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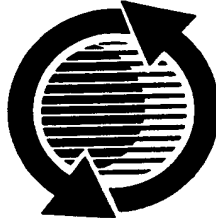
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Influence of the Velocity Near the Spark Plug on Early Flame Development

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ABSTRACT

The objective of this paper is to investigate how the velocity and turbulence within different locations close to the spark plug influence the combustion at individual cycles in a SI-engine.

2-D cycle-resolved laser doppler velocimetry (LDV) measurements have been done both inside the spark gap and around the spark tip to extract velocity information. The pressure in the cylinder was measured with a piezo-electric transducer connected to an A/D-card in a standard PC.

The velocity information was filtered to get "mean velocity" and "turbulence". The pressure signal was used in a one-zone heatrelease model to get different levels of mass fraction burned etc.

The results show a significant influence of both the "mean velocity" and the "turbulence" on the early part of the combustion when the velocity was measured close to the spark plug tip. The influence was less significant when the velocity was measured at some distance from the electrodes for both a pancake and a high squish combustion chamber. The correlation between the velocity close to the spark plug and the early flame development showed no dependence on the air-fuel ratio and a modest dependence on ignition timing.

INTRODUCTION

There is growing interest in running the spark ignition engine (SI-engine) with diluted mixtures. With diluted mixtures NO_x can be reduced because of the reduced maximum temperature level in the engine, and with a fast combustion chamber there is also a potential for increased efficiency. A stoichiometric mixture can be diluted with extra air (lean burn), recycled exhaust gases (EGR), water vapour or some other gas. The result is in all

cases an extra bulk gas that can absorb the heat from the combustion of the fuel [1].

The combustion is also affected however, when the mixture is diluted. The rate of combustion depends on the flame speed in the combustion chamber. This flame speed is considered to scale with the laminar flame speed and the turbulence level [1]. The laminar flame speed of the air/fuel-mixture is very sensitive to dilution, with a reduction to 20% or less of its maximum with extreme dilution [2]. It is therefore interesting to enhance turbulence if an engine should operate lean or with EGR. Combustion chambers and inlet manifolds which increase the turbulence level are in common use with lean burn engines [3],[4].

COMBUSTION STABILITY

When an engine is running at the dilution limit it experience a large variation in the combustion duration from cycle to cycle. This gives a fluctuating IMEP which in turn gives an unstable engine performance. The emissions of unburned hydrocarbons (HQ) are also increased at the dilution limit due to the extended duration of the combustion event. In some cycles the combustion is not completed before the exhaust valve opens (slow burn) and in some cycles the flame is extinguished as the pressure and temperature are reduced when the volume increases during the exhaust stroke (partial burn) [1]. It is very interesting to reduce this instability in the combustion event. If this could be achieved, the dilution limit would be moved to a somewhat higher level of dilution and the NO_x could then be reduced even more. To be able to do this the instability in the combustion event must be reduced.

Major causes of cycle to cycle instabilities - Three major causes are considered to give combustion instability [1]:

- 1: The total amounts of fuel, air and burned gas in the cylinder are not the same from cycle to cycle.
- 2: The charge is inhomogeneous. This is especially important close to the spark plug.
- 3: The flow field in the engine varies from cycle to cycle.

The relative importance of these three causes is not well understood but it is likely that the dominant cause changes depending on the operating parameters of the engine. This study will focus on the flow field effect.

EXPERIMENTAL APPARATUS

The engine - The measurements were made in a single cylinder engine based on a six-cylinder Volvo TD 102 diesel engine. Its main geometric properties are given in table 1. Optical access was provided by four rectangular quartz windows in a spacer between the cylinder head and engine block. To maintain a reasonable compression ratio the original piston was extended.

Displaced volume	1600 cm ³
Bore	120.65 mm
Stroke	140 mm
Connection rod	160 mm
Compression ratio	10:1

Table 1: Geometric properties of the engine.

The ignition power was supplied from a standard Transistorised Coil Ignition (TCI) system. The spark plug was a "semi-standard" Champion RC582YC with the electrodes extended to a 7.5 mm penetration into the combustion chamber.

