Linear Thermistor Networks for Temperature Measurements

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ABSTRACT

In this report a unified approach is given to the analysis of a one thermistor linear network. It is shown that an output function of the network is a linear function of a standard function $R_{th}^{pq}/(R_{th}^{pq} + R(T))$, where R(T) is the resistance of the thermistor at temperature T and R_{th}^{pq} is the Thevenin equivalent resistance with respect to the thermistor terminals. Further an upper limit of the self-heat error of the thermistor is given.

A design criterion is presented from which the circuit condition for optimum linearity may be calculated. The departure from linearity of an output function of the network is given for different temperature ranges. The unified approach is illustrated by analysing a one thermistor Wheatstone bridge. Experimental results from a constructed temperature transducer are also given.

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1. INTRODUCTION

Significant advantages may be obtained by using thermistors as temperature measurement elements because of their high temperature sensitivity, fast response, good stability, low cost, convenient ranges of operation resistances, and wide range of shapes and sizes. Outstanding features of the sensors are the high temperature sensitivity, fast response, and small size. However, the design of convenient measuring circuits for use with these elements is complicated by two facts, viz. the nonlinear characteristics and the resistance variations of the thermistors.

In section 2 it is shown that an output function of a one thermistor linear network is a linear function of a standard function.

$$R_{th}^{pq}/[R_{th}^{pq} + R(T)]$$

where R_{th}^{pq} is the Thevenin equivalent resistance with respect to the thermistor terminals and R(T) is the resistance of the thermistor at temperature T. Further an upper limit of the self-heat error of the thermistor is given. Universal design criteria may thus be developed.

In section 3 a design criterion from which the circuit condition for optimum linearity may be calculated appears. The departure from linearity of an output function of the network is given for different temperature ranges. The proposed criterion solves the problem of the nonlinear characteristics of the thermistors for a rather wide temperature range.

In section 4 a one thermistor Wheatstone bridge suitable for temperature measurement is analysed. Further a compensation network of a thermistor is discussed. The network is a solution to the problem of the resistance variations of the thermistors.

In section 5 finally experimental results from a constructed temperature transducer are presented.

2. A STANDARD OUTPUT FUNCTION OF A ONE THERMISTOR LINEAR NETWORK

In this section we show that the voltage across two arbitrary nodes in a network comprising linear resistors, a single thermistor, and zero-impedance voltage sources may be expressed as a linear function of a standard function $R_{th}^{pq}/[R_{th}^{pq} + R(T)]$, where R_{th}^{pq} is the Thevenin equivalent resistance with respect to the thermistor terminals and R(T) is the thermistor resistance at temperature T. Further an upper limit of the self-heat error of the thermistor is given. Universal design criteria for one thermistor linear networks may thus be developed.

Consider a network with nodes ℓ, m, n, o,..., node voltages V_ℓ, V_m, V_n, V_o,..., and branch currents $I_{\ell m}$, $I_{\ell n}$, $I_{\ell o}$, I_{mn}, ... The branch ℓm comprises the linear resistor $R_{\ell m}$ and the zero-impedance voltage source $E_{\ell m}$. The quantities $I_{\ell m}$ and $E_{\ell m}$ are positive if the current and the voltage respectively are directed from ℓ to m.

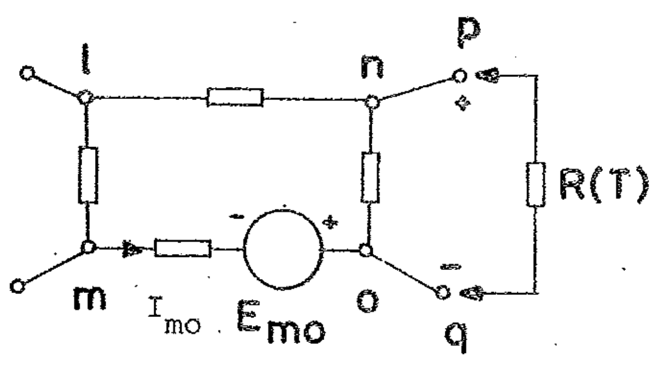

Fig. 2.1 - A general network comprising linear resistors and zero-impedance voltage sources.

