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Far-Field Orthogonality of Volume-Based Characteristic Modes for Real Materials

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***Abstract*–Two different solutions for the characteristic modes (CMs) of lossy structures were previously developed using induced volume currents. The first solution diagonalizes the scattering and perturbation matrices, guaranteeing far-field orthogonality at the cost of imaginary eigenvalues and eigencurrents. The second solution does not perfectly diagonalize these matrices, but maintains real-valued eigenvalues and eigencurrents. When these matrices are non-perfectly diagonalized the characteristic far-fields are no longer fully orthogonal to one another. However, the second formulation has not yet been investigated, and as such the effect of non-perfect diagonalization on the orthogonality of the far-field patterns is still unknown. For this reason, it is not clear if volume-based characteristic modes can be used for CM analysis (CMA) of lossy dielectric structures. This article evaluates the effect of different losses on the modal orthogonality of two dielectric resonant structures, and determines the practicality of using volume-based CMA.**

***Index Terms* — Antennas, Antenna Design, Characteristic Modes, VIE, Far-field Orthogonality.**

1. Introduction

The Theory of Characteristic Modes (TCM) [1] provides in-depth physical insights into the fundamental radiation properties of any structure. The resonant characteristics of each mode are defined by a set of characteristic attributes that are derived from the inherent orthogonal currents a structure supports. These orthogonal currents are traditionally found through an eigenvalue decomposition of a symmetric method-of-moments (MoM) impedance matrix. Conventionally, the impedance matrix is obtained for an object made from perfect electric conductors (PEC), but it is possible to solve for realistic materials using a symmetric surface integral equation (SIE) [2] or a symmetric volume integral equation (VIE) [3]. The characteristic modes (CM) of a symmetric SIE impedance matrix have were derived in [2] and evaluated in [4], but there are still many open questions for both SIE and VIE CM formulations. In [3], it was shown that the CMs of a dielectric, magnetic, or mixed-material object can be found using the VIE formulation. This formulation has multiple problems [3]: the first is that the CMs cannot be related back to the stored energy of the structure; the second is that, when losses are added, the scattering and perturbation matrices are not perfectly diagonalized.

In this article the problem associated with the non-perfect diagonalization of the VIE perturbation and scattering matrix is investigated. When an eigenvalue decomposition is used to define a set of orthogonal characteristic currents for a given lossy VIE impedance matrix, the far-fields as defined by those currents do not provide a perfectly orthogonal set of far-field patterns. The extent of this problem has never before been studied. In this article, two different dielectric resonant structures will be analyzed over six different losses, to determine the extent of the non-orthogonality of modal the far-fields across different losses.

2. Origin of non-orthogonal far-fields in VIE

CMs are defined by the set of orthogonal currents which can be used to determine any current induced by a given incident electric field. This set of currents can theoretically be viewed as the scattered divergent electric field which is induced by a given incident field. When this theory is applied to volumetric currents the electric field must be represented as the sum of both an incident electric field (E^i) on a volumetric element, as well as the scattered fields from other volumetric elements (E^s). It should be noted that this constitutive relationship, as defined by (1), is different than that defined by traditional SIE CM formulations [1].

$$J = \left(\omega\left(\varepsilon_2'' - \varepsilon_1''\right) + j\omega\left(\varepsilon_1' - \varepsilon_2'\right)\right)\left(E^i + E^s\right) \tag{1}$$

In (1), J is the induced currents, ω is the angular frequency, $\varepsilon_1 = \varepsilon_1' + j\varepsilon_1''$ is the permittivity outside a given basis tetrahedral, and $\varepsilon_2 = \varepsilon_2' + j\varepsilon_2''$ is the permittivity of a basis tetrahedral. The real part of the permittivity is associated with the stored energy within the dielectric and the imaginary part of the permittivity and is associated to the energy lost within the dielectric. This equation can be expressed in terms of impedance operators Z_v and Z_m, as described by

$$Z_v(J) = -E^s, \tag{2}$$

and

$$Z_m(J) = \left(\omega\left(\varepsilon_2'' - \varepsilon_1''\right) + j\omega\left(\varepsilon_1' - \varepsilon_2'\right)\right)^{-1}(J). \tag{3}$$

Hence, the induced current by a given incident field in the volume can be written as

$$\left(Z_v + Z_m\right)(J) = E^i. \tag{4}$$

The modes which perfectly diagonalize the impedance and scattering matrices can be found using (5), or (6) with the common terms canceled [3].

$$\left(Z_v + Z_m\right)\left(J_n\right) = \left(1 + j\lambda_n\right)\mathrm{Re}\left(Z_v\right)\left(J_n\right) \tag{5}$$

$$\left(\mathrm{Im}\left(Z_v\right) - jZ_m\right)\left(J_n\right) = \lambda_n\,\mathrm{Re}\left(Z_v\right)\left(J_n\right) \tag{6}$$

