w_{ice}, p_m, p_{em}, \omega_{ic}$ )
  - 32 algebraic equations



## Applications – Diesel Electric Powertrain

Control trajectories



- Mathematical problem formulation
  - Object function
  - $\min_{u(t)} \sum_{i=1}^4 (x_i(t_f) - x_i^{ref})^2 + \int_0^{t_f} \dot{m}^f dt$
  - subject to

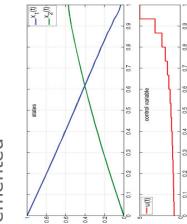
$$\begin{aligned} \dot{x}_1 &= f_1(x_2, x_3, u_1) \\ \dot{x}_2 &= f_2(x_1, x_2, x_4) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3, u_1, u_2) \\ \dot{x}_4 &= f_4(x_2, x_3, x_4, u_2) \end{aligned}$$

$$x_{lb,i} \leq x_i \leq x_{ub,i} \quad i = 1, \dots, 4$$

$$0 \leq u_1, u_2 \leq 1$$

- Processor:
  - 2xIntel Xeon CPU E5-2650
  - 16 cores @ 2.00GHz
  - OpenMP

Engine is accelerated only near the end of the time interval to meet the end constraints while minimizing the fuel consumption



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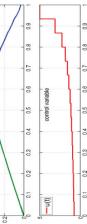
## Implementation Details – Current Status

- Realization with OpenModelica Environment
- Optimica prototype implementation is available
- Using Ipopt for solution process
- Necessary derivatives are numerically calculated
  - Gradients, Jacobians, Hessians, ...

- But: Complete tool chain not yet implemented

## Test Environment

- Processor:
  - 2xIntel Xeon CPU E5-2650
  - 16 cores @ 2.00GHz



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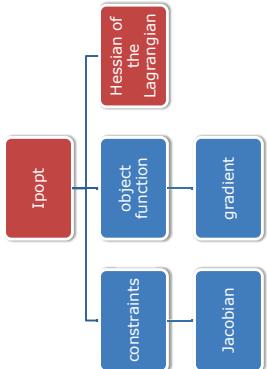
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## Implementation Details – Ipopt & Parallelization

- Schematic view of the required components of Ipopt



**Results - Diesel Electric Powertrain**

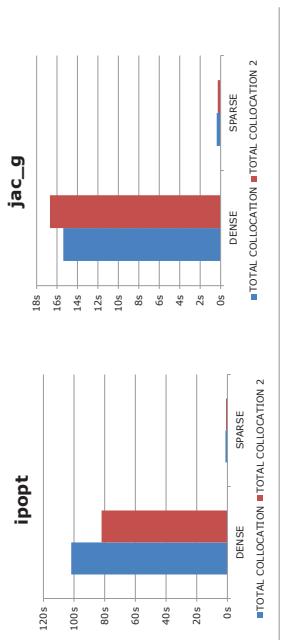
|         | Diesel  | MULTIPLE SHOOTING | MULTIPLE COLLOCATION | TOTAL COLLOCATION | TOTAL COLLOCATION 2 |
|---------|---------|-------------------|----------------------|-------------------|---------------------|
| 921, 6s | 921, 6s | 29519, 8s         | 9, 5s                | 15, 6s            |                     |

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## Implementation Details - Numerical Optimization

- Enormous speed-up when utilizing sparse Jacobian matrix
- Speed-up for the over-all optimization
  - Sparse-structure model independent



**Results - Sparse**

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## Results - Diesel Electric Powertrain

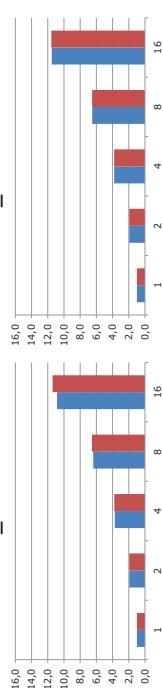
- Ipopt runs in serial mode
- Most execution time is elapsed in Jacobian calculation and solution process of the local discretization problem
- Reasonable speed-up
- Factors are non-optimal due to memory handling
  - Further investigations will be performed

**Results - Diesel Electric Powertrain**

Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica  
Bernhard Strohm, et. al.

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## MULTIPLE\_SHOOTING

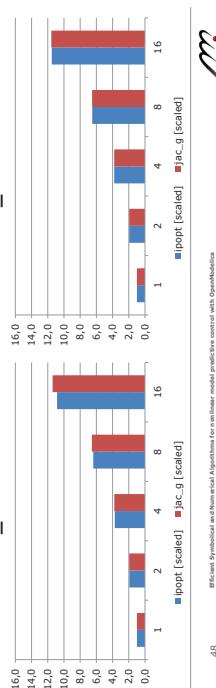


**MULTIPLE\_SHOOTING**

Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica  
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## MULTIPLE\_COLLOCATION



**MULTIPLE\_COLLOCATION**

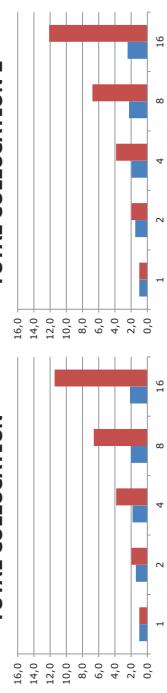
Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica  
Bernhard Strohm, et. al.

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## Results - Diesel Electric Powertrain

- Ipopt runs in serial mode
- Less execution time is elapsed in Jacobian calculation
- Reasonable speed-up for Jacobian calculation
- Factors are non-optimal due to memory handling
- Overall Speed-up increases with model complexity
  - Parallelizing of Ipopt necessary

## TOTAL COLLOCATION



49 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

## Example 1 – From the dark side (Francesco Casella)

50 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

51 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

## Example 2 – From the dark side (Francesco Casella)

52 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

optimization Example1A{

```
objective = a1/b1*(x1 - x10)^2 +
 a2/b2*(x2 - x20)^2 +
 a3/b3*(x3 - x30)^2,
```

...  
end Example1A;

optimization Example1B{

```
objective = f1 + f2 + f3,
```

...  
equation

```
f1 = a1/b1*(x1 - x10)^2;
f2 = a2/b2*(x2 - x20)^2;
f3 = a3/b3*(x3 - x30)^2;
```

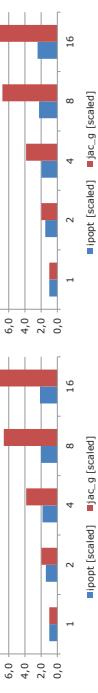
...  
end Example1B;

## Lessons learned

- Symbolic calculation of derivatives improve performance
  - Jacobian, Gradient, ...
- Utilizing sparsity pattern is crucial
- In serial mode total collocation methods are superior to multiple shooting/collation methods

- Parallelizing the algorithms performs better on multiple shooting/collation methods
- Symbolic transformation to ODE form is a key issue for the realization of an automatic tool chain

## TOTAL COLLOCATION 2



51 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

52 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

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80 Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenOpt

## Future work

- Implement complete tool chain in OpenModelica
- Automatic generation of simulation code based on Optimica
- Utilizing symbolically derived derivative information
  - Gradient, Jacobian, Hessian, ...
- Further improvements with appropriate scaling
- Exploiting parallel evaluation of the optimization method
- Advanced use of OMC symbolic machinery
  - Efficient handling of model dependent algebraic loops
- Generalization of NOCP problem formulation
  - e.g. time minimal optimization, parameter estimation
- Further testing on industrial-relevant problems

Thank you  
Questions?



Efficient Symbolic and Numerical Algorithms for nonlinear model predictive control with OpenModelica  
Borislava Douncheva, et. al.



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## MODELING SEEN AS PROGRAMMING

**Klaus Havelund, Jet Propulsion Laboratory, California Institute of Technology**

Code generation often requires the model to be concrete, at which point the distinction between model and code disappears from a philosophical point of view, and we are back in the situation where there is only code. If the purpose of the model is to be an alternative statement of the solution, which the code can be checked against, this approach fails to deliver that. Verification of code against model is challenging and suffers from computational complexity. Models can, however, be used for monitoring program execution. In this approach, often referred to as runtime verification, code is instrumented to emit events when executed. The generated execution trace (a sequence of events) is then monitored against the model, and if a discrepancy is detected according to the model, an error can be reported. Runtime verification can be performed during testing, either as the system executes, or post-mortem, by analyzing generated logs; or it can be performed during the actual operation of the software. We shall demonstrate an RV system called TraceContract, which in essence is an API in the high-level Scala programming language. The API offers a range of methods for writing models that are suited for trace analysis. This includes data parameterized state machines, temporal logic, and rule-based programming. Common for these techniques is the reliance on rewriting as the basis for the implementation. We argue that for certain forms of trace analysis, and modeling in general, the best weapon is a high level programming language augmented with constructs for temporal reasoning.

## Modeling Seen as Programming

Klaus Havelund

NASA JPL, California Inst. of Technology, USA

System Design meets Equation-based Languages

September 21, 2012



MSL



Landing



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## Acknowledgements

Part of the work described in this publication was carried out at Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

3.5 million lines of C code

A circular watermark is centered on the page. It contains the word "ERROR" in a large, bold, red sans-serif font. The background of the watermark is black, making the red text stand out. The watermark is set against a background of a repeating binary code pattern (0s and 1s) that covers the entire page.

Terminology

- model engineering = engineering models

Terminology

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## Terminology

## Terminology

- model engineering = engineering models
- model-based engineering
- mode-based programming

## Terminology

- model engineering = engineering models
- model-based engineering
- mode-based programming
- models, specifications used in software engineering (formal methods)

## Runtime verification

- Start with a system to monitor.

system

## Runtime verification

- Instrument the system to record relevant events.

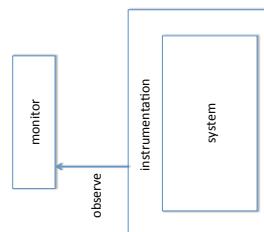
## Runtime verification

- Provide a monitor.



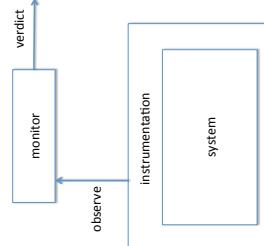
## Runtime verification

- Dispatch each received event to the monitor.



## Runtime verification

- Compute a verdict for the trace received so far.



## Runtime verification

- Possibly generate *feedback* to the system.

## Runtime verification

- We might possibly have synthesized monitor from a *property*.



## External versus internal DSL



```

COMMAND "STOP CAMERA" 1:22:50.001
COMMAND "ORIENT_ANTENNA_TOWARDS_GROUND" 2:22:50.101
requirements
COMMAND "ORIENT_ANTENNA_TOWARDS_GROUND" 3:22:52.021
relating events
across time
COMMAND "STOP_CAMERA" 4:22:55.011
SUCCESS "ORIENT_ANTENNA_TOWARDS_GROUND" 5:22:56.191
COMMAND "STOP_ALL" 6:23:01.101
FAIL("ORIENT_ANTENNA_TOWARDS_GROUND", 7:23:02.02)

```

## External versus internal DSL

- **External DSL**

- External DSL
  - ▶ small language typically with very focused functionality

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## External versus internal DSL

- **External DSL: LogScope**
  - ▶ small language typically with very focused functionality
  - ▶ specialized **parser**
  - ▶ pros:
    - \* can be optimally succinct
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- **Internal DSL: TraceContract**
  - ▶ an extension of an existing programming language
  - ▶ typically an **API** - using base language's features only
  - ▶ pros:
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## LogScope V1 syntax

```

A.1. LOGSCOPING_ABSTRACT
A.1.1. CLASS_ABSTRACT
CLASS_ABSTRACT --> NAME ::= (NAME
 | (NAME >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> L ::= (L
 | (L >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> C ::= (C
 | (C >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> I ::= (I
 | (I >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> T ::= (T
 | (T >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> P ::= (P
 | (P >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> R ::= (R
 | (R >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> S ::= (S
 | (S >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> M ::= (M
 | (M >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> O ::= (O
 | (O >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> D ::= (D
 | (D >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> F ::= (F
 | (F >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> G ::= (G
 | (G >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> H ::= (H
 | (H >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> J ::= (J
 | (J >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> K ::= (K
 | (K >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> L ::= (L
 | (L >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> N ::= (N
 | (N >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> Q ::= (Q
 | (Q >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> U ::= (U
 | (U >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> V ::= (V
 | (V >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> W ::= (W
 | (W >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> X ::= (X
 | (X >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> Y ::= (Y
 | (Y >--> CLASS_ABSTRACT))
CLASS_ABSTRACT --> Z ::= (Z
 | (Z >--> CLASS_ABSTRACT))

A.1.2. ATTRIBUTE
ATTRIBUTE --> NAME ::= (NAME
 | (NAME >--> ATTRIBUTE))
ATTRIBUTE --> P ::= (P
 | (P >--> ATTRIBUTE))
ATTRIBUTE --> R ::= (R
 | (R >--> ATTRIBUTE))
ATTRIBUTE --> S ::= (S
 | (S >--> ATTRIBUTE))
ATTRIBUTE --> T ::= (T
 | (T >--> ATTRIBUTE))
ATTRIBUTE --> U ::= (U
 | (U >--> ATTRIBUTE))
ATTRIBUTE --> V ::= (V
 | (V >--> ATTRIBUTE))
ATTRIBUTE --> W ::= (W
 | (W >--> ATTRIBUTE))
ATTRIBUTE --> X ::= (X
 | (X >--> ATTRIBUTE))
ATTRIBUTE --> Y ::= (Y
 | (Y >--> ATTRIBUTE))
ATTRIBUTE --> Z ::= (Z
 | (Z >--> ATTRIBUTE))

A.1.3. PROPERTY
PROPERTY --> NAME ::= (NAME
 | (NAME >--> PROPERTY))
PROPERTY --> P ::= (P
 | (P >--> PROPERTY))
PROPERTY --> R ::= (R
 | (R >--> PROPERTY))
PROPERTY --> S ::= (S
 | (S >--> PROPERTY))
PROPERTY --> T ::= (T
 | (T >--> PROPERTY))
PROPERTY --> U ::= (U
 | (U >--> PROPERTY))
PROPERTY --> V ::= (V
 | (V >--> PROPERTY))
PROPERTY --> W ::= (W
 | (W >--> PROPERTY))
PROPERTY --> X ::= (X
 | (X >--> PROPERTY))
PROPERTY --> Y ::= (Y
 | (Y >--> PROPERTY))
PROPERTY --> Z ::= (Z
 | (Z >--> PROPERTY))

A.1.4. CONTRACT
CONTRACT --> NAME ::= (NAME
 | (NAME >--> CONTRACT))
CONTRACT --> P ::= (P
 | (P >--> CONTRACT))
CONTRACT --> R ::= (R
 | (R >--> CONTRACT))
CONTRACT --> S ::= (S
 | (S >--> CONTRACT))
CONTRACT --> T ::= (T
 | (T >--> CONTRACT))
CONTRACT --> U ::= (U
 | (U >--> CONTRACT))
CONTRACT --> V ::= (V
 | (V >--> CONTRACT))
CONTRACT --> W ::= (W
 | (W >--> CONTRACT))
CONTRACT --> X ::= (X
 | (X >--> CONTRACT))
CONTRACT --> Y ::= (Y
 | (Y >--> CONTRACT))
CONTRACT --> Z ::= (Z
 | (Z >--> CONTRACT))

A.1.5. TRACE
TRACE --> NAME ::= (NAME
 | (NAME >--> TRACE))
TRACE --> P ::= (P
 | (P >--> TRACE))
TRACE --> R ::= (R
 | (R >--> TRACE))
TRACE --> S ::= (S
 | (S >--> TRACE))
TRACE --> T ::= (T
 | (T >--> TRACE))
TRACE --> U ::= (U
 | (U >--> TRACE))
TRACE --> V ::= (V
 | (V >--> TRACE))
TRACE --> W ::= (W
 | (W >--> TRACE))
TRACE --> X ::= (X
 | (X >--> TRACE))
TRACE --> Y ::= (Y
 | (Y >--> TRACE))
TRACE --> Z ::= (Z
 | (Z >--> TRACE))

A.1.6. LOG
LOG --> NAME ::= (NAME
 | (NAME >--> LOG))
LOG --> P ::= (P
 | (P >--> LOG))
LOG --> R ::= (R
 | (R >--> LOG))
LOG --> S ::= (S
 | (S >--> LOG))
LOG --> T ::= (T
 | (T >--> LOG))
LOG --> U ::= (U
 | (U >--> LOG))
LOG --> V ::= (V
 | (V >--> LOG))
LOG --> W ::= (W
 | (W >--> LOG))
LOG --> X ::= (X
 | (X >--> LOG))
LOG --> Y ::= (Y
 | (Y >--> LOG))
LOG --> Z ::= (Z
 | (Z >--> LOG))

```

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## LogScope V2 syntax

### Quote

```
rule_schema ::=
 modifier+ [" transition+ "]
 | modifier* ident ["(iden,*)"] ["(transition+ *)"]

modifier ::=
 "init" | "always" | "step" | "next" | "not"
 transition ::= pattern,* ">" pattern,*
pattern ::= [":"] ident ["(constraint,* *)"]
constraint ::=
 ident "*" range
 | range
```

### Hemingway & Hotchner, 1920ies:

If you are lucky enough to have lived in Paris as a young man, then wherever you go for the rest of your life, it stays with you, for Paris is a moveable feast.

