State Estimation in Power Networks I

A Literature Survey

van Overbeek, A. J. M.

1973

Document Version:
Publisher's PDF, also known as Version of record

STATE ESTIMATION IN POWER NETWORKS I
A LITERATURE SURVEY

A.J.M. van OVERBEEK

Report 7331 (C) November 1973
Lund Institute of Technology
Division of Automatic Control

STATE ESTIMATION IN POWER NETWORKS I

A literature survey

A.J.M. van Overbeek

ABSTRACT.

This report gives the results of a literature study on state estimation methods in power networks. First the steady state analysis of power networks is treated. From this analysis the state variables are defined: all complex bus voltages. It is pointed out that this state definition is different from the one used in control theory. In the presentation of the methods a distinction is made between estimators for the state at time-point t_k only using the measurements obtained at t_k and estimators that also use a priori knowledge about the state. All methods proposed for or in actual use belong to the latter category. These methods can be thought of as consisting of three families. The first two families are directly applying estimation theory, while the third one makes use of the physical properties of a power network. Finally one representative of each family is presented that has promising outlooks for further comparison. The actual comparison and results are the subjects of following papers.

This work has been done as partial fullfillment of the requirements for the Masters Degree in Electrical Engineering at the Eindhoven University of Technology, Eindhoven, The Netherlands.

CONTENTS.

1. INTRODUCTION.

1.1. The task of a power system.

When we talk about an electric power system, or power system for short, we mean the whole complex of plants, installations, and apparatus necessary to produce, transport and distribute the demanded electric energy to all customers. The task of such a power sytem can be formulated as follows:

> An electric power system must supply the varying active and reactive power demanded by consumers with satisfactory continuity and quality of service at a minimum price.

How do three elements in this definition, supply, varying demand and quality, look like in reality ?

The electricity is produced in generators driven by turbines. These can either be steam-, water-, or gasturbines. The majority of the turbines in the world are steamturbines. 400 to 600 MW is a normal capacity for a modern generator driven by a steamturbine. Heat sources for producing the necesarry steam can be coal, gas, oil or a nuclear reactor. To distinguish steam plants from hydro power plants they are also called thermal power plants. Gasturbines have a much smaller capacity and are used for fast stand-by and peak loads.

Plants will be located on sites with, for thermal plants, sufficient coòling water available or with favorable terrain conditions for hydro power plants. Usually these places differ from the ones where the load is concentrated: cities, industrial areas. Therefore it is necessary to transport large quantities of electric energy from the production to the consumption centers. There we'll have to distribute these large quantities to all individual customers. The means in use to do this are the transmission and distribution networks consisting of cables, air lines, switching centers and transformers. These networks operate at several hierachical voltage levels. The transmission network, taking care of the bulk transports, has the highest level. This voltage level is 400 to 800 kV(!) for the AC systems of today. In some cases High Voltage DC (HVDC) is used for bulk transportation.

The load variation, the second element in our formulation, can be about a factor two or more: the maximum load occurring during the early morning hours is about twice as large as the minimum load during the night. Of course this is dependent on the time of year and the geographical location. A power system must have sufficient production and transportation capacity to meet this maximum demand.

An annoying thing is that there is almost no way to store energy in the system. The demand has to be met instataneously. When the demand is varying the production must follow these variations. The largest variation takes place in the morning when within a few hours the demand goes from minimum to maximum. The system must be able to follow this rise while maintaining the required quality, the third element of our definition.

This quality has to do with the frequency in the system and the voltage level and form at the customers. The frequency is maintained within very close limits: 50 $\pm$ 0.01 Hz (60 $\pm$ 0.01 Hz for e.g. USA,Canada), while the voltage level is allowed to have larger variations: up to 10 %.

1.2. Reasons for state estimation.

The last element in our definition says that all of this has to be done at a minimum cost. Certain production units will be cheaper than others, certain lines will have higher losses than others. All of this has to be taken into account when deciding which part of the load is going to be produced where. This is the problem of load dispatch.

Now we'll first make a little excursion into the field of calculations. When we look on a timescale of minutes the demand is varying very slowly. For calculation purposes it is possible to assume that the network is in steady state. By doing so we disregard for example the energy that is stored in the generators as rotating and magnetic energy.

From here on we will mainly be concerned with the transmission network. The terms network and system are used interchangeably. The power plants are just sources of active and

reactive power and the load is lumped into loads concentrated at the main switching centers.

Steady state analysis is used in calculations for security purposes: e.g. how far is this line from its load limit ? what will happen with this line when an other one goes out of service ?, and in calculations concerning load dispatch.

In the 1950's, with the advent of the computer, a number of algorithms for off line use were developed to solve the so called load flow problem: how much energy is flowing in each line given this demand and this production schedule. With this problem solved one can calculate e.g. the line losses necessary in solving economic load dispatch problems. But let us go back to the reasons for state estimation.

Since the last century we have seen a doubling of the total demand every ten years. This has lead to ever increasing sizes of production units and to larger transmission networks operating at higher voltages and superimposed on the old ones. The whole system is getting more and more complex. It is of increasing importance to know the situation in the system at this moment for security reasons, dispatch purposes and local power plant control strategies.

The first steps to obtain this goal, the knowledge of the present situation or state of the system, were made by incorporating real time measurements in existing load flow algorithms. This proved to be unreliable, mainly because just as many measurements with their inherent uncertainties were used as there were state variables to determine. So one turned to other methods using more measurements to diminish the uncertainty in the knowledge of the state of the system.

The remainder of this report will be concerned with these last methods: state estimation methods. There are but a few state estimation methods tested in actual on line operation /16,20/[*]). Three stages are suggested in the on line application of an estimation algortihm or estimator. 1) use the estimator to obtain a reliable data base. 2) use this base for on line

[*]) See chapter 6: References.

security monitoring. 3) use this base for on line control. All reported applications are in the first and second stage.

1.3. Outline of this report.

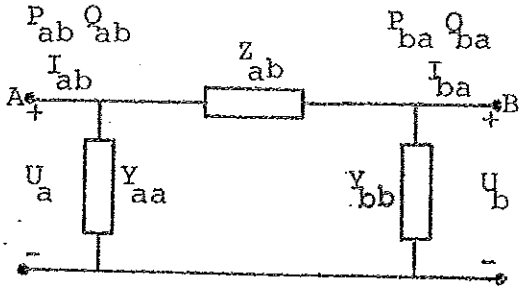
This report gives a review of the proposed methods as I found them in the literature. The analysis of the power system in steady state including the load flow problem is the subject of chapter 2. As a result of this analysis the state variables are defined. Before the presentation of the methods in chapter 4 a few general formula's will be derived from generalized least squares estimation theory in chapter 3. Both in chapter 3 and 4 a distinction is made between methods using only the information from the measurements and methods using a priori knowledge. The methods presented in chapter 4 can also be divided into three families. The first two families apply estimation theory directly. The first one uses batch processing of measurements, while the second one uses sequential measurement processing. In the third family use is made of the almost linear relation between the complex voltage difference across a line and the power flow in that line. In chapter 5 one representative of each family is presented that looks promising for further comparison.

2. STEADY STATE ANALYSIS IN POWER SYSTEMS.

2.1. The network model.

To obtain a mathematical model of the power network it is assumed that all three phase voltages and currents are symmetric. As a consequence of this assumption we only have to consider single phase values. That is why we can use the same model as in network theory consisting of nodes connected by branches.

In our model we have one ground node. All other nodes can be identified with buses in the real network. A branch between two nodes corresponds to the line(s) and/or cable(s) between the two corresponding buses. Such a branch is modelled by a Π-section (Fig. 2.1).



Z_{ab} : series impedance
Y_{aa} Y_{bb} : shunt admittances
U_a U_b : node voltages
I_{ab} I_{ba} : line currents
P_{ab} P_{ba} : active line flows
Q_{ab} Q_{ba} : reactive line flows

Fig. 2.1: Branch in network model.

The series impedance $Z_{ab} = R_{ab} + jX_{ab}$ consists of conductor resistance (R_{ab}) and inductance ($X_{ab} > 0$). The shunt admittances, $Y_{aa} = G_{aa} + jB_{aa}$ and $Y_{bb} = G_{bb} + jB_{bb}$ are lumped from the distributed admittance between line and ground. Usually these admittances are purely capacitive ($G_{aa} = G_{bb} = 0$ and $B_{aa} > 0, B_{bb} > 0$). For air lines these shunt admittances are about a factor 10^2 smaller than the series admittance ($1/Z_{ab}$). A consequence of our assumptions is that all results will have per phase values. The element values for the branches must also be single phase.

The variables we are interested in are summarized in table 2.1. The load flow problem is treated in the next section because it is both an example of steady state analysis and it gives a good introduction to the choice of state variables. We want to determine all the variables of interest of table 2.1 given a certain load and generation pattern.

At each bus (node):

V : Voltage magnitude
δ : Voltage angle
P_g : Real generated power
Q_g : Reactive generated power
P_d : Real power demand
Q_d : Reactive power demand

Complex voltage: $U = Ve^{j\delta}$

For each line (branch):

I_{ab} I_{ba} : Complex line current from A to B (B to A) at node A(B)
P_{ab} P_{ba} : Real line flow from A to B (B to A) at node A(B)
Q_{ab} Q_{ba} : Reactive line flow from A to B (B to A) at node A(B)

Table 2.1: The variables of interest.