The LDV system -The velocity measurements were performed with a 2-component DANTEC fibre-flow system. This system consists of a 5W Ar-ion laser, a transmitter that splits the multicolour laser beam into 514.5 and 488 nm wavelength components, frequency shifts half of each colour 40 MHz and leads the four resulting beams in optical fibres, a fibre optic probe which sends out the laser beams and collects the scattered light, two photomultipliers that convert the optical signal to electrical and two signal processors (BSA enhanced) which perform a real-time FFT to extract velocity information. The system is controlled by a standard 486/33 PC. The system is described in figure 1 and its main specifications are given in table 2.

The probe was translated with two traversing tables mounted on the engine. The resolution of the tables was

0.01 mm. As the probe was firmly mounted on the engine the relative motion between combustion chamber and probe was kept to a minimum.

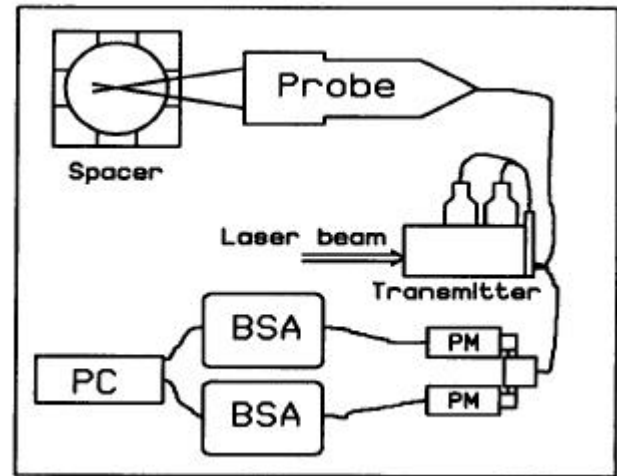


Figure 1: 2-component LDV-arrangement

wavelength	488 nm	514.5 nm
focal length	310 mm	310 mm
beam spacing	41.23 mm	40.87 mm
beam intersection-angle	3.804°	3.771°
beam diameter	4.3 mm	4.3 mm
fringe spacing	3.678 μm	3.911 μm
probe volume diameter	45 μm	48 μm
probe volume length	680 μm	724 μm

Table 2: LDV system specifications

Seeding - To get scattered light from the laser beam crossing some kind of particles must be added to the inlet air. There are a number of criteria which the seeding particles must fulfil:

- 1: The particle must be small enough to follow the gas flow with little lag. A diameter of less than 1 μm is in most cases satisfactory [5].
- 2: The particles must remain usable in the entire interesting part of the engine cycle.
- 3: The correct amount of particles should be present to give the wanted data rate.
- 4: The particles must not contaminate the windows of the combustion chamber.
- 5: The combustion in the engine should not be influenced by the particles.
- 6: The engine should not wear down due to particles in the lubricating oil.
- 7: The seeding equipment should be easy to handle.

There is no perfect seeding particle which fulfils all of these criteria. Several different types were tried. The first was DOP (dioctylphthalate) seeded from a TSI six-jet atomizer. These gave a good signal up to approx. 20° BTDC where the signal disappeared. The reason is thought to be evaporation due to the high temperature after the compression stroke. Tests with other liquids with high boiling temperature were therefore performed. The tested liquids were:

DOS (dioctylsebacate)
 DBP (dibutylphthalate)
 olive oil (three different kinds)
 corn oil
 heat-exchanger oil
 engine oil SAE 10-40
 gearbox oil SAE 90
 silicone oil (Dow 210H)
 polyalphaolefine (the base of synthetic oil)
 polyethyleneglycole 400
 glycerol

The only liquid that gave an improvement was the silicone oil. The products from the combustion of silicone gave however a milky layer on the windows and caused the spark plug to malfunction.

Seeding with a dry powder dispenser was also tried. Talc and carbon black were used as seeding material. The data rate was with this system very limited and the handling of the dispenser was awkward. TiO₂ and other commonly used dry particles were not tried as the potential wear of the cylinder liner and piston with a oil/TiO₂ mixture was considered to be too high.

As liquids evaporate, some kind of solid was wanted. Tests with salt and sugar mixed with water seeded from the TSI six-jet gave clogged windows and bad data rate. One usable seeding material was ordinary brewed coffee! It fulfilled most criteria. As coffee is an organic material it burned quite well so the windows stayed clean for approx. 0.5 -1 hour of running.

