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Active control of the pointing of a multi-terawatt laser

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The beam pointing of a multi-terawatt laser is stabilized on a millisecond time scale using an active control system. Two piezo mirrors, two position sensing detectors, and a computer based optimization program ensure that both near- and far-field are stable, even during single shot operation. A standard deviation for the distribution of laser shots of $2.6\,\mu$ rad is achieved. © 2011 American Institute of Physics. [doi:10.1063/1.3556438]

I. INTRODUCTION

In recent years, high power lasers have made tremendous progress and allowed scientists to access new and exciting fields of research. The advent of the chirped pulse amplification technique¹ allowed relativistic intensities to be reached even with compact laser systems. However, in order to fully utilize the potential of available high-power lasers, some quality issues still need to be addressed. In particular, the stability of laser parameters such as beam pointing, wavefront, and pulse energy are crucial in order for these lasers and their applications to benefit a broader community of scientific users. This paper addresses one of these issues, as the beam pointing of a multi-terawatt (TW) laser is actively stabilized.

The production of laser pulses of multi-TW peak power frequently causes the environment to become very noisy (electronically and mechanically). This makes the project particularly challenging in comparison with similar projects on more conventional lasers. In addition, experiments using multi-TW lasers usually require single-shot operation. Active beam pointing stabilization has been previously addressed for less powerful lasers, with high repetition rate, e.g., a 1 kHz Ti:sapphire femtosecond laser.² The principle used then is similar to ours, but it cannot operate in singleshot and only one piezo mirror was used, whereas in the work presented here single-shot operation is possible and two mirrors are used. More advanced schemes have also been demonstrated,^{3,4} but are not suitable to the present situation. In Ref. 3, the pointing stability is controlled using neural networks and a feedforward algorithm. However, this cannot be used in single-shot operation. On the other hand, the method described in Ref. 4 can be used in a single-shot mode, but since it is based on image rotation and sum-frequency generation it is not suitable for high power lasers.

The driving force for the present work was to improve the feasibility of electron acceleration experiments, with laser driven plasma waves in dielectric capillary tubes.^{5,6} Laser plasma wake fields can accelerate electrons to relativistic energies over a very short distance.⁷ However, in order to increase the energy of the electrons to the GeV range and above, the interaction length must be increased. With the help of

guiding structures, such as dielectric capillaries or plasma discharge channels, diffraction effects can be circumvented and plasma waves can be excited over several centimeters. The requirement of good laser beam pointing is crucial to achieve good guiding in this kind of structure. Good pointing stability is particularly important when using dielectric capillary tubes with inner diameter similar to the laser spot size to prevent damaging the capillary opening. In addition, the stability of the resulting electron beam depends on the pointing stability of the driving laser, among other parameters. Thus, in order to reach a good electron beam stability, the pointing of the driving laser beam must be controlled.

At the Lund Laser Centre, extensive work has been devoted to improve the pointing of its multi-TW laser. The first step was to improve it passively. The laser beam path was covered as much as possible to avoid air turbulences. Mechanical vibrations were also reduced. However, to further reduce remaining pointing instabilities, active control was necessary. This paper presents the active stabilization system developed at the Lund Laser Centre. The stabilization system was designed for single shot operation. Thus, corrections need to be made for both slow and fast effects that may have happened since the previous shot, which may be seconds, minutes, or even hours ago. The sources of pointing instabilities can be divided into three different types: (1) thermal drifts (on the time scale of minutes, hours), (2) air turbulence and slow mechanical vibrations (on the time scale of seconds, >10 ms), and (3) mechanical vibrations (>10 Hz). According to the Nyquist sampling theorem, in order to detect (and then correct) fluctuations with a given frequency, sampling has to be made at at least twice the frequency. The first type is, thus, very easily taken care of. Slow mirrors can be used to compensate for such drifts. The active stabilization system needs to be fast in order to correct for the two other types of fluctuations. The goal of our beam stabilization system is to take care of all of the first and the second type of instabilities and part of the third.

