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# Uncoupled Impedance Matching for Coupled Multi-Antenna Systems

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**Abstract**—Optimal impedance matching to a coupled antenna array requires implementation of a coupled load or matching network, leading to high complexity and in many cases narrow bandwidth. Research has therefore focused on the development of uncoupled matching networks with good performance, although such solutions have received little rigorous attention in the context of multiple-input multiple-output communication. This paper explores uncoupled impedance matching to the array *active* and *passive* impedances for multi-antenna communications and extends matching to the array active impedance to the case where the propagating field is specified stochastically. Simulations demonstrate the relative performance of the different matching techniques.

## I. INTRODUCTION

When using multi-antenna technology in compact mobile devices, care must be taken when matching the antenna to the transmitting or receiving network to minimize the coupling-induced degradation. Optimal solutions to this impedance-matching problem use coupled matching networks to compensate for the antenna coupling, with examples being the well-known optimal multiport conjugate match (MCM) for maximum power transfer [1] and the corresponding result for minimum noise figure [2]. Unfortunately, such coupled networks are typically complicated and often result in narrowband matching performance [3].

These observations motivate the identification of uncoupled matching networks that achieve near-optimal performance. This is typically accomplished by matching to the array self, *passive* [4], or *active* [5] impedance. However, for multiple-input multiple-output (MIMO) systems, the exploration of these uncoupled matching techniques has been limited [3], [6], [7]. This paper creates a common theoretical framework for exploring the behavior and performance of uncoupled impedance matching for maximum power transfer or minimum noise figure in MIMO communications, including the impact of the propagation channel. Furthermore, because the channel is generally time-variant, we extend the concept of matching to the active array impedance to the case of stochastically-specified fields. The analysis also determines the uncoupled matching condition that achieves optimal ergodic channel capacity using numerical optimization. Simulations illustrate that active impedance matching provides good beamforming

gain and therefore optimal MIMO capacity for small signal-to-noise ratio (SNR) or high antenna coupling, while passive matching is superior for high SNR and low coupling.

## II. SYSTEM MODEL

In this analysis, boldface lowercase and uppercase symbols denote vectors and matrices, respectively, while an overbar indicates a vector electromagnetic quantity. Consider the model for a coupled array of  $M$  antennas, with the  $m$ th antenna characterized by the open-circuit radiation pattern (pattern with all adjacent elements terminated in an open-circuit) as  $\bar{e}_m(\Omega)$  where  $\Omega = (\theta, \phi)$ . If the array receives an incident field denoted as  $\bar{p}_{\text{inc}}(\Omega)$ , the open-circuit voltage on the  $m$ th antenna is given as

$$v_{o,m} = \int_{\Omega} \bar{e}_m(\Omega) \cdot \bar{p}_{\text{inc}}(\Omega) d\Omega. \quad (1)$$

The antenna array and its termination are characterized by the impedance matrices  $\mathbf{Z}_A$  and  $\mathbf{Z}_L$ , respectively. The quantities  $\mathbf{v}$  and  $\mathbf{i}$  represent the voltage across and current through the terminations, respectively. It has been established that maximum power transfer from the antennas to the loads in a receiving system (or sources to the antennas in a transmitting system) is achieved using the MCM, or  $\mathbf{Z}_L = \mathbf{Z}_A^\dagger$ , where  $\{\cdot\}^\dagger$  indicates a conjugate transpose [1]. A corresponding result exists when the loads represent amplifiers and the objective is to minimize the receiver noise figure [2]. However, achieving these solutions requires that the terminating network itself be coupled, which can be challenging. The goal of this work is to explore design of optimal uncoupled terminations characterized by a diagonal impedance matrix  $\mathbf{Z}_L$ .

## III. MAXIMUM POWER TRANSFER

Consider first the goal of maximizing the power transferred from the receiving antennas to the loads. This is a useful objective for most transmitting systems as well as for receivers when the dominant noise is introduced after the front-end amplifiers. Because the impedance match for maximum power transfer is equivalent for the transmitter and receiver, we generally use the receiver architecture for the analysis. However, when appropriate, we will call upon reciprocity and consider the antennas at the transmitter.