### Quote

### Havelund 2012:

If you are lucky enough to have explored VDM as a young man, then wherever you go for the rest of your life, it stays with you, for VDM is a moveable feast.

### What is VDM?

- Combination of imperative and functional programming (data types, pattern matching, curried functions, lambda abstractions, side effects, loops, exceptions, )

- Design-by-contract: pre/post conditions + invariants
- Predicate subtypes
- Non-deterministic expressions (let x be such that P(x))
- First-order predicate logic as Boolean expressions: universal and existential quantification
- Sets, lists and maps as built-in data types
- VDM++ added object orientation (Nico Plat et. al)

## Chemical plant model in VDM versus Scala

Scala is a high-level unifying language

```
class Plant
instance variables
 o Alarms : set of Alarm;
 o Schedules : map Period to set of Expert;
 o Inv_PlantInGardens : schedule;

operations
 PlantInv: set of Alarm * map Period to set of Expert ==>
 PlantIn[cs, bool]
 return forall p in set alarm sch & sch(p) => {1} and
 (forall a in set cs & cs(a) &
 exists expert in set sch(p) &
 GetQualitO(expert) In set expert->GetQualitO);
 types
 public Period = token;
 operations
 public ExpertToAlg: Alarm * Period ==> Expert
 ExpertToAlg(p) ==>
 let expert in set schedule(p) be t;
 def ReqAlg(p) in set expert->GetQualitO
 return expert;
 pre in set alarm sch(p) and
 sch(p) != empty;
 post in set alarm sch(p) and
 let expert = RESULT
 in expert In set schedule(p) and
 GetQualitO(expert) => expert->GetQualitO;
 end

```

- Object-oriented + functional programming features
- Strongly typed with type inference
- Script-like, semicolon inference
- Sets, list, maps, iterators, comprehensions
- Lots of libraries
- Compiles to JVM
- Lively growing community

```
class Plant(alarms: Set[Alarm], experts: Set[Expert]): Map[Period, Set[Expert]] {
 schedule: Map[Period, Set[Expert]];
 def PlantInv(alarms: Set[Alarm], schedule: Map[Period, Set[Expert]]): Boolean =
 schedule.keySet forall { schedule(e) != Set(e) } &&
 setExpertCount(alarms) <= alarms.size &&
 schedule.keySet exists { expert =>
 expert->reqQualit ? expert.qualit
 }
}
def ExpertToAlg(p: Alarm, p: Period): Expert = {
 require(p >= 7 days && p > schedule.keySet)
 let expert in schedule(p) be t;
 ensure { expert->reqQualit == 7 days } &&
 expert->reqQualit == 7 days;
 expert;
}
ensuring { expert => expert->reqQualit == 7 days } &&
 expert->reqQualit == 7 days;
}
```

## Commands must succeed

- We are analyzing log files containing information about commands being issued, and their success and failure respectively.

### Requirement CommandMustSucceed

An issued command must succeed, without a failure to occur before then.

```
automaton CommandMustSucceed {
 always {
 Command(n,x) ==> RequireSuccess(n,x)
 }
}

hot RequireSuccess(name,number) {
 Fail(name,number) ==> error
 Success(name,number) ==> ok
}
```

## Property in LogScope

- For comparison we first show spec in the external DSL: LOGSCOPE.
- a hot state must be exited before end of log (non-final state).



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## Property in LogScope

- Using LogScope's temporal logic layer.



**pattern CommandMustSucceed:**

```
Command(n,x) =>
[
 ! Fail(n,x),
 Success(n,x),
]
```

### Property in TraceContract - looks very similar

- Uses partial functions: {case ... => ...} defined with pattern matching as arguments to DSL functions (*require* and *hot*) defined in *Monitor* class. *RequireSuccess* is a user-defined function representing a state.
- A quoted name, such as 'name' represents the *value* of that name.

```
class CommandMustSucceed extends Monitor[Event] {
 always {
 case Command(n,x) => RequireSuccess(n,x)
 }
}
```

```
def RequireSuccess(name: String, number: Int) =
 hot {
 case Fail('name', 'number') => error
 case Success('name', 'number') => ok
 }
}
```

## Events in TraceContract

- First we need to define the events we observe:
  - commands being issued, each having a name and a number
  - successes of commands
  - failures of commands
- Each event type sub-classes a type: Event
- case-classes allow for pattern matching over objects of the class

abstract class Event

```
case class Command(name: String, nr: Int) extends Event
case class Success(name: String, nr: Int) extends Event
case class Fail(name: String, nr: Int) extends Event
```

### Property in TraceContract - looks very similar

- Uses partial functions: {case ... => ...} defined with pattern matching as arguments to DSL functions (*require* and *hot*) defined in *Monitor* class. *RequireSuccess* is a user-defined function representing a state.
- A quoted name, such as 'name' represents the *value* of that name.

```
class CommandMustSucceed extends Monitor[Event] {
 require {
 case Command(n,x) => RequireSuccess(n,x)
 }
}
```

```
def RequireSuccess(name: String, number: Int) =
 hot {
 case Fail('name', 'number') => error
 case Success('name', 'number') => ok
 }
}
```

## Inlining the call of `RequireSuccess(n,x)`

- Since `RequireSuccess(n, x)` is a function, the call of it can be inlined.
- After all, this is "just" a program and standard program transformation works.
- The result is an interesting temporal logic like specification with an **un-named hot state**.

## Same property in LTL

- TRACECONTRACT also offers future time linear temporal logic (LTL).
- allowing to write events as formulas, negations, propositional formulas, and temporal.
- $\phi$  until  $\psi$  means:  $\psi$  must eventually hold, and until then  $\phi$  must hold.

```
class CommandMustSucceed extends Monitor[Event] {
 require {
 case Command(n, x) =>
 hot {
 case Fail('n', 'x') => error
 case Success('n', 'x') => ok
 }
 }
}
```

## Same property in LTL

- TRACECONTRACT also offers future time linear temporal logic (LTL).
- allowing to write events as formulas, negations, propositional formulas, and temporal.
- $\phi$  until  $\psi$  means:  $\psi$  must eventually hold, and until then  $\phi$  must hold.

```
class CommandMustSucceed extends Monitor[Event] {
 require {
 case Command(n, x) =>
 not(Fail(n, x)) until (Success(n, x))
 }
}
```

- note mix of Scala's pattern matching (to catch arguments of command) and LTL.**

## Same property in LTL

- TRACECONTRACT also offers future time linear temporal logic (LTL).
- allowing to write events as formulas, negations, propositional formulas, and temporal.
- $\phi$  until  $\psi$  means:  $\psi$  must eventually hold, and until then  $\phi$  must hold.

```
class CommandMustSucceed extends Monitor[Event] {
 require {
 case Command(n, x) =>
 not(Fail(n, x)) until (Success(n, x))
 }
}
```

## Success of power commands

$\leftarrow \square \triangleright \quad \leftarrow \Diamond \triangleright \quad \leftarrow \exists \triangleright \quad \leftarrow \forall \triangleright \quad \models \quad \Diamond \wedge \Box$

**Requirement PowerCommandSuccess**  
Power commands must succeed within 10 seconds.

$\leftarrow \square \triangleright \quad \leftarrow \Diamond \triangleright \quad \leftarrow \exists \triangleright \quad \leftarrow \forall \triangleright \quad \models \quad \Diamond \wedge \Box$

## Property in LogScope

- Defining and using Python predicates in LogScope.



```

class PowerCommands extends Monitor[Event] {
 def within(t1: Int, t2: Int, max: Int) = (t2 - t1) <= max

 require {
 case Command(n, x, t1) if n.startsWith("PWR") =>
 hot {
 case Success('n', 'x', t2) if within(t1, t2, 10000) => ok
 }
 }

 pattern PowerCommands:
 Command(n, x, t1) where { : n.startsWith("PWR") } =>
 Success(n, x, t2) where { : within(t1, t2, 10000) }
 }
}

```

10 first commands must succeed

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Counting: first 10 commands must succeed

- Code (here counting and testing on counter) can be mixed with logic.
- That is: increase counter and return LTL formula.

### Requirement First10CommandsMustSucceed

The first 10 issued commands must succeed, without a failure to occur before then.

```

class First10CommandsMustSucceed extends Monitor[Event] {
 var count = 0

 require {
 case Command(n, x) if count < 10 =>
 count = count + 1
 not(Fail(n, x)) until (Success(n, x))
 }
}

```

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## long sequence

## Property in LogScope

- Using LogSCOPE's sequence operator



### Requirement CommandSequence

Whenever a flight software command is issued, there should follow a dispatch and then exactly one success.

**No dispatch failure before the dispatch, and no failure between dispatch and success.**

### pattern CommandSequence:

```
Command(n,x) =>
```

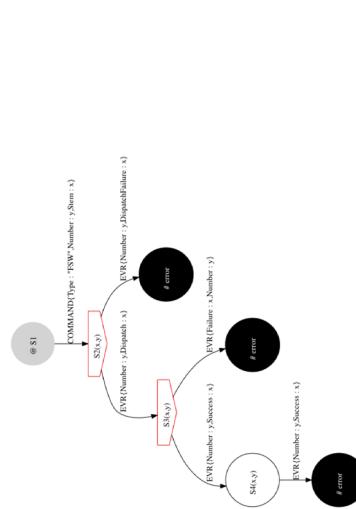
```
[! DispatchFailure(n,x),
 Dispatch(n,x),
 ! Fail(n,x),
 Success(n,x),
 ! Success(n,x)
]
```

## Same property in TraceContract

- TRACECONTRACT allows mixing of states.

```
class CommandSequence extends Monitor[Event] {
 require {
 case Command(n,x) =>
 hot {
 case DispatchFailure('n','x') => error
 case Dispatch('n','x') =>
 hot {
 case Fail('n','x') => error
 case Success('n','x') =>
 state {
 case Success('n','x') => error
 }
 }
 }
 }
 }
}
```

## Visualization of LogScope statemachine



Much more difficult to do with internal DSL such as TraceContract.

## Property that we cannot write in LogScope

- Antecedent (condition) containing multiple events.



## However we can write it in TraceContract

- TRACECONTRACT by just changing one of the state modifiers.

```

pattern CommandSequenceAsCondition:
[Command(n,x),
! DispatchFailure(n,x),
Dispatch(n,x)
] =>
[! Fail(n,x),
Success(n,x),
! Success(n,x)
]
class CommandSequence extends Monitor[Event] {
 require {
 case Command(n, x) =>
 state {
 case DispatchFailure('n', 'x') => error
 case Dispatch('n', 'x') =>
 hot {
 case Fail('n', 'x') => error
 case Success('n', 'x') =>
 state {
 case Success('n', 'x') => error
 }
 }
 }
 }
 }
 }
 }
}

```

Some notes from a notebook - before TraceContract

First, a spec in LogScope as it is:

```

monitor CommandsMustSucceed {
 always {
 COMMAND(name : x) => RequireSuccess(x)
 }
 hot RequireSuccess(command) {
 FAIL(name : command) => error
 SUCCESS(name : command) => ok
 }
}

```

We can try to eliminate the state requireSuccess by simply inlining it:

```

monitor CommandsMustSucceed {
 always {
 COMMAND(name : x) => hot {
 FAIL(name : x) => error
 SUCCESS(name : x) => ok
 }
 }
}

```

Alternation

**Requirement AlternatingCommandSuccess**  
Commands and successes should alternate.

TraceContract later offered this feature.

## State machine solution

### State machine solution - with next-states

```
class AlternatingCommandSuccess extends Monitor[Event] {
 property(s1)

 def s1: Formula =
 state {
 case Command(n, x) => s2(n, x)
 case _ => error
 }

 def s2(name: String, number: Int) =
 state {
 case Success('name', 'number') => s1
 case _ => error
 }
}
```

```
class AlternatingCommandSuccess extends Monitor[Event] {
```

```
 property(s1)
```

```
 def s1: Formula =
 next {
 case Command(n, x) => s2(n, x)
 }
```

```
 def s2(name: String, number: Int) =
 next {
 case Success('name', 'number') => s1
 case _ => error
 }
```

## A past time property

- Properties so far have been future time properties: from some event, the future behavior must satisfy some property.

- The following requirement refers to the past of some event (success).

### Requirement.SuccessHasAReason

A success must be caused by a previously issued command.

## TraceContract offers limited rule-based programming

- State logic and LTL cannot express this property.
- TRACECONTRACT offers a limited form of rule-based programming, were a fact  $f$  (sub-classing class  $Fact$ ) can be queried ( $f?$ ), created ( $f+$ ), and deleted ( $f-$ ). The result in the latter two cases is True.