If a lead containing a resistor R(T) is connected to the nodes p and q superposed currents and voltages are generated in the network. The Thevenin theorem states that these superposed currents and voltages are the same as the partial currents and voltages, caused by a zero-impedance voltage source, placed in the junction lead. The voltage of the source should equal the original voltage E_{th}^{pq} across the nodes and be directed as this will force the current. The partial currents and voltages may be calculated from the considered network with all internal voltage sources short-circuited and the junction lead comprising the thermistor and the source E_{th}^{pq} connected to the terminals p and q according to the principle of superposition. The network is found in Fig. 2.2.

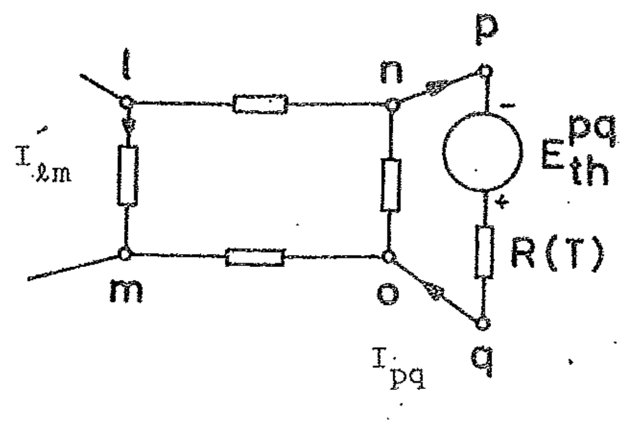

Fig. 2.2 - The network with all internal voltage sources short-circuited and the junction lead containing the thermistor and the source E_{th}^{pq} connected to the terminals p and q.

The current in the junction lead reaches

$$I_{pq} = \frac{E^{pq}_{th}}{R^{pq}_{th} + R(T)} \tag{2.1}$$

where R^{pq}_{th} is the Thevenin equivalent resistance with respect to the thermistor terminals. There exists a linear relationship between the current $I'_{\ell m}$ in the branch ℓm and the current I_{pq}.

This relationship is written

$$I'_{\ell m} = L^{pq}_{\ell m} \, I_{pq} \tag{2.2}$$

In the network of Fig. 2.1 the current in the branch ℓm with the thermistor connected across the terminals p and q is the sum of the current $I^0_{\ell m}$ existing before the connection of the lead and the current $I'_{\ell m}$. Thus we have according to Eq.(2.1) and (2.2)

$$I_{\ell m} = I^0_{\ell m} + L^{pq}_{\ell m} \frac{E^{pq}_{th}}{R^{pq}_{th} + R(T)} \tag{2.3}$$

The current in the branch ℓm is thus a linear function of the standard function F(T).

$$F(T) = \frac{R^{pq}_{th}}{R^{pq}_{th} + R(T)} \tag{2.4}$$

Since the choice of the branch ℓm is arbitrary the voltage across two arbitrary nodes in the network is also a linear function of the standard function, which completes the proof.

The passage of the current through a thermistor heats it thereby introducing an error in the value of the measured temperature. Remembering that the superposed current in the branch pq, previously discussed, is identical with the total current in the branch the power dissipated in the thermistor is

$$P \leq \frac{{E_{th}^{pq}}^2}{4R_{th}^{pq}} \qquad (2.5)$$

where equility is taken on if the resistance of the thermistor equals the Thevenin equivalent resistance with respect to the thermistor terminals. The self-heat error ΔT becomes

$$\Delta T \leq \frac{{E_{th}^{pq}}^2}{4\,R_{th}^{pq}} \qquad (2.6)$$

where δ is the dissipation constant of the thermistor. For a medium of large temperature range of operation the resistance of the thermistor takes on the value R_{th}^{pq} and the self-heat error reaches its upper limit. The maximum self-heat error tolerated limits the voltage sensitivity of the network.