II. SETUP

The basic setup for the active stabilization system consists of two piezo mirrors and two position sensing detectors (PSDs). Each PSD records the offset of the laser beam from

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FIG. 1. (Color online) Schematic view of the beam pointing stabilization system of the Lund Laser Centre multi-TW laser (details in the text). The inset shows a side view of the injection point of the reference beam. The system works in the following way: two piezo mirrors are compensating for fluctuations in the beam pointing, one controls the near-field and other the far-field. Right before a main laser pulse arrives, the mirrors are fixed while fast shutters close to protect the detectors from the high power laser pulse. Once the laser pulse has passed the shutters open and the system starts regulating again.

the desired position and provides a voltage proportional to this offset. A fast computer and a custom made program is used to read the signals from the detector and to send appropriate voltages to the piezo mirror. For each mirror two signals are sent: one for corrections in the horizontal direction and one for the vertical direction. In order to get the beam axis right and not only the far-field position, one piezo mirror regulates the near-field while the other one takes care of the far-field.

A schematic view of the active control setup is given in Fig. 1. The high-intensity Ti:sapphire based chirped pulse amplification laser system at the Lund Laser Centre delivers up to 40 TW onto the target, with a FWHM pulse duration down to 35 fs and a beam diameter before focusing of 50 mm. The laser repetition rate is 10 Hz. Therefore, it is not possible to use the laser beam itself as a reference, if we want to correct for fluctuations faster than 5 Hz. Another reference beam is necessary that follows the same path as the TW beam. For this purpose we use part of the oscillator beam. This beam is at 80 MHz, which on the time scale we want to correct for, can be considered as continuous. A pulse picker takes ten pulses per second from this oscillator beam to be amplified, but all the remaining pulses can be used. This beam is injected into the laser system, after the second amplification stage, and follows the same path as the main laser beam. Both beams are going through a spatial filter, placed right after the injection point, and the reference beam is aligned on an iris used for the alignment of the main beam. This ensures that from this point both beams are colinear.

The reference beam is apertured by an iris before entering into the spatial filter, and the diameter of the focused beam is significantly larger than the hole of the spatial filter. Fluctuations in the pointing of the reference beam prior to the filter are, therefore, not transmitted. As the spatial filter is not diffraction limited for the main beam, fluctuations of the main beam occurring before the spatial filter are transmitted.

The system compensates only for fluctuations experienced by the main beam after the spatial filter. Before the spatial filter, two slow piezo mirror systems using the 10 Hz beam itself as a reference are used to compensate for slow drifts and to keep the laser well aligned up to the spatial filter. The reference beam has a polarization perpendicular to the main beam polarization and the two beams can, therefore, be combined using a polarizing cube, where the main laser beam is transmitted and the reference beam is reflected (see inset in Fig. 1). Later in the laser chain, before the third and final amplification stage, a Pockels cell rotates the plane of polarization of the amplified pulse, while most of the time, between the laser shots, the reference beam is not affected. Thus, both beams have the same polarization further on.

The two-dimensional PSDs used in this setup are duolateral PSDs (from SiTek Electro Optics), which have four terminals, two on the back side and two on the front side, where the terminals on the back side are placed perpendicular to the terminals on the front side. The photoelectric current, generated by the incident light, flows through the device and the relationship between the currents on the four terminals gives the light spot position. Unlike quadrant sensors, which require overlap in all quadrants, this kind of PSD provides the position of any spot within the detector region, independent of beam shape, size, and power distribution. They have very good resolution and linearity. Their fast response (400 kHz bandwidth) gives them an advantage over CCD cameras.

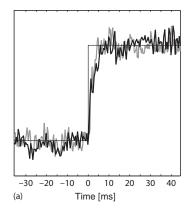
Two different types of piezo mirrors are used in the setup, as the beam has different size at the position of the mirrors. The first mirror (a 1 inch optic) sits in a piezoelectric mount (KC1-PZ from Thorlabs). The second mirror is a 4 inches optic and is mounted into a Piezo Tip/Tilt-Platform (S-340 from Physik Instrumente). Each mirror mount must be firmly mounted, or the movements of the piezo may drive the mirror mount itself into resonance. This is especially

important for the second mirror due to its large size and weight. The limiting factor of the hardware is the resonance frequency of the large piezo mirror, which is about 500 Hz.