## TraceContract offers limited rule-based programming

- State logic and LTL cannot express this property.
- TRACECONTRACT offers a limited form of rule-based programming, were a fact  $f$  (sub-classing class  $Fact$ ) can be queried ( $f?$ ), created ( $f+$ ), and deleted ( $f-$ ). The result in the latter two cases is True.

```
class SuccessHasAReason extends Monitor[Event] {
 case class Commanded(name: String, nr: Int) extends Fact
```

```
require {
 case Command(n, x) => Commanded(n, x) +
 case Success(n, x) =>
 if (Commanded(n, x) ?)
 Commanded(n, x) -
 else
 error
}
```

↳ Q, C

## Making monitors of monitors

- We can make this monitor simpler by using test-and-set:  $f ?=$ , for a given fact  $f$ , meaning: *return true iff. the fact  $f$  is recorded, delete the fact in any case.*

- We can create a new monitor which includes other monitors as sub-monitors. Useful for organizing properties.
- The semantics is the obvious one of conjunction: all monitors will get checked individually.

```
class SuccessHasAReason extends Monitor[Event] {
 case class Commanded(name: String, nr: Int) extends Fact
```

```
require {
 case Command(n, x) => Commanded(n, x) +
 case Success(n, x) => Commanded(n, x) ?=
```

↳ Q, C

```
class CommandRequirements extends Monitor[Event] {
 monitor(
 new CommandMustSucceed,
 new MaxOneSuccess,
 new SuccessHasAReason)
```

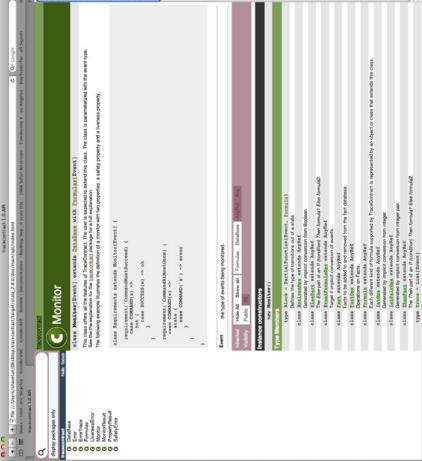
↳ Q, C

## Analyzing a complete trace (log analysis)

- To verify a trace, first create it, then instantiate monitor, and call `verify` method on monitor with trace as argument.

```
object TraceAnalysis extends Application {
 val trace: List[Event] =
 List(
 Command("STOP_DRIVING", 1),
 Command("TAKE_PICTURE", 2),
 Fail("STOP_DRIVING", 1),
 Success("TAKE_PICTURE", 2),
 Success("SEND_TELEMETRY", 42))
 val monitor = new CommandRequirements
 monitor.verify(trace)
}
```

## Result



## SuccessHasAReason property violated

Violating event number 5: Success(SEND\_TELEMETRY, 42)

Error trace:

```
1=Command(STOP_DRIVING, 1)
3=Success(SEND_TELEMETRY, 42)
```

## SuccessHasAReason property violated



## Alternatively: analyzing event by event (online monitoring)

- To verify a sequence of events: instantiate monitor, and call `verify` method on monitor for each event, and call `end()` if event flow terminates.

```
object TraceAnalysis extends Application {
 val monitor = new CommandRequirements
 monitor.verify(Command("STOP_DRIVING", 1))
 monitor.verify(Command("TAKE_PICTURE", 2))
 monitor.verify(Fail("STOP_DRIVING", 1))
 monitor.verify(Success("TAKE_PICTURE", 2))
 monitor.verify(Success("SEND_TELEMETRY", 42))
 monitor.end()
}
```

## Result



## ScalaDoc documentation of API

```
def eventually(e: Event) within Time(formula: Formula) within TimeDelta
def eventually(e: Event)(formula: Formula) within TimeDelta
 Eventually waits for a formula to be true.
 Eventually waits for a formula to not be true.
 def and not forall(p: Predicate, q: Predicate) within TimeDelta: Boolean
 Eventually waits for both predicates to be true.
 Eventually waits for both predicates to be false.
 def geteventually(p: Predicate) within TimeDelta: Boolean
 Returns the result of these predicates for this monitor.
 def thenReturn(location: Location) within TimeDelta
 Returns the location of a workspace.
 def within(T: Time) within TimeDelta
 def atRate(T: Time, n: Int): Predicate[Event](Breaker: Breaker) within TimeDelta
 A rate control for an event to generally reach a transition (required) between n and n + 1.
 def atRate(T: Time, condition: Boolean): Predicate[Event](Breaker) within TimeDelta
 A state condition for an event to generally reach a transition (required) between T and T + 1. The value reaches the break if the incoming event makes the condition evaluate to true.
 def when(condition: Boolean): Predicate[Event]
 If an event occurs, then this function is executed. If no event occurs, then nothing happens.
 def when(condition: Boolean)(f: Unit) within TimeDelta
 If an event occurs, then this function is executed. If no event occurs, then nothing happens.
 class Requirements extends Object(Event)
 requirements (condition: Boolean): Unit
 block (condition) {
 when (condition) then f()
 }
 }
}

Block partial function representing the transition heading out of the state.
```

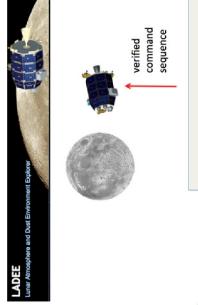
**return** .... the return formula.

**definition** classes **Formula**

```
def InternalEvents(symbol: String)(formula: String)(Time: Time): Unit
def ExternalEvents(symbol: String)(formula: String)(Time: Time): Unit
def eventually(formula: String)(Time: Time): Unit
 Used to await expectation of properties of external targets.
 def and not forall(formula: String)(Time: Time): Unit
 Machine control to expect several predicates.
 def when(formula: String)(Time: Time): Unit
 Machine control to expect a predicate.
 def when(formula: String)(Time: Time, condition: Boolean): Unit
 Machine control to expect a predicate with condition.
```

Note that in LTL+ required formulas:

## LADEE mission



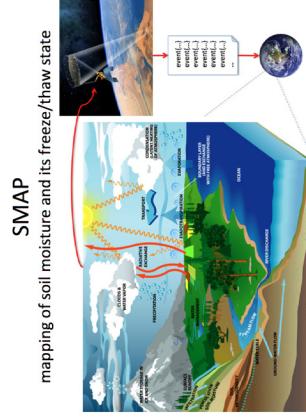
## GUI interface to TraceContract (LADEE mission)

| Flight Rules |                    |            |
|--------------|--------------------|------------|
| Requirement  | Flight Rule        | Block Rule |
| SAU          | SAU                | SAU        |
| CANIS        | CANIS              | CANIS      |
| OCRA         | OCRA               | OCRA       |
| FACCO        | FACCO              | FACCO      |
| TACCO        | TACCO              | TACCO      |
|              | Initial State File |            |
|              | Rule XML           |            |

| Absolute Time Command Sequence ATTS Files |                  |                  |
|-------------------------------------------|------------------|------------------|
| ATTS_0x001_1.aff                          | ATTS_0x002_1.aff | ATTS_0x003_1.aff |
| 11                                        | 11               | 11               |
| ATTS_0x004_1.aff                          | ATTS_0x005_1.aff | ATTS_0x006_1.aff |
| 10                                        | 10               | 10               |

↳ Q. C.

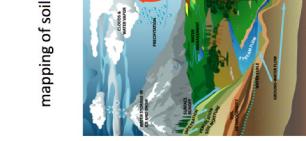
## SMAP mission



## SMAP

### mapping of soil moisture and its freeze/thaw state

↳ Q. C.

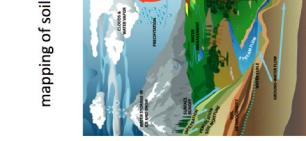


↳ Q. C.

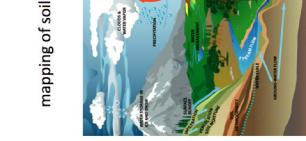
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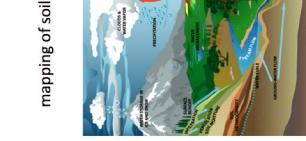
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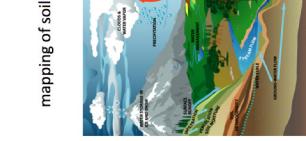
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↳ Q. C.



↳ Q. C.

## Definition of parameterized monitors

### Summary

```
class CommandSuccess(cmd: String, success: Boolean = true)
extends Monitor[Event] {
 require {
 case Command('cmd', number) =>
 hot {
 case Success('cmd', number) => success
 case Fail('cmd', number) => !success
 }
 }
 monitor(new CommandSuccess("STOP"))
}
```

- TRACECONTRACT is an API.
- Very expressive and convenient for programmers to use.
- For this reason mainly it has been adopted by practitioners.
- Has very simple implementation, which is easy to modify.
- Change requests are easy to process.
- It is, however, difficult to analyze a TRACECONTRACT specification since it fundamentally is a Scala program - requires some form of reflection or interaction with compiler.
- It will not be suitable for non-Scala programmers.



## VERIFICATION OF STIFF HYBRID SYSTEMS BY MODELING THE APPROXIMATIONS OF COMPUTATIONAL SEMANTICS

**Pieter J. Mosterman, MathWorks**

With the seemingly unbounded proliferation of computing power into most any engineered artifact, ever more 'smart' systems are being created. This increase of available smarts in engineered systems has given rise to a new field of innovation where unique value is derived from having intelligent systems interact in novel and unforeseen manners. With the physical world an intrinsic part of the interaction and the smarts being implemented in a networked information modality, also called cyber space, these innovative systems are referred to as Cyber-Physical Systems. Modeling cyber aspects, physics, and their nexus then plays a crucial role in the design of Cyber-Physical Systems. A pick-and-place machine is presented as a paradigmatic example of such Cyber-Physical Systems to illustrate the intricate interplay between cyber space and physics, which serves to motivate the importance of integrated heterogeneous modeling paradigms that support modeling, simulation, and analysis of combined physics, geometry, signal processing, and control aspects. At a macroscopic level, physics models often comprise differential and algebraic equations and these equations typically require computational approaches to derive solutions. Approximations introduced by the solvers that derive these solutions to a large extent determine the meaning of the models, in particular when continuous-time behavior interacts with discontinuities such as in so-called hybrid dynamic systems. In reasoning about models that are solved computationally it is therefore imperative to also model the solvers. This presentation outlines an approach to modeling numerical solver approximations to help reason about approximations and to enable verification of stiff hybrid dynamic systems.



# Modeling Approximation of Computational Semantics for Cyber-Physical System Design

**Pieter J. Mosterman**  
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In your opinion, what lasting legacy has YACC brought to language development?

YACC made it possible for many people who were not language experts to make little languages (also called domain-specific languages) to improve their productivity. Also, the design style of YACC - base the program on solid theory, implement the theory well, and leave lots of escape hatches for the things you want to do that don't fit the theory - was something many Unix utilities embodied. It was part of the atmosphere in those days, and this design style has persisted in most of my work since then.

Interview with Stephen C. Johnson in "The A-Z of programming languages: YACC."  
Computerworld, 08/07/2008  
[http://news.cnet.com/8301-3945\\_17A4-0f76-31509693E8E95c1](http://news.cnet.com/8301-3945_17A4-0f76-31509693E8E95c1)

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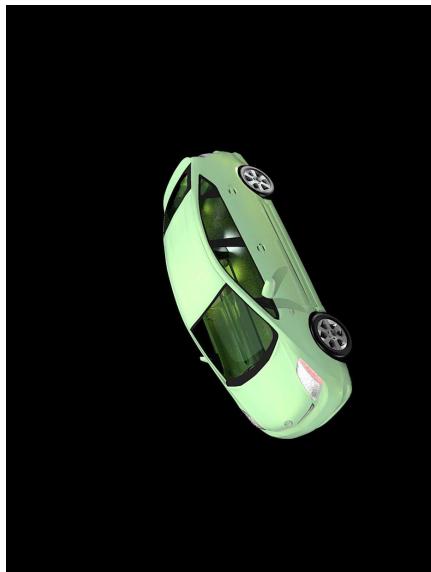
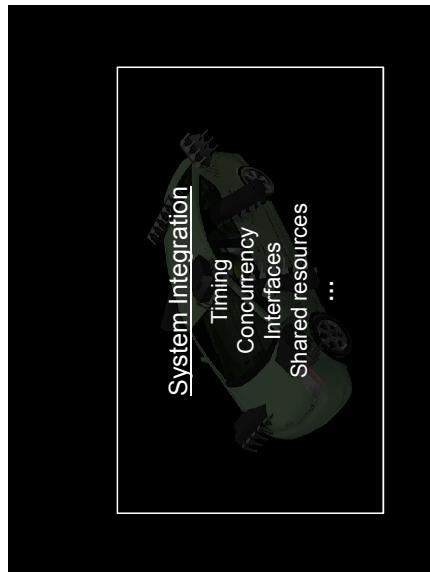
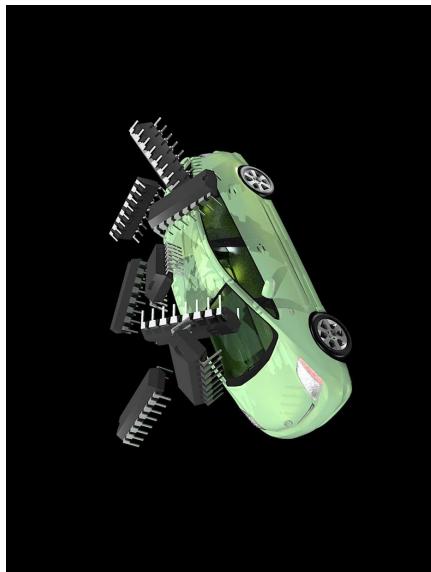
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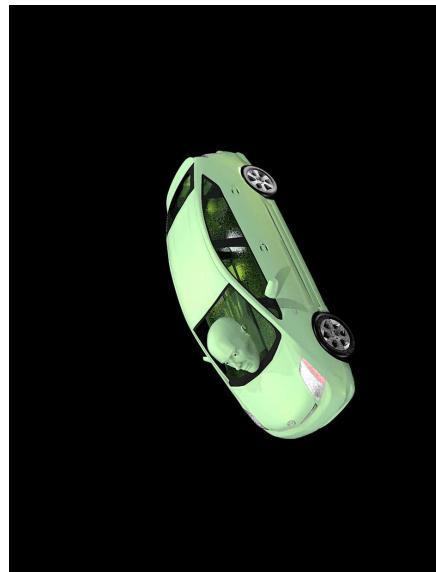


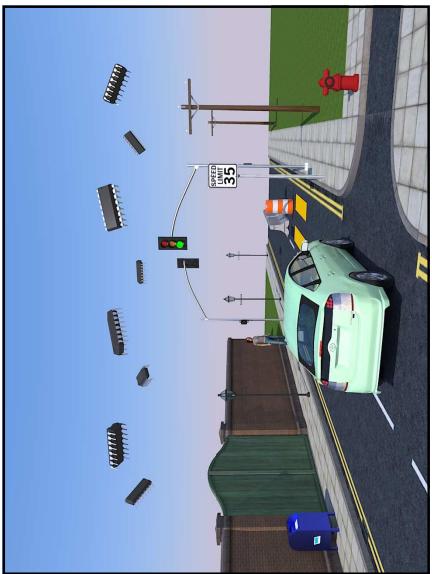
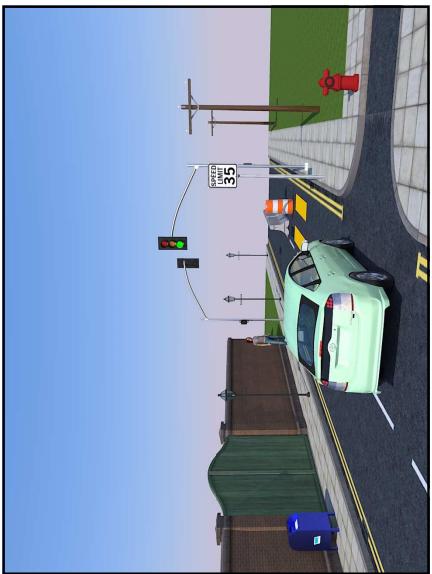
## Agenda

