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FINAL REPORT FOR PROJECT PROCESS
CONTROL 1970 - 1971
CONTRACT 70-337/U2 70 SWEDISH
BOARD FOR TECHNICAL DEVELOPMENT.

K. J. ÅSTRÖM

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LUND INSTITUTE OF TECHNOLOGY
DIVISION OF AUTOMATIC CONTROL

FINAL REPORT FOR PROJECT PROCESS CONTROL 1970 - 1971.
CONTRACT 70-337/U270 SWEDISH BOARD FOR TECHNICAL DEVELOPMENT

K.J. Åström

ABSTRACT

This report surveys the results of the process control project at the Division of Automatic Control at Lund Institute of Technology. The project covers system identification, adaptive control, computational control, real time computing, system theory, applications and technology assessment. During the year 1 book, 10 papers, 20 technical reports and 14 MS-theses have been published. These are reviewed in this report.

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1. INTRODUCTION

The project is part of a long range program in the area of process control [1]. Earlier results are reported in [2], [3] and [4]. A detailed program for the activity during 1970/71 is presented in [5].

All research at the Division of Automatic Control has been centered around the process control project. Contributions have been given by four research engineers supported by the Swedish Board for Technical Development (STU) as well as students and staff of the institute. The work has been organized so that three research engineers have acted as project leaders in the areas System Identification, Computational Control and Real Time Computing. One research engineer has provided programming support.

This plan has been followed in major respects. The thermal boiler project has thus been completed during this year. An unexpected result was that the insight obtained when making the linear models made it possible to derive simple nonlinear models that are valid over a wide operating range. Two major events have happened during the year.

Installation of a process computer.

Initiation of a PhD program.

The installation of the process computer was preceded by an extensive evaluation. The computer was installed on July 1st. The installation, the design of interface and experimental processes went smoothly and the computer has been running for about 3000 hours during the year. The access to the real time computer made it possible to initiate the new research program real time computing as was anticipated in the research program. The decision to start a graduate program has considerably broadened the base for

research. As part of their education most PhD students have participated in the project real time computing. A graduate course on systems identification was also given. In this course the graduate students were using the identification programs on real industrial data. This gave a good opportunity to try different methods on real data. It has significantly contributed to our knowledge about system identification. It is expected that this type of interaction between research and education will be even more extensive in the future.

The initiative by STU to install a secretary for liason with industry (kontaktsekreterare) at the Universities has provided a new channel for exchange of problems and results. During this year we have arranged two courses in collaboration with the liason secretary. A one week course on Hardware and Software for Real Time Control and a one day course on Stochastic Control Theory. In collaboration with Svenska Teknologföreningen, we have also run a one week course "Reglerteorier i omdaning" (New Concepts in Control Theory).

2. SYSTEM IDENTIFICATION

Process identification has been one of our major activities. The work is concentrated on off-line identification of both single input single output and multivariable systems, and on real time identification. Particular attention has been given to multivariable systems. Further computational experiences on industrial data have been gained. Programs for identification are implemented on the process computer. A first version of an Automatic identifier, is described in

Jönsson B., Konversativt datorprogram för processidentifiering (Computer Program for Automatic Process Identification). RE-91, Thesis report. February 1971.

Special problems like choice of sampling interval and input signals have been studied. Convergence properties of generalized least squares methods have been investigated. It has been shown that several solutions may exist. Identification of processes using a priori information on structure and parameter values has shown to be a promising tool to identify industrial processes. A survey of the status of the identification field has been given in

Åström K.J. and Eykhoff P., System Identification - A Survey. Automatica, Vol.7, pp. 123-162 (1971).

Off-line identification of single-input single-output systems has been extensively studied. In particular data from many different industrial processes have been investigated. Several methods have been used. Therefore a great deal of experience of identification of real data has been gained.

Linear models of such processes can easily be derived from experimental data by using available identification and analysis programs. However, in order to plan identification experiments and choose suitable models it seems to be of great value to have at least some experience of identification of real data. No such knowledge is yet collected and published anywhere.

Together with the well-known maximum likelihood method, methods using Fast Fourier Transform technique estimating impulse response and frequency response models have been applied to different data sets. Another method, frequently used, is a method estimating parameters of the state space model using a priori knowledge of the process. This means that the structure is assumed known together with some parameter values. The method is shortly described here and can directly be extended to identification of multivariable and even nonlinear systems. For these cases specific assumptions concerning the model structure are necessary.

Identification of state space models

Identification of the state space model is performed in the following way. Assume that the system equations are known except for a number of unknown parameters, α . Let us use the state space equations

$$\begin{aligned}\dot{x} &= f(x, u, \alpha, t) \\ y_m &= g(x, u, \alpha, t)\end{aligned}$$

as a model for the process. The parameters α are determined so that

$$V(y, y_m) = V(\alpha) = \int_0^T [y(t) - y_m(t, \alpha)]^2 dF(t)$$

is minimized. y is the process output and y_m the model output. With specific assumptions on the disturbances of the process and the assumption that the model has the correct structure, the estimate can be interpreted as a maximum likelihood estimate of α . Anyhow, it can always be interpreted as a minimization of the one-step-ahead prediction error. An example is shown in Fig.2.1.

A major problem for this identification technique is the minimization of the function, $V(\alpha)$, which most often is a function of several variables. It is difficult and time-consuming to compute gradients and second derivatives of the function. Therefore minimization algorithms using only function values have been used for most cases. However, the available algorithms have turned out not to have quite satisfactory convergence properties. It should be emphasized that for multivariable systems this method often leads to quite many unknown parameters. To make this identification method more feasible for practical purposes, more effective minimization algorithms must be developed.

Fast Fourier Transform

Fast Fourier Transform Technique has been used to estimate the weighting coefficients for a system. This method can be a useful tool for preliminary investigations to detect time lags and hidden feedback. An estimate of the error is also provided. The error depends on the number of data, the decreasing rate of the impulse response and on the power spectrum of the input.

Choice of input

The choice of input signal for spectral analysis is rather well understood. The connection between identification by such methods and by parametric methods on the other side may lead to a better understanding of how to choose input signals

for experiments intended for parametric identification. Comparison between different ways of performing spectral analysis showed that conventional spectral analysis with e.g. a spectral window of Hamming-type performed very well compared to methods using FFT, both in time and accuracy.

Choice of sampling interval

The problem of choosing sampling interval for identification has been discussed in

Gustavsson, I., Choice of Sampling Interval for Parametric Identification. Report 7103, April 1971. Division of Automatic Control, Lund Institute of Technology, Lund.

Specific assumptions on input signal, disturbances, criterion of optimality and the system structure led to optimum sampling rates or at least showed that a too fast sampling often is not worthwhile. A useful rule of thumb is to choose the sampling interval, ΔT according to

$$\Delta T = \min \{0.5T_{\min}, 0.5/\omega_r\}$$

where T_{\min} is the shortest time constant of the system and ω_r is the highest resonant frequency of the process. The choice of sampling interval is closely related to how a specific process can be modelled from measurements. Modes close together can often not be separated. Modes that are fast compared to the sampling rate can not be detected and slow modes will be treated as integrators and can hardly be separated from each other.

Generalized Least Squares

Convergence properties of generalized least squares estimates have been studied. Filtering of original data by an iteratively determined filter of fixed order is equivalent to maximizing the likelihood function for the problem. Asymptotically holds for small disturbances that a unique local maximum exists. For large disturbances on the other hand most often several local maxima exist.

Recursive algorithms

Preliminary comparisons between some recursive identification algorithms showed that the instrumental variables method was the best one for processes with correlated noise among the compared methods.

Real time identification

An algorithm for real time identification i.e. identification of processes with time varying parameters has been discussed in

Wieslander J., Real time identification Part I, Report 6908, November 1969. Division of Automatic Control, Lund Institute of Technology, Lund.

Lindeberg L., Kalmanfilter. Adaptiva Kalmanfilter. Reelltidsidentifiering (Kalman Filters. Adaptive Kalman Filters. Real Time Identification), Thesis report. July 1970.