For each node we have two real equations: the active and reactive power balance. However, we have six variables: $V, \delta, P_g, Q_g, P_d, Q_d$. At each bus we have to specify four of them to solve our problem.

Let us first look to the variables of table 2.1 in more detail. When we know all node voltages it is possible to compute all line currents and flows. An addition of a fixed amount to each δ does not change the solution. It is just a rotation of our chosen coordinates. So we'll have to choose at least one δ as reference. When we look to the problem from an operations point of view we can say that P_d and Q_d are always given. P_g and Q_g can be influenced by turbine and excitation regulation respectively.

2.2. The load flow problem.

In the actual solution of the load flow problem the demanded and generated at a bus are lumped together into the so called bus power: $S_a = P_a + jQ_a$, where $P_a = P_g - P_d$ and $Q_a = Q_g - Q_d$. Now we'll have to choose two of four variables to solve our problem. For physical reasons (see e.g. /1/) P_a is mostly dependent on δ and Q_a on V. Therefore we choose only one variable out of the group P_a, δ and one out of Q_a, V. This gives four possibilities, corresponding to the following bus types:

Type 1: P_a and Q_a given. The solution gives V and δ.

This is a load bus. All buses with no generation ($P_a = -P_d$, $Q_a = -Q_d$) belong to this category.

Type 2: P_a and V given. The solution gives Q_a and δ.
This is a generator bus. For most generators the reactive generation is determined by the automatic voltage control.

Type 3: V and δ given. The solution gives P_a and Q_a.
This is our reference bus. P_a and Q_a have to cover the line losses. These are unknown beforehand. Therefore this bus is also called slack or swing bus.

Type 4: Q_a and δ given. The solution give P_a and V.
This is an angle reference bus. It can be used together with a type 2 bus as an alternative to a type 3 bus.

In most problems there are type 1 and 2 buses with one type 3 bus as reference bus. The actual non linear equations are solved for the voltages (V and δ) by means of some iterative method, e.g. Newton-Raphson or Gauss-Seidel /1,2/.

From this example we see that with the knowledge of all node voltages we can compute the complete power flow in the network. From the node voltages we can calculate the line currents and line flows. With all line flows known we can calculate the net power injected at each bus, the bus power, by using the power balance equation.

2.3. Choice of state variables.

The state of a power system can be defined as:

> the minimum set of variables necesarry to compute all the other variables of interest in the network, given the network structure, network parameters and load characteristics.

From the previous section it will be clear that the node voltages form such a set. So for a network with N buses we have 2N - 1 state variables: the N voltage magnitudes V and the N - 1 angles δ. We'll have to define one voltage angle as reference (see previous section). An alternative way to represent the complex voltages is in rectangular coordinates: $U = e + jf$.
As state variables we then have the N e's and N - 1 f's.

When we drop the minimum in the definition it is possible to

form a different set of state variables: all complex line voltages plus at least one node voltage as reference to establish the voltage level in the system. This is of interest for one of the methods to be presented. But in the remainder of this paper we will stay with our set of nodal voltages.

One thing that should be stressed here, is that at the basis of this state definition lies our steady state assumption. This state does not give sufficient information to compute future states when system dynamics are involved. All the estimators to be presented want to estimate the above defined steady state. When they are following this slowly changing state in time they are sometimes called dynamic estimators or even dynamic static state estimators. This can be very confusing. A better word is maybe a tracking estimator: it keeps track of the slowly changing steady state.

3. GENERALIZED LEAST SQUARES ESTIMATION THEORY.

3.1. Introduction.

In this chapter a few general formula's, or better said first order approximations, will be derived from generalized least squares theory /6/. These formula's will be used in the next chapter with the presentation of the different estimation methods. Before we go on, first some definitions.

$\underline{y}$	: vector of measurements (real),	dimension m.
$\underline{x}$	: state vector (real),	dimension n.
$\hat{\underline{x}}$	: estimated state vector or estimate (real),	dimension n.
$\ldots^T$	: transpose of a vector or matrix.	

Table 3.1: Some definitions.

3.2. Estimation, general.

The basis of estimation theory is the measurement equation:

$$\underline{y} = g(\underline{x}) + \underline{e} \qquad (3\text{-}1)$$

The values we obtain from our measurement instruments won't be exact. This is represented by the error vector $\underline{e}$. The relationship between the exact or true values of the measurements and the state vector $\underline{x}$ is in general a non linear vector function g. In estimation we want to determine an estimate $\hat{\underline{x}}$ from our m noisy measurements which is best in some sense. We can distinguish two kinds of estimates: estimates based on $\underline{y}$ and estimates based on $\underline{y}$ and a priori knowledge of the estimate: $\bar{\underline{x}}$. It will be shown that the latter is a special case of the former.

3.3. Estimates based on $\underline{y}$.

Here "best in some sense" is defined as: $\hat{\underline{x}}$ is that value of $\underline{x}$ which minimizes the scalar criterion or loss function J.

$$J = \{\underline{y} - g(\underline{x})\}^T W \{\underline{y} - g(\underline{x})\} \qquad (3\text{-}2)$$

Note that $\underline{x}$ is not the actual state but the variabel with respect to which J has to be minimized given $\underline{y}$ and the function g. W in 3-2 is a m × m weighting matrix. A large value of W_{ii} means that we have much confidence in the value of the i-th measurement. In section 3.5. statistical interpretations of this weighting matrix will be treated. Putting the off diagonal ele-

ments equal to zero means that we assume no cross coupling between the measurements. It is possible to look upon J as a weighted error. For the minimum value of J the gradient with respect to $\underline{x}$ will be equal to $\underline{0}$.

$$\frac{\partial J}{\partial \underline{x}} = -2G^T(\underline{x})W\{\underline{y} - g(\underline{x})\} = \underline{0}$$
$$\text{with } G(\underline{x}_0) = \left.\frac{\partial g(\underline{x})}{\partial \underline{x}}\right|_{\underline{x} = \underline{x}_0} \tag{3-3}$$

$G(\underline{x}_0)$ is a m × n matrix: the Jacobian.
When we linearize equation 3-3 around a point $\underline{x}_0$ we'll get:

$$G^T(\underline{x}_0)W\{\underline{y} - g(\underline{x}_0) - G(\underline{x}_0)\{\underline{x} - \underline{x}_0\}\} = \underline{0} \tag{3-4}$$

In 3-4 $g(\underline{x})$ from 3-3 is approximated by its first order Taylor expansion around $\underline{x}_0$. This gives a first order approximation of our estimate given $\underline{y}$ and $\underline{x}_0$:

$$\boxed{\hat{\underline{x}} = \underline{x}_0 + \left(G^T(\underline{x}_0)WG(\underline{x}_0)\right)^{-1} G^T(\underline{x}_0)W\{\underline{y} - g(\underline{x}_0)\}} \tag{3-5}$$

3.4. Estimates based on $\underline{y}$ and $\bar{\underline{x}}$.

If we have $\bar{\underline{x}}$, an a priori value for our estimate, we can incorporate this in our criterion function:

$$J = \{\underline{y} - g(\underline{x})\}^T W_1 \{\underline{y} - g(\underline{x})\} + \{\bar{\underline{x}} - \underline{x}\}^T W_2 \{\bar{\underline{x}} - \underline{x}\} \tag{3-6}$$

W_1 is the same weighting matrix as W in section 3.3. With the weighting matrix W_2 we can express the confidence we have in our a priori estimate $\bar{\underline{x}}$. This problem can be brought back to the problem of the previous section by considering the a priori value of our estimate as extra measurements. We can define:

$$\underline{y}' = \begin{pmatrix} \underline{y} \\ \hline \bar{\underline{x}} \end{pmatrix} \quad g'(\underline{x}) = \begin{pmatrix} g(\underline{x}) \\ \hline \underline{x} \end{pmatrix} \quad W' = \left(\begin{array}{c|c} W_1 & 0 \\ \hline 0 & W_2 \end{array}\right) \quad G'(\underline{x}) = \begin{pmatrix} G(\underline{x}) \\ \hline I \end{pmatrix} \tag{3-7}$$

Substitution of these variables in 3-6 gives:

$$\hat{\underline{x}} = \underline{x}_0 + \left(G^T(\underline{x}_0)W_1G(\underline{x}_0) + W_2\right)^{-1}\left(G^T(\underline{x}_0)W_1\{\underline{y} - g(\underline{x}_0)\} + W_2\{\bar{\underline{x}} - \underline{x}_0\}\right) \tag{3-8}$$

By adding and subtracting $G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}$ the expression in the last pair of brackets of 3-6 can be written as:

$$G^T(\underline{x}_0)W_1\{\underline{y} - g(\underline{x}_0)\} + W_2\{\bar{\underline{x}} - \underline{x}_0\} =$$
$$= G^T(\underline{x}_0)W_1\{G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\} + \underline{y} - g(\underline{x}_0) - G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}\} +$$
$$+ W_2\{\bar{\underline{x}} - \underline{x}_0\} =$$
$$= \left(G^T(\underline{x}_0)W_1G(\underline{x}_0) + W_2\right)\{\bar{\underline{x}} - \underline{x}_0\} + G^T(\underline{x}_0)W_1\{\underline{y} - g(\underline{x}_0) -$$
$$- G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}\} \qquad (3\text{-}9)$$

