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A Modelica Library for Paper Machine Dryer Section Modeling
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Abstract

Following increased efforts during the last decade to formulate mathematical models for the paper drying process, this paper presents a Modelica library, `DryLib`, which enables users to rapidly develop complex models of paper machine dryer sections. In addition, parameter optimization, model reduction and moisture control by means of Non-Linear Model Predictive Control is treated. These applications have in common that they are based on numerical optimization schemes. Since the nature of the particular optimization problem dictates the requirements of the numerical code, the paper also serves as an illustration of the need for flexibility in terms of *i)* means for the user to express optimization problems, *ii)* and choice of numerical algorithms.

Key words
Paper Machine Dryer Section Modeling, Model Reduction, Parameter Optimization, Non-Linear Model Predictive Control

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1. Introduction

Detailed modeling of large scale plants has received increased industrial interest in recent years. This is due mainly to the availability of languages and tools enabling development of large, hierarchical and modular models. Using this methodology, the user is relieved from the burden of managing potentially cumbersome programming API:s for e.g. numerical simulation codes, and may instead focus on embedding expert knowledge using better suited abstractions.

There are several benefits of developing detailed models of large scale industrial plants. Typically, operation of industrial processes are costly, and it is therefore usually difficult to motivate extensive experiments, e.g. to evaluate control performance. Modeling is therefore attractive, since it enables a wide range of applications, including simulation, control design and evaluation, bottle-neck analysis and operator training, which may not be possible to implement on the real plant.

The topic of this paper is modeling, model reduction, parameter optimization and control of a paper machine drying section. The dryer section is the last part of the paper machine and consists of a large number of rotating steam heated cast iron cylinders. The moist paper is led around these cylinders and the latent heat of vaporization in the steam is used to evaporate the water from the web. When the steam releases its thermal energy it condenses into water which is drawn off by suction with a siphon and fed back to the boiler house. The cylinders are divided into separate dryer groups where the steam pressure can be individually controlled in each group. By adjusting the steam pressure in the dryer groups, and thereby the heat flow to the paper, the moisture in the paper web is controlled. The moisture ratio in the web is reduced from 1–1.5 kg water/kg dry substance when entering the dryer section to a final product of 0.03–0.1, i.e. a significant amount of water is removed in the dryer section.

To support and transport the paper web through the drying section, dryer fabrics are used. The dryer fabric is also used to press the web onto the cylinders to provide good thermal contact between the two surfaces.

The drying section is enclosed inside a drying hood. The main purposes of the hood are to create a controlled environment for the drying process, improve energy utilization, and also to establish good working conditions in the machine room. The exhaust air removes the evaporated water from the paper web while preheated dry air is added to the hood by the supply air.

Moisture is one of the most important quality parameters of the final paper product. It is essential to keep this property well regulated, both at steady-state and at state-transitions. A good model of the dynamics of drying is therefore vital for good moisture control. Based on the work [Slätteke, 2006], a Modelica library, `DryLib`, has been developed. `DryLib` implements the physical phenomena involved in the drying process, as well as convenient components and connectors which enables rapid development of dryer section models. An important feature of `DryLib` is its ability to express models which are scalable, in the sense that the complexity of the models can be easily changed. This feature is quite useful, since the need for granularity depends on the application – a high fidelity model may be suitable for simulation, whereas a course model capturing the main behavior may be appropriate for control design.

The present paper gives three main contributions. Firstly, the Modelica library

DryLib is presented. Secondly, important issues such as parameter optimization, model reduction and optimization based control schemes (Non-linear Model Predictive Control (NMPC)), are treated. Some of these topics have a general character, while others are dealing specifically with dryer section issues. Thirdly, the applications of the paper serves as examples of the wide range of relevant optimization problems that naturally follow the availability of high-fidelity models.

The paper is organized as follows. In Section 2 the physical model upon which DryLib is based, is presented. Section 3 deals with the structure and implementational details of DryLib. The Sections 4, 5 and 6 treats parameter optimization, model reduction and moisture control by means of non-linear MPC. In Section 7, the software used to solve the optimization problems presented in the paper is described. The paper ends with with conclusions and future work in Section 8.

2. Physical Modeling

2.1 Modeling of the Dryer Section

Mathematical modeling of cylinder drying started with the pioneer work [Nissan and Kaye, 1955]. An extensive review of drying models up to 1980 with some 130 references is given in [McConnell, 1980]. Many of these models have different objectives and are of different type. There are both static and dynamic models, and a majority of the models are first principles models but some describe black-box modeling of the dryer section. One mutual characteristic of the models is that they often focus on modeling the paper sheet and neglect the steam system. Consequently it is assumed that the steam pressure in the cylinders is a manipulated variable or that a collected data series of the steam pressures is used as an input to the model. This makes the model unsuitable for simulation of feedback control. The model described in this work includes the dynamics of the steam system and the inflow of steam is controlled by a steam valve. It is therefore possible to mimic the entire moisture loop of the feedback system in the paper machine.

The model library that is developed and used in this paper is built upon physical relations in terms of mass and energy balances, in combination with constitutive equations for the mass and heat transfer. The objective is to obtain a non-linear model that captures the key dynamical properties for a wide operating range. The core of the model is based on [Wilhelmsson, 1995] and [Slätteke and Åström, 2005], and it is also given in [Slätteke, 2006]. The model for the paper web is based on [Wilhelmsson, 1995] whereas the model for the cylinder, and steam system is taken from [Slätteke and Åström, 2005]. The model description given here is mainly given for completeness, but there are also some minor additions as compared to the description given in the above references.

While the physical behavior of the process is formulated using partial differential equations (PDE:s), numerical simulation require the PDE:s to be discretized in the spatial dimension(s). In this work, the paper process is discretized by partitioning the process into small control volumes where a mass and energy balance are defined for each volume. These control volumes are then put together so that the outflow of one becomes the inflow of the next. The precision of the model then depends on the size of the control volumes, where a finer discretization grid gives improved accuracy, but also increased computational complexity. In order to increase the clarity of the presentation, the indices identifying each individual control volume has been dropped. In Figures 2 and 3, however, the indices have been included to emphasize the discrete nature of the paper process model.

2.2 The Steam and Cylinder Process

Let q_s [kg/s] be the mass flow rate of steam into the cylinder, q_c [kg/s] the condensation rate, q_{bt} [kg/s] the blow through steam, and q_w [kg/s] the siphon flow rate. Also, let V_s [m^3] and V_w [m^3] be the volumes of steam and water, respectively, in the cylinder, and let ρ_s [kg/m^3] and ρ_w [kg/m^3] be the respective densities. The mass balances for water and steam are then

$$\begin{aligned} \frac{d}{dt}(\rho_s V_s) &= q_s - q_c - q_{bt} \\ \frac{d}{dt}(\rho_w V_w) &= q_c - q_w \end{aligned} \tag{1}$$

The energy balances for steam, water and metal are given by

$$\begin{aligned} \frac{d}{dt}(\rho_s u_s V_s) &= (q_s - q_{bt})h_s - q_c h_s \\ \frac{d}{dt}(\rho_w u_w V_w) &= q_c h_s - q_w h_w - Q_m \\ \frac{d}{dt}(m C_{p,m} T_m) &= Q_m - Q_p \end{aligned} \tag{2}$$

where Q_m [W] is the power supplied from the water to the metal, Q_p [W] is the power supplied from the metal to the paper, h_s [J/kg] is the steam enthalpy, h_w [J/kg] is the water enthalpy, m [kg] the mass of the cylinder shell, $C_{p,m}$ [J/(kg·K)] the specific heat capacity of the shell, T_m [K] the mean temperature of the metal, u_s [J/kg] and u_w [J/kg] are the specific internal energies of steam and water. From the thermodynamic definition of specific enthalpy, we get

$$\begin{aligned} h_s &= u_s + \frac{p}{\rho_s} \\ h_w &= u_w + \frac{p}{\rho_w}. \end{aligned} \tag{3}$$

