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CERTAINTY EQUIVALENCE AND AN IDENTIFICATION PROBLEM FROM ECONOMETRY

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An econometric model from Anderson and Taylor(1976) is considered. It is shown that with certainty equivalence control of such a simplified system, the parameter identification will not work.

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INTRODUCTION

Models of economy tend to be large. One method of handling that problem is to study one equation at a time, and simplify the equations as much as possible. But as shown below, this may intróduce artificial difficulties.

Anderson and Taylor(1976) consider an extremely simplified model under certainty equivalence control. They find that with this control law, the model parameters can not be estimated by least squares, but the control law converges to its desired value. This situation will now be analysed using the differential equation approach developed by Ljung(1977). It is shown why the parameter estimates do not, in general, converge to the true values, whereas the input does. However, the parameters can be made to converge if the model is slightly changed in a natural way.

PROBLEM STATEMENT

Consider the system

$$y(t) = b_1u_1(t-1) + b_2u_2(t-1) + e(t) \tag{1}$$

where $\{e(t)\}$ is a sequence of independent random variables with zero mean and variance σ^2. The parameters b_1 and b_2 are constant, but unknown. Only one of the control variables is used for control of the system. The second input, u_2, is not considered to be available for control of (1). It may e.g. be designated for control of some other equation in a larger model.

The control objective is to keep the output close to a prescribed reference value y_r. To do so, the unknown parameters b_1 and b_2 must be estimated. This is done with a least squares estimator, and the estimates are denoted $\hat{b}(t)$ and $\hat{b}_2(t)$. The certainty equivalence control rule then is

$$u_1(t) = \frac{y_r - \hat{b}_2(t)u_2(t)}{\hat{b}_1(t)} \tag{2}$$

Unfortunately, there is no feedback from the output in the control law (2). This is because the model (1) is very simplified. Normally, the output must also be allowed to depend on previous values of the output. This would then introduce feedback into the control law.

In the analysis it is assumed that $u_2(t)$ is constant, i.e. $u_2(t) = u_2$. The effects of a time-varying $u_2(t)$ will be discussed later on.

ANALYSIS

The estimation algorithm

To analyse the performance of the closed-loop system with least squares estimation, the ordinary differential equations approach of Ljung(1977) will be used. These ODEs describe the evolution of the parameter estimates, and equations for the recursive least squares estimator are needed in a special form. With

$$\theta = [b_1 \quad b_2]^T$$

$$\varphi(t) = [u_1(t-1) \quad u_2(t-1)]^T$$

the equations are

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{1}{t} \cdot R(t)^{-1} \varphi(t) \cdot [y(t) - \hat{\theta}(t-1)^T \varphi(t)] \tag{3}$$

$$R(t) = R(t-1) + \frac{1}{t} \cdot [\varphi(t)\varphi(t)^T - R(t-1) + \delta \cdot I] \tag{4}$$

The term $\delta \cdot I$ of (4) is added here to ensure the invertibility of $R(t)$. In normal least squares estimation $\delta=0$, but $\delta \neq 0$ may be used to avoid numerical difficulties. Equation (4) is normally written in terms of $P(t) = [t \cdot R(t)]^{-1}$, but the form (4) facilitates the analysis.

The differential equations

According to Ljung(1977) the equations (3)-(4) can be associated with the differential equations

$$\dot{\underline{\theta}} = E\{ \underline{R}^{-1} \varphi(\tau) \cdot [y(\tau) - \underline{\theta}^T \varphi(\tau)] \}$$

$$\dot{\underline{R}} = E\{ \varphi(\tau)\varphi(\tau)^T - \underline{R} + \delta \cdot I \}$$

The expectations shall be evaluated for fixed values of $\underline{\theta}$ and $\underline{R}$ and with the input from (2)

$$\underline{u} = \frac{y_r - \underline{b}_2 u_2}{\underline{b}_1} \tag{5}$$

Also let

$$\underline{\varphi} = [\, \underline{u}_1 \quad u_2]^T \tag{6}$$

The only random variable is then $y(\tau)$, which contains $e(\tau)$. The differential equations are

$$\begin{pmatrix} \dot{\underline{b}}_1 \\ \dot{\underline{b}}_2 \end{pmatrix} = \underline{R}^{-1} \begin{pmatrix} \underline{u}_1 \\ u_2 \end{pmatrix} \cdot \underline{\varepsilon} \tag{7}$$

$$\dot{\underline{R}} = \underline{\varphi}\,\underline{\varphi}^T - \underline{R} + \delta \cdot I \tag{8}$$

where

$$\underline{\varepsilon} = (b_1 - \underline{b}_1) \cdot \underline{u}_1 + (b_2 - \underline{b}_2) \cdot u_2 \tag{9}$$

Possible convergence points

In Ljung(1977) it is stated that the asymptotic behaviour of the identification algorithm is described by the solution to the differential equations (7)-(9). Only their stable stationary points are possible convergence points for the identification. With $u_2 \neq 0$ the only possibility is then $\varepsilon = 0$. With u_1 inserted this implies that the asymptotic estimates must satisfy ($\hat{b}_1 \neq 0$)

$$b_1(y_r - \hat{b}_2 u_2) = \hat{b}_1(y_r - b_2 u_2) \tag{10}$$

There is thus a whole line of stationary points of (7)-(9). The true values are fortunately on this line.

To check the stability, the ODEs (7)-(9) are linerized around these stationary points. The linerized system will have three eigenvalues in -1 (from equation (8)) and one in zero (with the eigenvector along the line of stationary points). The last eigenvalue will be

$$-\frac{b_1}{\underline{b}_1} (\underline{u}_1^2 + u_1^2)$$

For this to be negative, b_1 and its estimate must have the same sign. Since this is necessary for stability, the sign of b_1 will always be correctly estimated.

The above analysis explains the identification problems encountered by Anderson and Taylor(1976). It is also possible to answer their question if $\hat{b}_1$ will converge to b_1 w.p.1. The analysis shows that it will not.

It should be noted that the input u_1 will always converge to its desired value. From (10) it is obvious that

$$u_1 = \frac{y_r - \hat{b}_2 u_2}{\hat{b}_1} \rightarrow \frac{y_r - b_2 u_2}{b_1}$$

This agrees well with the findings of Anderson and Taylor. Thus, if the main concern is control, this method is probably good. But if the parameter estimates are wanted it cannot be used unchanged.

Modifications

It was stated in the introduction that the difficulties arise from the over-simplified model. It is noe easy to see why. The reason is, that the stationary points of (7) can be described by the simple equation $\underline{\varepsilon} = 0$ or (10). This in turn is caused by φ being a constant in the expectation defining the ODEs.

There are several ways to avoid the problem. They all assure a random φ in the expectation, either through u_1 or u_2 or both. In fact, they need not be random. It is sufficient if they change irregularly enough.

One possibility is to introduce dynamics in the model (1). This will make u_1 a feedback controller. Another possibility is to make frequent changes of the reference value or of the input u_2.

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