We conclude that for a given resistance temperature characteristic of a thermistor the linearity of an output of the network is essentially governed by a single parameter R_{th}^{pq}, which is the Thevenin equivalent resistance with respect to the thermistor terminals. The simple voltage divider circuit may thus be designed to yield as linear response as any other one thermistor linear network. The disadvantage of the divider circuit is that the slope and null point of the output function of the divider circuit simply cannot be changed. Another circuit should be searched for to obtain a convenient temperature transducer.

3. A DESIGN CRITERION OF A ONE THERMISTOR LINEAR NETWORK

The basic formula, relating the zero-power resistance of a thermistor to the temperature T in °K is

$$R(T) = R_0\, e^{B(\frac{1}{T} - \frac{1}{T_0})} \qquad (3.1)$$

where B is the material constant of the thermistor and R_0 is the zero-power resistance at temperature T_0. The material constant B increases slightly with increasing temperature. The formula (3.1) is the one obeyed by semi-conductors, whose temperature dependance of resistivity can be assumed to arise from the thermal excitation of carriers over a single energy gap.

From the previous section we know that the output function of a one thermistor linear network may be expressed as a linear function of a standard function (2.4).

By inserting Eq.(3.1) into (2.4) we obtain

$$F(T) = \frac{1}{1 + \frac{R_0}{R_{th}^{pq}}\, e^{B(\frac{1}{T} - \frac{1}{T_0})}} \qquad (3.2)$$

For any specified resistance ratio R_0/R_{th}^{pq} and resistance ratio temperature characteristic of a thermistor the function F(T) describes an "S" shaped curve. By permitting the ratio R_0/R_{th}^{pq} to assume a series of constant values, an entire family of "S" shaped curves are generated. Five members of the family, viz. R_0/R_{th}^{pq} = 2.00, 4.00, 8.00, 13,33, 25.00, are shown in the temperature range -20° C - - + 180° C. The material constant B is 3500° K and the temperature T_0 is 25° C.