The steam and water volumes add up to the total cylinder volume,

$$V = V_s + V_w \tag{4}$$

The energy flow to the metal is given by the heat transfer equation

$$Q_m = \alpha_{sc} A_{cyl} (T_s - T_m) \tag{5}$$

where α_{sc} [W/(m^2·K)] is the heat transfer coefficient from the steam-condensate interface to the centre of the cylinder shell, A_{cyl} is the inner cylinder area, and T_s the steam temperature. Experiments have shown that α_{sc} depends on both condensate thickness, machine speed, and the number of spoiler bars [Pulkowski and Wedel, 1988]. However, the condensate has a turbulent behavior and the heat transfer coefficient has proven to be difficult to model. Typical values ranges between 1000 and 4000 W/(m^2·K). The power flow to the paper is

$$Q_p = \alpha_{cp} A_{cyl} \eta (T_m - T_p) \tag{6}$$

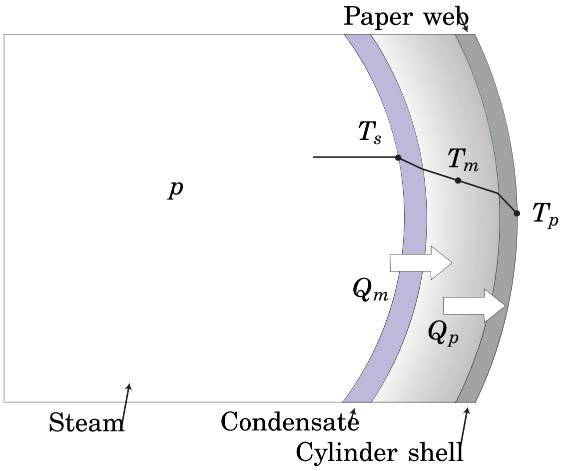


Figure 1 A piece of the cross-section of a drying cylinder, showing the steady-state temperature profile and energy flows.

where T_p [K] is the paper temperature, α_{cp} [W/(m^2·K)] the heat transfer coefficient from the cylinder shell to the paper, and η [·] is the fraction of dryer surface covered by the paper web. Figure 1 illustrates the heat flows in the steam and cylinder model. An empirical model for α_{cp} has been developed in [Wilhelmsson, 1995] where a linear relation with moisture ratio in the paper web, u [kg moisture/kg dry solids] is proposed. The relation is given by

$$\alpha_{cp}(u) = \alpha_{cp0} + \alpha_{cpk}u \tag{7}$$

where α_{cp0} varies between 200-500 W/(m^{2}K) and α_{cpk} has typical values in the range of 900-1200 W/(m^{2}K). It is well known that α_{cp} depends on other things, e.g. the web tension, and surface smoothness of both paper and cylinder, but these phenomena are omitted here. The energy flow from the part of the cylinder not covered by paper due to convection or radiation to ambient air has been reported to represent only 1-2% of the total energy flow and is therefore neglected, see [Wilhelmsson, 1995]. Since the steam flow to the cylinder cannot be manipulated directly, a valve model is also needed. From [Thomas, 1999] we have

$$q_s = C_v f_v(x_v)\sqrt{(p_{sh} - p)\rho_s}, \tag{8}$$

where C_v [m^2] is the valve conductance, x_v is the position of the valve stem and the function f_v is the valve characteristics called valve trim. The valve stem varies from 0 (minimum valve opening) to 1 (maximum valve opening). The supply pressure at the steam header is p_{sh}. We use equal percentage trim, since it is the most common characteristic in the process industry [Thomas, 1999]. This assumption gives

$$f_v(x_v) = R_v^{x_v - 1}. \tag{9}$$

where R_v is a constant known as the "rangeability" since it is the ratio between the maximum and minimum valve opening.

For simplicity, all steam within the cylinder cavity is assumed to be homogeneous, with the same pressure and temperature. We also assume that the steam in the cylinder is saturated. This means that the enthalpy, density, and temperature are functions of the pressure only. Fitting polynomials to the tabulated values for saturated steam in [Schmidt, 1969], gives

$$\begin{aligned} T_s &= 0.1723(\log p)^3 - 3.388(\log p)^2 \\ &\quad + 37.71 \log p + 124.5 \\ h_s &= (-0.07402(\log p)^4 + 2.887(\log p)^3 \\ &\quad - 39.58(\log p)^2 + 260 \log p + 1824) \times 10^3 \\ h_w &= (0.8842(\log p)^3 - 18.77(\log p)^2 \\ &\quad + 200 \log p - 748.5) \times 10^3 \\ \rho_s &= (0.005048p + 64.26) \times 10^{-3} \\ \rho_w &= -0.3136(\log p)^3 + 6.792(\log p)^2 \\ &\quad - 52.43 \log p + 1141 \end{aligned} \tag{10}$$

2.3 The Paper Web Process

The water and fiber content of the paper web are modeled by mass balances, whereas the temperature of the web is modeled by an energy balance. Starting with the mass balance of water, an expression defining the evaporation rate (condensation rate) between the paper surface and the surrounding air is needed. From [Wilhelmsson, 1995] we get the Stefan equation

$$q_{evap} = \frac{p_{tot} K M_w}{R_g T_p} \log \left(\frac{p_{tot} - p_{v,a}}{p_{tot} - p_{v,p}} \right), \tag{11}$$

where q_{evap} [kg/m^{2}s] is the evaporation rate, K [m/s] is the mass transfer coefficient, M_w [kg/mole] is the molecular weight of water, p_{tot} [Pa] the total pressure of the air, $p_{v,a}$ [Pa] the partial pressure for water vapor in the air, $p_{v,p}$ [Pa] the partial pressure for the water vapor at the paper surface, R_g [J/mole·K] the gas constant, and T_p [K] the paper temperature. The partial pressure $p_{v,a}$ is given by the moisture content of air, x [kg water vapor/kg dry air], and the total pressure, [Karlsson, 2000]

$$p_{v,a} = \frac{x}{x + 0.62} p_{tot}. \tag{12}$$

The vapor partial pressure at the paper surface is given by

$$p_{v,p} = \varphi p_{v0} \tag{13}$$

where p_{v0} [Pa] is the partial vapor pressure for free water, and is given by Antoine's equation

$$p_{v0} = 10^{\left(10.127 - \frac{1690}{T_p - 43.15}\right)} \tag{14}$$

As long as capillary transport can bring new water to the paper surface, the vapor partial pressure at the paper surface is equal to the partial pressure for free water. When the paper becomes dryer a correction factor called sorption isotherm,φ, is invoked which has a value between zero and one. In [Pettersson and Stenström, 2000] an investigation of some sorption isotherms found in the literature, is given. Many of those give a heat of sorption that goes to infinity as u goes to zero. This is physically unrealistic since the bond energy between the last fraction of water and a cellulose fiber must be finite. From [Heikkilä, 1993], a finite heat of sorption at the origin which matches the hydrogen bond energy

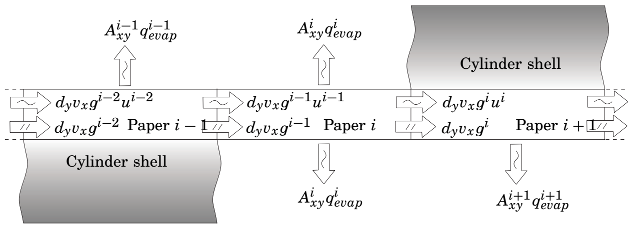


Figure 2 The mass transport for water and fiber in the paper web. The shaded areas represents cylinder walls. When the paper is in the transition between two cylinders (the free draw), evaporation occurs at both paper surfaces.

between water-fiber is given and is therefore found to be most appropriate. The sorption isotherm of a paper web depends on its composition and temperature. It is not very well investigated when compared to other materials, [Pettersson and Stenström, 2000], but [Heikkilä, 1993] gives an empirical expression for paper pulp,

$$\begin{aligned}\varphi = 1 - \exp(&-47.58u^{1.877} \\ &- 0.10085(T_p - 273.15)u^{1.0585})\end{aligned} \tag{15}$$

Now, let v_x [m/s] be the speed of the paper web, d_y [m] the width of the paper web, A_{xy} [m^2] the area of the dryer surface covered by paper, and g [kg/m^2] the dry basis weight. Then the mass balance of moisture for a paper sheet in contact with a cylinder can be written

$$\frac{d}{dt}(ugA_{xy}) = d_y v_x g_{in} u_{in} - A_{xy} q_{evap} - d_y v_x g u. \tag{16}$$

A similar mass balance for moisture in the free draws can derived from Figure 2, which shows a schematic picture of the mass flows in a paper sheet. Analogously, the mass balance for fiber in the paper web is given by

$$\frac{d}{dt}(gA_{xy}) = d_y v_x g_{in} - d_y v_x g. \tag{17}$$

To model the energy balance, introduce

$$C_{p,p} = \frac{C_{p,fiber} + uC_{p,w}}{1+u} \tag{18}$$

where $C_{p,p}$ [J/kg·K], $C_{p,fiber}$ [J/kg·K], and $C_{p,w}$ [J/kg·K] is the specific heat capacity of paper, fiber and water, respectively. As we can see, $C_{p,p}$ is a weighted sum of the heat capacities of the parts. From [Wilhelmsson, 1995] we have $C_{p,fiber} =$ 1256 J/(kg·K). Also, let T_p be the paper temperature and ΔH be the amount of energy needed to evaporate the water. Analogously to the discussion about the mass balance, if the web is wet enough this energy is equal to the latent heat of vaporization for free water. When the paper becomes dryer, however, an extra amount of energy ΔH_s (the heat of sorption) is necessary besides the latent heat of vaporization for free water. The heat of sorption can be derived from the

