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Abstract

Self-excited low frequency power oscillations in large electric power systems are a phenomenon, which have caused considerable problems by jeopardizing the operation of the systems. The Scandinavian power grid has had some serious incidents with power oscillations between, for example, southern Finland and southern Norway.

This thesis focuses on how to use power system stabilizers (PSS) to damp oscillations in large power systems. The problem of applying and tuning power system stabilizers is approached using multivariable control theory.

The control problem is decomposed into the three topics, modelling, finding feedback structure and tuning of the controller. A multimachine model suitable for design is presented. The design method is based on linear quadratic (LQ) regulator design. A new method to find a sparse feedback structure, which captures importants parts in a good control law, is presented. The feedback structure is not restricted, as it has traditionally been, to only local measurements. Two methods for tuning of a parametrized controller are discussed and exemplified. Finally, a design example for a 16 machine power systems is shown.

Key words
Power System Stabilizers, Multimachine model, Feedback structure, Incomplete state feedback

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Preface

This thesis is a symbiosis between the field of automatic control and power system engineering. Since the areas are non trivial the key to the problem have been knowledge in both areas. The recipe has been to read post graduate courses in automatic control at Lund Institute of Technology and power system courses at Denmark Institute of Technology in Lyngby to form a platform for the thesis work. Power systems are probably the most complicated system created by mankind, so the area can be a gold-mine for those who are interested in complicated problems. Anyhow, this thesis will show the flavours of different problems in multivariable power systems and hopefully get the reader interested in the combination power system and automatic control.

Acknowledgements

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A special thank goes to Sture Lindahl and Bo Eliasson at Sydkraft for their great interest and wise comments. Both have been invaluable sources of knowledge and support. I also would like to express my gratitude to David Hill, University of Newcastle, Australia, for starting the project in the right direction and for all later support. Also many thanks to Jan Rönne-Hansen and his staff at Denmark Institute of Technology for their hospitality and interesting courses. Thanks to Asea Brown Boveri for letting me have access to the simulation program Simpow, which has been a very good simulation tool. This thesis has been supported by Swedish National Energy Administration (Statens Energiverk) under project number 616 033.

Table of Contents

1. Introduction

This thesis focuses on a control problem in electric power systems called steady-state-stability. The following chapter gives an introduction to the problem and ends up with the objective of this thesis.

1.1 Power system stability

A power system consists of the power grid and the generating units. The power grid makes it possible to transfer the power from the production sites to the consumers. Today the power systems are large interconnected systems. The construction of the power grid is based upon a combination of technical, economical and environmental factors. Advantages offered by an electric power grid can be summerized as (Elgerd, 1983):

- It permits construction of larger and more economical generating units and the transmission of large blocks of energy from the generating sources to major load centers.
- It permits reduced reserve requirements by sharing of capacity between areas.
- It permits capacity savings from time zone and random diversity.
- It facilitates transmission of off-peak energy.
- It provides the flexibility to meet unforeseen emergency demands.

On the other hand, the main disadvantage is that large power systems are complex to analyse. But altogether the advantages are overwhelming and there are strong technical and economical motives for a power grid which connects generating units and loads into a large power system.

Two basic requirements can be posed on a power system (Anderson and Fouad, 1977):

- Quality demands on voltage and frequency.
- Maintain the integrity of the power system.

The quality demands are that voltage and frequency must be held within close tolerances of nominal values so that the consumers equipment may operate satisfactorily. Disturbances of various types can perturb the power system and this should not jeopardize the operation. The disturbances can be separated according to their time scale and magnitude. Typical disturbances are:

- Overvoltage caused by lightning, which is a very fast phenomenon with time scale with range from microseconds to milliseconds.
- Short circuits which can appear with duration of tens of a second.
- Loss of line or generating unit.
- Change in loads which is taking place all the time.

For all disturbances the system should maintain the integrity. This means that we have to analyse the behaviour of the power system for all possible situations. It is convenient to decompose the analysis in different parts depending on time scale and magnitude of the disturbances. One basic separation is to separate fast phenomenon like lightning, which demands electric field theory, from problems where we can assume that all quantities are sinusoidal. The latter problem class can be separated into four subclasses

- Long-term stability considers the time horizon of several minutes and the main issue is the balance between supply and demand of active power.
- Voltage stability is a recently discovered problem, which appears as severe and instantaneous voltage drop (Petersson, 1984). The reason is believed to be lack of reactive power in combination with unfavourable performance of voltage control. Voltage stability is not a fully understood phenomenon and is subject to research.
- Transient stability or Large Disturbance Stability (LDS) is the ability of the generators to remain synchronized when exposed to severe disturbances as large short circuits.
- Small disturbance stability (SDS), or steady-state stability, concerns smaller disturbances than LDS. Both LDS and SDS treat stability, in power system often called synchronous stability, the major difference is that SDS, per definition, uses linear theory and LDS has to use nonlinear theory.

Figure 1.1 gives an overview over different problem areas in power system.

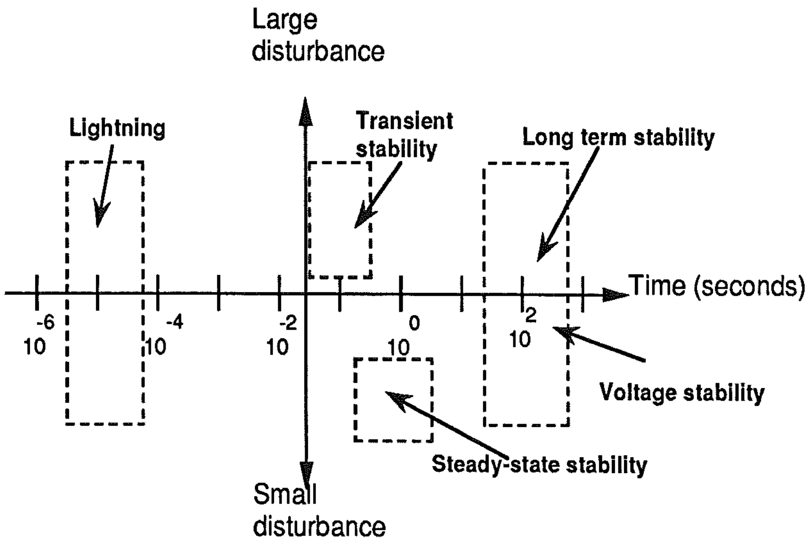


Figure 1.1 Examples of different problems in power systems. The x-axis shows the time scale and y-axis the magnitude of typical disturbances for each class of problems.

To fulfil the requirements on a power system each of these problems has to be solved. In the subsequent part of the thesis we will concentrate on the steady-state stability problem.

1.2 Steady-state stability

Steady-state instabilities result in self-excited power oscillations with low or negative damping. Since the oscillations are self-excited, start spontaneously and are not due to elements with highly nonlinear characteristics, the problem is classified as SDS. Low frequency oscillations (0.1-0.8 Hz) in power systems are a well known phenomenon which has caused considerable problems. The Nordic network (Nordel), which connects Sweden, Norway, Finland and a part of Denmark, has had some serious incidents (Lysfjord et al 1982, 1984). As a consequence the capacity of some tielines ($\approx$ network connections) is limited due to stability problems at heavy load. The problem is familiar all over the world, see for instance Yuan-Yih et al (1987).

Eigenvalues

Spontaneous power oscillations can be viewed as small perturbations around a stationary operation point. The analysis can therefore be handled by linear theory, see for example Anderson and Fouad (1977). Eigenvalues associated with self-excited power oscillations are poorly damped and are named electromechanical oscillations modes (EOM). The negative real part of the eigenvalues comes from resistance in load, network and damper windings on machines.

Usually EOM-eigenvalues are separated in three kinds (Larsen and Swann, 1981) according to frequency and how the machines participate in the oscillations

- Intertie (System wide, in range 0.1-0.8 Hz)
- Local (Single machine, in range 0.8-2 Hz)
- Interplant (Close units, in range 2-3 Hz)

The correspondence between type of mode and frequency range is not always true. In Sweden, for example, there exists remote hydro stations which oscillate as single machine modes in the lowest frequency band. Larsen and Swann point out that the most important modes are the system wide modes which should be given special attention when applying stabilizers.

Causes for power oscillations

The power oscillations are usually caused by a combination of

- High reactance on tielines
- High gains in automatic voltage regulators (AVR).
- Heavy loading, e.g. large load angles.
- Unfavourable load characteristics.

Historically the gain in the AVR has been increased from low values to improve the transient stability. The drawback is that eigenvalues associated with the EOM, tend to become less damped with increasing AVR gain, which can cause instability (Larsen and Swann, 1981). A number of ways to improve stability can be found in Petterson (1984). One simple and effective way is to decrease tie-line reactance by building a parallel line. However this is a very expensive solution.

1.3 Normal control structure

Open Loop

Each generator has two inputs which can be manipulated and at least two outputs on which there are requirements. Inputs are field voltage (U_f) and mechanical power (P_m), outputs are terminal voltage (V_t) and electric power (P_e). If we just consider one small generator, without any regulators and connected to a strong grid, typical responses to small steps look like Figure 1.2.

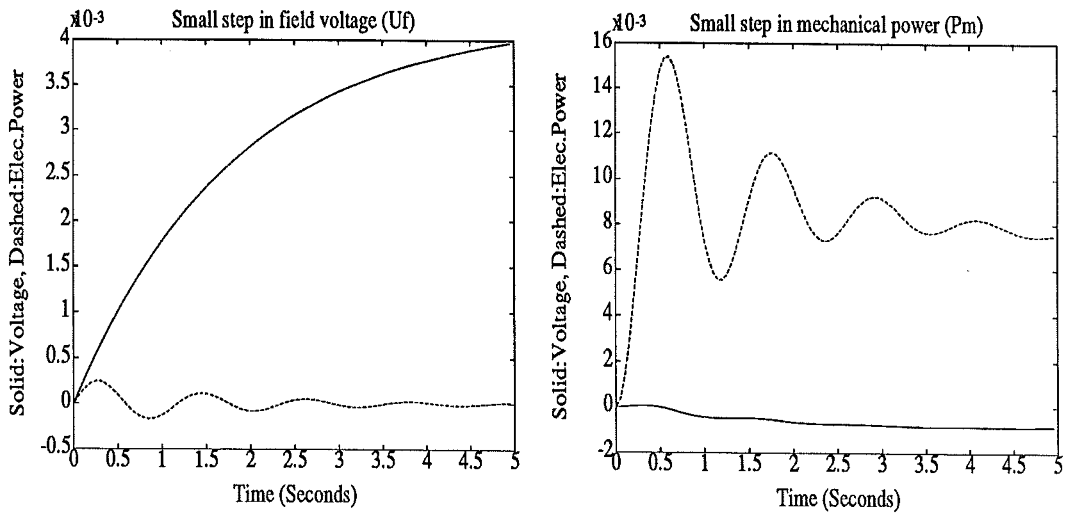


Figure 1.2 Simulated step responses for one generator without any regulators. The simulation was done with a linearised model according to Chapter 2.

Step responses show that U_f mainly influences V_t and that P_m mainly influences P_e. The normal way to close the loops is to use U_f to control V_t and P_m to control P_e. However, we also see that there is a weak dynamic interaction between U_f and P_e. The input variable P_m is considered to be an expensive control variable since change in mechanic power implies a change in energy flows and rotating masses which causes stress on mechanical parts. Therefore P_m is often used to control slow variations in P_e steady-state values. In contrast, new static exciters usually have fast dynamics which implies that field voltage U_f is a cheap and fast control signal as long as saturation not occurs.

Voltage regulation

The traditional control structure for voltage regulation is shown in Figure 1.3a. If the AVR gain is large the generator becomes sensitive to disturbances such as load changes, which can cause power oscillations as in Figure 1.3b.

Power system stabilizers

One way to improve steady-state-stability is to use power system stabilizers (PSS), which increase stability by adding a signal on the machine excitation system. In this way we use the dynamic interaction between U_f and P_e to damp out power oscillations. The control structure and response with PSS can be seen in Figure 1.4.

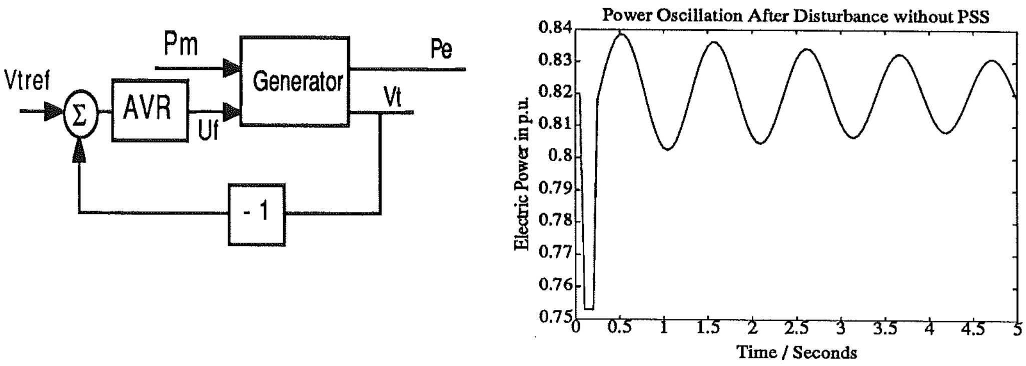


Figure 1.3 Control structure (a) and typical power oscillation (b) for a generator without PSS

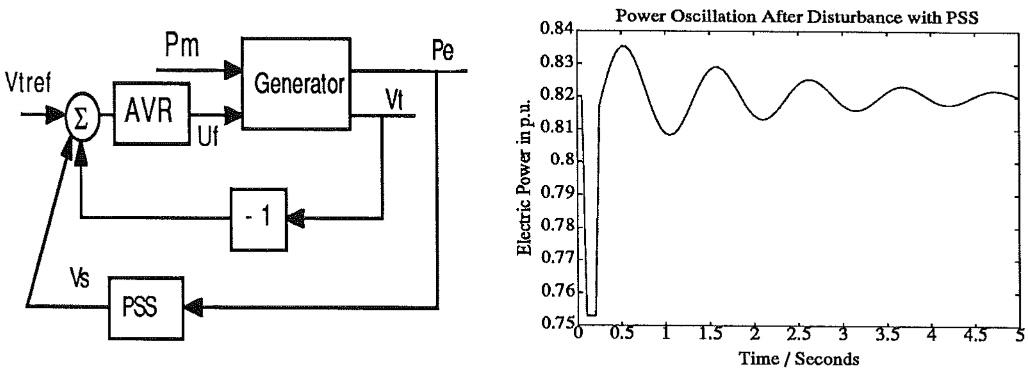


Figure 1.4 Control structure (a) and typical power oscillation (b) for a generator with PSS

As pointed out by DeMello in the discussion of Larsen and Swann (1981) the PSS often improves the transient stability as well. The hardware for the PSS is cheap, but to apply and tune the PSS demands more understanding of control theory.

Other signals than electric power can be used as input to the PSS, for example machine speed, frequency or field current (Larsen and Swann, 1981). All signals correspond to local states in the machine and the signals can be used to improve the stabilizer action. In Sweden it is common to use only electric power as input to the PSS and to estimate the speed by filtering the electric power. One reason is that electric power is easier to measure than speed. Another reason is the relatively simple relation between power and speed, which make it possible to accurately estimate the machine speed from power measurement.

1.4 The control problem

The evolution of the analysis and synthesis of the associated control problem, to apply and tune PSS, started in the early fifties by Heffron and Phillips (1952) who developed a basic single machine model. This model was generalized by Vournas and Flemming (1978) to the multimachine case. A classical paper was written by DeMello and Concordia (1969) where they showed that high AVR gain on a single machine can, under unfavourable circumstances, cause undamped power oscillations. A summary of the ideas established in the seventies can be found in Larsen and Swann (1981). In the eighties the attention has been driven towards a multi-input-multi-output (MIMO) problem restricted to local control (Wilson and Aplevich, 1986; Arnatovic, 1987; Huang et al, 1988). Adaptive approaches have been taken in Chang, Malik and Hope (1988).

The synchronous machine is constructed with an electric arrangement, called damper windings, which should damp out deviations from the synchronous speed. For oscillations with frequencies over 1 Hz the damper windings work quite well and damp power oscillations. In larger power systems there are often system wide oscillations with lower frequencies than 1 Hz. Hence, these oscillations cannot be damped by the damper windings and some other precautions must be taken. To solve this problem the power system stabilizers were introduced to complement the damper windings at low frequencies.

Much of the earlier work used single machine models, where the PSS-loop should give a torque component with right phase shift to damp power oscillations. In later work the emphasize has shifted over to multimachine models. However, a problem has always been the lack of models, which could be easily used. Anderson and Fouad (1977) and Vournas and Flemming (1978) have presented the equations, but not showed any implemented multimachine model, which they have used for design. The field of power system engineering has not exploited the benefits of a mathematical understanding of the model used to the same extent as other fields in engineering. Later years progress in computer science and numerical algebra has open the possibility to implement flexible multimachine models easily. Together with modern control software it is possible to explore the control problem and use multivariable control theory. A good control design must treat the problem as a multimachine problem. For example, we do not want to view the PSS as only a complement to the damper windings.

1.5 The objective of this thesis

With consideration to the comments above this thesis will focus on the following issues:

- Low frequency power oscillations is a MIMO problem and therefore must be approached by appropriate MIMO-theory. The first step is to have an adequate model, which exploits the possibilities of simplification given by the problem.
- Design methods proposed in literature seem to have a bias to eigenvalues/sensitivity methods. Often there is an implicit assumption about linearity between parameters and eigenvalues without considering the limitations. This thesis would like to balance this bias with some alternative methods.
- Linear Quadratic (LQ) regulator design is one of the best design methods for multivariable systems and can successfully be applied on this problem.
- To improve the stability margins against power oscillations it is important to damp both absolute deviation from the linearisation point and speed deviations relative to other machines.
- Point out the opportunity to transmit important signals, which can be used to improve steady-state stability. This can also be investigated by LQ-design.

- In the literature all analysis has been done on the open loop system. It seems better, or at least a good complement, to make a good global state feedback and to use this to analyse system properties.
- There are several ways to approximate a global control law to apply to practical constraints. All methods need some fiddling before giving satisfactory results and the approximations are of minor importance compared to the primary design they try to approximate.

1.6 Outline of the thesis

This report is divided into 8 chapters. Chapter 2 presents a multimachine model which is used in the subsequent chapters. Chapter 3 motivates, by criticism of publish methods, why we want to find new methods for the PSS design. The linear quadratic (LQ) regulator design method is presented in Chapter 4, where the importance of the loss matrices is emphasized. The design problem is separated into two parts. Chapter 5 treats the selection of a feedback structure and Chapter 6 the tuning of a parameterized controller. Finally, all loose ends will be put together in Chapter 7 which illustrates, by a large design example, how to use the ideas from Chapters 4, 5 and 6. The design is tested against the simulation package Simpow (Adielson, 1982). Chapter 8 contains the conclusions.

2. A Multimachine Model for Design

To make a design which takes into account the multimachine dynamics for a power system, it is essential to have a multimachine model. The goal of this chapter is to make a simplified linear multimachine model, which is accurate enough for design. This requires some calculations, which can appear to be a bit tedious for those who have a minor interest in power systems. To follow this chapter the reader ought to be acquainted with Parks transformation (Anderson and Fouad, 1977) and the $j\omega$-method. Comparison with a commercial simulation program shows that the model is suitable for design purposes.

2.1 Why another multimachine model?

This section motivates why we make a multimachine model and do not take one from the literature.

Why not use a multimachine model from a textbook?

Since the theory for the synchronous generator and electric components is well known, one can argue that it would be simple to write down a multimachine model. Multimachine models are not a standard subject in textbooks (Bergen, 1986; Elgerd, 1983). However, the theory for a multimachine model has been shown in Lindahl (1971) and in Anderson and Fouad (1977). From those two references we can also draw the conclusion that even a linearised multimachine model with simplifying assumptions becomes very complex. Because of the model complexity and heavy calculations it has been necessary to implement the models on a computer. This is a time consuming task if all programming has to be done in Fortran or Pascal. Therefore only large power companies with resources like Asea Brown Boveri have developed simulation packages with multimachine models. In the academic world in Scandinavia there have up to 1986, to the author's knowledge, not existed any linearised multimachine models in a form suitable for design. From the reports by Lysfjord et al (1982 and 1984) we also see that all design in the Scandinavian power grid has been done using linearised single machine models. The conclusion becomes that it is a considerable effort to get a linearised multimachine model implemented on a computer and this needs much more work than just to copy the equations from a textbook.

What is new with the proposed multimachine model?

What published articles have shown, for example Vournas and Flemming (1978), is the possibility and theory to make a multimachine model. However, there has been a major gap between the theory for multimachine models and the single machine models used by the power companies for PSS design, see Lysfjord et al(1982, 1984). The new idea is to make a multimachine model, which can be easily used and therefore bridges the gap between theoretical multimachine models and practically usable multimachine models. The implementation of the model in the interactive state-of-the-art matrix calculus program Matlab is supposed to be new. Matlab is an interactive program for matrix calculations with a very good user interface, see Moler et al (1987). The program allows the user to specify own functions and to use Matlab as a high level programming language. With a reasonable effort, it is possible to implement a multimachine model as a function in Matlab. In this way we can really use the model as a tool for design purposes and explore the problem. We can easily change parameters, load conditions, network configuration and number of machines and see how this influences the design.

2.2 Models for power systems

A power system can roughly be decomposed into three different parts

- Synchronous generators
- Loads
- The grid, or network, which connects generators and loads.

The complex system models are a set of nonlinear differential equations with algebraic constraints. To capture the steady-state-stability problem a linearised model is satisfactory. In our time scale, 0.1 - 2 Hz, we can assume that all quantities are sinusoidal with the net frequency (50 or 60 Hz) and use the $j\omega$-method in the calculations. Figure 2.1 shows an outline of a power system.

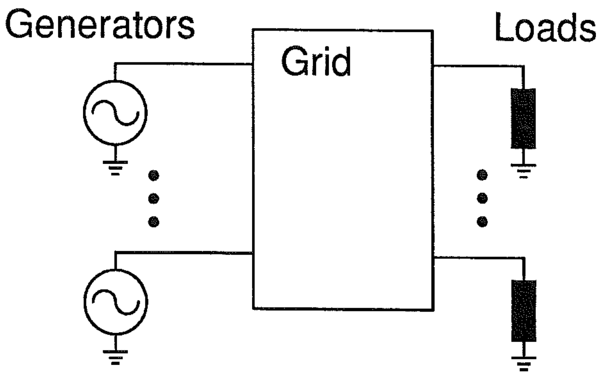


Figure 2.1 Principal outline of a power system. Generators and loads are connected by the grid.

Figure 2.2 shows a generating unit consisting of a synchronous generator driven by a turbine.

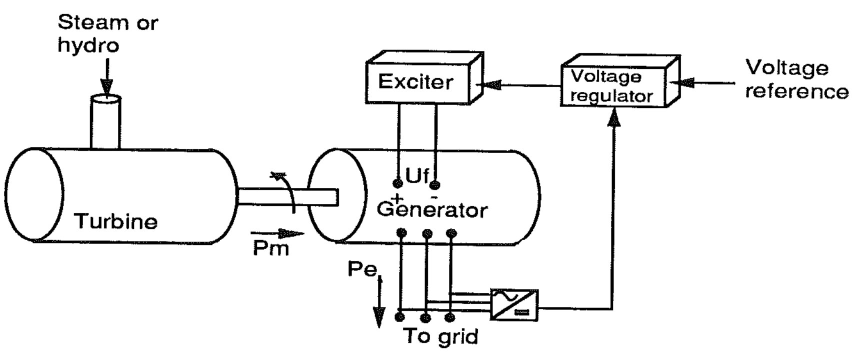


Figure 2.2 Principal drawing of a generating unit. Adopted from Elgerd (1983).

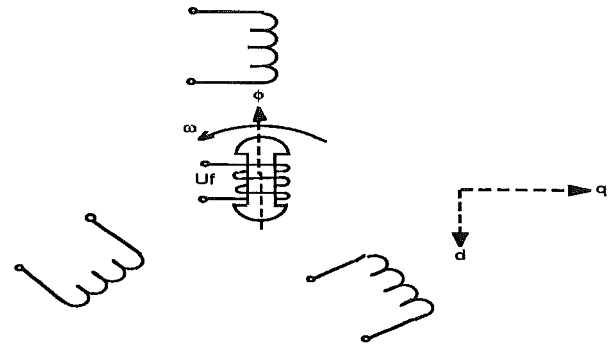


Figure 2.3 Principal drawing of a three phase synchronous generator without damper windings.

A brief description of Park's transformation and the d-q-system

Each synchronous generator looks like Figure 2.3. The rotating part is called the rotor and consists of magnetic material and the field winding. When the field winding is connected to a voltage source the current in the winding will give a magnetic flux ϕ which will rotate with the rotor speed ω. This rotating flux will induce a voltage with alternating polarity in the three fixed coils, called stator windings, and we have a three phase generator. When load is connected to the generator, current will float in the three fixed coils. This current will give a flux which will interfere with the field flux. Park's transformation is used to refer all fluxes and other quantities to a d-q-reference system which rotate with the field winding. The q-axis is orthogonal to the flux from the field winding and the d-axis is often oriented opposite the field flux. The transformation simplifies the calculations of the resulting flux and induced voltages in the stator windings. In this coordinate system the equations are time-invariant (time independent coefficients).

2.3 Assumptions in our model

Our model describes a power system with a number of synchronous generators with impedance loads. Equations for the synchronous machine are derived using the same assumptions as the Heffron-Phillips model (Anderson Fouad, 1977). These assumptions are

- Damper windings are not modelled explicitly.
- Small resistances are neglected in the phasor diagram.
- Balanced conditions are assumed and saturation is neglected.
- The deviation from synchronous speed is small.
- Some small terms in the voltage equations are neglected.

A list of all notations and units can be found in Appendix A.1.

Synchronous machine

There exists a number of candidate models for a synchronous machine depending on the degree of exactness (Heffron and Phillips, 1952; Vournas and Flemming, 1978; Hill and Bhatti, 1987; Andersson and Fouad, 1977; Yu, 1983). The model complexity needed depends very much what the model should be used for. A two state generator model is the lowest model order for open loop analysis of generator motion. If we also want to model the influence from the field winding we need one more state. Therefore we need at least three states per generator in a design model. For our design purpose a three state model gives enough accuracy (Malik Hope, 1988; deMello comments to Abdalla et al, 1984).

When we do simulations to check the design we can use a more detailed model. In the model we neglect the fast dynamics of the exciters (time constant$\approx$ 0.02 seconds) . The damper windings are modelled by a speed damping term D. We assume that the turbine governors are slow and therefore not active in the damping of power oscillations. This implies that the mechanic power is constant in our model.

Network

Our network, or grid, consists of impedances. We assume that our generators give sinusoidal voltage, e.g $u(t) = \hat{u}\sin(\omega t + \theta)$. It is a well known fact that in steady-state the currents in a linear impedance network driven by sinusoidal voltage sources will also have sinusoidal form. The time to reach the steady-state is determined by the ratios of resistances and reactances in the network. For power systems this time is very short compared to the time scale of the power oscillations and we neglect the short time before steady-state is reached. The network is then described by a set of algebraic relations between voltages and currents. If the algebraic relations are linearised around an operating point we see the coupling between the generators.

Loads

In general, the power consumption in a load is an arbitrary function of frequency (f) and voltage (v). A usual choice is to model the load to have a polynomial dependence of frequency and voltage, e.g.

$$P \sim f^k \cdot v^i \tag{2.1}$$

Here loads are modelled as impedances which corresponds to $k = 0$ and $i = 2$. The advantage with impedance loads is that all nodes without injection of current can be collapsed into an equivalent impedance matrix (Anderson Fouad, 1977).

The resulting model

The resulting multimachine model has $3n$ states, where n is the number of machines. If we set all speed dampings $D = 0$, we get two zero eigenvalues, which are not observable in electric power (P_e) or terminal voltage (V_t). They can be transformed away leaving $3n-2$ states. If we have some speed damping D not equal zero, which is the normal case, we only get one eigenvalue at zero. This corresponds to the fact that absolute values of angles cannot be observed in P_e and V_t. This is quite natural since electric power and voltage depend on differences in angles, not their absolute value. To control the absolute values of angles we need to control the rigid body motion of the power system and that is not the purpose with this design.

2.4 The equations

In this section the equations for the model are given. All calculations are done in the d-q-system given by Parks transformation.

The synchronous machine equations

Each generator is governed by two differential equations, the swing equation describing the mechanical rotation of the rotor and one for the induced voltage E'_q. From energy balance at steady-state we get the **linearised** swing equation for generator i

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i}{dt^2} + \frac{D_i}{\omega_R}\frac{d\delta_i}{dt} = P_{m_i} - P_{e_i} \tag{2.2}$$

Here H is a machine time constant, D machine damping, δ the angle from reference to the machine q-axis. The influence on E'_q from the exciter and the d-component of the current is given by

$$T'_{d0_i}\frac{dE'_{q_i}}{dt} = U_{f_i} - E'_{q_i} - (X_{d_i} - X'_{d_i})I_{d_i} \tag{2.3}$$

where the d-q-system is chosen so the q-axis lags the exciter flux by 90° and the d-axis lags the q-axis by 90°. This indicates that I_d is positive under inductive load. The active power from each generator is given by

$$\begin{aligned} P_{e_i} =& I_{t_i}V_{t_i}cos(\phi_i) = I_{d_i}V_{d_i} + I_{q_i}V_{q_i} = I_{d_i}(X_{q_i}I_{q_i}) + I_{q_i}(E'_{q_i} - X'_{d_i}I_{d_i}) \\ =& I_{d_i}I_{q_i}(X_{q_i} - X'_{d_i}) + I_{q_i}E'_{q_i} \end{aligned} \tag{2.4}$$

where I_t, V_t and ϕ are generator current, voltage and angle from current to voltage respectively for machine i. The terminal voltage is given by

$$V_{t_i} = \sqrt{V_{d_i}^2 + V_{q_i}^2} = \sqrt{(X_{q_i}I_{q_i})^2 + (E'_{q_i} - X'_{d_i}I_{d_i})^2} \tag{2.5}$$

The network

When many machines are connected in a power system they have a mutual influence of varying magnitude. The currents I_d and I_q, which are part of each generator's equations, are determined by other machines states and the network topology. If we assume that all loads in the network can be described by impedances, then the network yields the algebraic relation $I_t = YV_t$. Where I_t, V_t and Y are vectors of generator currents, voltages and the network admittance matrix respectively. By expanding, linearising and rearranging, the dependence of I_d and I_q can be expressed in δ and E'_q for small perturbations around an equilibrium point.

