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Hyades dynamics from *N*-body simulations: Accuracy of astrometric radial velocities from Hipparcos⋆

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Abstract. The internal velocity structure in the Hyades cluster as seen by Hipparcos is compared with realistic *N*-body simulations using the NBODY6 code, which includes binary interaction, stellar evolution and the Galactic tidal field. The model allows to estimate reliably the accuracy of astrometric radial velocities in the Hyades as derived by Lindegren et al. (2000) and Madsen et al. (2002) from Hipparcos data, by applying the same estimation procedure on the simulated data. The simulations indicate that the current cluster velocity dispersion decreases from 0.35 km s^{-1} at the cluster centre to a minimum of 0.20 km s^{-1} at 8 pc radius (2–3 core radii), from where it slightly increases outwards. A clear negative correlation between dispersion and stellar mass is seen in the central part of the cluster but is almost absent beyond a radius of 3 pc. It follows that the (internal) standard error of the astrometric radial velocities relative to the cluster centroid may be as small as 0.2 km s^{-1} for a suitable selection of stars, while a total (external) standard error of 0.6 km s^{-1} is found when the uncertainty of the bulk motion of the cluster is included. Attempts to see structure in the velocity dispersion using observational data from Hipparcos and Tycho-2 are inconclusive.

Key words. methods: N-body simulations – data analysis – techniques: astrometry – stars: kinematics – open clusters and associations: general – open clusters and associations: individual: Hyades

1. Introduction

The Hyades is the nearest rich open cluster and as such has played a fundamental role in astronomy as a first step on the cosmological distance ladder and as a test case for theoretical models of stellar interiors (Lebreton 2000). From the first use of the converging point method by Boss (1908) up to the use of pre-Hipparcos trigonometric parallaxes by van Altena et al. (1997), an important goal in astrometry has been the determination of an accurate distance to the cluster. With the advent of the Hipparcos Catalogue (ESA 1997) the Hyades lost its unique status for distance calibration, but as the depth and internal velocity field of the cluster were well resolved by Hipparcos, focus could instead be turned to its three-dimensional structure and kinematics (Perryman et al. 1998). A deeper understanding of the dynamics and evolution of the cluster should now be possible through detailed comparison with *N*-body simulations.

Thanks to the accurate Hipparcos measurements, the Hyades has recently acquired a completely new role as a practical standard in observational astrophysics: it is one of very few objects outside the solar system for which the accurate radial motion can be determined by geometric means, i.e. without using the spectroscopic Doppler effect. From a combination of Hipparcos parallaxes and proper motions, Madsen et al. (2002) obtained "astrometric radial velocities" for individual Hyades stars with a then estimated standard error of about 0.6 km s^{-1}. Currently the Hyades is the only cluster for which astrometric radial velocities are derived with individual accuracies better than 1 km s^{-1}, but the technique may be extended to many more objects with future space astrometry missions (Dravins et al. 1999b).

Astrometric radial velocities are important mainly because they make it possible to determine the *absolute* lineshifts intrinsic to the stars, through comparison with spectroscopic measurements. Such lineshifts are caused for instance by convective motions and gravitational redshift in the stellar atmospheres (Dravins et al. 1999a). Absolute lineshifts could previously only be observed in the solar spectrum, but are now within reach for a range of spectral types through the use of astrometric radial velocities. The present paper is part of a research programme at Lund Observatory in which absolute lineshifts are determined and used as a diagnostic tool in stellar astrophysics (Dravins et al. 1997, 1999b; Lindegren et al. 2000; Madsen et al. 2002; Gullberg & Lindegren 2002).

A major uncertainty in the astrometric radial velocities originates in the internal velocity dispersion of the cluster, which limits both the accuracy of the cluster motion as a whole, and that of the individual stars. A primary goal of the present investigation is to find out whether a better understanding of

⋆ Based on observations by the ESA Hipparcos satellite, and on the *N*-body code NBODY6 by Sverre Aarseth, publicly available at `ftp://ftp.ast.cam.ac.uk/pub/sverre/`

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the internal velocity structure of the cluster, obtained through N-body calculations, can be used to improve the accuracy of the astrometric radial velocities.

Section 2 briefly recalls the kinematic information, including astrometric radial velocities, that can be derived from Hipparcos data. Section 3 describes the model used to simulate the evolution of the cluster up to its present state, and its subsequent observation, as well as the main properties derived from the simulations. Implications for the accuracy of the astrometric radial velocities are discussed in Sect. 4, followed by a discussion of non-modelled effects in Sect. 5, and conclusions.

2. Cluster kinematics derived from astrometry

Since an ultimate aim of the present programme is to confront spectroscopic measurements of line shifts in stellar spectra with independent measurements of the stellar motions, it is essential that the kinematic data, including the radial velocities, are derived without using the spectroscopic Doppler effect. Dravins et al. (1999b) describe several methods to derive the radial motion of stars by purely geometric means, i.e. using astrometric data. Of these, the moving-cluster method has been successfully applied to several open clusters and OB associations, in particular the Hyades (Lindegren et al. 2000; Madsen et al. 2002). The principle of the moving-cluster method is very simple: let θ be the angular size of the cluster and R its distance. Assuming its linear size $R\theta$ to be constant, we have $\dot{R}\theta + R\dot{\theta} = 0$, where the dot signifies time derivative. Since R is known from trigonometric parallaxes, the astrometric radial velocity of the cluster follows as $\dot{R} = -R\dot{\theta}/\theta$.

In practice, several kinematic parameters are simultaneously estimated from the astrometric data of the cluster member stars, using the method of maximum likelihood (Lindegren et al. 2000). Some features of the method, relevant for the subsequent discussion, are recalled hereafter.

The estimated parameters include the common space velocity of the cluster ($\boldsymbol{v}_0$), the individual stellar parallaxes (π_i for star i), and the internal velocity dispersion (σ_v). The astrometric radial velocity of an individual star i is then calculated as $\widehat{v}_{\mathrm{r}i} = \boldsymbol{r}_i'\widehat{\boldsymbol{v}}_0$, where $\boldsymbol{r}_i$ is the unit vector towards the star and the caret $\widehat{\ }$ signifies estimated quantities. As part of the procedure, improved parallaxes $\widehat{\pi}_i$ are obtained for the individual stars. In the Hyades, these are 2–5 times more precise than the original Hipparcos parallaxes which have errors around 1–1.5 mas. The improvement results from a combination of trigonometric and kinematic parallaxes, where the latter follow from the proper-motion components *along* the cluster motion, which are inversely proportional to distance. The kinematically improved parallaxes allow a very precise mapping of the spatial structure of the cluster. The maximum likelihood estimate of σ_v is unfortunately biased. Instead the proper motions *perpendicular* to the cluster motion are used to estimate the velocity dispersion according to the method described in Lindegren et al. (2000), Appendix A.4. For each star, a goodness-of-fit statistic g_i is also obtained from the maximum-likelihood estimation (see Lindegren et al. 2000 for a thorough discussion of g_i). The statistic is primarily used to reject stars whose astrometric data do not fit the cluster model well enough; a rejection limit of $g_\mathrm{lim} = 15$ was normally used, although a stricter limit (10) or no limit at all (∞) were also tried. For the retained stars, the g_i values (which are then $\leq g_\mathrm{lim}$) could be regarded as a quality index, with a lower value indicating a better fit to the cluster model.

The error in the estimated astrometric radial velocity, $\widehat{v}_{\mathrm{r}i}$, has two parts. The first part is due to the error in the common space motion of the cluster, $\widehat{\boldsymbol{v}}_0$. Its uncertainty depends on global properties of the cluster such as its distance, angular extent, and richness, as well as on the accuracy of the astrometric data. The second part is due to the star's peculiar motion relative to the cluster centroid. This part depends only on the dispersion of the peculiar motions along the line of sight, which for a uniform, isotropic velocity dispersion equals σ_v. In most of the clusters for which the method has been applied, the main uncertainty comes from the first part, i.e. the error in the cluster's space motion. In the Hyades, however, the uncertainty in $\widehat{\boldsymbol{v}}_0$ is small enough (0.36 km s^{-1} along the line-of-sight; Madsen et al. 2002) that the total uncertainty in the astrometric radial velocities is dominated by the contribution from the internal velocity dispersion (0.49 km s^{-1} according to the estimate in the same source).