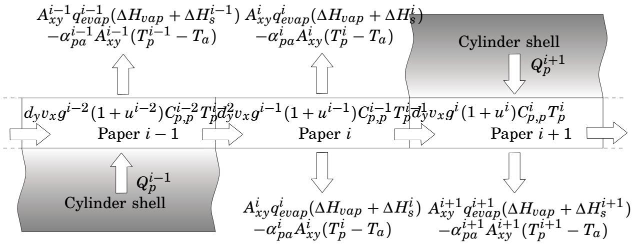


Figure 3 The energy balance of the paper web. The shaded areas represent cylinder walls. When the paper is in the transition between two cylinders (the free draw), energy flow to ambient air occurs at both paper surfaces.

sorption isotherm by thermodynamic theory and this relation is known as the law of Clausius-Clapeyron

$$\Delta H_s = -\frac{R_g}{M_w}\left(\frac{d(\log\varphi)}{d(1/T_p)}\right). \tag{19}$$

By applying this relation to (15), we obtain

$$\Delta H_s = 0.10085 u^{1.0585} T_p^2 R_g \frac{1-\varphi}{M_w \varphi} \tag{20}$$

The amount of energy required to evaporate water from the surface of the web is then given by

$$\Delta H = \Delta H_{vap} + \Delta H_s \tag{21}$$

where H_{vap} is the latent heat of vaporization for water, equal to 2260 kJ/kg (at atmospheric pressure). Furthermore, let the energy transport due to convection between the paper surface and the air be

$$Q_{conv} = \alpha_{pa} A_{xy}(T_p - T_a) \tag{22}$$

where α_{pa} [W/(m^2·K)] is the heat transfer coefficient from paper to air and T_a [K] the ambient air temperature. Since water is an incompressible medium, there is no pressure volume work on the surroundings, and we write the energy balance as a change in enthalpy. The energy balance of the paper web in contact with a cylinder is thus modeled as

$$\begin{aligned} \frac{d}{dt}(g(u+1)A_{xy}C_{p,p}T_p) &= d_y v_x g_{in}(1+u_{in})C_{p,p,in}T_{p,in} \\ &- A_{xy}q_{evap}(\Delta H_{vap} + \Delta H_s) - \alpha_{pa}A_{xy}(T_p - T_a) \\ &- d_y v_x g(1+u)C_{p,p}T_p + Q_p \end{aligned} \tag{23}$$

The energy balance for the free draws is similar, an can be formulated using the schematic illustration of energy flows shown in Figure 3.

3. `DryLib`

`DryLib` is implemented in the object-oriented modeling language Modelica, see [Modelica Association, 2005; Mattsson *et al.*, 1998; Fritzon, 2004]. Like any object-oriented programming language, Modelica provides the notions of classes, and instances, as fundamental abstractions. Properties like inheritance and abstract classes provide a structured approach to model structuring. Modelica also enables declarative programming, useful e.g. to express mathematical relations, as well as functional programming to express behavior in terms of algorithms. The advantages of main Modelica are *i)* it is built on a non-causal equation structure *ii)* it is possible to create model components that correspond to physical objects in the real world, in contrast to modeling techniques that require conversion to signal blocks *iii)* it permits mixing of physics with empirical models *iv)* it is easy to go from simple models to high fidelity models by graphical editing *v)* it is easy to build and exchange model libraries and *vi)* it is well suited for multi-domain modeling.

The objective of building the Modelica library `DryLib` has been to create a user friendly and extensible platform for modeling of paper machine dryer sections. In particular, the aim has been to design the library so that, at the user level, the appropriate level of model detail can be easily selected. The current implementation of `DryLib` contains a few examples of components where the level of detail can be specified by the user. More importantly, the library classes are designed to enable advanced users to add new behavior to key components in order to extend the functionality of the library. An important concept in the design process has been that of *model scalability*, which means that the granularity of the model behavior should be easy to change, without the need to re-build the model.

3.1 Hierarchical Structuring

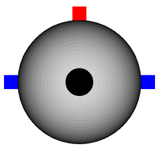
Having formulated a mathematical model for the paper machine dryer section, as presented in Section 2, the issues of structuring the equations into Modelica classes, and definition of interface classes (connectors) need attention. A paper machine dryer section model can be assembled using very few basic component types. In essence, there are only two fundamental entities, namely a steam heated cylinder and a sheet of paper. These two component types may then be combined, in large numbers, into a complete dryer section model. However, it is convenient to introduce additional hierarchical levels. As discussed above, the cylinders of a typical dryer section are organized into steam groups, in which a number of cylinders are operated at the same pressure. The introduction of steam groups into the library provides a convenient hierarchical level for the user, since many decisions regarding e.g. operating points and control design and evaluation are made at the steam group level. For basic usage of `DryLib`, it is also sufficient to utilize only classes defined at the steam group level in order to create a fully working dryer section model.

In order to increase the flexibility of the library, the boundary conditions of the physical entities have been factored out and modeled as separate classes. As an instructive example we consider a paper sheet, where the boundary conditions of the surfaces defining the sheet depends on the environment. For example, different boundary conditions are imposed on the surface if the paper is in contact with the air or a cylinder shell. The key to building a flexible Modelica libraries using this principle of separation is the design of generic connector classes. This topic will be discussed in detail below.

From a user's perspective, `DryLib` is intended to enable easy modeling of a dryer section. However, the user should remain in control of the implementational details of key components, e.g. paper sheets and cylinders. Also, advanced users should have the possibility to introduce new behavior of existing components. Two key features of Modelica have been used to satisfy these requirements. In the first case, extensive use of parametrized types (`replaceable/redeclare`) has been used to propagate type information downwards in the component hierarchy from the main user level (which is the steam group level) to lower level components. This strategy enables the user to easily select the appropriate level of detail for e.g. the cylinder dynamics. In the second case, inheritance has been used in order to simplify introduction of new component behavior. For the basic components such as cylinders and paper sheets, generic base classes have been introduced, which in turn serve as super classes for particular implementations. `DryLib` currently provides a few alternative implementations for key components, and additional behavior is easily added using the pre-defined base classes.

Connectors and Variable bindings The interface structure in `DryLib` is based on three connector classes. While the connectors for heat flow and mass flow (for connecting components with steam flow) are straight forward, the connector class for a paper surface deserves to be discussed. The paper web is modeled by separate mass balances for water and fiber, and an energy balance, as described above. Natural flow variables are thus mass flow of water and fiber, q_w [kg/s] and q_f [kg/s], and energy flow Q [W]. As for the potential variables, there are several feasible choices. However, since `DryLib` is likely to be used by domain experts in the field of paper drying, it was decided to use the standard variables within this domain. The natural choices are then moisture ratio, u [kg water/kg dry substance], dry basis weight, g [kg/m^2] and temperature T [°C].

A particular feature of Modelica that has been used to simplify the propagation of parameters and variables between components in `DryLib` is name look-up in the instance hierarchy (`inner/outer`). For example, the machine speed is used in various components, but is common for the entire dryer section. Implementation using `inner/outer` constructs is thus convenient. Examples of variables that may be assumed to be shared by the components of a steam group are ambient temperature and air moisture, which are also implemented using `inner/outer`.



Cylinder Models The (partial) cylinder base class `CylinderBase` contains mainly connector components and serves as a unifying class for particular implementations of dynamic behavior. The cylinder base class has two mass flow connectors corresponding to steam inlet and outlet, and one heat flow connector. Currently, `DryLib` contains two implementations of cylinder dynamics. The first implementation is based on Equations (1)-(5) and (10), whereas the second implementation is based on the simplified dynamics derived in [Slätteke, 2006], Chapter 4.

