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Restoring Characteristic Eigenvalues as Reactive Powers for Simple and Complex Media in Surface Integral Formulations

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Abstract—The Theory of Characteristic Modes (TCM) has recently been shown to be beneficial in solving a wide variety of complex electromagnetic problems. However, there are still open issues in using TCM to analyze objects which consist of simple or complex media. Either a volume integral equation (VIE) or a surface integral equation (SIE) is required to solve for the characteristic modes of these objects. Herein, we overview the important issue that the characteristic eigenvalues obtained from SIE formulations are not related to the modal reactive power, unlike the classical TCM definition. A recently proposed solution that restores the modal reactive power interpretation of characteristic eigenvalues is described. A numerical example is provided to demonstrate the differences between the SIE eigenvalues and the restored eigenvalues.

Keywords—Antenna analysis, characteristic modes, method of moments, Poynting's theorem.

I. INTRODUCTION

The Theory of Characteristic Modes (TCM) has been shown to provide detailed insights into a variety of complex electromagnetic problems, e.g., wideband antenna synthesis, multiple-input multiple-output (MIMO) antenna optimization, scattering problems, and object characterization [1]. More recently, this theory has been embraced by the electromagnetic community for aiding in the analysis of simple and complex media (lossless and lossy dielectric and magnetic materials). However, the theoretical basis for TCM as described in the classical paper [2] is not properly fulfilled when a surface integral equation (SIE) is used to solve for the characteristic modes (CMs) of penetrable media. Two problems are present when solving for CMs using the existing SIE methods [1]. The first problem is related to the presence of internal resonances in SIE CM solutions [3], and the second is related to the relationship between characteristic eigenvalue and modal reactive power. As the first problem has been solved using a variety of different methods [3]-[5], this paper focuses on the second problem. In particular, we first overview the problem that the characteristic eigenvalues from SIE formulations are not equal to the modal reactive powers. Then, we overview a recent proposal which address this problem [6]. This proposed method recalculates the characteristic eigenvalues based on the modal reactive power definition from Poynting's theorem. Finally, a numerical example is provided to demonstrate the significant differences between the SIE eigenvalues and the restored eigenvalues, which establishes the importance of the problem as well as the proposed solution.

II. EIGENVALUES IN TCM FORMULATIONS

The first TCM publication on solving for the CMs of penetrable objects obtains the CMs using a volume integral equation (VIE) [7]. In [7], Harrington *et al.* clearly state that the characteristic eigenvalues solved using a VIE method are not directly related to the reactive power of the corresponding CMs, as defined by Poynting's theorem. This indicates that the VIE eigenvalues are inconsistent with the classical definition of characteristic eigenvalues based on a perfect electric conductor (PEC) [2], which defines the eigenvalue as being directly related to the modal reactive power. Although this inconsistency was explicitly shown to be a problem for VIE CM solutions, the first SIE TCM paper [8] does not examine if a similar problem exists in SIE solutions. However, as detailed in [6], SIE eigenvalues are in fact different in meaning from both traditional eigenvalues for PEC objects as well as VIE eigenvalues. Moreover, it was proven that SIE eigenvalue is inconsistent with the definition of reactive power from Poynting's theorem, and as such it does not provide any direct insight into an object's resonant characteristics. Furthermore, papers [5] and [9] state that the traditional SIE TCM does not provide information into the modal reactive power. Therefore, they propose to modify other integral equations (IEs) to solve for the CMs of penetrable objects through using PEC or perfect magnetic conductor (PMC) as a specific boundary condition to represent the fields of a penetrable object. The equivalent currents on the PEC/PMC boundary allow the characteristic eigenvalue to again be related to modal reactive power [5], [9]. Consequently, these methods correctly solve for the equivalent characteristic currents and eigenvalues. Nevertheless, the imposed boundary condition presents some limitations on how this approach can be applied in antenna synthesis problems. Therefore, in some applications, it may be important to utilize a full SIE solution to solve the TCM problem.