Knutsson G.I., Jämförelse mellan tre olika metoder för reelltidsidentifiering (Comparison between Three Different Real Time Identification Methods), Thesis report. August 1970.

This algorithm has been implemented on the PDP-15 process computer at the institute. The code is fairly short (220 cells) and the execution time for a second order model is 23 ms. This algorithm has e.g. been used in the so called 'Automatic Identifier' program to produce parameter estimates on-line during the data acquisition. It has also been used together with a minimum variance regulator to form an adaptive controller. This is currently used in simulations of an auto-pilot. Adaptive controllers of this kind have been discussed in

Wieslander J. and Wittenmark B., An Approach
to Adaptive Control Using Real Time
Identification. Automatica, Vol 7.
(1971).

The real time identification algorithm requires some knowledge of the rate of change in the process parameters. This knowledge is generally not available. An algorithm as well as an off-line program to estimate such information is available and is given in

Wieslander J., Real Time Identification Part II.
Report 7111, September 1971. Division of
Automatic Control, Lund Institute of Tech-
nology, Lund.

Distributed parameter systems

Identification of systems governed by partial differential equations has been performed. A one dimensional thermal diffusion process with two inputs and seven outputs has been used as experimental object. The process has been modelled by linear models of ordinary differential or difference equations of finite order. It turns out that rather low order models, e.g. 5th order difference equations, were sufficient to describe the dynamics very well when the samp-

ling intervall was chosen 1/18 of the longest time constant. This could be a reasonable choice in practice. However, the order of the model will vary with the sampling rate.

Multivariable systems

Multivariable systems have been identified by estimating parameters of state space models. Processes investigated are e.g. a boiler, a mixer settler and the heat diffusion process. Other methods have also been used, e.g.

Valis J., Identification of Multivariable Linear Systems of Unknown Structure by the Prior Knowledge Fitting Method. Report 7005, Sept. 1970. Division of Automatic Control, Lund Institute of Technology, Lund.

Contacts with industry

Extensive contacts with industry have resulted in a large number of available sets of industrial data. Also collaboration in form of planning suitable identification experiments has taken place. Among the suppliers of data we only mention AB Isotopteknik, Sydsvenska Kraft AB, OECD Halden Reactor Project, AB Alfa-Laval and SAAB. Contacts have been established with other divisions of Lund Institute of Technology, e.g. Chemical Engineering and Architecture as well as with researchers at the Lund University Hospital. The collaboration with the Division of Architecture has resulted in two master theses.

Jensen L., Mätningar på luftkonditioneringsanläggning med återblandning. Identifiering av delsystem (Measurements on an Air Conditioning Plant with Recirculation. Identification of a Sub-System). RE-92, Thesis report. March 1971.

Frick B., Rumsmätningar och identifiering av rummets dynamik med maximum likelihoodmetodik (Temperature Measurements in a Room and Identification of the Dynamics of the Room Using the Maximum Likelihood Method). RE-89, Thesis report. November 1970.

It also initiated a new research project, control of interior climate, started in July 1970.

Heat exchanger dynamics have been studied in collaboration with Alfa-Laval AB. The results are given in

Lundström A. and Nilsson C., Undersökning av en plattvärmeväxlares dynamik: mätningar och identifiering (Investigation of the Dynamics of a Plate Heat Exchanger: Measurements and Identification). RE-90, Thesis report. February 1971.

The experiments performed at the thermal power station, Öresundsverket in Malmö, have also been analysed. A PhD thesis based on the analysis of this data is almost completed.

At Halden two new identification experiment series have been performed which were planned in collaboration with us. The data has not yet been analysed in detail. At the Division of Chemical Engineering experiments with a mixer settler have been performed. The data have been analysed with several methods, and the obtained models provide information of the dynamics of the process.

Miscellaneous

In connection with a course in identification for PhD students small projects in this area were carried out by the students. Some interesting problems were studied. E.g. it was clearly demonstrated that identification of systems with feedback,

using natural disturbances only, may need a tremendous amount of data to be accurate. A use of a priori knowledge of the structure at the process turned out to be very valuable in this case. The bias of least squares estimates, caused by measurement noise only, was also studied. Modelling of impulse responses for processes with recirculation was tried with parametric models. Unless special structure was assumed the result was rather high order models. It was encouraging to see that with the experience and the programming tools that have been accumulated and developed even unexperienced students could obtain good results in reasonable time.

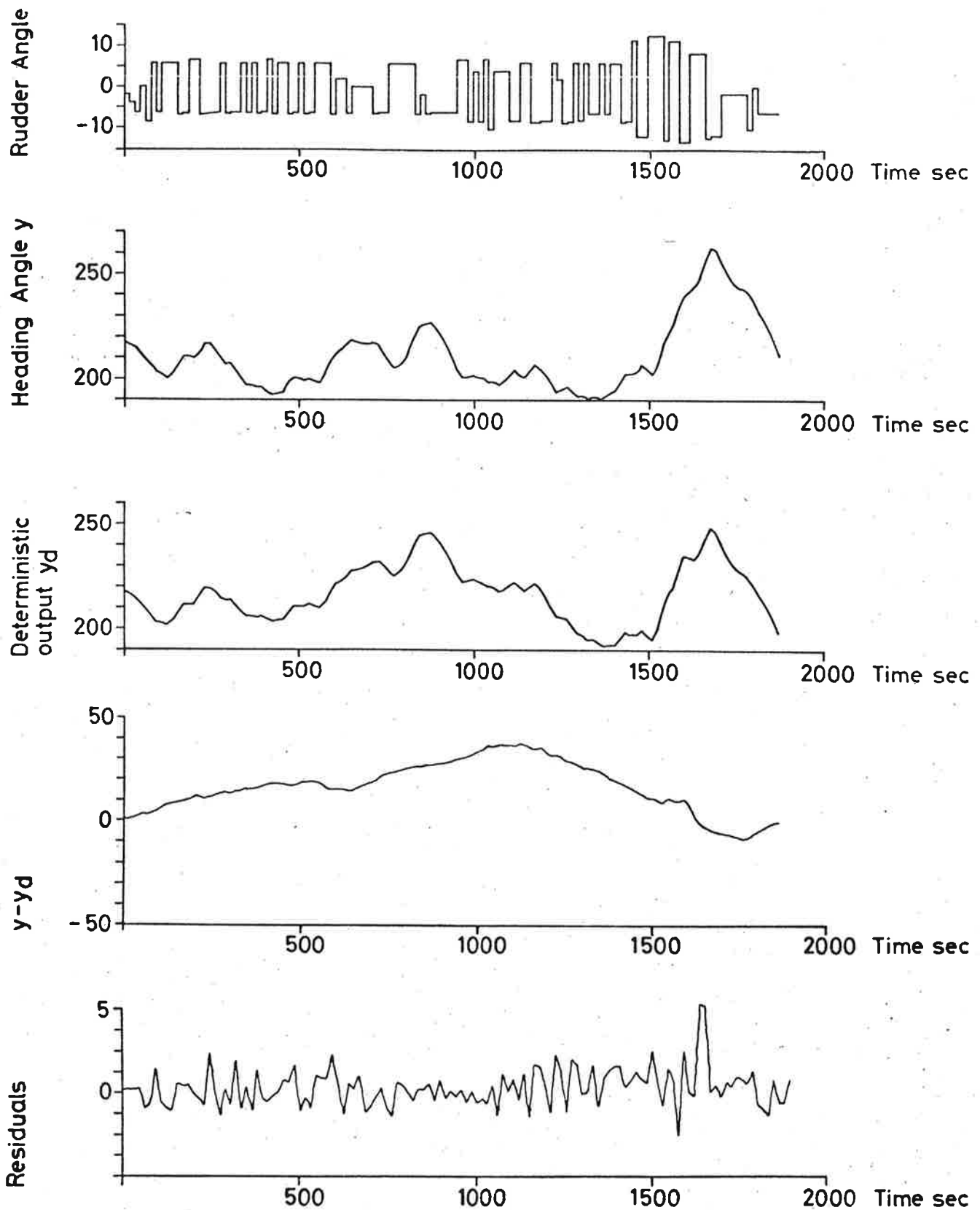


Fig.2.1. Identification of ship dynamics using the state space approach. In the figure the input, the rudder angle, and the output, the heading angle, are shown. In the lower part the results obtained for a third order model are given.