Paper Models The paper web base class contains essentially four paper connectors corresponding to the cross section areas and the upper and lower surfaces. This design enables separation of the actual paper web behavior, and the physical phenomena defined by the boundary conditions of the paper. The design also adds to the flexibility of the library, enabling e.g. easy extension to modeling of multi-ply paper drying. There are two particular implementations of paper web behavior. In the first implementation, the dynamics is included, whereas in the second implementation the balance equations are given as algebraic relations. The latter case is motivated by the fact that the

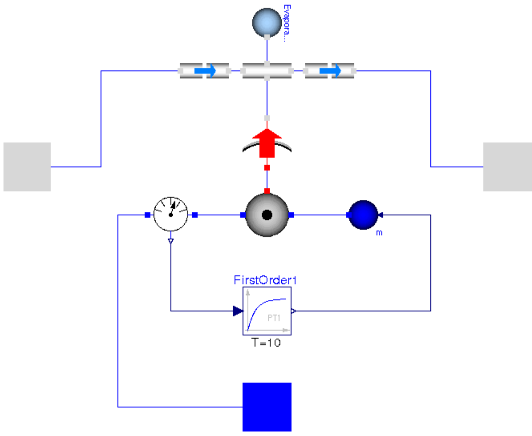
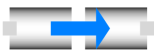
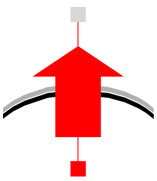


Figure 4 The component diagram for `CylinderUnit`.

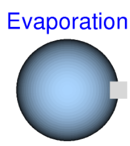
time constants of the paper web is small compared to the cylinder dynamics. Neglecting the fast dynamics of the paper may be attractive for applications where it is important to minimize the number of dynamical states.



Interfaces A key component is the connection of the cross section areas of two paper sheets. It is important to note that the single mechanism that drives the mass transport in the machine direction is the mechanical transportation of the paper, defined by v_x. Phenomena such as diffusion is neglectable given the high velocity of the paper through the dryer section, and is therefore not modeled. As a consequence, the mass and energy flows through the cross section area cannot be determined locally (compare e.g. mass transport driven by pressure gradients), but relies solely on the machine speed v_x. Using this arguments, implementation of the `PaperPaperInterface` class is straight forward and involves only encoding of appropriate terms of the right hand sides of Equations (16), (17) and (23).

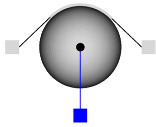


The interface between a steam cylinder and a paper surface is modeled by a the class `CylinderPaperInterface` which has one heat flow connector and one paper connector. The behavior of the class is defined by Equation (6), which implies that energy transport takes place but not mass transport.

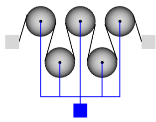


Much of the modeling effort in Section 2 was devoted to describing evaporation of water from the paper surface. This phenomena is encapsulated in the class `Evaporation`, which contains the associated equations ((11)-(15) and (20)-(22)) defining the mass and energy flows through the paper surface.

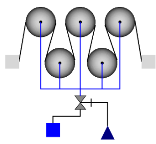
Steam Group Models The classes described above have the character of *specifying physical behavior*. We shall now turn our attention to classes which are mainly used as *structuring entities* in the sense that they introduce new hierarchical levels, and that they contain instances of behavior classes. Basic usage of `DryLib` may involve only classes introduced at this level.



In order to efficiently explore the strong repetitive character of a typical dryer section, the class `CylinderUnit` was introduced. As can be seen in Figure 4, this class combines a steam cylinder and a paper sheet which is attached to an evaporation component. While different cylinders may have different physical parameters, the structure of `CylinderUnit` is valid in most cases. A difficulty when modeling a steam cylinder is to determine the behavior governing the blow through steam and condensate flows. In this work, a simplified model which relates the input mass flow and the output mass flow by a first order system is used. Evaluation by simulation has shown that this model for the output model gives acceptable results, and also that the choice of time constant for the first order system is not critical.

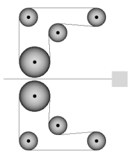


As noted above, a steam group is an important entity of a dryer section. In order to obtain increased flexibility, a steam group in `DryLib` is modeled by two classes – one for the actual cylinders and one for the associated control system. The class `CylinderArray` contains an arbitrary number of alternating `CylinderUnit` components and `Paper` components representing the free draws, and provides a convenient way to create large cylinder groups.

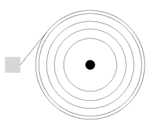


The actual control system typically consisting of a valve, a pressure sensor and a PID controller is encapsulated in the class `SteamGroup`, which also contains a `CylinderArray`, component representing the actual cylinders and the paper sheet. The `SteamGroup` class has four connectors corresponding to incoming and outgoing paper, the steam header and an input signal representing the reference value of the pressure controller.

Sources and Sinks Apart from the classes presented above, `DryLib` also contains classes which is used to drive a dryer section model, referred to as sources and sinks.



In a paper plant, there are several process steps preceding the dryer section. In particular, the wet end, consisting of the wire part and the press, is also considered to be part of the paper machine. Since these components are not included in `DryLib`, it is necessary to introduce a class which generates an output corresponding to the wet end. This mechanism is encapsulated in the class `PaperSource`. This class is equipped with a paper connector and simulates the incoming paper sheet given specifications for water and fiber mass flow and paper temperature.



Since the dryer section is the last part of the paper machine, the ending interface is straight forward and consists mainly of a paper connector which interface the last cylinder group of the dryer section.

3.2 PM7, Husum, Sweden

To demonstrate the capabilities of `DryLib`, a dryer section model corresponding to that of PM7 located at the M-real mill, Husum, Sweden, has been developed. The PM7 paper machine is a multi-cylinder machine producing copy paper. The dryer section of the machine is divided into a pre-dryer and an after dryer section with the surface sizing in the middle. The objective of the after-dryer section is only to dry the mixture added by the surface sizing and it cannot take care of

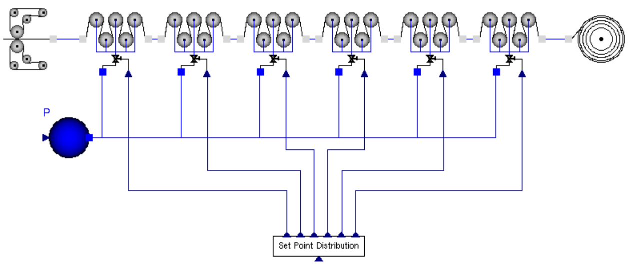


Figure 5 The top level of a complete dryer section model.

moisture problems from the pre-dryer section. Only the pre-dryer is modeled here. The PM7 drying cylinders are divided into six groups, consisting of one, two, two, three, ten and twelve cylinders respectively. For a detailed description of the plant, see [Ekvall, 2004].

In Figure 5, the top level of the PM7 dryer section model is shown, including six steam groups, a paper source, a paper sink, a mass flow source representing the steam header and a set point distribution for calculation of pressure set points for the groups. The final model consists of 7453 equations and 312 dynamical states when translated with Dymola.

3.3 Extensions

Possible extensions of `DryLib` can be sorted mainly into two categories. Firstly, the library may be extended by adding components modeling process equipment or physical phenomena not covered by the current implementation. For example, modeling of systems in direct connection with the dryer section, such as the condensate system, the steam production and the ventilation system would enable simulation of a larger part of the process. Also, adding this functionality would simplify connection of the dryer section model to models of other important parts of the paper machine, e.g. the press section, the wire section or other process units utilizing the same steam header.

Secondly, `DryLib` may be extended by introducing components which enables simulation of the drying process at an increased level of detail. The current design of `DryLib` is based on a particular choice of discretization of the underlying PDE:s (describing mass and energy transport), which yields a model with a reasonable level of detail, while maintaining acceptable simulation times. While this choice of discretization is suitable for analysis of moisture, temperature and pressure profiles in the machine direction, other applications may require different levels of detail. For example, in the work [Karlsson, 2005], the underlying PDE:s are discretized at a very high level of detail. This enables e.g. analysis of the risk of delamination in cartonboard manufacturing, as well as detailed study of moisture and temperature profiles, in the machine and thickness directions. Other applications, such as control design, may benefit from simple models capturing only the input output behavior of the system. This issue is addressed in Section 5, where a model reduction scheme is proposed to reduce the complexity of a dryer section model.

4. Parameter Optimization

It is desirable that the behavior of the model is similar to that of the real plant, in order for results obtained from using the model to be applicable on the plant. It is usually necessary to modify the original model to obtain a better match with measurement data. A common method to minimize the plant-model mis-match is to select one or more parameters of the model, and then tune these until a satisfactory model response is obtained. This procedure of tuning parameters while leaving the structure of the model unchanged is referred to as gray-box identification, see [Bohlin and Isaksson, 2003]. Parameter tuning may in simple cases be done by hand, but more complex problems requires structured methods for finding the parameter set which yields the best result. One such method is parameter optimization, which, in addition to selection of parameters to optimize, also includes definition of a performance criterion to minimize.