The detectors need to record both the near- and far-field positions of the reference beam without blocking the path for the main pulse as it arrives. The first detector, for the nearfield, monitors the zeroth order reflection from the first grating of the laser pulse compressor, placed under vacuum. The second detector monitors an image of the laser focal spot at the final interaction point. The first PSD controls a piezo mirror placed before the final stage of amplification. As this PSD monitors the near-field, a two lens system is used to project the transverse position of the full beam onto the PSD. The first lens (2 m focal length) focuses the beam in order to make the full beam small enough to fit on the PSD. The second lens (5 cm focal length) is used to image the position of the beam on the first lens onto the PSD. The second detector controls the large piezo mirror placed after the pulse compressor. PSD 2 is placed in the image plane of a lens collecting the light after the interaction point and, therefore, monitors the far-field. The lens is imaging the focal point onto the PSD with a magnification of two in order to increase the sensitivity. In many experiments using this kind of laser, the laser beam is focused into a gas jet. Part of the reference beam can then be collected after the focus point by a mirror and sent to the lens and the PSD. As the center part of the laser beam is not collected, the product of the interaction, e.g., an electron beam, can still pass through. For other types of experiments, other arrangement to image the focal spot of the reference beam might be needed. For the results presented below, the focusing optics had a focal length of 47.5 cm and the imaging lens a focal length of 20 cm. One should notice that the two piezo-controlled mirrors are far apart from each other in order to uncouple their actions.

In order for the active stabilization system to work efficiently, it must be optimized for speed. A properly adapted software must be use. We chose LABVIEW field-programmable gate array (FPGA) for its reliability, determinism, and parallelism. In addition, proportional-integral-derivative (PID) controllers were implemented in the code, which allows for faster corrections than simple proportional controllers. ¹⁰ The PID loop runs at a fast rate and corrections were sent to the mirrors at 4 and 2 kHz for the small and large piezo mirrors, respectively. These rates are higher than the resonance frequencies of the mirrors and, thus, the output signal is smoothened. In addition, in order to reduce the step-like behavior of the output signal, the program interpolates between the consecutive points of the output signal.

Filtering of the detector signal is also necessary to eliminate high frequency components above the resonance frequencies of the piezo mirrors. If fed to the piezo mirrors, these high frequency components drive the mirrors unstable, disturbing the stabilization. However, filter always introduces a time delay between the observed error and the correction. In order to operate the stabilization system at a fast rate, this delay must be minimized. This is done using low-pass finite impulse response digital filters. Such filters produce much shorter delays than conventional ones and have the advantage of producing the same delay for all frequencies. ¹¹



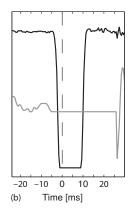


FIG. 2. (a) Typical step responses of the system. The gray and black curves are the signals recorded by the horizontal axis of the PSDs monitoring the far-field and near-field, respectively. In (b) the timing of the fast shutter used to protect the detectors is described. The black line shows the intensity recorded by the PSD and the gray line is the voltage sent to the piezo mirror. The dashed line corresponds to the time when the TW laser pulse arrives.

The time response of the system has been investigated by recording the error signal from the PSDs inside the loop. By frequency analysis of the open- and close-loop signals, it is found that frequency components up to 150 Hz for the near-field mirror and up to 100 Hz for the far-field mirror are successfully attenuated by the feedback loop. The response to set point changes was also investigated. A step of the order of the pointing fluctuations (corresponding to 6.8 μ rad for the far-field and 900 μ m for the near-field) was introduced in the set point value as shown in Fig. 2(a). We observe that it takes about 5 ms for the system to settle to the new set point values in both the far-field and the near-field. These are evidences that the system is able to act on a millisecond time scale.

In order to avoid the PSD to be destroyed by the high power laser shots, fast mechanical shutters (approximately millisecond response time, VS14 from Uniblitz) are protecting the PSDs when the main laser shots arrive. Each piezo mirror is regulating until just before the shutter closes and then holds the voltage, as described in Fig. 2(b). The delay between the laser shot and the end of regulation is very short, typically only a few ms, and the introduced error is, therefore, negligible. The timing between the shutter and the laser is shown in Fig. 2(b), where it is seen that the piezo mirrors are held at a fixed voltage 5 ms before the laser shot and the shutter starts closing 4 ms before the shot.