3. ADAPTIVE CONTROL

The research on adaptive control has been conducted along the lines given in the research program [1], i.e.,

Analysis of special adaptive systems.

Exploitation of real time identification algorithms.

Use of optimal stochastic control theory.

Special Adaptive Systems

A survey of existing adaptive control systems is given in the report

Wittenmark B., A Survey of Adaptive Control Methods. Report 7110, Sept. 1971. Div. of Aut. Control, Lund Inst. of Technology, Lund.

In this report particular attention has been given to the construction of examples which illustrate situations when it is preferable to use adaptive regulators. The examples are also useful for the purpose of testing different adaptive algorithms.

During the year 1970/71 we also had the pleasure to have professor A. Pearson as visiting professor for a period of 2 months. This significantly contributed to make us aware of the results obtained by using deterministic approaches to adaptive systems. It also contributed significantly to our understanding of the adaptive autopilot problem.

The phenomenon of 'turn off' reported earlier has been investigated further. The details are given in

Wittenmark B., On the Turn off Phenomenon in Adaptive Control. Report 7105, Sept. 1971. Div. of Aut. Control, Lund Inst. of Technology, Lund.

It turns out that the phenomenon can be explained through analysis of the linearized equations. The analysis results in a study of a linear stochastic difference equation with random coefficients.

Real time identification

A straightforward approach to the adaptive control is to assume separation of identification and control. Particularly simple algorithms are obtained if a least squares structure is assumed. Results obtained using this approach were presented at the 1970 IFAC Symposium on System Identification. The results have now been polished and published in

Wieslander J. and Wittenmark B., "An approach to Adaptive Control using Real time Identification" Automatica, Vol. 7, (1971).

Stochastic Control Theory

It is well-known that an adaptive problem can be formulated using stochastic control theory. This approach leads to a separation theorem where the optimal regulator consists of a nonlinear filter and a nonlinear regulator. It is also well-known that the regulator is highly unpractical since the state is of extremely high dimension (conditional probability distributions of all parameters and states). It is therefore of interest to investigate special cases where the conditional probability distributions can be characterized by few parameters. Linear systems whose parameters are gaussian random processes is such a special case. This includes also the practically important case of systems with constant but unknown parameters with a least squares structure. Unfortunately it does not include the models used to derive the minimum variance strategies because the disturbances are correlated. An analysis of the special case is presented in

Åström K.J. and Wittenmark B., "Problems of Identification and Control". J.Math. Analysis and Applications, 34, 90-113. (1971).

The analysis gives considerable insight into the connections between identification and control. In particular it is shown that if reasonably good initial parameter estimates can be obtained the simple-minded approach of least squares identification in combination with a separation assumption can give good results for the control of constant but unknown systems. It is also shown that such an approach might fail badly if the parameters are changing rapidly e.g. if the gain can change sign.

Systems with constant but unknown parameters

The control of systems with constant but unknown parameters is a practically important case. It corresponds e.g. to the problem of finding self-tuning regulators or the problem of automatic tuning which has a great practical relevance in process control. A break through in this problem has been made. It has been shown that the simple-minded approach for a model with least squares structure can be modified slightly to include also the case of correlated disturbances. This means for example that it is possible to design self-tuning minimum variance regulators. The convergence of the algorithms has not been shown. Under the assumption that the algorithm converges a complete characterization of the system obtained has been given. It has been shown that under reasonable assumptions the algorithm will converge to minimum variance strategies if it converges at all. Convergence has been shown by simulation. A simple case is shown in Fig. 3.1. A paper giving the main results has been submitted to the 4th IFAC Congress.

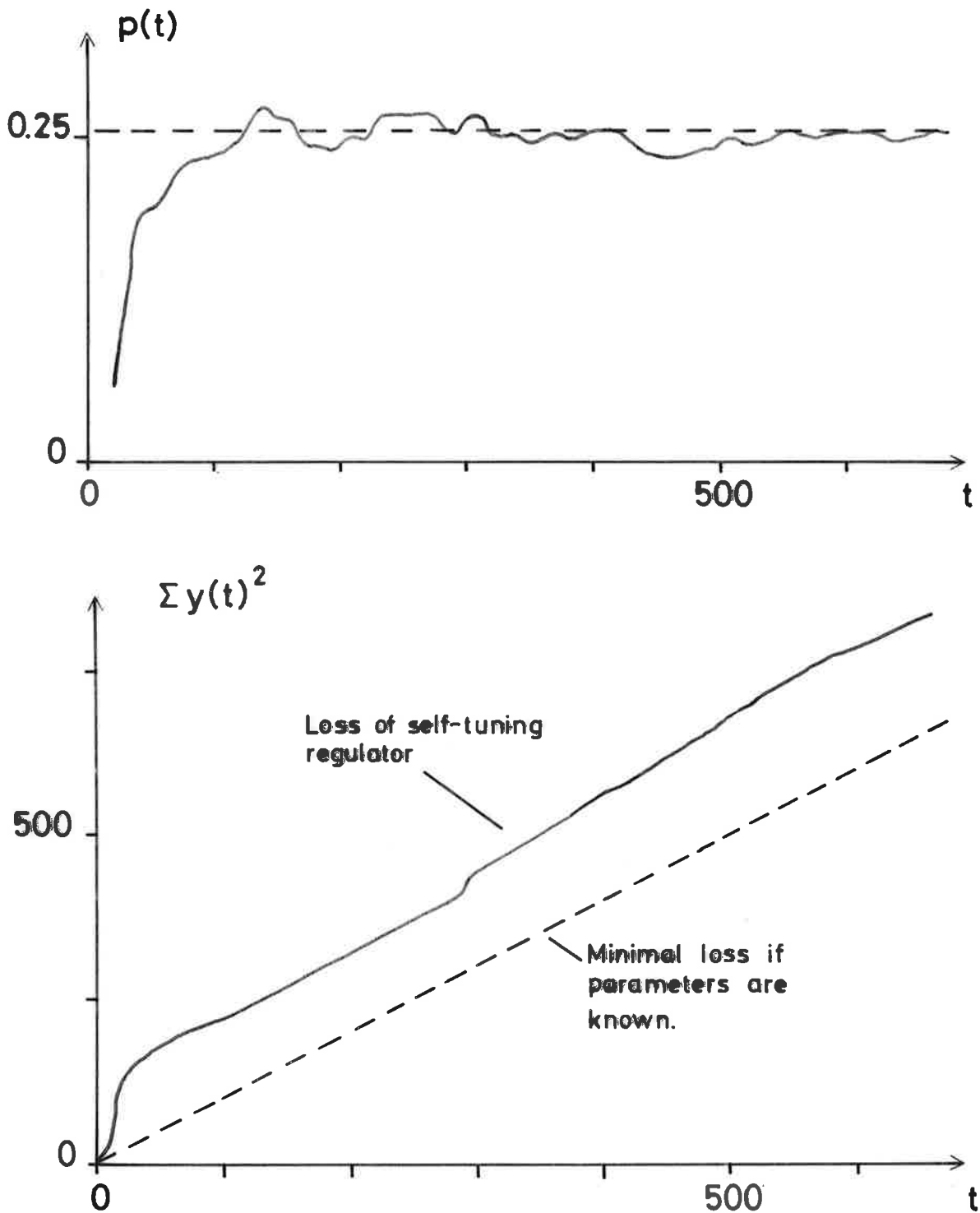


Fig. 3.1

Illustrates the self-tuning algorithm. The system $y(t) - 0.5y(t-1) = 5u(t-1) + e(t) + 0.8e(t-1)$ is controlled using the regulator $u(t) = -p(t)y(t)$. The minimal variance regulator is $u(t) = -0.26y(t)$. The upper curve shows how the gain adjusts to the optimal value and the lower curve shows the accumulated loss.