Algebraic relations

Consider a network with n connected generators. The currents in the network are governed by

$$I_t = YV_t \tag{2.6}$$

where

$$I_t = \begin{pmatrix} \bar{I}_{t_1} & \cdots & \bar{I}_{t_n} \end{pmatrix}^T$$

and

$$V_t = \begin{pmatrix} \bar{V}_{t_1} & \cdots & \bar{V}_{t_n} \end{pmatrix}^T$$

By substituting V_t with

$$V_t = V_q + V_d = E_q' - jX_d'I_d - jX_qI_q$$

where E_q', I_d, I_q are vectors of length n and X_d and X_q are diagonal matrices of order $n \times n$ with elements from the generators, we get

$$I_t = Y(E_q' - jX_d'I_d - jX_qI_q) \tag{2.7}$$

By dividing I_t in d-q-components and dividing Y in real and imaginary-parts we get

$$(I_d + I_q) = (G + jB)(E_q' - jX_d'I_d - jX_qI_q) \tag{2.8}$$

Rearranging gives

$$(I - BX_d' + jGX_d')I_d + (I - BX_q + jGX_q)I_q = (jB + G)E_q' \tag{2.9}$$

where I has been used for a unit matrix of order $n \times n$.

I_d, I_q, E_q' are vectors with complex elements which can be written as

$$\begin{aligned} I_d &= e^{j(\delta-\pi/2)}|I_d| \\ I_q &= e^{j\delta}|I_q| \\ E_q' &= e^{j\delta}|E_q'| \end{aligned}$$

where $|I_d| = (|I_{d_1}|, |I_{d_2}|, |I_{d_3}|)^t$, $|I_q|$, $|E_q'|$ in analog.

Introduce the notations

$$\begin{aligned} e^{j\delta} &= \operatorname{diag}\begin{pmatrix} e^{j\delta_1} & \cdots & e^{j\delta_n} \end{pmatrix} \\ &= \operatorname{diag}\begin{pmatrix} \cos\delta_1 & \cdots & \cos\delta_n \end{pmatrix} + j \cdot \operatorname{diag}\begin{pmatrix} \sin\delta_1 & \cdots & \sin\delta_n \end{pmatrix} \\ &= \cos(\delta) + j \cdot \sin(\delta) \end{aligned}$$

and $e^{j(\delta-\pi/2)} = -j \cdot \cos(\delta) + \sin(\delta)$

This together with rearranging yields

$$\begin{aligned} &\Big((I - BX_d')\sin\delta + GX_d'\cos\delta + j\Big(GX_d'\sin\delta + (BX_d' - I)\cos\delta\Big)\Big)|I_d| \\ &+ \Big((I - BX_q)\cos\delta - GX_q\sin\delta + j\Big(GX_q\cos\delta + (I - BX_q)\sin\delta\Big)\Big)|I_q| \\ &= \Big(\Big(G\cos\delta - B\sin\delta\Big) + j\Big(B\cos\delta + G\sin\delta\Big)\Big)|E_q'| \end{aligned} \tag{2.10}$$

By equating real and imaginary part we get two relations

$$
\begin{aligned}
f_1 =& \left(\left(I - BX'_d\right) \sin\delta + GX'_d \cos\delta \right) |I_d| \\
&+ \left(\left(I - BX_q\right) \cos\delta - GX_q \sin\delta \right) |I_q| \\
&- \left(G\cos\delta - B\sin\delta \right) |E'_q| = 0
\end{aligned}
\tag{2.11}
$$

and

$$
\begin{aligned}
f_2 =& \left(GX'_d \sin\delta + (BX'_d - I)\cos\delta \right) |I_d| \\
&+ \left(GX_q \cos\delta + (I - BX_q)\sin\delta \right) |I_q| \\
&- \left(B\cos\delta + G\sin\delta \right) |E'_q| = 0
\end{aligned}
\tag{2.12}
$$

Linearising algebraic relations

Linearising f_1 gives

$$
\begin{aligned}
\frac{\partial f_1}{\partial |I_d|} &= a_{11} = (I - BX'_d)\sin\delta_0 + GX'_d\cos\delta_0 \\
\frac{\partial f_1}{\partial |I_q|} &= a_{12} = (I - BX_q)\cos\delta_0 - GX_q\sin\delta_0 \\
\frac{\partial f_1}{\partial |E'_q|} &= b_{11} = -G\cos\delta_0 + B\sin\delta_0 \\
\frac{\partial f_1}{\partial \delta} &= b_{12} = \left((I - BX'_d)\cos\delta_0 - GX'_d\sin\delta_0 \right) \mathrm{diag}(|I_{d_0}|) \\
&\quad + \left((BX_q - I)\sin\delta_0 - GX_q\cos\delta_0 \right) \mathrm{diag}(|I_{q_0}|) \\
&\quad + \left(G\sin\delta_0 + B\cos\delta_0 \right) \mathrm{diag}(|E'_{q_0}|)
\end{aligned}
$$

Linearising f_2 gives

$$
\begin{aligned}
\frac{\partial f_2}{\partial |I_d|} &= a_{21} = GX'_d\sin\delta_0 + (BX'_d - I)\cos\delta_0 \\
\frac{\partial f_2}{\partial |I_q|} &= a_{22} = GX_q\cos\delta_0 + (I - BX_q)\sin\delta_0 \\
\frac{\partial f_2}{\partial |E'_q|} &= b_{21} = -B\cos\delta_0 - G\sin\delta_0 \\
\frac{\partial f_2}{\partial \delta} &= b_{22} = \left(GX'_d\cos\delta_0 + (I - BX'_d)\sin\delta_0 \right) \mathrm{diag}(|I_{d_0}|) \\
&\quad + \left(-GX_q\sin\delta_0 + (I - BX_q)\cos\delta_0 \right) \mathrm{diag}(|I_{q_0}|) \\
&\quad + \left(B\sin\delta_0 - G\cos\delta_0 \right) \mathrm{diag}(|E'_{q_0}|)
\end{aligned}
$$

Which leads to

$$
\begin{aligned}
a_{11}\Delta I_d + a_{12}\Delta I_q + b_{11}\Delta E'_q + b_{12}\Delta\delta &= 0\\
a_{21}\Delta I_d + a_{22}\Delta I_q + b_{21}\Delta E'_q + b_{22}\Delta\delta &= 0
\end{aligned}
$$

Which gives the matrix identity

$$
\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \Delta I_d \\ \Delta I_q \end{pmatrix} = - \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} \Delta E'_q \\ \Delta\delta \end{pmatrix}
$$

If A is invertible we get

$$
\begin{pmatrix} \Delta I_d \\ \Delta I_q \end{pmatrix} = - \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} \Delta E'_q \\ \Delta\delta \end{pmatrix}
$$

This can finally be stated as

$$
\begin{pmatrix} \Delta I_d \\ \Delta I_q \end{pmatrix} = \begin{pmatrix} F_{I_d E'_q} & F_{I_d \delta} \\ F_{I_q E'_q} & F_{I_q \delta} \end{pmatrix} \begin{pmatrix} \Delta E'_q \\ \Delta\delta \end{pmatrix} \tag{2.13}
$$

Linearised state space description

The equations for a generator with index i can be written as

$$
\begin{aligned}
\frac{d\delta_i}{dt} &= \omega_i \\
\frac{d\omega_i}{dt} &= -\frac{D_i}{2H_i}\omega_i + \frac{\omega_R}{2H_i}(P_{m_i} - P_{e_i}) \\
\frac{dE'_{qi}}{dt} &= \Big(u_{f_i} - E'_{q_i} - (X_{d_i} - X'_{d_i})I_{d_i}\Big)/T'_{d0_i} \\
P_{e_i} &= I_{d_i}I_{q_i}(X_{q_i} - X'_{d_i}) + I_{q_i}E'_{q_i} \\
V_{t_i} &= \sqrt{(X_{q_i}I_{q_i})^2 + (E'_{q_i} - X'_{d_i}I_{d_i})^2}
\end{aligned} \tag{2.14}
$$

Linearising and eliminating I_d and I_q give the linearised state space description

$$
\begin{aligned}
\frac{d}{dt}\begin{pmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \end{pmatrix} &= \begin{pmatrix} 0 & I & 0 \\ A_{\omega\delta} & A_{\omega\omega} & A_{\omega E'_q} \\ A_{E'_q\delta} & 0 & A_{E'_q E'_q} \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \end{pmatrix} \\
&\quad + \begin{pmatrix} 0 & 0 \\ 0 & B_{\omega P_m} \\ B_{E'_q u_f} & 0 \end{pmatrix} \begin{pmatrix} \Delta u_f \\ \Delta P_m \end{pmatrix} \qquad (2.15)\\
\begin{pmatrix} \Delta V_t \\ \Delta P_e \end{pmatrix} &= \begin{pmatrix} C_{V_t\delta} & 0 & C_{V_t E'_q} \\ C_{P_e\delta} & 0 & C_{P_e E'_q} \end{pmatrix} \begin{pmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \end{pmatrix}
\end{aligned}
$$

where $\Delta\delta$, $\Delta\omega$, Δu_f, ΔP_m , ΔV_t, ΔP_e are vectors of order $n \times 1$ and the blocks in the matrices are of order $n \times n$.

The matrix blocks in A are

$$
\begin{aligned}
A_{\omega\delta} =& -\operatorname{diag}(\frac{\omega_R}{2H_i}\frac{\partial P_{e_i}}{\partial I_{d_i}})F_{I_d\delta} - \operatorname{diag}(\frac{\omega_R}{2H_i}\frac{\partial P_{e_i}}{\partial I_{q_i}})F_{I_q\delta} \\
A_{\omega\omega} =& -\operatorname{diag}(\frac{D_i}{2H_i}) \\
A_{\omega E'_q} =& -\frac{\omega_R}{2H_i}(\operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{d_i}})F_{I_d E'_q} + \operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{q_i}})F_{I_q E'_q} + \operatorname{diag}(\frac{\partial P_{e_i}}{\partial E'_{q_i}})) \\
A_{E'_q\delta} =& \operatorname{diag}(\frac{\partial \dot{E}'_{q_i}}{\partial I_{d_i}})F_{I_d\delta} \\
A_{E'_q E'_q} =& -\operatorname{diag}(\frac{1}{T'_{d0_i}}) + \operatorname{diag}(\frac{\partial \dot{E}'_{q_i}}{\partial I_{d_i}})F_{I_d E'_q}
\end{aligned}
\tag{2.16}
$$

The matrix blocks in B are

$$
\begin{aligned}
B_{\omega P_m} &= \operatorname{diag}(\frac{\omega_R}{2H_i}) \\
B_{E'_q u_f} &= \operatorname{diag}(\frac{1}{T'_{d0_i}})
\end{aligned}
\tag{2.17}
$$

The matrix blocks in C are

$$
\begin{aligned}
C_{V_t\delta} =& \operatorname{diag}(\frac{\partial V_{t_i}}{\partial I_{d_i}})F_{I_d\delta} + \operatorname{diag}(\frac{\partial V_{t_i}}{\partial I_{q_i}})F_{I_q\delta} \\
C_{V_t E'_q} =& \operatorname{diag}(\frac{\partial V_{t_i}}{\partial I_{d_i}})F_{I_d E'_q} + \operatorname{diag}(\frac{\partial V_{t_i}}{\partial I_{q_i}})F_{I_q E'_q} + \operatorname{diag}(\frac{\partial V_{t_i}}{\partial E'_{q_i}}) \\
C_{P_e\delta} =& \operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{d_i}})F_{I_d\delta} + \operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{q_i}})F_{I_q\delta} \\
C_{P_e E'_q} =& \operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{d_i}})F_{I_d E'_q} + \operatorname{diag}(\frac{\partial P_{e_i}}{\partial I_{q_i}})F_{I_q E'_q} + \operatorname{diag}(\frac{\partial P_{e_i}}{\partial E'_{q_i}})
\end{aligned}
\tag{2.18}
$$

Where the matrices $F_{I_d\delta}$, $F_{I_q\delta}$, $F_{I_d E'_q}$, $F_{I_q E'_q}$ in general have off diagonal elements and the other matrices are diagonal.

The partial derivatives in (2.17) – (2.19) are

$$
\begin{aligned}
\frac{\partial P_{e_i}}{\partial I_{d_i}} &= I_{q0_i}(X_{d_i} - X'_{d_i}) \\
\frac{\partial P_{e_i}}{\partial I_{q_i}} &= I_{d0_i}(X_{q_i} - X'_{d_i}) + E'_{q0_i} \\
\frac{\partial P_{e_i}}{\partial E'_{q_i}} &= I_{q0_i} \\
\frac{\partial \dot{E}'_{q_i}}{\partial I_{d_i}} &= -\frac{(X_{d_i} - X'_{d_i})}{T'_{d0_i}} \\
\frac{\partial V_{t_i}}{\partial I_{d_i}} &= -\frac{(E'_{q0_i} - X'_{d_i} I_{d0_i})X'_{d_i}}{V_{t0_i}} \\
\frac{\partial V_{t_i}}{\partial I_{q_i}} &= \frac{X^2_{q_i} I_{q0_i}}{V_{t0_i}} \\
\frac{\partial V_{t_i}}{\partial E'_{q_i}} &= \frac{(E'_{q0_i} - X'_{d_i} I_{d0_i})}{V_{t0_i}}
\end{aligned}
\tag{2.19}
$$

Algebraic constraint in outputs from linearised model

The linearised model will contain an algebraic constraint due to power balance. Since no power can be stored in the grid or in the loads there must be balance between electric generation from the generators (P_{e_i}) and load consumption (P_{load_i}) (included with network losses). This gives

$$\sum_i P_{e_i} = \sum_i P_{load_i} \tag{2.20}$$

After linearising the voltage dependence of the loads we have

$$P_{load_i} = \alpha V_{t_i} \tag{2.21}$$

where α is a linearising constant. Then we can write an algebraic constraint due to the power balance as

$$\begin{pmatrix} P_{e_1} & \cdots & P_{e_n} & V_{t_1} & \cdots & V_{t_n} \end{pmatrix} \begin{pmatrix} h_1 \\ \vdots \\ h_{2n} \end{pmatrix} = 0 \tag{2.22}$$

2.5 Matlab implementation

A model of the described type with $3n$ states has been implemented as a user defined function in Matlab (Moler, Little and Bangert, 1987) and has been compared to the commercial computer package Simpow (Adielsson, 1982; Lindqvist, 1985). The user specifies parameters for generators, network data and initial conditions in the function. The program automatically calculates a state space description of the multimachine model. See Appendix A.2.

The user supplies the program the following data

1 General data
   - Number of machines.
   - The synchronous frequency (e.g. 50 or 60 Hz).
2 Machine data
   - Machine Rating: S_n
   - Parameters: X_d, X'_d, X_q, H, D, T'_{d0}
   - Initial values: V_{t0}, P_0, Q_0, θ_0 (angle from ref to V_{t0})
3 Network data Z_{ij} the complex impedance between nod i and j.
4 Load data $P_{0_{load}}$, $Q_{0_{load}}$

The program calculates

1 For each machine
   - δ_0, E'_{q0}, I_{d0}, I_{q0}
2 For each load Z_{series}
3 For the network
   - Admittance matrix Y
   - Susceptance matrix B
   - Conductance matrix G
4 From the linearised relations
   - $F_{I_dE'_q}$, $F_{I_qE'_q}$, $F_{I_d\delta}$, $F_{I_q\delta}$
5 Calculates for the linearised state space description
   - Partial derivatives
   - A-matrix
   - B-matrix
   - C-matrix

2.6 Case study to check model

This section describes a multimachine case study. The purpose was to compare the linearised multimachine model, implemented in Matlab, with Asea Brown Boveri's simulation package Simpow. Simpow has been used by Asea Brown Boveri engineers for several years as a design tool to analyse dynamic behaviour of power systems. Therefore there are good reasons to believe that Simpow gives accurate results. A generator model without damper windings were used in Simpow and the loads were modelled as impedances. See Appendix A.3 for more details.

Organization of case study

The case study was done in the following steps. (Optpow and Transta are parts of the simulation package Simpow.)

1 Specify input data to loadflow calculations.
2 Loadflow calculations in Optpow.
3 Simulation in Transta with small changes in voltage references.
4 Do postprocessing and editing to get time series for input (U_f) and the two outputs, voltage (V_t) and electric power (P_e), so they all can be read into Matlab.

5 Prepare the linearised Matlab model with parameters, initial data, load and network data.

6 Calculate linearised model in Matlab.

7 Read P_e, V_t, and U_f from Simpow into Matlab.

8 Give same input U_f to the linearised model and simulate P_e and V_t.

9 Compare results output P_e and V_t from linearised model and from Simpow.

The control signals, generator voltages and powers are shown in Figure 2.5, 2.6 and 2.7 respectively

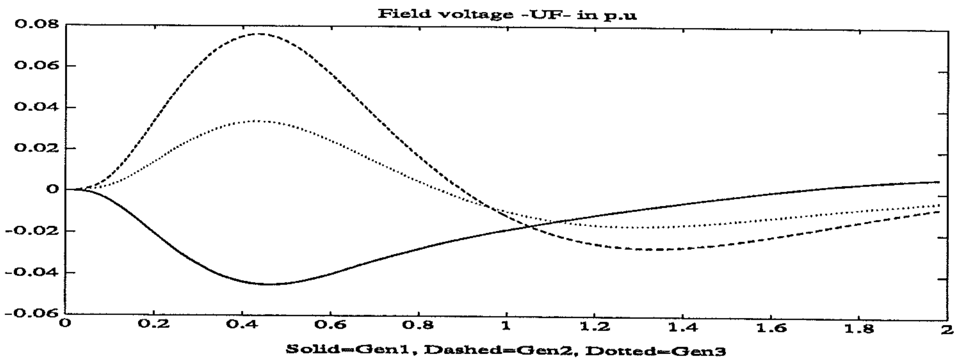


Figure 2.4 U_f - Field Voltage for Generator 1-3 in p.u.

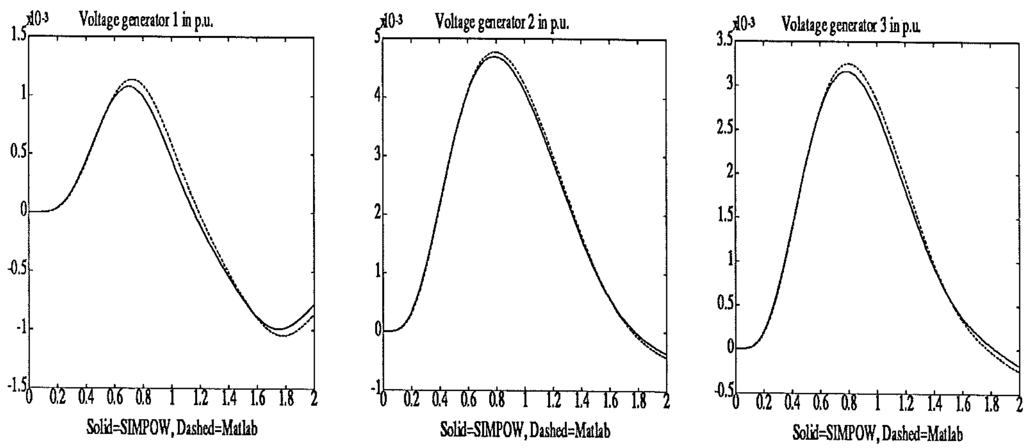


Figure 2.5 V_{t_1}, V_{t_2}, V_{t_3} - Terminal Voltage for Generator 1-3 in p.u.

The electric powers from the generators are shown in Figure 2.6.

2.7 Conclusions

Ocular inspection of the simulation results gives

- Terminal voltage V_t and electric power P_e correspond very well to the Simpow model.
- Both oscillation frequencies and phase correspond good.
- Damping corresponds to Simpow.

The primary goal of the design is to achieve good damping on power oscillations. Since the power oscillations in the model and Simpow correspond

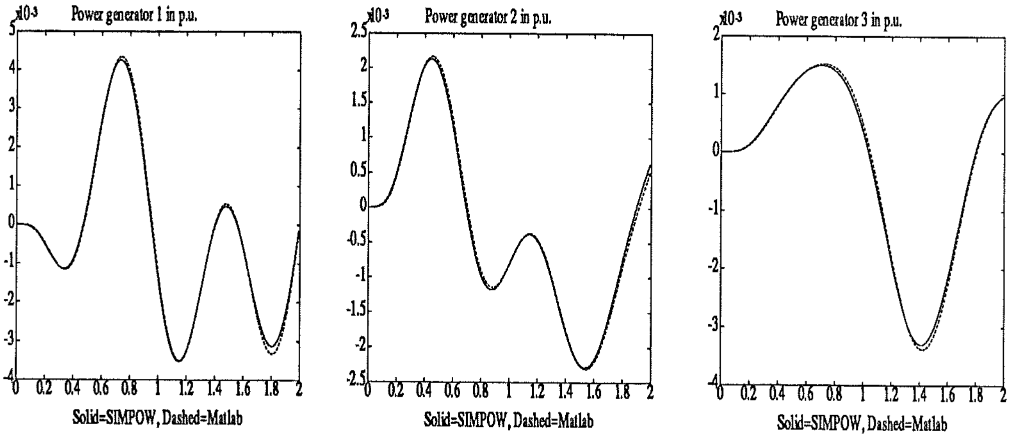


Figure 2.6 P_{e_1}, P_{e_2}, P_{e_3} - Electric Power for Generator 1-3 in p.u.

very good we can say that we have a model which can serve our design purpose. The model is of low order and captures the multimachine characteristics with a high degree of exactness and it is possible to do mathematical analysis with the model. In summary, we have an excellent model for our purposes.

3. Review of Today's Design Methods

The goal of this chapter is to point out drawbacks, or unfavourable characteristics, of design methods which have been published during recent years. The chapter should also serve as a motivation for new methods.

3.1 Design objectives

There is a need for a reliable technique to tune and find structure for PSS equipment together with determination of AVR gains. This technique must capture the multimachine characteristics and should also take into account the trade off between high AVR-gain and extra PSS-signals. In Wilson and Aplevich (1986) the following criteria for a good design procedure can be found

- The method should include important system dynamics for a multimachine power system.
- The method should be a co-ordinated design of stabilizers.
- The method should specify system performance in system variables.

I would like to add

- The design should be robust in the sense that stability properties should be preserved when
  - Load level changes.
  - Production dispatch changes.
  - Grid configuration changes.
  - The model has uncertainty.

3.2 In the literature proposed design methods

In this section various design methods are discussed.

Modal theory

In recent papers design methods based on eigenvalues and eigenvectors have become popular, so called modal theory (Arcidiacono et al, 1980). The eigenvectors which correspond to the EOM contain information about system properties. Magnitude and sign of the right eigenvector components can be used to characterize a mode. We can see which and how the machines in the power system swing for each mode. The eigenvector-eigenvalue technique can be refined by also using left eigenvectors. Left eigenvectors tell how initial states excite a mode. Together, right and left eigenvectors can be used to get direct information of how eigenvalues are influenced by a change in a matrix element (Wilkinson, 1965). This can be extended to eigenvalue sensitivity to stabilizer parameters (Vournas Papadias, 1987) to decide where to place stabilizers (de

Mello Nolan et al, 1980; Castro et al, 1988) and to iteratively tune stabilizers by tracking eigenvalues (Lefbere, 1983). The eigenvalues - eigenvector technique has even been used to do model reduction (Geeves, 1988).

Direct methods

A direct way to see where to place stabilizers is to add on a damping term at the different generators, one at the time, and see how the eigenvalues move. De Mello and Nolan (1980) use logarithmic increment as a measure of effect and Abdalla, Hassan and Tweig (1984) use normalized real part (NRP) as a measure. An interesting comment is made in DeMello, Nolan et al (1980) who point out that result from direct methods can differ compared to eigenvector methods, but do not explain why.

Root locus

One way to get a criterion for PSS siting and tuning is to calculate the initial derivative of root locus and to use this to place and tune stabilizers (Larsen and Swann, 1981; Arcidiancano, 1980).

Pole placement

Pole placement restricted to local control is a well known control problem (Konigorski, 1987). Applications to power systems have been done by Lim Eng et al (1985), Lim and Elangovan (1985) and Padiyar et al (1980).

3.3 Drawbacks of today's design methods

The drawbacks can be summarized as

No co-ordination between AVR and PSS

There is no co-ordination between high AVR-gain and PSS. The design methods above have the drawback to first accept as a rule of thumb high AVR-gain which causes steady-state problem, and then install PSS-equipment to correct the problem. A better way would be to do a co-ordinated design, which considers both the objectives of fast voltage control and preventing steady-state-problem.

Do not consider input energy

Another common drawback is that input energy is seldom or never taken into account when determining where to place and how to tune stabilizers. Unnecessarily large control signals can cause saturation.

Assume linearity between parameters and eigenvalues

A fundamental difficulty is that there is no linearity between feedback parameters and eigenvalues. Each parameter K_{ij}, in state feedback $u = -Kx$, appears linearly in the coefficients of the characteristic polynominal $p(s)$ for $A - BK$. However, this definitively does not imply linearity between parameters and the eigenvalues λ_i which fulfil $p(\lambda_i) = 0$. Then it is very doubtful if sequential methods, where one parameter at the time is tuned, will give accurate results. Because linearity does not hold, we cannot in general superpose results from changing one parameter at a time to multiparameter tuning.

There is usually an implicit assumption that we can linearise the eigenvalue dependence of parameters for small changes.

Discontinuities in eigenvalue sensitivity

The sensitivity of eigenvalues with respect to parameters can be discontinuous when the characteristic polynomial $p(s)$ has multiple roots. Hence, if two eigenvalue trajectories (root locus) cross each other, eigenvalue methods can provide useless and misleading information close to this point. This will be illuminated with two examples.

EXAMPLE 3.1—Complex poles become real
Consider the second order system (3.1) with a small ε.

$$\frac{dx}{dt} = \begin{pmatrix} -2+\varepsilon & -1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u \tag{3.1}$$

with the control law

$$u = -k \cdot x_1 + u_{ref}$$

This gives totally different $\frac{d\lambda}{dk}$ for arbitrarily small ε and $k = 0$. If we set $\varepsilon = 0.01$ we get two complex eigenvalues $\lambda_{1,2} = -0.9950 \pm 0.0999j$. Eigenvalue sensitivity with respect to k gives

$$\frac{d\lambda_{1,2}}{dk} = -0.500 \mp 4.981j$$

If we instead set $\varepsilon = -0.01$ we get two real eigenvalues $\lambda_1 = -1.1051$ and $\lambda_2 = -0.9049$. Eigenvalue sensitivity gives

$$\frac{d\lambda_1}{dk} = -5.5187$$
$$\frac{d\lambda_2}{dk} = 4.5187$$

The results with $\varepsilon = 0.01$ and $\varepsilon = -0.01$ are totally different and will be so for an arbitrarily small ε ! This indicates that we sometimes can get misleading results from eigenvalue sensitivity. Can we get the same type of results for complex modes? The answer is yes. □

EXAMPLE 3.2— Complex double poles
Consider the fourth order system (3.2) with a small ε.

$$\frac{dx}{dt} = \begin{pmatrix} -2 & -3 & -2 & -1+\varepsilon \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u \tag{3.2}$$

with the control law

$$u = k \cdot x_4 + u_{ref}$$

Totally different $\frac{d\lambda}{dk}$ are obtained for arbitrarily small ε. If we set $\varepsilon = -0.1$ and $k = 0$ all $\frac{d\lambda}{dk}$ become complex numbers with dominating imaginary part. If we instead set $\varepsilon = 0.1$ and $k = 0$ all $\frac{d\lambda}{dk}$ become complex numbers with dominating real part. In this case $\frac{d\lambda}{dk}$ give useless information and cannot be

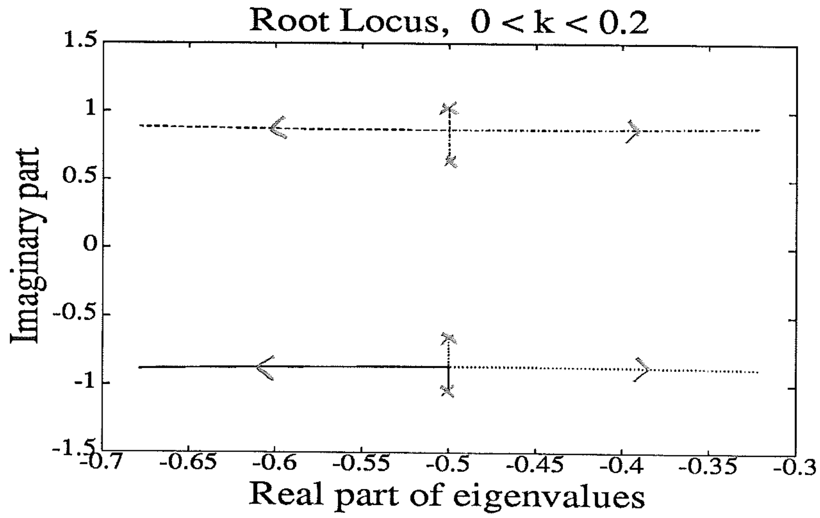


Figure 3.1 Root locus for system (3.2) with $\varepsilon = -0.1$ and feedback $u = k \cdot x_4$ and $0 < k < 0.2$ The four eigenvalues are solid, dashed, dotted, dash-dotted respectively and start to move parallel with the imaginary axis.

used for design purposes. The reason becomes obvious if we draw a root locus for system (3.2) with $\varepsilon = -0.1$ when $0 < k < 0.2$.

The direction of the root locus, which is equal to $\frac{d\lambda}{dk}$, changes abruptly after the double pole. First, it moves parallel with the imaginary axis which gives an imaginary $\frac{d\lambda}{dk}$, after the double poles it moves parallel with the real axis which gives a real $\frac{d\lambda}{dk}$. Hence, all methods, which use derivatives or differentiation of type $\frac{\Delta\lambda}{\Delta k}$ to find good stabilizer siting, can be dubious if two poles are close, or two root locus cross each other during tuning. □

Relevance for power systems? From the two examples we can only draw the conclusion that we sometimes can have discontinuities in eigenvalue sensitivity. Whether this can happen in power systems is hard to tell. Until the contrary is proved, we can not neglect this problem in power systems. When we have, say 200 machines, with 199 complex modes it seems likely that at least some branches of the root locus will cross each other during tuning.

