Dynamic Models for Electrical Heating Devices in Climate Control

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DYNAMIC MODELS FOR ELECTRICAL HEATING DEVICES IN CLIMATE CONTROL

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Abstract

Simple first order models for electrical heating devices are developed from construction data. These models are compared with models identified from data from specially made experiments. The main timeconstants and the static gains are roughly the same for the models derived in two different ways.

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1 INTRODUCTION

The main purpose with this report is to make a comparison between dynamic models for electrical heating devices based on construction data and based on measurements.

Simple linear first order models are developed from construction data in section 3. This is done although the radiation is T^4-dependent and that the airheater and the radiator consists of two parts with very different temperature.

In section 4 the least squares method and the maximum likelihood method have been used to identify discrete time models of first and second order from data. These are transformed into continuous time transferfunctions. The heated ceiling and the airheater can be regarded as first order system and the electrical radiator as a second order system.

Finally in section 5 a short comparison is made between the different models from section 3 and 4. The main time constant and the static gain turns out to be the same.

2 EXPERIMENTS

Experiments with control of room air temperature have been made with a fullscale testroom with different types of electrical heating devices. Further details about the room are given in Adamson (1969). The effect to the heating devices has been switched on and off to be able to determine dynamic models. Further details are given in Jensen (1973). Six different experiments have been used for identification of models and some of these are presented in figure 2.1 - 2.3. Three different types of used heating devices have been heated ceiling, air heater and radiators. Details about the heating devices are given in appendix 1 - 3 (A1-3). Introduce the following notations for the experiments:

Notation	Heating device	On effect
S2	heated ceiling	1 kW
S5	" "	1.8 kW
K2	air heater	1 kW
K5	" "	2 kW
R2	radiator	1 kW
R5	"	2 kW

The sampling interval was 1 minute in the above experiments and the number of samples was about 300.