Model parameter values can be determined in several ways. Some parameters are available in tables, and are not associated with uncertainty, whereas others may be determined from experiments. Mechanical systems may for example be disassembled and its components can be measured and weighted. Yet some parameters may be inherently hard to find accurate values for. In the dryer section model, typically heat transfer coefficients fall into this category.

When selecting parameters to optimize, parameters which are uncertain are attractive choices. However, it should be kept in mind, that the parameter optimization procedure does not necessarily produce the physically correct parameter values. Rather, the selected parameters are used to compensate for all types of model-data mismatch given a particular performance criterion. This implies that the actual parameter values obtained from optimization should not be interpreted as the true physical values, but rather those that achieves the best model-data match. On the other hand, it is usually desirable to ensure that parameters have physically feasible values.

4.1 Problem Definition

Setting up a parameter optimization problem requires insight into which aspects of the model are most important. In this case, both the dynamic and static model response is of importance. However, in a first step, only the static behavior has been considered. Specifically, cylinder and paper temperatures of the paper machine, as well as the output moisture, have been measured during stationary operation conditions. The aim of the optimization has been to improve the stationary response of the model in the sense that the difference between simulated temperatures and moisture and measured temperatures and moisture, should be minimized.

A reasonable cost function to minimize is then

$$\begin{aligned} J =&\gamma_{T_m} \sum_{i=1}^{N_{cyl}} (T_{m,i}^m - T_{m,i}^s)^2 + \\ &\gamma_{T_p} \sum_{i=1}^{N_{cyl}} (T_{p,i}^m - T_{p,i}^s)^2 + \gamma_u (u_{out}^m - u_{out}^2)^2 \end{aligned} \tag{24}$$

where N_{cyl} is the number of cylinders, super-script m indicates measured quantities, super-script s indicates simulated quantities and γ_{T_m}, γ_{T_p} and γ_u are

Table 1 Optimization parameters

Parameter	Nom.	Min.	Max.
α_{sc} [W/(m^{2}K)]	500	400	5000
K [m/s]	0.06	0.02	0.1
α_{p0} [W/(m^{2}K)]	400	200	1000
α_{con} [W/(m^{2}K)]	100	0	600

weights. While the measurement method used to determine cylinder temperatures is reliable, the measurements of paper temperatures should be regarded as uncertain. In particular, the paper temperature is varying considerably in the machine direction depending on the position, relative to a cylinder contact area, at which the measurement is done, [Slätteke, 2006]. Therefore, the weight γ_{T_p} was set to a small value. The moisture, on the other hand, is an important quality variable that should be matched with high accuracy. Accordingly, γ_u was set to high value.

Four parameters were selected for optimization: the heat transfer coefficient between steam and condensate in a cylinder α_{sc}, the mass transfer coefficient K, one of the parameters defining the heat transfer coefficient between the cylinder and the paper, α_{p0}, and finally the convection coefficient α_{con} .Table 1 summarizes nominal, maximum and minimum values for the parameters.

4.2 Solving the Problem

The minimization of (24) should be performed subject to the constraint constituted by the DAE representation of the model. Since the minimization is performed in stationarity, all derivatives may be set to zero, and the model is then represented by a purely algebraic constraint, $F(x, y, p) = 0$, where x is the state vector, y represents the algebraic variables and p are the parameters.

The optimization problem may now be written

$$\begin{aligned}
\min_{x,y,p} \; J = \min_{x,y,p} \; & \gamma_{T_m} \sum_{i=1}^{N_{cyl}} (T^m_{m,i} - T^s_{m,i})^2 + \\
& \gamma_{T_p} \sum_{i=1}^{N_{cyl}} (T^m_{p,i} - T^s_{p,i})^2 + \\
& \gamma_u (u^m_{out} - u^2_{out})^2 \\
\text{subject to} & \\
0 = & F(x, y, p)
\end{aligned} \tag{25}$$

The problem was solved by a custom made application coded in C, which is based on the `dsblock` interface for accessing the model description generated by Dymola, and the NLP code IPOPT, see [Wächter and Biegler, 2006], which is dedicated to solving large scale algebraic optimization problems. The software is described in detail in Section 7.

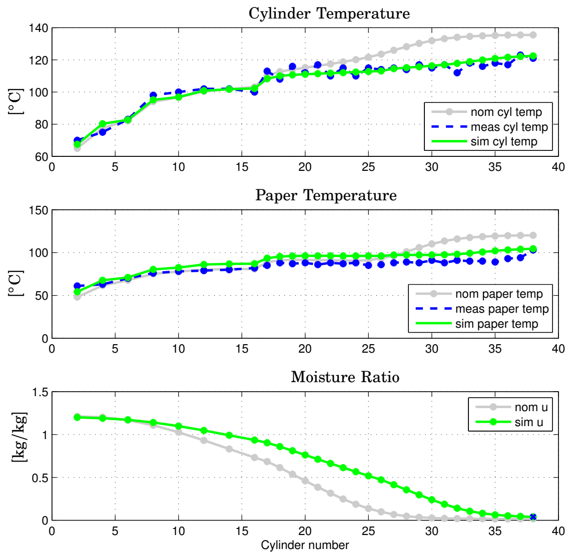


Figure 6 Stationary temperature and moisture profiles. The x-axis shows cylinder numbers.

4.3 Parameter Optimization Results

Solving the problem (25) yields an optimal cost of 277, compared to the cost 61869 for the nominal parameter values. The optimal temperature profiles are shown in Figure 6. For comparison, the nominal profiles are plotted. As can be seen, there is a significantly improved fit between simulated and measured responses. In particular, the output moisture in the nominal case is unrealistically low too early in the dryer section. It can also be noted that the fit of the cylinder temperature profile is better than that of the paper temperature profile. This phenomenon is expected, since the weight of the paper temperature errors was set to a low value.

The matching of the profiles can be improved further by introducing additional optimization parameters. This strategy is explored in [Åkesson and Ekvall, 2006] for a slightly different parameter optimization problem.

5. Model Reduction

Dryer section models built using `DryLib` results in large scale models, even though a sparse discretization scheme for mass and energy balances has been applied. For control design and evaluation purposes, however, a model describing the dynamic relationship between the inputs and the quality variables at the last free draw is usually sufficient. In practice, low order models (e.g. KLT-models with a gain, a time delay and a time constant) valid at a specific operating point are commonly used for dryer section control. In this section, a reduced model targeted towards moisture control design is developed. Since the moisture measurement signal available for feedback control is usually obtained at the end of the dryer section, the aim of the reduction scheme is to develop a simpler model, which captures the non-linear dynamical behavior relating the steam pressure reference signal, input moisture, input temperature and dry basis weight (from the press section) to output moisture. Accordingly, accurate simulation of the paper temperature and the moisture profile can be compromised in order to obtain a lower order model, which describes only the phenomenon of interest, i.e. the behavior of the moisture, accurately.

For linear systems, there exists methods which along with a reduced linear model also gives a bound on the maximum approximation error. The basic approach is usually to find a norm, where it is possible to actually solve the optimization problem resulting from posing a problem where the norm of the difference between the original and the reduced model is minimized. The most common method has historically been that of balanced truncation, where the Hankel norm is used to measure the distance between the models, see [Moore, 1981].

For non-linear systems, however, the situation is different in that there are few methods which offer a structured way of obtaining a lower order model and an upper bound for the approximation error. An additional complication in this case is that the underlying DAE is not easily accessible for e.g. coordinate transformations which is a common ingredient in model reduction schemes.

In the following, a method based on the equivalent dryer concept and optimization will be presented.

5.1 The Equivalent Dryer

In this paper, the structure of the dryer section will be exploited, in order to obtain a model of lower order. A previously reported concept is that of the equivalent dryer, which is described in [Rao *et al.*, 1994]. Instead of modeling each cylinder as a separate unit, the equivalent dryer concept suggests that one, larger cylinder can be used to approximate an entire steam group. This approach has several attractive features. *i)* It preserves the structure of the dryer section, since each steam group is replaced by its corresponding equivalent dryer, *ii)* each equivalent dryer has an intuitive physical interpretation *iii)* and the reduction potential is large, especially for large steam groups.