Co-ordination of inputs

A serious drawback of all methods is that they do not treat the problem as a multivariable problem. All methods are suitable for questions of the type,

- Given one input signal and a parametrized control law, how will small variations in **one** parameter influence each eigenvalue?

Both direct methods, root locus and modal theory consider one input at a time and from this try to draw conclusions how we should use the inputs in a multivariable design. The power oscillation problem is due to the fact that many machines swing together or against each other. To success in damping of oscillations we must co-ordinate the input signals. A relevant question is therefore instead

- Given a number of input signals, how shall they be used in a **co-ordinated** way to damp power oscillations?

Methods presented in this chapter have not answered the latter question but hopefully the following chapters will do better.

4. Linear Quadratic Regulator Design

The goal of this chapter is to show that Linear Quadratic (LQ) regulator design can be used successfully on the power oscillation problem. Suitable references are Anderson and Moore (1971) and Friedland (1986).

4.1 Presentation of the LQ method

This section presents the LQ design method. Consider the system (4.1) where $A \in R^{n \times n}$ and $B \in R^{n \times p}$

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4.1}$$

with initial conditions $x(0) = x_0 \neq 0$ and the control law

$$u(t) = -Kx(t) \tag{4.2}$$

Our problem is to find the optimal K in the sense that it minimizes the quadratic loss function J defined by

$$J = \int_0^{\infty} x^T(t)Qx(t) + u^T(t)Ru(t)\, dt \tag{4.3}$$

for all x_0. The matrix Q is the loss matrix for states and should be positive semi-definite to guarantee stability and the loss matrix for control signals, R, should be positive definite to prevent infinite gains. The solution K which is time independent is called the steady-state solution. If the system (4.1) is controllable there will exist such a solution. The solution K is given by solving S from the algebraic Riccati equation

$$0 = SA + A^TS + Q - SBR^{-1}B^TS \tag{4.4}$$

and calculating K from

$$K = R^{-1}B^TS \tag{4.5}$$

Furthermore, the minimal loss J_0 becomes

$$J_0 = x_0^TSx_0 \tag{4.6}$$

In most control design software packages there are fast routines, which solve K from (4.4) and (4.5). This reduces the design problem to the choice of loss matrices Q and R.

4.2 Application of LQ to power system problems

LQ techniques have been discussed as a candidate in Arnoutivic (1987), Lefebere (1983), Wilson and Aplevich (1986) and have many good features but unfortunately also drawbacks. Application of LQ has been done in Wilson Aplevich (1986) who used identification technique to make a single machine model that should capture the multimachine dynamics. Arnatovic (1987) uses the LQ design to find a good location for the closed loop poles and then applies projective control to come as close as possible to the desired spectra. A recent approach can be found in Huang et al (1988) who use parametric LQ to find a good feedback structure.

4.3 Credits and drawbacks of LQ control

Opinions about LQ design are divided and the argumentation can briefly be summarized as

Credits of LQ control

+ Is based on solid theory (e.g. Anderson and Moore, 1971) and is a standard subject in control textbooks (Åström and Wittenmark, 1984; Åström, 1970; Friedland, 1986).
+ Does trade off between input energy and pole placement.
+ Does trade off between feedback gains, in our case AVR-gains and PSS equipment.
+ Guarantees stable closed loop.

Drawbacks of LQ control

– LQ regulator design results in a global state feedback and to implement this we need all states available or a high order observer.
– The design method will not provide insight into the system properties.
– The user choice of weight matrices in the quadratic loss function is hard.
– Robustness is not explicit.

4.4 Loss matrices

To do LQ-design we need to choose the loss function J. In (4.3) we have to choose Q and R. The design method itself is then automatic, e.g. when we have chosen our loss matrices Q and R we just have to run a software package to get the optimal control law. This converts the design problem to specification of loss matrices which capture the design objectives. LQ design is an iterative design method where Q and R are the knobs to turn. The choice of loss matrices seems to be the most important issue to get a good control law. In the published papers about LQ-design in power system, only diagonal loss matrices have been considered (Wilson and Aplevich, 1986; Arnatovic, 1987; Huang et al, 1988). Since the choice of loss matrices is a very critical step in the design, this section will be dedicated to finding some alternative problem formulations, which result in not necessarily diagonal loss matrices.

Dimension free units in the model

To be able to compare states and control signals it is important to normalize the quantities. In our model all control signals and outputs are normalized in p.u. of nominal values. States which correspond to angles or their derivatives all have the unit electric *rad* or *rad/s*, which implies that the same type of states can be compared on different generators.

Transformation of states

When we do the design the objectives usually concern the output variables, like power and voltage. Furthermore the resulting control law ought to be feedback from physical outputs. Then it is convenient to transform our model so the variables we are interested in become states. This can be done if we have a minimal state model and choose as many linearly independent outputs as states in the model. In our case we have to remove one state in the model and choose $3n-1$ independent output variables. We can for example choose all generator voltages and speeds and all generator powers, but one. The excluded electric power is a linear combination of the other state variables.

Diagonal Elements

The easiest way to choose loss matrices is to choose diagonal matrices and this have been done in the published articles Wilson and Aplevich (1986), Arnatovic (1987), Huang et al (1988). The following rules of thumb can be given for the diagonal elements in loss matrices.

- The magnitude of Q_{ii} should be chosen according to the tolerated squared deviation of the states (Åström and Wittenmark, 1984).
- Punishment on states which are associated with speed have main influence on damping of oscillatory modes.
- The ratio $\sqrt{Q_{ii}/R_{ii}}$ is an estimation of the resulting regulator gain if x_i and u_i are normalized so $x_i/u_i \approx 1$ in steady state.

We will illuminate the rules by an example.

EXAMPLE 4.1—Diagonal loss matrices
Consider the power system in Figure 4.1 with parameters and load conditions as in Appendix B.1.

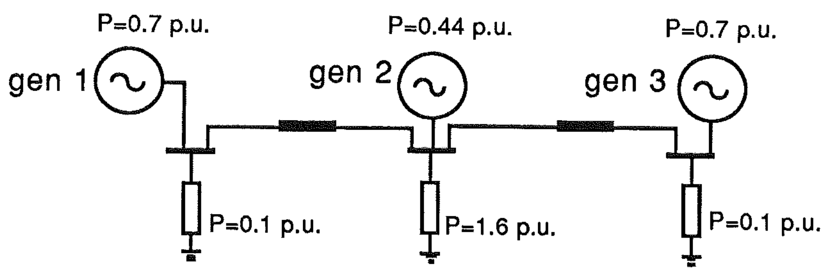


Figure 4.1 A three machine power system.

A linearised state space model for the power system, according to Chapter 2, will have the eigenvalues

$$\begin{aligned}
\lambda_{1,2} &= -0.103 \pm 7.37j \qquad (\zeta = 0.014)\\
\lambda_{3,4} &= -0.095 \pm 4.02j \qquad (\zeta = 0.024)\\
\lambda_5 &= -0.479\\
\lambda_6 &= -0.234\\
\lambda_7 &= -0.163\\
\lambda_8 &= -0.076
\end{aligned} \tag{4.7}$$

and the state vector is after transformation

$$x^T = \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_2} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \tag{4.8}$$

and P_{e_3} is

$$P_{e_3} = \begin{pmatrix} 0.17 & 3.39 & 0.17 & -1.00 & -1.07 & 0 & 0 & 0 \end{pmatrix} x \tag{4.9}$$

Our objective is to get voltage regulation with gain ≈ 30 and good damping of power oscillations. The gain specification gives that the ratio between the coefficients for V_t-terms and U_f-terms should be $30^2 \approx 1000$. Damping of oscillations corresponds to punishment of speed terms. We choose the loss function

$$\begin{aligned}
J = \int_0^\infty &(100V_{t_1}^2 + 100V_{t_2}^2 + 100V_{t_3}^2 + \omega_1^2 + \omega_2^2 + \omega_3^2\\
&+ 0.1U_{f_1}^2 + 0.1U_{f_2}^2 + 0.1U_{f_3}^2)dt
\end{aligned} \tag{4.10}$$

which corresponds to the loss matrices

$$\begin{aligned}
Q &= \text{diag}\begin{pmatrix} 100 & 100 & 100 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}\\
R &= \text{diag}\begin{pmatrix} 0.1 & 0.1 & 0.1 \end{pmatrix}
\end{aligned} \tag{4.11}$$

We can estimate the resulting AVR gain to $\sqrt{\frac{100}{0.1}} \approx 32$ and estimate the relative deviation between V_t and ω to $\sqrt{\frac{100}{1}} = 10$. Calculation of the optimal control law $u = -Kx$ gives

$$K = \begin{pmatrix} 29.3 & -8.1 & -1.3 & 13.20 & -3.00 & -2.01 & 0.31 & 0.58\\ -2.41 & 50.9 & -2.71 & 0.60 & 8.42 & -1.33 & -2.41 & -1.30\\ 1.15 & 32.8 & 31.2 & -12.3 & -15.7 & 0.48 & 0.33 & -2.00 \end{pmatrix} \tag{4.12}$$

and the eigenvalues of $A - BK$ are

$$\begin{aligned}
\lambda_{1,2} &= -2.55 \pm 7.20j \qquad (\zeta = 0.33)\\
\lambda_{3,4} &= -1.06 \pm 4.15j \qquad (\zeta = 0.25)\\
\lambda_{5,6} &= -3.82 \pm 2.83j \qquad (\zeta = 0.80)\\
\lambda_7 &= -3.86\\
\lambda_8 &= -3.51
\end{aligned} \tag{4.13}$$

where $\zeta = -\text{Re}(\lambda)/|\lambda|$. The response of an impulse at input number 1 with our linearised model looks like Figure 4.2. The simulation in Figure 4.2 shows that

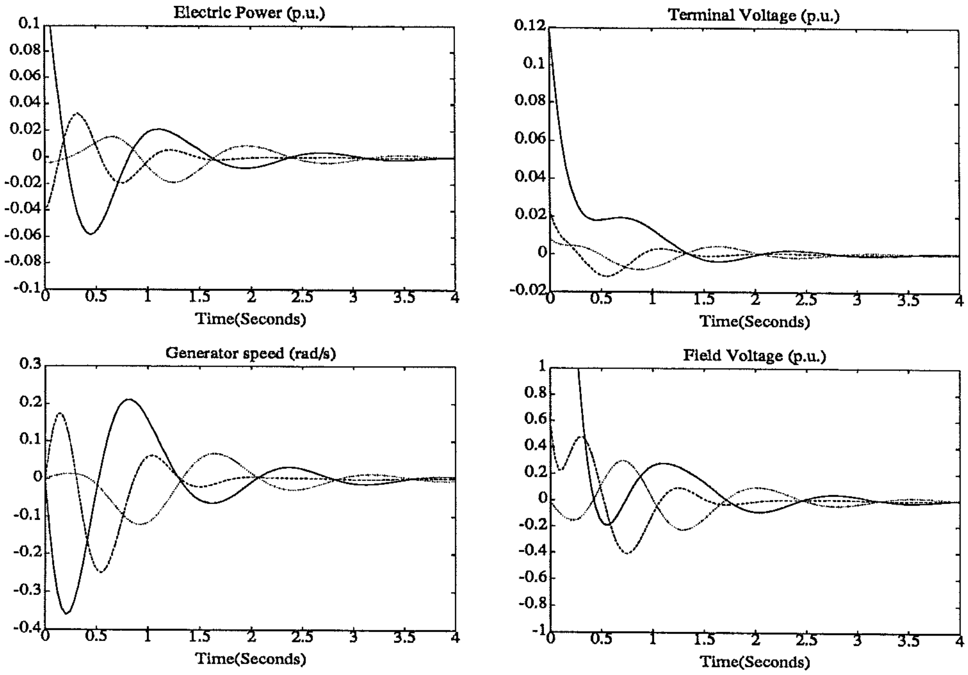


Figure 4.2 Simulated response to an impulse at input 1 with LQ-design (4.12). The simulation has been done in Matlab with the linearised model from Chapter 2. Solid=Gen1, Dashed=Gen2, Dotted=Gen3.

- Terminal voltages are well damped.
- Electric powers are mediocre damped.
- Machine speeds are poorly damped and swing against the linearisation point. See Figure 4.2
- Field voltages have, except for the impulse at number one, moderate amplitudes. □

Off Diagonal Elements

In Kailath (1980) it is said, "It is more art than science to choose loss matrices". This reflects the difficulty to give general rules for loss matrices. The choice is normally an iterative procedure which demands interactive software. One way to use the off diagonal elements in the loss matrix Q is to emphasize a certain mode. As mentioned, the speed components of the corresponding eigenvector tell how machines swing for this mode. To damp this mode we should punish this specific swing structure. Another way would be to punish relative speed. If speed terms are named ω_i, we should choose loss terms of the form $(\omega_i - \omega_j)^2$ for all i, j. An example illustrates this choice.

EXAMPLE 4.2— Loss matrices to punish relative speed

Consider the power system in Example 4.1. In this example we want to punish speed deviations relative other machines rather than speed deviations against the linearisation point. The only thing we change is the speed terms in loss function (4.10). We substitute the ω_i-terms in (4.10) with $(\omega_i - \omega_j)^2$, $i \neq j$

and we get

$$J = \int_0^\infty (100V_{t_1}^2 + 100V_{t_2}^2 + 100V_{t_3}^2 + (\omega_1 - \omega_2)^2 + (\omega_1 - \omega_3)^2 + (\omega_2 - \omega_3)^2 + 0.1U_{f_1}^2 + 0.1U_{f_2}^2 + 0.1U_{f_3}^2)dt \tag{4.14}$$

which corresponds to the block diagonal loss matrix Q

$$Q = \text{diag}\begin{pmatrix} Q_{V_t} & Q_{P_e} & Q_\omega \end{pmatrix} \tag{4.15}$$

with

$$Q_{V_t} = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{pmatrix}$$
$$Q_{P_e} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.16}$$
$$Q_\omega = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Calculation of the optimal control law $u = -Kx$ gives

$$K = \begin{pmatrix} 29.00 & -24.72 & -1.96 & 21.2 & -1.15 & -2.80 & 0.86 & 1.93 \\ -0.75 & 10.2 & -0.75 & 1.71 & 24.4 & 1.04 & -2.47 & 1.47 \\ 2.42 & 38.0 & 32.1 & -19.1 & -20.4 & 1.58 & 0.91 & -2.51 \end{pmatrix} \tag{4.17}$$

and the eigenvalues of $A - BK$ are

$$\begin{aligned} \lambda_{1,2} &= -\,3.78 \pm 7.94j \quad (\zeta = 0.43) \\ \lambda_{3,4} &= -\,1.87 \pm 4.24j \quad (\zeta = 0.40) \\ \lambda_5 &= -\,5.55 \\ \lambda_6 &= -\,3.01 \\ \lambda_7 &= -\,2.08 \\ \lambda_8 &= -\,0.09 \end{aligned} \tag{4.18}$$

Simulation of an impulse at input number 1 with our linearised model looks like Figure 4.3. The simulation in Figure 4.3 shows that

- Terminal voltages are well damped and the responses are similar to those in Figure 4.2.
- Electric powers are considerably better damped than in Figure 4.2. After 2 seconds the power oscillations in Figure 4.3 have nearly vanished compared to Figure 4.2 where the power oscillation still is observable.
- Machine speeds are now better damped than in Figure 4.2. It is very interesting to note that the oscillations in Figure 4.3 is not against the linearisation point, as in Figure 4.2, but rather against a slowly declining trajectory given by λ_8 in (4.18).
- Field voltages have, except for the impulse at number one, moderate amplitudes and no significant difference compared to Figure 4.2.

□

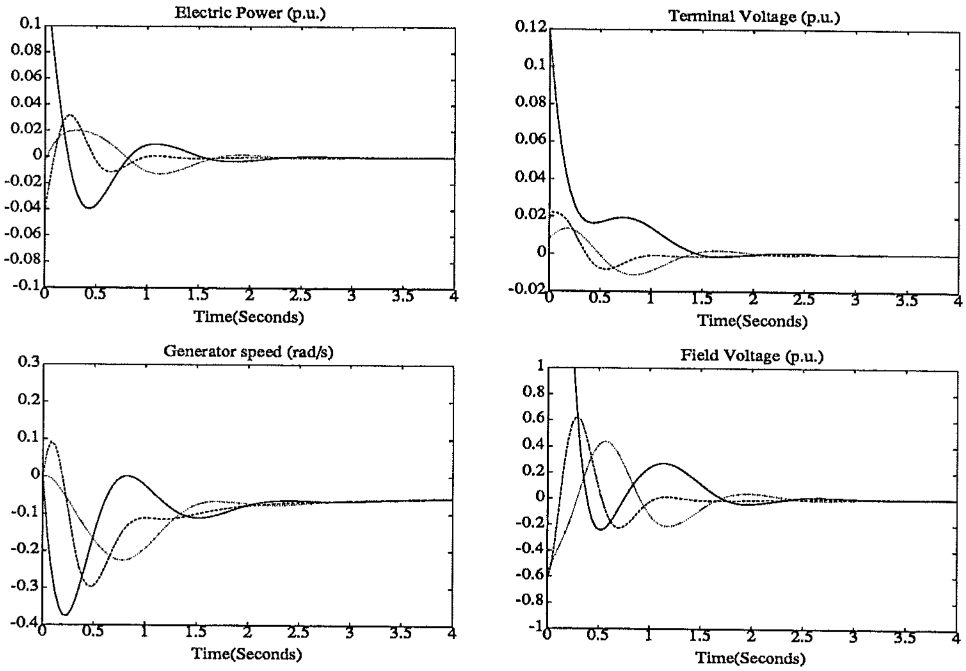


Figure 4.3 Simulated response to an impulse at input 1 with LQ-design (4.17) with off-diagonal loss matrices with damping of relative speed $(\omega_i - \omega_j)^2$. The simulations have been done in Matlab with the linearised model from Chapter 2. Solid=Gen1, Dashed=Gen2, Dotted=Gen3.

4.5 Comments on the results

The results from the two examples show that the LQ design is very dependent on loss matrices. When we choose to punish speed deviation between machines the damping on the critical eigenvalue $\lambda_{3,4}$ nearly increase by a factor 2. Both the specification of the loss function and the simulation shows that the control signal is influenced very little when the damping is increased. A major improvement to a very low cost!

Another interesting result is the speed behavior. In Example 4.2 when we only punish deviation relative other machines, the speed seems to settle down to another steady state point. The speed does not settle down, instead it will slowly decay with the time constant given by the eigenvalue λ_8. The eigenvalue λ_8 is mainly associated with the rigid body motion of all machines. In the latter example we only try to damp oscillations between machines and we do not change the eigenvalue associated with the rigid body motion of the system. One can say that in Example 4.2 we really specify a loss function that reflect what we want, namely to damp oscillations between machines. Since the loss function in Example 4.1, together with damping of oscillations, also tries to damp movements relative the operation point we use input energy on something else than the design objective. Therefore we can not achieve as good damping as in Example 4.2. An analog to a mechanical system can help the intuition.

Mechanical analog to a multimachine power system

A mechanical analog can be used as an aid in achieving a better feeling for the behavior of the electric system. This gives a nice interpretation of the two different loss functions and the resulting control laws.

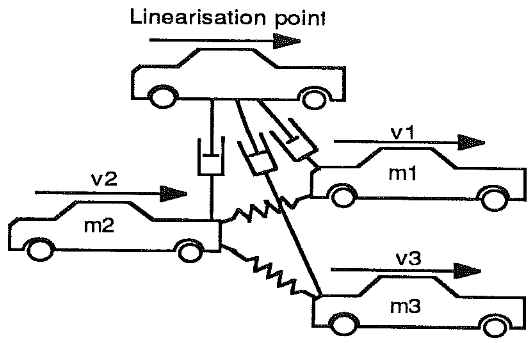


Figure 4.4 Mechanical analog to a multimachine power system with control design from Example 4.1.

Interpretation of Example 4.1 Consider Figure 4.4.

Figure 4.4 shows an interpretation of the loss function in Example 4.1. where we punish deviation from the linearisation point. The mechanical quantities speed v and mass m are analog to power system quantities generator speed ω, and inertia constant H respectively. The springs are analog to tielines. A spring stretched by a force corresponds to a tie line which transmits power. The positions of the cars determine the car which is pulling, in analog to load angels which determine the power flow in a power system. The design objective is to add damping towards the linearisation point. This is represented by the dampers connection between the cars and the linearisation point in Figure 4.4. When the system is perturbed the oscillations will be damped against the linearisation point.

Interpretation of Example 4.2 Consider now Figure 4.5.

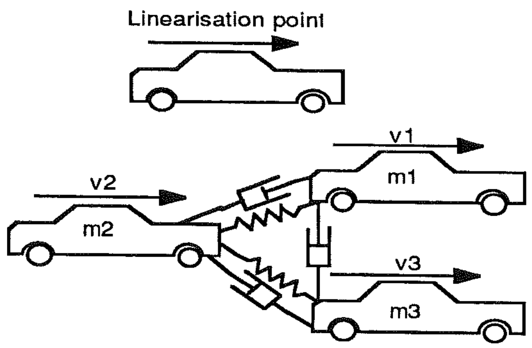


Figure 4.5 Mechanical analog to a multimachine power system with control design from Example 4.2

Figure 4.5 shows an interpretation of the loss function in Example 4.2. where we punish deviation relative other machines. The design objective is then only to add damping between the machines, not towards the linearisation point. This should be represented by the dampers connection between the cars in Figure 4.5. When the system is perturbed the oscillations between the cars will be damped, but not towards the linearisation point.

4.6 Conclusion about using LQ design methods

The LQ design method is a very good method to do a preliminary design. With preliminary we mean that we do not have to implement exactly the LQ controller. Instead the optimal LQ controller provides a control law that can be analysed and approximated to fulfil practical constraints like a sparse feedback structure. Furthermore it can be used to do analysis concerning what signals are important for a good design. It can give insight in what is possible to achieve in the best case. From the two examples we see that loss matrices are important. If our design objective is to damp the electromechanical oscillations modes (EOM) it seems better to choose a loss function which punish movement relative other machines rather than movement relative the linearisation point. I.e it is necessary to consider off diagonal loss matrices in (4.3).

5. Choice of Feedback Structure

The goal of this chapter is to present analysis tools to find a good feedback structure. We want to get deeper knowledge about the system properties that are important for the design. First we start to point out the problems and why we do not use proposed methods from the literature. Then we present a two step procedure to find the feedback structure. The first step determines the inputs which are important and the second step determines the feedback elements which are important for each input.

5.1 Control Structure

The special characteristics of a power system is that generators can be a long distance from eachother. Therefore the feedback at one machine has traditionally been restricted to local measurements. With novel technique it is possible to transmit signals over long distances in a reasonable short time. The time delay caused by the transmission can be neglected compared to the slow dynamics of the power oscillations. This opens new opportunities to add extra signals to PSS equipment if they have significance influence on damping.

The question arises, is it necessary to have all signals? Are there for each machine signals that are more important for stability than other? A first guess could be that the machine itself and its big neighbour machines are more important than small machines situated in remote areas.

From a control point of view the problem can be decomposed into two steps.

- Select a good feedback structure.
- Tune the feedback parameters.

In this chapter we concentrate on the first issue. A survey of problems for large scale systems with decentralized control can be found in Sandell et al (1978). The conclusion of this survey became "..the question of what structures are desirable for control of large scale systems has not been addressed in a truly scientific fashion". This also explains the lack of clean easy solutions. Katzberg and Johnson (1981) based on Katzberg (1977) has proposed one method, Brown and Vetter (1972) another. Both methods need heavy calculations which restrict them to small problem.

Because of the complexity of the problem it is more relevant to regard the methods below as guidelines to select a good feedback structure. This in combination with physical knowledge about the system will give the feedback structure. When we search for a good feedback structure we here assume static output feedback and look for important inputs.

5.2 Direct LQ-methods

One way to determine the feedback structure is to see how much the loss J (4.3) used in the LQ-design is changed by setting each feedback element $K_{ij} = 0$. This have been used in Huang et al (1988). The drawback is that we should either take away or use a feedback element. This implies that derivatives, which are valid for small changes, are no good indicators. As in Huang et al (1988) we have to do numerical minimization of the loss function J for every possible combination of feedback signals, to draw the right conclusions what signals are important. In the same article the authors propose a simplification to overcome the combinatoric problem. Instead of trying all possible combinations of taking out p feedback signals of m possible, they assume that we could use the signal we have used when we picked out $p-1$ from m signals. This give the algorithm; start to take 1 signal from m and then include this signal when choosing 2 signals and then include this 2 signals when choosing 3 and so on. Because of the complex nonlinear relation between J and the feedback parameters K_{ij} we cannot say anything quantitative about how this simplification influences the final choice of feedback structure. Take for example the system

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \tag{5.1}$$

with the quadratic loss $J = \int_0^\infty x^T Q x + u^T R u \, dt$ where $Q = I$ and $R = diag(1\ 1\ 10^{10})$. By inspecting the system we see that if we should only use one input signal, the best choice is obvious the expensive signal u_3 since this is the only input which makes the system controllable. This could, with the above simplification, lead to the conclusion that u_3 should be included when we choose two input signals. But the best choice of two signals is u_1 and u_2 because together they make the system controllable and are much cheaper signals than u_3. If power engineers read "apply PSS to machine number" instead of "using input number" the message becomes more apparent - superpositioning of results is in general not valid.

Why not sensitivity analysis?

The reason why we want to derive an alternative method to sensitivity analysis can be found in Chapter 2. Some of the reasons are

- It is uncertain to use derivatives, which are valid for small changes, on the open loop as a criterion for design. How a Root-Locus, as function of one parameter, moves initially seems to give very little information to decide a feedback structure from.
- Sensitivity analysis does not give an answer to how we should use the inputs in a co-ordinated way.

5.3 Which inputs are important?

This section presents a new method to find inputs which are important for a good design. The new method provides an analytical way to select the inputs which are critical in a good control design. No extensive calculations are needed. The basic calculation is only calculations of the eigenvalues and the eigenvectors of the system, which is a known relatively simple standard numerical problem.

Input energy as a guide to find important inputs

The idea is to use input energy as a measure how important each input is. When we have done the LQ design, an optimal trade off has been done between input energy and performance according to our loss function. An input, which uses large amounts of energy must be an important input for the design. The finesse is to use this information to find the important inputs in a feedback structure which is sparse but maintain much of the features of the optimal controller.

Because the final design should, in some sense, be closer to the optimal design than the open loop, it seems better to use the optimal design when investigating controller properties as the feedback structure. Note that this method starts with the global optimal design in contrast to other methods which start with the open system. We know that a good design uses certain inputs more than others. Open loop methods in contrast know that partial derivatives of parameters, in a fixed control law, at certain inputs are large and hope that there will be a correlation between this and important inputs in a working controller with tuned parameters.

We can express "the energy from input number i as function of x_0 ", $E_i(x_0)$, as

$$\begin{aligned} E_i(x_0) &= \int_0^\infty u_i(t)u_i(t)^H\,dt = \int_0^\infty (\sum_{j=1}^n x_j(t)k_{ij})(\sum_{j=1}^n x_j(t)k_{ij})^H\,dt \\ &= \int_0^\infty k_{i.}x(t)x(t)^H k_{i.}^T\,dt = k_{i.}\int_0^\infty x(t)x(t)^H\,dt\,k_{i.}^T \\ &= k_{i.}\int_0^\infty e^{\tilde{A}t}x_0x_0^H e^{\tilde{A}^Ht}\,dt k_{i.}^T \end{aligned} \tag{5.2}$$

Where $\tilde{A} = A - BK$, $x \in\ C^{n\times 1}$, $k_{i.} \in\ R^{n\times 1}$, and $k_{i.}$ is the i'th row in the feedback matrix K. The notation T stands for transpose and H for hermit transpose. If we denote $x_0 \cdot x_0^H = P$ we get

$$E_i = k_{i.}\int_0^\infty e^{\tilde{A}t}Pe^{\tilde{A}^Ht}\,dt\,k_{i.}^T \tag{5.3}$$

The integral $Z = \int_0^\infty e^{\tilde{A}t}Pe^{\tilde{A}^Ht}\,dt$ can be solved from the Lyapunov equation

$$\tilde{A}Z + Z\tilde{A}^H + P = 0 \tag{5.4}$$

for which there are good numerical solvers. However, we will for our purpose use another simpler way to calculate the input energy.

It is hard to choose just one initial condition x_0, which gives an E_i that accurate summarizes all system properties. We must be sure that the initial condition excite all eigenvalues in the system. By choosing different initial conditions, which excite just one eigenvalue at the time we can study how the inputs are used to damp each eigenvalue.

Input Energy for the closed loop with Optimal Controler (IEOC)

A good thing to investigate is the properties of the optimal feedback. If we choose the initial conditions so we excite one eigenvalue at the time we can see what inputs are active in damping out a certain mode. We need some sort of normalization to compare different modes. If we denote right eigenvectors of $\tilde{A}$ with v_j and left eigenvectors of $\tilde{A}$ with w_j the state vector can be expressed as

$$x(t) = \sum_j e^{\lambda_j t} v_j w_j^T x(0) \tag{5.5}$$

If we normalize $\|x(0)\|_2 = 1$ and $\|w_j\|_2 = 1$, which also determines $\|v_j\|_2$, the worst initial state for mode λ_j is $x(0) = w_j$ which make $w_j^T x(0) = 1$. Also assume that all eigenvalues are different, e.g. $\lambda_i \neq \lambda_j$, $i \neq j$. The contribution to $x(t)$ from this mode is

$$x_{\lambda_j}(t) = e^{\lambda_j t} v_j \tag{5.6}$$

The contribution from mode λ_j to input energy from u_i can be expressed as

$$\begin{aligned}
E_{ij} &= \int_0^\infty u_{i_{\lambda_j}}^2 \, dt \\
&= k_{i.} \int_0^\infty x_{\lambda_j}(t) x_{\lambda_j}^H(t) \, dt \, k_{i.}^T \\
&= k_{i.} \int_0^\infty e^{\tilde{A}t} v_j v_j^H e^{\tilde{A}^H t} \, dt \, k_{i.}^T \\
&= k_{i.} \int_0^\infty e^{\lambda_j t} v_j v_j^H e^{\lambda_j^H t} \, dt \, k_{i.}^T \\
&= k_{i.} v_j v_j^H k_{i.}^T \int_0^\infty e^{\lambda_j t} e^{\lambda_j^H t} \, dt \\
&= k_{i.} v_j v_j^H k_{i.}^T \frac{-1}{2\mathrm{Re}(\lambda_j)}
\end{aligned} \tag{5.7}$$

The last equality is obtained from the fact that $\tilde{A}$ is asymptotic stable which implies that $\mathrm{Re}\lambda_j < 0$.