On the other hand, the assumption of a constant and isotropic velocity dispersion throughout the cluster may be rather simplistic. Theoretically, one expects at least a variation with distance r from the centre of the cluster, and possibly also a variation with stellar mass due to the equipartition of kinetic energy. For instance, in a simple Plummer (1915) potential we have

$$\sigma_\mathrm{v}^2(r) = \frac{GM}{6\sqrt{r_\mathrm{c}^2 + r^2}} \tag{1}$$

(Gunn et al. 1988; Spitzer 1987), where M is the cluster mass and r_c the core radius ($\simeq$3 pc for the Hyades). According to Eq. (1), σ_v should decrease by one third as one moves two core radii away from the centre, and become even smaller further out in the cluster; but this trend is obviously broken at some distance by tidal forces. Clearly, these effects must be also reflected in the accuracy of the astrometric radial velocities. Attempts to measure the radial variation of dispersion in the Hyades from astrometry were inconclusive (Madsen et al. 2001), but N-body simulations could help to establish to what extent such variations exist in real clusters.

3. Dynamical simulation of the Hyades cluster

3.1. Previous N-body simulations

It is not new to use the Hyades as a comparison with N-body simulations. Aarseth (1977) discussed the dynamical relevance of the central binary 80 Tau (HIP 20995) in the context of binary formation and evolution in stellar systems as described by N-body simulations. Oort (1979) discussed the flattening of the Hyades parallel to the galactic plane by comparing observations with the N-body simulations by Aarseth (1973). Kroupa (1995c) simulated the evolution of star clusters and found excellent agreement between the models and the Hyades luminosity function, concluding that the initial conditions of

the cluster could to a large extent be reconstructed. An initial mass of the Hyades protocluster of some 1300 $M_\odot$ was suggested. Von Hippel (1998) used numerical simulations of clusters and data on Hyades white dwarfs, among others, to conclude that the white-dwarf mass fraction is relatively insensitive to kinematic evolution. Portegies Zwart et al. (2001) discussed the evolution of star clusters which were given initial conditions to represent open clusters, including the Hyades. A good model fit to the Hyades was obtained, thus illustrating the possibility to estimate the initial conditions for an observed star cluster.

What is new in the present study is that the three-dimensional kinematics of the Hyades is investigated through a direct comparison of the Hipparcos observations with a realistic N-body model, evolved till the present age of the cluster, as well as the objective to estimate the accuracy of the astrometric radial velocities from such a comparison.

3.2. Basic cluster data

Perryman et al. (1998) made a detailed study on the Hyades based on Hipparcos data and a compilation of spectroscopic radial velocities from the literature. They identified 197 probable member stars, which constitute the initial Hyades sample (Hy0) used for the present study. When comparing with the simulated cluster, only stars within 20 pc from the cluster centre are considered, due to the radial limitation in the N-body code (Sect. 3.3). Adopting the cluster centre of mass in equatorial coordinates, (+17.36, +40.87, +13.30) pc from Perryman et al. (1998), and using the kinematically improved parallaxes (Sect. 2), a subset of 178 stars (Hy0r) was found within a radius of 20 pc. The cluster has a general space velocity of (−5.90, +45.65, +5.56) km s^{-1} in equatorial coordinates (Madsen et al. 2002).

Perryman et al. (1998) note that a redetermination of membership with the above cited centre of mass will reduce the number of member stars outside 10 pc by 10 stars while keeping the same number of stars inside 10 pc. The true number of member stars in the Hy0r sample is then probably smaller than the 178 stars.

Hipparcos is nominally complete to $V \leq 7.3 + 1.1|\sin b|$ for spectral types later than G5 (or $B-V > 0.8$). However, it is known that the actual limit is somewhat fuzzy, due to photometric errors and other complications. Therefore, a conservative completeness limit of $V \leq 7$ mag is assumed for this study. Choosing a fainter completeness limit like e.g. $V \leq 8$ mag will, however, not significantly affect the outcome of the simulations as will be shown later (Table 1). The actual number of fainter Hyades members is not known. However, at least seven single white dwarfs have been found (e.g. Reid 1996), and this number can also be used as a constraint on the model.

Perryman et al. (1998) estimated the cluster age to be 625 ± 50 Myr, and this age is what is assumed in the following. It should be mentioned that in a more recent work by Lebreton et al. (2001), based on kinematically improved parallaxes from Dravins et al. (1997), only an upper limit of 650 Myr could be estimated due to the lack of a clear turn-off point (cf. top diagram in Fig. 1). In the same work they also estimated the metallicity to [Fe/H] = 0.14 ± 0.05 dex. The interstellar extinction is negligible: Taylor (1980) found only a very small colour excess $E(B-V) = 0.003 \pm 0.002$ mag.

From various studies, a large fraction of the stars are known to be binaries. In the compilation by Perryman et al. (1998), 75 of the 197 probable member stars were either identified as binaries in the Hipparcos Catalogue or previously known as spectroscopic binaries (their Table 2). Patience et al. (1998) found three new binaries from a speckle imaging survey of Hyades members, plus one marked as binary in the Hipparcos Input Catalogue (HIC; Turon et al. 1992), but not found by Hipparcos. In the Tycho Double Star Catalogue (Fabricius et al. 2002), an additional 21 binaries were identified. The eclipsing binary system HIP 17962 = V471 Tau (e.g. Werner & Rauch 1997, and references therein) must also be included in the list of Hyades binaries. We thus end up with 101 known binaries in the Hy0 sample, yielding a minimum multiplicity of 0.51 companions per primary. For the Hy0r sample (within 20 pc of the cluster centre) the minimum multiplicity is 0.53. To include some more binary statistics, binaries with periods $P < 10$ days have been taken from the compilations on the open–cluster database WEBDA[1].

The above values of the multiplicity are only lower limits to the true multiplicity, because of the difficulty to detect binaries in some intervals of separation ρ (or period P) and magnitude difference Δm (or mass ratio q). In restricted intervals, the searches can however be considered complete. For instance, Hipparcos probably detected practically all binaries with $0.2 < \rho < 2$ arcsec and $\Delta m < 2$; cf. Fig. 3.2.106 in vol. 1 of (ESA 1997), where 17 are found in Hy0r. Patience et al. (1998) observed a high fraction of Hyades stars that were also observed by Hipparcos. The 17 binaries they found with $0.1 < \rho < 1.07$ arcsec and $q \geq 0.4$ must therefore also be regarded as a nearly complete sample.

Hipparcos effectively observed for about 37 months (~3 years) spread over a period of nearly 4 years. This means that the proper motions of binaries may be significantly affected by the orbital motion of the photocentre, which must be taken into account when simulating the Hyades proper motions (Sect. 3.4). In order to reduce this effect in the observational analysis, proper motions from the Tycho-2 catalogue (Høg et al. 2000) have also been used, where available. In the solution for the cluster kinematics, the Tycho-2 proper motions yield slightly, but systematically smaller radial velocities (v_r(HIP) − v_r(Tycho-2) = +0.9 km s^{-1}) than do the Hipparcos data for the $g_{\rm lim} = 15$ sample (Madsen et al. 2002), which can be explained by the mean difference of −0.4 mas yr^{-1} of the proper motions in right ascension of what was considered the best sample. In declination, the mean difference of the proper motions is 0.0 mas yr^{-1}. Although the expected deviations between the Hipparcos and Tycho-2 Catalogues are generally under 0.5 mas (Urban et al. 2000), the result from the Hyades might reflect some subtle bias in the Tycho-2 proper-motion system. Since the Tycho-2 system of proper motions was effectively calibrated onto the Hipparcos system, greater confidence

[1] Available at `http://obswww.unige.ch/webda/`

should be put on the solution based on the Hipparcos data. The Tycho-2 data should therefore only be used to study the internal velocity structure of the cluster, where a possible bias is not important.