5.2 The Reduction Problem

At the steam group level, the reduction problem can be stated as *"Find the dimensions of* ***one*** *steam cylinder, including associated incoming and outgoing free draws and contact paper, which approximates as well as possible, the behavior of a given steam group"*. This qualitative objective needs, however, to be quantified, and specifically, the meaning of "as well as possible" should be given a mathematical interpretation. In principle, it should be possible to adopt the scheme commonly used for linear model reduction. The problem can then be stated as to minimize the maximum approximation error over the physical dimensions of the equivalent dryer cylinder. Solving this problem involves finding the solution to a dynamic optimization problem, where the search space consists of *i)*, the inputs (for generating the maximum error) and *ii)* the physical dimensions of the equivalent dryer. Since the dryer section model is very large, this approach does not seem attractive. Instead, a method based on physical insight will be used to formulate a tractable, yet challenging, reduction problem.

It is reasonable to assume that the main time constant of the steam group model is dominated by the dynamics of heating the cast iron cylinders. Clearly, the mass of the paper is neglectable compared to the mass of the steam cylinders. This means that when a reference step is applied to the pressure control loop of a cylinder group, the changes in the temperature of the paper sheet will be closely related to the temperature of the the cylinder shell. Consequently, since the drying process is driven by the heat transferred from the cylinders to the paper, it is reasonable to assume that associated variables, most importantly moisture, will be governed by the same time constant.

In line with this reasoning, we suggest that the dynamic and stationary response of the equivalent dryer cylinder may be treated separately. As for the dynamics, we assume that the mass and volume of the equivalent cylinder can be set to N_{cyl} times those of an individual cylinder in the steam group, where N_{cyl} is the number of cylinders. Now, simulation experiments reveal that the time constant of an equivalent cylinder, constructed based on this assumption, corresponds well to the time constant of the full steam group. However, the same result does not seem to hold for the stationary gains, where there is a significant mismatch. Intuitive ways to set the lengths of the free draw and contact papers, using the same reasoning as for mass and volume, does not produce acceptable results. A more sophisticated way of finding the physical dimensions and parameters is thus necessary.

5.3 Reduction of One Steam Group

A static model for a paper sheet in contact with a steam cylinder, can be formulated using algebraic versions of the dynamic mass and energy balances presented in Section 2. Assuming that the steam pressure, p, the input paper moisture, u_{in}, the input paper temperature, $T_{p,in}$, and the dry basis weight, g, are fixed and known, a system of five equations and five unknown can be derived. The unknowns of the system of equations are the energy flow from the cylinder to the paper, Q_p, the cylinder metal temperature, T_m, the mass flow of steam into the cylinder, q_s, the paper temperature, T_p and the paper moisture, u. The system of equations is then given by

$$\begin{aligned}
q_s &= \frac{\alpha_{cp}A_{cyl}\eta\alpha_{sc}(T_s - T_p)}{\alpha_{sc}h_s - \alpha_{sc}h_w + \alpha_{cp}\eta h_s - \alpha_{cp}\eta h_w} \\
Q_p &= \frac{\alpha_{cp}A_{cyl}\eta\alpha_{sc}(T_s - T_p)}{\alpha_{sc} + \alpha_{cp}\eta} \\
T_m &= \frac{\alpha_{cp}\eta T_p + T_s\alpha_{sc}}{\alpha_{sc} + \alpha_{cp}\eta} \\
0 &= d_y v_x g u_{in} - A_{xy}q_{evap} - d_y v_x g u \\
0 &= d_y v_x g(1 + u_{in})C_{p,p,in}T_{p,in} \\
&\quad - A_{xy}q_{evap}(\Delta H_{vap} + \Delta H_s) - \alpha_{pa}A_{xy}(T_p - T_a) \\
&\quad - d_y v_x g(1 + u)C_{p,p}T_p + Q_p
\end{aligned} \tag{26}$$

were $C_{p,p,in}$, ΔH_s and q_{evap} are functions of the unknowns T_p and u.

In a similar way, a static model for a paper sheet in the free draw can be formulated. The model is given by a system of equations containing two equations and the two unknowns u and T_p,

$$\begin{aligned}
0 &= d_y v_x g u_{in} - 2 * A_{xy}q_{evap} - d_y v_x g u \\
0 &= d_y v_x g(1 + u_{in})C_{p,p,in}T_{p,in} \\
&\quad - 2A_{xy}q_{evap}(\Delta H_{vap} + \Delta H_s) - 2\alpha_{pa}A_{xy}(T_p - T_a) \\
&\quad - d_y v_x g(1 + u)C_{p,p}T_p.
\end{aligned} \tag{27}$$

These systems of equations can then be put together to formulate a static model for a steam group.

As stated in the introduction of this section, the most important quality variable, at least for moisture control, is paper moisture. Therefore, a reasonable objective is to minimize the deviation between the moisture in the last free draw of the cylinder group, and the moisture in the outgoing free draw of the equivalent cylinder. In addition, as a secondary objective, it was decided to minimize the deviation in steam consumption. This objective was added since it may be desirable to limit the steam consumption during moisture control. Performing this minimization for a single operating point is not sufficient, however. In order to obtain a good fit over a wider operating range, a set of operating cases was introduced, over which the optimization was performed. Each case consists of a specification of the operating point in terms of steam pressure, input moisture, input temperature and basis weight. The cost function to be minimized, can now be be written as

$$J = \sum_{i}^{N_c} \gamma_u (u_{out,i} - u^r_{out,i})^2 + \gamma_{q_s} \left(\sum_{j}^{N_{cyl}} q_{s,j,i} - q^r_{s,i} \right)^2 , \tag{28}$$

where $u_{out,i}$ is the output moisture of the steam group in the i:th case, $u^r_{out,i}$ is the corresponding output moisture of the equivalent cylinder, N_c is the number of cases and N_{cyl} is the number of cylinders in the group. As for the steam flow, the squared sum of deviations between the total steam flow for the cylinder group and the equivalent dryer is penalized. γ_u and γ_{q_s} are weights representing the relative importance of a good match in moisture and steam flow respectively. The minimization of the criterion (28) is performed subject to the equations (26) and (27), which are repeated based on the number of cases, N_c, and the number of cylinders in the group N_{cyl}.

It remains to define the optimization parameters, over which the minimization of (28) is performed. Six parameters of the equivalent dryer were selected for optimization, namely the length of the free draws, the length of the contact paper, the heat transfer coefficient between steam and condensate, α_{sc}, the convection coefficient α_{pa} and the mass transfer coefficient K. The number of variables that are actually needed to obtain a good fit is not unambiguous, however. For small steam groups, or if few cases are used, some of the suggested optimization variables may well be fixed, without any increase in the approximation error. In fact, it is desirable to find an appropriate trade-off between the number of optimization variables and optimization performance, in order to avoid over-parametrization.

5.4 Reduction of a Dryer Section

A straight forward approach for deriving a reduced order dryer section would be to simply apply the method described in the previous section for each individual steam group. Recalling our main objective, which is to predict the moisture in the last free draw, this approach would not explore the full potential of the method. Instead, a larger optimization problem, incorporating all groups, may be formulated where most attention is given to minimizing the deviation of the last group. This means that *all* groups are reduced at the same time, and that the full reduction potential is used according to the main objective, which is to predict the final moisture. It may, however, be advantageous to include the deviations, with small weights, of all groups in the optimization criterion, in order to avoid a physically unrealistic model.

Initial optimization runs showed that the total length of the paper process in the reduced model was significantly larger than the total length of the paper process in the original model. This deficient of the reduced model may be suppressed by introducing a term in the optimization criterion penalizing the deviation of total paper process length between the original and the full model. This modification of the original problem resulted in a better match of the dynamic response, without a penalty in terms of degraded static match.

An additional modification of the problem concerning the matching of the steam flow rate was made in the final formulation. Since the *total* steam consumption of the dryer section is of interest, rather than the consumption of individual groups, the penalties on deviations in steam flows at the group level was replaced by single penalty on deviations in the total steam consumption.

The over all performance criterion can now be written

$$
\begin{aligned}
J_{tot} = &\sum_{i=1}^{N_c}\sum_{k=1}^{N_g}\gamma_{u,k}(u_{out,i,k}-u^r_{out,i,k})^2+\\
&\gamma_{q_s}\left(\sum_{k=1}^{N_g}\sum_{j=1}^{N_{cyl,k}} q_{s,i,j,k}-\sum_{k=1}^{N_g} q^r_{s,i,k}\right)^2+\\
&\gamma_l\left(\sum_{k=1}^{N_g}\sum_{l=1}^{N_{p,k}} l_{p,k,l}-\sum_{k=1}^{N_g}\sum_{l=1}^{3} l^r_{p,k,l}\right)^2
\end{aligned}
\tag{29}
$$

where N_g is the number of groups, $N_{p,k}$ is the number of paper segments in group k, $l_{p,k,l}$ is the length of a paper segment in the original model and $l^r_{p,k,l}$ is a length of a paper segment in the reduced model. In line with the arguments given above, $\gamma_{u,N_g} >> \gamma_{u,k}, k \neq N_g$.