In the calculation of (5.7) we have to calculate eigenvalues-eigenvectors and do some matrix multiplication which is easily done with numerical software.

EXAMPLE 5.1—IEOC for control law in Example 4.2
Consider the control law (4.17) from Example 4.2. We calculate the input energy E_{ij} for the three inputs u_i if we excite the two EOM to $A - BK$ and form Table 5.1. The calculations have been done in Matlab (Moler et al, 1987). Appendix C shows an implementation of a function in Matlab which calculates IEOC.

The table shows that input u_{f_2} is active in damping $\lambda_{1,2}$ and the inputs u_{f_1}, u_{f_3} are very active in damping $\lambda_{3,4}$. Since $\lambda_{3,4}$ is the dominant and critical eigenvalue we draw the conclusion that input u_{f_1} and u_{f_3} are important for a good design. □

Table 5.1 Input Energy for Optimal Controler (IEOC) from Example 4.2

IEOC	$\lambda_{1,2} = -3.78 \pm 7.93j$	$\lambda_{3,4} = -1.87 \pm 4.24j$
u_{f_1}	8.8	187.4
u_{f_2}	58.5	2.2
u_{f_3}	7.0	162.6

The next step is then to examine these two inputs more and see if it is sufficient with feedback from local variables or if we have to transmit signals between the generators.

5.4 Important feedback elements for an input

What we have so far is a method to find out important inputs. This section will give a method to pick out important feedback elements for each of these inputs. Steady-state stability problems are caused by unfavourable interaction between machines which results in oscillation of power. A good control law must contain components, which can prevent the oscillations. By investigating how a sinusoidal signal at one input influence the speed on other machines we can get information about the undesired interaction. Problem occur in general when one machine's input (field voltage) has large impact on another machine's speed and specially when the influence has nearly 180° phase shift compared to the own machines speed. Influence with 180° phase shift will force the two machines in different direction and is a potential danger for power oscillations between the two machines. To find the important feedback terms we will for each important input, at the important frequencies, search for interaction between the machine's own speed term and other machines speed terms. This will be done by inspection a selected column in $G(j\omega)$, were $G(s)$ is the transfer function from field voltages to speed terms and ω is the oscillation frequency of the critical EOM. In each selected column we look for complex numbers with large magnitude and 180° phase shift compared to the speed term with same index as the input (the diagonal element in $G(j\omega)$).

Interaction analysis

We calculate the transfer function from u_{f_j} to ω_i

$$G_{ij}(s) = C_\omega(sI - A)^{-1}B \tag{5.8}$$

for $s = j \cdot \text{Im}(\lambda_k)$ for the EOM and we get a complex matrix. Each column in the matrix shows the influence of a selected input. The complex elements in the column give magnification and phase shift on speed (in steady state) if a sinusoidal input is applied at the selected input. By plotting the interesting columns in the complex plane we can see the direction and amplitude of the influence of the selected input. We illustrate with an example.

EXAMPLE 5.2—Feedback elements for control law in Example 4.2
Consider the control law from Example 4.2. From Example 5.1 we know that input u_{f_1} and u_{f_3} are important to damp the critical EOM $\lambda_{3,4}$.

We calculate the matrix

$$G(j4.24) = \begin{pmatrix} 0.97 + 0.45j & 0.47 + 0.15j & -0.76 - 0.42j \\ 0.24 + 0.05j & -0.05 + 0.02j & 0.08 - 0.03j \\ -0.75 - 0.40j & 0.13 - 0.06j & 0.78 + 0.36j \end{pmatrix}$$

We then plot the first and third column in the matrix which corresponds to the influence from u_{f_1} and u_{f_3}. We get Figure 5.1 and 5.2.

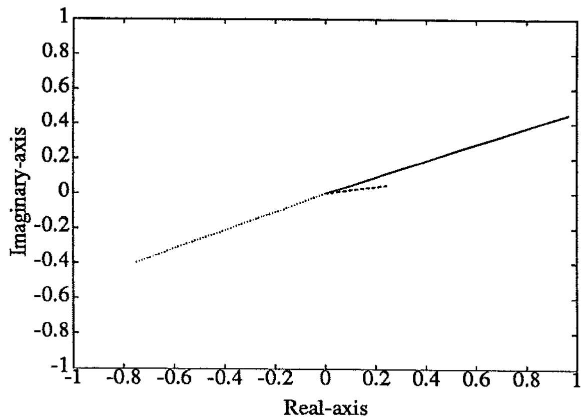


Figure 5.1 Influence from input number 1 to machine speed at the frequency 4.24 rad/s. Solid=Gen1, Dashed=Gen2, Dotted=Gen3.

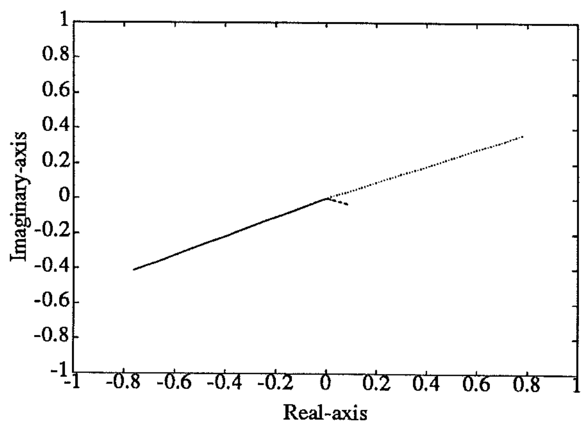


Figure 5.2 Influence from input number 3 to machine speed at the frequency 4.24 rad/s. Solid=Gen1, Dashed=Gen2, Dotted=Gen3.

Figure 5.1 shows that input number 1 will have main influence on ω_1 and ω_3 since the amplitudes are large. The direction tells us that u_{f_1} will influence ω_1 and ω_3 in totally different directions. For input number 3, in Figure 5.2, we draw the same type of conclusion. Since the power oscillations are due to the fact that machines swing against each other it seems important to have both feedback terms from generator 1 and generator 3 at these machines. Machine number 2 seems to be less important and we can use only local variables in the control law. A good control structure would then be to transmit information about speed (ω) and power (P_e) between generator 1 and 3 and use local measurements for voltage control. If we need a PSS at generator 2 we can use local variables since this machine is less important. We have now for this simple example the feedback structure $u = -\hat{K}y$, with

$$y^T = \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_2} & P_{e_3} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \tag{5.9}$$

and

$$\hat{K} = \begin{pmatrix} \hat{k}_{11} & 0 & 0 & \hat{k}_{14} & 0 & \hat{k}_{16} & \hat{k}_{17} & 0 & \hat{k}_{19} \\ 0 & \hat{k}_{22} & 0 & 0 & \hat{k}_{25} & 0 & 0 & \hat{k}_{28} & 0 \\ 0 & 0 & \hat{k}_{33} & \hat{k}_{34} & 0 & \hat{k}_{36} & \hat{k}_{37} & 0 & \hat{k}_{39} \end{pmatrix} \tag{5.10}$$

□

5.5 The procedure to find a feedback structure

We sum up and have the following procedure to find a good feedback structure.

Step 1 - LQ design. Do a LQ-design, which results in an optimal full state feedback, $u = -Kx$.

Step 2 - Find important inputs. Use the Input-Energy-Optimal-Controler (IEOC) criterion to find the important inputs u_i for each critical low frequency EOM.

Step 3 - Feedback elements for important inputs. For each important input u_i, see if there is(are) large interaction(s) with phase shift near 180° between own machine's speed term ω_i and some other machines speed term ω_j , $j \neq i$. If so, use communication between machine i and j, otherwise local PSS on machine i.

What we have got now is a method to choose a feedback structure, which captures the important parts for a good design and in the next chapter some tuning methods will be presented.

6. Tuning of an Incomplete State Feedback Controller

In this chapter we will concentrate on the question of tuning the controller. We know the elements k_{ij} in the feedback $u = -Kx$ which have to be zero and those elements we are allowed to tune. Two different tuning methods will be presented. The first is based on iterative numerical minimization of the LQ problem and the second uses least square approximation of a global feedback. Both methods have advantages and drawbacks and the use of each method is exemplified. Furthermore we exemplify how to combine the methods to use the best features of each.

6.1 Existing tuning methods

All tuning methods for incomplete state feedback is based on some type of approximation. The main difference between the published methods is the criterion they try to approximate. For example, Bengtsson and Lindahl (1974) try to approximate the subspace spanned by the eigenvectors to the full state feedback. Another approximation is presented in Konigorski (1987) where it is tried to approximate the characteristic polynomial for the incomplete state feedback so this should be close to the characteristic polynomial for the full state feedback. Both methods are iterative since there are some weights we have to fiddle with to achieve satisfactory results. Neither of the methods guarantees stability and this must be checked after the approximation. The best features of both methods are that they are easy to use and do not require extensive calculations.

6.2 Iterative solution of the reduced state feedback problem

This section presents a tuning method which uses iterative numerical minimization of the loss function in the LQ design. The reduced state feedback problem with a fixed structure is often called parametric LQ (PLQ).

Optimizing with fixed K-structure

With fixed K-structure we mean that the choice of measurement signals is fixed and that the regulator should be a static linear combination of those measurements. After having decided the structure of the feedback matrix K we can improve the reduced LQ-design by optimizing. The criterion to optimize is (4.3) for a system (4.1) with initial conditions $x(0) = x_0 \neq 0$. The control law is still $u = -Kx$ but here K is a matrix with some elements fixed to zero. When we have global feedback the resulting feedback minimizes the loss for all initial conditions. With incomplete state information we will not have the same result.

Solution by Gradient Method

The problem to find the optimal feedback has been treated by Mårtensson (1970) who gave equations for the derivatives to the loss function and proposed a numerical minimization algorithm.

A more efficient way to calculate the derivatives can be found in Geromel and Bernussou (1979). To get the gradient matrix we then have to solve $\frac{\partial J(K)}{\partial K}$ from the equations (6.1) – (6.3).

$$(A - BK)^T P + P(A - BK) + Q + K^T RK = 0 \tag{6.1}$$

$$(A - BK)L + L(A - BK)^T + V_0 = 0 \tag{6.2}$$

$$\frac{\partial J(K)}{\partial K} = 2(RK - B^T P)L \tag{6.3}$$

for those K such that $A-BK$ is an asymptotically stable matrix. It is assumed that the initial state vector x_0 is a random variable and $V_0 = E\{x_0 x_0^T\}$.

In order to obtain the gradient matrix for a given K, one has to solve two Lyapunov equations, which is a reasonable task even for large scale systems.

Features of parametric LQ

What are the drawbacks respectively strengths with parametric LQ? The three main drawbacks are

- Upper limit on system size. Even if the calculations are a reasonable task there is an upper limit on the system size due to computation time. For large systems the method does not become efficient since the computation time is proportional to the cube of the system size and the resulting tuning takes a lot of computer time.
- Can not incorporate the algebraic relation in the method. Since we have to exclude one electric power from the state vector we can not get a feedback term from this electric power. To exclude one P_e from the feedback can cause unsymmetry in the control law if we really need feedback from all electric powers.
- The dependence of the solution on initial conditions. There is an arbitraryness in the choice of initial conditions x_0 which influences the final control law. The approach of Geromel and Bernusson (1979) considers the initial state as a random variable and then minimizes the average value of the performance index. Two cases have been considered
  * x_0 is uniformly distributed over the n dimensional unit sphere; so $V_0 = E\{x_0 x_0^T\} = (1/n)I$
  * x_0 is Gaussian distributed with mean values $\bar{x}_0$ and covariance X_0. Hence $V_0 = E\{x_0 x_0^T\} = X_0 + \bar{x}_0 \bar{x}_0^T$

If these two cases are the most relevant for power systems is hard to tell. The mean value $\bar{x}_0$ and covariance X_0 can after some consideration represent initial conditions after typical disturbances. If we want to tune the PSS for a certain mode we must be sure to choose initial conditions that excite this specific mode.

The strength of the method is that it will decrease the loss function in each iteration step.

Implementation in Matlab

An implementation of PLQ has been done in Matlab. The implemented numerical algorithm uses Davidon-Fletcher-Powell's method (Luenberger, 1984) to do minimization of the loss function. The method converges reasonable fast and the computation time on a VAX 11/780 can be seen in Table 6.1.

Table 6.1 Computation time for numerical minimization on VAX 11/780

Number of machines	Model order	Computation time in minutes
3	8	5
6	17	44
12	35	480

Probably could the calculation times be cut considerably since no effort has been put into speeding up the calculations. We illustrate the use of the approximation method with an example

EXAMPLE 6.1—Parametric LQ

Consider the system from Example 4.2 with loss function (4.14). In Chapter 5 we found out that a good feedback structure will have local voltage control, local PSS on machine 2 and that the PSS on machine 1 and 3 need to communicate with each other. We also found out from the IEOC criterion that machine number 2 was less important than the two others to damp the low frequency EOM. Because of this we leave the electric power for machine number 2 out of the state vector. Hence, our state vector is after transformation

$$x^T = \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_3} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \tag{6.4}$$

Then K in the control law $u = -Kx$ will have the structure

$$K = \begin{pmatrix} k_{11} & 0 & 0 & k_{14} & k_{15} & k_{16} & 0 & k_{18} \\ 0 & k_{22} & 0 & 0 & 0 & 0 & k_{27} & 0 \\ 0 & 0 & k_{33} & k_{34} & k_{35} & k_{36} & 0 & k_{38} \end{pmatrix} \tag{6.5}$$

We choose V_0 in (6.2) so we excite voltage and speed terms since our design objective is to get a voltage regulation with good damping features.

$$V_0 = \text{diag}\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \tag{6.6}$$

After running the minimization function in Matlab we get the reduced feedback matrix

$$K = \begin{pmatrix} 28.8 & 0 & 0 & 24.6 & 3.4 & -2.37 & 0 & 1.95 \\ 0 & 92.0 & 0 & 0 & 0 & 0 & -4.91 & 0 \\ 0 & 0 & 29.0 & 2.82 & 22.0 & 1.72 & 0 & -2.24 \end{pmatrix} \tag{6.7}$$

and the eigenvalues of $A - BK$ are

$$\begin{aligned} \lambda_{1,2} &= -3.05 \pm 8.72j \quad (\zeta = 0.33) \\ \lambda_{3,4} &= -1.81 \pm 4.21j \quad (\zeta = 0.40) \\ \lambda_5 &= -13.49 \\ \lambda_6 &= -2.93 \\ \lambda_{7,8} &= -1.67 \pm 1.17j \quad (\zeta = 0.82) \end{aligned} \tag{6.8}$$

Simulation of the response to an impulse at input number 1 with the control law (6.7) looks like Figure 6.1.

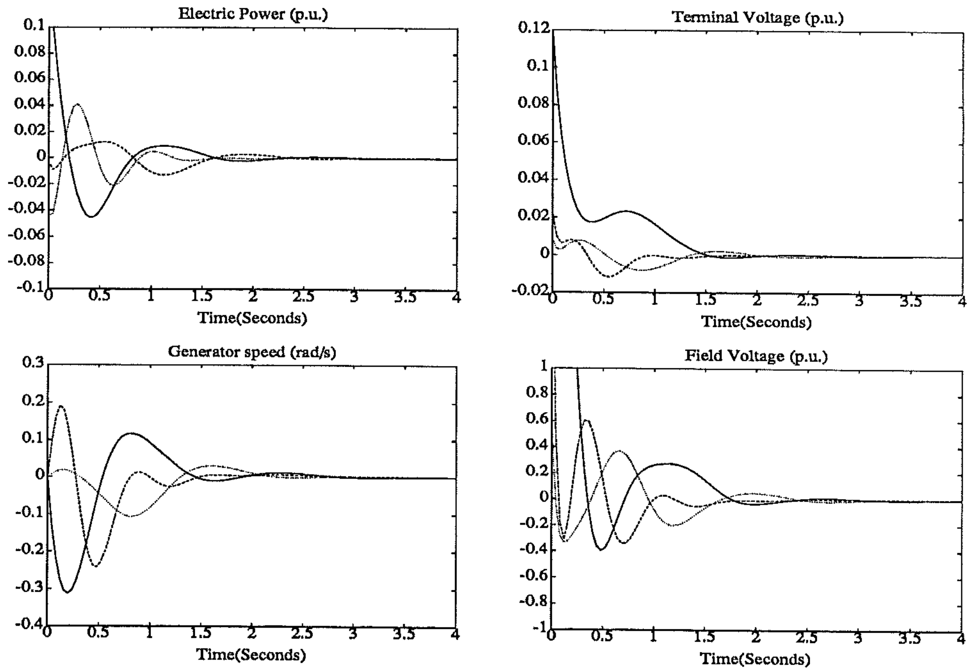


Figure 6.1 Simulated impulse response at input number 1 for parametric LQ design (6.7). The simulations have been done in Matlab with the linearised model from Chapter 2. Compare with Figure 4.3, which shows simulation with full state feedback.

Figure 6.1 shows the following

- Electric power has the same damping features as the full state feedback in Figure 4.3.
- Terminal voltages have no significant differences compared to full state feedback (Figure 4.3).
- Machine speeds have same damping features as in Figure 4.3. However, in Figure 6.1 the oscillations are damped against the linearisations point. Also there is a larger transient in machine numbers two's speed term in Figure 6.1 than in Figure 4.3.
- Field voltage for machine number two in Figure 6.1 has a transient with large amplitude.

The transients in machine speed and field voltage at machine number two must be caused by the high gain in feedback from voltage, see (6.7). The difference in speed oscillation compared to full state feedback can be explained by two reasons. The first reason is that we now have less free parameters in the LQ-optimization problem than in full state feedback. The second reason is that this control law is not optimal for all initial conditions and therefore we can not expect the control law to be optimal in the sense that it does not add speed damping towards the linearisation point. In conclusion we get a sparse feedback control law, which captures very much of the features of the full state feedback control law. A much simpler control law with nearly the same features! □

6.3 Least square approximation of feedback

This section presents a method to approximate the feedback together with the algebraic constraint.

The method

What we have after a standard LQ-design is a full state feedback control law $u = -Kx$ and one electric power, P_i which is not a state. We also know that P_i can be expressed in the state variables, which imply an algebraic constraint. The problem is to incorporate the algebraic constraint (2.22) and the state feedback control law to get a control law that can be used. Introduce the output vector y, defined by

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_{3n} \end{pmatrix} = \begin{pmatrix} V_{t_1} \\ \vdots \\ V_{t_n} \\ P_1 \\ \vdots \\ P_n \\ \omega_1 \\ \vdots \\ \omega_n \end{pmatrix} \tag{6.9}$$

The algebraic relation can be written

$$\begin{pmatrix} h_1 & \dots & h_{3n} \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_{3n} \end{pmatrix} = hy = 0 \tag{6.10}$$

Hence, the control law for each input u_i can be written

$$u_i = \begin{pmatrix} k_{i,1} & \cdots k_{i,k} & 0 & k_{i,k+1} & \cdots & k_{i,3n} \end{pmatrix} y + \mu_i hy = (k_{i.} + \mu_i h)y \tag{6.11}$$

Where μ_i is an arbitrary scalar and $k_{i.}$ is the i'th row in the matrix K. What we want is a control law $\hat{u}_i = -\hat{k}_{i.}y$ which is close to the optimal and also have small feedback gains. If we consider one input u_i with corresponding row $k_{i.}$ we can formulate a quadratic loss function

$$f_i = a_{i,1}\hat{k}_{i,1}^2 + \ldots + a_{i,3n}\hat{k}_{i,3n}^2 + (k_{i,1} + \mu_i h_1 - \hat{k}_{i,1})^2 + \ldots + (k_{i,3n} + \mu_i h_{i,3n} - \hat{k}_{i,3n})^2 \tag{6.12}$$

Where the a_{ij}'s are weight factors. If we want a feedback element $\hat{k}_{ij}$ to disappear we choose a large a_{ij}. By setting

$$\frac{\partial f_i}{\partial \hat{k}_{ij}} = 0$$
$$\frac{\partial f_i}{\partial \mu_i} = 0$$

we can analytically solve for $\hat{k}_{i.}$ and μ_i. The result is

$$\begin{pmatrix} \hat{k}_{i.}^T \\ \mu_i \end{pmatrix} = \begin{pmatrix} \text{diag}(a_{ij}) + I & -h^T \\ -h & hh^T \end{pmatrix}^{-1} \begin{pmatrix} k_{i.}^T \\ -hk_{i.}^T \end{pmatrix} \tag{6.13}$$

In our case we choose a_{ij} large for elements we would like to eliminate and small for others.

Features of least square approximation

What are the drawbacks respectively the strengths? The main drawback is that the method itself does not guarantee stability. This has to be checked separately after the approximation. The strength is that the iteration weights a_{ij} in (6.12) directly influence the feedback gains. This in combination with the easy incorporation of the algebraic constraints and a simple calculation without iterations are the main advantages. The difference with this problem formulation compared to the two methods mentioned in Section 6.1, is that this method iterate with weights on the the feedback gains and the two other methods iterate with weights on invariant subspaces and characteristical polynomial respectively.

An example of least square approximation

EXAMPLE 6.2—Least Square Approximation
Consider the system from Example 4.2 with the control law (4.17) and state vector (4.8).

$$x^T = \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_2} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix}$$

Our output vector y is

$$y^T = \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_2} & P_{e_3} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \tag{6.14}$$

In Chapter 5 we found out that a good feedback structure will have local voltage control, local PSS on machine 2 and that the PSS on machine 1 and 3 need to communicate with each other. Then $\widehat{K}$ in the control law $u = -\widehat{K}y$ will have the structure

$$\widehat{K} = \begin{pmatrix} \hat{k}_{11} & 0 & 0 & \hat{k}_{14} & 0 & \hat{k}_{16} & \hat{k}_{17} & 0 & \hat{k}_{19} \\ 0 & \hat{k}_{22} & 0 & 0 & \hat{k}_{25} & 0 & 0 & \hat{k}_{28} & 0 \\ 0 & 0 & \hat{k}_{33} & \hat{k}_{34} & 0 & \hat{k}_{36} & \hat{k}_{37} & 0 & \hat{k}_{39} \end{pmatrix} \tag{6.15}$$

We then choose the a_{ij}'s in (6.12) for each input u_i to

$$A = a_{ij} = \begin{pmatrix} 0.1 & 10 & 10 & 0.3 & 10 & 0.3 & 0.3 & 10 & 0.3 \\ 10 & 0.1 & 10 & 10 & 0.3 & 10 & 10 & 0.3 & 10 \\ 10 & 10 & 0.1 & 0.3 & 10 & 0.3 & 0.3 & 10 & 0.3 \end{pmatrix} \tag{6.16}$$

We calculate each row in $\widehat{K}$ from (6.13) and get the least square approximation $\widehat{K}$ of the control law (4.17) with the algebraic constraint given by (4.9). If we truncate small elements we get

$$\widehat{K} = \begin{pmatrix} 27.4 & 0 & 0 & 11.2 & 0 & -5.1 & -2.16 & 0 & 1.48 \\ 0 & 13.8 & 0 & 0 & 17.6 & 0 & 0 & -1.90 & 0 \\ 0 & 0 & 27.3 & -5.7 & 0 & 9.1 & 1.22 & 0 & -1.93 \end{pmatrix} \tag{6.17}$$

and the eigenvalues to the resulting closed loop system $A - B\widehat{K}C$, where $y = Cx$ are

$$\begin{aligned} \lambda_{1,2} &= -2.64 \pm 7.83j \quad (\zeta = 0.32) \\ \lambda_{3,4} &= -1.45 \pm 4.31j \quad (\zeta = 0.32) \\ \lambda_5 &= -4.15 \\ \lambda_6 &= -2.82 \\ \lambda_{7,8} &= -1.59 \pm 1.94j \quad (\zeta = 0.63) \end{aligned} \tag{6.18}$$

Simulation of an impulse at input number 1 with the control law (6.17) looks like Figure 6.2.

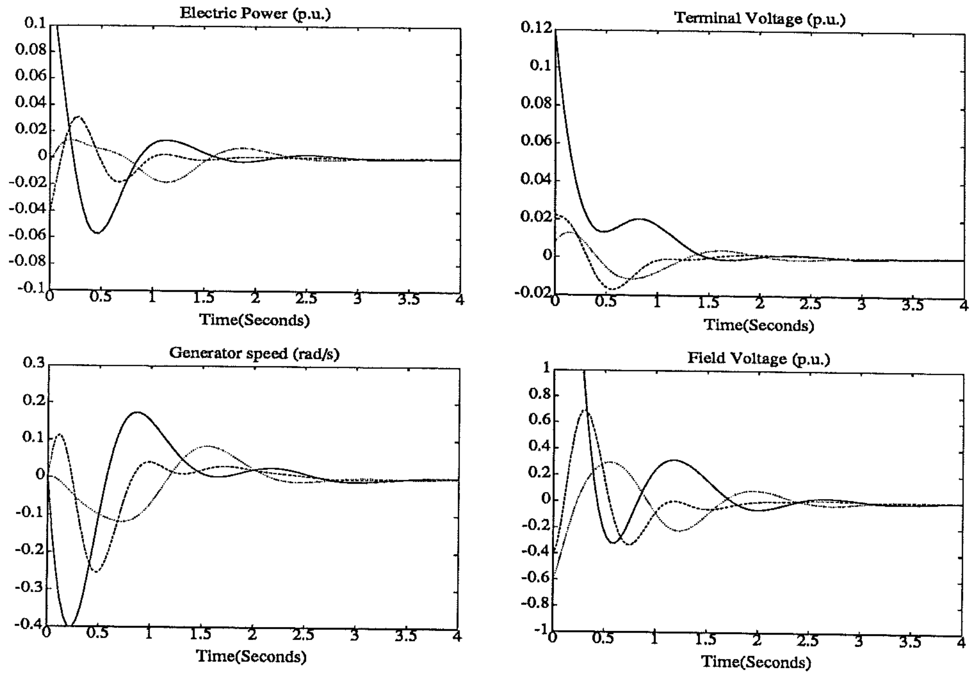


Figure 6.2 Simulated impulse response at input number 1 for least square approximation (6.17) of full state feedback (4.17). The simulations have been done in Matlab with the linearised model from Chapter 2. Compare with Figures 4.3 and 6.1. Solid=Gen1, Dashed=Gen2, Dotted=Gen3.

A comparison of least square approximation in Figure 6.2 with PLQ in Figure 6.1 shows the following

- The responses are very similar.
- Damping of speed and power are a little bit better in Figure 6.1 than in Figure 6.2.
- The initial transient in field voltage for machine number two is considerable smaller with least square approximation (Figure 6.2) than in PLQ (Figure 6.1). The reason must be the difference in magnitude of the feedback gain from V_{t_2}.

In conclusion, least square approximation gives a control law which capture the important parts and avoids high gains. □

6.4 Combination of PLQ and least square approximation

This section shows an example of how the two presented methods can be combined to squeeze the best out of each. We then do the following steps

- Do parametric LQ but keep full structure in the row in K which corresponds to the electric power which is not a state.
- Use the algebraic relation and least square approximation to approximate the full row in K to desired structure.
  We show an example

EXAMPLE 6.3—Combination of the two methods
Consider the system from Example 4.2 with the loss function (4.14) with the state- and output-vectors

$$\begin{aligned} x^T &= \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_3} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \\ y^T &= \begin{pmatrix} V_{t_1} & V_{t_2} & V_{t_3} & P_{e_1} & P_{e_2} & P_{e_3} & \omega_1 & \omega_2 & \omega_3 \end{pmatrix} \end{aligned} \tag{6.19}$$

First we do parametric LQ with the feedback structure K in $u = -Kx$.

$$K = \begin{pmatrix} k_{11} & 0 & 0 & k_{14} & k_{15} & k_{16} & 0 & k_{18} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & 0 & k_{27} & 0 \\ 0 & 0 & k_{33} & k_{34} & k_{36} & k_{37} & 0 & k_{38} \end{pmatrix} \tag{6.20}$$

After running parametric LQ in Matlab we get

$$K = \begin{pmatrix} 28.6 & 0 & 0 & 25.5 & 4.3 & -2.39 & 0 & 2.16 \\ -3.9 & 101 & -3.2 & -3.66 & -3.92 & 0 & -5.13 & 0 \\ 0 & 0 & 28.6 & 3.88 & 22.7 & 1.80 & 0 & -2.31 \end{pmatrix} \tag{6.21}$$

The next step is to use the least square approximation method on the second row. We do as in Example 6.2 with

$$a = \begin{pmatrix} 10 & 0.1 & 10 & 10 & 0.3 & 10 & 10 & 0.3 & 10 \end{pmatrix}$$

After truncation we get for u_{f_2}

$$u_{f_2} = \begin{pmatrix} 0 & 58.6 & 0 & 0 & 5.88 & 0 & 0 & -3.95 & 0 \end{pmatrix} y$$

The total control law can be expressed in y and we have $u = -\widehat{K}y$ with

$$\hat{K} = \begin{pmatrix} 28.6 & 0 & 0 & 25.5 & 0 & 4.3 & -2.39 & 0 & 2.16 \\ 0 & 58.6 & 0 & 0 & 5.88 & 0 & 0 & -3.95 & 0 \\ 0 & 0 & 28.6 & 3.88 & 0 & 22.7 & 1.80 & 0 & -2.31 \end{pmatrix} \tag{6.22}$$

The eigenvalues for the closed loop $A - B\widehat{K}C$, where $y = Cx$, are

$$\begin{aligned}
\lambda_{1,2} &= -2.24 \pm 9.29j \quad (\zeta = 0.23) \\
\lambda_{3,4} &= -1.85 \pm 4.31j \quad (\zeta = 0.39) \\
\lambda_5 &= -11.2 \\
\lambda_6 &= -2.78 \\
\lambda_{7,8} &= -1.42 \pm 1.28j \quad (\zeta = 0.74)
\end{aligned} \tag{6.23}$$

□

6.5 Conclusions

From the three Examples 6.1-6.3 we note the following

- The change in damping for the two EOM $\lambda_{1,2}$ and $\lambda_{3,4}$ is small if we compare (4.18) with (6.8), (6.18) and (6.23). At most the real part of $\lambda_{3,4}$ is decreased from -1.87 to -1.45 in the least square approximation.
- When we use parametric LQ in Example 6.1 and 6.3 the feedback term for voltage at input 2 becomes very large. The reason is believed to be that machine 2 has very little influence on the critical $\lambda_{3,4}$. Therefore can this voltage gain be increased without influencing $\lambda_{3,4}$.
- The feedback terms in least square approximation are in general smaller than numerical LQ .
- All approximation methods change the eigenvalue λ_8 in (4.18) considerably. This means that we also add some damping towards the linearisation point.