Some fluctuations are too dominant to be efficiently compensated for by the active control system. For example, the cryogenic cooler produces short but intense burst of high frequency vibrations that are difficult to handle because of their high frequency. We instead circumvent these vibration bursts. The cryogenic cooler is used to cool the crystal of the final stage of amplification. A pump circulates helium to the crystal at a rate of 1 Hz. During its operating cycle, when the piston reaches its stop, it makes the optical table vibrate. Given that the cycle consists of the piston going back and forth, there are two stops, giving a repetition frequency of 2 Hz for these bursts. After each shock, the optical table reacts and vibrates for some hundred milliseconds while damping the waves, as

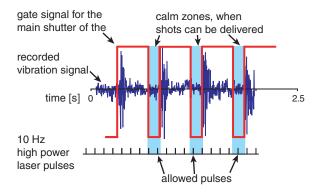


FIG. 3. (Color online) Filtering of the laser shots is necessary to avoid the short but intense burst of high frequency vibrations produced by the cryogenic cooler. The main shutter of the laser delivers shots only when the vibrations are minimal.

illustrated in Fig. 3. The vibrations are recorded and a gate signal is produced which allows the main shutter of the laser to deliver shots only when the vibrations are minimal. The calm period starts about 400 ms after the beginning of the shock and lasts 100 ms. This allows a laser shot to be delivered during this calm period. The dominant fluctuations are thus simply avoided. The control loop runs all the time but it is not able to regulate well during the shocks. However when the calm period starts, it has enough time to settle before the laser shot arrives. This mode of operation works well for single shot operation or repetition rate below 2 Hz. This is indeed the case for most types of ultrahigh intensity experiments. Modifications will be necessary if 10 Hz operation is required.

To summarize, the system works in the following way: two piezo mirrors are regulating and compensating for fluctuations in the beam pointing, one controls the near-field and one the far-field. When a high power laser shot is requested, the main shutter delivers a pulse during the calm time window of the cryogenic cooler. Right before the pulse comes, the voltages to the mirrors are fixed while fast shutters close to protect the detectors from the high power laser shot. Once the laser pulse has passed, the shutters open and the mirrors start regulating again.

III. EXPERIMENTAL RESULTS AND DISCUSSION

In order to assess the quality of the beam stabilization, signals recorded with the reference beam on the PSDs are not enough, actual laser shots must be recorded. To do so, a beam splitter was placed in the beam and the focal spot imaged by a 40 times microscope objective and recorded with a CCD camera. Two hundred consecutive laser shots in the high power configuration (the laser was attenuated, but everything else was running in the full power experimental conditions) were recorded with 3 s between each shot. The results are presented in Fig. 4. In the panels on the left, each point represents one laser shot. The origin (0,0) corresponds to the mean position of all the points. The positions recorded by the CCD camera are divided by the focal length of the laser focusing optics used and the scales are, therefore, shown in μ rad, which is

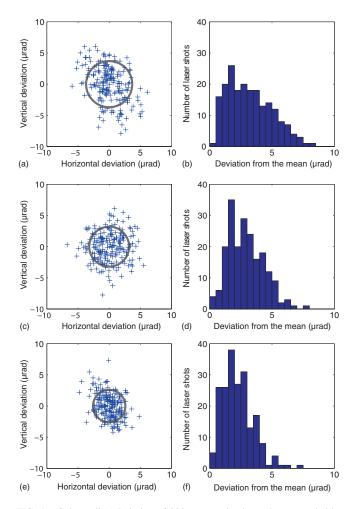


FIG. 4. (Color online) Pointing of 200 consecutive laser shots recorded by a microscope objective and a CCD camera. On the right side corresponding distributions of deviations from the mean. In (a) and (b) no active stabilization is used, in (c) and (d) the laser shots are gated in time in order to avoid firing when intense vibrations are produced by the cryogenic cooler in the laser amplifier. Finally, in (e) and (f), active stabilization is added with the help of two piezo mirrors. The circle represents the deviation corresponding to one standard deviation and its radius decreases for each step of the stabilization.

independent of the focusing optic used. In (a) no active stabilization is used. The beam pointing is, however, already fairly good thanks to all the work done on passive stabilization of the laser system. (c) Shows laser shots recorded with the gating system, delivering shots only in the calm section of the vibration cycle of the cryogenic cooler. (e) Presents shots with both gating and piezo mirror stabilization active. Each step of the stabilization shows an improvement in both the extremes and the mean of the distances between the laser shots.