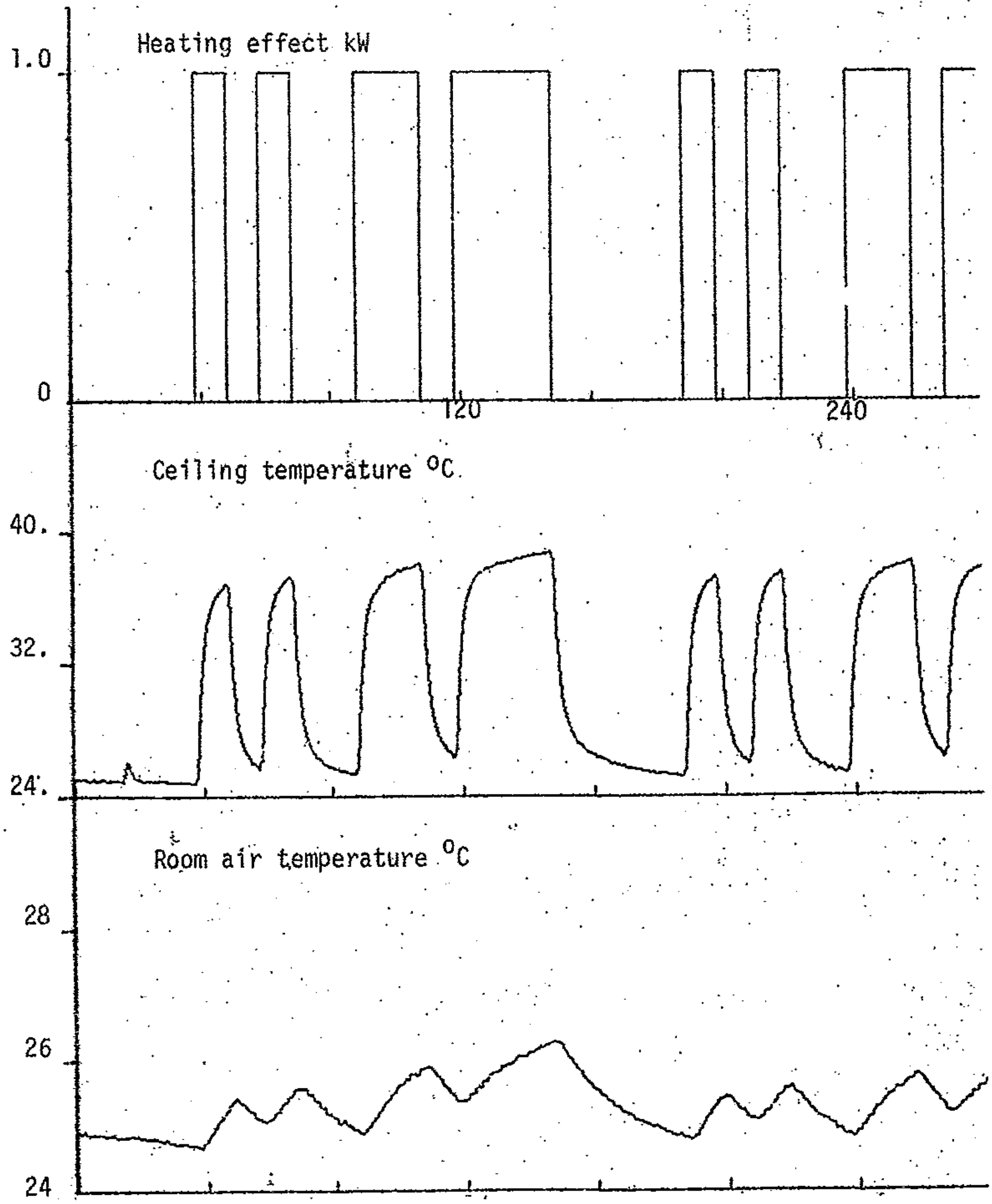

Figure 2.1 Experiment 1 (S2). Heated ceiling