The final choice was, however, a polystyrene-latex dispersion. Polystyrene can be obtained with a narrow size distribution and gives a very good SNR signal. The windows actually stay cleaner when seeding with polystyrene than without any seeding. This gave "unlimited" running time of the engine. The reason to this unexpected phenomenon is believed to be the contents of the original polystyrene dispersion. In the production of the dispersion tensides are used to make the polystyrene form the small spheres. The tensides are believed to clean the windows. The polystyrene however clogged the six-jet atomiser if the seeding was interrupted for more than 1 hour. The amount of seeding particles was also rather low from this the atomiser. The new atomisers used instead were 24 Hudson Up-Draft nebulizers. The mean size of the

liquid drops from these atomisers was 4-5 µm but as the dry weight of the polystyrene dispersion used was 1 % and the mean polystyrene particle size was 0.18 µm the resulting particle size was under 1 µm [6]. The Hudson atomisers were replaced after 4 hours of operation but as the cost of each was under \$2 the expense was low.

The Pressure measurement system - The pressure in the cylinder was measured with a AVL QC42 piezo-electric transducer connected to a Kistler 5001 charge amplifier. The charge amplifier voltage output was connected to a 486/33 PC clone with a Data Translation DT2823 100 kHz 16-bit A/D-card.

The control system - The ignition timing and skip-fire was controlled with a purpose-built system with 3 TC24 digital counter cards in a PC. Triggering signals to the LDV- and pressure-systems was also included in this system. Input signals to the control system were a sync-pulse (1 pulse per 2 revs), a TDC-pulse (1 pulse per rev) and a crank angle-pulse (5 pulses per crank angle degree). The A/D- card was triggered at every crank angle position in the last motored cycle and in the first cycle with combustion. The LDV-system required crank angle and Top Dead Centre pulses. To reduce the amount of data to be stored in the LDV-PC there was also an enable signal from the triggering system which disabled the BSA:s during times when the velocity was of low interest.

OPERATING CONDITIONS

The engine was run on natural gas which was fed to the engine through four pulsewidth-modulated solenoid valves. The contents of the gas used is given in table 3. The valves were controlled by a Intelligent Control IC5460 engine management system. To get a homogenous charge the mixing length from the solenoid valves to the engine was 3 in with a 16000 cm³ mixing tank in the middle. The influence of different amounts of fuel and air into the engine from cycle to cycle is therefore reduced.

Component	Vol. %	mass %
Methane	91.1	81.0
Ethane	4.7	7.9
Propane	1.7	4.2
n-Butane	1.4	4.7
Nitrogen	0.6	0.9
Carbon monoxide	0.5	1.2

Table 3: Contents of the natural gas used.

To reduce the influence of different amounts of residual gas the engine was run in a skip-fire mode in which the engine was fired for 3 cycles and then motored for 3 cycles. Tests with a longer motoring period showed

no difference in the measured parameters. Increasing the number of fired cycles heated the spacer as it was run uncooled. In this skip fire mode the pressure was measured in the last motored and in the first cycle with combustion and the LDV-system was enabled only in a crank angle interval between 20 degrees before ignition and TDC in the first cycle with combustion. This first cycle was thought to give small amounts of residual gases.

Several running conditions has been investigated. Engine speed has been varied between 600 and 1000 rpm, Air/fuel ratio between $\lambda=1$ and $\lambda=2$, ignition timing 30° to 10° BTDC. Two different combustion chambers were used: one slow burning chamber of the pancake type and one specially designed with a high level of squish. The high squish combustion chamber used a piston extender with two orthogonal 30 mm wide traces from window to window. The four remaining sectors of the bore were used as squish area. The total squish area was 44% of the piston area and the clearance height between the squish area and cylinder head was 1 mm. The high squish combustion chamber gave a significantly shorter combustion duration. The inlet port was from the standard diesel engine with a swirl ratio of approx. 2.8. The engine was operated during all runs with an inlet pressure of 1 atm. For some operation conditions, the probe volume was translated from the centre of the electrode gap out to a point where the correlation between velocity data and heatrelease was small. In each run approx. 200-250 cycles were collected. The exact number varied as the limitation was the BSA:s memory of 196588 velocity samples. Four different parameter variations are presented.