5.5 Solving the Optimization Problem

The resulting algebraic optimization problem is challenging, both due to its size and its non-linear character. The final problem consists of 9536 free variables and 9504 equality constraints, of which 8568 are non-linear. Efficient solution of large scale NLP problems of this type require state of the art numerical algorithms, exploring the sparse structure of the problem as well as analytical Jacobian and Hessian information.

The problem definition was programmed in AMPL, which is a language for mathematical programming, [Fourer *et al.*, 2003]. AMPL enables encoding of linear and non-linear algebraic optimization problems, using optimization oriented language constructs. The problem description, i.e. the AMPL code, is then executed within the AMPL tool, which in turn interfaces several numerical solvers. In this application, the NLP code KNITRO, [Waltz, 2005], has been used. The combination of AMPL and KNITRO is extremely powerful, since the AMPL interface to numerical solvers offers analytic evaluation of Jacobians and Hessians as well as sparsity information. This enables KNITRO to operate in its most efficient mode, resulting in acceptable execution times also for large systems. The reduction problem formulated in the previous section is solved in about 2-5 minutes, depending on configuration and initial starting point.

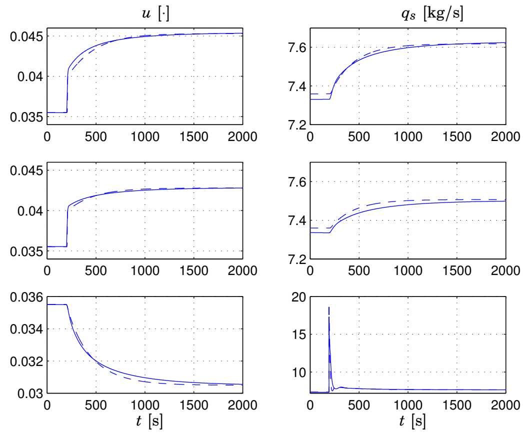


Figure 7 Step responses for moisture (left) and steam flow rate (right) of the original (dashed) and the reduced (solid) models. The responses corresponds to, from above, steps in input moisture, input dry basis weight and pressure reference, applied at 200 s.

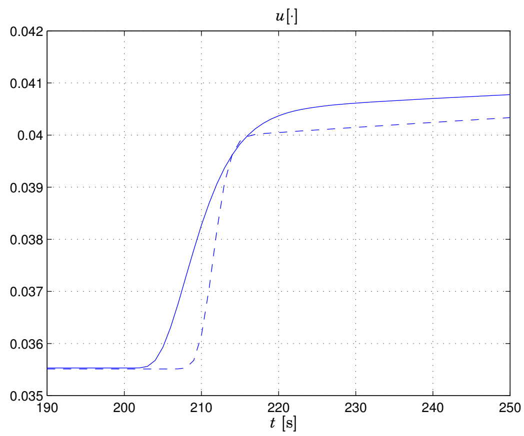


Figure 8 Response in output moisture, resulting from a step disturbance in the input dry basis weight. The dashed curve corresponds to the original model and the solid curve corresponds to the reduced model. The step disturbance is applied at t=200 s.

The proposed method has the distinct drawback of requiring complete re-encoding of the the model description. This was necessary, however, in order to enable utilization of the appropriate symbolical and numerical algorithms.

It is important to note, however, that the problem is non-convex, and that only local optimality can be expected. However, in this case, the solution to the reduction problem seemed to be robust with respect to different starting points. Also, the obtained solution is reasonable in the sense that the optimized parameter values lies within physically feasible limits.

5.6 Model Reduction Results

As mentioned above, a set of operating conditions need to be specified, in order to complete the problem formulation. Clearly, the operating range over which the reduced model is valid, is influenced by this choice. As the nominal case, values for steam pressures, input moisture, input temperature and dry basis

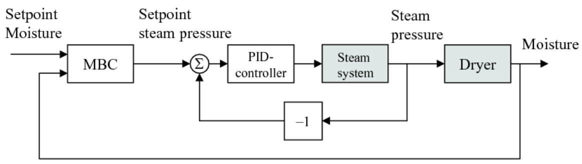


Figure 9 The moisture control cascade loop. The inner loop controls the steam pressure by manipulating a steam valve. The inner loop generally consists of a PID-controller and it gets its set point from some type of Model Based Control (MBC), commonly an IMC (Internal Model Control), a Dahlin controller, or a linear MPC (Model Predictive Control).

weight corresponding to a typical grade were chosen. Based on the nominal case, additional 35 cases were defined by varying the nominal parameters.

The result of the reduction procedure was evaluated by means of step responses in input moisture, dry basis weight and pressure set point, see Figure 7. As can be seen, there is a good match between the stationary responses of the original and reduced models. Also, the (slow) dominating time constant is captured well by the reduced system. However, the reduced model does not fully capture the fast transient behavior of the original model. The steep initial ascent in the step response, which is shown in Figure 8, is due to the transport delay of the model. Using the length of the paper web through the dryer section and the machine speed, the theoretical transport delay can be calculated. In this particular case, the delay is 12s, which is matched well by the original model, represented by the dashed curve. The reduced model, on the other hand, seem to have a smaller delay but a more smoothed initial response. This is, however, to be expected. The original model consists of a large number of paper components, which together forms a high dimension compartment system. The reduced model consists of significantly fewer segments, and cannot approximate the time delay with the same accuracy.

The original motivation for performing the model reduction was to obtain a model of lower complexity. Indeed, the reduced model has fewer dynamical states, namely 85, as compared to 318 for the original model. Also, the simulation time for a typical scenario was approximately 85% shorter for the reduced model.

6. NMPC of Output Moisture

The structure of the moisture control loop is depicted in Figure 9. It is usually the case that all cylinder groups are tied the same steam pressure set point which gives a single loop cascade control. It is common to let the MBC controller calculate the pressure set-point for the last steam group, p^{sp}, and then calculate the pressure set-points for the other groups as functions of p^{sp}. Using this method, it is straight forward to ensure that there is an monotonically increasing pressure profile in the machine direction, which is important since steam from high pressure groups is re-used to heat groups running at lower pressure.

The MBC controller is usually based on a low order linear model of the dryer section. While a well tuned controller works well at a given set-point, the non-linear character of the dryer section dynamics results in degraded performance if the set-point is changed. Since the plant is operated at several different set-points, corresponding to different grades, a traditional control system maintains several parameter sets for the MBC controller. Switching of controller parameters is then done after a grade change.

In this paper, we consider a different approach to moisture control. Based on the reduced non-linear dryer section model derived in Section 5, a basic Non-Linear Model Predictive Control (NMPC) scheme is implemented. The main benefit of using a non-linear model in the control design is that the operating range of the controller may be increased. In addition, successful implementation of a controller which achieves good performance in a wide operating range may serve as a unifying strategy for stationary and transition (grade change) control, whereas common practice today is to use separate controllers for these two control modes.

A realistic implementation of an MPC controller consists of tree main parts – reference target calculation, state estimation and solution of the optimal control problem. In this paper, the problem of solving the optimal control problem is addressed. The resulting controller is evaluated under the assumption of full state information.

6.1 Model Predictive Control

MPC refers to a family of controllers which are based on the receding horizon principle. At each sample, a finite horizon open loop optimal control problem is solved, and the first part (corresponding to the first sample) of the resulting optimal control profile is applied to the plant. At the next sample, the procedure is repeated and a new optimal control problem with the horizon shifted one sample is solved. Thereby the name receding horizon control. Two of the most important advantages of using MPC is that it works well for MIMO plants and that it takes state and control bounds into account *explicitly*. However, an MPC controller, including the on-line solution of an optimization problem (at least in the case of a non-linear model), is computationally demanding, which makes application to processes with fast dynamics troublesome. During the last decade, MPC has emerged as a major control strategy, mainly in the process industry, see [Qin and Badgwell, 2003] for an overview.

MPC comes in many flavors. The theory for MPC based on LTI models is well developed. The linear case has particularly attractive features in that the arising optimization problem is convex, and the availability of stability results. For non-linear systems, the situation is somewhat different. While stability results exist, see [Mayne *et al.*, 2000], the problem of solving the arising optimal control problem is complicated, since the problem is in general non-convex, which means that global optimality cannot be guaranteed. Still, several algorithms exit, and non-linear MPC has received increased industrial interest during the last few years.

