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Abstract—Adverse weather conditions can cause deterioration of wireless channels, leading to reduced link capacities. Unlike in other types of networks, in radio-based wireless mesh networks, the link capacities depend not only on the prevailing conditions, but also on interfering transmissions, as well as the transmission power and modulation and coding schemes used. This leads to increased difficulty in modelling and mitigating partial link failures, such as those caused by unfavourable weather. In this paper we present optimisation models for reconfiguration of routing and traffic demand volumes in wireless mesh networks, taking into account transmission power and modulation and coding schemes. We consider both a network planning application and online optimisation in response to failure states as they occur. We conduct a numerical study on a realistic network topology, showing the feasibility of our models for practical implementation.

I. INTRODUCTION

A common problem in wireless networks is the variability of the radio channel, leading to varying link capacities over time. While channel variation also occurs on a small time scale, in this paper we focus on changes that occur at larger time scales, on the order of an hour or longer. Degradation of link capacity at this scale can be caused for example by changes in the weather, such as precipitation resulting in lowered achievable data rates, especially at higher frequencies [1].

In the case of large time scale changes, it is possible to mitigate the effects of channel degradation by reconfiguration of the network. However, the configuration of a wireless mesh network is more complex than that of, for example, free-space optical networks or fixed-link networks, as have been considered in previous work on weather-related failures. Because of interference, the capacity of radio links is dependent on the constellation of active links, as well as their chosen modulation and coding schemes (MCSs) and transmission power.

In this paper, we apply linear and mixed-integer programming to optimise the transmission power and MCS for each link, along with link scheduling, in order to minimise the reduction in service during partial or total link failures. We then apply our model in a numerical study to a realistic wireless network with various traffic loads and investigate its performance. We consider two different implementation scenarios for our models: a network planning application, in which the network configuration is optimised for predicted future failure states; and an online application, in which optimisation is performed for states as they occur, or shortly in advance, for example by exploiting weather forecasts.

The rest of this paper is organised as follows. Section II discusses related work in this area. In Section III, we describe our system model and optimisation formulations. Then, in Section IV, we present our numerical study and results. Finally, in Section V, we conclude this paper and discuss directions for future work.

II. RELATED WORK

Two key characteristics of weather-inflicted failures in wireless networks are their locality, and the prevalence of partial, rather than total, failures. Region-based failures have been studied in [2], and extended to consider multiple failure regions in [3]. This allows for modelling of spatially correlated failures instead of considering only network connectivity or k-connectivity. In our models in this paper, we use a physical interference model that captures the effects of nearby links on each other, as well as mitigation of these through optimisation of both MCS and transmission power. In our numerical study, we consider failure states based on real-world weather data, where links fail in geographically limited regions.

In [4], the notion of probabilistic region failures was introduced, and in [5], three different measures of survivability were presented for wireless mesh networks with such regional failures. We focus particularly on the third of these, namely the expected percentage of total flow delivered after a failure. For each failure state we consider, we maximise the service level, that is, the proportion of the nominal rate of the traffic demands that can be delivered while in the failure state. Further, we consider the case where the network has a limit on its average power across all states. We consider known failure states based on historical data, rather than probabilistic failures, however, failure states in our model are weighted according to their historical prevalence, which corresponds to an estimate of future likelihood of occurrence of each state.

A MIP model for routing configuration and dynamic antenna alignment in wireless mesh networks with regional failures caused by adverse weather was given in [6]. Here, separate channels are chosen for intersecting links in order to avoid interference. We however adopt a full physical interference model, allowing us to capture the effects of even distant links, which may collectively interfere with an ongoing transmission on a given link, even if their individual contributions to interference are small. This also allows us to model adjustment of transmission power and MCS, which

can have benefits in energy usage, interference reduction, or improved capacity, depending on which values are chosen.

Transmission power and MCS are also critical to consider when addressing partial link failures in radio networks. An increase in signal attenuation can be counteracted by dropping to an MCS with a lower rate, increasing transmission power, or both, but increasing transmission power also increases the interference to other links. Multiple partial link failures have been studied for microwave networks in [7], and for free space optical networks in [8]–[11], however the links in such networks do not interfere with each other.