The conclusion becomes that if we emphasize a fast approximation method which can be used on large systems with a fair degree of accuracy we should use least square approximation. If we on the other hand are interested in more accurate tuning on the EOM and can accept time consuming calculations we should choose parametric LQ. One drawback with parametric LQ is that we have to exclude one output variable from the state vector and therefore we can not get a setting on the feedback from the excluded variable. The problem can be overcome by a combining the two methods.

7. Design for a power system with 16 Machines

The goal of this chapter is to illustrate the ideas from previous chapters by making a design for a large multimachine system. The power system consists of sixteen machines connected by tie-lines in the configuration shown in Figure 7.1.

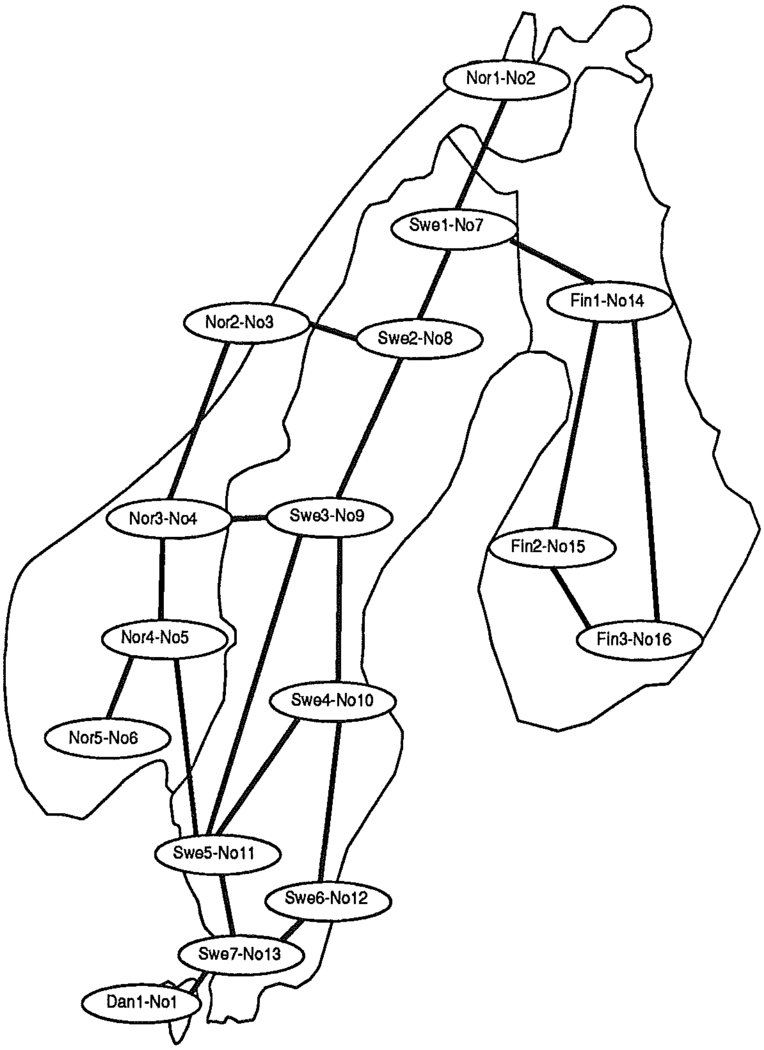


Figure 7.1 Sixteen machine model of the Nordel power system

The sixteen machine power system is a model of the Nordel power system, which connects Sweden, Norway, Finland and a part of Denmark. Each machine in the model represents several, maybe hundreds, generators in the true system. The Nordel system has a total capacity of approximately 70 GW and has over the years had severe problems with undamped power oscillations (Lysfjord et al, 1982 and 1984). Machine parameters, network data and initial conditions for the model can be found in Appendix D.1.

Design procedure for AVR and PSS gains

The design procedure is divided into the following steps:

1 Make multimachine model and simulate with only voltage regulators. To see the effect of damper windings, do simulation with respectively without damper windings. Sort out the critical low frequency eigenvalues which are not damped by the damper windings.

2 Make first a LQ-design where the choice of loss matrices is done without any á priori knowledge about important feedback signals.

3 Decide feedback structure according to Section 5.5.

4 If the feedback structure is very sparse do a new LQ-design, which only punish the parts in the feedback structure.

5 Do approximation of the global feedback from LQ-design to fit decided structure.

6 Simulate the design and compare with the result in the first step. If the result is not satisfactory go back to step 2.

From Lysfjord et al (1982 and 1984) we get the design specification, AVR-gain ≈ 30, the signal V_s in Figure 1.4 should be limited in magnitude to 0.05 p.u. and the feedback parameters from power and speed should be limited so V_s will not saturate for small deviations. From Appendix 6 in Lysfjord (1982) we can estimate normal values for our state feedback terms from P_e and ω to be 15 and -1.5 respectively with the units in our model.

7.1 The Model

From the initial conditions and parameters in Appendix D.1 we make a linearised multimachine model according to Chapter 2. The model will after transformation have 47 states, 16 inputs and 48 outputs.

Eigenvalues without PSS

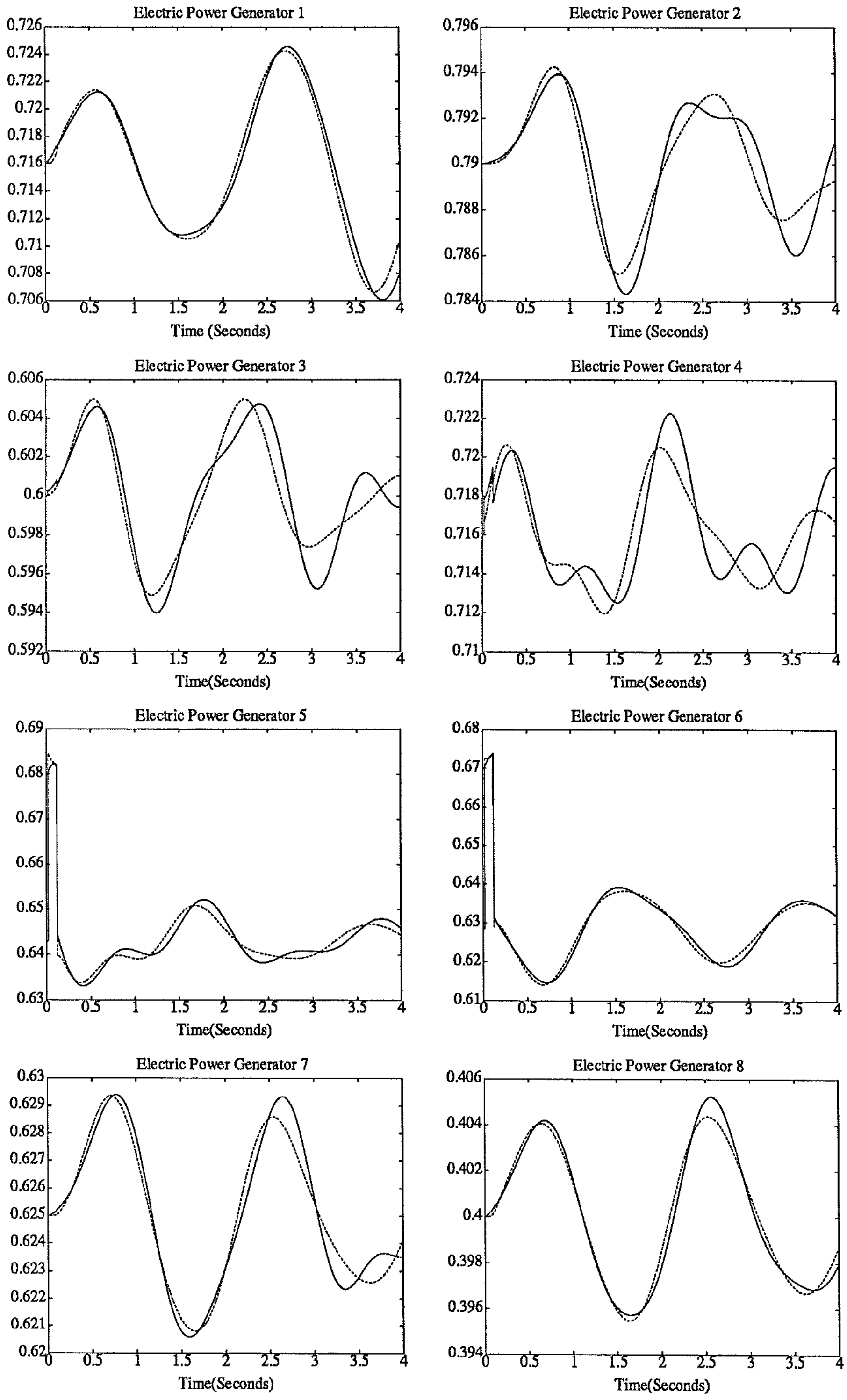
If we calculate the eigenvalues of the linearised model with only AVR feedback with gain 30, we get

$$\begin{array}{rr}
\lambda_{1,2} = -0.11 \pm 8.01j & \lambda_{3,4} = -0.14 \pm 7.84j \\
\lambda_{5,6} = -0.32 \pm 7.33j & \lambda_{7,8} = -0.43 \pm 7.09j \\
\lambda_{9,10} = -0.19 \pm 6.82j & \lambda_{11,12} = -0.41 \pm 6.71j \\
\lambda_{13,14} = -0.26 \pm 6.63j & \lambda_{15,16} = -0.20 \pm 6.19j \\
\lambda_{17,18} = -0.26 \pm 6.00j & \lambda_{19,20} = -0.34 \pm 5.43j \\
\lambda_{21,22} = -0.42 \pm 5.36j & \lambda_{23,24} = -0.27 \pm 4.65j \\
\lambda_{25,26} = -0.15 \pm 3.89j & \lambda_{27,28} = -0.10 \pm 3.08j \\
\lambda_{29,30} = -0.13 \pm 2.02j & \lambda_{31} = -6.27 \\
\lambda_{32} = -6.03 & \lambda_{33} = -5.75 \\
\lambda_{34} = -5.25 & \lambda_{35} = -5.02 \\
\lambda_{36} = -4.87 & \lambda_{37} = -4.62 \\
\lambda_{38} = -4.11 & \lambda_{39} = -3.10 \\
\lambda_{40} = -2.99 & \lambda_{41} = -2.47 \\
\lambda_{42} = -1.80 & \lambda_{43} = -1.75 \\
\lambda_{44} = -1.31 & \lambda_{45} = -1.03 \\
\lambda_{46} = -0.66 & \lambda_{47} = -0.01
\end{array} \tag{7.1}$$

7.2 Simulation with AVR

To find out a typical behaviour of the system and study the effect of the damper windings we simulate it with the simulation package Simpow (Adielsson, 1982; Lindkvist, 1985). The simulation result can be found in Figure 7.2.

Power oscillations for machine 1-8 without PSS



Power oscillations for machine 9-16 without PSS

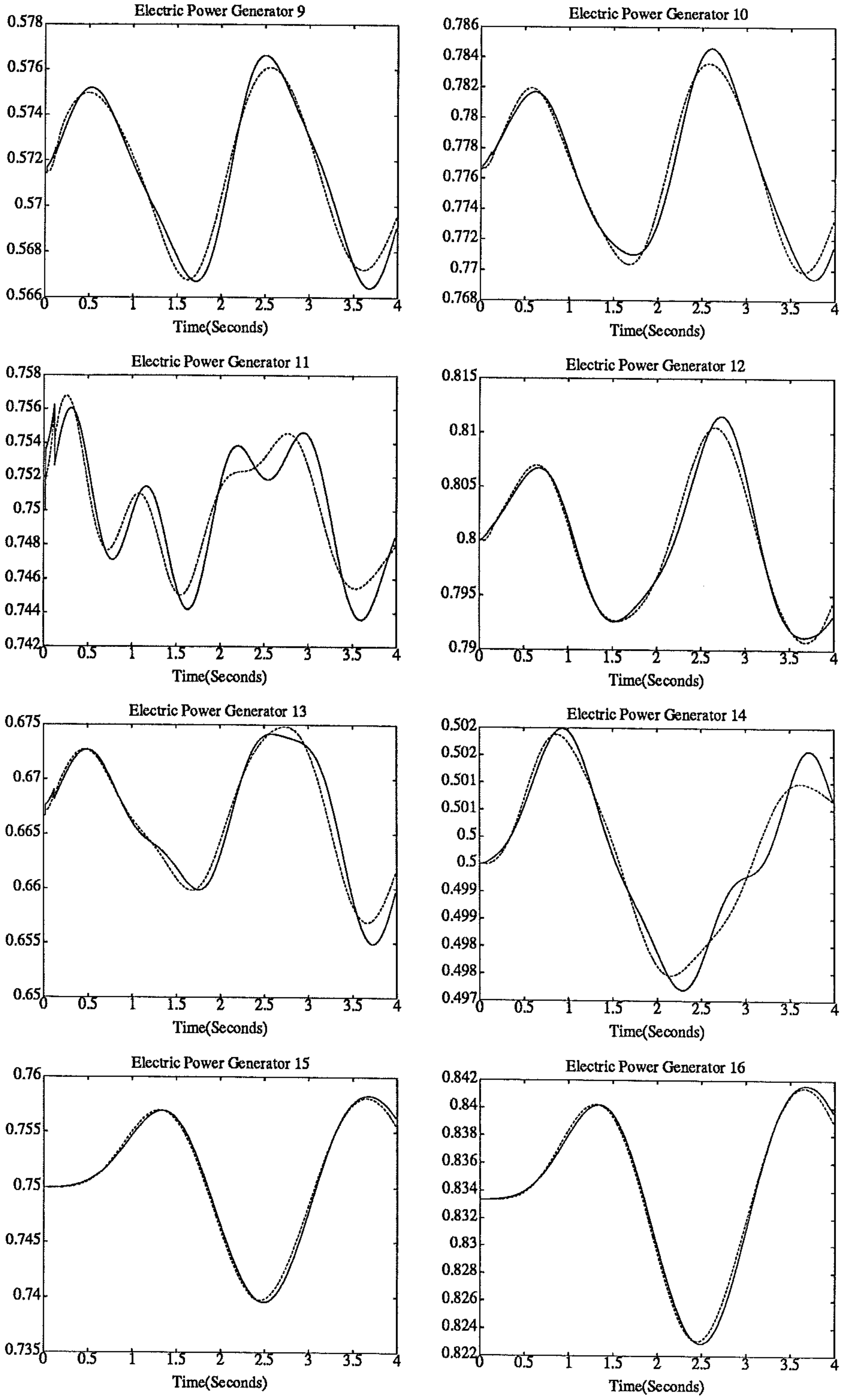


Figure 7.2 Power oscillations in the Nordel network after disturbance at machine five and six. The generators are simulated with/without damper windings (Solid/Dash) in Simpow.

Effect of damper windings

When we compare the simulation result with and without damper windings we see that the damper windings damp out the higher frequencies in the oscillations. The low frequency oscillations are not affected by the damper windings.

Critical modes

From this we conclude that the low frequency modes below approximately 5 rad/s must be damped by PSS. In the design we should concentrate on the modes

$$\begin{aligned}
\lambda_{23,24} &= -\,0.27 \pm 4.65j & (\zeta = 0.06)\\
\lambda_{25,26} &= -\,0.15 \pm 3.89j & (\zeta = 0.04)\\
\lambda_{27,28} &= -\,0.10 \pm 3.08j & (\zeta = 0.03)\\
\lambda_{29,30} &= -\,0.13 \pm 2.02j & (\zeta = 0.07)
\end{aligned} \tag{7.2}$$

7.3 LQ-design

Here we do a full LQ design which will be used to find a good feedback structure.

Weight matrices

We choose our loss matrices as a compromise between the two strategies illustrated by (4.10) and (4.14). When we only weight speed deviation between machines (4.14) the matrix Q_ω will have all row sums equal to zero, e.g. $\sum_j Q_{\omega_{ij}} = 0$, $\forall\, i$. Specification of speed damping for machine i against the linearisation point as in (4.14) corresponds to a row sum $\sum_j Q_{\omega_{ij}} > 0$. Our compromise will then have $\sum_j Q_{\omega_{ij}} = 0.01$ for a small damping of speed deviation towards the linearisation point and the non diagonal elements are $Q_{\omega_{ij}} = -0.49/15$, so $Q_{\omega_{ii}} = 0.50$. We also have to remember that Q should be symmetric and positive semidefinite. Then we choose the loss matrices Q and R in (4.3) to

$$\begin{aligned}
R &= \mathrm{diag}\begin{pmatrix} 0.1 & \cdots & 0.1 \end{pmatrix}\\
Q &= \mathrm{diag}\begin{pmatrix} Q_{V_t} & Q_{P_e} & Q_\omega \end{pmatrix}
\end{aligned} \tag{7.3}$$

with

$$\begin{aligned}
Q_{V_t} &= \mathrm{diag}\begin{pmatrix} 100 & \cdots & 100 \end{pmatrix}\\
Q_{P_e} &= \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & 0 & \vdots \\ 0 & \cdots & 0 \end{pmatrix}\\
Q_\omega &= \begin{pmatrix} 0.5 & -0.49/15 & \cdots & -0.49/15 & -0.49/15 \\ -0.49/15 & 0.5 & -0.49/15 & \cdots & -0.49/15 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -0.49/15 & -0.49/15 & -0.49/15 & \cdots & 0.5 \end{pmatrix}
\end{aligned} \tag{7.4}$$

Eigenvalues

The design software gives the optimal K in $u = -Kx$ and the eigenvalues of $A - BK$ are

$$
\begin{array}{rr}
\lambda_{1,2} = -1.22 \pm 8.06j & \lambda_{3,4} = -1.42 \pm 8.04j \\
\lambda_{5,6} = -1.37 \pm 7.53j & \lambda_{7,8} = -1.40 \pm 7.26j \\
\lambda_{9,10} = -1.84 \pm 6.96j & \lambda_{11,12} = -1.39 \pm 6.86j \\
\lambda_{13,14} = -1.51 \pm 6.55j & \lambda_{15,16} = -1.90 \pm 6.27j \\
\lambda_{17,18} = -1.60 \pm 6.06j & \lambda_{19,20} = -1.58 \pm 5.83j \\
\lambda_{21,22} = -1.24 \pm 5.53j & \lambda_{23,24} = -1.57 \pm 4.82j \\
\lambda_{25,26} = -2.28 \pm 3.83j & \lambda_{27,28} = -2.14 \pm 2.66j \\
\lambda_{29,30} = -2.95 \pm 1.53j & \lambda_{31} = -6.96 \\
\lambda_{32} = -5.03 & \lambda_{33} = -4.67 \\
\lambda_{34} = -5.67 & \lambda_{35} = -4.33 \\
\lambda_{36} = -4.09 & \lambda_{37} = -3.98 \\
\lambda_{38} = -3.09 & \lambda_{39} = -2.33 \\
\lambda_{40} = -1.98 & \lambda_{41} = -1.81 \\
\lambda_{42} = -1.63 & \lambda_{43} = -1.46 \\
\lambda_{44} = -1.11 & \lambda_{45,46} = -0.69 \pm 0.05j \\
\lambda_{47} = -0.48 &
\end{array}
\tag{7.5}
$$

The relative damping for the four low frequency EOM are

$$
\begin{pmatrix} \zeta_{23,24} & \cdots & \zeta_{29,30} \end{pmatrix} = \begin{pmatrix} 0.31 & 0.51 & 0.62 & 0.88 \end{pmatrix}
$$

Compare with (7.2).

7.4 Feedback structure

IEOC

Consider the full state feedback from the LQ design given by (7.3) and (7.4). We calculate the input energy E_{ij} for the inputs u_i if we excite the four EOM of $A - BK$ with frequencies below 5 rad/s and form Table 7.1. The elements have been rounded to integers. The table shows that inputs u_{f_5}, u_{f_6}, $u_{f_{15}}$ and $u_{f_{16}}$ are very active in damping $\lambda_{29,30}$ and $\lambda_{27,28}$. We also see that u_{f_3} and u_{f_4} give most input energy to eigenvalue $\lambda_{25,26}$ and that u_{f_1}, u_{f_2} and u_{f_7} are most active in damping $\lambda_{23,24}$. We can now concentrate on these inputs and search for the interaction in the system for the critical EOM.

Interaction

We know that low frequency power oscillations are caused by interaction between machines. The control law must capture this interaction in the feedback terms. For the critical inputs at the different oscillation frequencies we will now search for important interaction in the system. We do as in section (5.6) and plot magnitude and phase shift for the column in $G(j\omega)$ which corresponds to important inputs. The result can be found in Figure 7.3 – 7.6.

Table 7.1 Input Energy for Optimal Controler with LQ design (7.3) and (7.4)

IEOC	$\lambda_{23,24}$	$\lambda_{25,26}$	$\lambda_{27,28}$	$\lambda_{29,30}$
u_{f_1}	31	4	25	48
u_{f_2}	14	0	4	47
u_{f_3}	2	46	4	16
u_{f_4}	2	41	4	55
u_{f_5}	0	0	81	1414
u_{f_6}	1	2	123	830
u_{f_7}	15	0	2	19
u_{f_8}	2	0	2	5
u_{f_9}	4	1	50	85
$u_{f_{10}}$	0	1	26	10
$u_{f_{11}}$	1	2	12	20
$u_{f_{12}}$	2	2	14	21
$u_{f_{13}}$	7	1	8	12
$u_{f_{14}}$	0	0	5	270
$u_{f_{15}}$	3	1	69	726
$u_{f_{16}}$	2	0	46	486

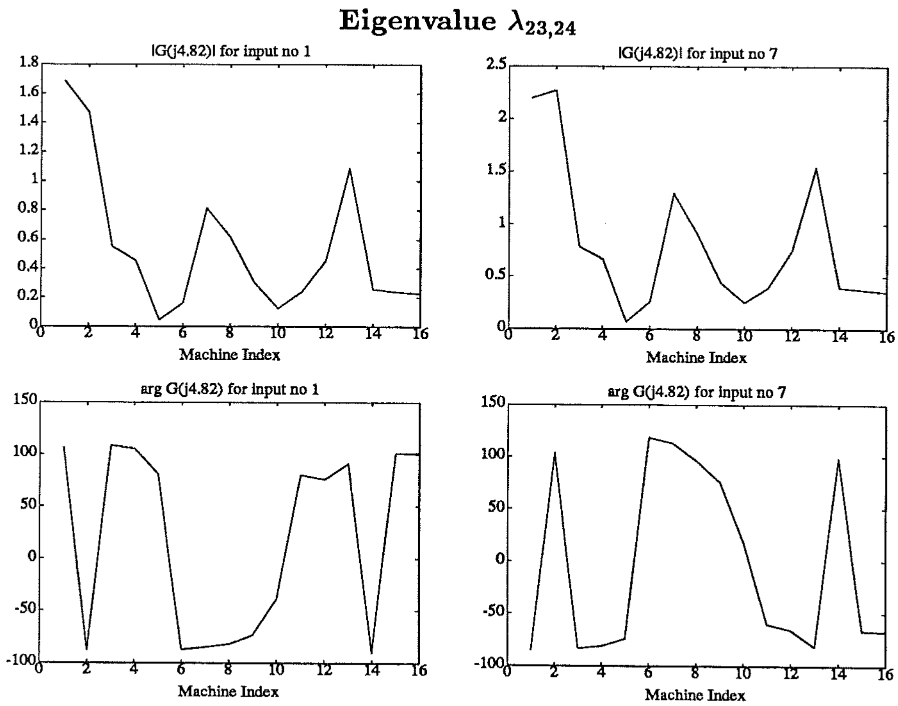


Figure 7.3 Influence on machine speed from u_{f_1} and u_{f_7} at ω=4.82 rad/s

Eigenvalue $\lambda_{25,26}$

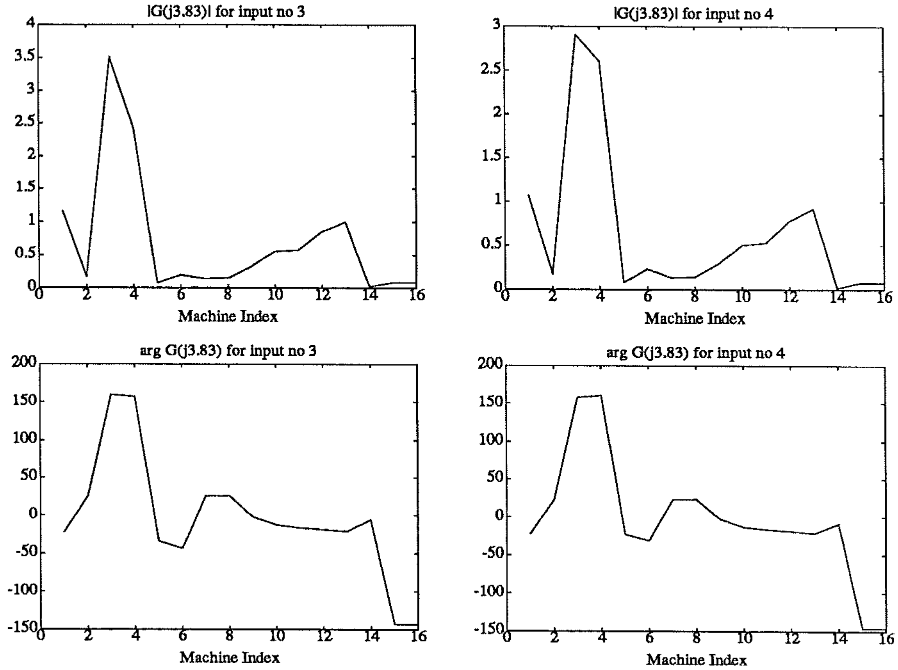


Figure 7.4 Influence on machine speed from u_{f_3} and u_{f_4} at ω=3.83 rad/s

Eigenvalue $\lambda_{27,28}$

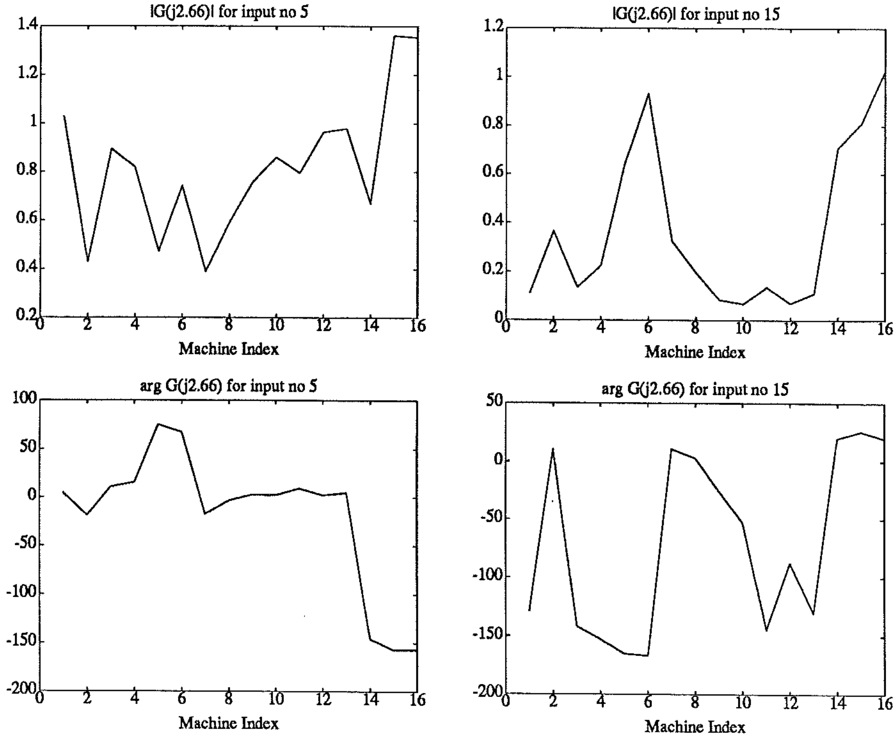


Figure 7.5 Influence on machine speed from u_{f_5} and $u_{f_{15}}$ at ω=2.66 rad/s

Eigenvalue $\lambda_{29,30}$

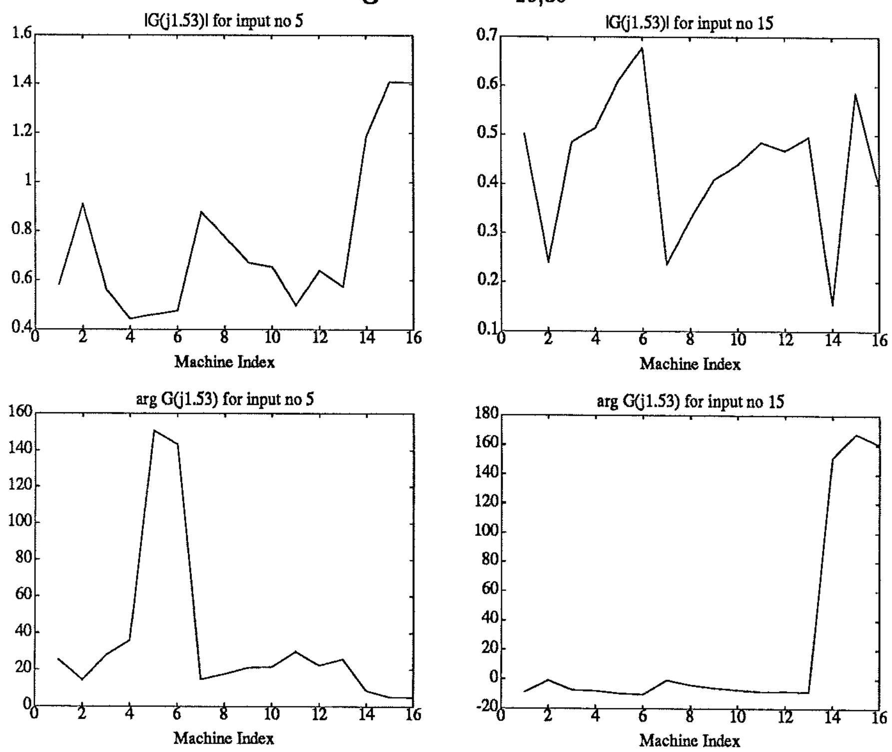


Figure 7.6 Influence on machine speed from u_{f_5} and $u_{f_{15}}$ at ω=1.53 rad/s

Conclusions

What are the conclusions about the feedback structure? First we know the critical eigenvalues and for these we know the inputs which are important in a good design. Moreover we have plotted the interaction for the important inputs. The reason for low frequency power oscillations is the fact that machines, or groups of machines, interact by swinging towards each other. To find "good" feedback variables for the important inputs, we should for each input look for influence on speed terms on other machines which have big magnitude and nearly 180° phase shift compared to the own machine's speed term.