On the right-hand side in Fig. 4, the distribution of the laser shots as function of their distance from the mean position is presented. This gives a better understanding of the effect of each step of the stabilization. We see that the gating of the shots already reduce the extreme points. The active stabilization further reduces it and no laser shots are more than 7.3 μ rad away from the center. If we calculate the standard deviation of the distribution, we also observe a clear improvement: 3.7 μ rad for (a), 3.2 μ rad in (c), and 2.6 μ rad in (e), which corresponds to an overall improvement of 30%. At the beginning of this project, i.e., before passive stabilization, the

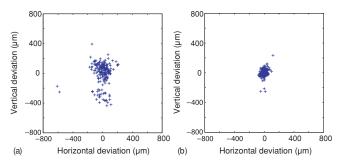


FIG. 5. (Color online) Near-field position of the reference beam for 200 consecutive shots recorded by a CCD camera imaging the position of the beam on the focusing optics. In (a) no active stabilization is used and in (b) the active stabilization is added

mean was 6.7 μ rad. It shows the importance of doing passively what can be done first. It is interesting to compare the obtained pointing fluctuations with the focal spot size. In the experiments reported in Refs. 5 and 6, using a f=1.5 m focusing mirror, the focal spot size was 40 μ m (radius of the first minimum of an Airy-like pattern). The mean of the resulting pointing fluctuations with this focusing optics would be 3.9 μ m. The resulting pointing fluctuations, thus, correspond to only 10% of the focal spot size.

When only the second piezo mirror is regulating, the standard deviation is $2.8~\mu rad$, which is similar to the value obtained with two piezo mirrors. This is expected as the CCD is monitoring the position of the beam in the focal plane, i.e., the far-field, and the second piezo mirror regulates only the far-field. In this case, the laser beam is hitting the right point in the focal plane but not necessarily along the correct axis. The use of two mirrors improves the overall pointing stability. This further shows that the use of two piezo mirrors in this configuration does not produce any interference effect.

The stability of the near-field was also investigated. To do so, a lens was placed after the focal spot to image the position of the beam on the parabolic mirror onto the CCD camera. To avoid problems with varying intensity profile, we assess the near-field stability of the reference beam instead of the main beam. Figure 5 shows the effect of the active stabilization on the position of the reference beam on the focusing optics, i.e., the near-field. In (a) there is no stabilization active, whereas in (b) the system is regulating. It is clearly observed that the near-field position of the beam is well stabilized and that the reference beam can be used to keep the main beam axis stable, as its intensity profile is very stable.

Depending on the requirements of the experiment, one might not need any active stabilization or one might want to use only one mirror. When only far-field stability is needed. For guiding experiments in capillaries, one mirror is enough in order not to damage the entrance of the capillary and it was used in this configuration in Refs. 5 and 6 with PSD 1 acting as a far-field detector. With two mirrors it should be possible to achieve even better guiding. In gas jet experiments for electron acceleration, pointing stabilization is not crucial for the generation of the electron beams. However, in order to obtain an electron beam with good pointing stability, the laser pointing stability of both the near- and far-field is important. Experiments on solid target (for example, target normal

sheath acceleration on flat foils) might not require very high pointing stability, however, for mass-limited targets one may need fast active stabilization with at least one mirror.

IV. CONCLUSION AND OUTLOOK

A system that actively stabilizes both the near-field and the far-field beams pointing of a multi-TW laser on the few millisecond time scale is introduced. The work presented above shows that a beam stabilization system can be implemented on an existing TW laser system without modifying its layout despite the high power laser pulses delivered and the resulting noisy environment due to the production of such pulses. Laser shots can be contained within a circle of 7.3 μ rad radius with a standard deviation of 2.6 μ rad. In addition, both the near- and far-field are stabilized. The results on monomode guiding in dielectric capillaries presented in Refs. 5 and 6 were possible thanks to this work. This opens prospects for further experiments, where the beam pointing is an important parameter.

Of course, this stabilization system can further be improved. Sources of high frequency vibrations should be isolated better from the laser system. The implementation of a feed-forward loop could predict the position of the beam in the gap between the shutter closing and the laser shot. When designing a new laser system, issues related to beam pointing can be taken into account from the start, making the implementation of such stabilization system much easier.

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