In order to model interference, we make use of compatible sets (c-sets), first introduced in [12], which are sets of links that can transmit simultaneously. Extensions for modulation and coding schemes, and transmission power control were given in [13] and [14], respectively. Since our models are inherently non-compact, we apply column generation to find the c-sets needed for the optimal solutions. In each c-set, we generate the links to be activated as well as the MCS and transmission power for each link.

III. SYSTEM MODEL

A. Problem setting

We consider a wireless mesh network characterised by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$. A number of different unicast demands use the network, with a single, given route for each demand. The demands are elastic, that is, their data rates may be reduced in response to adverse network conditions, however, in our optimisation problem our objective will be to minimise this reduction in demand rates.

At any given time, the network may be in any one of a number of possible failure states. Each state specifies the failure level of each link in the network. We employ a physical interference model to determine whether successful transmission is possible on a given link (possibly with simultaneous transmissions on other links), with a given transmission power and modulation and coding scheme. Successful transmission can occur when a minimum signal-to-interference-to-noise ratio (SINR) condition is satisfied, with this minimum threshold depending on the MCS used. Failure states for links are then characterised by the path loss specific to each link. A high path loss corresponds to poorer channel conditions on the link. In general, link failures are partial, that is transmission is still possible but the data rate may be reduced. However, if the path loss becomes too high, data reception may not be possible on that link even with maximum transmission power, the lowest available MCS, and no interfering transmissions. In this case we can consider the link to have failed completely.

This model of link failure states differs to previous work in, for example, FSO networks (as discussed in Section II), since the capacity of radio links depends not only on the prevailing environmental conditions, but also on concurrent, interfering transmissions. This means that failure states cannot be expressed only as a coefficient of the nominal link capacity, but must instead be closely integrated with the interference model used. For this, we will use *compatible sets* (c-sets) to describe sets of links, with accompanying transmission power coefficients, MCSs, and state-dependent path loss exponents, that are able to successfully transmit data simultaneously at a specified rate.

We will consider two related optimisation problems, one for use in network planning, and one for configuration of the network in response to, or in anticipation of, failure states as they occur. In both problems, we take as the objective to maximise the minimum relative rate over all traffic demands and all states. Here, relative rate refers to the proportion of the demand traffic rate that can be realised relative to the specified requested rate for the demand. For example, a demand that requests a rate of 5 Mb/s but, in a given state, only achieves a rate of 2.5 Mb/s, will have a relative rate of 0.5. In order to combine the demand rates in each state, we take a weighted average over all states, where the weights are chosen by the network operator. These could, for example, be based on the probability distribution over the occurrence of the different states.

For each state, we consider the lowest relative rate achieved by any demand in that state. We will call this minimum relative rate the *service level* for the state. We then find the c-sets, along with the proportion of time the network should use each c-set, that maximise the weighted sum of the service levels across all states. At the same time, we require that the network use at most a specified power, averaged over the state distribution, and that all demands receive at least some minimally acceptable rate in all states.

Here, we will not explicitly consider link scheduling, as this depends heavily on the specific medium access protocol to be used. In the case of TDMA, our optimised c-set durations may be directly realised to within some quantisation error that will depend on the number of c-sets, the distribution of their durations, and the length of the TDMA frame. In the case of CSMA-based protocols, there will be further performance losses due to contention, and the optimal time proportions may only be able to be realised statistically. However, even in this case, the values given by our models may be used to provide an upper bound on the network's performance.

B. Notation

In our formulations, we will make use of the following notation:

- $\mathcal{S}$: set of failure states
- $\mathcal{D}$: set of demands
- $\mathcal{C}$: set of c-sets
- $\mathcal{M}$: set of MCSs
- $b(a), e(a)$: beginning and end of arc $a \in \mathcal{A}$, respectively ($a = (b(a), e(a))$)
- $w(s)$: weight of state $s \in \mathcal{S}$, e.g. probability of being in state s
- $f(d)$, $d \in \mathcal{D}$: data rate [Mb/s] of demand d
- $P(d)$, $d \in \mathcal{D}$: fixed route used by demand $d \in \mathcal{D}$ ($P(d) \subseteq \mathcal{A}$)
- $\mathcal{D}(a)$: set of demands whose routes contain arc $a \in \mathcal{A}$; $\mathcal{D}(a) := \{d \in \mathcal{D} : a \in P(d)\}$