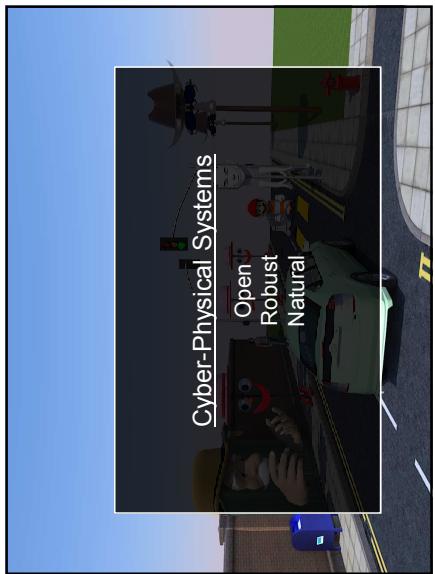
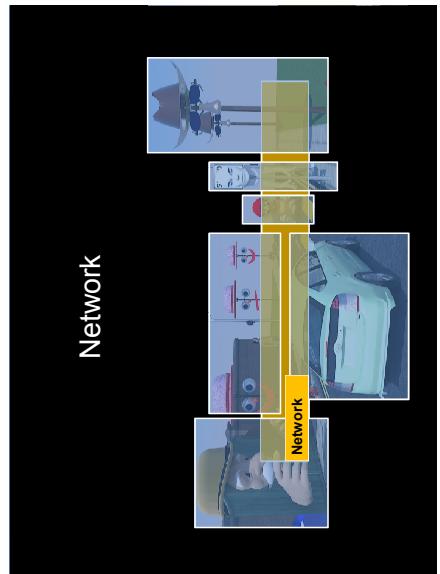
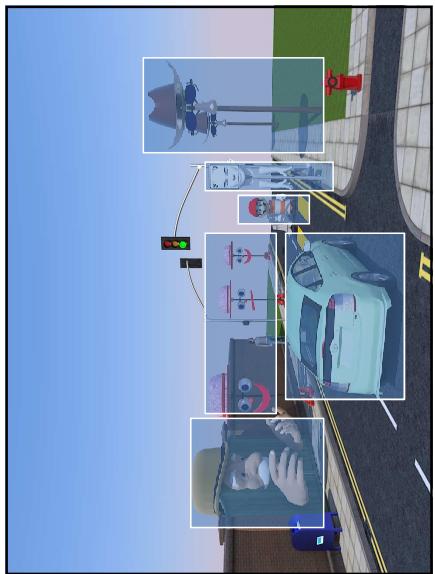
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- Modeling cyber-physical systems
- Modeling approximations
- A solver model for control synthesis
- Conclusions

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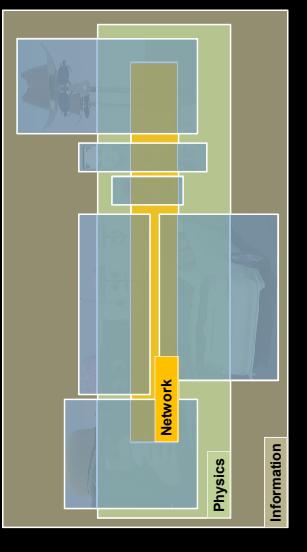




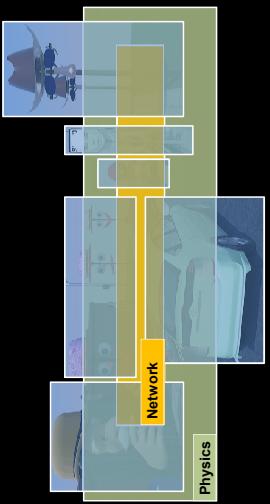




## Information

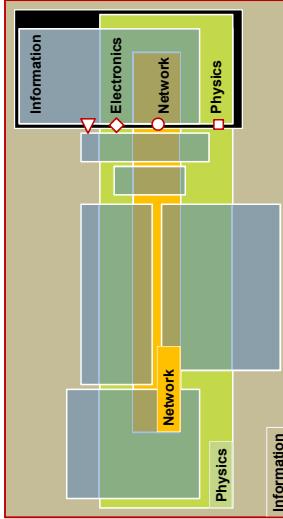


## Physics



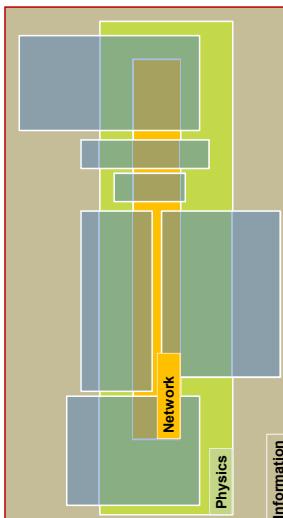
## Cyber-physical systems

MathWorks®



## Cyber-physical systems

MathWorks®

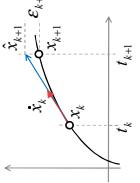


24

23



 **Numerical integration**



Euler: step  $h$  in time along  $\dot{x} = f(x, t)$

$$\hat{x}_e(t_{k+1}) = x(t_k) + \dot{x}(t_k)h_k$$

$$v(t) = \frac{dp(t)}{dt}$$

Maxwell:  $i(t) = \frac{dq(t)}{dt}$

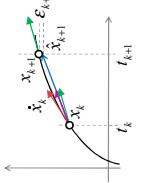
An ideal oscillator:

$$i(t) = C \frac{dv(t)}{dt}$$

$$v(t) = L \frac{di(t)}{dt}$$

29

 **Numerical integration**



Euler: step  $h$  in time along  $\dot{x} = f(x, t)$

$$\hat{x}_e(t_{k+1}) = x(t_k) + \dot{x}(t_k)h_k$$

Trapezoidal: average the end points

$$\hat{x}_t(t_{k+1}) = x(t_k) + \frac{\dot{x}(t_k) + \dot{x}(t_{k+1})}{2}h_k$$

Taylor series expansion for error analysis

$$x(t_{k+1}) = x(t_k) + \frac{\dot{x}(t_k)}{1!}h_k + \frac{\ddot{x}(t_k)}{2!}h_k^2 + O(h_k^3)$$

$$\epsilon_t(t_{k+1}) = \frac{\ddot{x}(t_k)}{2!}h_k^2$$

Change step size based on estimate:  $\hat{x}_e(t_{k+1}) - \hat{x}_t(t_{k+1}) \approx \frac{\ddot{x}(t_k)}{2!}h_k^2$

30

 **Modeling a physical system**



Capacitor:  $V = \frac{q}{C}$

Inductor:  $I = \frac{P}{L}$

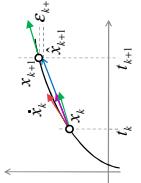
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 **Numerical integration**



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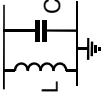
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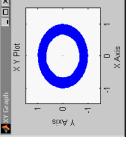
When  $x(t)/y$  changes little,  $h_k$  can be large!

31

 Sophisticated solver ... ?

- Let's compute a solution to an ideal oscillator

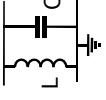


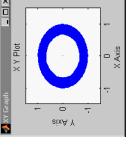


33

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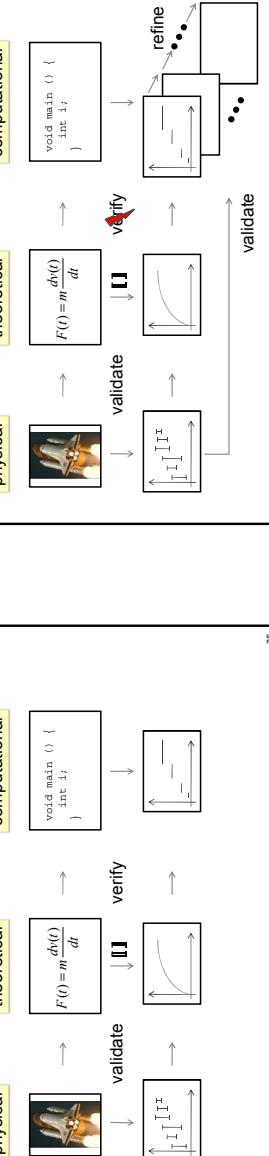




- We can make the error small ... but only locally!
- It accumulates for long time behavior
- So, ... how come we can engineer today's complex systems?!

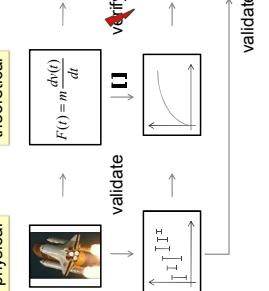
34

 The models in engineering an embedded system



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 The models in engineering an embedded system



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## Desiderata of an execution engine model

- Declarative
  - Nonimplementation details
- Stateless
  - State explicitly formulated (e.g., as input)
- Function composition

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## Agenda

- Cyber-physical systems
- Modelling cyber-physical systems
- ▶ ▶ ▶ ▶ ▶ ▶
- Modelling approximations
- A solver model for control synthesis
- Conclusions

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## A declarative formalism with fix-point semantics

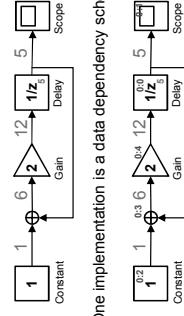
A LATTICE-THEORETICAL FIXPOINT THEOREM  
AND ITS APPLICATIONS  
ALFRED TARSKI  
*Pacific J. Math.*, 5 (1955), 265-399

*Pacific J. Math.*, 5 (1955), 265-399

ALFRED TARSKI

*Pacific J. Math.*, 5 (1955), 265-399

- Repeated application of a monotonically increasing partial function converges to a fixed point



40



## A declarative formalism with fix-point semantics

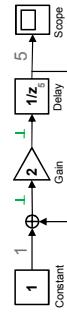
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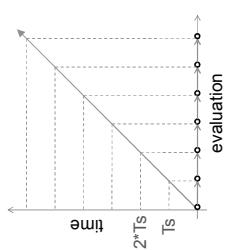


39



## Can we use this framework to define a variable-step solver?

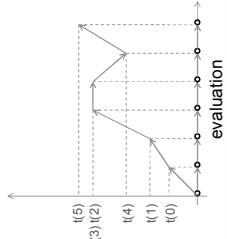
- Separate
  - Time (explicit)
  - Evaluations (ordered)
- Time as a function of evaluations



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## Can we use this framework to define a variable-step solver?

- Separate
  - Time (explicit)
  - Evaluations (ordered)
- Time as a function of evaluations
  - Step is variable
  - Step may be 0
  - Step may be negative
  - Time may recede



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## The two stages of a stream based functional solver

### Euler integration

$$y_e(e) = \begin{cases} \sum_{i=1}^e u(i)h(i) & \text{if } odd(e) \\ y_e(e-1) & \text{otherwise} \end{cases}$$

### Trapezoidal integration

$$y_e(e) = \sum_{i=1}^e \frac{(u(i-1) + u(i))h(i-1)}{2}$$

$$d(e) = \sum_{i=1}^e \frac{(u(e-3) + u(e-2))h(e-3)}{2} - u(e-2)h(e-2)$$

### Error computation

$$|u(e-2)h(e-2)| < tol$$

45



## The two stages of a stream based functional solver

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46



## The two stages of a stream based functional solver

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45



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46



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47



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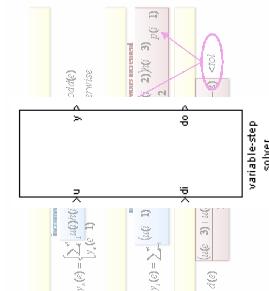
48



## The two stages of a stream based functional solver



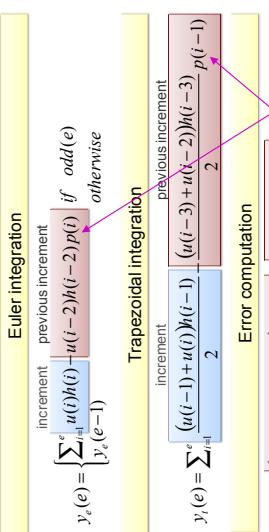
## The two stages of a stream based functional solver



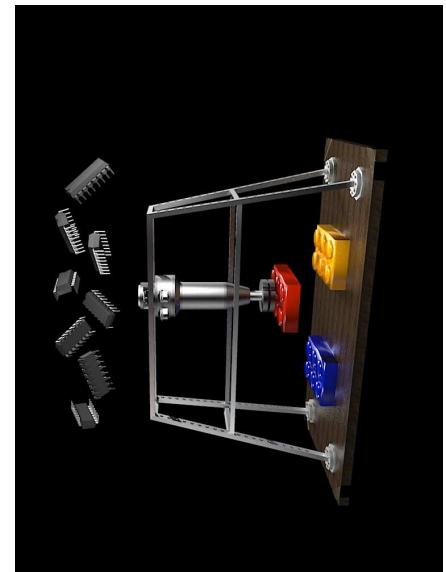
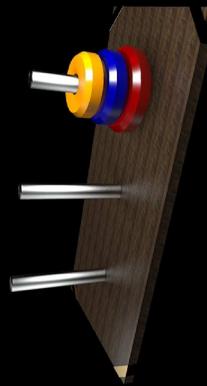
## The two stages of a stream based functional solver



## The two stages of a stream based functional solver



## Towers of Hanoi

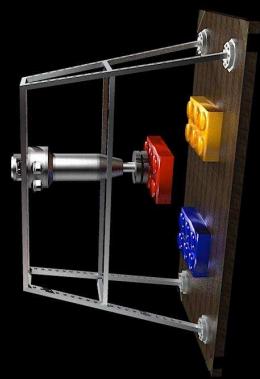


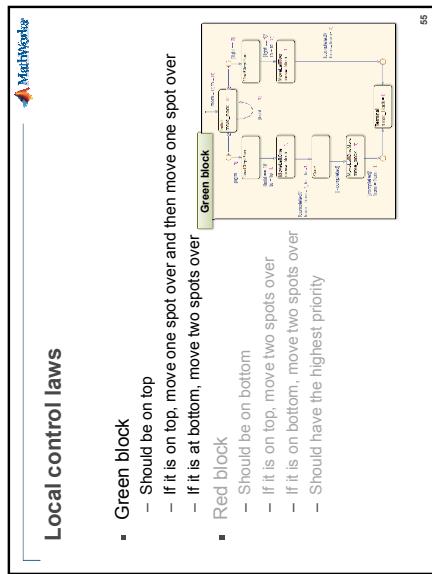
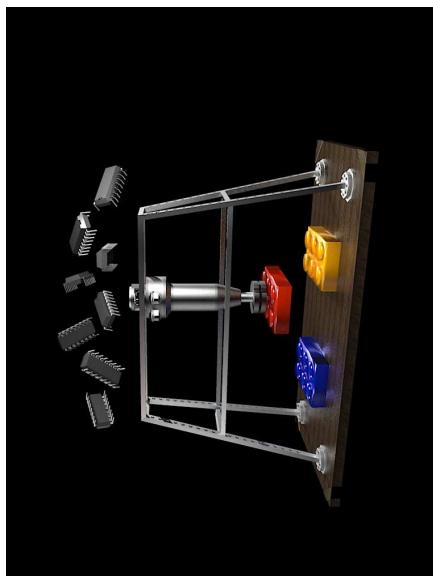
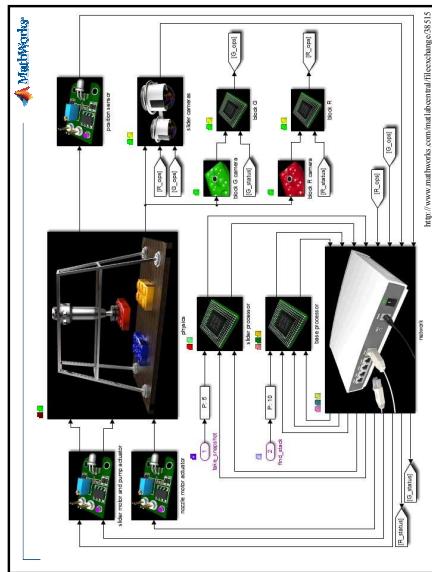
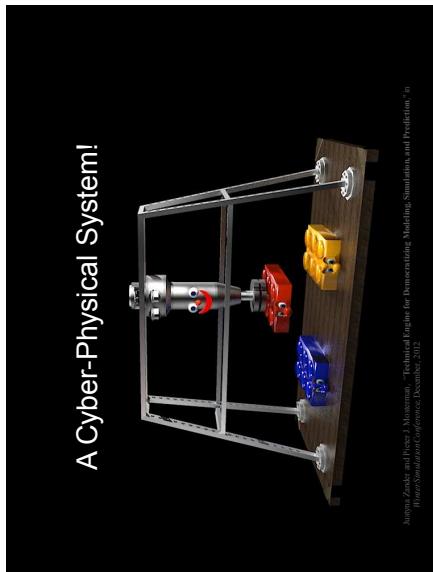
## Agenda

- Cyber-physical systems
- Modelling cyber-physical systems
- Modelling approximations
- A solver model for control synthesis
- Conclusions

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## A Cyber-Physical System?