### A. Known Fields: Active Impedance Match

Maintaining maximum power transfer for an uncoupled termination is possible for a specific incident field using the notion of the active impedance match, the goal of which is to maintain the voltages and currents associated with the optimally-terminated network. This is accomplished through the relations

$$\mathbf{Z}_L \mathbf{i} = \mathbf{Z}_A^\dagger \mathbf{i} = \mathbf{Z}_{\text{act}}^* \mathbf{i} \quad \mathbf{v}_o = \underbrace{(\mathbf{Z}_L + \mathbf{Z}_A)}_{\mathbf{Y}^{-1}} \mathbf{i}, \quad (2)$$

where  $\mathbf{Z}_{\text{act}}$  is the diagonal active impedance and  $\mathbf{v}_o$  is the open-circuit voltage across the antenna terminals. These expressions can be solved to obtain

$$Z_{L,m} = \frac{[\mathbf{Z}_A^\dagger \mathbf{Y}_C \mathbf{v}_o]_m}{[\mathbf{Y}_C \mathbf{v}_o]_m}, \quad (3)$$

where  $\mathbf{Y}_C = (\mathbf{Z}_A^\dagger + \mathbf{Z}_A)^{-1}$  and  $[\cdot]_m$  indicates the  $m$ th element of the vector within the brackets. The power transferred to the load for the specific value of  $\mathbf{v}_o$  matches that achieved assuming a multiport conjugate match, although naturally the performance falls off as the incident field and therefore  $\mathbf{v}_o$  change.

This solution is useful for static scenarios or when a tunable matching network can adapt to the changing propagation conditions. However, in most cases, such expectations are unreasonable. We can therefore generalize the solution to the case where the incident field is specified stochastically. Specifically, if we consider a set of  $P$  incident fields, with the  $p$ th field generating the load current  $\mathbf{i}^{(p)}$ , we seek instead to compute the load impedance that minimizes the objective

$$\gamma_z = \frac{1}{P} \sum_{p=1}^P \sum_{m=1}^M \left| Z_{L,m} i_m^{(p)} - Z_{\text{act},m}^{(p)*} i_m^{(p)} \right|^2, \quad (4)$$

where  $\mathbf{Z}_{\text{act}}^{(p)}$  is the active impedance for the  $p$ th excitation. In the limit as  $P$  becomes infinite, the solution is

$$Z_{L,m} = \frac{[\mathbf{Z}_A^\dagger \mathbf{C}]_{mm}}{C_{mm}}, \quad (5)$$

where  $\mathbf{C} = \mathbf{Y} \mathbf{K}_o \mathbf{Y}^\dagger$  and  $\mathbf{K}_o = \text{E}\{\mathbf{v}_o \mathbf{v}_o^\dagger\}$  is the open-circuit covariance that can be computed from the radiation patterns and the power angular spectrum (PAS) of the incident field [8]. Since  $\mathbf{Z}_L$  appears in the expression for  $\mathbf{C}$ , the solution must be obtained through iteration.

### B. Unknown Fields: Passive Impedance Match

Systems often operate in a wide variety of scenarios, making it impossible to impedance match even to the stochastic nature of the fields. To design an uncoupled load that works for all possible incident fields, we can simply perform the active match discussed above when the PAS of the incident field is  $1/4\pi$ , meaning the field arrives from all directions. One advantage of this approach is that for this scenario of full angle spread,  $\mathbf{K}_o \propto \mathbf{R}_A$ , which is the mutual resistance of the

coupled array [2]. This allows computation of the load using knowledge only of the array impedance matrix.

An alternate approach is to match to the array passive impedance [4], which is the impedance seen looking into each antenna port with all ports terminated in their load impedance and therefore independent of the incident field. This is the goal of the *input impedance match* [3] suggested for MIMO implementation, and a closed-form expression for the matching impedances exists for the case of two identical antennas [3]. For larger or inhomogeneous arrays, we can use an iterative solution procedure. Specifically, we excite the array with a voltage  $v_s$  on the  $m$ th port and a set of terminations on the other ports, and we subsequently compute the input impedance seen looking into the coupled array from the  $m$ th port. The load impedance for the  $m$ th port is then chosen as the conjugate of this input impedance, which can be expressed as

$$Z_{L,m} = \left\{ \frac{[\mathbf{Z}_A \mathbf{Y}]_{mm}}{Y_{mm}} \right\}^*. \quad (6)$$

Since  $\mathbf{Z}_L$  appears in the expression for  $\mathbf{Y}$ , this equation must be solved iteratively.

### C. Computational Results

To evaluate the performance of the different matching networks, we use the procedure outlined in [9] to determine the ergodic capacity (based on 250 random channel realizations) for a MIMO system with coupled receive antennas but uncoupled transmit antennas. In all computations, the PAS follows a truncated Gaussian function in elevation centered at  $\theta = 90^\circ$  and with an angle spread of  $10^\circ$  and a Laplacian function in azimuth with an angle spread of  $40^\circ$ . Because undesired superdirective solutions can emerge, we suppress these solutions by introducing antenna loss, with the efficiency of each element specified as  $\mu = 0.97$  [10].