DATA PROCESSING

Cycle-resolved analysis of the gas velocity -For each component, the velocity was low and high-pass filtered with the moving window technique [7] to extract "mean velocity" and "turbulence". In the moving window procedure the mean value of the velocity samples within a specified crank angle window is considered to represent the mean velocity. The RMS-value within the window represents the level of turbulence. In the calculation of the RMS-value, the main velocity trend in the window was eliminated by means of extracting a linear curve fit to the data before the RMS-calculation was performed. No attempt was made to get "true" turbulence as in [8]. The moving window technique requires a cut-off frequency to be chosen. This frequency determines how much of the velocity that should be considered to be turbulence and how much should be considered mean velocity transients during the engine cycle. If a small window is used, there is a problem with the data rate. The uncertainty of the RMS value can be estimated according to

$$unc = \pm 1.96 \frac{1}{\sqrt{2N}}$$

With 95 % confidence level 1.96 becomes 1.96 . Table 4 gives the uncertainty for different number of data points within the window and the required data rate with a one and six crank angle degrees wide window at 700 rpm. The average data rate was 40-70 kHz giving between 60 and 100 data points within a 6 degrees window. The resulting uncertainty in the RMS calculation can be considered to be high but it is hard to achieve a better cycle resolved turbulence estimate.

Data points in window	Uncertainty in RMS [%]	Required data rate window=1° [kHz]	Required data rate window=6° [kHz]
30	25	126	21
50	20	210	35
100	14	420	70
200	10	840	140

Table 4: Uncertainty and required data rate for the RMS-calculation. 700 rpm.

If the window is wide, the assumption must be made that the flow does not change within this period. As there is an uncertainty the velocity data were processed with the windows 1, 2, 4, 6, 8 and 10 crank angle degrees which at 700 rpm represent a cut-of frequency of 4200, 2100, 1050, 700, 525 and 420 Hz. Figure 2 shows individual velocity registrations of the LDV-system, "mean velocity" and "turbulence" for one individual cycle with a crank angle window of 1 degree. Figure 3 shows the same cycle with a crank angle window of 10 degrees.

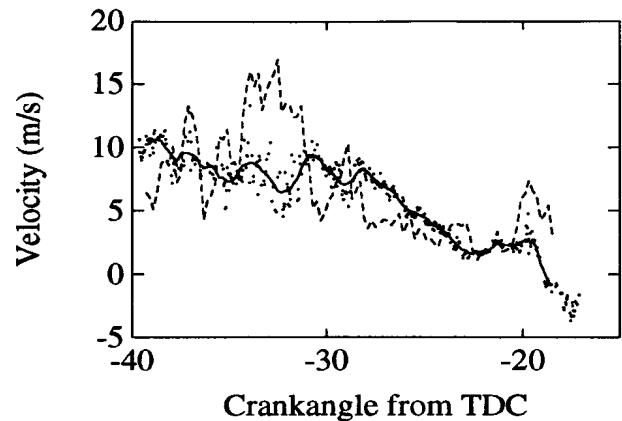


Figure 2: Horizontal velocity registrations with 1 crank angle degree window. Solid line indicates mean velocity and dashed indicates RMS value. The RMS values are magnified by a factor ten.

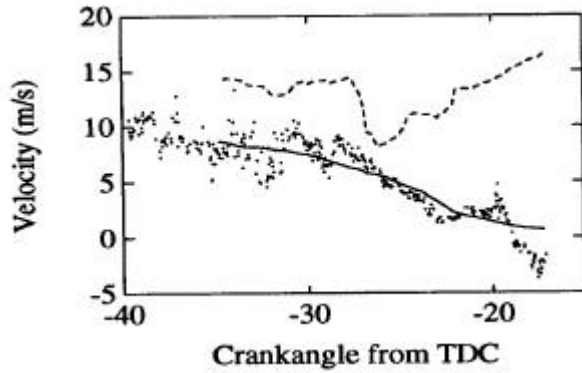


Figure 3: Horizontal velocity registrations with 10 crank angle degrees window. Solid line indicates mean velocity and dashed indicates RMS value. The RMS values are magnified by a factor ten.

One-zone heatrelease model - To extract information on the early flame development, a cycle-resolved heatrelease calculation was performed. The assumptions in this model are the following:

- 1: The ratio of specific heats $\gamma = C_p/C_v$ is a linear function of temperature.
- 2: The rate of heat transferred to the walls is proportional to the temperature difference between gas and wall and to the exposed area at a specific crank angle position.
- 3: Blow-by is small.
- 4: Crevice effects are neglected.