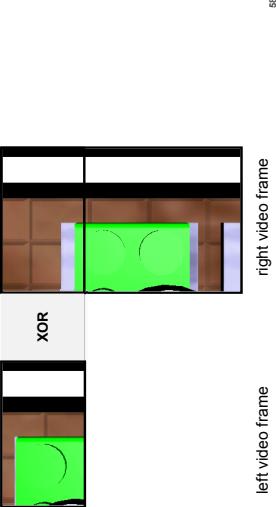




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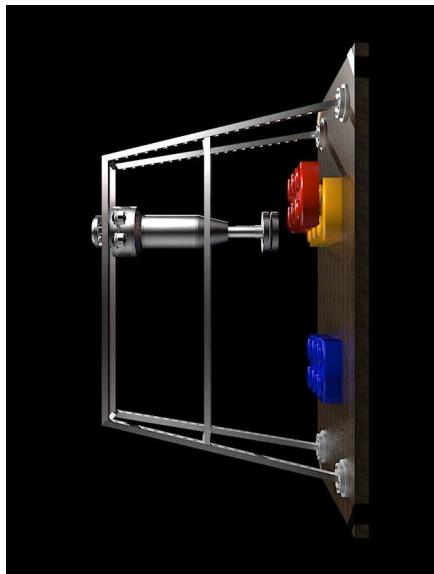
### Stereoscopic analysis to find the stack of blocks

- Multiple values at one time step



left video frame      right video frame  
XOR

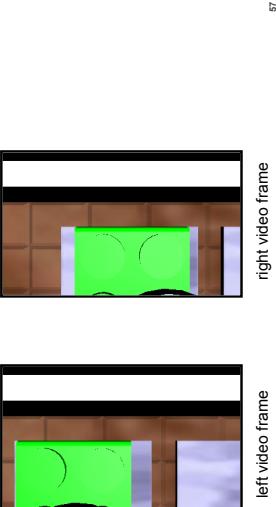
58



 MathWorks

### Stereoscopic analysis to find the stack of blocks

- Multiple values at one time step



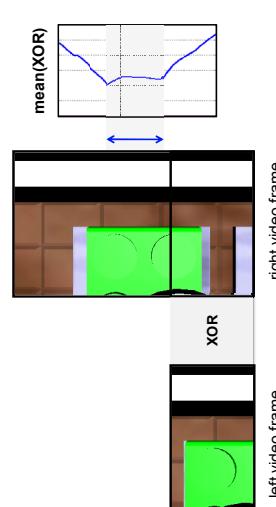
left video frame      right video frame  
XOR

57

 MathWorks

### Stereoscopic analysis to find the stack of blocks

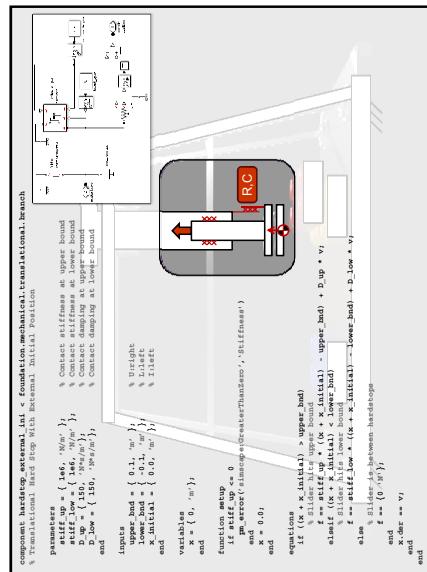
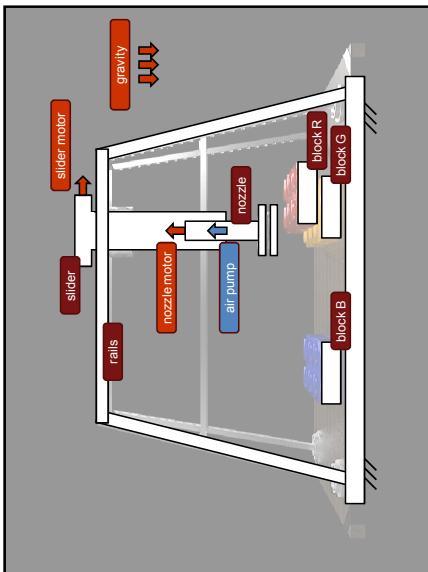
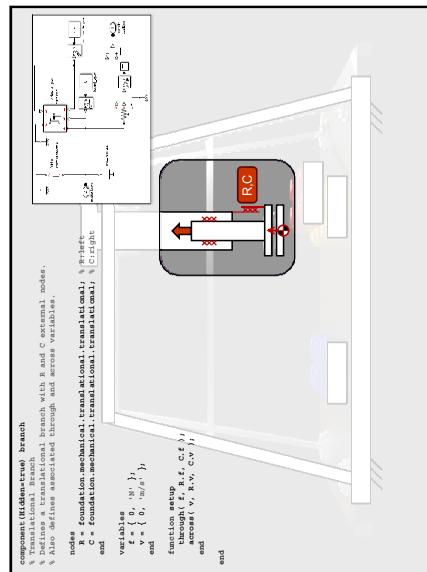
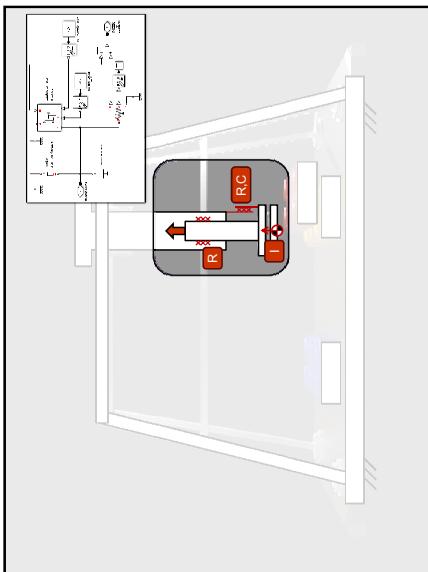
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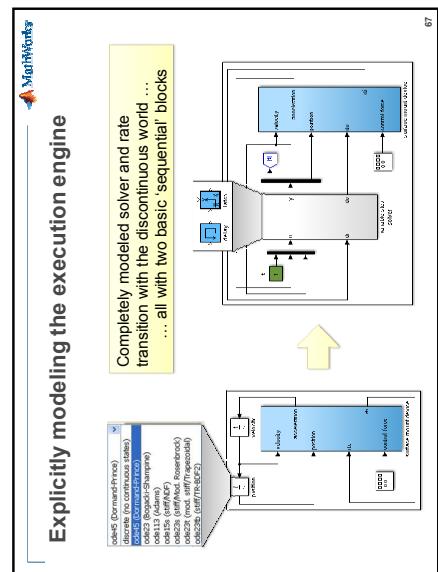
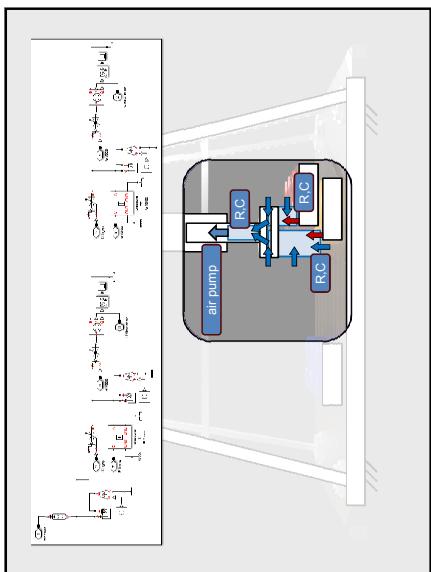
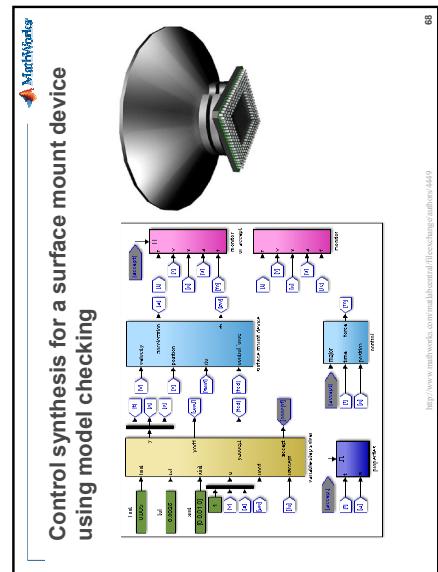
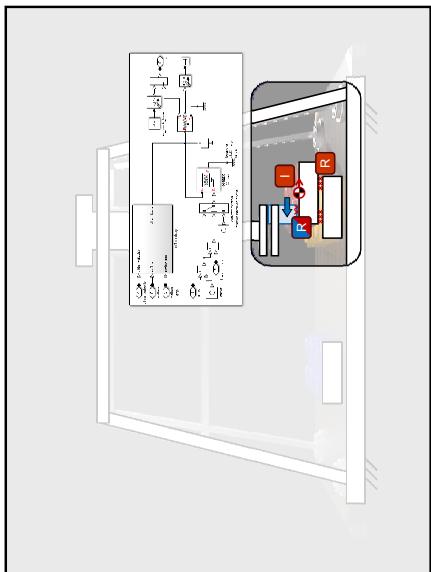


left video frame      right video frame  
XOR

mean(XOR)

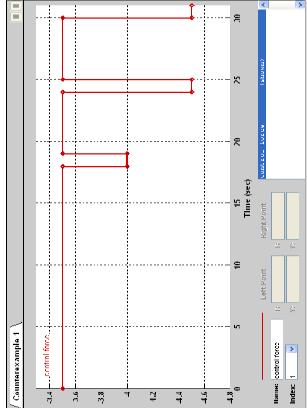
59







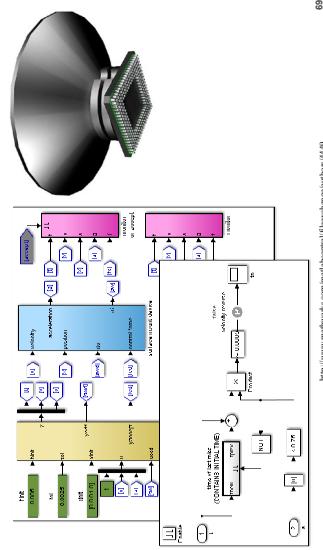
## Model checking to generate a counterexample



Peter J. Mosterman, Justyna Zander, Grigorie Hamon, and Ben Demele, "A Computational Model of Time for Soft/Hybrid Systems Applied to Control Synthesis," in *Control Engineering Practice*, vol. 20, no. 1, pp. 2-13, January 2012.



## Control synthesis for a surface mount device using model checking



<http://www.mathworks.com/matlabcentral/fileexchange/36100>



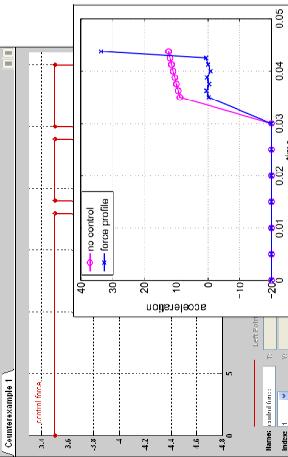
## Characteristics of the semantic domain

- Declarative
  - Purely functional (no side effects)
- Ordered evaluations
- Untimed
  - Time as explicit function,  $t(e)$
  - Time is not strictly increasing
- Broadly applicable to dynamic systems
  - Differential equations, difference equations, discrete events

Peter J. Mosterman, Justyna Zander, Grigorie Hamon, and Ben Demele, "Towards Computational Hybrid System Semantics for Time-Based Block Diagrams," in *2nd IFAC Conference on Analysis and Design of Hybrid Systems (ADHS10)*, A. Giua, C. Mahadevan, M. Silva, and I. Zaytsev (eds.), pp. 376-385, Zaragoza, Spain, September 16-18, 2010, plenary paper.

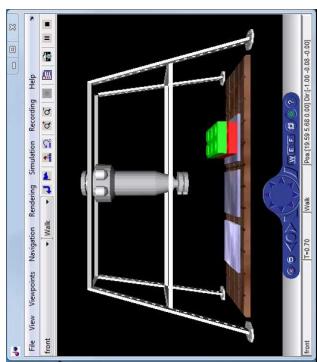


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## Scenarios—emerging behavior



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## Agenda

- Cyber-physical systems
- Modeling cyber-physical systems
- Modeling approximations
- A solver model for control synthesis
- Conclusions



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## Conclusions

- Today's systems are open
  - Interact across various modalities
- Computational models include a variety of semantics
  - Many interacting approximations
- We should understand our computational methods
- Model solvers
  - A functional stream-based approach
  - Formalize computational semantics of the execution engine
- Exploit the abstraction
  - Computational methods for analysis, design, and synthesis
- Bring disciplines together
  - Engineering, Computer Science, Physics, Mathematics

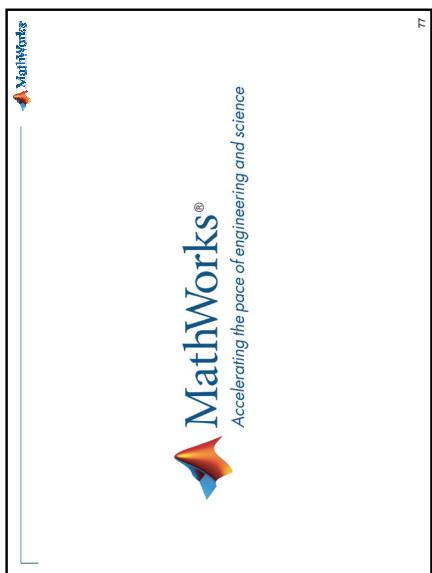
75

## Acknowledgments

Justyna Zander  
Harvard University  
SimulatedWay, Berlin  
  
Hans Vangheluwe  
Universiteit of Antwerp  
McGill University

Many thanks for their continuing collaboration!

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## ALGORITHMIC DIFFERENTIATION: SENSITIVITY ANALYSIS AND THE COMPUTATION OF ADJOINTS

**Andrea Walther, Institut für Mathematik Universität  
Paderborn**



Algorithmic differentiation: Sensitivity analysis and the computation of adjoints. The provision of exact and consistent derivative information is important for numerous applications arising from optimisation purposes as for example optimal control problems. However, even the pure simulation of complex systems may require the computation of derivative information. Implicit integration methods are prominent examples for this case. The talk will present the technique of algorithmic (or automatic) differentiation (AD) to compute exact derivative information for function evaluations given as computer programs. This includes a short overview of the history of AD and a description of the main variants of AD, namely the forward mode to compute sensitivities and the reverse mode for the provision of adjoints. A discussion of complexity estimates follows yielding the important cheap gradient result. Then several aspects closely connected with the computation of sensitivity and adjoint information are emphasised. This covers also the structure exploitation in time and space. Some examples stemming optimal flow control problems illustrate the presented aspects.