To damp $\lambda_{23,24}$ we know that u_{f_1}, u_{f_2} and u_{f_7} are important. From Figure 7.3 we see that the influence on ω_1 and ω_7 has nearly 180° difference in phase shift. We draw the conclusion that PSS on machine 1 and 7 with communication is important to damp $\lambda_{23,24}$. We also choose a local PSS on machine 2.

For $\lambda_{25,26}$ we know that u_{f_3} and u_{f_4} are important and from Figure 7.4 we see that the influence on ω_3 and ω_4 has no difference in phase shift. From this we draw the conclusion that local PSS on machine 3 and 4 are important to damp $\lambda_{25,26}$.

If we look at $\lambda_{27,28}$ and $\lambda_{29,30}$ we know that input 5, 6, 15 and 16 are important. Here we show the Figures for u_{f_5} and $u_{f_{15}}$. From Figure 7.5 and 7.6 we see that there is a 180° phase shift between influence on ω_5 and ω_{15}. This leads to the conclusion that communication between, at least one of machine 5, 6, to one of machine 15, 16 is important to damp $\lambda_{29,30}$. We choose communication between machine 5 and 15.

The structure is then

$$
\begin{aligned}
U_{f_1} &= -\, k_{11}V_{t_1} - k_{12}P_{e_1} - k_{13}P_{e_7} - k_{14}\omega_1 - k_{15}\omega_7 \\
U_{f_2} &= -\, k_{21}V_{t_2} - k_{22}P_{e_2} - k_{23}\omega_2 \\
U_{f_3} &= -\, k_{31}V_{t_3} - k_{32}P_{e_3} - k_{33}\omega_3 \\
U_{f_4} &= -\, k_{41}V_{t_4} - k_{42}P_{e_4} - k_{43}\omega_4 \\
U_{f_5} &= -\, k_{51}V_{t_5} - k_{52}P_{e_5} - k_{53}P_{e_{15}} - k_{54}\omega_5 - k_{55}\omega_{15} \\
U_{f_6} &= -\, k_{61}V_{t_6} - k_{62}P_{e_6} - k_{63}\omega_6 \\
U_{f_7} &= -\, k_{71}V_{t_1} - k_{72}P_{e_1} - k_{73}P_{e_7} - k_{74}\omega_1 - k_{75}\omega_7 \\
U_{f_{15}} &= -\, k_{81}V_{t_{15}} - k_{82}P_{e_5} - k_{83}P_{e_{15}} - k_{84}\omega_5 - k_{85}\omega_{15} \\
U_{f_{16}} &= -\, k_{91}V_{t_{16}} - k_{92}P_{e_{16}} - k_{93}\omega_{16}
\end{aligned}
$$

The other machines have only local feedback from voltage.

The nine largest generators, ordered according to their rated power, are: 9, 5, 6, 10, 11, 15, 7, 16, 12. If we had choosen to place the PSS at the nine largest generators we have had a different result. The PSSs at generators 1 (Denmark), 2 (Norway), 3 (Norway) and 4 (Norway) should instead have been placed at generators 9,10,11,12 (all Sweden).

7.5 Tuning and approximation

In this section we should make an approximation of the global state feedback to fit a sparse feedback structure. From the previous section we know the important parts. Because of computational aspects we choose to use the least square approximation method described in Chapter 6. We do the approximation on a LQ design where the loss function emphasizes the important parts.

LQ with only important parts in loss function

We know the important machines and their interaction. From this we can modify the loss function to mostly punish parts which we have in our feedback structure. When we have communication between machines we punish both relative and absolute speed deviation and for local PSS we only punish absolute speed deviation. This implies that we may use the loss function

$$
\begin{aligned}
J = \int_0^\infty \Big(& 100V_{t_1}^2 + \cdots + 100V_{t_{16}}^2 \\
& + 0.49(\omega_1 - \omega_7)^2 + 0.49(\omega_5 - \omega_{15})^2 \\
& + 0.01(\omega_1^2 + \omega_5^2 + \omega_7^2 + \omega_{15}^2) \\
& + 0.50(\omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_6^2 + \omega_{16}^2) \\
& + 0.1U_{f_1}^2 + \cdots + 0.1U_{f_{16}}^2 \Big)\, dt
\end{aligned}
\tag{7.6}
$$

Calculation of the optimal K give the EOM eigenvalues to $A - BK$

$$\begin{array}{ll}
\lambda_{1,2} = -0.76 \pm 7.95j & \lambda_{3,4} = -0.59 \pm 7.92j \\
\lambda_{5,6} = -0.78 \pm 7.42j & \lambda_{7,8} = -0.71 \pm 7.14j \\
\lambda_{9,10} = -1.96 \pm 6.97j & \lambda_{11,12} = -1.04 \pm 6.72j \\
\lambda_{13,14} = -0.84 \pm 6.35j & \lambda_{15,16} = -1.98 \pm 6.29j \\
\lambda_{17,18} = -2.07 \pm 6.19j & \lambda_{19,20} = -0.83 \pm 5.78j \\
\lambda_{21,22} = -1.17 \pm 5.57j & \lambda_{23,24} = -1.77 \pm 5.01j \\
\lambda_{25,26} = -2.47 \pm 3.77j & \lambda_{27,28} = -1.36 \pm 2.79j \\
\lambda_{29,30} = -4.16 \pm 2.38j &
\end{array} \tag{7.7}$$

and

$$\begin{pmatrix} \zeta_{23,24} & \dots & \zeta_{29,30} \end{pmatrix} = \begin{pmatrix} 0.33 & 0.55 & 0.51 & 0.87 \end{pmatrix}$$

Where we note that only the four low frequency eigenvalues have good damping.

Least squares approximation

We use the least square approximation (6.12) and (6.13) from chapter 6 with the a_{ij} 10 or 0.1 and after truncation the control law $\hat{u} = \widehat{K}y$, becomes

$$\begin{aligned}
U_{f_1} &= -27.1V_{t_1} - 5.4P_{e_1} + 4.1P_{e_7} + 0.86\omega_1 - 0.28\omega_7 \\
U_{f_2} &= -27.1V_{t_2} - 10.7P_{e_2} + 1.48\omega_2 \\
U_{f_3} &= -26.7V_{t_3} - 12.1P_{e_3} + 1.60\omega_3 \\
U_{f_4} &= -26.5V_{t_4} - 12.3P_{e_4} + 1.63\omega_4 \\
U_{f_5} &= -28.2V_{t_5} - 11.6P_{e_5} + 0.35P_{e_{15}} + 1.82\omega_5 - 0.81\omega_{15} \\
U_{f_6} &= -27.8V_{t_6} - 11.4P_{e_6} + 1.86\omega_6 \\
U_{f_7} &= -25.5V_{t_7} + 1.8P_{e_1} - 9.3P_{e_7} - 0.75\omega_1 + 1.37\omega_7 \\
U_{f_8} &= -21.9V_{t_8} \\
U_{f_9} &= -20.5V_{t_9} \\
U_{f_{10}} &= -24.9V_{t_{10}} \\
U_{f_{11}} &= -25.4V_{t_{11}} \\
U_{f_{12}} &= -26.2V_{t_{12}} \\
U_{f_{13}} &= -19.9V_{t_{13}} \\
U_{f_{14}} &= -24.7V_{t_{14}} \\
U_{f_{15}} &= -27.3V_{t_{15}} + 3.5P_{e_5} - 7.3P_{e_{15}} - 0.15\omega_5 + 1.41\omega_{15} \\
U_{f_{16}} &= -27.1V_{t_{16}} - 6.2P_{e_{16}} + 1.19\omega_{16}
\end{aligned} \tag{7.8}$$

Eigenvalues

The EOM eigenvalues to $A - B\widehat{K}C$ is then

$$\begin{array}{ll} \lambda_{1,2} = -0.45 \pm 7.87j & \lambda_{3,4} = -0.15 \pm 7.85j \\ \lambda_{5,6} = -0.44 \pm 7.37j & \lambda_{7,8} = -2.03 \pm 7.17j \\ \lambda_{9,10} = -0.44 \pm 7.08j & \lambda_{11,12} = -0.80 \pm 6.68j \\ \lambda_{13,14} = -2.05 \pm 6.63j & \lambda_{15,16} = -1.71 \pm 6.42j \\ \lambda_{17,18} = -0.63 \pm 6.24j & \lambda_{19,20} = -0.63 \pm 5.78j \\ \lambda_{21,22} = -1.26 \pm 5.73j & \lambda_{23,24} = -1.25 \pm 5.13j \\ \lambda_{25,26} = -2.05 \pm 3.87j & \lambda_{27,28} = -1.00 \pm 2.75j \\ \lambda_{29,30} = -3.38 \pm 1.93j & \end{array} \tag{7.9}$$

and

$$\begin{pmatrix} \zeta_{23,24} & \dots \zeta_{29,30} \end{pmatrix} = \begin{pmatrix} 0.33 & 0.54 & 0.44 & 0.90 \end{pmatrix}$$

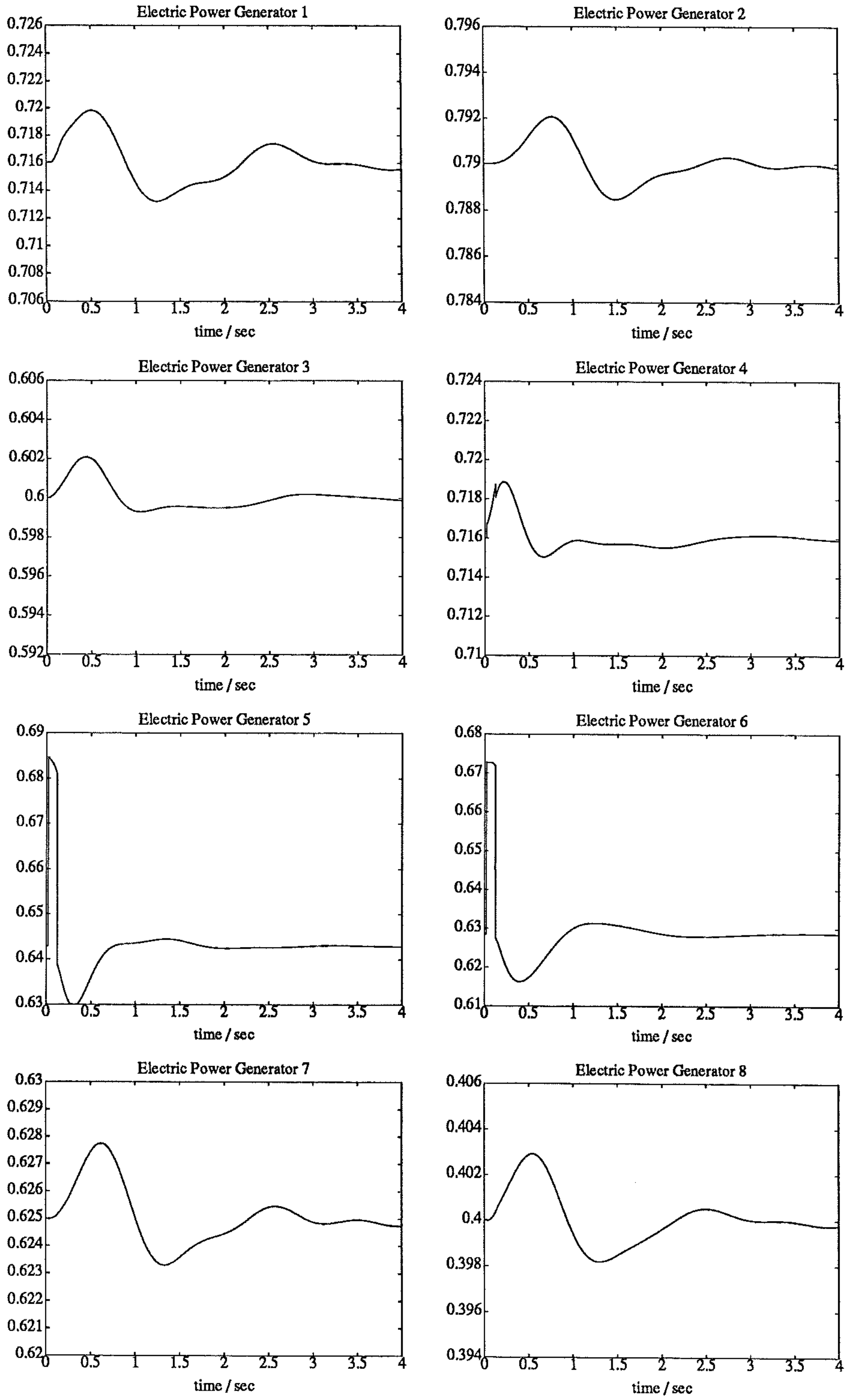
Which still have fair damping on the low frequency EOM.

7.6 Simulation with Simpow

We do simulation in Simpow and choose a PSS with machine speed and electric power as inputs. The feedback gains have been scaled to fit Simpows units. All simulation data can be found in Appendix D.2. The disturbance is identical to the previous without PSS in Figure 7.2. In the following Figures we show electric power for all machines. Furthermore speed (ω), PSS signal (V_S), field voltage (U_f) and terminal voltage (V_t) are shown for machines 1, 3, 5 and 15.

A comparison between Figure 7.2 and Figure 7.7 shows that the power oscillations in general are much better damped with PSS and in particular the low frequency oscillations. If we for example look at generator number 8 we have changed the power oscillations from a very poor behaviour in Figure 7.2 to a well damped behaviour in Figure 7.7. We also see that higher frequencies, which were not emphasized in the design, turn up in Figure 7.7. As seen for generator 11, these higher frequencies are effectivelly damped by the damper windings. Inspection of the PSS signal V_S and control signal u_f in Figure 7.10 and 7.11 respectively, shows that both signals have moderate amplitudes as indicated by the feedback elements. With a simple control law, which uses small control signals, we have succeeded in the design objective.

Power oscillations for machines 1-16 with PSS



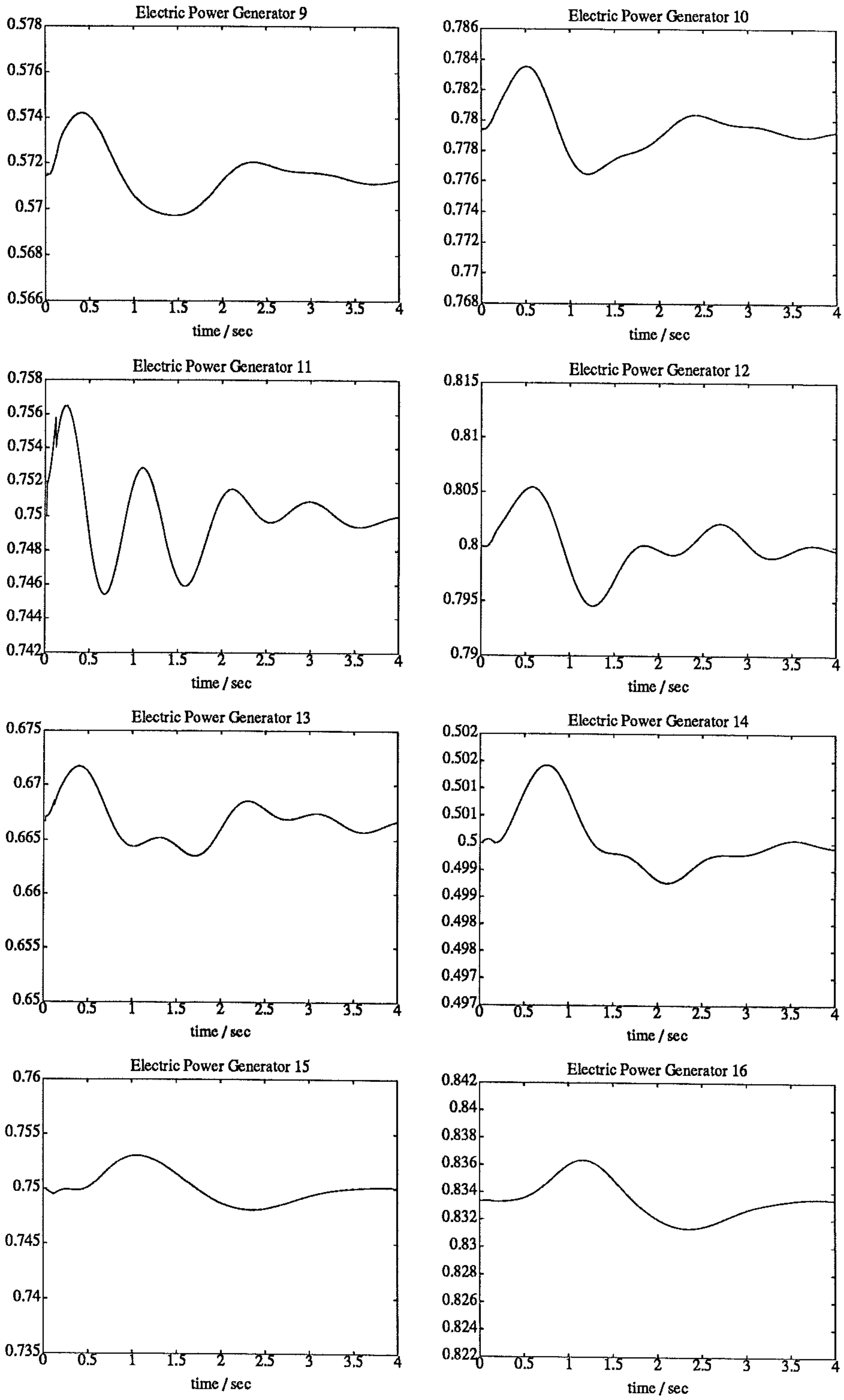


Figure 7.7 Power oscillation in the Nordel network after disturbance at machines five and six. Strategic generators have PSS after design (7.8) and are simulated with damper windings in Simpow. Compare with Figure 7.2.

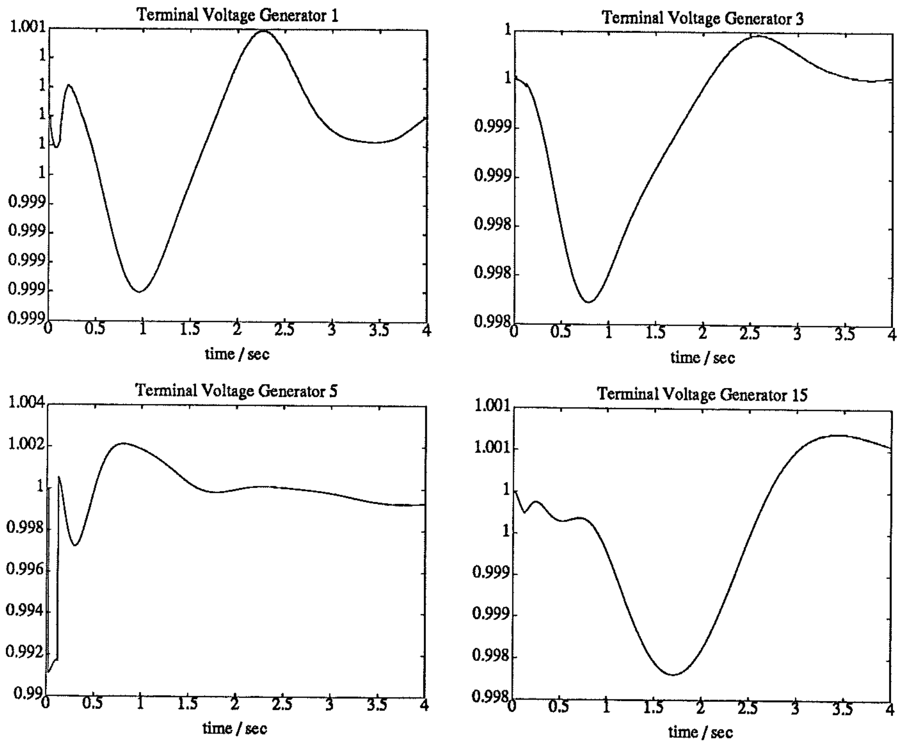


Figure 7.8 Terminal voltage V_t for machines 1, 3, 5, 15 with design (7.8)

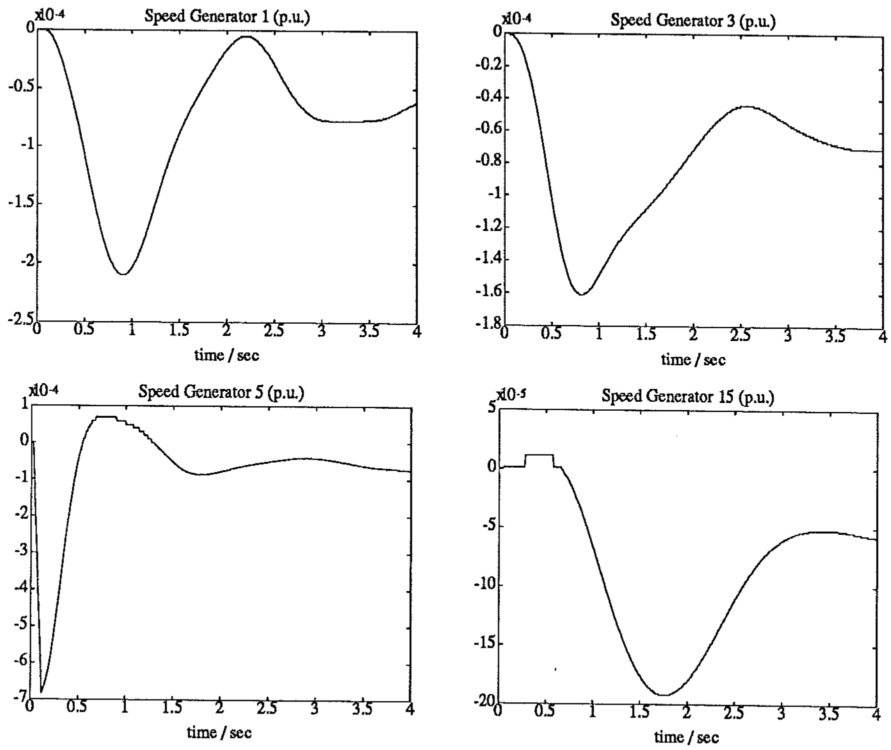


Figure 7.9 Machine speed ω for machines 1, 3, 5, 15 with design (7.8)

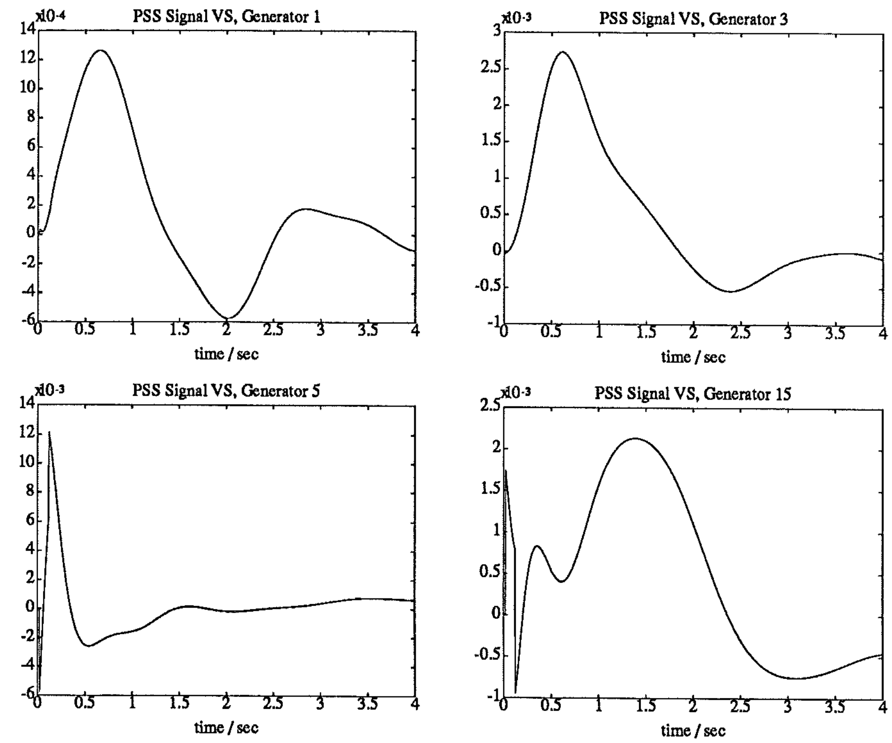


Figure 7.10 PSS signal V_S for machines 1, 3, 5, 15 with design (7.8)

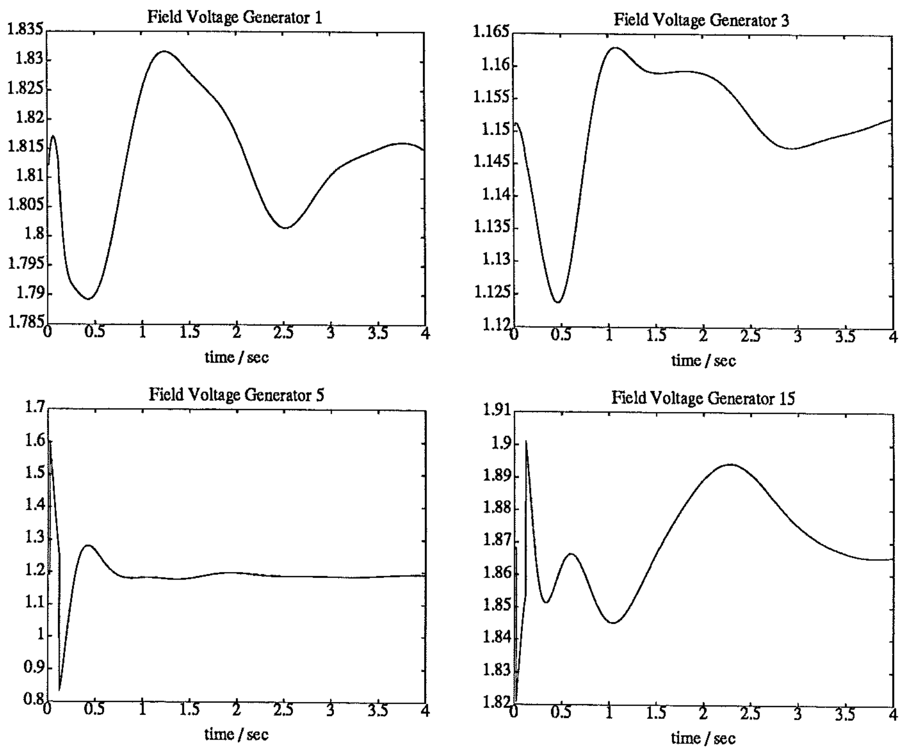


Figure 7.11 Control signal U_f for machines 1, 3, 5, and 15 with design (7.8)

The load models importance

The same simulation has been done were all loads were modelled as constant power loads. This is a very complicated load model since the power demand is independent of frequency and voltage. In (2.1) both $k = 0$ and $i = 0$, which is a very difficult load characteristic for power system stability problems. The result can be seen in Appendix D.3 and shows that the good damping features are lost. This shows the importance of the load models. Our design is done for a model with impedance loads and will therefore make very poor under such unfavourable circumstances as constant power loads. Much of the idea with a PSS is to vary with the voltage so the load on the generator is changed in a way that damp power oscillations. With constant power loads we do not have this possibility since all loads are independent of both voltage and frequency. The design is not robust for this load case. Basicly since the design was done on totally different premises on load characteristics.

The loads in a real power system are a mix of all kinds. One large part is electrical heating and lights, which can be modelled as impedances. Constant power loads, as computers and electronic devices, are usually a smaller fraction of the total loads. Then it seems very likely that the response in a real power system would more look like Figure 7.7 (impedance loads) than Appendix D.3 (constant power loads).

7.7 Conclusions

This example shows that a relative simple control law can capture the most important parts for a good design. The damping of the low frequency oscillation is considerably better with the PSS stabilizers. In Figure 7.7 we also see that other oscillation modes with higher frequencies show up. But these are not a big problem since they are damped by the damper windings. A comparison between Figures 7.2 and 7.7 shows that the low frequency oscillations below 1 Hz are very well damped. This shows that we have succeeded in the objective to damp the low frequency oscillations if all loads are impedances. When we simulate with constant power loads we get a totally different behaviour and see the importance of the load characteristics.

8. Conclusions

8.1 Summary

The objective of this thesis has been to present methods to apply and tune power system stabilizers in multimachine systems. The problem has been approached from a control point of view. From this point of view the PSS-problem is a multivariable problem, which has been decomposed in the following parts:

- Make a multimachine model.
- Find feedback structure for controller.
- Tune the controller.

A multimachine model suitable for design has been presented and verified in Chapter 2. After a review of today's methods, the Linear Quadratic regulator design method has been presented as a PSS design method. The choice of loss matrices in the LQ design has been emphasized. Chapter 5 has presented a procedure to find a sparse feedback structure, which captured important parts in a good control law for a multivariable system. Chapter 6 has presented two tuning methods for an incomplete state feedback controller. Finally, Chapter 7 has illustrated how to use the presented ideas in a PSS-design for a 16 machine power system.

8.2 What is new?

Implementation of a multimachine model

To the author's knowledge there has not existed any linearised multimachine model which has been used for design purpose in neither the academic world in Scandinavia nor in the power industry in Scandinavia. Here we do all steps in a multimachine design for the Nordic power system. Even though there have been large economical motives, all previous designs for the Nordel system have been done using single machine models.

Choice of loss function in LQ-design

The choice of non diagonal loss matrices has not, to the author's knowledge, been published. This choice can improve the design when we have interaction between dominant machines.

Input Energy Optimal Controler

The IEOC is supposed to be new. Intuitively it seems to be a much better approach to see what we can achieve with a good design (LQ) and use this as an analysis tool.

Global signals can improve PSS action

Transmitting signals between generators can successfully be used in a PSS control law, see design (6.17). Especially if we want to damp oscillations between two machine groups. A good compromise is to transmit one or two signals for each low frequency eigenvalue which caused problem. A control law of the type

$$u_{f_i} = -\alpha V_{t_i} + \beta_1(P_{e_i} - P_{e_j}) + \beta_2 P_{e_i} + \gamma_1(\omega_i - \omega_j) + \gamma_2\omega_i \qquad (8.1)$$

can be used, where the relative part should punish a specific mode and the absolute part all oscillations towards the linearisation point.