Substitution of 3-9 in 3-8 gives the result:

$$\hat{\underline{x}} = \bar{\underline{x}} + \left(G^T(\underline{x}_0)W_1G(\underline{x}_0) + W_2\right)^{-1}G^T(\underline{x}_0)W_1\{\underline{y} - g(\underline{x}_0) - G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}\} \qquad (3\text{-}10)$$

Because 3-10 is derived as a special case of 3-5 we obtain the term $\underline{y} - g(\underline{x}_0) - G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}$. This term can be seen as a first order approximation of $\underline{y} - g(\bar{\underline{x}})$. The matrix to be inverted, $G^T W_1 G + W_2$, has dimension $n \times n$. For some applications it is useful to have 3-10 in a form which involves the inversion of a $m \times m$ matrix, for example when we want to use this formula for processing single measurements ($m = 1$). This can be done by applying the matrix inversion lemma:

$$\hat{\underline{x}} = \bar{\underline{x}} + W_2^{-1}G^T(\underline{x}_0)\left(G(\underline{x}_0)W_2^{-1}G^T(\underline{x}_0) + W_1^{-1}\right)^{-1}\{\underline{y} - g(\underline{x}_0) - G(\underline{x}_0)\{\bar{\underline{x}} - \underline{x}_0\}\} \qquad (3\text{-}11)$$

3.5. Statistical interpretations, relations with filter theory.

For the derivation of formula's 3-5, 3-10 and 3-11 we didn't make any assumptions concerning the statistics of the process. This section wants to give some interpretations if we do know something about the statistics of our process.

If we know that the error $\underline{e}$ is a zero mean random vector with covariance R then it is possible to determine the covariance of the estimation error $P_{\hat{\underline{x}}} = E\{\{\hat{\underline{x}} - \underline{x}\}\{\hat{\underline{x}} - \underline{x}\}^T\}$. E is the expectation operator. In the linear case $g(\underline{x}) = G\underline{x}$ where G is a constant matrix. Then formula 3-5 becomes:

$$\hat{\underline{x}} = \left(G^T WG\right)^{-1}G^T W\underline{y} \qquad (3\text{-}12)$$

Then $P_{\hat{\underline{x}}}$ is:

$$P_{\underline{\hat{x}}} = \left(G^T WG\right)^{-1} G^T WRWG \left(G^T WG\right)^{-1} \tag{3-13}$$

If W is chosen to be R^{-1} then 3-13 simplifies to:

$$P_{\underline{\hat{x}}} = \left(G^T R^{-1} G\right)^{-1} \tag{3-14}$$

The choice $W = R^{-1}$ means that the larger the uncertainty of a measurement (large element in R) the smaller the weight we place on the measurement. It can be shown that this choice gives the minimum mean square error in the estimate of all linear unbiased estimates.

In practice R will contain off diagonal terms. Voltage and power measurements make use of the same voltage transformer or the same communication channel, for example. For calculation purposes W is in most cases assumed to be diagonal. Equation 3-13 tells us something about the quality of the estimate in that case.

For the linear case equation 3-10 is:

$$\underline{\hat{x}} = \underline{\bar{x}} + \left(G^T W_1 G + W_2\right)^{-1} G^T W_1 \{\underline{y} - G\underline{\bar{x}}\} \tag{3-15}$$

By choosing $W_1 = R^{-1}$ and $W_2 = P_{\underline{\bar{x}}}^{-1}$, Where $P_{\underline{\bar{x}}}$ is the covariance of the a priori estimation error $E\{\{\underline{\bar{x}} - \underline{x}\}\{\underline{\bar{x}} - \underline{x}\}^T\}$, the inverse of W' from equation 3-7 is equal to the covariance of the extended observation error. Then it follows from 3-14 and 3-7 that:

$$P_{\underline{\hat{x}}} = \left(G^T R^{-1} G + P_{\underline{\bar{x}}}^{-1}\right)^{-1} \tag{3-16}$$

This can also be written as:

$$P_{\underline{\hat{x}}} = P_{\underline{\bar{x}}} - P_{\underline{\bar{x}}} G^T \left(GP_{\underline{\bar{x}}} G^T + R\right)^{-1} GP_{\underline{\bar{x}}} \tag{3-17}$$

Till here we have only talked about first and second order moments. When we assume Gaussian densities, which are completely specified by their first and second order moments, the minimum mean square estimate is also the maximum likelihood estimate. In that case the quantity $\underline{y} - G\underline{\bar{x}}$ in equation 3-15 is independent from $\underline{\bar{x}}$. This quantity, the innovation, contains the new information about the estimate.

The results in this section were derived for the linear case. They can also be derived from linear filtering theory /3,4,5/. For corresponding results in the non linear case see /4,6/.

4. STATE ESTIMATION METHODS IN POWER NETWORKS.

4.1. Introduction.

We start again with the measurement equation $\underline{y} = g(\underline{x}) + \underline{e}$. The next section deals with questions as: -what kind of measurements are available in power networks ?, -what is the vector function $g(\underline{x})$?, -how does $G(\underline{x})$ look like ?, - How do we form our weighting matrix W ? The measurements will be available at discrete points in time. Every t_k we will have a new measurement vector $\underline{y}_k = \underline{y}(t_k)$. We can divide the estimation methods in two classes. Class 1, treated in section 4.3., consists of methods in which the estimate $\underline{\hat{x}}_k$ of the state at t_k is only based on $\underline{y}_k$: the measurements obtained at t_k. In the methods of class 2, the subject of section 4.4., also some form of a priori knowledge about the estimate is taken into account. Most of the methods used in or proposed for practical applications belong to class 2. By means of this distinction it is easier to explain some of the properties of the proposed methods. In the following table some subscripts are defined which will be used in the formula's of this chapter.

k: time argument.
i: iteration counter for iterative methods.
j: measurement counter: y_j is the j-th measurement of the vector $\underline{y}$.

Table 4.1: Subscripts.

4.2. The measurement equation.

The following measurements are used in power networks:

type	measurement
1	voltage magnitude at a bus.
2	line current magnitude at a line terminal.
3	active line flow at a line terminal.
4	reactive line flow at a line terminal.
5	net injected active power at a bus.
6	net injected reactive power at a bus.

Table 4.2: Measurements in power networks.

The measurements of type 5 and 6 can be considered as a heritage of the load flow problem from chapter 2. They can also be con-

sidered as composite measurements. The net injected power at a bus is the sum of all outgoing line flows. Usually the measurements are telemetered to the operations center where they are further processed. When there is not sufficient transmission capacity available it is possible to lump the line flow measurements into injection measurements.

The explicit relationships necessary to compute $g(\underline{x})$ and $G(\underline{x})$ are given in the appendix for both the rectangular and the polar representation of the state vector. Before an estimation algorithm can use these relationships it must know the topology of the network and all the element values given in fig. 2.1 for all branches.

One thing that must be stressed is that the matrix G will be extremely sparse. When there are N buses and m measurements G will have dimension $m \times (2N - 1)$. N can be of the order of 50 or more while m usually is about 1.2 to 1.5 larger as the number of state variables. The quotient number of measurements/number of state variables is the redundancy of the measurement system. It should be larger than one, otherwise it does not make much sense to apply estimation methods (see chapter 1). It is very easy to understand why G is so sparse. For type 1 measurements only the two elements which have to do with the voltage at that bus are not equal to zero. For the measurements of type 2,3, and 4 only the four elements that have to do with the voltages of the two buses the branch is connecting are not equal to zero. For the measurements of type 5 and 6 the only non zero elements are the elements that have to do with the voltage of the bus itself and with the voltages of the buses to which that bus is directly connected by a branch. It is very important to exploit this sparsity in the calculations in order to save computing time.

The last element of the measurement equation, the error $\underline{e}$, can be modelled as:

$$e_j = \{\alpha(\text{full scale value}) + \beta(\text{reading})\}v_j \qquad (4\text{-}1)$$

where v_j is a random variable $N(0,1)$. The coefficient α represents the fixed errors of transducers, A/D converters, transmission

channels, etc., while β corresponds to the reading dependent errors. For power and injection measurements (types 3,4,5,6) α and β are of the order 0.01. For voltage and current measurements β is usually set equal to zero. β has to do with the watt and var transducers that are used for power measurements. Except for this random component there can also be a constant bias component in the error. If a large bias is suspected, this bias may also be estimated. We can choose the weighting factor for the j-th measurement (diagonal W matrix) as:

$$W_{jj} = \{\alpha(\text{full scale value}) + \beta(\text{reading})\}^{-1} \qquad (4\text{-}2)$$

4.3. Estimation methods based on $\underline{y}_k$ only.

The first method is just a straight forward application of formula 3-5. We start in some point $\underline{\hat{x}}_{k,0}$ and obtain a first order approximation of the estimate $\underline{\hat{x}}_{k,1}$. Using this point as our new reference point we do a new estimate, etc. till we converge to some point: our estimate $\underline{\hat{x}}_k$. The formula for this iterative method is:

$$\underline{\hat{x}}_{k,i+1} = \underline{\hat{x}}_{k,i} + \left(\underline{G}^T(\underline{\hat{x}}_{k,i})WG(\underline{\hat{x}}_{k,i})\right)^{-1} \underline{G}^T(\underline{\hat{x}}_{k,i})W\{\underline{y} - g(\underline{\hat{x}}_{k,i})\} \qquad (4\text{-}3)$$