In addition to the Hy0r sample (which thus includes all 178 probable members within a radius of 20 pc from the cluster centroid), the following samples are also discussed: Ty0r, which is the same as Hy0r but with proper motions from Tycho-2 replacing those in the Hipparcos Catalogue; Hy1r, which is the subset of 85 stars in Hy0r for which there is no known indication of multiplicity; and Ty1r, which is the same as Hy1r but with Tycho-2 proper motions.

It has been suggested that there might be systematic errors in the Hipparcos parallaxes for at least some open clusters (Pinsonneault et al. 1998). The discussion shall not be repeated here, but it should just be stated that there is a general consensus that the mean Hyades parallax is not affected by any correlation between positions and parallaxes (Narayanan & Gould 1999; van Leeuwen 2000; Lindegren et al. 2000). This problem, if it exists, has been neglected in the simulations.

3.3. N-body model of the Hyades cluster

The dynamical evolution of a Hyades-type open star cluster was simulated using the well-known N-body code NBODY6 (Aarseth 1999, 2000). The code incorporates algorithms to deal with stellar (including binary) encounters (Mikkola & Aarseth 1993, 1996, 1998) and stellar evolution (Hurley et al. 2000). For the present study, no modifications were made to the code. Some of the non-modelled effects are discussed in Sect. 5.

External perturbations are represented by a fixed, galactic tidal field. The cluster is assumed to move in a circular orbit at the present distance of the Sun from the galactic centre. The angular velocity is $\Omega = A - B$, where $A = 14.4\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ and $B = -12.0\,\mathrm{km\,s^{-1}\,kpc^{-1}}$ are Oort's constants. This gives rise to tidal forces plus a Coriolis force (cf. Chandrasekhar 1942, Ch. 5.5).

To set up the initial cluster configuration, stars are randomly picked from the initial mass function (IMF) described by Kroupa et al. (1993), until the required total particle number has been reached. Binaries are included as described below. Stars are initially deployed randomly in a Plummer potential (Plummer 1915; Spitzer 1987) with virial radius $r_{\rm v} = 4$ pc. During the evolution of the cluster, stars are kept in the simulation as long as they are within two tidal radii ($\simeq$21–23 pc). The simulation is run until the cluster reaches an age of 625 Myr.

The reason for choosing one single age was to have a fixed parameter for comparing different model realisations. The age uncertainty is not important regarding the conclusions about the current dynamics since the cluster has been relaxed for quite a while.

Binaries are generated by randomly pairing stars picked from the IMF. This gives an almost uniform distribution in the logarithm of the mass ratio ($\log q$), i.e. a strong preference for small q, similar to what has been observed for G-dwarf systems (Duquennoy & Major 1991). The semimajor axis (a) is selected from a uniform distribution in $\log a$ with an upper cut-off at 3000 AU (Quist & Lindegren 2000). The period distribution is afterwards generated by NBODY6 based on the modelling by Kroupa (1995a, 1995b) with minimum period 1 day, and binaries merged if $a \leq 10\ R_\odot$. The initial distribution of eccentricities e is assumed to be thermal, i.e. with a probability density function $2e$ (Kroupa 1995b).

The only free model parameters are thus the total particle number and the initial binary fraction (or multiplicity). Their determination is discussed in Sect. 3.5.

3.4. Transformation to observables

From NBODY6, the luminosity and temperature is obtained for each star. These parameters are transformed to the observational plane ($B-V, M_V$) using Kurucz's colour tables (e.g., Kurucz 1979; Buser & Kurucz 1992) for [Fe/H] = 0.10. Johnson's V is used instead of the Hipparcos magnitude Hp, because of the lack of adequate transformations for the latter. For binaries, the combined colour and magnitude are calculated and plotted in order to get results that are directly comparable with Hipparcos data. In view of the very small interstellar reddening (Sect. 3.2), $E_{B\text{-}V} = 0.0$ is assumed.

When comparing the simulated and observed HR diagrams it should be borne in mind that the theoretical models and colour transformations may produce non-negligible errors. Observed discrepancies for the Hyades amount to some 0.05 mag in $B-V$ or 0.3 mag in M_V in the cool end of the main sequence (Castellani et al. 2001). No (empiric) corrections for this effect have, however, been made.

In order to mimic the real Hyades cluster, as observed by Hipparcos, the simulated present-day cluster is "observed" from the same distance as the real Hyades and given the same centroid velocity relative the Sun. Small errors in the "observed" V magnitudes (standard deviation 0.0015 mag) are introduced, and parallaxes and proper motions, including observational errors, are generated following the same procedure as in Lindegren et al. (2000). The simulated sample includes all stars brighter than the completeness limit $V = 7$, plus a random selection of the fainter stars matching the real sample in the number of stars per magnitude interval. It is assumed that the Hyades stars in the Hipparcos Catalogue with $V > 7$ mag are not subject to any selection effects, although it cannot be ruled out due to a sometimes impenetrable selection procedure of Hipparcos objects in open clusters (Mermilliod & Turon 1989).

Binaries receive different treatments depending on the magnitude difference (Δm), period (P), and angular separation (ρ), in order to simulate how they were treated in the Hipparcos data analysis (see Sect. 1.4.2 in the Hipparcos Catalogue). Here, $\Delta m = M_{V2} - M_{V1}$, where subscripts "1" and "2" refer to the primary and secondary components. For certain combinations of these parameters, Hipparcos effectively observed the motion of the photocentre of the system. In the remaining cases the centre of mass were observed. The former systems include those with $P \simeq 0.1$ to 20 years and $\rho \geq 10$ mas, or $P > 10$ years and $\rho \leq 100$ mas; the short-period binaries ($P <$ few months), which may deviate from a single-star solution (the "stochastic" (X) solution), although they may have been detected as

binaries by Hipparcos; and systems with Hipparcos magnitude difference $\Delta Hp > 4$ mag. For these systems, the component velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are combined into a single velocity of the photocentre,

$$\boldsymbol{v}_{\rm ph} = (1-\xi)\boldsymbol{v}_1 + \xi\boldsymbol{v}_2\,, \tag{2}$$

where

$$\xi = \frac{I_2}{I_1+I_2} = \frac{1}{1+10^{0.4\Delta m}} \tag{3}$$

is the fractional intensity of the secondary. I is the intensity for each component given by $I \propto 10^{-0.4M_V}$. For these systems, a single proper motion is derived from $\boldsymbol{v}_{\rm ph}$.

The resulting simulated astrometric data are subject to exactly the same maximum-likelihood estimation procedure as was used for the real cluster (Lindegren et al. 2000). In particular, astrometric radial velocities and kinematically improved parallaxes are derived for the individual stars or binaries. The error in the estimated parallaxes is in the range 0.2–1.0 mas (an error of 0.5 mas corresponds to approximately 1 pc in the cluster centre). The improved parallaxes are used to compute distances from the cluster centre, which allow to count the number of stars within a certain radius. Furthermore, for any subsample of the stars, the velocity dispersion can be estimated from the proper-motion residuals (Sect. 2).

3.5. Model fitting

In order to tune the model parameters, it is necessary to make several simulations for the same parameter values but using different initialisations of the random number generator. The average of the different random realisations is then compared with the observational data, and the input parameters adjusted accordingly. The quantities to be compared are the radial distribution of the stars, their total number above a given magnitude limit, and binary statistics. Also the number of giants (defined as $M_V < 1$ and $B-V > 0.5$) and the number of single white dwarfs are used to constrain the model.