- t_c: proportion of time of using $c \in \mathcal{C}$ in one frame
- $B(c, a)$, $a \in \mathcal{A}$: rate [Mb/s] of arc a in c-set $c \in \mathcal{C}(a)$
- $\mathcal{C}(a) \subseteq \mathcal{C}$: family c-sets containing arc $a \in \mathcal{A}$
- $\mathcal{C}(s) \subseteq \mathcal{C}$: family of c-sets in state $s \in \mathcal{S}$
- $\mathcal{C}(a, s) \subseteq \mathcal{C}$: family of c-sets containing arc $a \in \mathcal{A}$, in state $s \in \mathcal{S}$
- $\mathcal{V}(c) \subseteq \mathcal{V}$: set of nodes active in c-set $c \in \mathcal{C}$
- $\mathcal{A}(c) \subseteq \mathcal{A}$: set of arcs active in c-set $c \in \mathcal{C}$
- P: upper bound on the power to be used on average by the network
- F: lower bound on proportion of demand rate to be realised in each state.

C. Master Problem

We formulate the master problem as follows.

$$\max \quad \sum_{s \in \mathcal{S}} w(s) x^s \tag{1a}$$

$$[\pi_a^s \geq 0] \quad \sum_{c \in \mathcal{C}(a,s)} B(a,c) t_c^s \geq F(a) x^s, \\ a \in \mathcal{A}, s \in \mathcal{S} \tag{1b}$$

$$[\lambda^s] \quad \sum_{c \in \mathcal{C}(s)} t_c^s = 1, \qquad s \in \mathcal{S} \tag{1c}$$

$$[\mu^s \geq 0] \quad x^s \leq 1, \qquad s \in \mathcal{S} \tag{1d}$$

$$[\tau^s \geq 0] \quad x^s \geq F, \qquad s \in \mathcal{S} \tag{1e}$$

$$[\varphi \geq 0] \quad \sum_{s \in \mathcal{S}} w(s) \sum_{c \in \mathcal{C}(s)} P(c) t_c^s \leq P \tag{1f}$$

$$x^s \in \mathbb{R}_+, \qquad s \in \mathcal{S} \tag{1g}$$

$$t_c^s \in \mathbb{R}_+, \qquad c \in \mathcal{C}, s \in \mathcal{S}, \tag{1h}$$

where $F(a) = \sum_{d \in \mathcal{D}(a)} f(d)$, and $P(c) = \sum_{v \in \mathcal{V}(c)} \eta(v, c) G(v)$. $F(a)$ thus gives the total requested rate on arc $a \in \mathcal{A}$, while $P(c)$ gives the power used for all arcs when c-set $c \in \mathcal{C}$ is in operation. The total power for c is found by summing over the power used by each node $v \in \mathcal{V}$ active in c, where the node power is given by its power control coefficient $\eta(v, c)$ (determined using the pricing problem that we will define in Section III-E) multiplied by the node's maximum usable power $G(v)$. The variables in square brackets on the left-hand side denote the dual variables for each constraint, which will be used to define the dual problem in Section III-D.

The objective function (1a) maximises the weighted average of the service level over all states. Constraint (1b) ensures that, on each arc, the demands' traffic is served at at least the service level x^s. $B(a, c)$ describes the achievable rate on arc $a \in \mathcal{A}$ when applying c-set $c \in \mathcal{C}$, while t_c^s describes the proportion of time that c is applied while the network is in state $s \in \mathcal{S}$. The left-hand side thus gives an average rate for arc a over all the c-sets used in state s. The right-hand side is then the total requested rate on arc a multiplied by the service level.