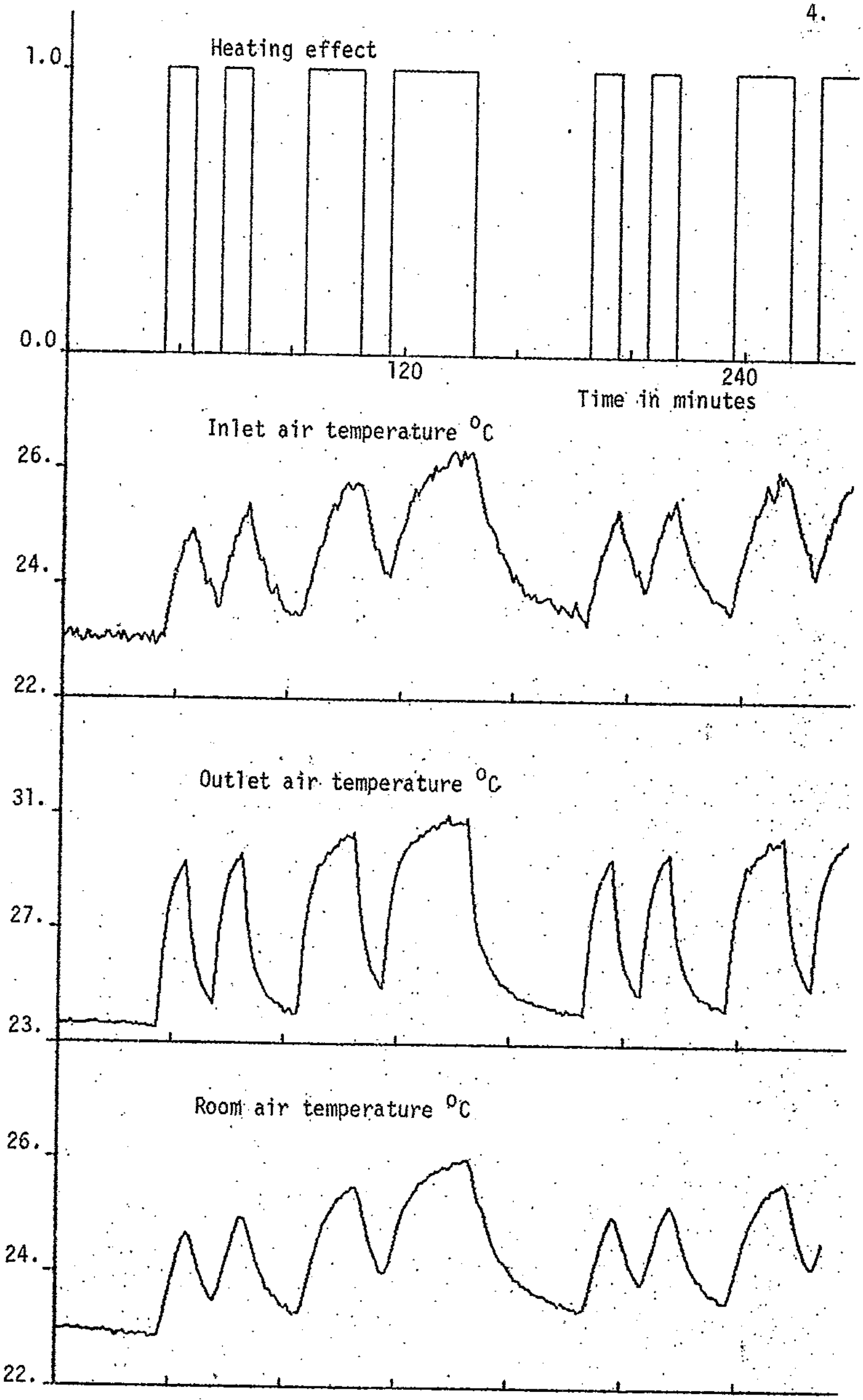

Figure 2.2 Experiment 3 (K2). Heating with heated air.

