LUND UNIVERSITY

Modeling of Electricity Distribution Networks and Components.

Status Report 1997-11-30 for Elforsk project 3153

Möllerstedt, Erik; Mattsson, Sven Erik; Bernhardsson, Bo

1997

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Möllerstedt, E., Mattsson, S. E., & Bernhardsson, B. (1997). *Modeling of Electricity Distribution Networks and Components. Status Report 1997-11-30 for Elforsk project 3153*. (Technical Reports TFRT-7570). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
3

General rights
Unless other specific re-use rights are stated the following general rights apply:
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.
- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: https://creativecommons.org/licenses/

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

ISSN 0280–5316
ISRN LUTFD2/TFRT--7570--SE

Modeling of Electricity Distribution Networks and Components

– Status Report 1997-11-30 for Elforsk Project 3153

Erik Möllerstedt
Sven Erik Mattsson
Bo Bernhardsson

Department of Automatic Control
Lund Institute of Technology
December 1997

Department of Automatic Control
Lund Institute of Technology
Box 118
S-221 00 Lund Sweden

Document name
INTERNAL REPORT

Date of issue
971205

Document Number
ISRN LUTFD2/TFRT--7570--SE

Author(s)
Erik Möllerstedt
Sven Erik Mattsson
Bo Bernhardsson

Supervisor

Sponsoring organisation
Elforsk AB and Nutek

Title and subtitle
Modeling of Electricity Distribution Networks and Components — Status report 1997-11-30 for Elforsk project 3153

Abstract

This is a status report for the Elfork project 3153, *Modeling of Electricity Distribution Networks and Components.* The project focuses on modeling and simulation of distribution networks, which are very complex with many non-linear loads and switching devices. A new method for calculation of harmonics at steady-state is presented. The method is based on harmonic balance. Since the level of distortion in distribution networks is limited, we assume that deviations from the nominal voltage affect the current harmonics linearly. The models consist of a nominal current spectrum and a Jacobian matrix, which can be pre-calculated. The method is non-iterative and the networks are solved using linear algebra, it supports aggregation and modularization and the use of model-libraries. The more recent results include a procedure for obtaining models from real measurements. The report describes the procedure and demonstrates how it can be used to deduce a model for a light dimmer. The plots show that the method gives good results.

Key words
distribution networks, modeling, harmonics, steady state analysis, harmonic balance

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title
0280–5316

ISBN

Language
English

Number of pages
19

Recipient's notes

Security classification

The report may be ordered from the Department of Automatic Control or borrowed through:
University Library 2, Box 3, S-221 00 Lund, Sweden
Fax +46 46 222 44 22 E-mail ub2@ub2.lu.se

1. Introduction and Status

This is a status report for the project *Modeling of electricity distribution networks and components*, which is financed by Elforsk AB as project 3153. The aim of the project is to develop new methods to analyze and simulate electrical distribution networks. Characteristic for these systems is that they are very complex and widespread, and contain numerous nonlinear and switching devices.

We have developed a new method to model nonlinear distribution networks. The model structure implies that solving networks is a non-iterative and fast procedure without any convergence problems. The method supports aggregation and reuse of results, and the parameters of the models can be obtained through simulation, real measurements, or analytical calculations. The method is outlined in Section 2. A more detailed describtion is to be found in the previous status report, [Möllerstedt *et al.*, 1997b].

The more recent results include a procedure for obtaining models from real measurements. Section 3 describes the procedure and Section 4 demonstrates how it can be used to deduce a model for a light dimmer. The plots show that the method gives good results.

Results have been presented at the IMACS 1997 World Congress for Mathematical modeling and simulation, in Berlin, [Möllerstedt *et al.*, 1997c], and a paper is accepted for presentation at the IEEE Conference on Decision and Control in San Diego, 1997, [Möllerstedt *et al.*, 1997a].

The method will constitute the basis of a licentiate thesis by Erik Möllerstedt and the thesis will be finished early 1998.

2. Description of the Method

Distribution systems for electric power are very complex. They consist of very many components, many of which are nonlinear or switching. This means that analysis and simulation of such systems are very complicated. Typical configurations, like an office building or a shopping mall, consist of hundreds of computers, fluorescent lamps, air conditioners, etc. It is impossible to model such networks in detail. There is a need for methods to aggregate nonlinear loads.