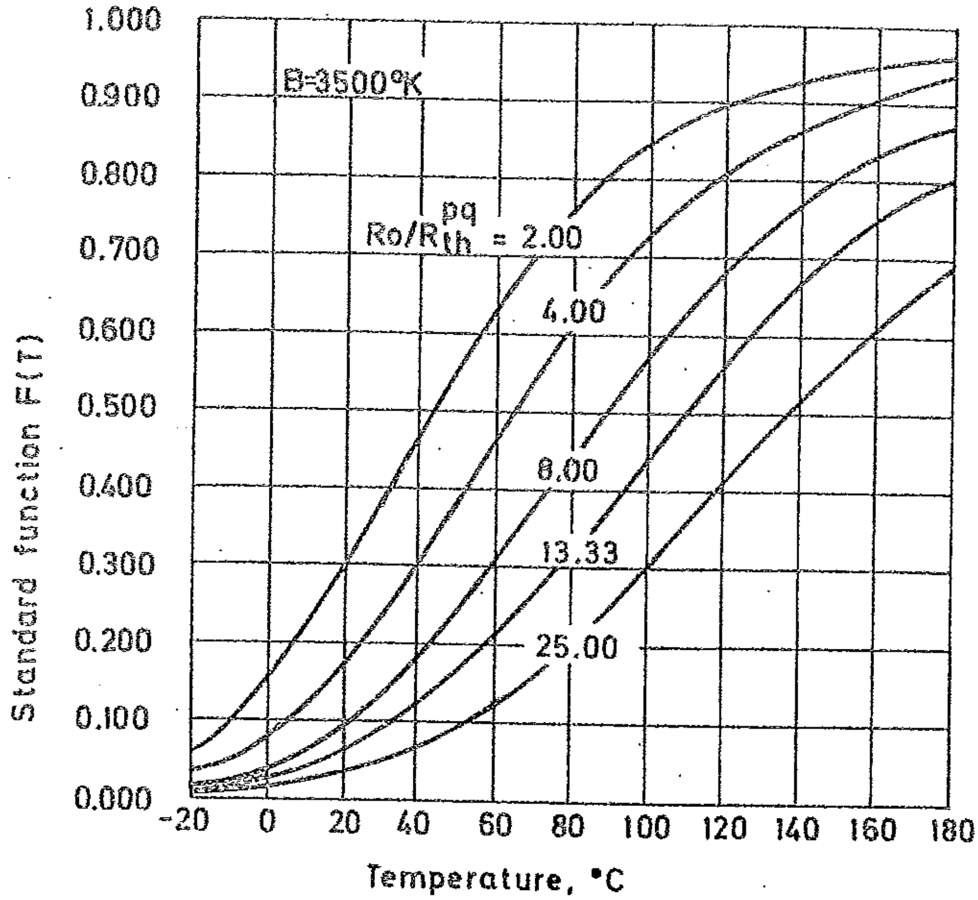

Fig. 3.1 - The standard function F(T) in the temperature range -20° C - $+180^{\circ}$ C for $T_0 = 25^{\circ}$ C.

The curves in Fig. 3.1 exhibit a long section of almost constant slope in the vicinity of the inflection points.

Let T_1, T_2, and T_m denote temperatures such that

$$T_1 \leq T_m \leq T_2$$

$$T_m = (T_1 + T_2)/2 \qquad (3.3)$$

and denote the output function of the general network

$$f(T) = f_1 F(T) + f_0 \qquad (3.4)$$

where f_0 and f_1 are constants. The proposed criterion suggests that the resistance R_{th}^{pq} should be chosen to equate the increments of the output function in the temperature ranges $T_1 \leq T \leq T_m$ and $T_m \leq T \leq T_2$, i.e.

$$f(T_m) - f(T_1) = f(T_2) - f(T_m) \tag{3.5}$$

where $T_1 \leq T \leq T_2$ is the specified temperature range. The value of R_{th}^{pq} upon application of the matched increment criterion is

$$R_{th} = \frac{R(T_1) - \frac{\Delta R_1}{\Delta R_2} R(T_2)}{\frac{\Delta R_1}{\Delta R_2} - 1} \tag{3.6}$$

where

$$\begin{aligned} \Delta R_1 &= R(T_1) - R(T_m) \\ \Delta R_2 &= R(T_m) - R(T_2) \end{aligned} \tag{3.7}$$

If the temperature T_1, T_m, and T_2 fulfil condition (3.3) it is shown in Appendix A that

$$R(T_1) \geq \frac{\Delta R_1}{\Delta R_2} R(T_2) \tag{3.8}$$

The condition (3.8) guarantees that the resistance R_{th}^{pq} is nonnegative.

The maximum departure from linearity of an output function is calculated for different temperature ranges using the matched increment and a least squares criterion. The linearity error is thereby defined by

$$\Delta f(T) = f(T) - 2\,\frac{T-T_m}{T_2-T_1}\,f_a \qquad (3.9)$$

where f_a is the output amplitude. The resistance temperature characteristic of the thermistor is calculated from Eq.(3.1) where

$$T_0 = 25^\circ\ C$$
$$B = 3500^\circ\ K$$
$$R_0 = 2000\ \Omega \qquad (3.10)$$

First the matched increment is considered and the parameters f_0 and f_1 are chosen such that the linearity errors vanish for the temperatures T_1 and T_2. The resistance R_{th}^{pq} and the linearity error appear in Table 3.1 for different temperature ranges. For rather wide temperature ranges the proposed criterion yields a small linearity error. The linearity error reaches 4 % at a temperature range of 100° C.

Then a least square criterion is considered. The set of parameters f_0, f_1, and R_{th}^{pq} which minimizes the quadratic criterion

$$\min_{f_0,\, f_1,\, R_{th}^{pq}} \sum_i \{\Delta f(T_i)\}^2 \qquad (3.11)$$

in 101 equidistant points in the temperature range $T_1 \leq T \leq T_2$ is calculated. The resistance R_{th}^{pq} and the linearity error appear in Table 3.1 for different temperature ranges.