The algorithms can be conveniently implemented on a process computer. Simple versions require about 30 FORTRAN statements and the corresponding code requires about 450 memory locations. A significant effort will be required to clean up all details, in particular in order to obtain algorithms which cover the practical constraints.

An attempt to use these ideas in a practical case is described in

Mårtensson I. and Weibull A., "Styrning av robot med modifierad minimalvarians styrlag". (Control of a missile using a modified minimum variance regulator). Thesis report, RE-88. November, 1970.

A study has also been initiated to use similar strategies on the design challenge for an adaptive autopilot which was given at the Joint Automatic Control Conference, Atlanta, Georgia, 1970.

Learning Systems

An exploratory study of principles for learning systems has also been initiated as an independent study project for a PhD student.

4. COMPUTATIONAL CONTROL

The research work has been concentrated on two main activities. One is trying to make optimal control theory a useful tool for designing control systems, and involves development of computational methods based on existing algorithms. The other main activity is trying to find new theoretical approaches to the optimal control problem, in order to develop more efficient algorithms and numerical methods. This domain also includes further development of the general purpose program library.

The program library, originally developed for the computer CDC 3600 in Uppsala, has now been transferred to the computer UNIVAC 1108 at Lund University Computing Centre. Updating of programs and development of new ones, has been continuously going on during the year.

A great part of the program library has also been transferred to the process computer PDP-15/35 at the institute. Because of the differences between a medium size process computer as PDP-15/35 and a typical off-line computer as UNIVAC 1108, the program library for PDP-15 has been modified to be efficient on a medium size process computer. Several subroutines for real time control of processes have also been included in the PDP-15 program library.

Both the UNIVAC 1108 program library and the PDP-15 program library can now be considered to be of such extent that they very efficiently facilitate the work in the different research areas. A paper

Hagander P., Numerical Solution of $A^T S + SA + Q = 0$,
Report 6920, October 1969. Division of Automatic
Control, Lund Institute of Technology, Lund.

has been written and accepted for publication in the Information Sciences.

Optimal Control of Linear Systems

The research has been directed towards problems connected with the implementation of optimal linear feedbacks.

One of the problems that arise when the full optimal solution is implemented, is the necessity to measure all the state variables of the system. A method to compute linear feedback laws when just some of the state variables can be measured, is described in the report

Mårtensson K., Suboptimal Linear Regulators for Linear Systems with Known Initial State Statistics, Report 7004, July 1970. The Division of Automatic Control, Lund Inst. of Technology, Lund.

The method is based upon the assumption that the initial state of the system is a normally distributed random variable with known mean value and known covariance. When the measurable states have been specified, the algorithm computes the linear feedback which minimizes the expected value of a quadratic lossfunction.

The method has been successfully applied to in

Olsson K-E. and Åkerlund M., Tillämpning av linjärkvadratisk teori på kraftsystem. (Application of Linear-Quadratic Theory to Power Systems). Thesis Report, RE-94, April 1971.

where the regulation of a synchronous power generator connected to an infinite bus is studied.

Optimal Control of Non-linear Systems

The development of computational methods for optimal control of non-linear systems, has constituted a great part of the activity. Different possibilities to apply optimal control theory to a specific industrial process have been investigated in cooperation with the Swedish company ASEA. The process is a container terminal illustrated in Fig.4.1.

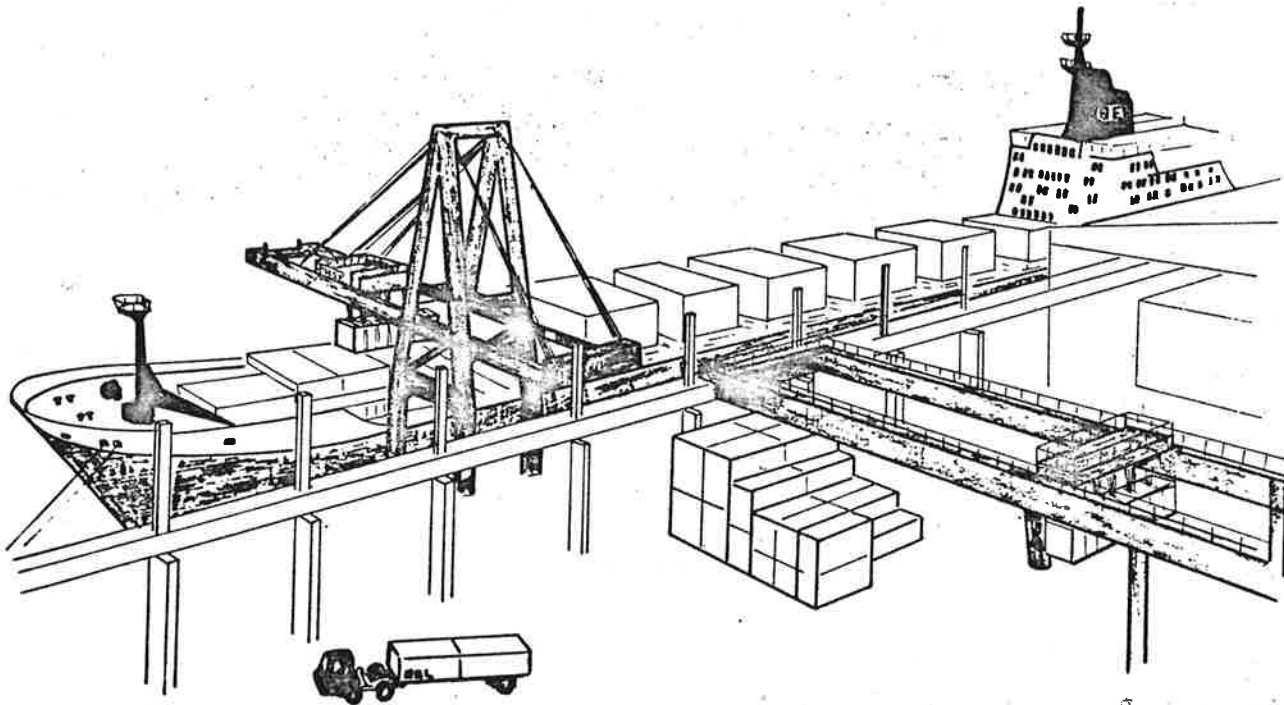


Fig. 4.1

When unloading the ship, the time-consuming elements are the crane operations from the boat to the truck and from the truck to the storage area. The following part of the process has then investigated. What crane strategy should be used to take the container from the truck to a predetermined storage position in minimum time, and is it possible to compute these control strategies in real time?

Fig. 4.2.