The finally adopted (protocluster) model comprises 200 single stars and 1200 binaries, i.e. an initial multiplicity of 0.86 companions per primary. The total initial mass is 1100–1200 $M_\odot$. This is slightly less than previous estimates of 1200–1500 $M_\odot$ (Reid 1993) or 1300 $M_\odot$ (Kroupa 1995c). This smaller initial mass was found necessary in order to correctly reproduce the number of observed stars with the given IMF. The true initial mass of the Hyades is probably higher due to non-modelled mass loss (Sect. 5). According to the simulations, the total current mass of the Hyades stars is $\simeq$460 $M_\odot$ with a tidal radius of $\simeq$11 pc. Observationally, Reid (1992) made the estimation 410–480 $M_\odot$ while Perryman et al. (1998) estimated 400 $M_\odot$ in their Hipparcos study of the cluster.

An example of the observational HR diagram for one of the model realisations is shown in Fig. 1, together with the corresponding observed diagram for the Hyades cluster. In addition to the standard deviation introduced in V, a standard deviation of 0.01 in $B-V$ is also introduced in the model HR diagram to make the colour distribution appear more realistic. This standard deviation includes both observational errors and the effects of peculiar stars, stellar rotation, etc. Apart from the previously mentioned possible discrepancy in the cool end of the main sequence, and the fact that the giant stars are too red in the simulations, the general agreement is reasonable. The precise colours of the giants are, however, irrelevant in the context of this study.

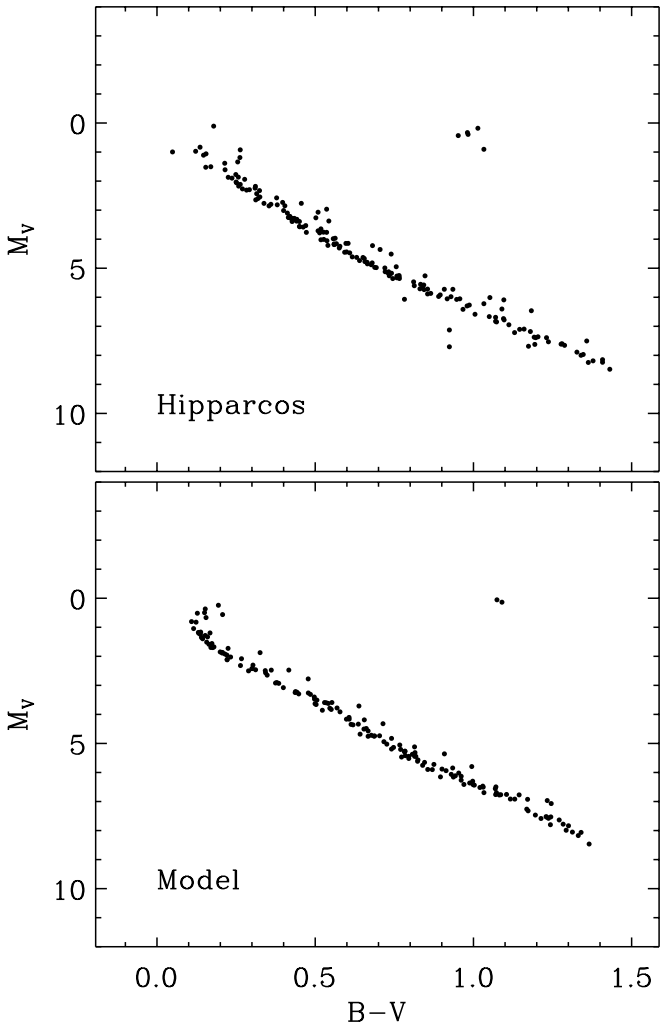

Fig. 1. The observational Hertzsprung-Russell diagram for the Hyades cluster, based on Hipparcos data (top), and for one of several realisations of the cluster model (bottom). In both cases the kinematically improved parallaxes ($\bar{\pi}$) are used.

Table 1 shows some statistics computed from this model, after evolution to an age of 625 Myr and transformation to the observables, together with the corresponding observed numbers. From Table 1 it appears that the distribution of stars with radial distance and apparent magnitude in the Hyades is well reproduced by the model cluster. The number of stars decreases when we go from the constraints based on the true parallaxes to the constraints based on the estimated parallaxes, and the number decreases even further when we use the observed parallaxes. This is a result of observational errors affecting the parallaxes, and mostly for the smallest sphere $r \leq 3$. In fact, the resemblence in the three columns is so good that it shows the modelling of the errors are in accordance with reality.

Table 1. The number of stars from Hipparcos ($N_{\rm HIP}$) and the mean of 20 model realisations ($N_{\rm model}$) for certain constraints based on distance from the cluster centre r and magnitude V. Numbers after ± show the dispersion among the 20 realisations. r is calculated using either observed, estimated and true parallaxes. The latter are, of course, not known for the real cluster. Note that the term "observed" in the table means both real and simulated observations. Giants are defined as stars with $B-V > 0.5$ and $M_V < 1$. Note that white dwarfs are too faint to appear in the Hipparcos observations of the Hyades, but since they are produced in the simulations, their number can be compared with the minimum number from other observations.

	observed parallaxes:		estimated parallaxes:		true parallaxes:
constraint	$N_{\rm HIP}$	$N_{\rm model}$	$N_{\rm HIP}$	$N_{\rm model}$	$N_{\rm model}$
$r \leq 20$ pc	173	166.7 ± 9.6	178	166.7 ± 9.7	167.6 ± 10.0
$r \leq 10$ pc	134	146.5 ± 9.3	143	149.5 ± 10.4	153.1 ± 9.9
$V \leq 8$ mag ($r \leq 10$ pc)	83	79.6 ± 6.1	88	81.3 ± 6.9	82.4 ± 6.5
$V \leq 7$ mag ($r \leq 10$ pc)	57	60.6 ± 6.1	58	61.8 ± 7.0	62.7 ± 6.4
$V \leq 8$ mag ($r \leq 3$ pc)	30	30.3 ± 5.8	38	37.0 ± 6.5	47.8 ± 8.4
$V \leq 7$ mag ($r \leq 3$ pc)	24	24.1 ± 4.4	29	30.2 ± 5.1	39.0 ± 6.3
Giants ($r \leq 20$ pc)	5	5.1 ± 2.0	5	5.1 ± 2.0	5.1 ± 2.0
single white dwarfs	≥7†				8.5 ± 2.7

† From Reid (1996).

Table 2. The number of Hyades binaries in the Hipparcos Catalogue ($N_{\rm HIP}$) compared with the number from the mean of several random realisations of the adopted cluster model ($N_{\rm model}$). The value after ± is the dispersion around the mean value among the different realisations.

Constraint	$N_{\rm HIP}$	$N_{\rm model}$
$r \leq 20$ pc:		
binaries, all	≥95	137.8 ± 8.8
binaries, $0.2 < \rho < 2''$, $\Delta m < 2$	17	13.7 ± 3.7
binaries, $0.1 < \rho < 1''$, $q > 0.4$	17	10.3 ± 3.2
binaries, $P < 10$ days	≥9	3.6 ± 1.6

The underabundance of stars in the models relative to the observations in the range $10 < r < 20$ pc can be explained by an overestimation of stars outside 10 pc by Perryman et al. (1998). They argued that using another centre of mass in the Hyades would lead to fewer stars in the halo (Sect. 3.2).

It has been much more difficult to reproduce the observed binary statistics (Table 2). Bright binaries with high mass ratio or small magnitude difference are underproduced. Even if every star in the protocluster were assumed to be a binary (multiplicity 1.0), the model would still predict too few binaries of these characteristics. The observed sample also has significantly more known short-period binaries ($P < 10$ days) than obtained in the simulations. These discrepancies indicate that the model distributions in mass ratio and/or semi-major axis would need some adjustment. Alternatively, a higher initial mass leading to more binaries with the required properties could be an explanation assuming non-modelled mass loss of preferentially low mass stars. However, the discrepancies are not dramatic and for the present study it was preferred not to change the relevant code in NBODY6.

Since the initial multiplicity must be very high to fit the observed binary statistics without being in contradiction with the observed number of Hyades member stars, the degree of degeneracy between the two free input parameters (initial particle number and initial multiplicity) is small.