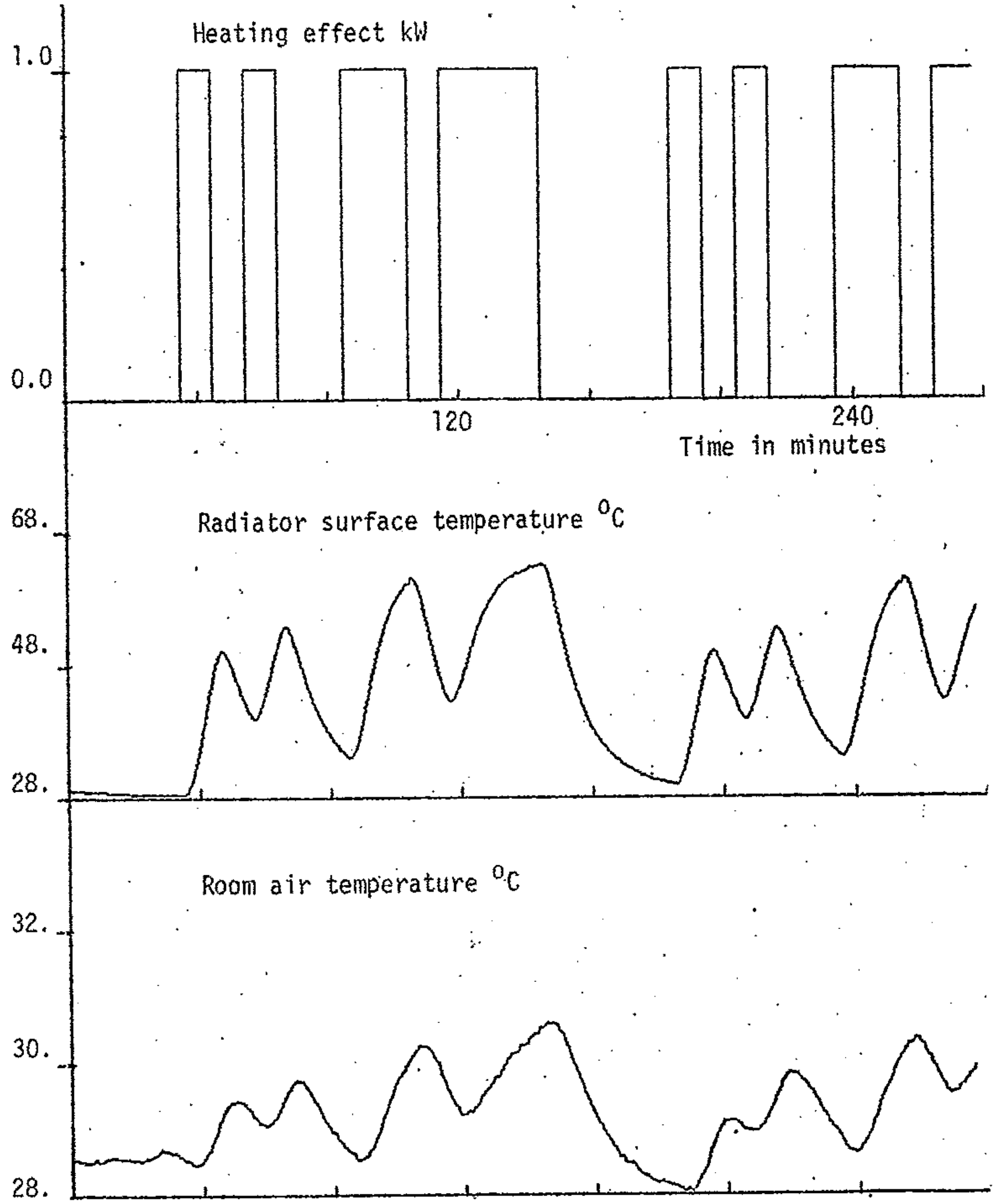

Figure 2.3 Experiment 5 (R2). Heating eith electrical radiators

3 MODELS BASED ON HEATBALANCE EQUATIONS AND CONSTRUCTION DATA

Heattransferprocesses can always be described with a set of partial differential equations, but in many cases a model with lumped parameters and of low order can describe the process well. First and second order models will be developed for the different heating devices in this section.

3.1 A model for heated ceiling

The heated ceiling consists of several thin resistance film strips imbedded in plastic film, which covers the whole ceiling. Between the concrete in the ceiling and the "heated ceiling film" there is a isolation of a thickness of 150 mm. Further details - see A1.

One way to describe the temperature on the surface of the heated ceiling film is to add all heat capacity into one (C) and add all heattransfernumbers into one (k). It is possible to describe the heated ceiling temperature x(t) with a simple heatbalance equation, a linear first order differential equation. The inputs are effect $u_1(t)$ and the roomair temperature $u_2(t)$.

The heatbalance equation becomes:

$$C\frac{dx(t)}{dt} = k(u_2(t) - x(t)) + u_1(t) \qquad (3.1)$$

The values C and k have been estimated in A1 to the following

C = 2600 Joule/^{o}C

k = 72.5 W/^{o}C

One can now easily compute the transferfunction between the output x(t) and the two inputs $u_1(t)$ and $u_2(t)$.

$$G_1(s) = \frac{1.}{Cs + k} = \frac{1/k}{C/ks + 1}$$

$$G_2(s) = \frac{k}{Cs + k} = \frac{1.}{C/ks + 1}$$

The static gains K_1 and K_2 and the time constant T can be computed with data from A1 to:

$T = C/k = 36$ sec

$K_1 = 1/k = 0.014\ {}^{o}C/W$

$(K_2 = 1.0)$

3.2 A model for air heater

The air heater consists of several resistance barrs inserted in a constant air stream in an air duct. Further details - see A2. The model which is of interest, is from the inputs effect $u_1(t)$ and inlet air temperature $u_2(t)$ to the output $x_2(t)$ the outlet airtemperature. The temperature of the heated surface $x_1(t)$ are not of interest. The heat is transfered to the air directly from the resistance bar and indirectly from the air duct heated by radiation from the resistance bar. Therefore the temperature $x_1(t)$ is supposed to be an equivalent temperature for both the resistance bar and the air duct.