This is just a steepest descent method. In each iteration we calculate the gradient of the loss function in the last obtained point and go in the negative direction of the gradient to obtain our new estimate. The gain factor is $\left(\underline{G}^T(\underline{\hat{x}}_{k,i})WG(\underline{\hat{x}}_{k,i})\right)^{-1}$. When we choose $W = R^{-1}$ then this gain is equal to the first order approximation of the covariance of the estimation error $(\underline{\hat{x}}_{k,i} - \underline{x})$. In the implementation of 4-3 we make use of the facts that $\underline{G}^T\underline{W}\underline{G}$ is a sparse, symmetric, positive definite matrix. We solve the system of linear equations:

$$\left(\underline{G}^T(\underline{\hat{x}}_{k,i})WG(\underline{\hat{x}}_{k,i})\right)\Delta\underline{\hat{x}}_{k,i+1} = \underline{G}^T(\underline{\hat{x}}_{k,i})W\Delta\underline{y}_{k,i+1} \qquad (4\text{-}4)$$

$$\text{with } \Delta\underline{\hat{x}}_{k,i+1} = \underline{\hat{x}}_{k,i+1} - \underline{\hat{x}}_{k,i}\ ,\ \Delta\underline{y}_{k,i+1} = \underline{y}_k - g(\underline{\hat{x}}_{k,i})$$

by means of a matrix square root method. The sparsity can be exploited in the updating of $\underline{G}^T\underline{W}\underline{G}$ and $\underline{G}^T\underline{W}$.

There are several simplifications possible to reduce the

computational effort. These can all be interpreted as different choices of the gain factor in the steepest descent method.

- Do not compute a new gain factor at each iteration but keep it constant after one or a few iterations. For the matrix square root method this means that we use the same triangularized version of $G^T W G$ during the remainder of the iterations. This gives a very large reduction in computing time (see chapter 5).
- Take only the diagonal elements of $G^T W G$ through the iterations. Put the off diagonal elements equal to zero.
- Hold the diagonal elements constant after a few iterations. The off diagonal elements are put equal to zero.
- Make use of the fact that the real power is mostly dependent on δ and the reactive power on V /1/. In this way we can write $G^T W G$ as a block diagonal matrix and split the problem into two problems of lower dimension. Note that this is not possible when there are line current measurements in $\underline{y}$ (compare equations A-45 till A-48 in the appendix).

All these simplifications will increase the number of iterations, but each iteration takes less time as one iteration of type 4-3. Primary references for this method: /9,10,11,24/. Also: /7,8,13, 17,21/.

The method of formula 4-3 may be classified as batch processing of the measurements. In 4-5 the formula's are given for the sequential processing of the measurements. If W is a diagonal matrix then, after processing all m measurements, the result will be exactly equal to the result obtained by batch processing. First some definitions:

$q_j(\underline{x})$	measurement function for the j-th measurement y_j.
$G_j(\underline{x}_0) = \left.\frac{\partial q_j(\underline{x})}{\partial \underline{x}}\right\|_{\underline{x}=\underline{x}_0}$	: the j-th row of $G(\underline{x}_0)$.
$W_{1,j}$	weighting factor for the j-th measurement (scalar).
$W_{2,j}$	weighting matrix for the a priori estimate when processing the j+1-st measurement (n × n matrix).

Table 4.3: Definitions for sequential measurement processing.

With these definitions we have the following set of formula's

to obtain $\underline{\hat{x}}_{k,i+1}$ from $\underline{\hat{x}}_{k,i}$:

$$\begin{aligned}
&\underline{\hat{x}}_{k,j+1} = \underline{\hat{x}}_{k,j} + K_{j+1}\{y_{k,j+1} - q_{j+1}(\underline{\hat{x}}_{k,i}) - G_{j+1}(\underline{\hat{x}}_{k,i})(\underline{\hat{x}}_{k,j} - \underline{\hat{x}}_{k,i})\} \\
&K_{j+1} = W_{2,j}^{-1} G_{j+1}^T(\underline{\hat{x}}_{k,i})\left(G_{j+1}(\underline{\hat{x}}_{k,i}) W_{2,j}^{-1} G_{j+1}^T(\underline{\hat{x}}_{k,i}) + W_{1,j+1}^{-1}\right)^{-1} \\
&W_{2,j+1}^{-1} = W_{2,j}^{-1} - K_{j+1} G_{j+1}(\underline{\hat{x}}_{k,i}) W_{2,j}^{-1} \qquad (4\text{-}5) \\
&\text{with } \underline{\hat{x}}_{k,0} = \underline{\hat{x}}_{k,i} \ , \ W_{2,0}^{-1} = kI \text{ (k large)}
\end{aligned}$$

If W in 4-3 is a diagonal matrix then $G^T W G$ can be written as : $G_1^T W_{1,1} G_1 + G_2^T W_{1,2} G_2 + \ldots\ldots + G_m^T W_{1,m} G_m$. By repeatedly applying the matrix inversion lemma on the in this way written matrix $G^T W G$ it is possible to derive 4-5 from 4-3. See also /5/. From this derivation it follows directly that $W_{2,0} = \underline{0}$. Then $W_{2,0}^{-1}$ is infinite. In practice this is approximated by choosing this matrix to be a diagonal matrix with large diagonal elements.

Note the similarity of 4-5 with 3-11 and 3-17 with $\underline{\bar{x}} = \underline{\hat{x}}_{k,j}$, $\underline{x}_0 = \underline{\hat{x}}_{k,i}$, $W_2^{-1} = W_{2,j}^{-1}$ and $P_{\underline{\hat{x}}} = W_{2,j+1}^{-1}$. We can interpret $\underline{\hat{x}}_{k,j}$ as the estimate obtained after using the information of j measurements. $W_{2,j+1}^{-1}$ is then the covariance of the estimation error after processing j + 1 measurements. This is equal to the covariance of the a priori estimation error when processing the j + 2-nd measurement. $W_{2,0}^{-1}$ is infinite means that we don!t know anything about the a priori estimate before processing the first measurement. This is exactly the same as in 4-3. We only use the information of the measurements obtained at t_k.

When we interpret $\underline{\hat{x}}_{k,j}$ as the estimate after processing j measurements we can ask ourselves: wouldn't it be better to relinearize at our new estimate as soon as we have it available. In other words, relinearize after each measurement. When we do this we get in the i + 1-st iteration (cf. 3-11 with $\underline{\bar{x}} = \underline{x}_0 = \underline{\hat{x}}_{k,j}$):

$$\begin{aligned}
&\underline{\hat{x}}_{k,j+1} = \underline{\hat{x}}_{k,j} + K_{j+1}\{y_{k,j+1} - q_{j+1}(\underline{\hat{x}}_{k,j})\} \\
&K_{j+1} = W_{2,j}^{-1} G_{j+1}^T(\underline{\hat{x}}_{k,j})\left(G_{j+1}(\underline{\hat{x}}_{k,j}) W_{2,j}^{-1} G_{j+1}^T(\underline{\hat{x}}_{k,j}) + W_{1,j+1}^{-1}\right)^{-1} \\
&W_{2,j+1}^{-1} = W_{2,j}^{-1} - K_{j+1} G_{j+1}(\underline{\hat{x}}_{k,j}) W_{2,j}^{-1} \qquad (4\text{-}6) \\
&\text{with } \underline{\hat{x}}_{k,0} = \underline{\hat{x}}_{k,i} \ , \ W_{2,0}^{-1} = kI \text{ (k large)}
\end{aligned}$$

A disadvantage seems to be the updating of g and G with every measurement, but only the elements that have to do with the j + 1-st measurement need updating. What becomes important with this method is the order of measurement processing. The ordering of the measurements should be such that each following measurement contains as much new information about the estimate as possible.

Also for this method simplifications are proposed. These all have to do with the choice of the weighting matrix W_2.

- Diagonalize $W_{2,1}^{-1}$. This gives a considerabel saving in the updating of W_2^{-1} and the evaluation of K.
- Use a constant W_2^{-1}. This only eliminates the updating of W_2^{-1}. In the computation of K we still have to handle the complete m × m matrix.
- Use a constant diagonal W_2^{-1}.

All these simplifications mean that we place less and/or wrong weights on the a priori estimate $\underline{\hat{x}}_{k,j}$. The first one is the most promising for practical application.

Primary references for this method: /13,14,15/. Also: /7,11,21,22/.

This concludes the description of the estimation methods based on $\underline{y}_k$ only. The two presented methods, the iterative method with batch processing and the method with sequential processing and relinearization after each measurement will be extended in the next section.