Constraint (1c) ensures the proportions of time applied to the c-sets used in each state form a distribution, that is, sum to 1. Constraint (1d) caps the service level at 1, since we are not able to serve the demands at higher rates than those requested. Constraint (1e) gives the minimum service level for all demands and states. Finally, constraint (1f) imposes a limit on the average power used by the network across all states. On the left-hand side, we take the average power used in each state across the c-sets applied in that state, weighted by their time proportions, and then take the weighted average over all states. This constraint connects the different states and makes them dependent on each other, which increases the difficulty of the problem. In Section III-F, we will give a decomposed version of the problem for cases where such a linking constraint is not required.

D. Dual problem

Since the master problem is not compact — there are exponentially many possible c-sets in relation to the size of the network — we will apply column generation to find the c-sets that are required for an optimal solution. To do so, we first take the dual of the master problem, and then use it to define a pricing problem for the column generation. Since the master (and dual) problem is linear, optimality is assured when solving the master using the generated c-sets.

The dual problem is given by

$$\min \quad \varphi P + \sum_{s \in \mathcal{S}} (\lambda^s + \mu^s - F \tau^s) \tag{2a}$$

$$\sum_{a \in \mathcal{A}} F(a) \pi_a^s + \mu^s - \tau^s \geq w(s), \qquad s \in \mathcal{S} \tag{2b}$$

$$\sum_{a \in \mathcal{A}(c)} B(a,c) \pi_a^s - w(s) p(c) \varphi \leq \lambda^s, \\ s \in \mathcal{S}, c \in \mathcal{C}(s) \tag{2c}$$

$$\pi_a^s, \mu^s, \tau^s, \varphi \in \mathbb{R}_+ \qquad a \in \mathcal{A}, s \in \mathcal{S} \tag{2d}$$

$$\lambda^s \in \mathbb{R} \qquad s \in \mathcal{S}. \tag{2e}$$

Constraint (2c) can be used to define the pricing problem. For each c-set $c \in \mathcal{C}$ in the list of c-sets generated so far, there will be one instance of this constraint corresponding to c. The goal of the pricing problem is then to find a new c-set that maximally breaks this constraint.

E. Pricing problem

The pricing problem defines the conditions for a compatible set. Since each state $s \in \mathcal{S}$ has its own family of c-sets $\mathcal{C}(s)$ that it uses, at each column generation iteration, we can solve a separate pricing problem for each state. This makes each pricing problem simpler. The pricing problem for each state is as follows.

$$\max \quad \sum_{a \in \mathcal{A}} \pi_a^s \left(\sum_{m \in \mathcal{M}} B(a, m) y_a^m \right) \\ - w(s) \varphi \sum_{v \in \mathcal{V}} G(v) \eta_v X_v \tag{3a}$$

$$X_v = \sum_{m \in \mathcal{M}} x_v^m, \qquad v \in \mathcal{V} \tag{3b}$$

$$Y_a = \sum_{m \in \mathcal{M}} y_a^m, \qquad a \in \mathcal{A} \tag{3c}$$

$$\sum_{a \in \delta(v)} Y_a \leq 1, \qquad v \in \mathcal{V} \tag{3d}$$

$$\sum_{a \in \delta^+(v)} y_a^m = x_v^m, \qquad v \in \mathcal{V}, m \in \mathcal{M} \tag{3e}$$

$$N + \sum_{v \in \mathcal{V} \setminus \{b(a), e(a)\}} G(v) p(v, e(a)) \eta_v X_v \leq \\ \frac{1}{\gamma(m)} G(b(a)) p(b(a), e(a)) \eta_{b(a)} + M(1 - y_a^m), \\ a \in \mathcal{A}, m \in \mathcal{M} \tag{3f}$$

$$\eta_v \leq 1, \qquad v \in \mathcal{V} \tag{3g}$$

$$X_v, Y_a \in \mathbb{B}, \qquad v \in \mathcal{V}, a \in \mathcal{A} \tag{3h}$$

$$x_v^m, y_a^m \in \mathbb{B}, \qquad v \in \mathcal{V}, a \in \mathcal{A}, m \in \mathcal{M} \tag{3i}$$

$$\eta_v \in \mathbb{R}_+, \qquad v \in \mathcal{V}. \tag{3j}$$

Binary decision variables X_v and Y_a, for node $v \in \mathcal{V}$ and arc $a \in \mathcal{A}$, respectively, indicate whether v and a are to be included in the c-set, that is, actively transmitting when the c-set is in use. The corresponding binary decision variables x_v^m and y_a^m indicate whether or not MCS $m \in \mathcal{M}$ is to be used for this transmission, where $\mathcal{M}$ is the set of all available MCSs. The continuous decision variable, η_v gives the power control coefficient for node $v \in \mathcal{V}$, with $\eta_v = 1$ indicating that node v transmits with full power, and $\eta_v = 0$ indicating that node v does not transmit at all.