Consider the case where the PAS of the incident field arrives at endfire for an array of two half-wave dipole elements. Figure 1 plots the average capacity as a function of dipole separation at two different SNR levels for the passive impedance match, the active impedance match to the PAS, and the active impedance match for uniformly distributed PAS. Results for the optimal MCM and for the termination generated using numerical optimization to maximize the ergodic capacity are also included. As can be seen, for low SNR, the active impedance match is the optimal uncoupled termination, since this matching technique maximizes power transfer by maximizing the beamforming gain appropriate for the scenario. However, for higher SNR, matching to the passive impedance is optimal for all but the smallest antenna separations. This can be explained by recognizing that capacity is a function not only of the power received but also of the relative quality of the channel eigenmodes, particularly for high SNR. In this case, the beamformer obtained for the active match to the PAS enhances one communication mode at the expense of the other, reducing the capacity. The match to the passive impedance

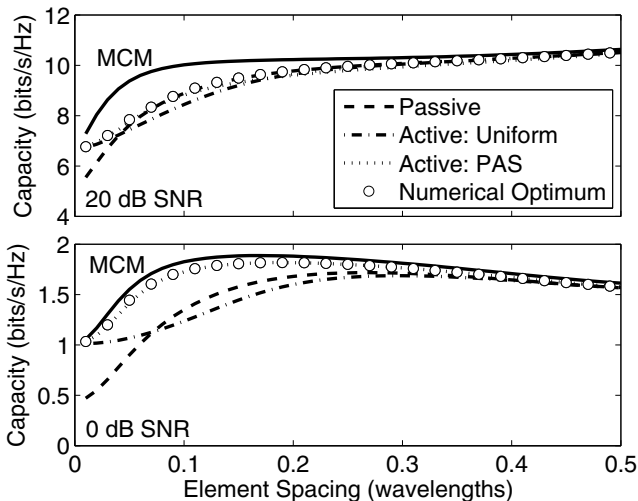


Fig. 1. Average capacity of a two-element dipole array with a single Laplacian cluster arriving at array endfire for two different SNR values.

more effectively equalizes the two communication modes so that they can both be exploited.

This observation can be reinforced by examining the eigenvalues of the covariance of the voltages across the loads for the scenario considered in Fig. 1 with an SNR of 20 dB. Figure 2 plots these results. For small separation, the active match to the PAS is optimal (consistent with the results of Fig. 1), since it maximizes the beamforming gain. However, once the separation becomes larger, the optimal termination sacrifices beamforming gain (which maximizes the dominant eigenvalue) in favor of some equalization of the two eigenvalues in order to increase average capacity.

#### IV. MATCHING FOR NOISE FIGURE

A discussion on optimal impedance matching would be incomplete without considering impedance matching for practical receivers where the front-end amplifiers represent a dominant noise source. In this situation, some amplifier noise is coupled between ports due to the antenna coupling, and the matching between the antennas and the amplifiers directly controls the front-end noise figure and system capacity.

##### A. Matching Implementation

The challenge of identifying an uncoupled termination that achieves minimum receiver noise figure is that it is based on a theory of optimally *mismatching* the antennas to the front-end amplifiers. In this case, rather than simply consider the uncoupled termination, we introduce a matching network between the antennas and amplifiers using the model and theory derived in [11]. For the sake of conciseness, we only indicate that the goal of the matching network is to transform the antenna impedance such that it appears to the amplifiers as the optimal noise figure termination with diagonal impedance matrix  $\mathbf{Z}_{\text{opt}}$ . Naturally, this precise condition is only satisfied either for a specific incident field or for a coupled matching network.

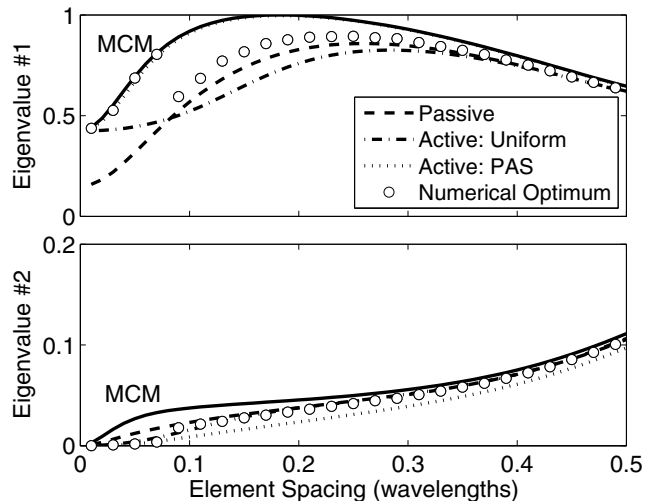


Fig. 2. Load voltage covariance eigenvalues for a two-element dipole array with a single Laplacian cluster arriving at array endfire for SNR = 20 dB.