4.4. Estimation methods based on $\underline{y}_k$ and a priori knowledge of $\underline{\hat{x}}_k$.

The first three methods of this section can be seen as an extension of the ones presented in the previous section. The last method is different from all others. It makes use of the fact that the relation between line flows and line voltages (line voltage = the voltage difference across a line) is much more linear than the relation between line flows and bus voltages. All methods of this section make use of the fact that the state is changing slowly by putting as an initial estimate for $\underline{\hat{x}}_{k+1}$: $\underline{\hat{x}}_{k+1} = \underline{\hat{x}}_k$. Also in this section, the inverse of the weighting matrix W_2 will be called P and W_1^{-1} will be called R. Only in special cases this P, respectively R, matrix may be interpreted as

the covariance matrix of the a priori estimation error, respectively measurement error.

The first method is just the batch method of 4-3. We start with $\underline{\hat{x}}_k$ as our initial estimathe for $\underline{\hat{x}}_{k+1}$ and hope that one iteration will be sufficient to obtain our new estimate:

$$\underline{\hat{x}}_{k+1} = \underline{\hat{x}}_k + \left(G^T(\underline{\hat{x}}_k) R^{-1} G(\underline{\hat{x}}_k)\right)^{-1} G^T(\underline{\hat{x}}_k) R^{-1} \{\underline{y}_{k+1} - g(\underline{\hat{x}}_k)\} \qquad (4\text{-}7)$$

Compare this with 3-10 with $\underline{\bar{x}} = \underline{x}_0 = \underline{\hat{x}}_k$ and $W_2 = \underline{0}$. Also here we can keep $G^T W G$ constant. Here constant means constant during a certain period of time. For example calculate a new estimate every 30 seconds and a new triangularized version of $G^T R^{-1} G$ every 20 minutes. Note that the right-hand side of equation 4-4 is recomputed at every t_k. A decision for relinearization can be based on the value of the loss function 3-2 and/or on the distance between the current estimate and the linearization point. Of course it is possible to iterate the same measurement set more than once, especially when we relinearize. Compare this with the iterated extended Kalman filter /4/.

It is possible to formulate a sequential version of 4-7 making use of either 4-5 or 4-6. Using 4-5 gives:

$$\begin{aligned}
&\underline{\hat{x}}_{k+1,j+1} = \underline{\hat{x}}_{k+1,j} + K_{j+1}\{\underline{y}_{k+1,j+1} - g(\underline{\hat{x}}_{k+1,j})\} \\
&K_{j+1} = P_j G_{j+1}(\underline{\hat{x}}_k)\left(G_{j+1}(\underline{\hat{x}}_k) P_j G_{j+1}^T(\underline{\hat{x}}_k) + R_{j+1}\right)^{-1} \\
&P_{j+1} = P_j - K_{j+1} G_{j+1}(\underline{\hat{x}}_k) P_j \\
&\text{with } \underline{\hat{x}}_{k+1,0} = \underline{\hat{x}}_k \ , \ P_0 = kI \ (k \text{ large})
\end{aligned} \qquad (4\text{-}8)$$

In the second method we choose $\underline{\bar{x}}$ of 3-10 as $\underline{\hat{x}}_k$ and P_k^{-1} as the weighting matrix for our a priori estimate. This gives:

$$\underline{\hat{x}}_{k+1} = \underline{\hat{x}}_k + \left(G^T(\underline{\hat{x}}_k) R^{-1} G(\underline{\hat{x}}_k) + P_k^{-1}\right)^{-1} G^T(\underline{\hat{x}}_k) R^{-1} \{\underline{y}_{k+1} - g(\underline{\hat{x}}_k)\} \qquad (4\text{-}9)$$

The sequential version of 4-9 is the same as 4-8 only with $P_{k+1,0} = P_k$. Also for this sequential method the simplifications described in the previous section: diagonal P, constant P, and constant diagonal P, are possible. A problem with this method

is that P^{-1}, or $\left(G^T R^{-1} G + P^{-1}\right)^{-1}$ from 4-9, tends to become singular. This is caused by putting to much weight on the a priori estimate.

The previous two methods can be considered as special cases of the third method. This method makes use of the following "model" for the state:

$$\underline{x}_{k+1} = \underline{x}_k + \underline{v}_k \tag{4-10}$$

$\underline{v}_k$ is a random vector with zero mean and covariance Q_k. This model gives $\underline{\hat{x}}_k$ as the expected value for $\underline{\hat{x}}_{k+1}$ and $P_k + Q_k$ as the a priori expected value for P_{k+1}.
With these values 3-11 becomes:

$$\begin{aligned}
\underline{\hat{x}}_{k+1} &= \underline{\hat{x}}_k + K_{k+1}\left\{\underline{y}_{k+1} - g(\underline{\hat{x}}_k)\right\} \\
K_{k+1} &= \left(P_k + Q_k\right) G^T(\underline{\hat{x}}_k)\left(G(\underline{\hat{x}}_k)\left(P_k + Q_k\right) G^T(\underline{\hat{x}}_k) + R\right)^{-1} \\
P_{k+1} &= \left(P_k + Q_k\right) - K_{k+1} G(\underline{\hat{x}}_k)\left(P_k + Q_k\right)
\end{aligned} \tag{4-11}$$

Q can be looked upon as a kind of memory measure. The larger Q, the larger the covariance of the state noise, the less important the value of $\underline{\hat{x}}_k$ is in the evaluation of $\underline{\hat{x}}_{k+1}$. The two extreme cases $Q = \underline{0}$ and $Q = \underline{\infty}$ correspond to 4-7 and 4-9 respectively. Compare this with what is said after equation 4-5.
Usually Q will be chosen to be a diagonal matrix. It remains a problem to determine good values for the elements of Q. They must have something to do with the maximum rate of change of the state. An other way to look at Q is as artificially introduced noise in the previous method to prevent P from getting singular. Then Q should be determined experimentally.

The sequential version of this method with relinearization after each measurement is given below:

$$\begin{aligned}
\underline{\hat{x}}_{k+1,j+1} &= \underline{\hat{x}}_{k+1,j} + K_{k+1,j+1}\left\{\underline{y}_{k+1,j+1} - g(\underline{\hat{x}}_{k+1,j})\right\} \\
K_{k+1,j+1} &= P_{k+1,j} G_{j+1}^T(\underline{\hat{x}}_{k+1,j})\left(G_{j+1}(\underline{\hat{x}}_{k+1,j}) P_{k+1,j} G_{j+1}^T(\underline{\hat{x}}_{k+1,j}) + R_{j+1}\right)^{-1} \\
P_{k+1,j+1} &= P_{k+1,j} - K_{k+1,j+1} G_{j+1}(\underline{\hat{x}}_{k+1,j}) P_{k+1,j}
\end{aligned} \tag{4-12}$$

with $\underline{\hat{x}}_{k+1,0} = \underline{\hat{x}}_k$, $P_{k+1,0} = P_k + Q_k$

Also here the constant P, diagonal P, and constant diagonal P variations are possible. The version with diagonal P is one of the methods in actual use /15,16/.
References for the first method 4-7 are: /9,10,11,12/and/24/. Primary references for the second method are/13,14/and the already mentioned/15/and/16/. Also:/7,11,21,22,23,25/.

The last method of this section is also a method that is in use now /20/. As said before this method builds on the almost linear relation between line voltage and line flow. The complex line flow $S_{ab} = P_{ab} + jQ_{ab}$ (see fig. 2.1) is equal to:

$$S_{ab} = P_{ab} + jQ_{ab} = U_a I_{ab}^{*}$$
$$= U_a \left(\frac{(U_a - U_b)^{*}}{z_{ab}^{*}} + y_{aa}^{*} U_a^{*} \right) \quad (*: \text{ complex conjugate}) \qquad (4\text{-}13)$$

U_a appears quadratic in 4-13 while $U_a - U_b$, the line voltage, appears linearly. Further we have to realize that with a change in the state of the power network the procentual change of the line voltage will be much larger than the procentual change in the bus voltage. This method uses only line flow and at least one voltage measurement for references purposes. From the line flow measurements(it is assumed that both real and reactive line flow are available) new measurement variables are calculated: the line voltages. These new measurements are used to estimate the state variables. Since the new measurements are dependent on the state variables the estimate has to be obtained in an iterative way.