## Outline

Algorithmic differentiation:  
Sensitivity analysis and  
the computation of adjoints

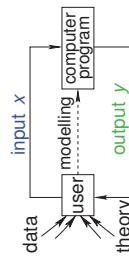
Andrea Walther  
Institut für Mathematik  
Universität Paderborn

LCCC Workshop on Equation-based Modelling

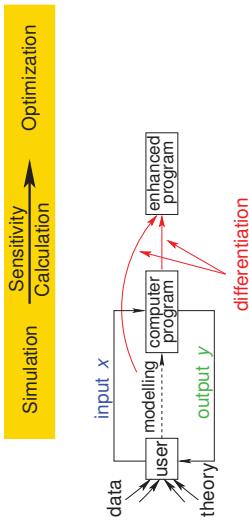
September 19–21, 2012

- Introduction
- Basics of Algorithmic Differentiation (AD)
  - The Forward Mode
  - The Reverse Mode
- Structure-Exploiting Algorithmic Differentiation
  - Time Structure Exploitation
  - Time and Space Structure Exploitation
- Conclusions

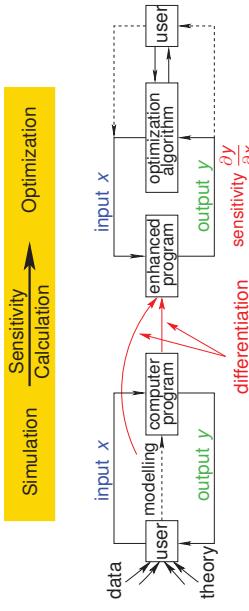
## Computing Derivatives



## Computing Derivatives



## Computing Derivatives



## Finite Differences

Idea: Taylor-expansion,  $f : \mathbb{R} \rightarrow \mathbb{R}$  smooth then

$$\begin{aligned}
 f(x+h) &= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \dots \\
 \Rightarrow f(x+h) &\approx f(x) + hf'(x) \\
 \Rightarrow Df(x) &= \frac{f(x+h) - f(x)}{h}
 \end{aligned}$$

## Finite Differences

Idea: Taylor-expansion,  $f : \mathbb{R} \rightarrow \mathbb{R}$  smooth then

$$\begin{aligned}
 f(x+h) &= f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \dots \\
 \Rightarrow f(x+h) &\approx f(x) + hf'(x) \\
 \Rightarrow Df(x) &= \frac{f(x+h) - f(x)}{h}
 \end{aligned}$$

- simple derivative calculation (only function evaluations!)

- inexact derivatives

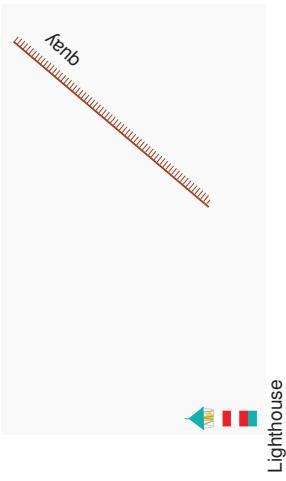
- computation cost often too high

$$F : \mathbb{R}^n \rightarrow \mathbb{R} \Rightarrow \text{OPS}(\nabla F(x)) \sim (n+1)\text{OPS}(F(x))$$

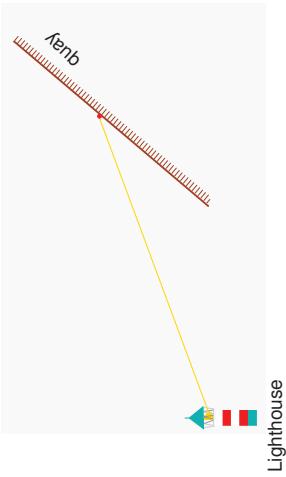




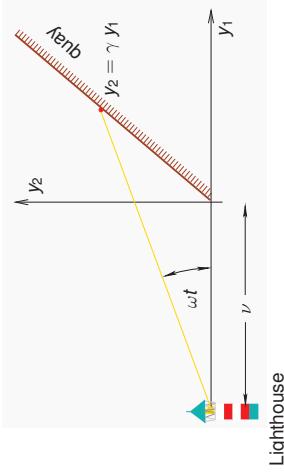
## The “Hello-World”-Example of AD



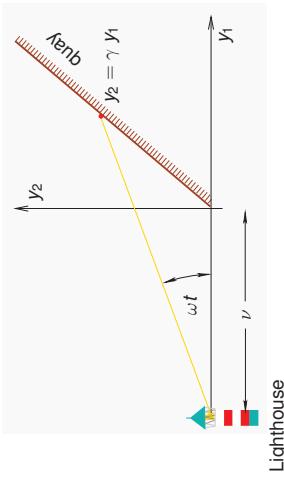
## The “Hello-World”-Example of AD



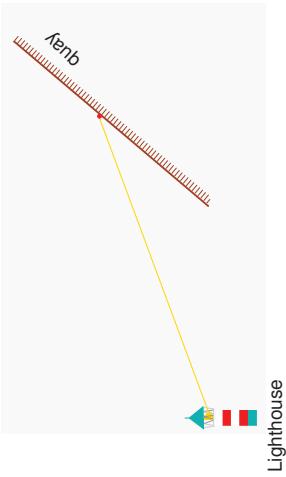
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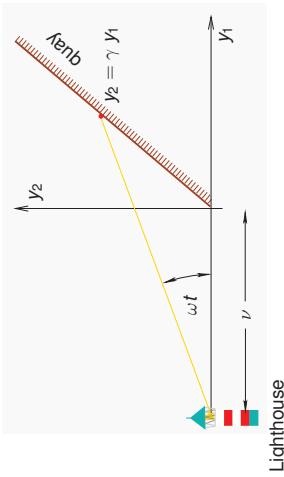
## The “Hello-World”-Example of AD



## The “Hello-World”-Example of AD



## The “Hello-World”-Example of AD



$$y_1 = \frac{\nu \tan(\omega t)}{\gamma - \tan(\omega t)} \quad \text{and} \quad y_2 = \frac{\gamma \nu \tan(\omega t)}{\gamma - \tan(\omega t)}$$

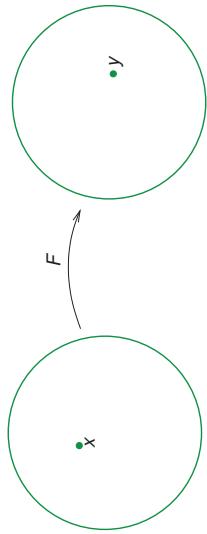
## Evaluation Procedure (Lighthouse)

|                                                    |                         |                         |                 |
|----------------------------------------------------|-------------------------|-------------------------|-----------------|
| $v_{-3} = x_1 = \nu$                               | $v_{-2} = x_2 = \gamma$ | $v_{-1} = x_3 = \omega$ | $v_0 = x_4 = t$ |
| $v_1 = v_{-1} * v_0 \equiv \varphi_1(v_{-1}, v_0)$ |                         |                         |                 |
| $v_2 = \tan(v_1) \equiv \varphi_2(v_1)$            |                         |                         |                 |
| $v_3 = v_2 - v_2 \equiv \varphi_3(v_2, v_2)$       |                         |                         |                 |
| $v_4 = v_{-3} * v_2 \equiv \varphi_4(v_{-3}, v_2)$ |                         |                         |                 |
| $v_5 = v_4 / v_3 \equiv \varphi_5(v_4, v_3)$       |                         |                         |                 |
| $v_6 = v_5 * v_{-2} \equiv \varphi_6(v_5, v_{-2})$ |                         |                         |                 |
| $y_1 = v_5$                                        |                         |                         |                 |
| $y_2 = v_6$                                        |                         |                         |                 |

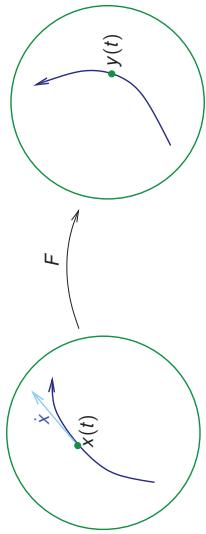
$$y_1 = \frac{\nu \tan(\omega t)}{\gamma - \tan(\omega t)}$$

$$y_2 = \frac{\gamma \nu \tan(\omega t)}{\gamma - \tan(\omega t)}$$

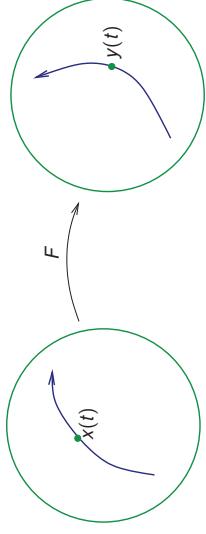
## Forward Mode of AD



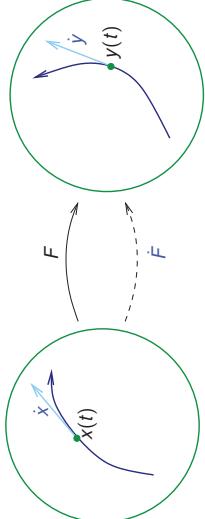
## Forward Mode of AD



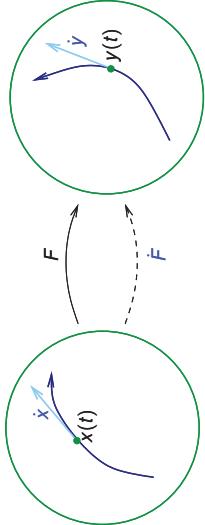
## Forward Mode of AD



## Forward Mode of AD



## Forward Mode of AD



$$\dot{y}(t) = \frac{\partial}{\partial t} F(x(t)) = F'(x(t)) \dot{x}(t) \equiv F(x, \dot{x})$$

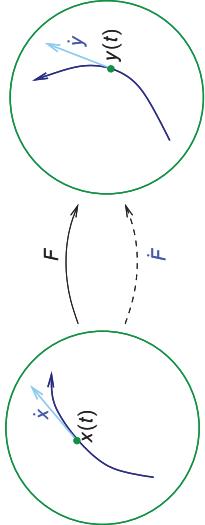
## Forward AD (Lighthouse Example)

|                      |                      |                                       |
|----------------------|----------------------|---------------------------------------|
| $v_1 = v_{-1} * v_0$ | $v_1 = v_{-1} * v_0$ | $v_1 = v_{-1} * v_0 + v_{-1} * v_0$   |
| $v_2 = \tan(v_1)$    | $v_2 = \tan(v_1)$    | $v_2 = v_1 / \cos(v_1)^2$             |
| $v_3 = v_{-2} - v_2$ | $v_3 = v_{-2} - v_2$ | $v_3 = v_{-2} - v_2$                  |
| $v_4 = v_{-3} * v_2$ | $v_4 = v_{-3} * v_2$ | $v_4 = v_{-3} * v_2 + v_{-3} * v_2$   |
| $v_5 = v_4 / v_3$    | $v_5 = v_4 / v_3$    | $v_5 = (v_4 - v_3 * v_5) * (1 / v_3)$ |
| $v_6 = v_5$          | $v_6 = v_5$          | $v_5 = v_5$                           |
| $v_7 = v_5 * v_{-2}$ | $v_7 = v_5 * v_{-2}$ | $v_7 = v_5 * v_{-2} + v_5 * v_{-2}$   |
| $y_1 = v_6$          | $y_1 = v_6$          |                                       |
| $y_2 = v_7$          | $y_2 = v_7$          |                                       |

## Forward AD (Lighthouse Example)

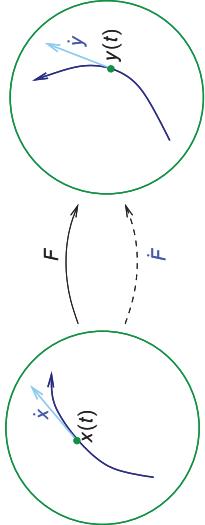
|                         |                      |                                       |
|-------------------------|----------------------|---------------------------------------|
| $v_{-3} = x_1 = \nu$    | $v_{-3} = x_1 = \nu$ | $v_{-3} = \dot{x}_1$                  |
| $v_{-2} = x_2 = \gamma$ | $v_{-2} = \gamma$    | $v_{-2} = \dot{x}_2$                  |
| $v_{-1} = x_3 = \omega$ | $v_{-1} = \omega$    | $v_{-1} = \dot{x}_3$                  |
| $v_0 = x_4 = t$         | $v_0 = t$            | $v_0 = \dot{x}_4$                     |
|                         |                      |                                       |
| $v_1 = v_{-1} * v_0$    | $v_1 = v_{-1} * v_0$ | $v_1 = v_{-1} * v_0 + v_{-1} * v_0$   |
| $v_2 = \tan(v_1)$       | $v_2 = \tan(v_1)$    | $v_2 = v_1 / \cos(v_1)^2$             |
| $v_3 = v_{-2} - v_2$    | $v_3 = v_{-2} - v_2$ | $v_3 = v_{-2} - v_2$                  |
| $v_4 = v_{-3} * v_2$    | $v_4 = v_{-3} * v_2$ | $v_4 = v_{-3} * v_2 + v_{-3} * v_2$   |
| $v_5 = v_4 / v_3$       | $v_5 = v_4 / v_3$    | $v_5 = (v_4 - v_3 * v_5) * (1 / v_3)$ |
| $v_6 = v_5$             | $v_6 = v_5$          | $v_5 = v_5$                           |
| $v_7 = v_5 * v_{-2}$    | $v_7 = v_5 * v_{-2}$ | $v_7 = v_5 * v_{-2} + v_5 * v_{-2}$   |
| $y_1 = v_6$             | $y_1 = v_6$          |                                       |
| $y_2 = v_7$             | $y_2 = v_7$          |                                       |

## Forward Mode of AD



$$\dot{y}(t) = \frac{\partial}{\partial t} F(x(t)) = F'(x(t)) \dot{x}(t) \equiv F(x, \dot{x})$$

## Forward Mode of AD



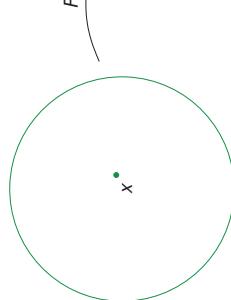
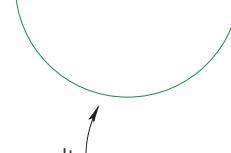
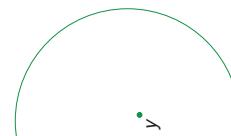
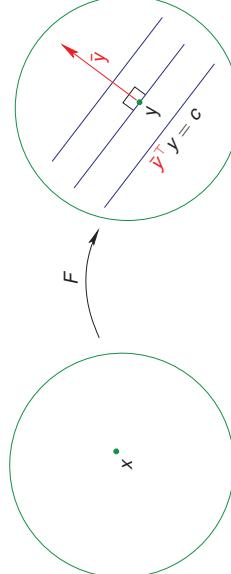
## Forward AD (Lighthouse Example)

|            |                |                  |                                           |
|------------|----------------|------------------|-------------------------------------------|
| $v_{-3} =$ | $x_1 = \nu$    | $\dot{v}_{-3} =$ | $\dot{x}_1$                               |
| $v_{-2} =$ | $x_2 = \gamma$ | $\dot{v}_{-2} =$ | $\dot{x}_2$                               |
| $v_{-1} =$ | $x_3 = \omega$ | $\dot{v}_{-1} =$ | $\dot{x}_3$                               |
| $v_0 =$    | $x_4 = t$      | $\dot{v}_0 =$    | $\dot{x}_4$                               |
| $v_1 =$    | $v_{-1} * v_0$ | $\dot{v}_1 =$    | $\dot{v}_{-1} * v_0 + v_{-1} * \dot{v}_0$ |
| $v_2 =$    | $\tan(v_1)$    | $\dot{v}_2 =$    | $\dot{v}_1 / \cos(v_1)^2$                 |
| $v_3 =$    | $v_{-2} - v_2$ | $\dot{v}_3 =$    | $\dot{v}_{-2} - \dot{v}_2$                |
| $v_4 =$    | $v_{-3} * v_2$ | $\dot{v}_4 =$    | $\dot{v}_{-3} * v_2 + v_{-3} * \dot{v}_2$ |
| $v_5 =$    | $v_4 / v_3$    | $\dot{v}_5 =$    | $(\dot{v}_4 - \dot{v}_3 * v_5) * (1/v_3)$ |
| $v_6 =$    | $v_5$          | $\dot{v}_6 =$    | $\dot{v}_5$                               |
| $v_7 =$    | $v_5 * v_{-2}$ | $\dot{v}_7 =$    | $\dot{v}_5 * v_{-2} + v_5 * \dot{v}_{-2}$ |
| $y_1 =$    | $v_6$          | $\dot{y}_1 =$    | $v_6$                                     |
| $y_2 =$    | $v_7$          | $\dot{y}_2 =$    | $\dot{v}_7$                               |