	Matched increment criterion		Least square criterion	
Temp range	R_{th}^{pq}	max linearity error	R_{th}^{pq}	max linearity error
^{o}C	Ω	%	Ω	%
24-26	1418	0.0026	1418	0.0026
20-30	1419	0.063	1419	0.0063
15-35	1424	0.25	1423	0.25
0-50	1457	1.6	1451	1.5
0-100	601	4.4	593	4.2

Table 3.1 - The resistance R_{th}^{pq} and the linearity error calculated from the matched increment criterion and a least square criterion for different temperature ranges.

The difference between the linearity errors of the matched increment criterion and the least square criterion is small. Table 3.1 thus shows that the linearity error of a one thermistor linear network is essentially governed by a single parameter R_{th}^{pq}.

In practice any precise information of the resistance temperature characteristic of the thermistor is seldom available and a poor estimation of the resistance R_{th}^{pq} may be obtained. Therefore it is of great importance that the condition stated as optimal can be checked simply and a calibration procedure developed to adjust the resistance R_{th}^{pq} in a nonoptimal situation. The calibration errors are reduced by utilizing the whole temperature range $T_1 \leq T \leq T_2$ to perform the calibration. When a calibration procedure is used it can be shown that the output function is insensitive to slow perturbations of the nominal resistance temperature characteristic of the thermistor. The highest frequency component of the perturbation should be less or equal to $1/(2(T_2-T_1))$. The matched increment criterion fulfils the requirements given above as distinguished by the criteria mentioned in [1] and [2].

4. A WHEATSTONE BRIDGE CIRCUIT

A circuit which finds widespread use for precision resistance measurements is the Wheatstone bridge. Such circuits are commonly used in conjunction with thermistors in the design of accurate temperature measurement and control equipments. A typical one thermistor bridge is shown in Fig. 4.1.

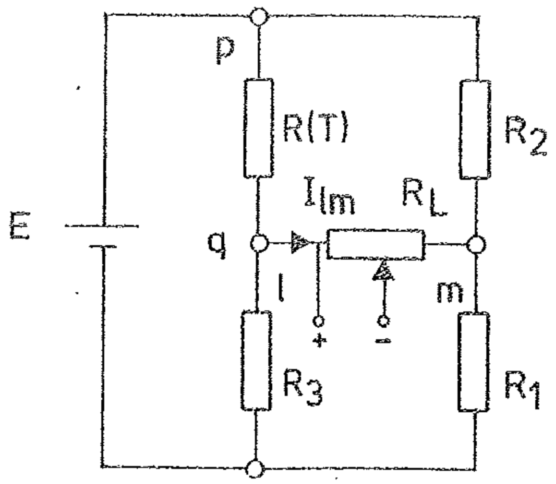

Fig. 4.1 - A one thermistor Wheatstone bridge

According to Eq(2.3) the current $I_{\ell m}$ is given by

$$I_{\ell m} = I^{0}_{\ell m} + L^{pq}_{\ell m} \frac{E^{pq}_{th}}{R^{pq}_{th} + R(T)} \qquad (4.1)$$

where $I^{0}_{\ell m}$ is the current in the branch ℓm when the branch pq is open, $L^{pq}_{\ell m}$ is defined by Eq.(2.2), E^{pq}_{th} is the Thevenin equivalent voltage and R^{pq}_{th} is the Thevenin equivalent resistance with respect to the thermistor terminals. The current $I^{0}_{\ell m}$ is given by

$$I^{0}_{\ell m} = - \frac{R_1}{R_1 + R_3 + R_L} \; \frac{E}{R_2 + \frac{R_1(R_3 + R_L)}{R_1 + R_3 + R_L}} \qquad (4.2)$$