We start with our original loss function:

$$J = \{\underline{y} - g(\underline{x})\}^{*T} W \{\underline{y} - g(\underline{x})\} \qquad (W \text{ diagonal}) \qquad (4\text{-}14)$$

Note that the scalar loss function is defined for complex measurements. When we call our new measurements $\underline{z}$ and our new measurement function $g'(\underline{x})$ then we can write the loss function as:

$$J = \{\underline{z} - g'(\underline{x})\}^{T} D \{\underline{z} - g'(\underline{x})\}^{*} \qquad (4\text{-}15)$$

What is the new weighting matrix D ?
First of all the relation between the line voltages and the bus voltages is linear:

$$g'(\underline{x}) = A\underline{x} \qquad (4\text{-}16)$$

The relation between $\underline{y}$ and $\underline{z}$ (see 4-13) is in vector form:

$$\underline{Y} = H^{*}(\underline{z}^{*} + \underline{k}^{*}) \qquad (4\text{-}17)$$

with $H_j = \dfrac{U_j}{Z_{ab,j}}$, H diagonal matrix, $k_j = Z_{ab,j}\, Y_{aa,j}\, U_j$

U_j is the bus voltage at the line terminal where we take our line flow measurement. From 4-17 we can formulate the relationship between $g(\underline{x})$ and $g'(\underline{x})$:

$$g(\underline{x}) = H^{*}(g'(\underline{x})^{*} + k^{*}) \qquad (4\text{-}18)$$

Substitution of 4-17 and 4-18 in 4-14 gives:

$$\begin{aligned} J &= \{H(\underline{z} - g'(\underline{x})\}^{T} W\{H^{*}(\underline{z} - g'(\underline{x}))^{*}\} \\ &= \{\underline{z} - g'(\underline{x})\}^{T} HWH^{*}\{\underline{z} - g'(\underline{x})\}^{*} \end{aligned} \qquad (4\text{-}19)$$

Because W is a diagonal matrix D is also a diagonal matrix:

$$D_j = W_j \left| \frac{U_j}{Z_{ab,j}} \right|^2 \qquad (4\text{-}20)$$

Now the assumption is made that D is constant. For U_j we use the nominal value. Here we make use of the relatively small changes in bus voltages that occur under normal operation.
The relation between $\underline{z}$ and $\underline{y}$ is:

$\underline{z} = H^{-1}\underline{y}^{*} - \underline{k}$ or in component form:

$$z_j = \frac{Z_{ab,j}}{x_j}\, y_j^{*} - Z_{ab,j}\, Y_{aa,j}\, x_j \qquad (4\text{-}21)$$

Since it is impossible to determine the bus voltages when we only know the differences between the bus voltages we need at least one voltage measurement to establish the voltage level in our system. This is obtained by splitting $g'(\underline{x}) = A\underline{x}$ in:

$$A\underline{x} = A_g \underline{E}_g + B\underline{E} \qquad (4\text{-}22)$$

$\underline{E}_g$ is our vector with known reference voltages (usually one). $\underline{E}$ is the vector of unknown voltages we want to estimate. With these formula's we can state our estimation problem as minimize J with respect to $\underline{E}$

$$J = \{\underline{z} - A_g \underline{E}_g - B\underline{E}\}^{T} D\{\underline{z} - A_g \underline{E}_g - B\underline{E}\}^{*} \qquad (4\text{-}23)$$

The solution is the linear least squares estimate of 3-15:

$$\hat{\underline{E}} = \left(\underline{B}^T DB\right)^{-1} \underline{B}^T D\{\underline{z} - A_g \underline{E}_g\} \qquad (4\text{-}24)$$

Note that B is only dependent on the network structure and only contains elements that are either 1 or 0. $\underline{B}^T DB$ can be saved in triangularized form as long as the topology of the network does not change!
To obtain our estimate at t_{k+1} from the one at t_k we can now formulate the following iterative method:

$$\underline{z}_{k+1,i+1} = H^{-1}_{k+1,i+1} \underline{Y}^{*}_{k+1} - \underline{k}_{k+1,i+1}$$

$$\hat{\underline{E}}_{k+1,i+1} = \left(\underline{B}^T DB\right)^{-1} \underline{B}^T D\{\underline{z}_{k+1,i+1} - A_g \underline{E}_g\}$$

$$\underline{x}^T_{k+1,i+1} = (\underline{E}^T_{k+1,i+1} \mid \underline{E}^T_g)$$

with $\underline{x}_{k+1,0} = \underline{x}_k$ (4-25)

$H^{-1}_{k+1,i+1}$ is a diagonal matrix with elements:

$$H^{-1}_{k+1,i+1,j} = \frac{z_{ab,j}}{x_{k+1,i+1,j}}$$

$\underline{k}_{k+1,i+1}$ is a vector with elements:

$$k_{k+1,i+1,j} = z_{ab,j}\, Y_{aa,j}\, x_{k+1,i+1,j}$$

One can say that this method exploits the special properties of power networks. It is also possible to look at this method as a change of state variables as opposed to a change of measurement variables. The calculation of the old state variables (bus voltages) from the new ones (line voltages plus one reference voltage) includes the use of pseudo inverses. This probably leads to the same formula's. This method could also have been treated in section 4.3. Since it is so different from the other presented methods and since it uses $\underline{x}_k$ as initital estimate for $\underline{x}_{k+1}$ just like the first method of this section it was preferred to treat it here.
References for this method: /18,19/and/20/.

4.5. Other methods.

Of course it is possible to formulate still many more methods. All the methods presented use a first order approximation of the

loss function. One can think of a Fletcher-Powell method using two terms in the Taylor expansion of J or of some random search method based directly on the loss function. Also one can try to do real dynamic estimation. But all these other methods run into difficulties when we try to implement them due to the high dimensionality and/or the non linear nature of the problem. The presented methods are a representative sample of the ideas on state estimation in power networks as I found them in the literature.

5. THREE PROMISING METHODS FOR FURTHER COMPARISON.

5.1. Introduction.

The methods presented in chapter 4 can be divided into three families. The first and second family consist of methods based directly on the estimation methods presented in chapter 3. In the first family batch processing of measurements is used while in the second family the measurements are processed sequentially. The third family makes use of the physical behavior of power networks. It consists in fact of one member: the line flow method.

If we want to choose one of the methods from chapter 4 for application in a real situation we need some comparative data. At the moment there are only a few comparisons available /7,21/. /7/ is more a demonstration of the feasibility of these methods and gives almost no comparative data. /21/ compares a generalized load flow method, a least squares method (4-7) and a tracking method (the sequential version of 4-7, given in 4-8). So it does not cover all our families.

When we want to make comparisons one of the things to be taken into consideration is the computing time each algorithm needs. Based on a rough estimate of the needed computing time a choice is made of one representative from each family that looks promising for further comparison since it is impossible to compare all methods. Section 5.2. presents the three methods and section 5.3. gives the estimated computing times. The actual comparison and the results are going to be the subject of following reports.

5.2. The three methods.

Method A: Batch processing with constant gain.

This is the method given in 4-7 and implemented as in 4-4:

$$\left(G^T(\underline{x}_l)R^{-1}G(\underline{x}_l)\right)\Delta\underline{x}_{k+1} = G^T(\underline{x}_k)R^{-1}\Delta\underline{y}_{k+1}$$

$$\Delta\underline{x}_{k+1} = \underline{x}_{k+1} - \underline{x}_k \qquad \Delta\underline{y}_{k+1} = \underline{y}_{k+1} - g(\underline{x}_k) \qquad (5\text{-}1)$$

$\underline{x}_l$: last linearization point.

In this method one of the things worth investigating is how to

make the relineariztion decision.

Method B: Sequential processing with relinearization after each measurement.

This is exactly the method presented in 4-12 with diagonal P:

$$\underline{x}_{k+1,j+1} = \underline{x}_{k+1,j} + K_{k+1,j+1}\{Y_{k+1,j+1} - g(\underline{x}_{k+1,j})\}$$

$$K_{k+1,j+1} = P_{k+1,j} G_{j+1}^T(\underline{x}_{k+1,j})\left(G_{j+1}(\underline{x}_{k+1,j}) P_{k+1,j} G_{j+1}^T(\underline{x}_{k+1,j}) + R_{j+1}\right)^{-1}$$

$$P_{k+1,j+1} = P_{k+1,j} - \mathrm{diag}\{K_{k+1,j+1} G_{j+1}(\underline{x}_{k+1,j}) P_{k+1,j}\} \qquad (5\text{-}2)$$

with $\underline{x}_{k+1,0} = \underline{x}_k$, $P_{k+1,0} = P_k + Q_k$

Here one of the interesting things is the choice of Q. It should be interesting to have a V,δ representation of the state vector and different values for the corresponding elements in Q. Another one is the relationship between this P and the real covariance of the estimation error.

Method C: The line flow method.

This is the last presented method in 4-25.

Note that here all variables are complex.

$$\underline{z}_{k+1,i+1} = H_{k+1,i+1}^{-1} Y_{k+1}^{*} - \underline{k}_{k+1,i+1}$$

$$H_{k+1,i+1,j}^{-1} = \frac{Z_{ab,j}}{\underline{x}_{k+1,i+1,j}}$$

$$k_{k+1,i+1,j} = Z_{ab,j} Y_{aa,j} \underline{x}_{k+1,i+1,j} \qquad (5\text{-}3)$$

$$\hat{\underline{E}}_{k+1,i+1} = \left(B^T D B\right)^{-1} B^T D\{\underline{z}_{k+1,i+1} - A_g \underline{E}_g\}$$

$$\underline{x}_{k+1,i+1}^T = (\underline{E}_{k+1,i+1}^T \,|\, \underline{E}_g^T) \qquad \underline{x}_{k+1,0} = \underline{x}_k$$

The influence of the choice of reference voltage and the influence of the error in this voltage measurement on the estimate are a few interesting things to look at. An other one is the inclusion of other measurements in this method.

5.3. Estimated computing times per iteration.

The estimated computing times are given in table 5.1. For each multiplication or addition a time of 5 μsecs is taken. Only line flow measurements are considered. This is done to make a comparison with method C possible. This is not so far from the general case since voltage and line current measurements require less operations while bus powers require more. It is assumed that as many variables as possible are updated at the same time. In other words multiplications with zero are avoided.

	n = 20 m = 30		n = 50 m = 100		n = 100 m = 200		n = 200 m = 300	
	msec	msec	msec	msec	sec	msec	sec	msec
method A	26	9	260	38	1.83	125	13.9	450
method B		9		23		45		90
method C		6		31		112		424

Table 5.1: Estimated computing times.