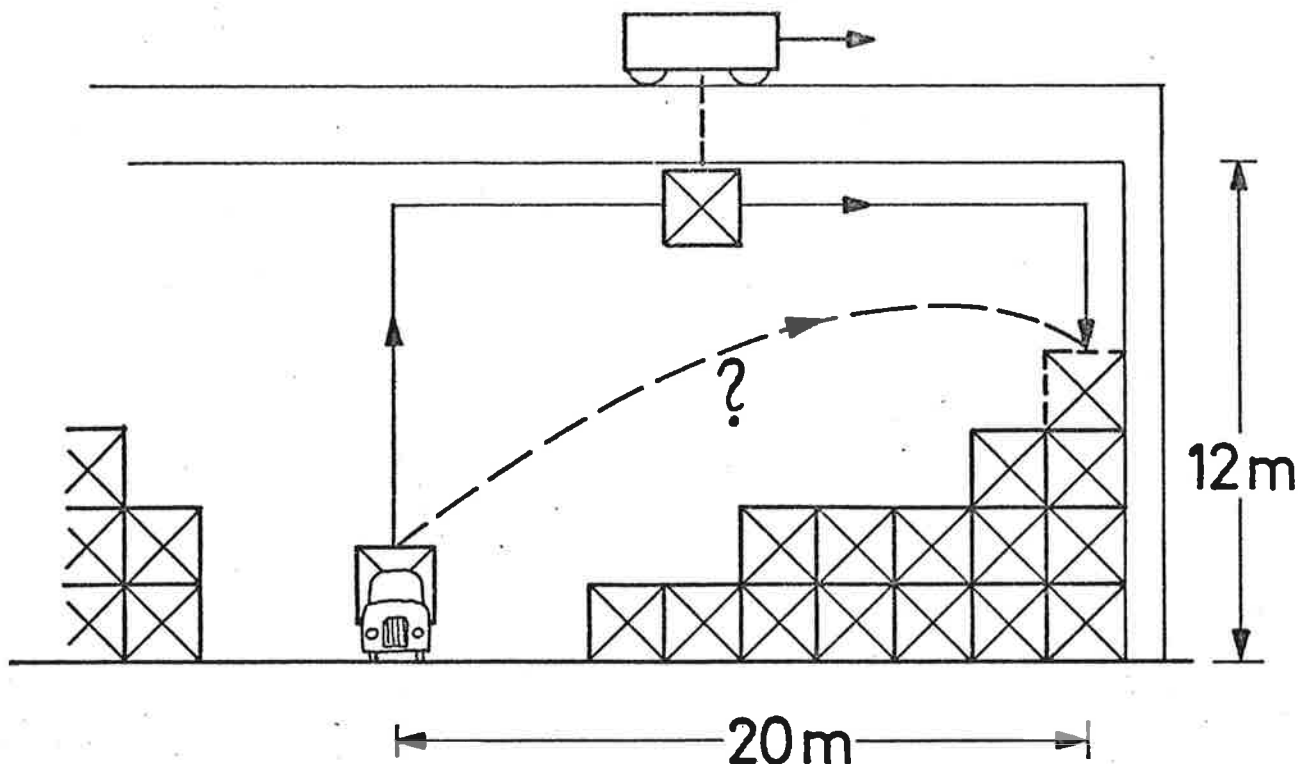


Fig 4.2.

A non-linear sixth order mathematical model of the process has been derived out of basic physical relations. The control variables are the accelerations of the traversing motor and of the winding motor, and they are limited for constructional reasons. Further, the traversing velocity and the winding velocity are constrained, and also the geometric profile of the stored containers must be taken into account. Summarizing, the problem thus constitutes a rather complex optimal control problem, but the complexity could probably be considered as typical for real industrial processes.

To compute the optimal strategies, it was found necessary to develop new computational methods. These will be presented in forthcoming reports, as well as the overall solution of the container handling problem.

New Methods for Constrained Function Optimization

Different possibilities to solve ordinary function optimization problems have been investigated. There are two main reasons why function optimization is of interest in optimal control theory. One reason is that the computational program

often requires a function optimization subprogram, e.g. to carry out the minimization of the Hamiltonian. Another reason is the analogy between function optimization and dynamic optimization. Since dynamic optimization can be considered as a generalization to a more general space, it is of great importance to investigate different possibilities for ordinary function optimization. A new approach to the constrained function optimization problem is presented in

Mårtensson K., Methods for Constrained Function Minimization, Report 7101, March 1971.
Division of Automatic Control, Lund
Institute of Technology, Lund.

Further work will consider the possibility to generalize these results to the dynamic optimization problem, and thereby derive new computational algorithms.

5. REAL TIME COMPUTING

Lack of understanding of real time computing processes is a factor which contributes significantly to the cost of a process control system. The purpose of the project is to develop an understanding of real time computing processes. Another purpose of the project is also to provide graduate students with a feeling for practical problems by doing real experiments. During this year a working laboratory was built up both with respect of hardware and software.

Hardware

The real time computing laboratory is based on the following equipment.

PDP 15/35 16 k core 256 k disk etc.
 Interface 16A/D 4 D/A 18 logical in/out etc.
 Line Printer
 Display Oscilloscope (Tektronix 611)
 Inkjet plotter
 Thermal diffusion process
 Mechanical device (ball balancing)
 (Servo mechanism)
 (Small analog computer)

The PDP-15 was delivered in July 1970 and has since then been used extensively. (Approximately 3000 hours in the period July 1970 - June 1971). The interface containing analog input/output etc. was designed and built at the institute using pre-fabricated logic modules. It contains also interface - circuits for the line printer and the

display. The line printer is a second-hand unit that originally comes from the SMIL-computer in Lund. It has been repaired and interfaced to the PDP by the Division. The mechanical device consists of an aluminum bar on which a steel ball is rolling. It is possible to measure the position of the bar and the ball. The bar is rotated by a motor controlled from the computer. It is then possible to try different computer algorithm to control the ball position. The servo mechanism is in principle of the same nature but with different dynamic properties.

Software

The programming efforts have during the year been directed primarily towards the following areas.

Interface programs

Plotter routines for the display

Experiment start-up and documentation program

Mathematical routines

Regulators (PID, dead-beat, minimum variance)

Identification routines (real time identification, recursive least squares).

Kalman filter

Interactive programs

The interface programs are e.g. routine to handle the D/A and A/D converter. Device handlers for the line printer, display and inkjet plotter also belong to this area. A special kind of these programs are the FORTRAN-callable routines used to plot text and diagrams on the display. A special version of the text editing system program that uses the display has been designed.

An interactive program for start up and documentation of

experiments has been written. It performs also the data acquisition and data saving during the experiment. It has been used by all on-line experiments with regulators, Kalman filters etc.

In building a library of mathematical programs it was soon realized that if certain elementary computations was programmed in assembly level language a time saving of a factor 3 was obtained. All mathematical library programs for matrix handling, equation solving etc. are written in FORTRAN and use this assembly routine when applicable. Typical times are 220 ms for matrix multiplication of 10x10 matrices. Solving a 10:th order linear equation system takes 500 ms.

Routines for conversion from continuous to sampled system representation as well as routines for simulation of linear systems are also included. Some experiences from this work are available in

Hagander P., Källström C. and Åström K.J., Real Time Computing I - Implementing Linear Filters. Report 7106(B), May 1971. The Division of Automatic Control, Lund Institute of Technology, Lund.

During the autumn of 1970 a 'project group' in the institution was using the thermal diffusion process for experiments with PID, dead-beat and minimum variance regulators. This has led to the program system described in

Borisson U. and Holst J., Real Time Computing II - Minimum Variance Control on Process Computer, Report 7108(B), September 1971. The Division of Automatic Control, Lund Inst. of Technology, Lund.

For process identification, routines for least-squares-type identification are available. They have been used to identify a simple process in the presence of noise of various intensity. This was done during the spring course in identification. They have also been used in the 'automatic identifier' program that has been mentioned before (RE-91).

Presently the maximum likelihood method is being implemented on the PDP. Early results on computing time are promising in spite of the relatively low floating point speed of the machine.

A Kalman filter has been programmed. The complete time-variable multiple input - multiple output equations have been used. A test of the residuals so that erroneous measurement values may be detected and discarded is also included. This filter has been used on the thermal diffusion process with success. The computing time for a 10:th order system with 10 measurement signals is approximately 2.5 sec.

During the year to come, linear-quadratic control on the thermal process as well as the mechanical device will be implemented. The work on interactive programs, started with the experiment control and 'automatic identifier' programs will continue. Today a first version of an interactive package for synthesis and simulation of control systems is being designed. The future development of this package as well as for the automatic identifier will pose some interesting problems of how man - machine interaction is to be implemented to solve problems in the field of automatic control.

6. APPLICATIONS

The effort in applications has also in this year been greater than was anticipated in the research program. It has been comparatively easy to obtain real data from industrial processes. It has, however, not been easy to find cases where multivariable control, adaptive control and optimal control can be tried in practice. It is expected that this situation may improve when we can establish feasibility of these techniques in the laboratory. The major effort has been devoted to the thermal boiler project which is now completed. Some work has also been done in power systems in connection with the PhD program.