In distribution networks, the loads are connected in parallel, see Figure 1. The networks are said to be radial. This means that the voltage is the same for all loads, except for the small voltage drops caused by line losses. Furthermore, there are norms and standards, [Friman, 1994], that limit the allowed voltage distortion. Consequently, two observations can be made

1. The nominal voltage, v_0, is known (e.g., 230 V, 50 Hz),
2. The voltage distortion is limited.

The method we have developed is a variant of harmonic balance [Gilmore and Steer, 1991; Kundert and Sangiovanni-Vincentelli, 1986], and uses these observations. In harmonic balance, current and voltage are described by truncated Fourier series

$$
\begin{aligned}
i(t) &= \sum_{k=1}^{N} A_k \cos k\omega t + B_k \sin k\omega t \\
I &= [\, A_1 \quad \dots \quad A_N \quad B_1 \quad \dots \quad B_N \,]^T \\
v(t) &= \sum_{k=1}^{N} a_k \cos k\omega t + b_k \sin k\omega t \\
V &= [\, a_1 \quad \dots \quad a_N \quad b_1 \quad \dots \quad b_N \,]^T .
\end{aligned}
\tag{1}
$$

As we only allow small deviation from the nominal voltage, it is reasonable to assume a linear relationship between the Fourier coefficient vectors of the current and the voltage

$$
I = I_0 + Y(V - V_0), \tag{2}
$$

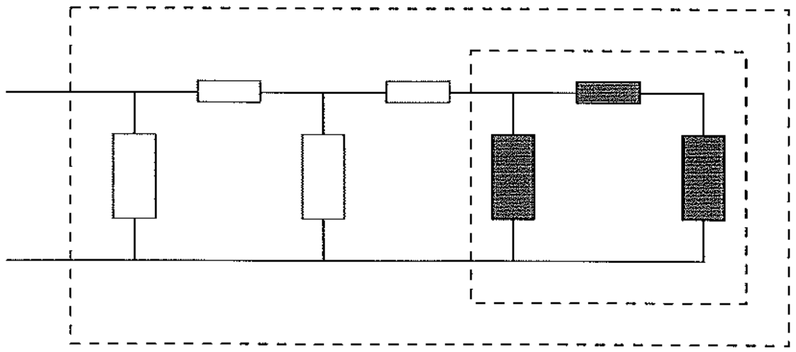

Figure 1 In radial networks, the loads are connected in parallel.

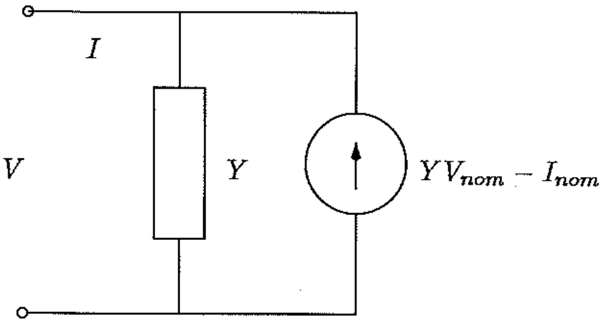

Figure 2 The model can be interpreted as a Norton equivalent, with an admittance matrix, Y, and a current source.

where I_0, the nominal current spectrum, contains the Fourier coefficients of the current when the voltage is nominal. The admittance matrix Y is a matrix that describes how the current spectrum is affected by changes in the voltage spectrum. Each column in Y describes the change in the current spectrum when a small cosine or a sine component of a certain harmonic frequency is added to the nominal voltage. In Figure 2, it is shown how the model can be interpreted as a Norton equivalent.

The dimensions of Y and I_0 depend on how many terms that are considered in the truncated Fourier series, that is, the integer N in equation (1). The size can be reduced if it is known that certain frequencies are not present in a network. In many applications it is reasonable to assume that the loads are symmetric, thus only odd harmonics are present.

The linearization implies that aggregating loads and solving networks is a noniterative procedure that is done with linear algebra. Long computational times, and convergence problems are avoided. Subresults can be stored in model libraries, and reused in other applications. The models can be obtained through simulations, analytical calculations or through real measurements.

3. Obtaining Models from Measurements

This section describes an experimental procedure for estimating the two parameters I_0 and Y in the model (2). The procedure is straightforward, but there are some problems that must be considered when treating data from non-linear systems.

3.1 The Estimation Procedure

Assume, for simplicity, that all harmonic frequencies up to order N are considered in the model. The model parameters can be estimated with some kind of least squares fitting of measurement data. However, as mentioned in Section 2, the columns of Y describe how the nominal current is affected when a cosine or sine voltage of a certain frequency is added to the nominal voltage. This leads to a very straightforward way to estimate the model.