It is very interesting to see the amount of time saved by using a constant gain in method A. The right-hand times with method A are the times using a constant gain. Method A and C can still be improved if we exploit the sparsity in the triangularization process. This is in fact what is done in method B. There we treat each measurement separately and change only the elements that need to be changed.

6. REFERENCES.

Power systems:

1. Elgerd,O.I.; Electrical Energy Systems Theory, McGraw Hill, New York, 1971, Chapter 1,3,7.
2. Stagg,G.W. and El-Abiad,A.H.; Computer Methods in Power Systems Analysis, McGraw Hill, New York, 1968, Chapter 8.

Estimation theory:

3. Åström,K.J.; Introduction to Stochastic Control Theory, Academic Press, New York, 1970, pp 210-233.
4. Jazwinski,A.H.; Stochastic Processes and Filtering Theory, Academic Press, New York, 1970, pp 194-218 and 266-281.
5. Sorenson,H.W.; Kalman Filtering Techniques. In: Leondes,C.T.(editor); Advances in Control Systems, Vol. 3, Academic Press, New York, 1966, pp 219-292.
6. Swerling,P.; Modern State Estimation Methods from the Viewpoint of the Method of Least Squares, IEEE Transactions on Automatic Control, Vol. AC-16, No 6, December 1971, pp 707-719.

State estimation methods in power networks:

7. Handschin,E.; Real Time Data Processing Using State Estimation in Electric Power Systems. In: Handschin,E.(editor); Real Time Control of Electric Power Systems, Elsevier, Amsterdam, 1972, Chapter 2.
8. Gamm,A.Z.; Power System Instantaneous State Estimation as a Problem in Nonlinear Programming, Elektrichestvo, Vol. 3 (1972)No 9, 1-7-1972. In English in: Electric Technology USSR, Vol. 3(1972), Selected Papers from Elektrichestvo, Nos 7-8-9, 1972, Pergamom Press, London, March 1973.
9. Schweppe,F.C.; Power System Static-State Estimation, Part I: Exact Model, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 1, January 1970, pp 120-125.
10. Schweppe,F.C. and Rom,D.B.; Power System Static-State Estimation, Part II: Approximate Model, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 1, January 1970, pp 125-130.

11. Schweppe,F.C.; Power System Static-State Estimation, Part III: Implementation, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 1, January 1970, pp 130-135.

12. Masiello,R.D. and Schweppe,F.C.; A Tracking Static State Estimator, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-90, No 3, May/June 1971, pp 1025-1033.

13. Larson,R.E., Tinney,W.F., Peschon,J.; State Estimation in Power Systems, Part I: Theory and Feasibility, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 3, March 1970, pp 345-352.

14. Larson,R.E., Tinney,E.F., Hajdu,L.P., Piercy,D.S.; State Estimation in Power Systems, Part II: Implementation and Applications, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 3, March 1970, pp 353-363.

15. Debs,A.E., Larson,R.E.; A Dynamic Estimator for Tracking the State of a Power System, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 7, September/October 1970, pp 1670-1676.

16. Debs,A.E., Larson,R.E., Hajdu,L.P.; On-Line Sequential State Estimation for Power Systems, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/3.

17. Stagg,G.W., Dopazo,J.F., Klitin,O.A., VanSlyck,L.S.; Techniques for the Real-Time Monitoring of Power Systems Operations, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 4, April 1970, pp 545-555.

18. Dopazo,J.F., Klitin,O.A., Stagg,G.W., VanSlyck,L.S.; State Calculation of Power Systems from Line Flow Measurements, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-89, No 7, September/October 1970, pp 1698-1708.

19. Dopazo,J.F., Klitin,O.A., VanSlyck,L.S.; State Calculation of Power Systems from Line Flow Measurements, Part II, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, No 1, January/February 1972, pp 145-151.

20. Dopazo,J.F., Klitin,O.A., Sasson,A.M., VanSlyck,L.S.; Real-Time Load Flow for the AEP System, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/8,

21. Ariatti,F., Castagnoli,V., Marzio,L., Ricci,P.; Methods for Electric Power System State Estimation: Comparative Analysis of Computing Simulations Based upon Generalized Load Flow, Tracking, and Least Square Methods, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/10.

22. Bubenko,J.A., Petterson,L.O.; State Estimation in Electric Power Systems Using Filtering Techniques, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, 1973, paper 3.3/6.

23. Kobayashi,H., Ninagawa,S., Tsujujama,K., Tamura,Y., Narita,S.; Dynamic State Estimation Techniques Applied to Security Control Problems, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/2.

24. Geominne,P., Van Miegroet,P., Waha,J.P.; Le Projet Descartes, Contributions à l'Augmentation de la Securité dans les Reseaux de Transport, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/9.

25. Wolters,E.J.; Results of a Numerical and Hybrid Simulation of a Power System Tracking State Estimator with an Emphasis on Line Current Measurements, Proceedings of the Fourth Power Systems Computation Conference, Vol. 3, Department of Electrical and Electronic Engineering, Queen Mary College, University of London, London, 1973, paper 3.3/3.

APPENDIX: RELATIONSHIPS BETWEEN MEASUREMENT AND STATE VARIABLES.

This appendix contains the necessary formula's to compute $g(\underline{x})$ and $G(\underline{x})$ in both rectangular and polar coordinates. The following variables are used:

- the complex bus voltage $U_a = e_a + jf_a = V_a e^{j\delta_a}$
- the complex line current I_{ab}
- the real line flow P_{ab}
- the reactive line flow Q_{ab}
- the complex line flow $S_{ab} = P_{ab} + jQ_{ab}$
- the complex injected bus current I_a
- the real injected bus power P_a
- the reactive injected bus power Q_a
- the complex injected bus power $S_a = P_a + jQ_a$
- the line series admittance $Y_{ab} = G_{ab} + jB_{ab} = |Y_{ab}|e^{j\theta_{ab}}$
- the line shunt admittance $y_{ab} = g_{ab} + jb_{ab} = |y_{ab}|e^{j\theta'_{ab}}$

The line series admittance is the inverse of the line series impedance of fig. 2.1: $Y_{ab} = \frac{1}{Z_{ab}}$. If there is no branch between A and B then $Y_{ab} = Y_{ba} = 0$. The line shunt admittance y_{ab} corresponds to the shunt admittance Y_{aa} of fig. 2.1. It is the shunt admittance of the line from A to B at terminal A. The measurement types used are those of table 4.1.

Rectangular coordinates.

type 1: Voltage magnitude at bus A.

$$V_a = \sqrt{e_a^2 + f_a^2} \qquad \text{(A-1)}$$

$$\frac{\partial V_a}{\partial e_a} = \frac{e_a}{\sqrt{e_a^2 + f_a^2}} \qquad \text{(A-2)}$$

$$\frac{\partial V_a}{\partial f_a} = \frac{f_a}{\sqrt{e_a^2 + f_a^2}} \qquad \text{(A-3)}$$

type 2: Line current magnitude at line terminal A.

$$I_{ab} = (U_b - U_a)Y_{ab} + U_a y_{ab} \tag{A-4}$$

$$I_{ab} = c_{ab} + jd_{ab}$$

$$c_{ab} = (e_a - e_b)G_{ab} - (f_a - f_b)B_{ab} + e_a g_{ab} - f_a b_{ab} \tag{A-5}$$

$$d_{ab} = (f_a - f_b)G_{ab} + (e_a - e_b)B_{ab} + f_a g_{ab} + e_a b_{ab}$$

$$|I_{ab}| = \sqrt{c_{ab}^2 + d_{ab}^2} \tag{A-6}$$

$$\frac{\partial |I_{ab}|}{\partial e_a} = \frac{c_{ab}(G_{ab} + g_{ab}) + d_{ab}(B_{ab} + b_{ab})}{\sqrt{c_{ab}^2 + d_{ab}^2}} \tag{A-7}$$

$$\frac{\partial |I_{ab}|}{\partial e_b} = \frac{-c_{ab} G_{ab} - d_{ab} B_{ab}}{\sqrt{c_{ab}^2 + d_{ab}^2}} \tag{A-8}$$

$$\frac{\partial |I_{ab}|}{\partial f_a} = \frac{-c_{ab}(B_{ab} + b_{ab}) + d_{ab}(G_{ab} + g_{ab})}{\sqrt{c_{ab}^2 + d_{ab}^2}} \tag{A-9}$$

$$\frac{\partial |I_{ab}|}{\partial f_b} = \frac{c_{ab} B_{ab} - d_{ab} G_{ab}}{\sqrt{c_{ab}^2 + d_{ab}^2}} \tag{A-10}$$

type 3 and 4: Active and reactive line flow at line terminal A.