The simulations could in principle be "inverted" to derive an age, by for instance stopping the modelling when the realisations appear similar to observed structural or dynamical features in the Hyades. But the non-modelled effects leading to mass loss during the dynamical evolution will be a major uncertainty (Sect. 5).

3.6. Observed kinematics versus simulated data

3.6.1. Dispersion versus cluster radius

In a Plummer potential, the velocity dispersion decreases with cluster radius according to Eq. (1). At some radius, however, the relation is expected to break down when the stars have left the cluster potential and become subject to the Galactic field. In the following this possible structure is investigated.

The various observed samples (Hy0r, Hy1r, Ty0r, Ty1r), as well as the different realisations of the adopted cluster model, are analysed by means of the maximum-likelihood method mentioned in Sect. 2. The samples are divided according to distance (r) from the cluster centroid in order to determine if there is a radial variation of the kinematics. The ranges in r have not been chosen at random: 3 pc is approximately the core radius while 10 pc is approximately the tidal radius. Table 3 summarises the results for the number N of stars (or systems) and the estimated velocity dispersion $\widehat{\sigma}_{\rm v}$.

The analysis method includes the rejection procedure designed to "clean" the cluster membership described in Sect. 2 with the goodness-of-fit statistic g_i calculated for each star. For the model simulations, no results are given for $g_{\rm lim} = \infty$ because of their sensitivity to run-away stars. In the observed

Table 3. The number of stars (N) and observed velocity dispersion $\widehat{\sigma}_{\rm v}$ in four intervals of distance r from the Hyades cluster centre, as estimated from the proper-motion residuals in the Hipparcos and Tycho-2 catalogues and the kinematically improved parallaxes. The Hy0r sample is the "full" sample with 178 stars within 20 pc radius. Hy1r is the same sample but with all known binaries removed. The Ty0r sample was created from Hy0r by replacing Hipparcos proper motions with Tycho-2 ones, where available. Ty1r is the same sample but with all known binaries removed. The last columns marked "Model" give the average number of stars and dispersions from 20 realisations of the adopted cluster model. $\widehat{\sigma}_{\rm v}$ is the dispersion estimated as for the real cluster, while $\overline{\sigma}_{\rm v}$ is the "true" dispersion in the model, calculated from the three-dimensional peculiar velocities relative the cluster centroid.

$g_{\rm lim}$	Hy0r		Hy1r		Ty0r		Ty1r		Model		
	N	$\widehat{\sigma}_{\rm v}$	N	$\widehat{\sigma}_{\rm v}$	N	$\widehat{\sigma}_{\rm v}$	N	$\widehat{\sigma}_{\rm v}$	$\langle N\rangle$	$\langle\widehat{\sigma}_{\rm v}\rangle$	$\langle\overline{\sigma}_{\rm v}\rangle$
						$r < 3$ pc:					
∞	55	0.70 ± 0.08	20	0.32 ± 0.08	60	0.39 ± 0.05	20	0.22 ± 0.08			
15	51	0.42 ± 0.06	21	0.30 ± 0.08	57	0.30 ± 0.04	21	0.20 ± 0.07	54.8	0.45 ± 0.07	0.33 ± 0.02
10	45	0.21 ± 0.05	20	0.26 ± 0.08	52	0.24 ± 0.04	18	0.22 ± 0.08	48.7	0.32 ± 0.05	0.32 ± 0.02
						$3 < r < 6$ pc:					
∞	56	0.83 ± 0.09	30	0.34 ± 0.08	58	0.39 ± 0.05	28	0.28 ± 0.07			
15	53	0.47 ± 0.06	27	0.33 ± 0.08	58	0.39 ± 0.05	27	0.28 ± 0.08	50.6	0.44 ± 0.10	0.28 ± 0.01
10	43	0.22 ± 0.05	25	0.28 ± 0.08	52	0.30 ± 0.05	28	0.29 ± 0.07	45.1	0.30 ± 0.06	0.28 ± 0.01
						$6 < r < 10$ pc:					
∞	31	0.86 ± 0.13	10	0.36 ± 0.12	23	0.46 ± 0.09	13	0.51 ± 0.12			
15	25	0.49 ± 0.09	10	0.36 ± 0.12	20	0.24 ± 0.07	12	0.37 ± 0.11	25.5	0.41 ± 0.10	0.25 ± 0.02
10	20	0.29 ± 0.09	11	0.34 ± 0.12	20	0.18 ± 0.07	9	0.20 ± 0.10	22.6	0.28 ± 0.07	0.25 ± 0.02
						$10 < r < 20$ pc:					
∞	35	1.26 ± 0.16	25	1.20 ± 0.18	34	1.23 ± 0.16	23	1.26 ± 0.19			
15	29	0.49 ± 0.09	21	0.40 ± 0.10	24	0.38 ± 0.08	16	0.31 ± 0.10	13.2	0.40 ± 0.13	0.26 ± 0.02
10	24	0.25 ± 0.07	18	0.33 ± 0.10	24	0.33 ± 0.08	17	0.29 ± 0.09	11.6	0.26 ± 0.10	0.27 ± 0.03

sample such cases were already removed by Perryman et al. (1998). It should be noted that the cleaning process successively reduces the estimated internal velocity dispersion, because the latter is based on the proper-motion residuals, which are also reflected in g_i. This is most clearly seen for the Hy0r sample at all radii, and for the other samples at $r > 10$ pc. The reason that there seems to be more stars for e.g. Ty1r at $g_{\rm lim} = 15$ than $g_{\rm lim} = \infty$ for certain ranges in r is that kinematically improved parallaxes have been used to calculate the distance from cluster centre. Since it is a different solution for each $g_{\rm lim}$, the kinematically improved parallaxes may change slightly.

Kinematically, one cannot in general distinguish between actual non-member stars and member stars with a deviating space motion. The most probable reason for a member star not to follow the common space motion of the cluster is that it is a binary in a non-modelled orbit. As explained in Sect. 3.2, this effect should be greater for the samples based on the Hipparcos proper motions than when using the Tycho-2 data. Comparing the results for Hy0r and Ty0r as function of $g_{\rm lim}$ suggests that binaries are the main cause for deviating proper motions out to $r \simeq 10$ pc, while for the greater radii they are partly caused by actual non-members.

The last two columns in Table 3 show the estimated and true dispersions from 20 realisations of the model. It appears that $g_{\rm lim} = 10$ yields a correct estimation of the dispersion, while $g_{\rm lim} = 15$ leads to an over-estimation of $\sigma_{\rm v}$. Using $g_{\rm lim} = 10$, the cluster as a whole (inside 20 pc) yields a dispersion of 0.23 ± 0.02 km s^{-1}, with no clear dependence on r. The model cluster yields a slightly larger value (0.30 km s^{-1}) and shows a 20% decrease from the centre outwards. It should be noted that two of the 20 models yield estimated values as small as the observations ($\leq$0.23 km s^{-1}). The dispersions $\overline{\sigma}_{\rm v}$ characterise the stars in the simulated Hyades sample, and not the total number of stars in the cluster. Due to the limiting magnitude, stars with masses less than 0.5–0.6 $M_\odot$ do not contribute to the velocity dispersions in the table, just as with the observations.

Madsen et al. (2001) found some rather large radial variations of the velocity dispersion in the Hyades, but could not conclude whether the structure was real or not. From the present simulations it is concluded that the observed structure is probably spurious: similar variations (of either sign) can be seen in some of the model realisations, although they are absent in the average of the realisations.

Hitherto in studies of open clusters, only in the Pleiades has an indication of a relationship between r and (the tangential component of) $\sigma_{\rm v}$ been found (van Leeuwen 1983). In the globular cluster M 15, however, a velocity dispersion decreasing from the centre out to 7 arcmin and then increasing was found by Drukier et al. (1998). They interpreted it as an indication of heating of the outer part of the cluster by the galactic tidal field. But how the minimum at 7 arcmin was related to the

tidal radius or other quantities remained unclear. Heggie (2001) argued that heating might be an incorrect interpretation since the effect can also be seen in N-body simulations of star clusters moving under influence of a steady tidal field (cf. Giersz & Heggie 1997). In the models here, the same trend is seen, with a minimum in the $r - \sigma_{\rm v}$ relation just inside 10 pc (the mean tidal radius of the models is between 10 and 11 pc).