Least square approximation of LQ

The least square approximation with incorporating of the algebraic constraint is supposed to be new. Even though it is a very simple idea it gives satisfactory results.

8.3 Strengths and weaknesses

The thesis presents the main parts in a multimachine design and shows how to use this for a design on a large power system. With a reliable commercial simulation package (Simpow) we verify the result that we can get a good damping of the low frequency modes.

The design concept to transmit signals has not been tested in the field. No practical experiment with a full size multimachine power system has been done. Also we have not considered the robustness problem. This is more a limitation of the scope of this thesis and the robustness issue should need a separate thesis. Furthermore we have seen that load models are important for the design. The resulting design works well on load characteristic which it was designed for, but not on constant power loads. A fundamental question is if the PSS-idea works at all for constant power loads?

8.4 Future work

Future work would be to try out the design concept in laboratory with some synchronous machines and then on a larger full size system. A way to do full scale experiments under controlled conditions is to use the method presented in Degn and Östrup (1988). The method is to enter a small sinusoidal signal in the excitation system and see how the induced power ocillations are damped by the PSSs.

Another topic is to see how various simplifying assumptions influence the design, e.g load and machine modelling. An interesting direction for further work is to investigate the influence of load characteristics on the control law. Is it more important to transmit signals when we have constant power loads? To do this we have to refine the multimachine model with different load models.

Another topic is to incorporate robustness aspects in the design to guarantee good damping for different operation conditions. The question how to find a good feedback structure in multivariable system is very complicated and needs more research.

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Appendices

A1 - Notations and units in multimachine model

Notations and units

S_n	Nominal generator 3-phase VA-power	VA
P_m	Mechanical power	p.u. of S_n
P_e	Electrical power	p.u. of S_n
H	Inertia time constant, W_k/S_n	s
W_k	kinetic energy in machine at ω_R	Nm
ω_R	Rated electrical angular velocity	rad/s
δ	Load angle from reference to generators q-axis	rad
D	Speed damping term	p.u. / (rad/s)
T'_{d_0}	d-axis transient open circuit time constant	s
E'_q	Stator emk on q-axis, prop. to flux from field winding	p.u. of V_{t_n}
V_{t_n}	Nominal terminal (generator) voltage	V
V_t	Terminal voltage	p.u. of V_{t_n}
I_{t_n}	Nominal terminal (generator) current	A
I_t	Terminal current	p.u. of I_{t_n}
Z_n	Base impedance $V_{t_n}^2/S_n$	ohm
X_d	d-axis synchronous reactance	p.u. of Z_n
X_q	q-axis synchronous reactance	p.u. of Z_n
X'_d	d-axis transient reactance	p.u. of Z_n
V_d	d-axis componet of V_t	p.u. of V_{t_n}
V_q	q-axis componet of V_t	p.u. of V_{t_n}
I_d	d-axis componet of I_t	p.u. of I_{t_n}
I_q	q-axis componet of I_t	p.u. of I_{t_n}
U_{f_n}	Field voltage corresponding to V_{t_n} with no load	V
U_f	Field voltage	p.u. of U_{f_n}
ϕ	Angle between I_t and V_t	rad

A2 - Multimachine model

Linearised model - implemented in Matlab

```
function [A,B,C]=case9();
%function [A,B,C]=case9();
%
% version 89-01-23
%
% Generates a linearised multimachine system from the
% machinedata, loaddata and network specified below
% The machines have different bases
%======================================================
%
%  This case have nine machines
%
%======================================================
% Programmers data
Debugg=0;
Offset= 0*pi/180;
%
%======================================================
%  General data
%
f=50; % Hz
OmegaR=2*pi*f  ;
NoOfMachines=9 ;
SBas= 100   ; %  MVA
UBas= 10          ; % kV
IBas=SBas/(sqrt(3)*UBas) ; % kA
ZBas=(UBas^2)/SBas  ; %  ohm
YBas=1/ZBas   ;  % siemens , mho
%======================================================
%======================================================
% Machine data
%
%--------------------------------------------------------------------
MachineNo= 1 ;
SBasM(MachineNo)=100    ; % MVA
UBasM(MachineNo)=10     ; % kV
IBasM(MachineNo)=SBasM(MachineNo)/(sqrt(3)*UBasM(MachineNo))     ; % kA
ZBasM(MachineNo)=UBasM(MachineNo)^2/SBasM(MachineNo)     ; % ohm

% All machine parameters and initialvalues refer to each machine base
%====================================================================


% Parameters:
            Xd(MachineNo)=       1.0   ;% p.u.
            Xdprim(MachineNo)=   0.2   ; % p.u.
            Xq(MachineNo)=       0.8   ; % p.u.
            H(MachineNo)=        3.1   ; %
            D(MachineNo)=        0.000; % p.u. / rad/s
            Td0prim(MachineNo)=  3.0   ; % s

% InitialValues:
            Vt0(MachineNo)=      1.00 ;% p.u.
            P0(MachineNo)=       0.610058; % p.u.
            Q0(MachineNo)=       0.254013;  % p.u.
            Teta0(MachineNo)=    0.00*pi/180 +Offset;      % rad
%--------------------------------------------------------------------

%--------------------------------------------------------------------
MachineNo=2 ;
SBasM(MachineNo)=200    ; % MVA
UBasM(MachineNo)=10     ; % kV
IBasM(MachineNo)=SBasM(MachineNo)/(sqrt(3)*UBasM(MachineNo))     ; % kA
ZBasM(MachineNo)=UBasM(MachineNo)^2/SBasM(MachineNo)     ; % ohm
```

```
.
.
.
and so  on
.
.
.


%-----------------------------------------------------------------------
%-----------------------------------------------------------------------
MachineNo=9  ;
SBasM(MachineNo)=900  ; % MVA
UBasM(MachineNo)=10    ; % kV
IBasM(MachineNo)=SBasM(MachineNo)/(sqrt(3)*UBasM(MachineNo))     ; % kA
ZBasM(MachineNo)=UBasM(MachineNo)^2/SBasM(MachineNo)     ; % ohm
% All machine parameters and initialvalues refer to each machine base
%
% Parameters:
             Xd(MachineNo)=      1.3   ;   % p.u.
             Xdprim(MachineNo)=  0.25   ;   % p.u.
             Xq(MachineNo)=      1.0  ;   % p.u.
             H(MachineNo)=       5.1   ;   %
             D(MachineNo)=       0.000  ;    % p.u. / rad/s
             Td0prim(MachineNo)= 9.1  ;  % s

% InitialValues:
             Vt0(MachineNo)=     1.000   ;       % p.u.
             P0(MachineNo)=      0.86/9    ;       % p.u.
             Q0(MachineNo)=      0.199705/9     ;     % p.u.
             Teta0(MachineNo)=   -32.7906*pi/180+Offset;      % rad
%-----------------------------------------------------------------------

%
% End Machine data
%=======================================================

%======================================================
% Network data  in p.u.   of system base
%
j=sqrt(-1)       ;
%
%
% Just fill in element in upper half of matrix
Big=1/eps      ;
z=Big* ones(NoOfMachines) ;

z(1,2)=    ( 0.011    ) + j*( 0.21  )    ;
z(1,3)=    ( 0.012    ) + j*( 0.31  )     ;


z(2,3)=    ( 0.013  ) + j*( 0.32  )    ;
z(2,4)=    ( 0.020    ) + j*( 1.24  )    ;

z(3,7)=    ( 0.021    ) + j*( 1.37  )    ;

z(4,5)=    ( 0.014    ) + j*( 0.45  )    ;
z(4,6)=    ( 0.015    ) + j*( 0.46  )    ;

z(5,6)=    ( 0.016    ) + j*( 0.56  )    ;

z(6,8)=    ( 0.022    ) + j*( 1.68  )    ;

z(7,8)=    ( 0.017    ) + j*( 0.78  )    ;
z(7,9)=    ( 0.018    ) + j*( 0.79  )    ;

z(8,9)=    ( 0.019    ) + j*( 0.89  )    ;

%
%  Get symmetric matrix
```

```
Z=triu(z)+triu(z).';

%
% End Network data
%==========================================================
%==========================================================
% Load data
% load connected to generatorbuses      in system base
%
P0Load(1)= 0.20 ;
Q0Load(1)= 0.26 ;

P0Load(2)= 0.60 ;
Q0Load(2)= 0.24 ;

P0Load(3)= 1.00 ;
Q0Load(3)= 0.22 ;

P0Load(4)= 0.30 ;
Q0Load(4)= 0.20 ;

P0Load(5)= 0.85 ;
Q0Load(5)= 0.18 ;

P0Load(6)= 1.30 ;
Q0Load(6)= 0.16 ;

P0Load(7)= 0.25 ;
Q0Load(7)= 0.14 ;

P0Load(8)= 0.95 ;
Q0Load(8)= 0.12 ;

P0Load(9)= 1.45 ;
Q0Load(9)= 0.10 ;

%
% End Load data
%==========================================================
% --- No further input data needed --
%==========================================================
% Calculation of initialvalues for generators
% in p.u. of each machine base

S0=sqrt(P0.*P0+Q0.*Q0) ;
It0=S0./Vt0 ;
SinPhi=Q0./S0 ;



Iq0=P0 ./ sqrt(Vt0.* Vt0+2*Vt0.*It0.*Xq.*SinPhi+It0.*Xq.*It0.*Xq   ) ;

Id0=sign(Q0).*sqrt(It0.*It0-Iq0.*Iq0) ;

Eqprim0=sqrt( Vt0.*Vt0 - (Xq.*Iq0).^2 ) + Xdprim .*Id0 ;

Delta0= Teta0+ asin( Xq.*Iq0./ Vt0 ) ;

%
% End calculation of initialvalues for generators
%==========================================================

%====================================================================
%   Calculation of load impedances   in system p.u.
%
Xparallell=Vt0.^2 ./ Q0Load ;

Rparallell=Vt0.^2 ./ P0Load ;

Xserie=Rparallell.^2 .* Xparallell ./ ( Rparallell.^2 + Xparallell.^2 ) ;
```

```
Rserie=Xparallell.^2 .*Rparallell ./ ( Rparallell.^2 + Xparallell.^2 )    ;

Zload=Rserie+j*Xserie ;

%
%  End calculation of load impedances
%=====================================================================
%=====================================================================
% Calculation of admittance-matrix
% in system p.u.

Y=zeros(NoOfMachines) ;


for i=1:NoOfMachines
  for k=1:NoOfMachines
    if i ~= k
      Y(i,k)= - 1/Z(i,k) ;
    end
  end
end


for i=1:NoOfMachines
  Y(i,i)=1/Zload(i) - sum(Y(i,:))  ;
end
Z=inv(Y) ;
B=imag(Y) ;
G=real(Y) ;
%
%    End calculation of admittance matrix
%======================================================================
%=====================================================================
%  Linearisation of algebraic relation
%
%
CosDelta = diag( cos(Delta0)  )       ; % dimension free
SinDelta = diag( sin(Delta0)  )       ; %    -- " --
Xdprim   = diag( Xdprim )             ; % in p.u. of each machine base
Xd       = diag(Xd)                   ; %    -- " --
Xq       = diag( Xq)                  ; %    -- " --
Id0      = diag(Id0 )                 ; %    -- " --
Iq0      = diag(Iq0)                  ; %    -- " --
Eqprim0  = diag(Eqprim0)              ; %    -- " --
Vt0      = diag(Vt0)                  ; %    -- " --
One      =eye(NoOfMachines)           ;
%
ZBasM=diag(ZBasM);          % ohm
IBasM=diag(IBasM)*1000;     % A
UBasM=diag(UBasM)*1000;     % V


%
% To linearise, we must first convert all quantities  to have same units
%  V, A, ohm
% and then  to prefered basunit
%
XdprimSI=Xdprim*ZBasM   ;      % ohm
XdSI=Xd*ZBasM           ;      % ohm
XqSI=Xq*ZBasM           ;      % ohm
Id0SI=Id0*IBasM         ;      %  A  per phase
Iq0SI=Iq0*IBasM         ;      %  A  per phase
Vt0SI=Vt0*UBasM/sqrt(3)  ;            %  V  phase voltage
Eqprim0SI=Eqprim0*UBasM/sqrt(3)    ;  %  V  phase voltage
BSI=YBas*B                ;     % Siemens
GSI=YBas*G                ;     % Siemens
%
 a11=(One-BSI*XdprimSI)*SinDelta + GSI*XdprimSI*CosDelta;
 a12=(One-BSI*XqSI)*CosDelta     - GSI*XqSI*SinDelta    ;
 a21=GSI*XdprimSI*SinDelta+(BSI*XdprimSI-One)*CosDelta  ;
 a22=GSI*XqSI*CosDelta+(One-BSI*XqSI)*SinDelta          ;
```

```
%
 b11= - GSI*CosDelta+BSI*SinDelta ;
%
 b12Id0=( (One-BSI*XdprimSI)*CosDelta-GSI*XdprimSI*SinDelta )*Id0SI;
 b12Iq0=( (BSI*XqSI-One)*SinDelta-GSI*XqSI*CosDelta )* Iq0SI ;
 b12Eqprim0=( GSI*SinDelta + BSI*CosDelta )* Eqprim0SI ;
 b12=b12Id0+b12Iq0+b12Eqprim0 ;
%
 b21= -BSI*CosDelta-GSI*SinDelta ;
%
 b22Id0=(GSI*XdprimSI*CosDelta+(One-BSI*XdprimSI)*SinDelta )*Id0SI  ;
 b22Iq0=(-GSI*XqSI*SinDelta+(One-BSI*XqSI)*CosDelta )* Iq0SI ;
 b22Eqprim0=( BSI*SinDelta - GSI*CosDelta )* Eqprim0SI ;
 b22=b22Id0+b22Iq0+b22Eqprim0 ;
%
  F= -([ a11 a12 ; a21 a22]) \ [ b11 b12; b21 b22] ;
 NoOfM=NoOfMachines;
%
 FIdEqprim=F(1:NoOfM,1:NoOfM) ;
 FIdDelta=F(1:NoOfM,NoOfM+1:2*NoOfM) ;
%
 FIqEqprim=F(NoOfM+1:2*NoOfM,1:NoOfM);
 FIqDelta=F(NoOfM+1:2*NoOfM,NoOfM+1:2*NoOfM);
%

%
% Go from S.I units to machine units
%
 UBasMPhase=UBasM*(1/sqrt(3))   ;

 FIdEqprim=inv(IBasM)*FIdEqprim*(UBasMPhase) ;
 FIdDelta=inv(IBasM)*FIdDelta ;
%%
 FIqEqprim=inv(IBasM)*FIqEqprim*(UBasMPhase) ;
 FIqDelta=inv(IBasM)*FIqDelta ;
%
%  End linearisation of algebraic relation
%
%====================================================================


%===================================================================
%   Linearised state space formulation
%        .
%        x  = A x + B u
%        y  = C x
%                                  t
% where  x = [ Delta Omega Eqprim ]
%                     t
%        u=  [ Uf  Pm ]
%                   t
%        y=  [ Vt Pe ]
% A consist of 9 blocks, B of 6 blocks, C of 6 blocks
% indexed with D W Eq  respectivly
%
OneOverTd0prim=diag(   Td0prim.^(-1) );
dPdId=Iq0*(Xq-Xdprim)           ;
dPdIq=Id0*(Xq-Xdprim)+Eqprim0 ;
dPdEq=Iq0;
dEqdId= -OneOverTd0prim*(Xd-Xdprim);


% make blocks
AblockDD   = zeros(NoOfMachines)  ;
AblockDW   = eye(NoOfMachines)    ;
AblockDEq  = zeros(NoOfMachines)  ;

AblockWD   = - diag( OmegaR./(2*H) ) * (dPdId * FIdDelta + dPdIq*FIqDelta)  ;
AblockWW   = - OmegaR*diag(D./(2*H))      ;
% new
AblockWEq  = -diag(OmegaR./(2*H)) * (dPdId*FIdEqprim+dPdIq*FIqEqprim+dPdEq);
```

```
AblockEqD  =   dEqdId *FIdDelta  ;
AblockEqW  = zeros(NoOfMachines)  ;
AblockEqEq = - OneOverTdOprim* eye(NoOfMachines) + dEqdId*FIdEqprim  ;

BblockDEfd    =  zeros(NoOfMachines) ;
BblockWEfd    =  zeros(NoOfMachines) ;
BblockEqEfd   =  OneOverTdOprim      ;

BblockDPm=zeros(NoOfMachines) ;
BblockWPm=diag(OmegaR./(2*H)) ;
BblockEqPm=zeros(NoOfMachines);

OneOverVt0= diag(diag(Vt0).^(-1) );
dVdIq = Xq*Xq*Iq0*OneOverVt0  ;
dVdId = - (Eqprim0-Xdprim*Id0)*Xdprim*OneOverVt0 ;
dVdEq =   (Eqprim0-Xdprim*Id0)*OneOverVt0 ;


CblockVD   = dVdId*FIdDelta + dVdIq*FIqDelta ;
CblockVW   =  zeros(NoOfMachines) ;
CblockVEq  = dVdId*FIdEqprim + dVdIq*FIqEqprim + dVdEq ;
%

CblockPD   = dPdId*FIdDelta+dPdIq*FIqDelta ;
CblockPW   = zeros(NoOfMachines) ;
CblockPEq  = dPdId*FIdEqprim +dPdIq*FIqEqprim +dPdEq ;


%
%
A=[AblockDD   AblockDW    AblockDEq
   AblockWD   AblockWW    AblockWEq
   AblockEqD  AblockEqW   AblockEqEq ] ;

B=[BblockDEfd  BblockDPm
   BblockWEfd  BblockWPm
   BblockEqEfd BblockEqPm ] ;

C=[ CblockVD CblockVW CblockVEq
    CblockPD CblockPW CblockPEq] ;
%
%  End linearised state space formulation
%=====================================================================
```

A3 - Test case to compare model with Simpow

Input file to loadflow calculation in Optpow

```
CASE9 NINE MACHINE CASE
OPTPOW LOADFLOW DATA
**
NODES
BUS1 UB 10
BUS2 UB 10
BUS3 UB 10
BUS4 UB 10
BUS5 UB 10
BUS6 UB 10
BUS7 UB 10
BUS8 UB 10
BUS9 UB 10
END
LINES
BUS1 BUS2 TYPE 11 R=0.011  X=0.21
BUS1 BUS3 TYPE 11 R=0.012  X=0.31
BUS2 BUS3 TYPE 11 R=0.013  X=0.32
BUS4 BUS5 TYPE 11 R=0.014  X=0.45
BUS4 BUS6 TYPE 11 R=0.015  X=0.46
BUS5 BUS6 TYPE 11 R=0.016  X=0.56
BUS7 BUS8 TYPE 11 R=0.017  X=0.78
BUS7 BUS9 TYPE 11 R=0.018  X=0.79
BUS8 BUS9 TYPE 11 R=0.019  X=0.89
BUS2 BUS4 TYPE 11 R=0.020  X=1.24
BUS3 BUS7 TYPE 11 R=0.021  X=1.37
BUS6 BUS8 TYPE 11 R=0.022  X=1.68
END
LOADS
BUS1 P=20  Q=26
BUS2 P=60  Q=24
BUS3 P=100 Q=22
BUS4 P=30  Q=20
BUS5 P=85  Q=18
BUS6 P=130 Q=16
BUS7 P=25  Q=14
BUS8 P=95  Q=12
BUS9 P=145 Q=10
END
POWER
BUS1 TYPE NODE RTYP=SW  U=10 FI=0
BUS2 TYPE NODE RTYP=UP  P=70  U=10
BUS3 TYPE NODE RTYP=UP  P=74  U=10
BUS4 TYPE NODE RTYP=UP  P=76  U=10
BUS5 TYPE NODE RTYP=UP  P=78  U=10
BUS6 TYPE NODE RTYP=UP  P=80  U=10
BUS7 TYPE NODE RTYP=UP  P=82  U=10
BUS8 TYPE NODE RTYP=UP  P=84  U=10
BUS9 TYPE NODE RTYP=UP  P=86  U=10
END
END
```

Output from loadflow calculation in Optpow

```
% CASE 9
 BUS1      (   1)   1.00000     P.U.
                    10.0000     KV
                    0.00000     DEGREES
        POWER FROM                                    MW          MVAR
                     PROD BUS1                     61.0058      25.4013
                     LINE BUS1     BUS2       0   -16.5917      0.578894
                     LINE BUS1     BUS3       0   -24.4141      0.198022E-01
```

```
                  LOAD BUS1         0            -20.0000      -26.0000

BUS2     (   1)   1.00000      P.U.
                  10.0000      KV
                 -2.00038      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS2                      70.0000       25.8663
                  LINE BUS1    BUS2         0    16.5613      -1.15769
                  LINE BUS2    BUS3         0   -12.7501      0.257333
                  LINE BUS2    BUS4         0   -13.8112     -0.965979
                  LOAD BUS2         0            -60.0000      -24.0000

BUS3     (   1)   1.00000      P.U.
                  10.0000      KV
                 -4.34064      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS3                      74.0000       25.3186
                  LINE BUS1    BUS3         0    24.3426      -1.86755
                  LINE BUS2    BUS3         0    12.7290     -0.777757
                  LINE BUS3    BUS7         0   -11.0716     -0.673264
                  LOAD BUS3         0            -100.000      -22.0000

BUS4     (   1)   1.00000      P.U.
                  10.0000      KV
                 -11.8502      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS4                      76.0000       23.9244
                  LINE BUS4    BUS5         0   -21.4257     -0.367612
                  LINE BUS4    BUS6         0   -38.3472      -2.14591
                  LINE BUS2    BUS4         0    13.7729      -1.41088
                  LOAD BUS4         0            -30.0000      -20.0000

BUS5     (   1)   1.00000      P.U.
                  10.0000      KV
                 -17.3800      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS5                      78.0000       19.8665
                  LINE BUS4    BUS5         0    21.3614      -1.69877
                  LINE BUS5    BUS6         0   -14.3614     -0.167724

BUS6     (   1)   1.00000      P.U.
                  10.0000      KV
                 -21.9914      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS6                      80.0000       21.6455
                  LINE BUS4    BUS6         0    38.1259      -4.63960
                  LINE BUS5    BUS6         0    14.3284     -0.987433
                  LINE BUS6    BUS8         0   -2.45430     -0.184702E-01
                  LOAD BUS6         0            -130.000      -16.0000

BUS7     (   1)   1.00000      P.U.
                  10.0000      KV
                 -13.0568      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS7                      82.0000       23.4050
                  LINE BUS7    BUS8         0   -25.1580      -1.93589
                  LINE BUS7    BUS9         0   -42.8877      -6.45681
                  LINE BUS3    BUS7         0    11.0457      -1.01229
                  LOAD BUS7         0            -25.0000      -14.0000

BUS8     (   1)   1.00000      P.U.
                  10.0000      KV
                 -24.3543      DEGREES
         POWER FROM                                 MW            MVAR
                  PROD BUS8                      84.0000       15.9764
                  LINE BUS7    BUS8         0    25.0498      -3.03016
                  LINE BUS8    BUS9         0   -16.5027     -0.863480
                  LINE BUS6    BUS8         0    2.45298     -0.827322E-01
                  LOAD BUS8         0            -95.0000      -12.0000

BUS9     (   1)   1.00000      P.U.
```

```
                10.0000      KV
               -32.7906      DEGREES
POWER FROM                                       MW          MVAR
                 PROD BUS9                     86.0000      19.9705
                 LINE BUS7      BUS9      0    42.5492     -8.40348
                 LINE BUS8      BUS9      0    16.4508     -1.56698
                 LOAD BUS9        0           -145.000     -10.0000
```

Input file to simulation with Transta

```
CASE 9  NINE MACHINE CASE
TRANSTA SIMULATION DATA
**
CONTROL DATA
TEND=2 METHOD=2 HFIX=0.010
END
GENERAL
REF GEN1 FN 50
END
LOADS
BUS1 SW=1 DPC=0 DPI=1
BUS2 SW=1 DPC=0 DPI=1
BUS3 SW=1 DPC=0 DPI=1
BUS4 SW=1 DPC=0 DPI=1
BUS5 SW=1 DPC=0 DPI=1
BUS6 SW=1 DPC=0 DPI=1
BUS7 SW=1 DPC=0 DPI=1
BUS8 SW=1 DPC=0 DPI=1
BUS9 SW=1 DPC=0 DPI=1
END
SYNCHRONOUS MACHINES
GEN1 BUS1 TYPE=3A SN=100  UN=10 H=3.1  RA=0.0001 XA=0.001
      XD=1.0    XQ=0.80  XDP=0.20
      TDOP=3.0  VREG=1   TURB=1
GEN2 BUS2 TYPE=3A SN=200  UN=10 H=3.2  RA=0.0001 XA=0.001
      XD=1.1    XQ=0.9   XDP=0.25
      TDOP=3.3  VREG=2 TURB=1
GEN3 BUS3 TYPE=3A SN=300   UN=10 H=3.3  RA=0.001  XA=0.001
      XD=1.2  XQ=1.1  XDP=0.3
      TDOP=3.8  VREG=3  TURB=1
GEN4 BUS4 TYPE=3A SN=400  UN=10 H=4.3  RA=0.0001 XA=0.001
      XD=1.5    XQ=1.4   XDP=0.40
      TDOP=5.5  VREG=4   TURB=1
GEN5 BUS5 TYPE=3A SN=500  UN=10 H=4.0  RA=0.0001 XA=0.001
      XD=1.4    XQ=1.25  XDP=0.30
      TDOP=6.0  VREG=5 TURB=1
GEN6 BUS6 TYPE=3A SN=600   UN=10 H=3.7  RA=0.001  XA=0.001
      XD=1.3  XQ=1.1  XDP=0.2
      TDOP=5.9  VREG=6  TURB=1
GEN7 BUS7 TYPE=3A SN=700  UN=10 H=3.1  RA=0.0001 XA=0.001
      XD=1.4    XQ=0.90  XDP=0.25
      TDOP=7.1  VREG=7   TURB=1
GEN8 BUS8 TYPE=3A SN=800  UN=10 H=4.1  RA=0.0001 XA=0.001
      XD=1.5    XQ=1.2   XDP=0.30
      TDOP=8.1  VREG=8 TURB=1
GEN9 BUS9 TYPE=3A SN=900   UN=10 H=5.1  RA=0.001  XA=0.001
      XD=1.3  XQ=1.0  XDP=0.25
      TDOP=9.1  VREG=9  TURB=1
END
REGULATORS
1 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=10 T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=1
2 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=9  T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=2
3 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=8  T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=3
4 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=7  T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=4
5 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=8  T1=0.0 T2=0.0 T3=0.200 T4=0.200
```

```
    REFTAB=5
6 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=9  T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=6
7 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=9  T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=7
8 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=10 T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=8
9 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=11 T1=0.0 T2=0.0 T3=0.200 T4=0.200
    REFTAB=9
101 TYPE=PSS2 TE=0 TW=2.0  KA=0.000 KDW=0 VSMAX=1 VSMIN=-1
102 TYPE=PSS2 TE=0 TW=2.0  KA=0.000 KDW=0 VSMAX=1 VSMIN=-1
103 TYPE=PSS2 TE=0 TW=2.0  KA=0.000 KDW=0 VSMAX=1 VSMIN=-1
END
TURBINES
1 TYPE=ST1 GOV 101 TC 0.05 KH 1.0 TR 0.05
101 TYPE=SG2 YMAX=1 YMIN=0 YPMAX=0.10 YPMIN=-0.10 K=0
             T1=100 T2=0 TY=0.1
END

TABLES
1 TYPE=0  F 0.0 1.00 0.08  0.98 0.5  1.00  3.0 1.00
2 TYPE=0  F 0.0 1.00 0.10  1.04 0.5  1.00  3.0 1.00
3 TYPE=0  F 0.0 1.00 0.12  1.02 0.5  1.00  3.0 1.00
4 TYPE=0  F 0.0 1.00 0.08  1.02 0.5  1.00  3.0 1.00
5 TYPE=0  F 0.0 1.00 0.10  0.96 0.5  1.00  3.0 1.00
6 TYPE=0  F 0.0 1.00 0.12  1.04 0.5  1.00  3.0 1.00
7 TYPE=0  F 0.0 1.00 0.08  1.02 0.5  1.00  3.0 1.00
8 TYPE=0  F 0.0 1.00 0.10  1.04 0.5  1.00  3.0 1.00
9 TYPE=0  F 0.0 1.00 0.12  0.96 0.5  1.00  3.0 1.00
END
END
```

B1 - Data for Example 4.1

Network data

```
CASE3 THREE MACHINE CASE
OPTPOW LOADFLOW DATA
**
NODES
BUS1 UB 10
BUS2 UB 10
BUS3 UB 10
END
LINES
BUS1 BUS2 TYPE 11 R=0.05   X=0.80
BUS2 BUS3 TYPE 11 R=0.05   X=0.80
END
LOADS
BUS1 P=10  Q=5
BUS2 P=160 Q=10
BUS3 P=10  Q=10
END
POWER
BUS1 TYPE NODE RTYP=UP  P=70  U=10
BUS2 TYPE NODE RTYP=SW  U=10  FI=0
BUS3 TYPE NODE RTYP=UP  P=70  U=10
END
END
```

Operation conditions

```
CASE3 THREE MACHINE CASE
OPTPOW LOADFLOW DATA
BUS1      (   1)   1.00000     P.U.
                   10.0000     KV
                   28.3199     DEGREES
        POWER FROM                                     MW           MVAR
                    PROD BUS1                        70.0000       16.2110
                    LINE BUS1      BUS2          0  -60.0000      -11.2110
                    LOAD BUS1         0             -10.0000      -5.00000

BUS2      (   1)   1.00000     P.U.
                   10.0000     KV
                 -0.278681E-05 DEGREES
        POWER FROM                                     MW           MVAR
                    PROD BUS2                        43.7257       47.1891
                    LINE BUS1      BUS2          0   58.1372      -18.5945
                    LINE BUS2      BUS3          0   58.1372      -18.5945
                    LOAD BUS2         0             -160.000      -10.0000

BUS3      (   1)   1.00000     P.U.
                   10.0000     KV
                   28.3199     DEGREES
        POWER FROM                                     MW           MVAR
                    PROD BUS3                        70.0000       21.2110
                    LINE BUS2      BUS3          0  -60.0000      -11.2110
                    LOAD BUS3         0             -10.0000      -10.0000
```