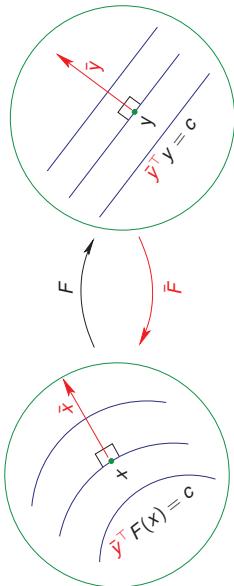
```
void d1_f(double* x, double* d1_x, double* y, double* d1_y)
{
 //\$ad indep x d1_x
 //\$ad dep y d1_y
 {
 double v[2];
 double w1[0] = 0;
 ...
 double w1_5 = 0;
 ...
 double d1_w1_5 = d1_x[2];
 d1_w1_1 = d1_x[3];
 w1_0 = x[2];
 w1_1 = x[3];
 d1_w1_2 = w1_1*d1_w1_0 + w1_0*d1_w1_1;
 w1_2 = w1_0*w1_1;
 d1_w1_3 = 1/(cos(w1_2)*cos(w1_2)) * d1_w1_2;
 w1_3 = tan(w1_2);
 ...
 }
 using ddc 1.0 (U. Naumann, RWTW Aachen)
}
```

## Reverse Mode of AD

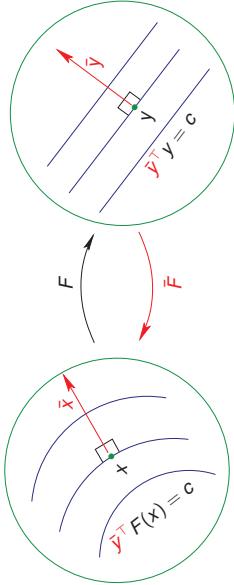
## Reverse Mode of AD



## Reverse Mode of AD



## Reverse Mode of AD



$$\bar{x}^T \equiv \bar{y}^T F'(x) = \nabla_x \langle \bar{y}^T F(x) \rangle \equiv \bar{F}(x, \bar{y})$$

```

V_-3 = x1; V_-2 = x2; V_-1 = x3; V_0 = x4;
V_1 = V_-1 * V_0; V_2 = tan(V_1);
V_3 = V_-2 - V_2; V_4 = V_-3 * V_2;
V_5 = V_4 / V_3; V_6 = V_5 * V_-2;
V_6 = V_5 * V_2; y1 = V_6;
y2 = V_6; y2 = V_6;

V_6 = V_1; V_6 = V_6;
V_6 += V_6 * V_-2; V_2 += V_6 * V_5;
V_6 += V_6 / V_3; V_3 -= V_6 * V_5 / V_3;
V_-3 += V_4 * V_2; V_2 += V_4 * V_-3;
V_-2 += V_3; V_2 -= V_3;
V_1 += V_2 / cos(V_1);
V_1 += V_2 * V_0; V_0 += V_1 * V_-1;
x4 = V_0; x3 = V_-1; x2 = V_-2; x1 = V_-3;

```

```

void b1_f(int& bmode, 1, double*x, double*b1_x, double*y, double*b1_y)
{
 // Sad indep x bmode_1, double*x, double*b1_x, double*y, double*b1_y
 // Sad dep y b1_x
 { double v1[2];
 double v2[2];
 double w1_0 = 0;
 double w1_1 = 0;
 double w1_2 = 0;
 int save cs_c = 0;
 if (bmode == 1 == 1) { // augmented forward section
 cs_c_0 = 0;
 cs_c = cs_c_0;
 fds[fdsc_c] = v0[0];
 fdsc_c = fdsc_c + 1;
 v0[0] = tan(x[2]*x[3]);
 }
 fds[fdsc_c] = fds_c + 1;
 fds_c = fds_c + 1;
 while (cs_c < save cs_c) { // reverse section
 cs_c = cs_c - 1;
 if (cs_c <= 0) {
 fdsc_c = fdsc_c - 1;
 v1[0] = x[1];
 y[1] = y[0];
 w1_1 = w1_0;
 b1_w1_2 = b1_y[1];
 b1_w1_0 = w1_1 * b1_w1_2;
 b1_y[0] = b1_y[0] + b1_w1_1;
 b1_x[1] = b1_x[1] + b1_w1_0;
 }
 }
 }
}

```

## ... and the real code generated by dcc 1.0

```

void b1_f(int& bmode, 1, double*x, double*b1_x, double*y, double*b1_y)
{
 // Sad indep x bmode_1, double*x, double*b1_x, double*y, double*b1_y
 // Sad dep y b1_x
 { double v1[2];
 double v2[2];
 double w1_0 = 0;
 double w1_1 = 0;
 double w1_2 = 0;
 int save cs_c = 0;
 if (bmode == 1 == 1) { // augmented forward section
 cs_c_0 = 0;
 cs_c = cs_c_0;
 fds[fdsc_c] = v0[0];
 fdsc_c = fdsc_c + 1;
 v0[0] = tan(x[2]*x[3]);
 }
 fds[fdsc_c] = fds_c + 1;
 fds_c = fds_c + 1;
 while (cs_c < save cs_c) { // reverse section
 cs_c = cs_c - 1;
 if (cs_c <= 0) {
 fdsc_c = fdsc_c - 1;
 v1[0] = x[1];
 y[1] = y[0];
 w1_1 = w1_0;
 b1_w1_2 = b1_y[1];
 b1_w1_0 = w1_1 * b1_w1_2;
 b1_y[0] = b1_y[0] + b1_w1_1;
 b1_x[1] = b1_x[1] + b1_w1_0;
 }
 }
 }
}

```

```

V_-3 = x1; V_-2 = x2; V_-1 = x3; V_0 = x4;
V_1 = V_-1 * V_0; V_2 = tan(V_1);
V_3 = V_-2 - V_2; V_4 = V_-3 * V_2;
V_5 = V_4 / V_3; V_6 = V_5 * V_-2;
V_6 = V_5 * V_2; y1 = V_6;
y2 = V_6; y2 = V_6;

V_6 = V_1; V_6 = V_6;
V_6 += V_6 * V_-2; V_2 += V_6 * V_5;
V_6 += V_6 / V_3; V_3 -= V_6 * V_5 / V_3;
V_-3 += V_4 * V_2; V_2 += V_4 * V_-3;
V_-2 += V_3; V_2 -= V_3;
V_1 += V_2 / cos(V_1);
V_1 += V_2 * V_0; V_0 += V_1 * V_-1;
x4 = V_0; x3 = V_-1; x2 = V_-2; x1 = V_-3;

```

```

V_-3 = x1; V_-2 = x2; V_-1 = x3; V_0 = x4;
V_1 = V_-1 * V_0; V_2 = tan(V_1);
V_3 = V_-2 - V_2; V_4 = V_-3 * V_2;
V_5 = V_4 / V_3; V_6 = V_5 * V_-2;
V_6 = V_5 * V_2; y1 = V_6;
y2 = V_6; y2 = V_6;

V_6 = V_1; V_6 = V_6;
V_6 += V_6 * V_-2; V_2 += V_6 * V_5;
V_6 += V_6 / V_3; V_3 -= V_6 * V_5 / V_3;
V_-3 += V_4 * V_2; V_2 += V_4 * V_-3;
V_-2 += V_3; V_2 -= V_3;
V_1 += V_2 / cos(V_1);
V_1 += V_2 * V_0; V_0 += V_1 * V_-1;
x4 = V_0; x3 = V_-1; x2 = V_-2; x1 = V_-3;

```



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Basics of Algorithmic Differentiation      The Reverse Mode

## Conclusions: Basic AD

- ▶ Evaluation of derivatives with working accuracy  
(Griewank, Kulshreshtha, Walther 2012)
- ▶ Forward mode:  $\text{OPS}(F'(x)\dot{x}) \leq c\text{OPS}(F), \quad c \in [2, 5/2]$
- ▶ Reverse mode:  $\text{OPS}(\bar{y}^\top F'(x)) \sim c\text{OPS}(F), \quad c \in [3, 4]$

→ Gradients are cheap  $\sim$  Function Costs!!

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## Conclusions: Basic AD

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- ▶ Forward mode:  $\text{OPS}(F'(x)\dot{x}) \leq c\text{OPS}(F), \quad c \in [7, 10]$
- ▶ Reverse mode:  $\text{OPS}(\bar{y}^\top F'(x)) \sim c\text{OPS}(F), \quad c \in [2, 5/2]$
- ▶ MEM( $\bar{y}^\top F'(x)$ )  $\sim c\text{OPS}(F), \quad c \in [3, 4]$

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Basics of Algorithmic Differentiation      The Reverse Mode

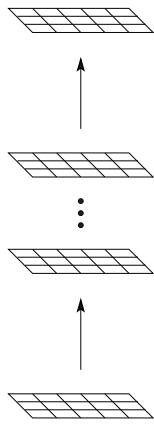
## Automatic Differentiation by Overloading in C++

- ▶ **ADOL-C version 2.3**
- ▶ available at COIN-OR since May 2009
- ▶ interface to ColPack (Purdue University) and Ipopt (COIN-OR)
- ▶ recent developments
  - ▶ improved computation of sparsity pattern for Hessians
  - ▶ handling of MPI-parallel codes
  - ▶ handling of GPU-parallel codes
  - ▶ future plans
    - ▶ generalized derivatives for nonsmooth functions
    - ...

**Questions:** Structure Exploration!!  
Time-stepping, sparsity, fixed point iteration, ...



## Calculating Adjoints

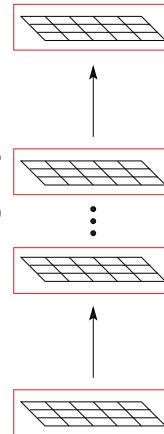


Integration of forward solution:

$$y_{i+1} = F(y_i, u_i), \quad i = 1, \dots, I$$

Integration of adjoint  $\bar{y}_{i-1} = \bar{F}(\bar{y}_i, \bar{u}_i, y_i)$ ,  $i = I, \dots, 1$ ?

## Calculating Adjoints



Integration of forward solution:

$$y_{i+1} = F(y_i, u_i), \quad i = 1, \dots, I$$

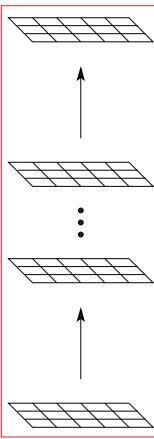
Integration of adjoint  $\bar{y}_{i-1} = \bar{F}(\bar{y}_i, \bar{u}_i, y_i)$ ,  $i = I, \dots, 1$ ?

Time Structure Exploitation

Memory requirement?? Computing time ?? Adjoint ??



## Calculating Adjoints

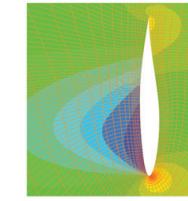


Integration of forward solution:

$$y_{i+1} = F(y_i, u_i), \quad i = 1, \dots, I$$

Integration of adjoint  $\bar{y}_{i-1} = \bar{F}(\bar{y}_i, \bar{u}_i, y_i)$ ,  $i = I, \dots, 1$ ?

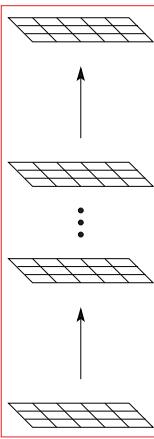
## Calculating Adjoints



- ▶ Example:  
Shape Optimization  
in Aerodynamics
- ▶ Target: Minimize drag



## Calculating Adjoints

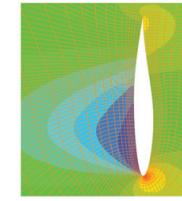


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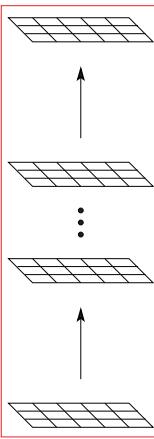
## Pseudo Time-dependent Problems



- ▶ Example:  
Shape Optimization  
in Aerodynamics
- ▶ Target: Minimize drag



## Calculating Adjoints

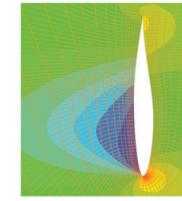


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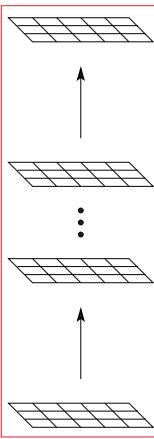
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## Calculating Adjoints

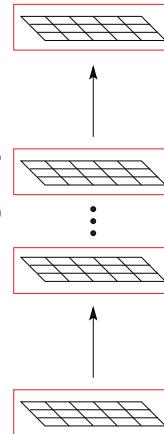


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## Calculating Adjoints



Integration of forward solution:

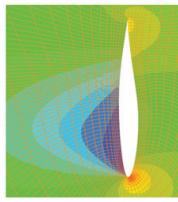
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Time Structure Exploitation

Memory requirement?? Computing time ?? Adjoint ??

## Pseudo Time-dependent Problems



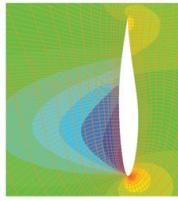
- ▶ Example:  
Shape Optimization  
in Aerodynamics

- ▶ Target: Minimize drag

### Approaches:

- ▶ Exploitation of fixed point structure  
⇒ reverse accumulation of gradient (Christiansen 1991)  
⇒ TIME(gradient)/TIME(target function)  $< 9$   
(Gauger, Walther, Özkeya, Moldenhauer 2012)

## Pseudo Time-dependent Problems



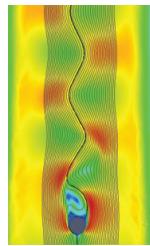
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(Gauger, Walther, Özkeya, Moldenhauer 2012)
- ▶ One-Shot Optimization  
⇒ again adjoint of only one time step required  
N. Gauger, A. Griewank, E. Özkeya

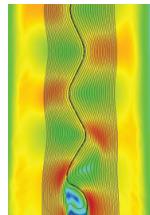
## Real Time-dependent Problems



- ▶ Example:  
Transient flows

- ▶ Target: Minimize drag/turbulence

## Real Time-dependent Problems



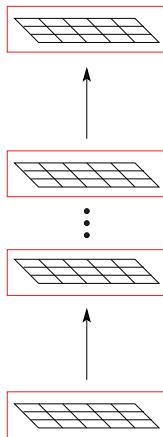
- ▶ Example:  
Transient flows

- ▶ Target: Minimize drag/turbulence

Approaches: Checkpointing in all variations, adjoint of one time step

- ▶ PDE-based optimization: Windowing  
Berggren, Meidner, Vexler, ...
- ▶ Binomial Checkpointing  
Griewank, Walther, Sternberg, Stumm, Moin, ...
- ▶ in general for AD: subroutine oriented checkpointing  
OpenAD, Tapenade

## Calculating Adjoints II



Integration of forward solution:

$$y_{i+1} = F(y_i, u_i), \quad i = 1, \dots, I$$

$$\text{Integration of adjoint } \bar{y}_{i-1} = \bar{F}(\bar{y}_i, \bar{u}_i, y_i), \quad i = I, \dots, 1?$$

Time Structure Exploitation

Memory requirement?? Computing time ??

Adjoint ?? Computing time ??