3.6.2. Dispersion versus stellar mass

Theoretically we should also expect a decreasing velocity dispersion with higher mass, or correspondingly lower absolute magnitude, due to equipartition of kinetic energy. This should in turn lead to dynamical mass segregation, with the massive stars more concentrated to the centre of the cluster. This effect may have been seen in IC 2391 (Sagar & Bhatt 1989) and Praesepe (Holland et al. 2000). Perryman et al. (1998) found a clear mass segregation in the Hyades from the number density of stars in various mass groups as a function of distance from the centre. Direct searches by Lindegren et al. (2000) and Madsen et al. (2001) for a relation between the observed velocity dispersion and mass (or absolute magnitude), did however prove inconclusive. Evidence of any equipartition of kinetic energy is best sought among the stars in the core of the cluster (Inagaki & Saslaw 1985). For the present study, a limiting radius of 3 pc is therefore used. This is approximately the core radius of the Hyades.

In the Hipparcos Catalogue, often only the common absolute magnitude for a binary is available, and not the absolute magnitudes for both components. Since it is the mass that is interesting, only the samples without known binaries should be used, to ensure a reasonably unique correspondence between absolute magnitude and mass. In the simulated samples, binaries with a difference in absolute magnitude between the combined absolute magnitude of the two components in the binary and the primary component of more than 0.1 mag have been removed. This is the simplest way to simulate the hy1r sample.

The remaining stars with $r < 3$ pc in the hy1r sample are separated in four intervals of absolute magnitude, with divisions at $M_V = 2.1$, 3.4, and 5.4 mag, approximately corresponding to the masses 1.8, 1.4, and 1.0 $M_\odot$. The estimated dispersions in these intervals are 0.17 ± 0.13, 0.20 ± 0.11, 0.24 ± 0.11 km s^{-1}, and no solution for the last interval. The uncertainties are too large to allow any firm conclusion, although the expected trend is there. For comparison, the simulations gave an average dispersion going from 0.28 to 0.36 km s^{-1} in the same intervals.

3.6.3. Other determinations of the dispersion

Several studies of the velocity dispersion of the Hyades have been performed during the years. In a detailed discussion by Gunn et al. (1988), who performed a spectroscopic investigation of the cluster, a mean dispersion of 0.23 km s^{-1} was derived from a Plummer model. Their result agreed with the velocity dispersion obtained from the most precise spectroscopic radial velocities in their Hyades sample. However, it is important to note that the result of 0.23 km s^{-1} is dependent on the estimated M and $r_{\rm c}$, where the mass is the major uncertainty. Perryman et al. (1998) also used a Plummer model and got 0.21 km s^{-1} for the central velocity dispersion. Again this value was derived by estimating the mass and the core radius. Compared to this work the values are 50% lower, but can be explained by the uncertainty in the estimation of the masses. Makarov et al. (2000) used Tycho-2 proper motions to discuss the velocity dispersion of the Hyades, and found the velocity dispersion to be 0.32 km s^{-1} for the stars with the most precise proper motions. If known spectroscopic binaries were removed, the velocity dispersion decreased to 0.22 km s^{-1}. The last value agrees well with the value obtained with Tycho-2 proper motions in Table 3.

4. Accuracy of astrometric radial velocities

From the cluster simulations and subsequent application of the maximum-likelihood method (Sect. 2) the astrometric radial velocities are estimated for the individual stars (or systems), $\widehat{v}_{{\rm r}i}$. Of course, the true radial velocities $\overline{v}_{{\rm r}i}$ are also known directly from the simulation. Thus the estimation errors $\Delta_{ij} = \widehat{v}_{{\rm r}i} - \overline{v}_{{\rm r}i}$ are known. Here, index j is used to distinguish the different realisations of the cluster model. With $\langle\,\rangle_k$ denoting an average over index k, the following statistics are computed:

$$\Delta_j = \left\langle \Delta_{ij} \right\rangle_i \tag{4}$$

is the "cluster bias" in realisation j (i.e., the common error for all stars in the cluster);

$$\epsilon_{\rm int} = \left\langle (\Delta_{ij} - \Delta_j)^2 \right\rangle_{ij}^{1/2} \tag{5}$$

is the "internal standard error" of the astrometric radial velocities (i.e., the dispersion of the individual values around the cluster bias); and

$$\epsilon_{\rm tot} = \left\langle \Delta_{ij}^2 \right\rangle_{ij}^{1/2} \tag{6}$$

is the "total standard error" of the astrometric radial velocities (i.e., including the cluster bias). Clearly $\epsilon_{\rm int}$ is the relevant statistic for the precision of *relative* astrometric radial velocities within a given cluster, while $\epsilon_{\rm tot}$ is relevant for the accuracy of *absolute* astrometric radial velocities. Both $\epsilon_{\rm int}$ and $\epsilon_{\rm tot}$ can be computed for various subsets depending on observable quantities such as the goodness-of-fit measure g_i, radial distance r, and mass or absolute magnitude. An interesting question is whether it is possible to observationally define subsets with reduced $\epsilon_{\rm int}$ or $\epsilon_{\rm tot}$.

The results presented below are based on solutions using the rejection limit $g_{\rm lim} = 15$, although the results for $g_{\rm lim} = 10$ are very similar. Any conclusions from these simulations are also applicable to the astrometric radial velocities published in Madsen et al. (2002).

4.1. Standard errors versus goodness-of-fit

In Fig. 2 (top) the internal and total standard errors of the astrometric radial velocities are shown versus the goodness-of-fit g_i.

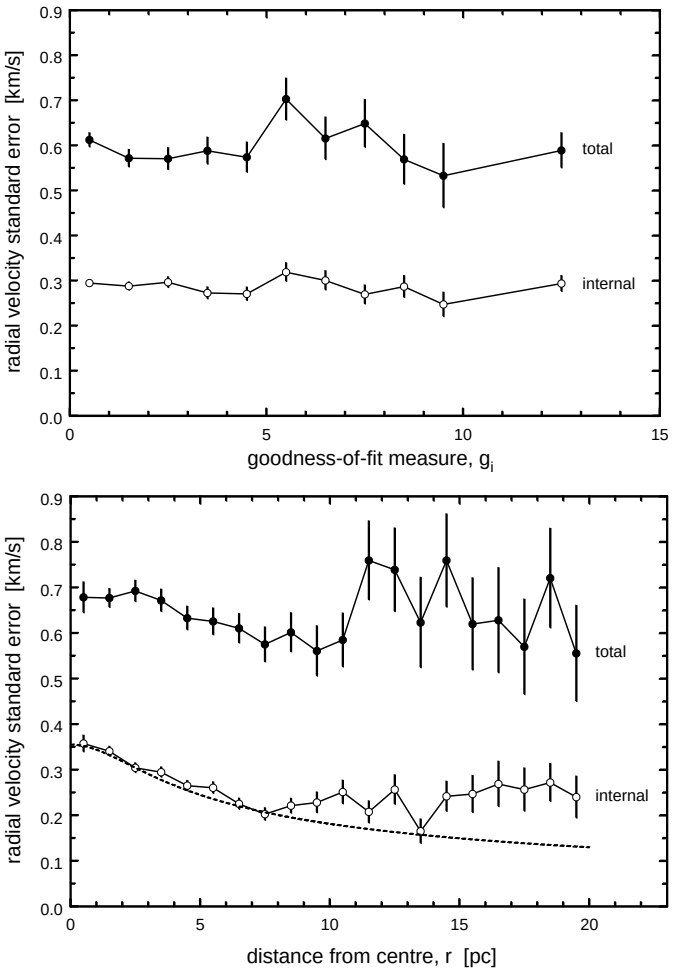

Fig. 2. Standard errors of the astrometric radial velocities as function of the goodness-of-fit measure g_i (top) and distance from the cluster centre r (bottom). Open circles show the internal standard errors $\epsilon_{\rm int}$ (i.e., for the relative velocities within the cluster); filled circles show the total standard errors $\epsilon_{\rm tot}$ (i.e., for the absolute velocities). The dashed line is the expected relation from the Plummer model.