$G(v)$ is the maximum transmission power of node v, and $p(v, w)$ represents the path loss from node $v \in \mathcal{V}$ to node $w \in \mathcal{V}$, such that $\eta_v G(v) p(v, w)$ will be the total received power at w from v. $B(a, m)$ is the data rate achievable on arc a when using MCS m. In general, this may vary from arc to arc, for example if nodes have different numbers of antennas allowing for multiple spatial streams, but in the case where all nodes have identical hardware, the rate will only depend on the chosen MCS. For each MCS $m \in \mathcal{M}$, an SINR threshold $\gamma(m)$ must be reached for m to be used on a given arc. N is the noise level, which we take to be equal for all arcs.

For a given optimal solution to the dual problem, the optimal values of variables π_a^s and φ are taken and used to define the objective function (3a) for the pricing problem. The form of the objective mirrors the dual constraint (2c), except that we explicitly define the arc rate $B(a, c)$ and c-set power $p(c)$ in terms of the pricing problem decision variables y_a^m, η_v, and X_v. If the optimal pricing problem objective exceeds λ^s, thus breaking constraint (2c), a new c-set has been found and should be added to $\mathcal{C}(s)$.

Constraints (3b) and (3c) ensure that only one MCS is chosen for each active node and arc, respectively, and that if the node or arc is not active, no MCS is chosen. Constraint (3d) requires that at most one arc incident to any give node is active in the c-set, that is, the node may not receive and send at the same time, and all transmissions are unicast. Constraint (3e) forces each node $v \in \mathcal{V}$ to use the same MCS as its active incident arc, and also makes sure that the node is active if and only if one of its incident arcs is active.

Constraint (3f) gives the SINR condition to receive successfully on arc $a \in \mathcal{A}$ using MCS $m \in \mathcal{M}$, with the transmission power defined by $\eta_{b(a)}$. The left-hand side gives the noise and interference terms, while the right-hand side gives the signal relative to the threshold $\gamma(m)$. M is a constant used to cancel the constraint when arc a is not active or does not use MCS m, and is given by $\max_{a \in \mathcal{A}} \{ \frac{1}{\hat{\gamma}} G(b(a)) p(b(a), e(a)) \}$, where $\hat{\gamma} = \min_{m \in \mathcal{M}} \{ \gamma(m) \}$. The last constraint (3g) ensures that each node transmits with at most its maximum power.

The bilinearities $\eta_v X_v$ may be resolved by defining auxiliary variables $z_v = \eta_v X_v$, $z_v \in \mathbb{R}_+$, and adding the following constraints to the pricing problem.

$$z_v \leq \eta_v; \; z_v \leq X_v; \; z_v \geq X_v + \eta_v - 1, \quad v \in \mathcal{V}. \quad (4)$$

The objective should then be replaced with maximising

$$\sum_{a \in \mathcal{A}} \pi_a^s \left(\sum_{m \in \mathcal{M}} B(a, m) y_a^m \right) - w(s) \varphi \sum_{v \in \mathcal{V}} G(v) z_v \quad (5)$$

and similarly for constraints (3f).