The parameters in the model are tuned in against the last motored cycle in the skip-fire sequence until the apparent heat released during motoring is below a maximum level (in most cases approx. 5 J). As it is the very early flame which is most sensitive to changes in the flow field, it is desirable to detect the heat released as early as possible [9]. The measurement noise does, however, set a minimum level which in this case was approx. 10-15 J. This corresponds to 0.5 % of the total energy released of the combustion. Figure 4 shows the rate of heatrelease for some 30 cycles when the engine is running lean ($\lambda=1.84$) and figure 5 shows the accumulated heat released for the same cycles. To give an indication of how the early flame development and hence the heatrelease look like the early heatrelease of the same cycles are plotted in figure 5 and 6. The ignition system was in this case triggered at 20° BTDC and the 0.5% heat released was detected at approx 10° BTDC. In figure 7 the level of 0.5% heat released is indicated with a horizontal line.

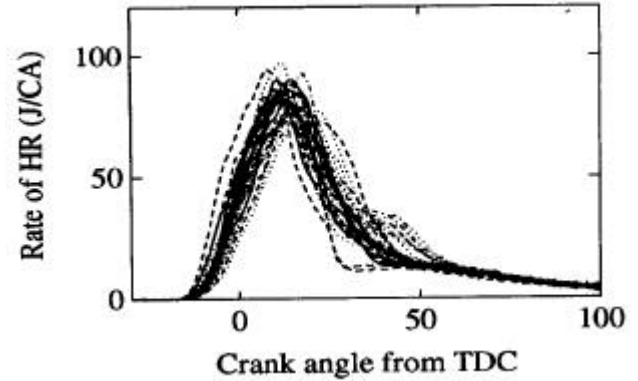


Figure 4: Rate of heatrelease for 30 cycles. $\lambda=1.84$.

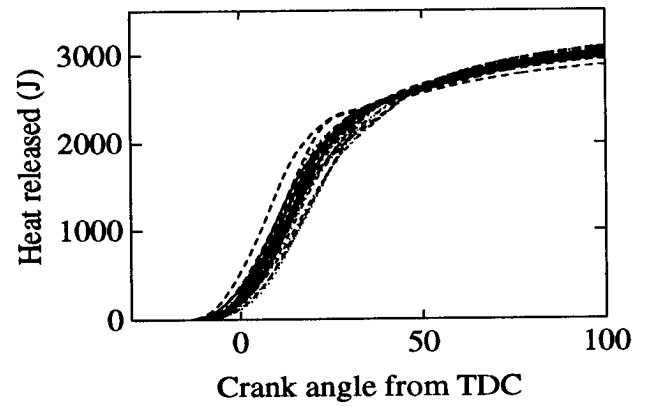


Figure 5: Accumulated heat released for 30 cycles. $\lambda=1.84$.

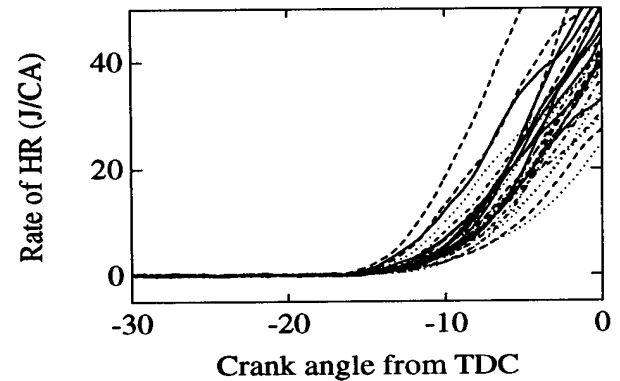


Figure 6: Rate of heatrelease in the early phase of combustion.