The absence of any significant trend shows that g_i is not a useful criterion for selecting "good" astrometric radial velocities. Even stars with $g_i > 10$ are not worse than the rest in terms of radial-velocity precision. This somewhat counter-intuitive result can be understood if the line-of-sight component of the peculiar velocities is statistically independent of the tangential component. This is obviously the case for truly random motions, but one might expect that large proper-motion errors caused by photocentric motion in binaries should be correlated with large errors in the radial component.

4.2. Standard errors versus radius

The bottom part of Fig. 2 shows the internal and total standard errors of the astrometric radial velocities versus the distance r from the cluster centre. In this case the standard errors clearly decrease from the centre out to 7–8 pc radius, after which they seem to increase again.

The initial decrease (for $r < 8$ pc) is roughly in agreement with the Plummer model in Eq. (1) for $M \simeq 460\ M_\odot$ and $r_{\rm c} \simeq 2.7$ pc.

4.3. Standard errors versus mass and absolute magnitude

In Fig. 3, the internal standard errors of the astrometric radial velocities are plotted versus the true masses of the stars or systems (top) and versus the absolute magnitudes (bottom). The sample is divided at 3 pc (see Sect. 3.6.2). Inside 3 pc there is a clear difference in the velocity dispersion between the highest masses and 1 $M_\odot$, although not as much as for a full equipartition of kinetic energy ($\sigma_{\rm v} \propto M^{-1/2}$). The effect is much smaller outside of 3 pc. The velocity dispersion also seems to decline again for stars with masses less than 1 $M_\odot$.

The effect can still be seen when the dispersion is plotted versus absolute magnitude instead of mass (Fig. 3, bottom), although the trend is less clear because of the many binary systems, for which there is no unique correspondence between system mass and total luminosity.

Together with the results of the previous section we can conclude that the practical minimum for the internal error of the astrometric radial velocities in the Hyades is around 0.20 km s^{-1}, which is achieved for stars at an intermediate distance (≃2–3 core radii) from the cluster centre. At that distance there is little equipartition of kinetic energy, so it does not matter much if more or less massive stars are selected.

5. Non-modelled effects

The validity of the conclusions above depends critically on the realism of the N-body simulations. A number of non-modelled effects, and their possible impact on the results, are briefly considered below.

Time-dependent tidal field: When star clusters move through the galactic disk, they are subject to tidal shocks, and shock heating from the bulge. These effects are important to consider here since they increase the random motion of the stars. For globular clusters it has been found that tidal shocks accelerate significantly both core collapse and evaporation (Gnedin et al. 1999).

In the case of open clusters, Bergond et al. (2001) estimated that those with high-z oscillations lose some 10–20% of the mass integrated over the lifetime of the cluster, mainly in low-mass stars, through disk-shocking. The Hyades have a low vertical velocity ($W = 6$ km s^{-1} relative to the LSR), and therefore only oscillates with an amplitude of about 50 pc in z. Since this is small compared with the thickness of the disk, the disk-crossings should not cause much additional heating. The radial oscillations in the galactic plane, having an amplitude of 2 kpc, may be more important. The present N-body model assumes that the cluster moves in a circular galactic orbit. Thus it cannot be excluded that it underestimates the mass loss by perhaps some 5–10% of the initial mass. Preferentially, the lowest-mass stars leave the cluster, forming tidal tails (Combes et al. 1999). Although this would slightly affect the estimation

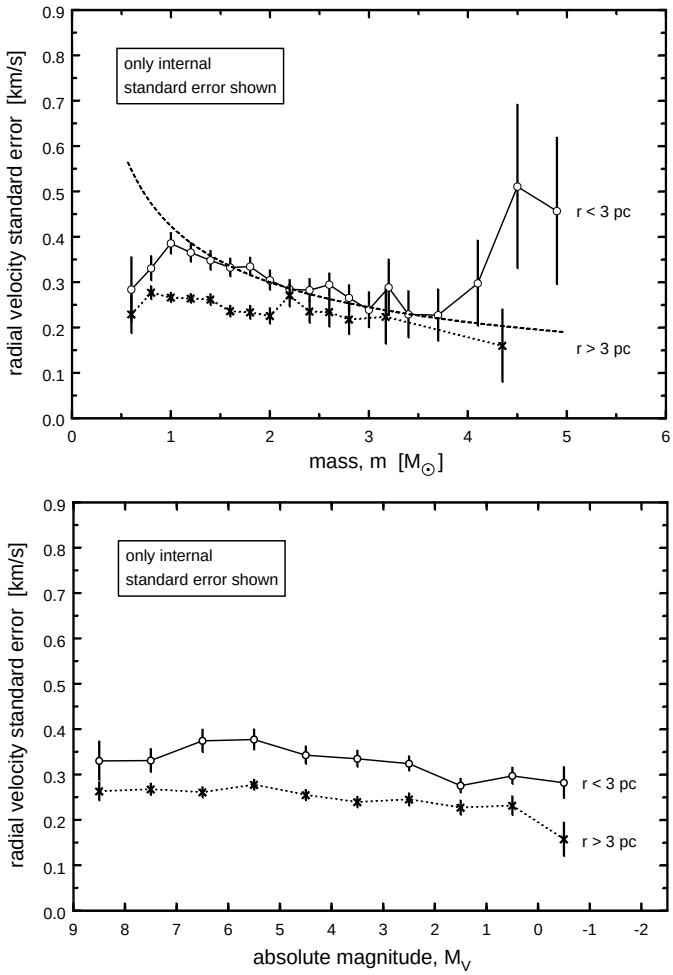

Fig. 3. The internal standard error of the astrometric radial velocities, $\epsilon_{\rm int}$, as function of stellar mass (top) and absolute magnitude (bottom). Circles refer to stars inside 3 pc of the cluster centre, crosses to those outside 3 pc. For binaries, m is the total mass of the system and M_V the total absolute magnitude. The dashed line is the curve $\sigma_{\rm v} \propto M^{-1/2}$.

of the velocity dispersion, it would have only a very small effect on the number of observed stars of spectral type earlier than M0.

Molecular clouds: Terlevich (1987) studied open cluster *N*-body models with initially 1000 particles and moving in a circular orbit at 10 kpc from the galactic centre (i.e., assumptions comparable with this work). She concluded that the timescale for encounters with giant molecular clouds is of the same order of magnitude as the present age of the Hyades. Since such an encounter would probably be catastrophic, it can be assumed that the Hyades have not been exposed to such a meeting. More abundant are encounters with smaller interstellar clouds. They will not shorten the lifetime of open clusters significantly but may contribute to the tidal heating of the outer regions in a given cluster. Wielen (1975) stated that gravitational shocks due to interstellar clouds will produce a significant flattening (up to 1:2) of the halo of the cluster perpendicular to the galactic plane. For the Hyades the flattening is 1:1.5 (Perryman et al. 1998). Since the galactic tidal field is also contributing to the flattening, it is doubtful if the Hyades have had any but minor interactions with interstellar clouds.

Perryman et al. (1998) examined the possibility that the Hyades recently experienced an encounter with a massive object causing a tidal shear in the outer regions of the cluster, but excluded it based on the impulsive approximation (Spitzer 1958; Binney & Tremaine 1987). Lindegren et al. (2000) included more velocity components in their model to test for non-isotropic dilation, and concluded that if such an effect existed it had to be higher than 0.01 km s^{-1} pc^{-1} to be detected with Hipparcos data. Effects from a tidal heating are thus not detectable in the Hyades with current astrometric precision.