Thermal Power Plants 13.2.1967

The analysis of the experiments performed on Öresundsverket in collaboration with Sydsvenska Kraft AB have now been completed. The study has resulted in complete models for the drum boiler turbine unit. Detailed linear models for given operating conditions as well as nonlinear models covering a wide operating range have been obtained. The study has also given valuable insight into the problems of modeling and identification of industrial processes. In particular a technique for numerical modelbuilding has been completed. The study now consists of a description of the field measurements, derivation of models from construction data, identification of linear difference equation models and state models and comparison and evaluation of the different models. The agreement between the field tests and the construction data models is very good although no attempt has been made to adjust the model parameters to improve the fit. The discrepancies which still exist have been explained. This analysis has given insight into the physical phenomena that are of major importance in the model. The models obtained by identification agree

of course even better with the data. In typical cases the error in predicting the state have been decreased by a factor of 2-5 in comparison with the construction data models. The data has been completely exploited in the sense that the residuals are now dominated by effects that are due to imperfect experiments e.g. nonsimultaneous sampling and imperfect input signals. It is therefore believed that no significant improvement can be obtained without further experiments. An illustration is given in Fig. 6.1.

The insight into the dynamics of the drumboiler - turbine unit that has been obtained when developing the linear models have made it possible to derive simple nonlinear models which can describe the behaviour of the system over a wide operating range. This model is described in

Åström K.J. and Eklund K., A Simplified Non-linear Model of a Drum Boiler - Turbine Unit, Report 7104, April 1971. The Division of Automatic Control, Lund Inst. of Technology, Lund.

This model is useful to describe a thermal power plant when analysing a complete power system.

Power Systems

A power system is a good example of a large multivariable system. With the changing structure of power generation in Sweden there is also a significant interest in the control of power systems. In collaboration with Vattenfall AB we have therefore initiated a study to investigate the feasibility of applying modern control theory to the problem of controlling a power system.

Modern control theory makes it possible to handle multivariable systems described by a set of first order differential equations on state space form

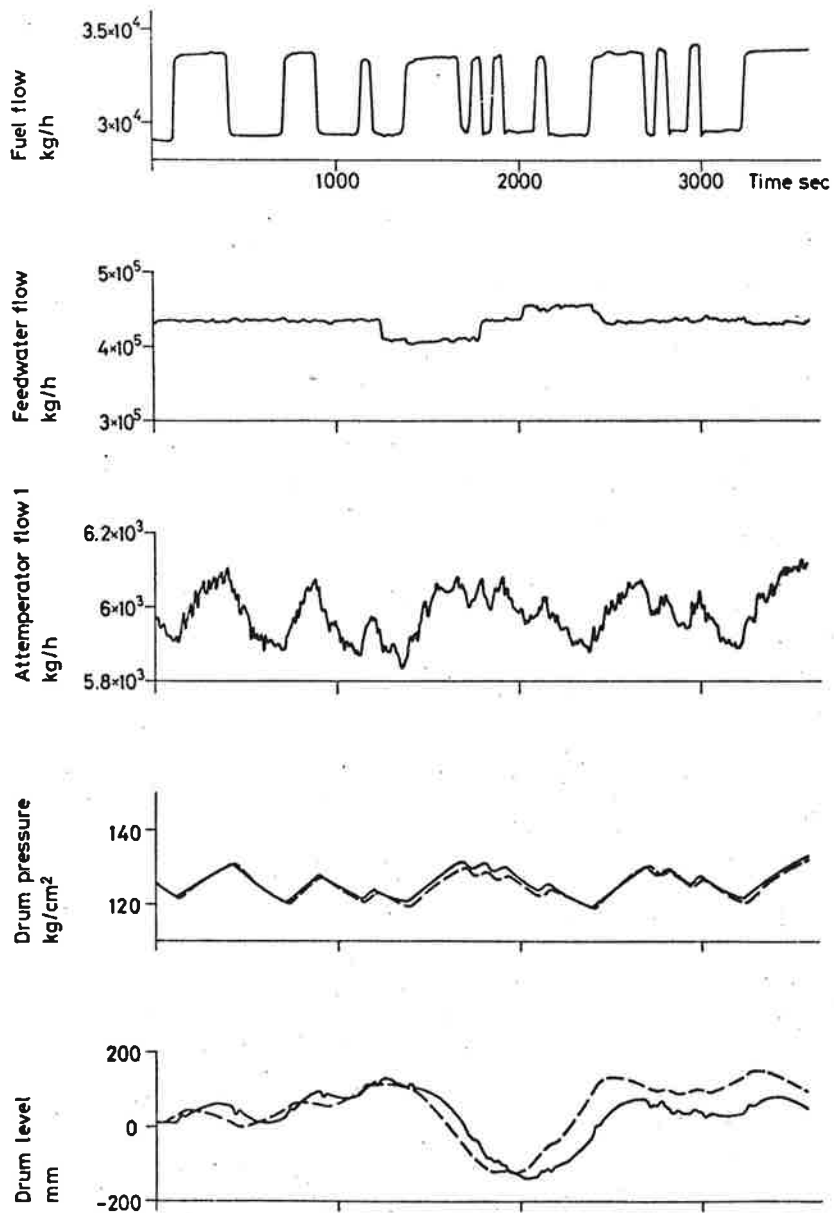


Fig. 6.1

Comparison of measured boiler (solid) and identified model (dashed) responses.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad (6.1)$$

As the matrices A and B become large it is desirable to use digital computers to compute the matrices (6.1). A major effort has been devoted to study the basic physical laws for a multimachine power system. A computer program has been developed for the modeling of a multimachine power system.

Process identification is an alternative to obtain models for power systems. Data from an experiment, performed by Dr. K.N. Stanton of Purdue, has been analyzed using spectral analysis, maximum likelihood method and structural identification. Preliminary results show that only a minor part of the frequency variations can be explained with a first order model, but a significant part can be explained with a fifth order model.

Application of linear regulator theory to a power system consisting of one generator connected to an infinite bus is given in

Olsson K.E. and Åkerlund M., Tillämpning av linjärkvadratisk teori på kraftsystem, (Application of linear control theory to a power system), Thesis Report RE-94, April 1970.

where the optimal control law is compared with suboptimal regulators when the feedback is taken from a few state variables only. Digital simulation shows that the behaviour of the regulator is not affected by the nonlinearities.

Control of Indoor Environment

During the year the main effort has been to make simple linear dynamic models describing how the air-temperature in a room is influenced by the temperature of the heating-

system or by its used energy, by outdoor-temperature and air-temperatures of adjoining rooms. Temperature-measurements were available from experiments with either radiant-, convective- or radiator-heating.

The models do not intend to describe the whole complicated process consisting of heat radiation, heat convection and heat conduction, but is an attempt to see how simple models could be used to describe the dynamics of a room for regulation purposes. A very interesting thing was to find out if data like masses, areas, specific-heat-constants, convective-heat-transfer-constants and heat-conduction-constants could be used directly in the models.

The best result was achieved with a third order model with room-air, walls and floor-ceiling as the three states. During the work it was clear that it would not be possible to make dynamic models from construction data.

Models achieved with the Maximum-Likelihood method contained a direct term, which was due to the sampling-interval of 20 minutes and a fast time constant. So to be able to get better models in the time-domain of minutes new experiments had to be done and have taken place during May and June, 1971.

The dynamics of an airduct have also been studied. Data from three experiments with different means and amplitudes were used. The input-output-relation was determined to be of second order with the Maximum-Likelihood method. A model based on a partial differential equation was tested with a good result. Only one parameter namely the convective-heat-transfer-constant between the air and the ductwall was let free in the identification and the final value was within the limits.

Thermal Process

A one-dimensional thermal conduction process is one of the major experimental processes used in the real time control laboratory. The process has been completed and interfaced with the process computer. A description of the process is given in

Leden B., Design of a One Dimensional Heat Diffusion Process, Report 7010, December 1970. The Division of Automatic Control, Lund Institute of Technology, Lund.

The process is an example of a distributed parameter system, and a good vehicle into the research on control of such processes. It has been used for identification studies where it has been shown that the diffusion process can in fact be modeled by a low order model. The thermal process has also been used for DDC control and for Kalman filtering experiments.

Miscellaneous

A mathematical model for the ecology of a lake has been given in

Broqvist S., Matematisk modell för sjön Trummen
(Mathematical Model of the Lake Trummen)
Thesis Report RE-87, October 1970.