F. Problem Decomposition

In the master problem, constraint (1f) couples the different states, meaning that during column generation, at each iteration the dual is solved for all states, and one instance of the pricing problem is solved for each state. Finally, the master problem is also solved for all states together. However, if the network does not require the average power limitation defined by constraint (1f), or any other similar coupling constraint, then the master problem may be decomposed into a subproblem for each state $s \in \mathcal{S}$, given by

$$\max \quad x \quad (6a)$$
$$[\pi_a \geq 0] \quad \sum_{c \in \mathcal{C}(a)} B(a, c) t_c \geq F(a) x, \quad a \in \mathcal{A} \quad (6b)$$
$$[\lambda] \quad \sum_{c \in \mathcal{C}} t_c = 1, \quad (6c)$$
$$[\mu \geq 0] \quad x \leq 1 \quad (6d)$$
$$[\tau \geq 0] \quad x \geq F \quad (6e)$$
$$t_c \in \mathbb{R}_+, \quad c \in \mathcal{C} \quad (6f)$$
$$x \in \mathbb{R}_+. \quad (6g)$$

The dual problem is then

$$\min \quad \lambda + \mu - F\tau \quad (7a)$$
$$\sum_{a \in \mathcal{A}} F(a) \pi_a + \mu - \tau \geq 1 \quad (7b)$$
$$\sum_{a \in c} B(a, c) \pi_a \leq \lambda, \quad c \in \mathcal{C} \quad (7c)$$
$$\pi_a \in \mathbb{R}_+, \quad a \in \mathcal{A} \quad (7d)$$
$$\lambda \in \mathbb{R}. \quad (7e)$$

The pricing problem for each state $s \in \mathcal{S}$ will be as in formulation (3), except the objective will be replaced with

$$\max \quad \sum_{a \in A} \pi_a \left(\sum_{m \in \mathcal{M}} B(a, m) y_a^m \right), \quad (8)$$

which must exceed λ in order to add a new c-set.

IV. NUMERICAL STUDY

A. Test data and parameters

In order to test the performance of our model, described in Section III, we conducted a numerical study using the network topology shown in Figure 1. This network instance is based on that used in [10] and [11], but adapted to suit a radio network instead of a free-space optical (FSO) network as in that work. By using this network, we could also make use of the accompanying partial link failure state data, which was based on real weather data collected for the Paris metropolitan area. In total, 206 partial link failure states were determined based on weather data for one year. For our study, however, we used only the first 10 states, which collectively accounted for 6802 hours, or approximately 78% of the time. To determine the weights for each state, the number of hours in which that weather state was observed was divided by 6802, the total number of hours for all 10 states.

To adapt the network instance to be suitable for a radio network, we reduced the link distances by a factor of 50 (so, for example, a 10 km FSO link would become a 200 m radio
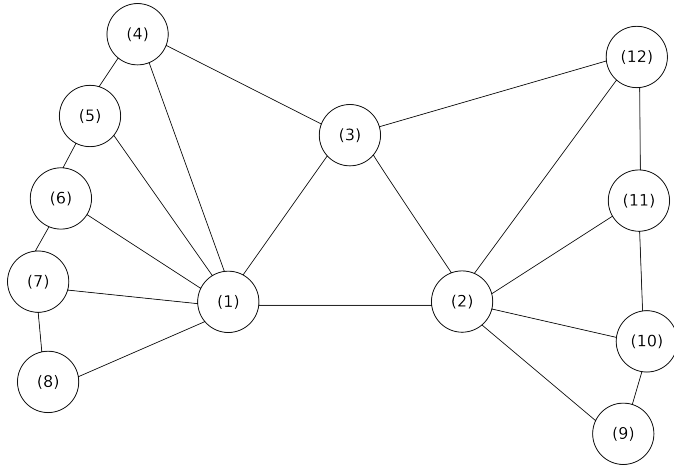
Fig. 1. Network topology for the numerical study