By rearranging terms in Eq.(4.2) we conclude

$$I_{\ell m}^{0} = -\frac{R_1}{X_b} E \tag{4.3}$$

where

$$X_b = R_1R_2 + (R_1 + R_2)(R_3 + R_L) \tag{4.4}$$

Remembering that the total voltage across the bridge is E the Thevenin equivalent voltage E_{th}^{pq} is

$$E_{th}^{pq} = E - R_3 I_{\ell m}^{0} \tag{4.5}$$

A current I_{pq} in the branch pq gives rise to a current $I_{\ell m}$ in the branch ℓm where

$$I_{\ell m} = \frac{R_3}{R_3 + R_L + \frac{R_1R_2}{R_1+R_2}} I_{pq} \tag{4.6}$$

We have

$$L_{\ell m}^{pq} = \frac{R_3(R_1+R_2)}{X_b} \tag{4.7}$$

according to Eq.(2.2) and (4.6).

By inspection of Fig. 4.1 we conclude that the Thevenin equivalent resistance R_{th}^{pq} is

$$R_{th}^{pq} = \frac{R_3(R_L + \frac{R_1R_2}{R_1+R_2})}{R_3 + R_L + \frac{R_1R_2}{R_1+R_2}} \tag{4.8}$$

Eq.(4.8) may be rewritten as

$$R_{th}^{pq} = X_a/X_b \tag{4.9}$$

where

$$X_a = R_1R_2R_3 + R_3R_L(R_1 + R_2) \tag{4.10}$$

A lengthy but straight-forward calculation now gives

$$I_{\ell m} = K_1 \frac{R_{th}^{pq}}{R_{th}^{pq} + R(T)} E + K_2E \tag{4.11}$$

where

$$K_1 = R_2R_3/X_a + R_1/X_b$$

$$K_2 = -R_1/X_b \tag{4.12}$$

The output voltage $V_{\ell m}$ is finally given by

$$V_{\ell m} = R_LI_{\ell m} \tag{4.13}$$

The resistance temperature characteristic of the thermistor and the specified temperature range determine the resistance R_{th}^{pq} according to Eq.(3.6) and (3.7). Hence for optimal linearity the Thevenin equivalent resistance with respect to the thermistor terminals should equal R_{th}^{pq}. In order to obtain zero output voltage $V_{\ell m}$ at the temperature T_1 we choose

$$R_1 = R_3$$
$$R_2 = R(T_1) \qquad (4.14)$$

Further we choose

$$R_L = a\,\frac{R_1R_2}{R_1+R_2} \qquad (4.15)$$

The condition for optimal linearity then becomes

$$R_3 = \frac{(a+2)R_2R_{th}^{pq}}{(a+1)R_2-R_{th}^{pq}} \qquad (4.16)$$

according to Eq.(4.8), (4.14), and (4.15).

The parameter a may be chosen to meet requirements on the full scale deflection or the output impedance of the network. In order to simplify the adjustment of the full scale deflection a potentiometer R_L,yielding a variable attenuation,may be used instead of the load resistance R_L as indicated in Fig. 4.1.

The compensation network in Fig. 4.2 is often used to account for the resistance variations of the thermistors or to reduce the self-heat errors of the thermistors. In the former case the resistors R_s and R_p are included in the thermistor probe. Thereby an interchangeable probe is obtained.If the change of the resistance temperature characteristic of a thermistor included in a compensation network is taken into consideration, the analysis presented above still applies for the Wheatstone bridge circuit.

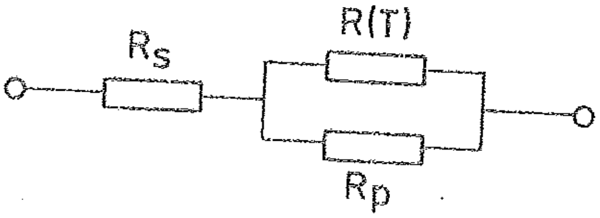

Fig. 4.2 - A compensation network of a thermistor

5. CONSTRUCTION OF A TEMPERATURE TRANSDUCER

The general design consideration has been used to construct a transducer recording room temperature. Thereby the temperature range was specified to 15° C - 35° C. The thermistor selected was a YSI-thermistor, no 44007, which had a nominal resistance of 5000 Ω and a temperature coefficient of -4.5 %/$^{\circ}$C at 25° C. The dissipation constant was 1 mW/$^{\circ}$C and the time constant was 10 sec in still air. The YSI-thermistors offer the exclusive feature of interchangeability within 1 % or 0.5 % over a wide temperature range.