This work was carried out in collaboration with the Institutes of Plant Ecology and Limnology.

Application of linear quadratic control theory to the design of a ship steering system is described in

Ekdahl K. and Henriksson I., Om regulatorer för maximalekonomisk styrning av fartyg (On Regulator for Economical Optimal Control of Ships), Thesis Report RE-83, July 1970.

In connection with this work some experiments to determine plant dynamics were also performed. These data has then been analysed and it has been shown that hydrodynamic derivatives can be conveniently determined using identification techniques. (Fig. 2.1)

A ball rolling system has also been designed as was described in section 5. The electromechanical servos used in regular teaching have also been modified and interfaced to our real time computer. Concentration variations in laminar flow are investigated in

Åström K.J., Dynamics of Concentration Variations in Laminar Tube Flow, Report 7102(B), April 1971.

7. SYSTEM THEORY

As was stated in the introduction the doctorate program has made it possible for us to broaden the basis for our research. In connection with the courses given, several problems have been encountered which have initiated research.

Riccāti equations and related topics

In linear systems theory work has been done which relates the Riccati equation to factorization of some linear operators into a product of a causal and an anticausal part, reported in

Hagander P., Linear Control and Estimation using Operator Factorization. Report 7114. Division of Automatic Control, Lund Institute of Technology, Lund.

The filtering prediction and smoothing problems as well as the linear quadratic control problems can generally be formulated in linear function spaces. Using basic vector space lemmas like the projection theorem their solution is obtained as Fredholm type II integral equations, which however, are difficult to solve directly.

Operator factorization by the Riccati equation now separates the problem into two Volterra operator equations corresponding to conventional linear dynamic systems, the anticausal one having reversed time.

The Kalman filter equations and the linear regulator problem appear as special cases. Even the smoothing results, which are difficult to prove by conventional methods in the continuous case, are easily obtained.

The work gives further insight and a unified deduction of the major results in linear dynamic systems. A correspondence between the inversion of a linear dynamic system and a well-known matrix inversion identity is presented in

Hagander P., Inverse dynamic systems by an operator inversion identity. Report 7109(B), Sept. 1971.

Multivariable Systems

One important problem for multivariable systems is to maintain satisfactory control over some selected variables even if the system is subject to relatively large external disturbances. If the disturbances can be modelled as dynamical systems they can be effectively compensated by introducing dynamical elements in the control functions. The structure of such control functions has been investigated as well as effective means of synthesis. One possible application area is PI-control of multivariable systems. A report is under preparation.

Many modern design techniques require a state space description of the system. An algorithm realizing a transfer-function matrix in state space form has been derived. The realization is on the Jordan Canonical form and is of minimal order.

8. TECHNOLOGY ASSESSMENT

It might be of interest to users in society¹⁾ to know where the results developed can be applied and what the state of the art is. It is also our hope that such a discussion can contribute to further interactions between our research group and users.

1. System Identification and Modeling

Many programs and much experience from analysis of real data is available.

2. Control of Linear Multivariable Noisy Systems

This is believed to be a good area for applying the results of modern control theory. The theory is available including a significant amount of general purpose programs. Industrial experience of single input single output systems is available. It would be of extreme interest to us to become involved with a real application.

3. Adaptive Control

It is believed that the self-tuning algorithms developed can have many applications. So far only single input single output systems have been considered. All results now available are based on computer simulation. Feasibility in laboratory experiments is not established. Several problems exist e.g. including program aspects, sensitivity to large parameter changes.

4. Optimal Control

Computational methods have been developed, and have proved efficient to a large number of different kinds of optimal

1) I consider Industry as part of society

control problems. It is now considered important and necessary to examine and develop the applicability to complex industrial processes. Probably there are many problems well suited for a rather straightforward application of optimal control theory, e.g. production scheduling and different kinds of batch processes.

9. REFERENCES

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LIST OF PERSONNEL

Professor	Karl Johan Åström
Universitetslektor	Gustaf Olsson
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Forskningsingenjör	Leif Andersson
"	Karl Eklund
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"	Ulf Borisson
"	Per Hagander
"	Sture Lindahl
"	Lennart Ljung
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Ingenjör	Rolf Braun
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"	Torkel Glad
"	Jan Holst
"	Bo Leden
"	Uno Nävert
"	Jan Sternby
"	Torsten Söderström
Tekniskt Biträde	Marianne Steinertz
Sekreterare	Eva Schildt

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- Åström, K.J.: "Introduction to Stochastic Control Theory".
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International Journal of Control 11, 973-985. (1970).

TECHNICAL REPORTS

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- Report 7004 Mårtensson, K.: Suboptimal Linear Regulators for Linear Systems with Known Initial-state Statistics, July, 1970.
- Report 7005 Valis, J.: Identification of Multivariable Linear Systems of Unknown Structure by the Prior Knowledge Fitting Method, September, 1970.
- Report 7006 Åström, K.J. and Eykhoff, P.: System Identification, September, 1970.
- Report 7007 Pettersson, B.: Production Control of a Pulp and Paper Mill, September, 1970.
- Report 7008 Åström, K.J.: Final Report for Project Process Control 1.7.1969 - 30.6.1970. Contract 69-631/U489, Swedish Board for Technical Development, October, 1970.
- Report 7009 Leden, B.: Linear Temperature Scales from One Thermistor Reciprocal Networks, December, 1970.
- Report 7010 Leden, B.: The Design of One Dimensional Heat Diffusion Process, December, 1970.
- Report 7011 Olsson, G.: Master Theses in Automatic Control 1965 - 1970, November, 1970.

- Report 7101 Mårtensson, K.: Methods for Constrained Function Minimization, March, 1971.
- Report 7102 Åström, K.J.: Dynamics of Concentration
(B) Variations in Laminar Tube Flows, April, 1971.
- Report 7103 Gustavsson, I.: Choice of Sampling Interval for Parametric Identification, April, 1971.
- Report 7104 Åström, K.J. and Eklund, K.: A Simplified Nonlinear Model of a Drum Boiler Turbine Unit, April, 1971.
- Report 7105 Wittenmark, B.: On the Turn off Phenomenon in Adaptive Control, September, 1971.
- Report 7106 Hagander P., Källström, C. and Åström, K.J.:
(B) Real Time Computing I Implementing Linear Filters, May, 1971.
- Report 7108 Borisson, U. and Holst, J.: Real Time Computing
(B) II Minimal Variance Control on Process Computer, September, 1971.
- Report 7109 Hagander, P.: Inversion of a Dynamical System
(B) by an Operator Identity, September, 1971
- Report 7110 Wittenmark, B.: A Survey of Adaptive Control Methods, September, 1971.
- Report 7111 Wieslander, J.: Real-Time Identification Part II, September, 1971.
- Report 7114 Hagander, P.: Linear Control and Estimation using Operator Factorization, July, 1971.

MS THESES

- RE-82 Lindberg, Lave: Kalmanfilter. Adaptiva Kalmanfilter. Reelltidsidentifiering (Kalman Filters. Adaptive Kalman Filters. Real-Time Identification), juli 1970.
- RE-83 Ekdahl, Kurt, Henriksson, Ingvar: Om regulatorer för maximalekonomisk styrning av fartyg (On Regulator for Economical Optimal Control of Ships), juli 1970.
- RE-84 Andersson, Leif: Numerisk behandling av Kalmans ekvationer för tidsdiskreta system (Numerical Considerations on Implementing Discrete Time Kalman Filters), augusti 1970.
- RE-85 Kristofersson, Martin: Analys av en andra ordningens allmän kvadratisk differential-ekvation (Analysis of a General Quadratic Second Order Differential Equation), augusti 1970.
- RE-86 Knutsson, Gert Ingvar: Jämförelse mellan tre olika metoder för reelltidsidentifiering (Comparison between Three Different Real Time Identification Methods), augusti 1970.
- RE-87 Broqvist, Stig: Matematisk modell för sjön Trummen (Mathematical Model of the Lake Trummen), oktober 1970.
- RE-88 Mårtensson, Ingmar, Weibull, Anders: Minimalvariansstyrning av robot (Minimal Variance Control of a Missile), nov. 1970.
- RE-89 Frick, Bengt: Rumsmätningar och identifiering av rummets dynamik med maximum likelihoodmetodik (Temperature Measurements in a Room and Identification of the Dynamics of the Room Using the Maximum Likelihood Method), november 1970.