6.2 Dynamic Optimization

Traditionally, optimization problems incorporating constraints imposed by dynamic systems have been addressed by dynamic programming, [Bellman, 1957] or the maximum principle, [Pontryagin *et al.*, 1962]. During the last two decades, however, a new family of methods, referred to as direct methods have emerged. These methods are based on discretization of the original optimization formulation, transforming the infinite dimensional problem into a finite dimensional one. The discretized problem is then solved by means of algebraic non-linear programming.

There are two main approaches to direct discretization. Simultaneous methods are based on full discretization of the control and state spaces, yielding very large NLP to solve, see [Biegler *et al.*, 2002] for an overview. There exist, however, efficient solvers for this type of problems. Sequential methods, on the other hand, are based on discretization of the control space only, resulting in a smaller number of parameters in the resulting NLP, see [Vassiliadis, 1993]. Optimization of Dymola models has previously been considered in the work [Franke *et al.*, 2003], where the Simulink interface provided with Dymola was used to access the model. The main difference between the approach used in [Franke *et al.*, 2003] and this work lies in the methods of accessing the model, where the `dsblock` interface has the advantage of offering evaluation of an analytical Jacobian.

In order for a (gradient based) NLP algorithm to have fast convergence, it is important to provide to the algorithm not only the cost function, but also its gradient with respect to the optimization parameters. Calculation of high accuracy gradients for dynamical systems generally involves calculation of the state sensitivities with respect to parameters. This can be done by integration of the sensitivity equations. By exploring that the sensitivity equations have the same Jacobian as the original DAE, integration can be done efficiently.

The algorithm used to solve the dynamic optimization problem described in this section is a straight forward implementation of a sequential single shooting algorithm, see [Vassiliadis, 1993].

6.3 State Estimation

There are two main approaches to state estimation for non-linear systems, namely the extended Kalman filter, [Anderson and Moore, 1979], and receding horizon estimation, [Rao *et al.*, 2003]. The problem is challenging, and also computationally demanding, especially for large systems. Solution the filtering problem is not treated in this paper, although the problem must be solved in order to apply the MPC scheme to a real plant.

6.4 The Optimal Control Problem

An integral part of an NMPC controller is the formulation of the open loop optimal control problem to be solved in each sample. Since the aim of the control scheme in this application is to control the moisture ratio, it is natural to penalize deviations from the target moisture. The control trajectory in the optimization problem, p^{sp}, is parametrized by a piece-wise constant function with N_u segments. In order to avoid violent control moves, which may introduce disturbances in the steam system, a term penalizing the deviation between two successive control moves is introduced in the cost function. In addition, there are hard limits acting on the control variable. This yields the following optimization problem

$$\begin{aligned}
&\min_{\hat{p}_i^{sp}} \int_0^{T_f} \gamma_u (u_{out}^{sp} - \hat{u}_{out}(t))^2 dt + \sum_{i=0}^{N_u-1} \gamma_p (\Delta \hat{p}_i^{sp})^2 \\
&\text{subject to} \\
&F(x, \dot{x}, y, p^{sp}) = 0 \text{ (DAEdynamics)} \\
&466 \text{ kPa} \leq p^{sp} \leq 596 \text{ kPa (control constraint)}
\end{aligned} \tag{30}$$

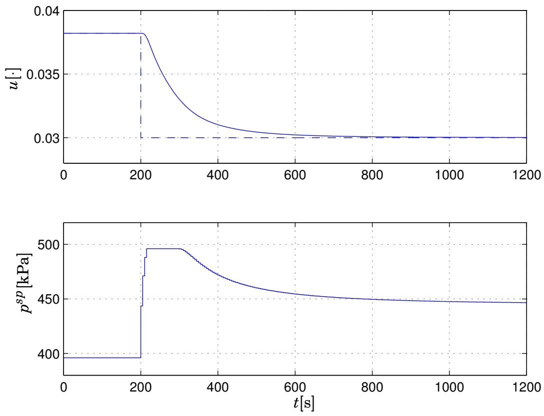


Figure 10 Step response of the NMPC controller.

where T_f is the prediction horizon, u_{out}^{sp} is the target moisture, $\hat{u}_{out}(t)$ is the predicted moisture profile, $\hat{p}_i^{sp}$ is the predicted pressure set point trajectory and $\Delta\hat{p}_i^{sp} = \hat{p}_i^{sp} - \hat{p}_{i-1}^{sp}$. γ_u and γ_p are weights. In the simulation, the parameters were set to $\gamma_u = 10000$, $\gamma_p = 0.01$, $N_u = 4$, and the samplig interval was 5 s.

6.5 Results

A simulation where the NMPC controller is applied to the reduced dryer section model is shown in Figure 10. In the simulation, a reference step, from $u_{out}^{sp} = 0.038$ to $u_{out}^{sp} = 0.03$ is applied at $t = 200$ s. As can be seen, the moisture reaches the desired set-point, while the control signal respects the specified constraints.

An important, and often limiting, factor when using MPC controllers, is the execution time for solving the on-line optimization problem. In this case, execution times ranged from 10 s to 80 s, with a mean of 13.5 s. Typically, execution times are longer when reference changes and disturbances occur, while shorter and more predictable execution times are obtained during stationary operation. Assuming a sampling interval of $h = 5$ s, it is clear that the execution times must be decreased. There are several approaches to reducing execution times, e.g. modifying the lengths of the control and prediction horizons, reducing the complexity of the model or using a more efficient optimization algorithm. This is, however, beyond the scope of this paper.

7. Software Tools

The dryer section model has been implemented, as mentioned above, in Modelica and Dymola. The parameter optimization problem and the NMPC problem, however, were solved by integrating several software packages into a custom application, which utilized the C-code representing the model generated by Dymola.

The software packages used in the development of the custom application are:

- a C programming interface to access routines generated by Dymola, `dsblock`. Using this interface, custom applications can be developed for e.g. simulation or like in this case, optimization. The interface provides basic routines

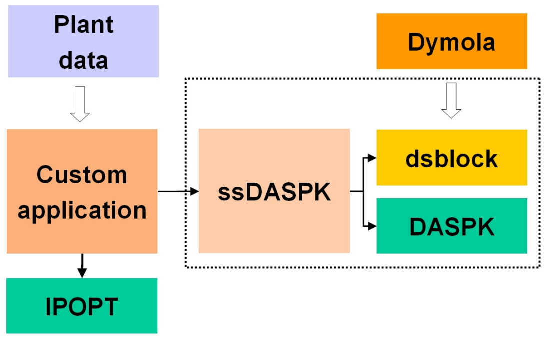


Figure 11 Software application structure.

for obtaining information about model parameter and initial state values, evaluation of the right side of the resulting ODE (DAE) and the associated Jacobian.

- a DAE-solver, DASPK 3.1 [Maly and Petzold, 1996]. This code solves DAE:s as well as calculates sensitivities required for optimization. The code is written in Fortran and was translated to C using `f2c`.
- an NLP-code, IPOPT [Wächter and Biegler, 2006]. This code implements a primal-dual interior point method and was used to solve the NLP resulting from the parameter optimization and NMPC problems.
- a package for managing the communication between the Dymola C interface and DASPK, which has been developed in order to enable simplified development of optimization applications based on models generated by Dymola. This package, in the following referred to as ssDASPK, provides e.g. simulation and sensitivity calculation for use in custom applications.

These packages were compiled and linked with the code representing the model generated by Dymola, into an application which was used to set up and solve the particular optimization problems. The structure of the applications is shown in Figure 11.

8. Summary and Conclusions

In this paper, modeling, model reduction, parameter optimization and NMPC control design for a paper machine dryer section has been considered. It has been demonstrated how Modelica models of high complexity can be used for purposes other than simulation. The resulting optimization problems are challenging and require state of the art numerical solvers. In particular, solution of the model reduction problem, which has more than 9000 free variables, is dependent on algorithms exploring the problem structure. Our experience from this project is that there is no single tool or software that can address all problems arising in simulation and optimization. Rather, in order to solve problems effectively, it is essential that Modelica tools are designed to be interfaced with software for solution of optimization problems. In general, it is highly desirable that software for complex systems is provided with interfaces so that they can be combined.

There are several possible extensions of the paper. The `DryLib` library may be extended as outlined in Section 3, and the parameter optimization scheme would benefit from including also time series data. Regarding the model reduction scheme, it may be desirable to derive models with further reduced complexity valid over a wide operating range. Finally, the NMPC scheme outline in Section 6 needs to be further elaborated in order to be applicable to the real plant.

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