TABLE I
DISTANCES BETWEEN NODES

	1	2	3	4	5	6	7	8	9	10	11	12
1	–	200.0	176.40	200.0	204.6	164.0	180.0	248.0	264.0	320.0	232.0	378.0
2	200.0	–	176.40	309.4	270.2	264.0	312.2	444.4	164.4	160.0	132.0	179.20
3	176.40	176.40	–	162.8	154.0	174.0	242.2	452.4	333.8	367.2	204.8	207.2
4	200.0	309.4	162.8	–	56.6	118.0	138.4	371.8	414.6	473.2	341.6	370.4
5	204.6	270.2	154.0	56.6	–	72.0	98.0	329.0	362.0	424.6	305.0	360.6
6	164.0	264.0	174.0	118.0	72.0	–	72.0	278.0	312.0	380.0	282.0	366.0
7	180.0	312.2	242.2	138.4	98.0	72.0	–	230.2	358.0	434.0	350.6	440.4
8	248.0	444.4	452.4	371.8	329.0	278.0	230.2	–	391.2	479.0	481.4	622.8
9	264.0	164.4	333.8	414.6	362.0	312.0	358.0	391.2	–	88.8	181.0	363.0
10	320.0	160.0	367.2	473.2	424.6	380.0	434.0	479.0	88.8	–	177.0	344.6
11	232.0	132.0	204.8	341.6	305.0	282.0	350.6	481.4	181.0	177.0	–	181.4
12	378.0	179.2	207.2	370.4	360.6	366.0	440.4	622.8	363.0	344.6	181.4	–

link). We also adjusted the data rates to use the modulation and coding schemes available in 802.11ac, but kept the same relative traffic demands between each pair of nodes as in [10], [11], which were based on real population data. Each demand was thus scaled by 10^{-4}, e.g. a 10 Gbps demand for the FSO network becomes instead a 1 Mbps demand for the radio network (in our lowest traffic load scenario). We call this the base demand level, which was then multiplied by a scaling factor that we varied in our experiments. The distances and base demands for the network are shown in Tables I and II.

Specifying link failure states is also more complicated for a radio network than for an FSO network, due to the interference between nearby links. In our model, links may also change their transmission power and MCS to account for either adverse channel conditions or interference. As such, the failure state data also needed to be modified to fit our model. This was done by mapping failure percentages from the FSO network to path loss exponents in our model. In the states we considered, four failure levels were present: 100% (total link failure), 50%, 25%, and 0% (fully working). We used the following path loss exponents to represent these failure levels: 3.9 for 100%, 3.6 for 50%, 3.4 for 25%, and 3.0 for 0%. These values were chosen such that links' available rates

TABLE II
BASE DEMANDS (MBPS)

	1	2	3	4	5	6	7	8	9	10	11	12
1	–	7.438	0.540	0.426	0.499	0.502	0.524	0.474	0.541	0.526	0.487	0.382
2	7.438	–	0.540	0.426	0.499	0.502	0.524	0.474	0.541	0.526	0.487	0.382
3	0.556	0.556	–	0.023	0.034	0.035	0.036	0.033	0.038	0.037	0.029	0.023
4	0.460	0.460	0.025	–	0.027	0.028	0.029	0.026	0.030	0.030	0.022	0.001
5	0.539	0.539	0.037	0.029	–	0.036	0.037	0.034	0.039	0.037	0.034	0.026
6	0.542	0.542	0.037	0.029	0.036	–	0.037	0.034	0.039	0.038	0.034	0.026
7	0.566	0.566	0.039	0.030	0.037	0.037	–	0.034	0.040	0.039	0.035	0.028
8	0.422	0.422	0.026	0.018	0.026	0.027	0.028	–	0.029	0.028	0.023	0.018
9	0.468	0.468	0.029	0.020	0.029	0.029	0.031	0.028	–	0.031	0.026	0.020
10	0.455	0.455	0.028	0.020	0.028	0.029	0.030	0.027	0.031	–	0.025	0.020
11	0.502	0.502	0.030	0.020	0.031	0.031	0.033	0.029	0.034	0.033	–	0.021
12	0.394	0.394	0.023	0.016	0.024	0.024	0.026	0.023	0.027	0.026	0.021	–

TABLE III
MODULATION AND CODING SCHEMES

MCS	Rate (Mbps)	SINR threshold
0	6.5	1.58
1	13.0	3.16
2	19.5	7.94
3	26.0	12.59
4	39.0	31.62
5	52.0	63.10
6	58.5	100.00

with the highest achievable MCS would reduce with successive failure levels. In concrete terms, using the parameter values we applied in our experiments, a 200 m link without interference and transmitting at full power can achieve MCS 6, with a data rate of 58.5 Mbps, when using a path loss exponent of 3.0. For an exponent of 3.4, this drops to MCS 4 with 39.0 Mbps, for 3.6 it becomes MCS 3 with 26 Mbps, and for 3.9, MCS 0 with 6.5 Mbps. Longer and shorter links will of course be affected differently.