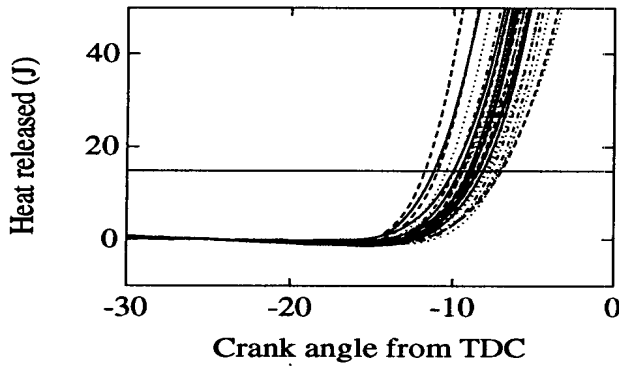


Figure 7: Accumulated heat released in the early phase of the combustion. The horizontal line indicates the level of 0.5% of the total heat released.

Determination of the correlation between velocity and heatrelease data - As the velocity parameters the value of the "mean velocity" and "turbulence" vectors were chosen. The combustion parameter was chosen to the crank angle position at which 0.5% of the total heat was released. The level of 0.5% was a compromise between the desire to detect a small flame size and the noise in the pressure measurements and heatrelease calculations.

To find out what relationship there is between the velocity parameter and the 0.5% position a multiple correlation coefficient (R) was calculated. R can be considered to be an extension of the normal correlation coefficient but in the calculation of R more than one variable can be used to explain the variance of the 0.5% position [10]. Appendix A relates R to the normal correlation coefficient. R was calculated as follows with the first and second orders of the velocity parameter as explaining variables.

1. A polynomial of second order was fitted to the data containing 200 cycles of velocity and combustion information. This gave the three coefficients A, B, C.
2. The value of the polynomial was calculated for the "turbulence" at each cycle i giving a estimate of the crank angle position at 0.5% heat released from the "turbulence". This gave a equation of the form

$$A_{0.5,i} = A u'_i(\theta, \Delta\theta)^2 + B u'_i(\theta, \Delta\theta) + C$$

were

θ = crank angle position of window center

$\Delta\theta$ = crank angle window used

$u_i(\theta, \Delta\theta)$ = "turbulence" at cycle i

$A_{0.5,i}$ = estimate of $CA_{0.5,i}$ from $u'_i(\theta, \Delta\theta)$

A typical data set is shown in figure 8 with the second order polynomial fitted to the data.

3. The normal correlation coefficient between this estimate and the true crank angle position of 0.5 % HR was calculated. This correlation coefficient is equivalent with the multiple correlation coefficient R from the multi-variable regression analysis [10].

4. Calculation of the correlation coefficient was performed using u' determined with the moving windows centre at -5 to +5 crank angle degrees from ignition. The maximum level of R within this period is considered to give an estimate of how much the early flame development is influenced by u' . The same procedure was then used with the "mean velocity" as the velocity parameter.

Figure 9 shows the coff.coeff. for the different windows used. A wide window means that the coeff. has a smoother curve and that it ends earlier. The data was lost approx. 5° after ignition as the seeding particles were burned.

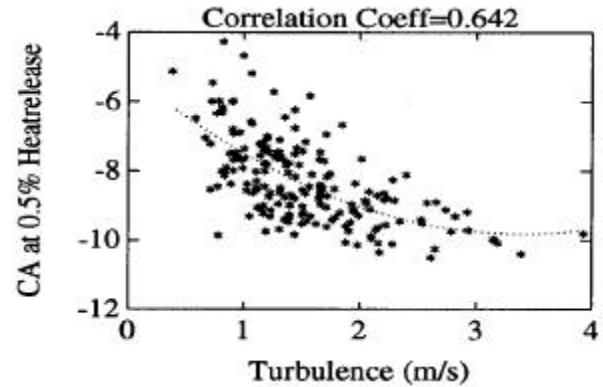


Figure 8: Data set with high correlation coefficient.

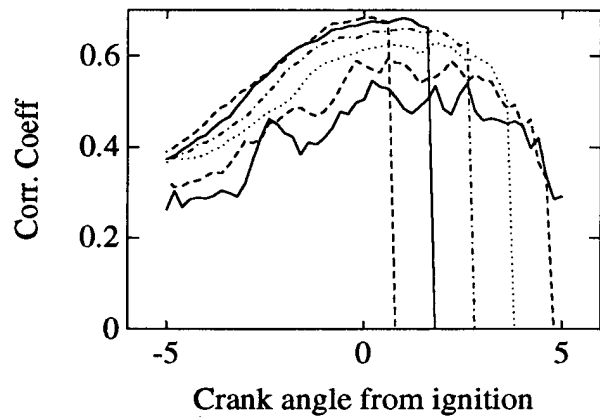


Figure 9: Coff.coeff = f (crank angle relative ignition) for the turbulence calculated at the crank angle position and the duration of the 0-0.5% heat released. The crank angle windows 1 to 10 degrees were used. The loss of data due to combustion ends the signal early for a window of 10 degrees and later for smaller windows.