## Optimisation for Nanooptics

Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

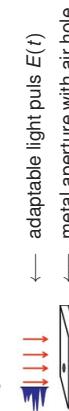
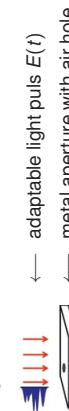
Generic configuration:



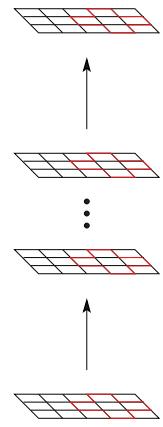
## Optimisation for Nanooptics

Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

Generic configuration:



## Calculating Adjoints II



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Time and Space Structure Exploitation

Memory requirement?? Computing time ??

# UNIVERSITÄT PADERBORN

Structure-Exploring Algorithmic Differentiation Structure in Time and Space  
Die Universität der Informationsgesellschaft

# Optimisation for Nanooptics

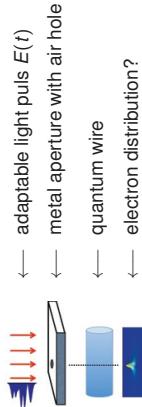
Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

Generic configuration:



Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

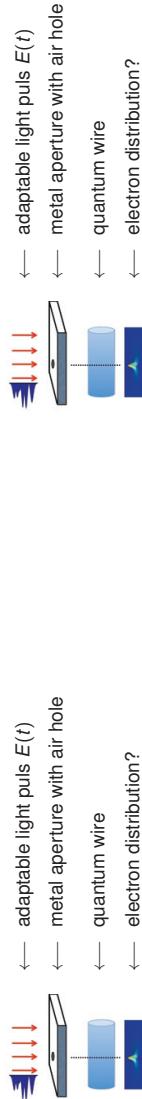
Generic configuration:



# Optimisation for Nanooptics

Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

Generic configuration:



# Optimisation for Nanooptics

Cooperation with T. Meier, M. Reichelt, Dep. Physik, Uni Paderborn

Generic configuration:



$$\text{Light puls: } E(t) = \sum A_i \exp\left(-\left(\frac{t-t_i}{\Delta t_i}\right)^2\right) \cos(\omega_i t + \phi_i)$$

$$\text{with } E(t) = \sum A_i \exp\left(-\left(\frac{t-t_i}{\Delta t_i}\right)^2\right) \cos(\omega_i t + \phi_i) \quad \text{Parameter: } A_i, \phi_i \Rightarrow 60!$$

## Nanooptics: Optimisation

so far: Genetic algorithms

- Now: L-BFGS and efficient gradient computation
  - AD coupled with hand-coded adjoints
  - Checkpointing (160 000 time steps!!)

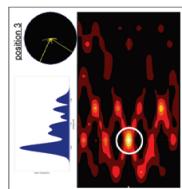
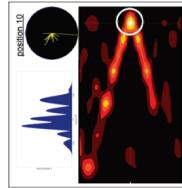
$\Rightarrow \text{TIME}(\text{gradient})/\text{TIME}(\text{target function}) < 7$  despite of checkpointing!

## Nanooptics: Optimisation

so far: Genetic algorithms

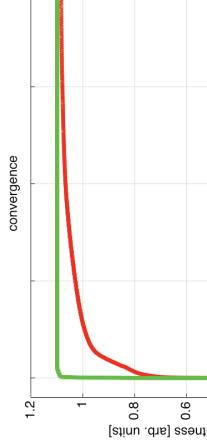
- Now: L-BFGS and efficient gradient computation
  - AD coupled with hand-coded adjoints
  - Checkpointing (160 000 time steps!!)

$\Rightarrow \text{TIME}(\text{gradient})/\text{TIME}(\text{target function}) < 7$  despite of checkpointing!



- excite
  - at same position
  - at same time
  - with same energy
- optimize
  - for same  $L_{\text{ext}}$
  - for same  $L_{\text{int}}$
  - for different positions

## Nanooptics: Comparison



## Conclusions

- Basics of Algorithmic Differentiation
- Efficient evaluation of derivatives with working accuracy
- Discrete Analogons of sensitivity and adjoint equation
- Theory for basic modes complete, advanced AD?

(Walther, Reichelt, Meier 2011)

## Conclusions

- ▲ Basics of Algorithmic Differentiation
  - ▲ Efficient evaluation of derivatives with working accuracy
    - ▲ Discrete Analogons of sensitivity and adjoint equation
    - ▲ Theory for basic modes complete, advanced AD?
  - ▲ Structure exploitation indispensable
- ▲ Consistent adjoint information? Efficient implementation?  
Suitable combination of continuous and discrete approach!

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- ▲ Consistent adjoint information? Efficient implementation?  
Suitable combination of continuous and discrete approach!



## FUNCTIONAL DEVELOPMENT WITH MODELICA

### Stefan-Alexander Schneider, Schneider System Consulting

In the early phase of the product development, it is crucial to quickly and accurately evaluate a systems overall performance in order to fully define and optimize viable system and functional architectures. The presentation explains the development steps for an embedded controller. Typically, the behavior of a dynamic system (plant and controller) is in general too complex to treat by theory or formulas. Several simulation methods have been established for analyzing such systems.

The presented virtual integration method allows to model and simulate the entire system, and thus the validation of the design decisions in an early phase of the development. This approach is conducted on a model in equation based languages to gain knowledge about the (intended) real system behavior. Such an abstraction typically allows to focus on the main properties and their effects of the studied multi-domain system.

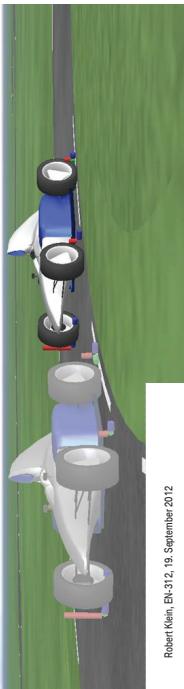
The new approach of virtual integration is demonstrated for the development of a control algorithm for an embedded controller. The entire system – both the plant and the control components – is designed with the modeling language Modelica. All necessary activities are presented for the role of the function developer and explained for the example traffic light controller for a simple intersection.

The virtual integration method usually combines components that require specific domain solvers for mechanical, electrical, etc. components, and, consequently, is based on the co-simulations, described in [1, 4]. There is a rather large literature on the Vee-Model and systems engineering, see e.g. [5, 3, 2]. For more general introduction see, e.g., [6, 7].

- 
- [1] Stefan-Alexander Schneider Andreas Maier. Grundlagen, Methoden und Anwendungen in Modellbildung und Simulation. Tagungsband ASIM 2011, 2011.
  - [2] R. Haberfellner, Olivier L. de Weck, E. Fricke, and S. Vössner. Systems Engineering – Grundlagen und Anwendungen. Orell Füssli Verlag, Zurich, 12th edition edition, January 2012. ISBN 978-3-85743-998-8.
  - [3] Richard Harwell. Systems Engineering, A Way of Thinking, A Way of Doing Business, Enabling Organized Transition from Need to Product, 1997. [Online; August 1997].
  - [4] H. Palm Stefan-Alexander Schneider, B. Schick. Virtualization, Integration and Simulation in the Context of Vehicle Systems Engineering. In Embedded World 2012 Exhibition & Conference Proceedings. Weka Fachmedien, 2012.
  - [5] Tim Weilkiens. Die Rolle des Systems Engagements.
  - [6] Wikipedia. Systems Engineering – Wikipedia, the free encyclopedia, 2012. [Online; Status 13 May 2012].
  - [7] Wikipedia. V-Modell – Wikipedia, Die freie Enzyklopädie, 2012. [Online; Stand 29. März 2012].

## AGENDA.

- Problem Definition
- Virtual System Prototyping
  - Virtual System Design
  - Virtual System Integration
- Test of Numerical Stability
- Design Evaluation



Robert Klein, EH-312, 19 September 2012

## VIRTUAL SYSTEM PROTOTYPING.

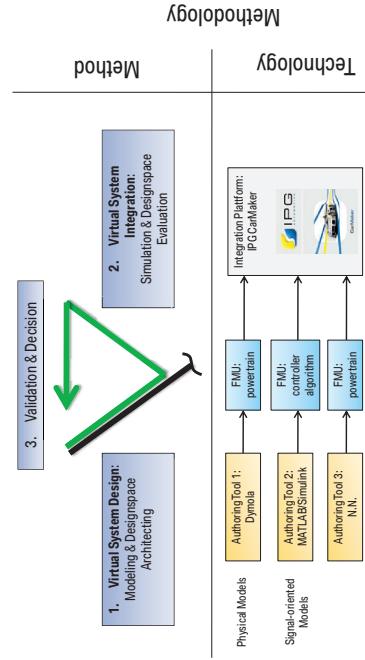
RESULTS OF THE MASTER THESIS ROBERT KLEIN  
DR. STEFAN-ALEXANDER SCHNEIDER, BMW GROUP AND  
PROF. DR. HERBERT PALM, HOCHSCHULE MÜNCHEN.

**BMW**  
GROUP

## PROBLEM DEFINITION.

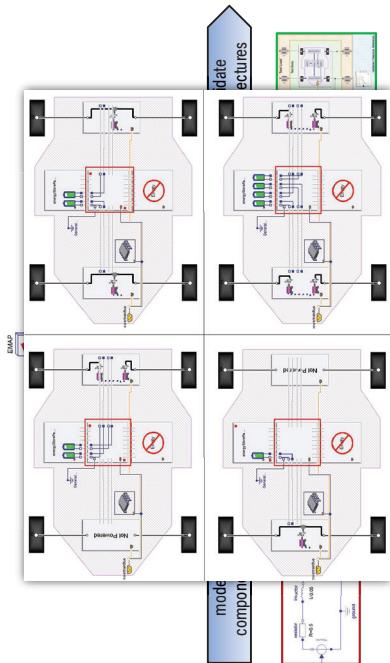


## VIRTUAL SYSTEM PROTOTYPING.



## VIRTUAL SYSTEM DESIGN.

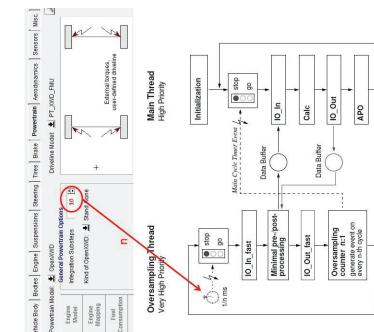
## VIRTUAL SYSTEM INTEGRATION.



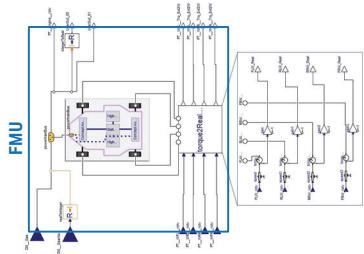
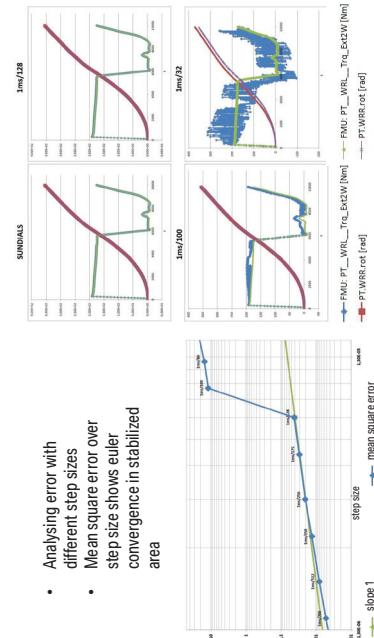
## VIRTUAL SYSTEM INTEGRATION.

4. Exporting FMU using the Modelon FMI Toolbox
5. Integrating the powertrain model using the OpenXWD framework

- Analysing error with different step sizes
- Mean square error over step size shows euler convergence in stabilized area



## TEST OF NUMERICAL STABILITY.



1. Definition of FMU with signal-oriented, CarMaker-specific interfaces
2. Converting physical interfaces into signal-oriented interfaces with modelica sensor component
3. Synchronizing axes with modelica speed component

C:\...\e...

C:\...\e...

C:\...\e...

C:\...\e...

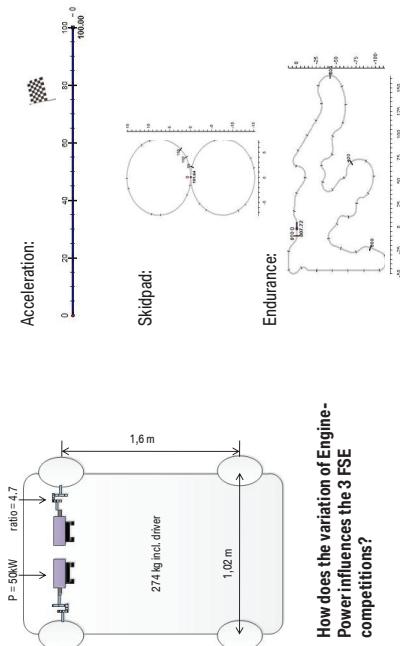
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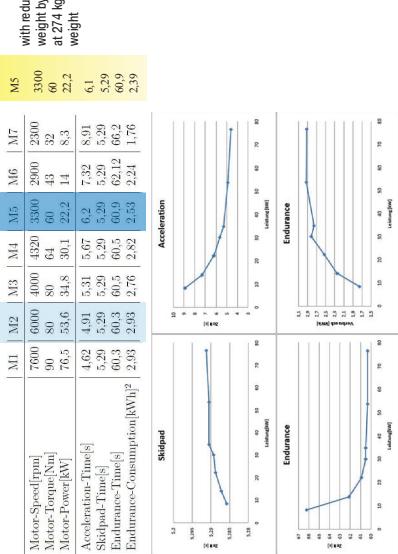
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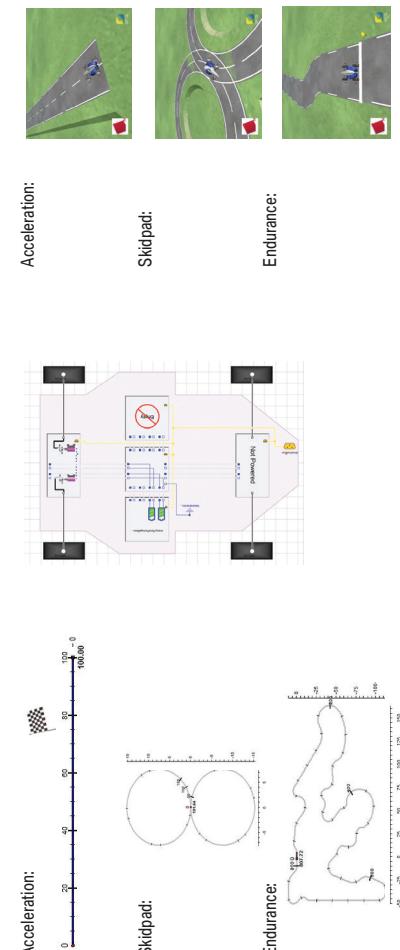
## DESIGN EVALUATION.



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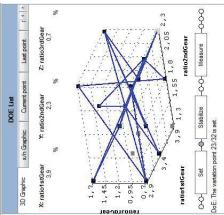


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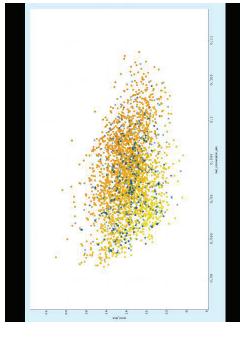


Acceleration:  
Skidpad:  
Endurance:

## PROSPECT.



- Systematic Evaluation with Design of Experiments
- Localize optimized Trade-Offs



Energy consumption

Source: UPM, EU 91/10, Commission Directive 2011/73

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Design of Experiments, Dr. S. V. T. C. Commissaris, MSc

THANK YOU.



Fakultät für Elektrotechnik  
und Informationstechnik



Darmstadt, 10.07.2013, Dr. Svenja Lüdtke

Schm... 19