In [6], a post-processing method of correctly determining the characteristic eigenvalues of penetrable objects using a full SIE solution which consists of both electric and magnetic currents was proposed. This solution utilizes the method presented in [3] to remove the SIE internal resonances from the CM solution. Once the internal resonances are removed, the characteristic electric and magnetic currents are used to correctly solve for the reactive power of each mode. This is achieved using Poynting's theorem, which states that the electric and magnetic currents can solve for the radiated (P_{rad}), dissipated (P_d), and reactive power ($2\omega(W_m - W_e)$) of an object, where ω is the angular frequency, W_m is the magnetic energy, W_e is the electric energy. The real part of Poynting's theorem is related to the radiated and dissipated power, and the imaginary part is related to the reactive power. Using these insights, (1) and (2) were derived in [6] to solve for the characteristic eigenvalues of a CM solution. In particular, equation (1) should be used for forced symmetric SIE formulations, such as using the Poggio-Miller-Chan-Harrington-Wu-Tsai (PMCHWT) SIE formulation in TCM problems. On the other hand, equation (2) is meant for symmetric SIE formulations. In these equations, **J** is the electric current density, **E** is the electric field intensity, **M** is the magnetic current density, **H** is the magnetic field intensity, S defines the surface of the object $(\bullet)_n$ denotes the nth CM, and $(\bullet)^R$ or $(\bullet)^I$ define if the quantity is real or imaginary, respectively.

$$\lambda_n = \frac{\iint\limits_{S}\left(\mathbf{J}_n^R \bullet \mathbf{E}_n^I - \mathbf{M}_n^R \bullet \mathbf{H}_n^I\right)ds}{\iint\limits_{S}\left(\mathbf{J}_n^R \bullet \mathbf{E}_n^R + \mathbf{M}_n^R \bullet \mathbf{H}_n^R\right)ds} = \frac{2\omega\left(W_m - W_e\right)}{P_{rad} + P_d} \qquad (1)$$

$$\lambda_n = \frac{\iint\limits_{S}\left(\mathbf{J}_n^R \bullet \mathbf{E}_n^I + \mathbf{M}_n^I \bullet \mathbf{H}_n^R\right)ds}{\iint\limits_{S}\left(\mathbf{J}_n^R \bullet \mathbf{E}_n^R + \mathbf{M}_n^I \bullet \mathbf{H}_n^I\right)ds} = \frac{2\omega\left(W_m - W_e\right)}{P_{rad} + P_d} \qquad (2)$$

III. NUMERICAL EXAMPLE

A simple dielectric object is used to illustrate the difference between the restored eigenvalues (1) that are directly related to the modal reactive powers and the traditional SIE eigenvalues described by (20) in [2]. In this example, the penetrable object is a dielectric cylinder with a height of 250 mm, and a radius of 100 mm. It is made of a lossless dielectric material with a relative permittivity of $\varepsilon_r = 10$. The TCM SIE problem was solved using a forced symmetric PMCHWT impedance matrix. The operator form of the equations was implemented in matrix form by using the Rao-Wilton-Glisson (RWG) basis functions in the method of moments (MoM) computations and the internal resonances were removed in post-processing using [3]. The eigenvalues for the first 7 CMs (λ_1-λ_7) were calculated using both (1) and the traditional eigenvalue definition, and shown in Fig. 1. As can be seen, there are substantial differences between the two sets of eigenvalues. Notably, all the traditional eigenvalues (not related to reactive power) are capacitive below their resonances, whereas several of the restored eigenvalues are inductive below their resonances.

IV. CONCLUSION

This paper overviews one problem related to analyzing simple and complex media using SIE TCM formulations. In particular, traditional SIE formulations incorrectly solve for the TCM eigenvalue. However it has been shown that the correct eigenvalue can be restored using a post-processing method. Furthermore, the differences between the correct eigenvalues and the incorrect eigenvalues are illustrated using a simple numerical example. This example clearly demonstrates that the two sets of eigenvalues are significantly different, e.g., they store different types of energies.
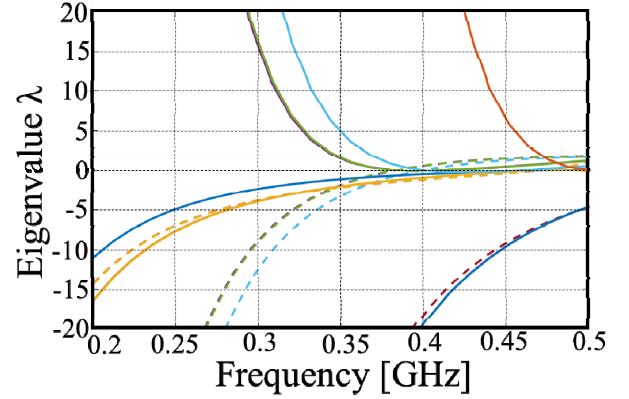

Fig. 1. Eigenvalues of lossless cylinder based on both Poynting's theorem definition from (1) (solid lines) and TCM definition (dashed lines).

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