The modulation and coding schemes for 802.11ac are shown in Table III, along with their corresponding SINR thresholds used in our experiments. The necessary SINR threshold for a given MCS will differ for different hardware; the ones we have used are from [15]. The other parameters used in our experiments are shown in Table IV. Four experiments were performed, with increasing demand scaling factors in order to increase the total load on the network.

B. Results and discussion

The results of our numerical study are shown in Table V. As would be expected, the final objective function value obtained from solving the master problem decreases with the demand scaling factor: the network becomes more loaded and is unable to meet the demands with the same level of service,

TABLE IV
PARAMETERS FOR THE NUMERICAL STUDY

Parameter	Value
Noise (N)	-111 dB
Maximum transmission power ($G(v)$)	20 mW
Demand scaling factor	1.0, 1.5, 2.5, 3.0

TABLE V
NUMERICAL STUDY RESULTS

S	M	I	C	T_D [s]	T_P [s]	T_M [s]	T [s]
1.0	0.997	44	171	0.23	8068.80	0.01	8069.04
1.5	0.798	39	384	0.21	32114.14	0.01	32114.36
2.5	0.479	47	424	0.39	39982.06	0.02	39982.47
3.0	0.399	40	376	0.25	34809.70	0.01	34809.96

S: demand scaling factor, M: master problem objective, I: number of pricing problem iterations, C: total number of c-sets (for all states, includes 78 initial c-sets), T_D: solution time for the dual problem (all iterations), T_P: solution time for the pricing problem (all iterations), T_M: solution time for the master problem, T: total solution time.

especially in more challenging failure states. The number of c-sets generated, as well as the time spent on iterations of the pricing problem, also increases. This is because when the network is lightly loaded, some states may reach full service. When this occurs, no more c-sets can be generated that break the corresponding dual constraint for these states, and the pricing problem executes rapidly with objective 0. In all cases, both the dual and master problem had very short solution times, which is to be expected as these are linear problems, whereas the pricing problem is a mixed-integer problem and took much longer to solve.

Overall, the solution times were acceptable for practical applications. For network planning, solution times on the order of hours, as seen here, are easily accommodated. More states could also be added, which would linearly increase the overall solution time, since the majority of the time is spent in the pricing problem, where we have a separate problem for each state.

For online optimisation using the decomposed version of our models given in Section III-F, the optimisation would be performed upon entering a new failure state. In this case, we need only solve for a single state, so the times shown in the table for the pricing problems can be reduced by approximately a factor of 10 (since we used 10 states in our experiments). The resulting solution time is then on the order of one hour. This is an acceptable duration, if a state will last for several hours, and especially if states can be predicted, as is typically the case for weather conditions, since the problem may then be solved in advance. The solution time can also be reduced further by saving c-sets from previous solutions, potentially reducing the number of iterations needed. Another way to reduce solution times is by stopping the c-set generation after some smaller number of iterations, for example when the change in the optimal dual solution becomes negligible over several consecutive iterations. This will then give a feasible, albeit typically very close to optimal, solution. This is a well known phenomenon in column generation, discussed for a similar c-set generation context in [16].

V. Conclusion

In this paper, we have presented mixed-integer programming models for multiple partial link failures in wireless mesh networks. Our models maximise the proportion of traffic demands that can be met in each failure state, while meeting an overall power limit for the network. We achieve this using a realistic physical interference model, and optimising both transmission power, and modulation and coding schemes for each link.

To evaluate our models, we have conducted a numerical study using failure states based on real-world weather data with a realistic network topology. The solution times for our optimisation problems were on the order of hours, for a network planning application, or approximately one hour, for an online optimisation use case in which the network adapts to new states as they occur. These times are acceptable for practical implementation and use of the models.

In future work, more extensive experiments can be carried out to more thoroughly evaluate our models. In particular, experiments based on real radio networks with detailed channel models would be especially valuable. Testing with more scenarios and with different sized networks would also provide a more complete picture of the performance of our models.

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