Given the different matching techniques outlined in Section III, the challenge is to determine the mechanism for specifying the characteristics of the matching network to achieve the goal of minimum noise figure. In the context of the theory presented in [11], we have found that the following sequence of steps produces the best results for each of the techniques:

- 1) Use the theory outlined in Section III to design the load  $\mathbf{Z}_L$  that achieves maximum power transfer.
- 2) Given this load, assume that the active impedance seen looking into the antenna terminals is  $\mathbf{Z}_{\text{act}} = \mathbf{Z}_L^*$ .
- 3) Using the theory in [11], design the uncoupled matching network that transforms this uncoupled antenna active impedance to  $\mathbf{Z}_{\text{opt}}$ .

This approach works for the active impedance matching techniques as well as the passive impedance match. However, rather than use this approach for the active impedance match assuming a uniform PAS, we instead use the theory in [2] that presents a similar solution achieving the goal of minimum noise figure for this PAS.

##### B. Example Computation

Computations performed using this theory generally demonstrate that the basic observations and conclusions made in connection with the results presented in Section III-C apply to the case of optimal noise matching. As an example, consider again our two-element dipole array with the Laplacian cluster arriving at array broadside. The transistors forming the amplifiers have noise parameters  $R_n = 3.5\Omega$ ,  $Z_0 = 50\Omega$ ,  $F_{\text{min}} = 2.5$  dB, and  $\Gamma_{\text{opt}} = 0.475\angle 166^\circ$  (optimal reflection coefficient). Furthermore, the amplifiers have an input impedance of  $50\Omega$  (see [11] for a detailed discussion on how these are used in the design and simulations).

Figure 3 plots the capacity resulting from this analysis for two different values of SNR, where the “Minimum Noise”

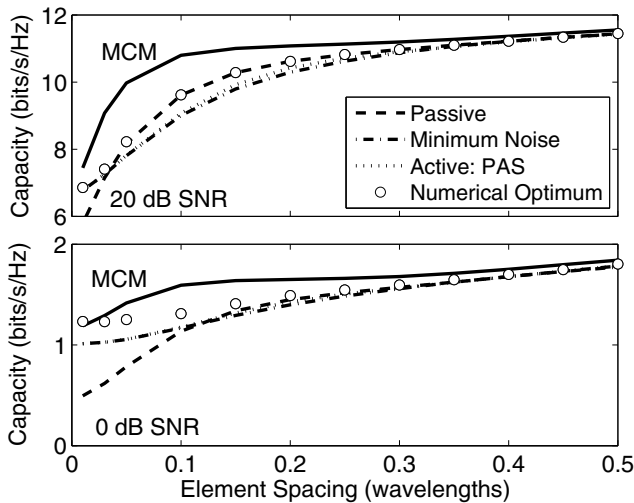


Fig. 3. Average capacity as a function of element spacing for a PAS described by a single Laplacian cluster arriving on a two-element array at broadside for different matching conditions and two different SNR levels.

match is that obtained from [2]. As can be seen, under this procedure, the conclusions for high SNR are similar to those obtained during the analysis of matching for maximum power transfer. Specifically, when the SNR is high, the matching to the active impedance leads to optimal performance only for high coupling, while matching to the passive impedance is optimal elsewhere.

However, when the SNR is low, the numerically optimum solution outperforms all other uncoupled solutions for high antenna coupling. Closer investigation of this case reveals that, despite the problem symmetry that suggests identical matching on each of the antenna ports, the numerical solution provides asymmetric matching to achieve these results. In fact, when the optimization is constrained to produce symmetric matching, the numerically-optimized solution follows the analytical curves as observed in all other capacity plots in this paper. For this case of low SNR, it appears there is benefit to degrading the quality on one output port to allow improvement of the quality on the other. We also point out that the port selected to achieve improved performance is arbitrary. Given the symmetric nature of this problem, it is currently unclear as to how to develop a closed-form matching strategy to achieve this behavior. However, it is important to recognize that this case of high coupling and 0 dB SNR is impractical for most realistic communication scenarios.

## V. CONCLUSIONS

This paper uses a common framework to develop and analyze active and passive uncoupled impedance matching techniques for coupled array antennas. Specifically, it discusses

matching to the array active impedance for deterministic and stochastically-specified electromagnetic fields, and shows that such active matching in effect creates a beamformer that maximizes received power. It also discusses a previously-proposed technique for matching to the antenna passive impedance, also referred to as input impedance matching, known to be optimal in certain circumstances. Simulation results of MIMO capacity using different propagation environments demonstrate that for low SNR or high coupling, active matching outperforms passive matching due to the associated beamforming gains. However, for moderate coupling or high SNR, passive impedance matching enables better use of the multiple propagation environment communication modes. The discussion concludes by demonstrating the application of the matching techniques for minimizing the system noise figure.

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