However, this decomposition may result in complex eigenvalues (λ_n) and characteristic currents (J_n). In additional, the impedance matrix as found using the method described in [5] makes it difficult to split the full impedance matrix into the two individual impedance matrix operators Z_v and Z_m. For these two reasons, it is obvious that the related impedance matrices should remain a single matrix, and an eigenvalue decomposition of the resulting linked matrix could be used to determine the CMs of the full structure as is described by

$$\left(\mathrm{Im}\left(Z_v\right)+\mathrm{Im}\left(Z_m\right)\right)\left(J_n\right)=\lambda_n\left(\mathrm{Re}\left(Z_v\right)+\mathrm{Re}\left(Z_m\right)\right)\left(J_n\right). \quad (7)$$

The specific eigenvalue decomposition that is applied in (7) maintains mode-to-mode current orthogonality, but the scattering matrices are not diagonalized as shown by (64) in [3]. This causes the far-fields to become non-orthogonal by an amount equal to $\langle J_m, \mathrm{Re}(Z_m) J_n \rangle$. As can be seen in (2), $\mathrm{Re}(Z_m)$ is based solely on the loss of the structure and will increase at a rate related to the loss and the amount of current for any given tetrahedral basis function. As the matrices are not easily separable, the non-orthogonality relationships can instead be quantified by the envelope correlation coefficient (ECC) of the far-field patterns for any two modes [6].

3. Effect of loss on volume based characteristic modes

To better understand the effect of loss on the far-field orthogonality of a structure, two different structures were analyzed over six different levels of losses. The first structure was a dielectric sphere with a radius of 150 cm, dielectric constant of $\varepsilon_r = 10$ and center frequency of 250 MHz, with greater than 13,000 basis functions. The second structure was a dielectric cylinder with a radius of 135 cm, height of 300 cm, dielectric constant of $\varepsilon_r = 10$ and center frequency of 350 MHz, with greater than 18,000 basis functions. Both structures had at least one mode near resonance ($|\lambda_n| < 0.5$) at the center frequency. These structures were simulated over six different dielectric losses, with loss tangents tan δ = 0, 0.005, 0.01, 0.025, 0.05, and 0.1. The first twelve CMs of each structure, for each loss level, were obtained and their electric far-field patterns were correlated with one another to obtain the ECC between all modes. The electric far-field patterns were calculated over a sphere at 250 meters for every 10° in both theta and phi. This final ECC calculation resulted in a 12 × 12 correlation matrix, with each off-diagonal element representing how a given mode correlates to a different mode of the same structure, for a given level of loss.

The highest ECC of a given mode with all other modes is shown in Fig. 1 for each of the two structures. In this figure, modes of different loss levels (of a given structure) are mapped to one another based on a weighted tracking function which utilizes the eigenvalue, far-field correlation, and current correlation. It is noted that the 2D plot only illustrates the maximum ECC between a given mode and all other modes, and it does not provide information regarding whether one mode is correlated to more than one other mode. These simulations show that for low loss structures, the correlation is small when evaluated for a limited number of modes (i.e., 12 in this case). All the 12 evaluated modes for each structure were found to fall in the range of $|\lambda_n| < 1000$.

As the loss of the structure increases, the modal correlation increases significantly. However, no clear trend is observed between the two structures in the maximum ECC over different modes and loss levels. This is because the two structures have significantly different modes and mode mappings. The cylinder's mode-to-mode correlation was found to be proportional to the square of the loss, whereas modes 2 and 12 of the sphere are fully correlated for all losses above tan δ = 0.025, and not correlated for all tan $\delta \leq 0.01$. Therefore, it can be seen that the correlation is dependent on the loss, the amount of current in any given region of the structure, and the specific shape of the structure. From this simple analysis, it can be concluded that if the CMs of lossy structures are computed using the VIE formulation, low losses (e.g., tan $\delta \leq 0.01$) are not expected to have a significant effect on the mode-to-mode correlation. However, the correlation should be evaluated for higher loss structures.

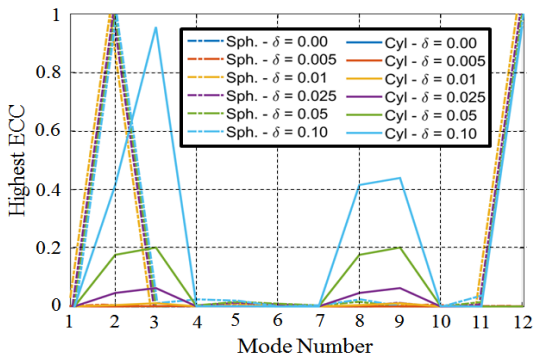

Fig. 1. Highest ECC of volume-based characteristic modes. (Modes are skewed in the y-axis to help visualize each mode)

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