EXPERIMENTAL RESULT

Dependence on measurement position with high squish combustion chamber - A series of runs was made with different positions of the probe volume relative to the spark plug electrodes. The engine was operated with ignition at 20 °BTDC, WOT, 700 rpm, $\lambda=1.0$ and 1.8. Figure 10 gives the geometry of the spark plug and the co-ordinate system for the measurements. The co-ordinates were $X=0.0$, $Y=0.0$, $Z=-0.43$ mm when nothing else is stated. Figure 11 to 22 shows the maximum correlation coefficient between the "mean velocity" and "turbulence" to the 0-0.5% heat released for $\lambda=1.0$ and $\lambda=1.8$. Each position was measured twice. This gives some indication of the uncertainties of the measurements.

As the probe volume was translated in the X-direction and the Y and Z components of velocity were measured (figure 11-14) there is a detectable trend in both the "mean velocity" and the "turbulence" correlation when the engine is running lean. The correlation for mean velocity is reduced from 0.4 to 0.2 when the velocity was measured 4 mm from the spark plug. This gives an indication of the integral length scale in this combustion chamber. When the engine was running at $\lambda=1.0$ this trend was not as clear. There is however a large scatter in the measurements. When the probe volume was translated in the Y direction (figure 15-18) there is a more detectable trend in both "mean velocity" and "turbulence". This can depend on the fact that the translation was made in the same direction as the velocity measurement was performed in. When the X and Z velocity components were measured it was not possible to measure in the spark plug gap as the side electrode was blocking the laser beams. The trend of lower corr. coeff. is however evident also in this case. Here the reduction of the corr. coeff. is more clear for the stoichiometric chase. The maximum correlation coefficient achieved is however at the same level.

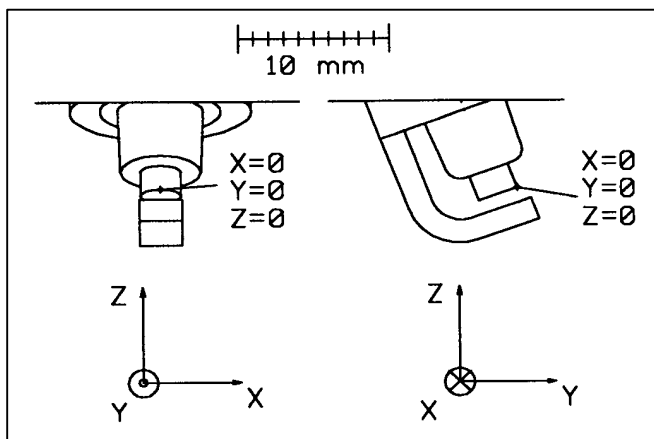


Figure 10: Geometry of the spark plug used and the co-ordinate system.

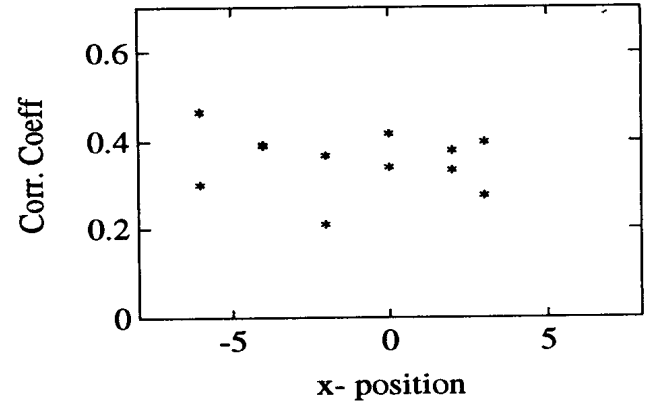


Figure 11: Max. Corr. coeff. = $f(x\text{-position})$ for "mean velocity". $\lambda=1.0$. Crank angle window = 6°
Y and Z velocity components measured.