The full scale deflection and the maximum self-heat tolerated were specified to 100 mV and 0.02° C respectively. The resistance R_{th}^{pq} was calculated from the resistance temperature characteristic of the thermistor according to Eq.(3.6) and (3.7). The value of R_{th}^{pq} was 3823 Ω.

The complete scheme of the constructed transducer appears in Fig. 5.1.

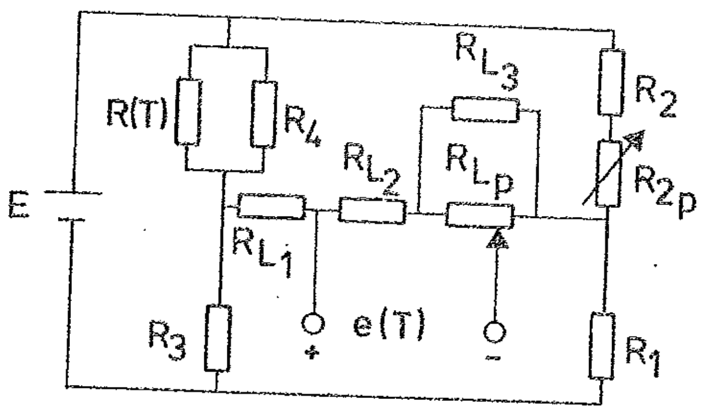

Fig. 5.1 - A complete scheme of a temperature transducer
Resistors: $R_1 = R_3 = 7.00$ kΩ (0.1 % precision resistors)
$R_2 = 4.7$ kΩ (1 % metal film resistor)
$R_4 = 20.0$ kΩ (0.1 % precision resistor)

R_{L_1} = 3.3 kΩ (1 % metal film resistor)

R_{L_2} = 6.2 kΩ (1 % metal film resistor)

R_{L_3} = 10 kΩ-100kΩ (1 % metal film resistor chosen to meet the requirements on the load resistance, i.e. $R_{L_1} + R_{L_2} + R_{L_3} \cdot R_{L_p}/(R_{L_3}+R_{L_p})$ = 11.43 kΩ)

Potentiometers: R_{2_p} = 2 kΩ (helipot 89 P)

R_{L_p} = 2 kΩ (helipot 89 P)

Battery: E = 1.35 V (standard mercury cell)

The potentiometers R_{2_p} and R_{L_p} are used to adjust the null voltage and the full scale deflection of the transducer respectively. The resistor R_{L3} should be chosen to meet requirements on the load resistance R_L. The resistor R_4 reduces the maximum self-heat error of the transducer. The battery is a standard mercury cell. The output voltage e(T) is

$$e(T) = 0.05(T-15)$$

where T is the body temperature of the thermistor in $^{\circ}C$.

The measured departure from linearity of the transducer at various temperatures and a continuous curve showing the expected departure from linearity appear in Fig. 5.2. The thermistor was immersed in a temperature bath and the temperature of the bath measured with a mercury-in-glass thermometer of accuracy 0.02° C. The maximum departure from linearity reaches 0.05° C which is in accordance with Table 3.1.