$$S_{ab} = U_a I_{ab}^{*} = (e_a + jf_a)(c_{ab} - jd_{ab}) \tag{A-11}$$

$$P_{ab} = e_a c_{ab} + f_a d_{ab} \tag{A-12}$$

$$Q_{ab} = f_a c_{ab} - e_a d_{ab} \tag{A-13}$$

$$\frac{\partial P_{ab}}{\partial e_a} = (2e_a - e_b)G_{ab} + f_b B_{ab} + 2e_a g_{ab} \tag{A-14}$$

$$\frac{\partial P_{ab}}{\partial e_b} = -e_a G_{ab} - f_a B_{ab} \tag{A-15}$$

$$\frac{\partial P_{ab}}{\partial f_a} = (2f_a - f_b)G_{ab} - e_b B_{ab} + 2f_a g_{ab} \tag{A-16}$$

$$\frac{\partial P_{ab}}{\partial f_b} = -f_a G_{ab} + e_a B_{ab} \tag{A-17}$$

$$\frac{\partial Q_{ab}}{\partial e_a} = f_b G_{ab} + (-2e_a + e_b) B_{ab} - 2e_a b_{ab} \qquad \text{(A-18)}$$

$$\frac{\partial Q_{ab}}{\partial e_b} = -f_a G_{ab} + e_a B_{ab} \qquad \text{(A-19)}$$

$$\frac{\partial Q_{ab}}{\partial f_a} = -e_b G_{ab} + (-2f_a + f_b) B_{ab} - 2f_a b_{ab} \qquad \text{(A-20)}$$

$$\frac{\partial Q_{ab}}{\partial f_b} = e_a G_{ab} + f_a B_{ab} \qquad \text{(A-21)}$$

type 5 and 6: Injected active and reactive power at bus A.

$$S_a = U_a I_a^* = U_a \sum_{\substack{i=1 \\ i\neq a}}^{N} I_{ai}^* = \sum_{\substack{i=1 \\ i\neq a}}^{N} S_{ai} \qquad \text{(A-22)}$$

$$I_a = c_a + jd_a$$

$$c_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} c_{ai} \qquad \text{(A-23)}$$

$$d_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} d_{ai}$$

$$P_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} P_{ai} \qquad \text{(A-24)}$$

$$Q_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} Q_{ai} \qquad \text{(A-25)}$$

$$\frac{\partial P_a}{\partial e_i} = -e_a G_{ai} - f_a B_{ai} \qquad i\neq a$$

$$= e_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (G_{aj} + g_{aj}) + f_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (B_{aj} + b_{aj}) + c_a \qquad i=a \qquad \text{(A-26)}$$

$$\frac{\partial P_a}{\partial f_i} = -f_a G_{ai} + e_a B_{ai} \qquad i \neq a$$

$$= f_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (G_{aj} + g_{aj}) - e_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (B_{aj} + b_{aj}) + d_a \qquad i=a \qquad \text{(A-27)}$$

$$\frac{\partial Q_a}{\partial e_i} = -f_a G_{ai} + e_a B_{ai} \qquad i \neq a$$

$$= f_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (G_{aj} + g_{aj}) - e_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (B_{aj} + b_{aj}) - d_a \qquad i=a \qquad \text{(A-28)}$$

$$\frac{\partial Q_a}{\partial f_i} = e_a G_{ai} + f_a B_{ai} \qquad i \neq a$$

$$= -e_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (G_{aj} + g_{aj}) - f_a \sum_{\substack{j=1 \\ j\neq a}}^{N} (B_{aj} + b_{aj}) + c_a \qquad i=a \qquad \text{(A-29)}$$

In A-26 till A-29 i = 1,.....,N.

Polar coordinates.

type 1: Voltage magnitude at bus A.

$$\frac{\partial V_a}{\partial V_a} = 1 \qquad \text{(A-30)}$$

$$\frac{\partial V_a}{\partial \delta_a} = 0 \qquad \text{(A-31)}$$

type 3 and 4: Active and reactive line flow at line terminal A.

$$S_{ab} = U_a I_{ab}^{*} = U_a(U_a - U_b)^{*} y_{ab}^{*} + V_a^2 y_{ab}^{*} \qquad \text{(A-32)}$$

$$P_{ab} = -V_a V_b |Y_{ab}| \cos(-\theta_{ab} + \delta_a - \delta_b) + V_a^2 \{ |y_{ab}| \cos(-\theta'_{ab}) + |Y_{ab}| \cos(-\theta_{ab}) \} \qquad \text{(A-33)}$$

$$Q_{ab} = -V_a V_b |Y_{ab}| \sin(-\theta_{ab} + \delta_a - \delta_b) + V_a^2 \{ |y_{ab}| \sin(-\theta'_{ab}) + |Y_{ab}| \sin(-\theta_{ab}) \} \qquad \text{(A-34)}$$

$$\frac{\partial P_{ab}}{\partial V_a} = -V_b|Y_{ab}|\cos(-\theta_{ab}+\delta_a-\delta_b) + 2V_a\{|Y_{ab}|\cos(-\theta'_{ab}) + |Y_{ab}|\cos(-\theta_{ab})\} \quad \text{(A-35)}$$

$$\frac{\partial P_{ab}}{\partial V_b} = -V_a|Y_{ab}|\cos(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-36)}$$

$$\frac{\partial P_{ab}}{\partial \delta_a} = V_a V_b|Y_{ab}|\sin(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-37)}$$

$$\frac{\partial P_{ab}}{\partial \delta_b} = -V_a V_b|Y_{ab}|\sin(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-38)}$$

$$\frac{\partial Q_{ab}}{\partial V_a} = -V_b|Y_{ab}|\sin(-\theta_{ab}+\delta_a-\delta_b) + 2V_a\{|Y_{ab}|\sin(-\theta'_{ab}) + |Y_{ab}|\sin(-\theta_{ab})\} \quad \text{(A-39)}$$

$$\frac{\partial Q_{ab}}{\partial V_b} = -V_a|Y_{ab}|\sin(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-40)}$$

$$\frac{\partial Q_{ab}}{\partial \delta_a} = -V_a V_b|Y_{ab}|\cos(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-41)}$$

$$\frac{\partial Q_{ab}}{\partial \delta_b} = V_a V_b|Y_{ab}|\cos(-\theta_{ab}+\delta_a-\delta_b) \quad \text{(A-42)}$$

type 2: Line current magnitude at line terminal A.

$$I_{ab} = \frac{S^*_{ab}}{U^*_a} \quad \text{(A-43)}$$

$$|I_{ab}| = \frac{\sqrt{P^2_{ab}+Q^2_{ab}}}{V_a} \quad \text{(A-44)}$$

$$\frac{\partial|I_{ab}|}{\partial V_a} = \frac{P_{ab}\frac{\partial P_{ab}}{\partial V_a} + Q_{ab}\frac{\partial Q_{ab}}{\partial V_a}}{|I_{ab}|V_a^2} - \frac{|I_{ab}|}{V_a} \quad \text{(A-45)}$$

$$\frac{\partial|I_{ab}|}{\partial V_b} = \frac{P_{ab}\frac{\partial P_{ab}}{\partial V_b} + Q_{ab}\frac{\partial Q_{ab}}{\partial V_b}}{|I_{ab}|V_a^2} \quad \text{(A-46)}$$

$$\frac{\partial I_{ab}}{\partial \delta_a} = \frac{P_{ab}\frac{\partial P_{ab}}{\partial \delta_a} + Q_{ab}\frac{\partial Q_{ab}}{\partial \delta_a}}{|I_{ab}|V_a^2} \qquad \text{(A-47)}$$

$$\frac{\partial I_{ab}}{\partial \delta_b} = \frac{P_{ab}\frac{\partial P_{ab}}{\partial \delta_b} + Q_{ab}\frac{\partial Q_{ab}}{\partial \delta_b}}{|I_{ab}|V_a^2} \qquad \text{(A-48)}$$

type 5 and 6: Injected active and reactive power at bus A.

$$S_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} S_{ai} \qquad \text{(A-22)}$$

$$P_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} P_{ai} \qquad \text{(A-24)}$$

$$Q_a = \sum_{\substack{i=1 \\ i\neq a}}^{N} Q_{ai} \qquad \text{(A-25)}$$

$$\frac{\partial P_a}{\partial V_i} = -V_a|Y_{ai}|\cos(-\theta_{ai}+\delta_a-\delta_i) \qquad i\neq a$$

$$= \sum_{\substack{j=1 \\ j\neq a}}^{N} \frac{\partial P_{aj}}{\partial V_a} \qquad i=a \qquad \text{(A-49)}$$

$$\frac{\partial P_a}{\partial \delta_i} = V_a V_i|Y_{ai}|\sin(-\theta_{ai}+\delta_a-\delta_i) \qquad i\neq a$$

$$= \sum_{\substack{j=1 \\ j\neq a}}^{N} \frac{\partial P_{aj}}{\partial \delta_a} \qquad i=a \qquad \text{(A-50)}$$

$$\frac{\partial Q_a}{\partial V_i} = -V_a|Y_{ai}|\sin(-\theta_{ai}+\delta_a-\delta_i) \qquad i\neq a$$

$$= \sum_{\substack{j=1 \\ j\neq a}}^{N} \frac{\partial Q_{aj}}{\partial V_a} \qquad i=a \qquad \text{(A-51)}$$

$$\frac{\partial Q_a}{\partial \delta_i} = V_a V_i |Y_{ai}| \cos(-\theta_{ai} + \delta_a - \delta_i) \qquad i \neq a$$

$$= \sum_{\substack{j=1 \\ j \neq a}}^{N} \frac{\partial Q_{aj}}{\partial \delta_a} \qquad i = a \qquad \text{(A-52)}$$

In A-49 till A-52 i = 1,.....,N.