- RE-90 Lundström, Anders, Nilsson, Christer: Undersökning av en plattvärmväxlarens dynamik: mätningar och identifiering (Investigation of the Dynamics of a Plate Heat Exchanger: Measurements and Identification), februari 1971.
- RE-91 Jönsson, Bengt: Konversativt datorprogram för processidentifiering (Computer Program for Automatic Process Identification), februari 1971.
- RE-92 Jensen, Lars: Mätningar på luftkonditioneringsanläggning med återblandning. Identifiering av delsystem (Measurements on an Air Conditioning Plant with Recirculation. Identification of a Sub-System), mars 1971.
- RE-93 Hagbjer, Lennart: Utmatning på plotter (Programs for Plotter Output on an Ink Jet Plotter), april 1971.
- RE-94 Olsson, Karl-Erik, Åkeflund, Mats: Tillämpning av linjärkvadratisk teori på kraftsystem (Application of Linear Quadratic Theory on Power Systems), april 1971.
- RE-95 Andersson, Bo, Nordenskjöld, Kjell: Samkörning av motorer (Control of Motors driving common load), June, 1971.

COURSES AND SEMINARS

Besides the internal seminars, given at the Division a number of graduate courses and seminars have been offered.

Courses

The following courses have been given by invited lecturers, in cooperation with other departments at the university and by the personnel at the Division

Process Identification (Div. of Automatic Control)

Finite Dimensional Linear Systems (Div. of Automatic Control).

Introduction to Stochastic Control Theory (Div. of Automatic Control).

Minimization of Functions of Finite Number of Variables (Dept. of Mathematics in cooperation with Div. of Automatic Control).

Hardware and Software for Real Time Computers (Guest professor, Dr James D. Schoeffler, Case Western Reserve Univ., Cleveland, Ohio).

Functional Analytic Methods in Stability Theory (Guest professor, Dr. P. Falb, Brown Univ., Providence, R.I.)

Seminars

Invited Swedish and foreign lecturers have contributed with the following seminars.

Mr K. Todd, Computer Consultant at Cemente AB, Malmö.

"Control of a Cement Kiln", September 25, 1970.

Professor E.I. Jury, Univ. of California, Berkeley.
"A New Look into Problems of System Theory I",
October 26, 1970.

Professor E.I. Jury, Univ. of California, Berkeley.
"A New Look into Problems of System Theory II",
October 27, 1970.

Professor C. Penescu, Institute of Automatic Control,
Bukarest, Rumania.
"A Method for Stochastic Process Identification",
October 27, 1970.

Professor C. Penescu, Institute of Automatic Control,
Bukarest, Rumania.
"Computer Control of Large Power Systems", October 29,
1970.

Mr T. Wikland, TUAB, Stockholm.
"Systemanalys", (System Analysis), November 6, 1970.

Dr W.L. Root, Univ. of Michigan, Ann Arbor, Mich.
"Some General Theory of System Representations and
Measurements", November 17, 1970.

Mr S. Strandh, Museum of Technical History, Stockholm.
"Några drag ur reglerteknikens utveckling", (Some
Features of the Development of the Automatic Control),
November 26, 1970.

Dr B. Pettersson, Billerud AB, Grums.
"Produktionsstyrning", (Production Planning), December
2, 1970.

Mr E. Persson, ASEA, Västerås.
"Metoder för syntes av reglersystem", (Synthesis
Methods for Control Systems), December 10, 1970.

Mr E. Persson, ASEA, Västerås.
"Några intressanta tillämpningar av frekvensanalytiska
metoder", (Some Interesting Applications of Fre-
quency Domain Methods), December 10, 1970.

Mr E. Persson, ASEA, Västerås.

"Synpunkter på syntes med hjälp av datorer", (Some Aspects of Computer Aided Synthesis), December 11, 1970.

Mr E. Persson, ASEA, Västerås.

"Optimering av produkter och system", (Optimization of Products and Systems), December 11, 1970.

Professor Y.C. Ho, Harvard University, Cambridge, Mass.

"Stochastic Singular Control Problems", January 12, 1971.

Professor Y.C. Ho, Harvard University, Cambridge, Mass.

"Information Pattern and Decentralized Control Systems" January 12, 1971.

Professor E.A. Pearson, Brown Univ, Providence, R.I.

"Industrial Organization", April 4, 1971.

Professor E.A. Pearson, Brown Univ. Providence, R.I.

"Urban Systems", April 21, 1971.

Professor E.A. Pearson, Brown Univ. Providence, R.I.

"Environmental Systems". April 23, 1971.

Professor H.Cramér, Former Chancellor of the Swedish Universities; Stockholm.

"Användningar av Hilbertrymdens teori på stokastiska processer I", (Application of Hilbert Space Theory on Stochastic Processes I), April 28, 1971.

Professor H. Cramér, Former Chancellor of the Swedish Universities, Stockholm.

"Användningar av Hilbertrymdens teori på stokastiska processer II", (Application of Hilbert Space Theory on Stochastic Processes II), April 30, 1971.

Professor H. Cramér, Former Chancellor of the Swedish Universities, Stockholm.

"Användningar av Hilbertrymdens teori på stokastiska processer III", (Application of Hilbert Space Theory on Stochastic Processes III), May 5, 1971.

Mr. C. Dahlström, ASEA, Västerås.

"Hur industrin idag löser sina reglerproblem", (How the Industry solves its own Control Problems Today), May 7, 1971.

Professor E.A. Pearson, Brown Univ, Providence, R.I.

"Parameter Adaptive Control Systems I", May 12, 1971.

"Parameter Adaptive Control Systems II", May 14, 1971.

"Impulse Response Identification using fast Fourier Transform Techniques", May 19, 1971.

Mr B. Sjöberg and Mr. D. Folkesson, SAAB-SCANIA, Linköping.

"Central Dator i Flygplan 37", (The Central Computer of the Aircraft 37), June 8, 1971.

Mr A. Spang, General Electric, USA.

"Comparison and Structure of three Real Time Operating Systems for Process Control", June 9, 1971.

Dr E. Angel, Univ. of Southern California, Los Angeles.

"Nonclassical Approaches to Classical Boundary-Value Problems: Basic Principles and Examples", July 7, 1971.

"Invariant Imbedding, Dynamic Programming and Partial Differential Equations I", July 14, 1971.

"Invariant Imbedding, Dynamic Programming and Partial Differential Equations II", July 15, 1971.

"Invariant Imbedding, Dynamic Programming and Partial Differential Equation III", July 15, 1971.

The course by Dr J. Schoeffler was organized by the secretary for liason with industry (kontaktsekreterare) at the Lund University.

The main topics were the organization of hardware and its impact on real time programming as well as the implementation of real time executives.

The course was intended to constitute, together with some hands-on experience on the PDP-computer, a course in the PhD program.

Dr Peter Falb visited the Division during June 1971 and gave a course on "Functional Analytic Methods in Stability Theory". The object of the course was to present a stability theory for input output problems using functional analytic methods. Dr Falb himself has contributed significantly to the development of the theory and presented extensions of Nyquist's criterion to the non-linear time-varying case.

Dr E. Angel visited the Division during July and August 1971. He gave a lecture series on "Numerical Solution of Partial Differential Equations", a topic which is related to the research projects going on at the Division.

Especially, Dr. Angel concentrated on the numerical solution of equations of the type

$$u_{xx} + u_{yy} + g(u) = 0$$

where the boundary conditions are given. He showed that after discretization it is possible to formulate the problem as a known problem of control. It can be formulated as an optimization problem and dynamic programming can be applied. Dr. Angel has shown in this research that it is possible to avoid the 'curse of dimensionality' by different methods.