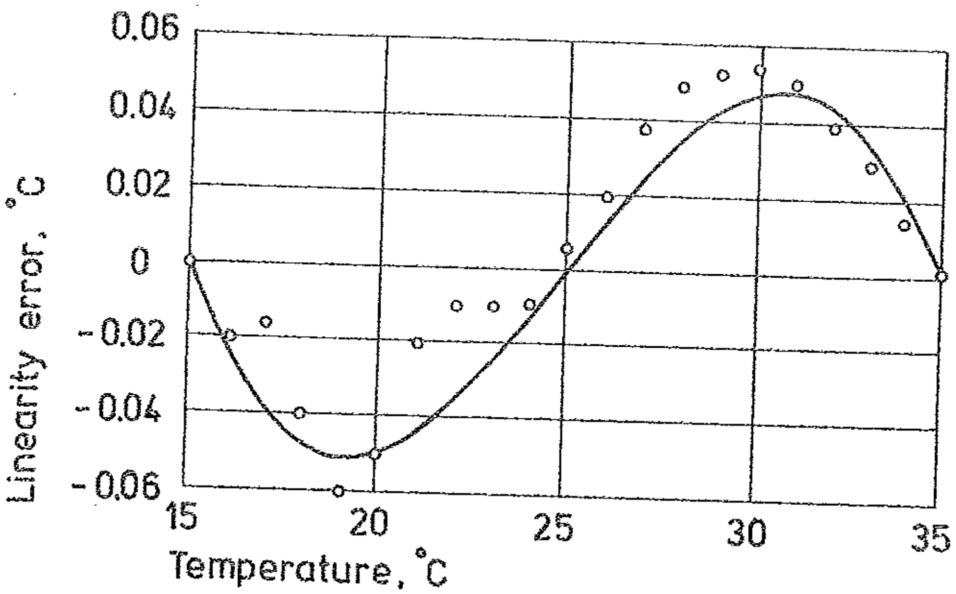

Fig. 5.2 - Measured and theoretical linearity error of a temperature transducer.

6. REFERENCES

[1] W.R. Beakly: "The Design of Thermistor Thermometers with Linear Calibration", Journal of Scientific Instruments, Vol. 28, pp.176 - 179, June, 1951.

[2] M. Oppenheim, R.M. Sapoff "The Design of Linear Thermistor Networks". IEEE International Convection Record, Part 8, 1964.

APPENDIX A

Let T_1, T_m, and T_2 denote temperatures such that

$$T_1 \leq T_m \leq T_2$$
$$T_m = (T_1+T_2)/2 \qquad (A.1)$$

The resistance of a thermistor as a function of the temperature T in ^{o}K is given by Eq.(3.1). Putting T_0 equal to T_2 in this equation the resistance $R(T_1)$, $R(T_m)$, and $R(T_2)$ at the temperatures T_1, T_m, and T_2 become

$$R(T_1) = R_2\, e^{B(\frac{1}{T_1} - \frac{1}{T_2})}$$

$$R(T_m) = R_2\, e^{B(\frac{1}{T_m} - \frac{1}{T_2})}$$

$$R(T_2) = R_2 \qquad (A.2)$$

We have

$$R(T_1) - \frac{\Delta R_1}{\Delta R_2} R(T_2) =$$

$$= \frac{R_2 e^{B(\frac{1}{T_1} - \frac{1}{T_2})}}{e^{B(\frac{1}{T_m} - \frac{1}{T_2})} - 1} \{e^{B\cdot\frac{T_2-T_m}{T_2 T_m}} + e^{-B\frac{T_m-T_1}{T_1 T_m}} - 2\}$$

$$\geq \frac{R_2 e^{B(\frac{1}{T_1} - \frac{1}{T_2})}}{e^{B(\frac{1}{T_m} - \frac{1}{T_2})} - 1} \{e^{B \frac{T_2 - T_m}{T_1 T_m}} + e^{-B \frac{T_m - T_1}{T_1 T_m}} - 2\} \qquad (A.3)$$

according to Eq.(3.6) and (3.7).

Consider the function

$$f(x) = e^x + e^{-x} - 2 \qquad (A.4)$$

where

$$x = \frac{T_2 - T_m}{T_1 T_m} \qquad (A.5)$$

A short calculation gives

$$f(0) = 0$$

$$f'(x) \geq 0, \quad x \geq 0 \qquad (A.6)$$

Eq. (A.1), (A.3), (A.4), (A.5), and (A.6) now give

$$R(T_1) \geq \frac{\Delta R_1}{\Delta R_2} R(T_2) \qquad (A.7)$$

which completes the proof.