Generator data

```
CASE 3  THREE MACHINE CASE-   EXAMPLE 1
TRANSTA SIMULATION DATA
**
CONTROL DATA
TEND=2.0 METHOD=2 HFIX=0.01
END
```

```
GENERAL
REF GEN1 FN 50
END
LOADS
BUS1 SW=1 DPC=0 DPI=1
BUS2 SW=1 DPC=0 DPI=1
BUS3 SW=1 DPC=0 DPI=1
END
SYNCHRONOUS MACHINES
GEN1 BUS1 TYPE=3A SN=100   UN=10 H=6.0  RA=0.005  XA=0.01
      XD=1.0      XQ=0.8   XDP=0.3
      TDOP=6.0   VREG=1   TURB=1
GEN2 BUS2 TYPE=3A SN=100  UN=10 H=3.00  RA=0.005  XA=0.01
      XD=1.0    XQ=0.8    XDP=0.3
      TDOP=4.00  VREG=2 TURB=1
GEN3 BUS3 TYPE=3A SN=100   UN=10 H=7.00  RA=0.005   XA=0.01
      XD=1.0    XQ=0.8    XDP=0.3
      TDOP=6.0   VREG=3  TURB=1
END
```

C - Design software, Matlab functions

Input Energy Optimal Controler (IEOC)

```
function [E,Lambda]=IEOC(A,B,K,LowOmega,HighOmega)
%function [E,Lambda]=IEOC(A,B,K,LowOmega,HighOmega)
% calculate   the input energy for each eigenvalues to A-BK which have
% LowOmega < imag(eigenvalue)  < HighOmega
%
% E(i,j)= Energy from input u_i for eigenvalue Lambda_j
%-----------------------------------------------------
n=max(size(A));
m=(n+1)/3;
Aprim=A-B*K ;
[v,d]=eig(Aprim);
d=diag(d);
w=inv(v);
% normalize left eigenvectors
for j=1:n
  w(j,:)=w(j,:)/norm(w(j,:))                           ;
end
v=inv(w);
%

for j=1:n
  ImagPart=imag(d(j));
  if  (ImagPart < HighOmega)  & (ImagPart > LowOmega)
    Lambda=[Lambda d(j)]
    vvH=real(v(:,j)*(v(:,j)'));
    for i=1:m
      newE(i)=K(i,:)*vvH*(K(i,:)')*(-1/(2*real(d(j)))) ;
    end
    E=[E newE'];
  end
end;
```

D1 - Data for 16 machine model of Nordel

Network data

```
CASE16 16  MACHINE CASE
 OPTPOW LOADFLOW DATA
**
GENERAL
SN=100
END
NODES
BUS1 UB 400
BUS2   UB 400
BUS3   UB 400
BUS4   UB 400
BUS5   UB 400
BUS6   UB 400
BUS7   UB 400
BUS8   UB 400
BUS9   UB 400
BUS10  UB 400
BUS11  UB 400
BUS12  UB 400
BUS13  UB 400
BUS14  UB 400
BUS15  UB 400
BUS16  UB 400
END
LINES
BUS1 BUS13 TYPE 11 R=0.0010  X=0.020
BUS2 BUS7 TYPE 11 R=0.0050  X=0.100
BUS3 BUS4 TYPE 11 R=0.0035  X=0.070
BUS3 BUS8 TYPE 11 R=0.0090  X=0.180
BUS4 BUS5 TYPE 11 R=0.0080  X=0.160
BUS4 BUS9 TYPE 11 R=0.0080  X=0.160
BUS5 BUS6 TYPE 11 R=0.0015  X=0.030
BUS5 BUS11 TYPE 11 R=0.0020  X=0.040
BUS7 BUS8 TYPE 11 R=0.0005  X=0.010
BUS7 BUS14 TYPE 11 R=0.0020  X=0.040
BUS8 BUS9 TYPE 11 R=0.0005  X=0.010
BUS9 BUS10 TYPE 11 R=0.0005  X=0.010
BUS9 BUS11 TYPE 11 R=0.0010  X=0.020
BUS10 BUS11 TYPE 11 R=0.0030  X=0.060
BUS10 BUS12 TYPE 11 R=0.0010  X=0.020
BUS11 BUS13 TYPE 11 R=0.0010  X=0.020
BUS12 BUS13 TYPE 11 R=0.0015  X=0.030
BUS14 BUS15 TYPE 11 R=0.0025  X=0.050
BUS14 BUS16 TYPE 11 R=0.0035  X=0.070
BUS15 BUS16 TYPE 11 R=0.0020  X=0.040
END
LOADS
BUS1  P=1800 Q=10
BUS2  P=800 Q=10
BUS3  P=1400 Q=10
BUS4  P=1800 Q=10
BUS5  P=5500 Q=10
BUS6  P=3500 Q=10
BUS7  P=1500 Q=10
BUS8  P=500 Q=10
BUS9  P=2000 Q=10
BUS10  P=5800 Q=10
BUS11  P=4000 Q=10
BUS12  P=1000 Q=10
BUS13  P=1700 Q=10
BUS14  P=1100 Q=10
BUS15  P=2800 Q=10
BUS16  P=2600 Q=10
END
```

```
POWER
BUS1 TYPE NODE RTYP=UP  P=1790  U=400
BUS2 TYPE NODE RTYP=UP  P=790   U=400
BUS3 TYPE NODE RTYP=UP  P=1500  U=400
BUS4 TYPE NODE RTYP=UP  P=1790  U=400
BUS5 TYPE NODE RTYP=UP  P=4500  U=400
BUS6 TYPE NODE RTYP=UP  P=4400  U=400
BUS7 TYPE NODE RTYP=UP  P=2500  U=400
BUS8 TYPE NODE RTYP=UP  P=1000  U=400
BUS9 TYPE NODE RTYP=UP  P=4000  U=400
BUS10 TYPE NODE RTYP=SW  U=400   FI=0
BUS11 TYPE NODE RTYP=UP  P=3000  U=400
BUS12 TYPE NODE RTYP=UP  P=2000  U=400
BUS13 TYPE NODE RTYP=UP  P=1000  U=400
BUS14 TYPE NODE RTYP=UP  P=1000  U=400
BUS15 TYPE NODE RTYP=UP  P=3000  U=400
BUS16 TYPE NODE RTYP=UP  P=2500  U=400
END
END
```

Operation conditions

```
CASE16 16  MACHINE CASE

 OPTPOW LOADFLOW DATA

---------------------------------------------------------------------------

BUS1      (   1)   1.00000     P.U.
                   400.000     KV
                   -3.17847    DEGREES

        POWER FROM                                    MW           MVAR
                   PROD BUS1                        1790.00       10.5101
                   LINE BUS1      BUS13        0    10.0000      -0.510053
                   LOAD BUS1          0            -1800.00      -10.0000

---------------------------------------------------------------------------

BUS2      (   1)   1.00000     P.U.
                   400.000     KV
                   26.3726     DEGREES

        POWER FROM                                    MW           MVAR
                   PROD BUS2                        790.000       10.5503
                   LINE BUS2      BUS7         0    9.99999      -0.550276
                   LOAD BUS2          0            -800.000      -10.0000
---------------------------------------------------------------------------
BUS3      (   1)   1.00000     P.U.
                   400.000     KV
                   17.5743     DEGREES

        POWER FROM                                    MW           MVAR
                   PROD BUS3                        1500.00       12.6100
                   LINE BUS3      BUS4         0   -135.644       0.326294
                   LINE BUS3      BUS8         0    35.6444      -2.93633
                   LOAD BUS3          0            -1400.00      -9.99997
---------------------------------------------------------------------------

BUS4      (   1)   1.00000     P.U.
                   400.000     KV
                   12.1251     DEGREES

        POWER FROM                                    MW           MVAR
                   PROD BUS4                        1790.00       31.4553
                   LINE BUS3      BUS4         0    135.000      -13.2059
                   LINE BUS4      BUS5         0   -133.937      -7.73829
                   LINE BUS4      BUS9         0    8.93631      -0.511071
                   LOAD BUS4          0            -1800.00      -10.0000
```

```
---------------------------------------------------------------------------

BUS5     (   1)  1.00000      P.U.
                 400.000      KV
               -0.212956      DEGREES

        POWER FROM                                          MW          MVAR

                   PROD BUS5                             4500.00       197.229
                   LINE BUS4     BUS5           0      132.497      -21.0600
                   LINE BUS5     BUS6           0      887.759      -167.100
                   LINE BUS5     BUS11          0     -20.2562      0.930367
                   LOAD BUS5        0                 -5500.00      -10.0000
---------------------------------------------------------------------------

BUS6     (   1)  1.00000      P.U.
                 400.000      KV
                 15.3820      DEGREES


        POWER FROM                                          MW          MVAR
                   PROD BUS6                             4400.00       87.7119
                   LINE BUS5     BUS6           0     -900.000      -77.7119
                   LOAD BUS6        0                 -3500.00      -9.99998

---------------------------------------------------------------------------


BUS7     (   1)  1.00000      P.U.
                 400.000      KV
                 26.9472      DEGREES


        POWER FROM                                          MW          MVAR
                   PROD BUS7                             2500.00       9.13297
                   LINE BUS2     BUS7           0     -10.0050      0.449974
                   LINE BUS7     BUS8           0     -989.548      0.394750
                   LINE BUS7     BUS14          0    -0.447124      0.223160E-01
                   LOAD BUS7        0                 -1500.00      -10.0000

---------------------------------------------------------------------------


BUS8     (   1)  1.00000      P.U.
                 400.000      KV
                 21.2681      DEGREES


        POWER FROM                                          MW          MVAR
                   PROD BUS8                             1000.00       140.518
                   LINE BUS3     BUS8           0     -35.7595      0.633866
                   LINE BUS7     BUS8           0      984.652      -98.3152
                   LINE BUS8     BUS9           0     -1448.89      -32.8363
                   LOAD BUS8        0                 -500.000      -10.0000

---------------------------------------------------------------------------


BUS9     (   1)  1.00000      P.U.
                 400.000      KV
                 12.9467      DEGREES

        POWER FROM                                          MW          MVAR
                   PROD BUS9                             4000.00       410.264
                   LINE BUS4     BUS9           0     -8.94272      0.382881
                   LINE BUS8     BUS9           0      1438.39      -177.200
                   LINE BUS9     BUS10          0     -2247.54      -141.834
```

```
                LINE BUS9     BUS11     0  -1181.91      -81.6125
                LOAD BUS9       0          -2000.00      -10.0000
```

```
BUS10    (   1)   1.00000      P.U.
                  400.000      KV
               -0.707345E-17   DEGREES

     POWER FROM                                    MW          MVAR
                PROD BUS10                       3106.43      423.423
                LINE BUS9     BUS10     0        2222.18     -365.320
                LINE BUS10    BUS11     0       -19.6861     0.867524
                LINE BUS10    BUS12     0        491.077     -48.9701
                LOAD BUS10      0               -5800.00     -10.0000
```

```
BUS11    (   1)   1.00000      P.U.
                  400.000      KV
                -0.678264      DEGREES

     POWER FROM                                    MW          MVAR
                PROD BUS11                       3000.00      205.240
                LINE BUS5     BUS11     0        20.2479     -1.09484
                LINE BUS9     BUS11     0        1167.87     -199.102
                LINE BUS10    BUS11     0        19.6744     -1.10050
                LINE BUS11    BUS13     0       -207.796      6.05741
                LOAD BUS11      0               -4000.00     -10.0000
```

```
BUS12    (   1)   1.00000      P.U.
                  400.000      KV
                  5.66460      DEGREES

     POWER FROM                                    MW          MVAR
                PROD BUS12                       2000.00      23.0185
                LINE BUS10    BUS12     0       -493.512     0.259292
                LINE BUS12    BUS13     0       -506.488     -13.2778
                LOAD BUS12      0               -1000.00     -10.0000
```

```
BUS13    (   1)   1.00000      P.U.
                  400.000      KV
                 -3.06359      DEGREES

     POWER FROM                                    MW          MVAR
                PROD BUS13                       1000.00      87.9446
                LINE BUS1     BUS13     0       -10.0010     0.490001
                LINE BUS11    BUS13     0        207.364     -14.7006
                LINE BUS12    BUS13     0        502.637     -63.7341
                LOAD BUS13      0               -1700.00     -10.0000
```

```
BUS14    (   1)   1.00000      P.U.
                  400.000      KV
                  26.9369      DEGREES

     POWER FROM                                    MW          MVAR
                PROD BUS14                       1000.00      17.2130
                LINE BUS7     BUS14     0       0.447120    -0.223962E-01
                LINE BUS14    BUS15     0        93.4346     -6.87152
                LINE BUS14    BUS16     0        6.11833     -0.319087
                LOAD BUS14      0               -1100.00     -9.99998
```

```
BUS15    (   1)  1.00000     P.U.
                 400.000     KV
                 29.6244     DEGREES

        POWER FROM                                        MW          MVAR
                   PROD BUS15                          3000.00      4.46921
                   LINE BUS14     BUS15          0  -93.6540      2.48290
                   LINE BUS15     BUS16          0  -106.346      3.04788
                   LOAD BUS15       0              -2800.00     -9.99997
--------------------------------------------------------------------------------

BUS16    (   1)  1.00000     P.U.
                 400.000     KV
                 27.1829     DEGREES

        POWER FROM                                        MW          MVAR
                   PROD BUS16                          2500.00      17.2826
                   LINE BUS14     BUS16          0  -6.11964      0.292812
                   LINE BUS15     BUS16          0   106.120     -7.57539
                   LOAD BUS16       0              -2600.00     -10.0000
--------------------------------------------------------------------------------
```

D2 - Simulation data for design simulation in Simpow

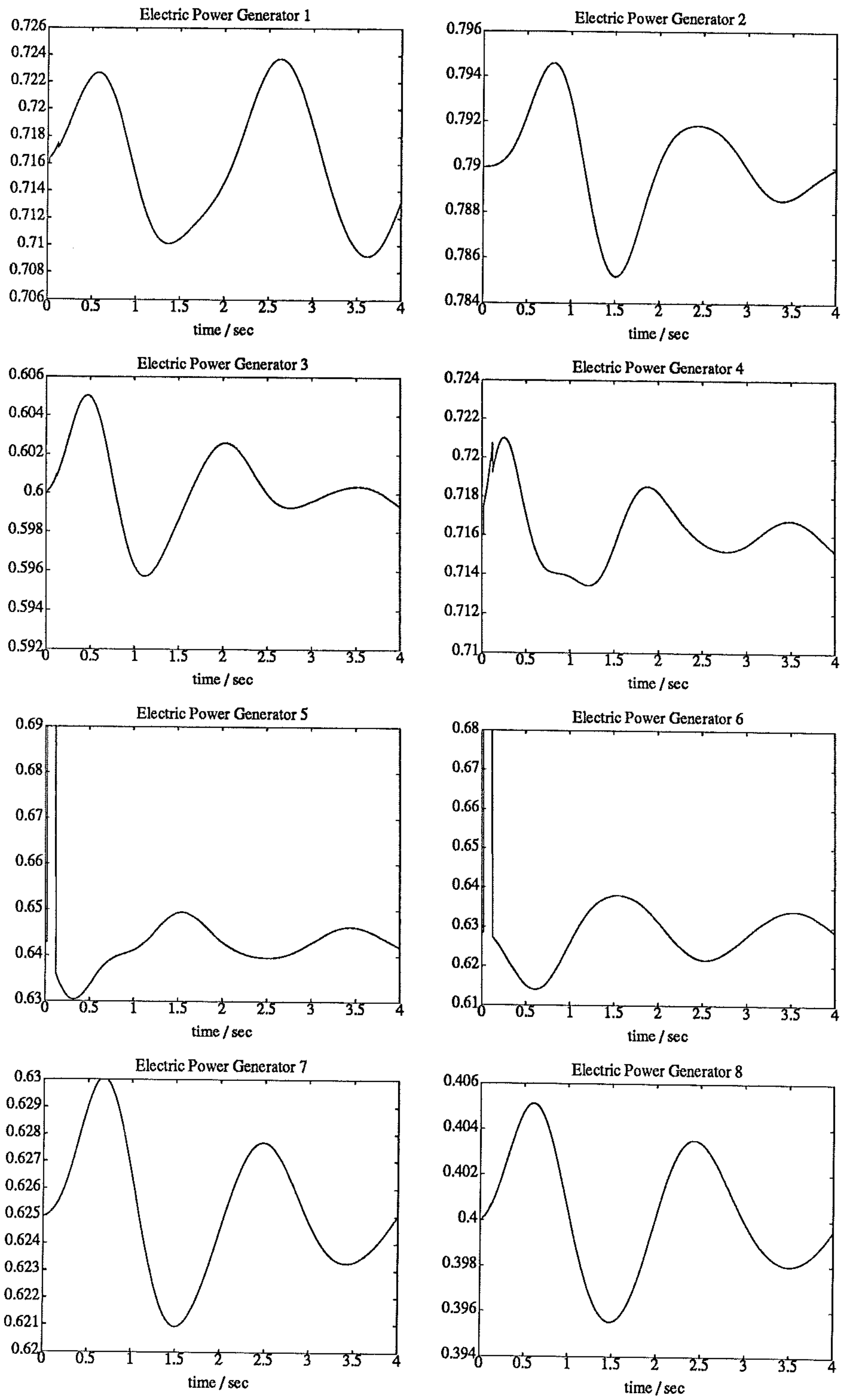
```
CASE 16 C - 16 MACHINE, CASE C - GLOBAL PSS  ON STRATEGIC MACHINES
TRANSTA SIMULATION DATA
**
CONTROL DATA
TEND=4.0 METHOD=2 HFIX=0.01
DEND=0.001
END
GENERAL
REF GEN1 FN 50
END
LOADS
BUS1B SW=1 DPC=0 DPI=1
BUS2B SW=1 DPC=0 DPI=1
BUS3B SW=1 DPC=0 DPI=1
BUS4B SW=1 DPC=0 DPI=1
BUS5B SW=1 DPC=0 DPI=1
BUS6B SW=1 DPC=0 DPI=1
BUS7B SW=1 DPC=0 DPI=1
BUS8B SW=1 DPC=0 DPI=1
BUS9B SW=1 DPC=0 DPI=1
BUS10B SW=1 DPC=0 DPI=1
BUS11B SW=1 DPC=0 DPI=1
BUS12B SW=1 DPC=0 DPI=1
BUS13B SW=1 DPC=0 DPI=1
BUS14B SW=1 DPC=0 DPI=1
BUS15B SW=1 DPC=0 DPI=1
BUS16B SW=1 DPC=0 DPI=1
END
SYNCHRONOUS MACHINES
GEN1 BUS1A TYPE=2A SN=2500 UN=400 H=5.0 RA=0.0049 XA=0.01
      XD=2.10    XQ=2.0  XDP=0.30
      XDB=0.2  XQB=0.2   TDOB=0.040    TQOB=0.080
      TDOP=7.0  VREG=1   TURB=1
GEN2 BUS2A TYPE=2A SN=1000  UN=400 H=3.0  RA=0.0040 XA=0.01
      XD=1.00   XQ=0.60   XDP=0.20
      XDB=0.15  XQB=0.15   TDOB=0.050    TQOB=0.150
      TDOP=5.00  VREG=2 TURB=1
GEN3 BUS3A TYPE=2A SN=2500   UN=400 H=3.00  RA=0.0041  XA=0.01
      XD=1.0  XQ=0.6  XDP=0.20
      XDB=0.15   XQB=0.15    TDOB=0.050   TQOB=0.150
      TDOP=5.0  VREG=3  TURB=1
GEN4 BUS4A TYPE=2A SN=2500 UN=400 H=3.00 RA=0.0049 XA=0.01
      XD=1.0     XQ=0.60  XDP=0.20
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=4   TURB=1
GEN5 BUS5A TYPE=2A SN=7000  UN=400 H=3.00  RA=0.0040 XA=0.01
      XD=1.0    XQ=0.6   XDP=0.2
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=5 TURB=1
GEN6 BUS6A TYPE=2A SN=7000   UN=400 H=3.00  RA=0.0041  XA=0.01
      XD=1.0  XQ=0.6  XDP=0.20
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=6  TURB=1
GEN7 BUS7A TYPE=2A SN=4000 UN=400 H=4.00 RA=0.0049 XA=0.01
      XD=1.0     XQ=0.6  XDP=0.2
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=7   TURB=1
GEN8 BUS8A TYPE=2A SN=2500  UN=400 H=4.00  RA=0.0040 XA=0.01
      XD=1.0    XQ=0.60  XDP=0.2
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=8 TURB=1
GEN9 BUS9A TYPE=2A SN=7000   UN=400 H=4.00  RA=0.0041  XA=0.01
      XD=1.0  XQ=0.6  XDP=0.2
      XDB=0.15  XQB=0.15    TDOB=0.050    TQOB=0.150
      TDOP=5.0  VREG=9  TURB=1
GEN10 BUS10A TYPE=2A SN=4000 UN=400 H=5.0 RA=0.0049 XA=0.01
```

```
      XD=2.1    XQ=2.0  XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=10   TURB=1
GEN11 BUS11A TYPE=2A SN=4000  UN=400 H=5.00  RA=0.0040 XA=0.01
      XD=2.1   XQ=2.0   XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=11 TURB=1
GEN12 BUS12A TYPE=2A SN=2500   UN=400 H=6.00  RA=0.0041  XA=0.01
      XD=2.1  XQ=2.0  XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=12  TURB=1
GEN13 BUS13A TYPE=2A SN=1500 UN=400 H=6.0 RA=0.0049 XA=0.01
      XD=2.1    XQ=2.0  XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=13   TURB=1
GEN14 BUS14A TYPE=2A SN=2000  UN=400 H=3.00  RA=0.0040 XA=0.01
      XD=1.0   XQ=0.6  XDP=0.2
      XDB=0.15  XQB=0.15   TDOB=0.050   TQOB=0.150
      TDOP=5.0  VREG=14 TURB=1
GEN15 BUS15A TYPE=2A SN=4000   UN=400 H=6.00  RA=0.0041  XA=0.01
      XD=2.1  XQ=2.0  XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=15  TURB=1
GEN16 BUS16A TYPE=2A SN=3000   UN=400 H=6.00  RA=0.0041  XA=0.01
      XD=2.1  XQ=2.00  XDP=0.3
      XDB=0.20  XQB=0.20   TDOB=0.040   TQOB=0.080
      TDOP=7.0  VREG=16  TURB=1
END
REGULATORS
1 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=27.1  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        NSWS=2 SWS=111 171
2 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=27.1  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        SWS=22
3 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=26.7  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        SWS=33
4 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=26.5  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        SWS=44
5 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=28.2  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        NSWS=2 SWS=55  515
6 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=27.8  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        SWS=66
7 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=25.5  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        NSWS=2 SWS=71 77
8 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=21.9  T1=0.1 T2=0.1 T3=0.1 T4=0.1
9 TYPE=BBC1   UEMAX=6 UEMIN=-6 K=20.5  T1=0.1 T2=0.1 T3=0.1 T4=0.1
10 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=24.9  T1=0.1 T2=0.1 T3=0.1 T4=0.1
11 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=25.4  T1=0.1 T2=0.1 T3=0.1 T4=0.1
12 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=26.2  T1=0.1 T2=0.1 T3=0.1 T4=0.1
13 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=19.9  T1=0.1 T2=0.1 T3=0.1 T4=0.1
14 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=24.7  T1=0.1 T2=0.1 T3=0.1 T4=0.1
15 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=27.3  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        NSWS=2 SWS=1515 155
16 TYPE=BBC1  UEMAX=6 UEMIN=-6 K=27.1  T1=0.1 T2=0.1 T3=0.1 T4=0.1
        SWS=1616
111 TYPE=PSS1 TP=0 TW=0 KP=-0.20 KW=-9.9 TF=0.0 TW1=4  TW2=4 PREG BUS1A BUS1B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN1
171 TYPE=PSS1 TP=0 TW=0 KP= 0.10 KW=3.2 TF=0.0 TW1=4  TW2=4 PREG BUS7A BUS7B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN7
22 TYPE=PSS1 TP=0 TW=0 KP=-0.40 KW=-17.1 TF=0.0 TW1=4  TW2=4 PREG BUS2A BUS2B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN2
33 TYPE=PSS1 TP=0 TW=0 KP=-0.45 KW=-18.7 TF=0.0 TW1=4  TW2=4 PREG BUS3A BUS3B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN3
44 TYPE=PSS1 TP=0 TW=0 KP=-0.47 KW=-19.4 TF=0.0 TW1=4  TW2=4 PREG BUS4A BUS4B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN4
55 TYPE=PSS1 TP=0 TW=0 KP=-0.41 KW=-20.2 TF=0.0 TW1=4  TW2=4 PREG BUS5A BUS5B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN5
515 TYPE=PSS1 TP=0 TW=0 KP=0.02 KW=9.06 TF=0.0 TW1=4 TW2=4 PREG BUS15A BUS15B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN15
66 TYPE=PSS1 TP=0 TW=0 KP=-0.45 KW=-21.0 TF=0.0 TW1=4  TW2=4 PREG BUS6A BUS6B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN6
77 TYPE=PSS1 TP=0 TW=0 KP=-0.27 KW=-16.8 TF=0.0 TW1=4  TW2=4 PREG BUS7A BUS7B
```

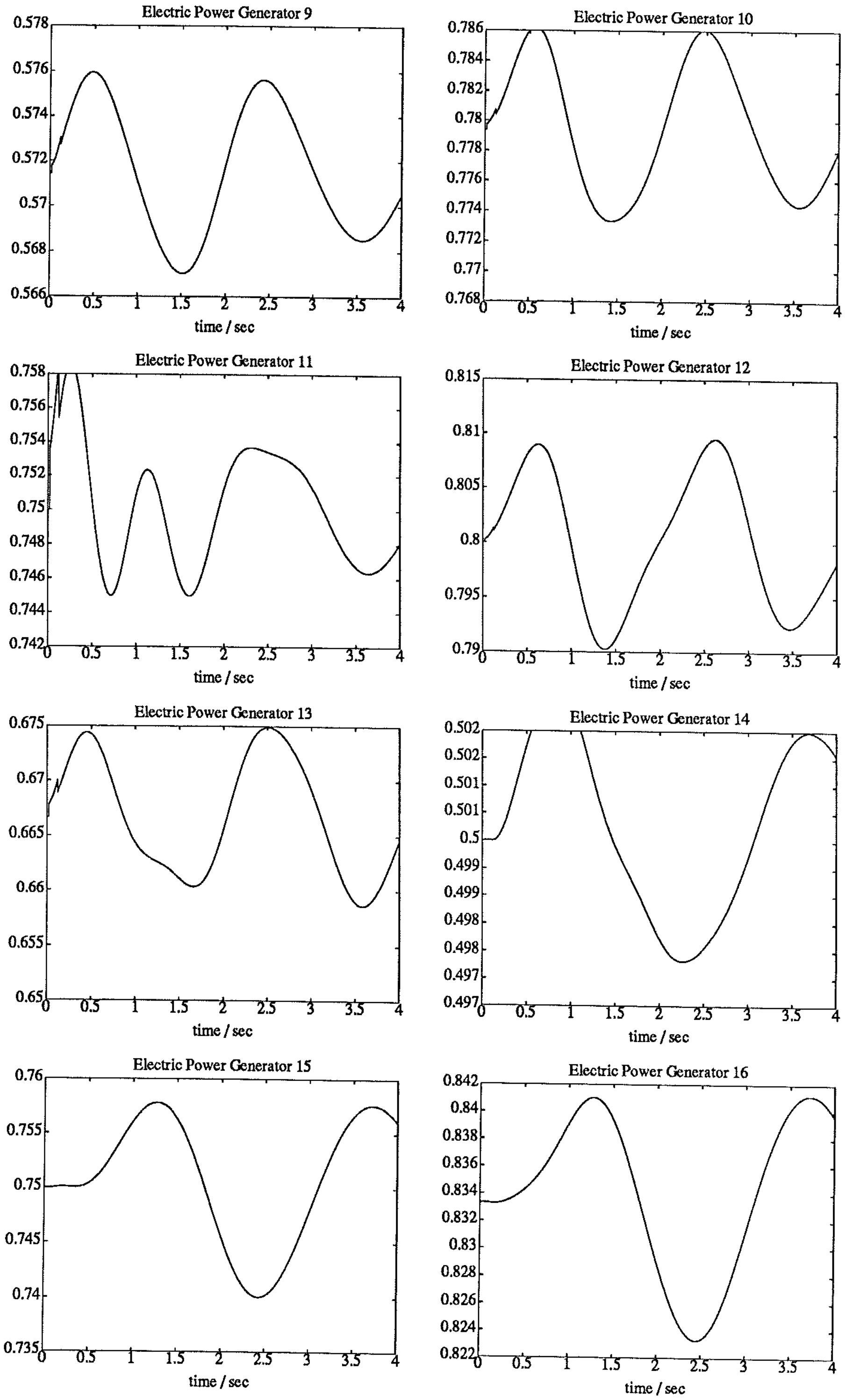
```
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN7
71 TYPE=PSS1 TP=0 TW=0 KP=0.11 KW=9.2  TF=0.0 TW1=4 TW2=4 PREG BUS1A BUS1B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN1
1515 TYPE=PSS1 TP=0 TW=0 KP=-0.27 KW=-16.2 TF=0.0
          TW1=4 TW2=4 PREG BUS15A BUS15B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN15
155 TYPE=PSS1 TP=0 TW=0 KP=0.07 KW=1.75 TF=0.0 TW1=4 TW2=4 PREG BUS5A BUS5B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN5
1616 TYPE=PSS1 TP=0 TW=0 KP=-0.23 KW=-13.8 TF=0.0
          TW1=4 TW2=4 PREG BUS16A BUS16B
        VSMIN=-0.05 VSMAX=0.05 T1=0.1 T2=0.1 T3=0.1 T4=0.1 WREG=GEN16
END
TURBINES
1 TYPE=ST1 GOV 101 TC 0.05 KH 1.0 TR 0.05
101 TYPE=SG3 YMAX=1 YMIN=0  K=0.01 T1=45  T2=5 T3=0
END
FAULTS
FEL5 TYPE 3PSG NODE BUS5A  R=200.0 X=200.0
FEL6 TYPE 3PSG NODE BUS6A  R=200.0 X=200.0
END
RUN INSTRUCTION
AT 0.02 INST CONNECT FAULT FEL5
AT 0.02 INST CONNECT FAULT FEL6
AT 0.12 INST DISCONNECT FAULT FEL5
AT 0.12 INST DISCONNECT FAULT FEL6
END

END
```

D3 - Power oscillations with constant power loads

Electric power from generators with constant power loads



Electric power from generators with constant power loads



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