The same approximations are made as before and one gets two coupled differential equations; one for $x_1(t)$ and one for $x_2(t)$. The air temperature is assumed to be same between the bars as in the outlet. The heat balance equations are:

$$C_1\frac{dx_1(t)}{dt} = k_{12}(x_2(t) - x_1(t)) + u_1(t) \tag{3.2}$$

$$C_2\frac{dx_2(t)}{dt} = k_{12}(x_1(t) - x_2(t)) + q(u_2(t)-x_2(t)) \tag{3.3}$$

With data from A2 one gets

C_1 = 3200 Joule/^oC
C_2 = 124 Joule/^oC
k_{12}= 13. W/^oC
q = 181 W/^oC

The heat capacity of the air (C_2) is small in comparison with the heat capacity in the resistance bars (C_1). If C_2 is neglected, then it is possible to eliminate $x_2(t)$ in equation (3.3) and get a new one

$$C_1\frac{dx_1(t)}{dt} = -a\ x_1(t) + u_1(t) + a\ u_2(t) \tag{3.4}$$

where

$$a = \frac{k_{12}\ q}{(k_{12} + q)}$$

The transferfunctions between the output $x_2(t)$ and the inputs

$u_1(t)$ and $u_2(t)$ can be computed from the transferfunctions between the temperature of the bars $x_1(t)$ and the inputs $u_1(t)$ and $u_2(t)$ and the simplified equation (3.3), which now is the following static equation.

$$x_2(t) = \frac{k_{12}}{(k_{12} + q)} \; x_1(t) + \frac{q}{(k_{12} + q)} \, u_2(t) \qquad (3.5)$$

The wanted transferfunctions are

$$G_{x_2/u_1}(s) = \frac{1}{(C_1/a \; s + 1)q}$$

$$G_{x_2/u_2}(s) = \frac{a}{k_{12}} + \frac{a}{(C_1/A + 1)q}$$

still

$$a = \frac{k_{12} \; q}{(k_{12} + q)}$$

The static gains K_1 and K_2 and the time constant are obtained as

$K_1 = 1/q = 0.0055 \; W/^oC$
$K_2 = 1.$
$T = C_1/a = 195 \; sec$
$a = 16.4 \; W/^oC$

3.3 A model for radiator

A radiator consists mainly of two parts: the inner part with the resistance wires in ceramic pipes hold together by some small amount of steel and the outer part which is heated by the inner part through radiation and indirectly by convection. All radiation and the main part of the convection to the room comes from the outer part.

To describe the dynamics of the radiator in full detail would give nonlinear high order model. The radiation is t^4 dependent. The same simple model as for the heated ceiling will be used. Details about the radiator is given in A3. The total heat capacity and the total heat transfer coefficient between the surroundings and the radiator are given below:

$C = 3725$ Joule/^oC
$k = 9.3$ W/^oC

This gives the following timeconstant and static gains:

$T = C/k = 400$ sec
$K_1 = 1/k = 0.107$ $^oC/W$
$(K_2 = 1.0)$

4 MODEL DERIVED FROM EXPERIMENTS

Two types of models will be given in this section. The temperature of the heating devices are influenced by the used effect and the surrounding temperature. The surrounding temperature does not change much in comparison with the effect and the temperature of the heating devices. It is then natural to study one model with only the effect as input and one model with both the effect and the surrounding temperature as inputs.