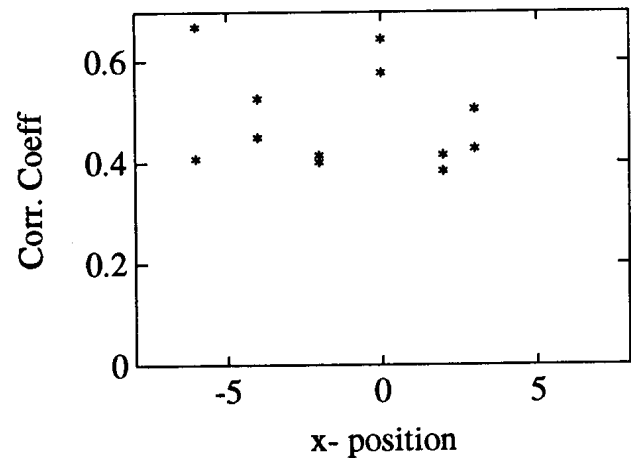


Figure 12: Max. Corr. coeff. = $f(x\text{-position})$ for "turbulence".
 $\lambda=1.0$. Crank angle window = 6°
Y and Z velocity components measured.

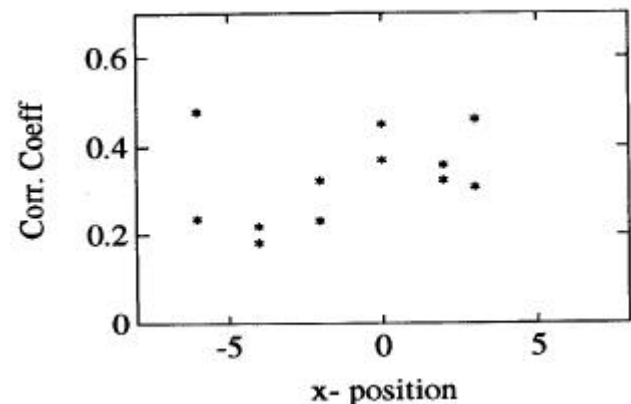


Figure 13: Max. Corr. coeff. = $f(x\text{-position})$ for "mean velocity". $\lambda=1.8$. Crank angle window = 6°
Y and Z velocity components measured.

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APPENDIX A:

THE RELATIONSHIP BETWEEN THE NORMAL AND MULTIPLE CORRELATION COEFFICIENT

Throughout this paper the multiple correlation coefficient R has been used. This correlation coefficient is derived from the linear regression analysis and is defined as

$$R^2 = \frac{\text{Square sum due to regression}}{\text{Total square sum}} = \frac{\sum (\hat{Y}_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2}$$

were

Y_i = general variable which has a variance to be explained.
 \bar{Y} = mean value of Y .

\hat{Y}_i = an estimation of Y obtained from one or several explaining variables X with some model.

R^2 measures the "proportion of total variation about the mean \bar{Y} explained by the regression" and is often called "the percentage variation explained" [10].

The normal correlation coefficient r_{XY} is defined as

$$r_{XY} = \frac{\text{Covar}(X, Y)}{\{V(X)V(Y)\}^{1/2}} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\left[\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2 \right]^{1/2}}$$

It can be shown that in general the multiple correlation coefficient R is equal to the normal correlation coefficient between Y and its estimate [10].

$$r_{Y\hat{Y}} = R$$

This relationship has been used in the calculation of R to reduce the computer time.

It can also be shown [10] that if the estimation of Y , in the regression analysis, is received by fitting a straight line between X and Y then

$$r_{XY} = R = (\text{sign of } b_1)(R^2)^{1/2}$$

b_1 is here given from the model for a straight line fit

$$Y = b_0 + b_1 X$$

If several explaining variables are used to find causes to the variation in Y then r_{XY} can no longer be used to give an indication of how well the variance of Y is explained. R is a measure of how well this is done. As multiple variables are used for explaining the variance the corresponding measure of how good this is done is called the multiple correlation coefficient (R).

If a more complicated model like

$$Y = b_0 + b_1 X + b_2 X^2$$

are used to explain the variance of Y the resulting correlation coefficient (R) will be higher than r_{XY} if and only if the extra variables can give an extra explanation. If the straight line is included in the more complicated model then

$$r_{XY} \leq R = r_{Y\hat{Y}}$$