Brown dwarfs: Despite extensive searches, no single-star brown dwarf (BD) candidate has been found in the Hyades (Reid & Hawley 1999; Gizis et al. 1999; Dobbie et al. 2002). Reid & Hawley (1999) found that the lowest-mass Hyades candidate star (LH 0418+13) has a mass of 0.083 $M_\odot$, placing it very close to the hydrogen-burning limit. The only promising candidate brown dwarf in the Hyades is the unresolved companion in the short-period system RHy403 (Reid & Mahoney 2000). Of course, the faintness of these substellar objects make them hard to observe, but still, the conclusion seems to be that the number today is quite small.

Adams et al. (2002) performed extensive simulations with a modified version of NBODY6 to model the brown dwarf population in open clusters, and concluded that the effects of different brown-dwarf populations were minimal, leaving the dynamics of the cluster largely unchanged.

The IMF in the version of NBODY6 used here cannot produce brown dwarfs, so this must be considered when defining the initial binary fraction. The IMF for brown dwarfs, or substellar masses, is very uncertain. Kroupa (2001) argues that a power-law value of $\alpha = 0.3 \pm 0.7$ is the most reasonable. Since stellar masses with $M < 0.08\ M_\odot$ are not produced in the code, one must represent the star–BD binary systems either as single stars or by overproducing binaries with secondary components slightly above the BD limit. Thus an initial binary fraction of 86% was assumed, which corresponds approximately to unity if brown dwarfs had been included. Based on the investigations of Adams et al. (2002), and considering that Hipparcos did not observe stars less massive than M0 stars in the Hyades, the above approximation should be sufficient for the present purpose.

Cluster rotation: Gunn et al. (1988) did a comprehensive study of the rotation of the Hyades, but had to conclude that it was at most of the same size as their statistical error. Nonetheless they stated that their results *suggested* a cluster rotation, but not higher than 0.015 km s^{-1} pc^{-1}.

Perryman et al. (1998) did a thorough study of the velocity residuals and concluded that they were consistent with a non-rotating system and the given observational errors. Lindegren et al. (2000) tested the Hyades for rotation by assuming solid-body rotation parameters, but found that it was too small to be detected, setting an upper limit of 0.01–0.02 km s^{-1} pc^{-1}. If this upper limit should equal the true rotation of the Hyades, then the effect is non-negligible at 10 pc compared to the internal error. But there seems to be nothing in the present study suggesting such a rotation.

But probably the solid-body assumption is too simple. In the globular cluster ω Centauri, Merritt et al. (1997) found that only at small radii could the rotation be approximated by a solid-body. Beyond that the rotation falls off. Einsel & Spurzem (1999) did theoretical investigations on the influence of rotation on the dynamical evolution of collisional stellar systems, that could explain the findings by Merritt et al. (1997). In fact, it seems that only inside the half-mass radius is it reasonable to talk about a solid-body rotation (cf. Kim et al. 2002).

Although it is unlikely that the cloud in which the Hyades formed had zero angular momentum, there currently exists no certain measure of the rotation. In the model, it is instead assumed that the effects are sufficiently small and can be ignored.

Expansion: During the evolution of a cluster parts of it expand and parts of it contract. Under the assumption that the relative expansion rate equals the inverse age of the cluster, Dravins et al. (1999b) estimated that an isotropic expansion of the Hyades would lead to a bias in the astrometric radial velocity of 0.07 km s^{-1} of the centroid velocity. This is completely negligible and any expansion effects have been ignored.

To summarise, it appears that none of these non-modelled effects would affect the results very significantly. While the modelling of tidal fields and brown dwarfs could be improved, the possible effect of cloud encounters remains an uncertainty which cannot easily be included in the modelling of a specific cluster such as the Hyades. Although NBODY6 allows encounters with interstellar clouds, the option has not been used in the present study.

6. Conclusions

A dynamical model of the Hyades cluster, based on N-body simulations using the NBODY6 code, has been fitted to the astrometric information available in the Hipparcos and Tycho-2 catalogues in order to study the accuracy of astrometric radial velocities. The number of stars as function of magnitude, their three-dimensional distribution, and the distribution of proper motions have been adequately reproduced by the model, as well as basic binary statistics. No spectroscopic radial velocities have been used in the present study (except for the initial membership determination by Perryman et al. 1998) meaning that the results should be directly comparable with the astrometrically determined radial velocities of Hyades stars by Lindegren et al. (2000) and Madsen et al. (2002).

From the simulations it is concluded that the velocity dispersion of the Hyades decreases from $\sigma_{\rm v} \simeq 0.35$ km s^{-1} at the centre of the cluster to nearly 0.2 km s^{-1} at 7–8 pc from the centre. Outside the tidal radius of 10–11 pc, the dispersion slightly increases again. Compared with previous studies of the velocity dispersion in the centre of the Hyades, the results here indicate a somewhat higher value.

The internal velocity dispersion contributes to the random errors of the astrometric radial velocities with the same magnitude. This is significantly less than the $\sigma_{\rm v} = 0.49$ km s^{-1} estimated in Madsen et al. (2002) directly from the Hipparcos observations. This discrepancy can be understood with reference to Table 3 as an *overestimation* from the observed data when the less strict rejection limit $g_{\rm lim} = 15$ was used. Thus the previous estimate of the internal standard error (due to the dispersion) can now be almost halved.

In fact, stars with an expected velocity dispersion as low as 0.20 km s^{-1} can be selected for studies that compare astrometric and spectroscopic radial velocities in order to disclose astrophysical phenomena causing spectroscopic line shifts. However, it should be remembered that the total standard error, including the uncertainty of the motion of the cluster centroid, is still of order 0.55–0.65 km s^{-1} (Fig. 2, bottom), in agreement with the previous estimate.

Attempts to see a radial dependence of the velocity dispersion with Hipparcos and Tycho-2 astrometry have been inconclusive. The observed relation is essentially flat for the most optimal sample. Given the uncertainty of the estimated velocity dispersions when the stars are divided into radial shells, this result is not surprising. Similar examples can be found in the simulations. Only when the mean relation is computed from several realisations of the cluster model do the variations become clear. In particular, it appears that the structure of dispersion/radius relation reported by Madsen et al. (2001) does not reflect typical dynamical properties of the cluster, but could result by chance or from some (unknown) mechanism related to the photocentric motions of undetected binaries.

The fit has yielded an estimate of the initial cluster mass of 1100–1200 $M_\odot$ and of the initial multiplicity, which appears to be very high (possibly near 100%, if brown-dwarf companions are included). The current cluster mass is estimated to be $\simeq$460 $M_\odot$ with a tidal radius of $\simeq$11 pc and a mean velocity dispersion within $r < 3$ pc of 0.32 km s^{-1}.

Some of the differences between observations and simulations could be due to some of the non-modelled features discussed in Sect. 5, which would lead to a higher initial particle number in the model and which might also solve some of the discrepancies noted in the binary statistics. The development of numerical tools such as NBODY6 to include e.g. a time-dependent tidal field would allow an improved realism of the Hyades model, and to study the effect on the accuracy of astrometric radial velocities from assumed negligible contributions to the velocity field with respect to the Hipparcos precisions.

The method used to estimate astrometric radial velocities discussed in Sect. 2 cannot eliminate of the error contribution from the internal dynamics of the cluster, no matter how precise the astrometry might be. The velocity dispersion therefore sets a fundamental limit on the accuracy of astrometric radial velocities, and as a consequence the results from the simulations presented here also apply to planned astrometric space missions such as GAIA (Perryman et al. 2001), even though it has been Hipparcos observations of the Hyades that have been simulated.

The Hipparcos and Tycho-2 catalogues contain the best available astrometry to study the internal velocity structure of the nearest open cluster, the Hyades. To study it in greater detail, even better astrometry is needed. The GAIA mission, in combination with improved N-body simulations, will make it possible to observe directly the internal velocity field of the Hyades, and give us insight in the kinematics of the Hyades in particular and open clusters in general.

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