The least squares method and the maximum likelihood method have been used to derive models from data. The heated ceiling and the air heater turns out to be of first order and the radiator of second order. The different continuous time transferfunctions are given in table 4.1 and 4.2. Only the interesting part effect to heating device temperature is given for the models with two inputs. The second order radiator models have all got non real poles. Instead only the real part of the poles is used to compute the timeconstant.

Table 4.1

Least squares identification

Static gain and timeconstant for the modelpart effect to heating device temperature

Experiment	Model one input		Model two inputs	
	K	T	K	T
	^{o}C/kW	min	^{o}C/kW	min
S2	12.2	2.21	11.4	1.84
S5	11.4	1.93	10.7	1.68
K2	6.33	4.29	5.04	2.98
K5	6.33	3.98	5.16	2.84
R2	69.6	5.7	72.4	7.2
R5	61.8	5.3	68.4	8.9

Table 4.2

Maximum likelihood identification

Static gain and timeconstant for the model part effect to heating device temperature

Experiment	Model one input		Model two inputs	
	K	T	K	T
	^{o}C/kW	min	^{o}C/kW	min
S2	12.2	2.33	11.5	1.81
S5	11.3	2.06	10.7	1.72
K2	6.30	4.31	5.09	3.04
K5	6.32	4.02	5.24	2.94
R2	-	-	65.2	8.0
R5	-	-	55.2	8.6

5 COMPARISON AND REMARKS

In this section a short comparison is made between the computed and measured timeconstants and static gains for the different heating devices. Average values are used from the different LS and ML models. All model parameters are given in table 5.1.

Table 5.1

Heating device	Method	Timeconstant sec	Static gain $^{o}C/kW$
Heated ceiling	computed	36	14.
"	LS	115	11.
"	ML	119	11.
Air heater	computed	195	5.5
"	LS	211	5.7
"	ML	215	5.7
Radiator	computed	400	107.
"	LS	510	68.
"	ML	500	60.

5.1 Heated ceiling

The measured and computed values differ very much. The simple model must be rather valid because of the small thickness of the heated ceiling (only 0.14 mm). Instead may the termocouple, which has been used as sensor, cause the higher measured timeconstants. A simple calculation shows that the heat capacity in the heated ceiling is rather small in comparison with the termocouples. Also the heat conductance in the termocouple wire may cause a delaying cooling of the termocouple junction. From simulations of the temperature in a termocouple junction connected to a surface with a temperature rise as a first order system with a timeconstant of 40 seconds showed that the sensored temperature indicated a system with timeconstant 1 to 3 minutes. This shows that the choice of the sensor has not been carefully made.

5.2 Air heater

The difference is rather small between the measured and computed values. The static gain can be very accuratey computed if the air flow is well known.

5.3 Radiator

The computed static gain is rather large. The temperature difference between the radiator and the surrounding is about 70 ^{o}C at 1 kW input.

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A1 Data heated ceiling

Mark ESWA

Power 360 W

Voltage 230 V

Type 6x36/3 H813

Plastic film

Length	3.5 m
Width	0.45 m
Thickness	0.13 mm
Density	1000. kg/m^3
Weight	0.2 kg
Specific heat	1680 Joule/kg ^oC
Heat capacity	336 Joule/^oC
Heat conductivity	0.25 W/m ^oC

Resistance film

Length	3.5 m
Width	0.34 m
Thickness	0.017 mm
Density	10. kg/m^3
Weight	0.2 kg
Specific heat	190 Joule/kg ^oC
Heat capacity	38 Joule/^oC
Heat conductivity	50 W/m ^oC

Total heat capacity

The whole heated ceiling consists of 7 elements

$C = 2620$ Joule/^oC

Total heat transfer coefficient

Convection with heat transfer coefficient $h = 0.5\ W/^{o}C\ m^2$ gives

$$k_{con} = 0.5 \cdot 11. = 5.5\ W/^{o}C$$

Linearized T^4 dependant radiation gives

$$k_{rad} = 67.\ W/^{o}C$$

Together

$$k_{tot} = k_{con} + k_{rad} = 72.5\ W/^{o}C$$

A2 Data air heater

Mark Backer

Power	3 kW (1 + 1 + 1 kW)
Voltage	380/220 V
Type	VBT Nr 1424 G

Resistance wire

Neglected

Keramics (M_gO)

Length	1.6 m
Outer diameter	8.1 mm
Inner diameter	1.0 mm
Density	3400 kg/m^3
Weight	0.275 kg
Specific heat	875 Joule/kg ^oC
Heat capacity	239 Joule/^oC
Heat conductivity	25 W/m ^oC

Pipe

Length	1.6 m
Outer diameter	9.1 mm
Inner diameter	8.1 mm
Density	7800. kg/m^3
Weight	0.17 kg
Specific heat	460 Joule/kg ^oC
Heat capacity	82 Joule/^oC
Heat conductivity	48 W/m ^oC

Air duct walls

Area	1.0 m^2
Thickness	0.8 mm
Density	7800. kg/m^3
Weight	6.25 kg
Specific heat	460 Joule/kg ^oC
Heat capacity	2880 Joule/^oC
Heat conductivity	48 W/m ^oC

Total heat capacity

C_1 = 3200 Joule/^oC

Air in the heater

Height	0.4 m
Width	0.4 m
Length	0.6 m
Density	1.29 kg/m^3
Weight	0.124 kg
Specific heat	1000 Joule/kg ^oC
Heat capacity	124 Joule/^oC

C_2 = 124 Joule/^oC

Air flow

Flow	0.140 m^3/sec
Density	1.29 kg/m^3
Specific heat	1000 Joule/kg ^oC

q = 181 W/^oC

Total heat transfer coefficient

The heat transfer coefficient h varies from 10 to 100 $W/m^2\ ^oC$ for air at forced convection. If h is large then is the heat mainly transferred directly between the heater and the air. The total heattransfer coefficient for the convection from the heater and the air duct has been computed from steady state conditions for h = 10, 50 and 100 $W/m^2\ ^oC$ to 8.1, 18.1 and 16.8 $W/^oC$.

$$k_{12} = 18.1\ W/^oC$$

A3 Data radiator

Mark ADAX

Power	1 kW
Voltage	220 V
Type	VP2 No 70969

Resistance wires

Length	1.9 m
Diameter	0.5 mm
Number	9
Density	8300 kg/m^3
Weight	0.110 kg
Specific heat	460 Joule/kg ^oC
Heat capacity	50 Joule/^oC
Heat conductivity	14.6 W/m ^oC

Keramic pipes

Total length	5.3 m
Outer diameter	7.4 mm
Inner diameter	3.7 mm
Density	2600 kg/m^3
Weight	0.45 kg
Specific heat	500 Joule/kg ^oC
Heat capacity	225 Joule/^oC
Heat conductivity	1.96 W/m ^oC

Inner metal part

Area	0.24 m^2
Thickness	1.0 m^2
Density	7800 kg/m^3
Weight	1.9 kg
Specific heat	460 Joule/kg ^oC
Heat capacity	870 Joule/^oC
Heat conductivity	48 W/m ^oC

Outer metal part

Area	0.72 m^2
Thickness	1.0 mm
Density	7800. kg/m^3
Weight	5.6 kg
Specific heat	460 Joule/kg ^oC
Heat capacity	2580 Joule/^oC
Heat conductivity	48 W/m ^oC

Total heat capacity

inner part	2580 J/^oC
outer part	1145 J/^oC
total	3725 J/^oC

Total heat transfer coefficient

Convective part

surface 0.72 m^2
specific heat transfer coefficient h = 10 $W/m^2\ ^oC$

Ah_{con} = 7.2 $W/^oC$

Radiative part

corresponding heat transfer coefficient at 75 ^oC difference between heating temperature and surrounding temperature

$A\ h_{rad}$ = 4.3 $W/^oC$

Total heat transfer coefficient

k = 11.5 $W/^oC$