Let $I_{\widehat{a}_i}$ be a column vector that contains the Fourier coefficients of the current when a voltage, $\widehat{a}_i \cos i\omega t$ is added to the nominal voltage. Similarly, $I_{\widehat{b}_i}$ is the current vector when the added voltage is $\widehat{b}_i \sin i\omega t$.

$$
\begin{aligned}
v(t) &= a_0 \cos \omega_0 t + \widehat{a}_i \cos i\omega t \\
&\quad \Rightarrow i(t) = [\cos 1\omega t \quad \cos 2\omega t \quad \ldots \quad \sin 1\omega t \quad \sin 2\omega t \quad \ldots]\, I_{\widehat{a}_i} \\
v(t) &= a_0 \cos \omega_0 t + \widehat{b}_i \sin i\omega t \\
&\quad \Rightarrow i(t) = [\cos 1\omega t \quad \cos 2\omega t \quad \ldots \quad \sin 1\omega t \quad \sin 2\omega t \quad \ldots]\, I_{\widehat{b}_i}.
\end{aligned}
$$

The Y-matrix is then obtained in the following way

$$
\begin{aligned}
Y &= [\, \tfrac{dI}{da_1} \quad \tfrac{dI}{da_2} \quad \ldots \quad \tfrac{dI}{db_1} \quad \tfrac{dI}{db_2} \quad \ldots] \\
&= \left[\tfrac{I_{\widehat{a}_1} - I_0}{\widehat{a}_1} \quad \ldots \quad \tfrac{I_{\widehat{b}_1} - I_0}{\widehat{b}_1} \quad \ldots \right].
\end{aligned}
$$

The method requires that the voltage contain only the fundamental frequency and one harmonic frequency:

$$
v(t) = a_0 \cos \omega_0 t + \widehat{a}_i \cos i\omega t.
$$

This can be hard to achieve, especially if the loads to be modeled are nonlinear. If the voltage source is not stiff, and there is distortion in the current, the voltage will become distorted too. However, if the deviations from the nominal voltage and current spectra. V_0, and I_0, are columns in matrices, $\widehat{V}$, and $\widehat{I}$, the following relation holds

$$
\begin{aligned}
\widehat{V} &= [\, V_1 - V_0 \quad V_2 - V_0 \quad \ldots \quad V_{2n} - V_0\,] \\
\widehat{I} &= [\, I_1 - I_0 \quad I_2 - I_0 \quad \ldots \quad I_{2n} - I_0\,] \\
\widehat{I} &= Y\widehat{V}
\end{aligned}
$$

This shows that if the column vectors of $\widehat{V}$ are linearly independent, then $\widehat{V}$ is invertible and Y can be obtained, even if the voltage cannot be shaped according to Equation (3.1). One problem is that the amplitudes of the voltage deviations must be chosen large enough to get $\widehat{V}$ well conditioned and to overcome measurement noise. As the element in Y are derivatives of the current coefficients, and describe the local behavior in the neighborhood of the nominal voltage, increased voltage deviations imply poorer estimate of these derivatives and, thus, the elements in Y. Another problem with a non-stiff voltage source is that all frequencies that are apparent in the voltage must be considered in the estimation experiment. This means that even though the configuration of the network implies that, for instance, there cannot be a fifth harmonic component in the signals, this frequency has to be considered in the model during estimation. The model can be reduced afterwards, to exclude the fifth harmonic, by means of model reduction techniques. However, when estimating parameters it is necessary to have a model that includes all frequencies that appear in the measurements. Unfortunately, more measurements are needed since there are more parameters to estimate.

3.2 Spectral Analysis

The Fourier coefficients of the voltages and currents are calculated from the sampled time domain signals using the discrete Fourier transform, see Appendix A.

The choice of window for the Fourier transform depends on several factors, for instance noise and disturbances, and also the how close to perfect periodicity the signals are. The importance of windows to avoid spectral leakage is obvious, as the fundamental frequency is very dominant. The amplitudes of the harmonic frequencies are normally just a few percents of the fundamental frequency amplitude. Thus, the spectral leakage between harmonic frequencies must be much less than one percent. As multiplication of a window in the time domain is equivalent to a convolution with the Fourier transform of the window in the frequency domain, this means that the amplitude of the Fourier transform of the window for frequencies $\pm m f_0$, where m is an integer, must be very small.

3.3 Comparison with Estimation of Linear Loads

Frequency response analysis is often a convenient way to obtain models of linear systems. A sinusoidal signal is applied to the system, and amplitude and phase shift is measured as a function of frequency. For nonlinear systems, a single frequency input does not result in a single frequency output. An applied sinusoidal signal will at steady state affect all harmonic frequencies. When linearizing the system around the nominal 50 Hz component, we measure how much a small voltage superposed to the nominal voltage affects the nominal current spectrum. This means that we have to consider all harmonic frequencies for each frequency of the added voltage. The current variation depends on both frequency and phase of the superposed voltage.

When sampling a continuous time signal, an anti-aliasing filter must be used to avoid aliasing problems. A filter always affects the amplitude and the phase of the signals. When estimating linear systems, this does not cause any problems, because both inputs and outputs are affected in the same way. With nonlinear loads, however, the signals contain many frequencies at the same time. As the filter effects are different for frequencies, the dynamics of the filter must be known and compensated for.

4. An Example: A Light Dimmer

In this section, the method described in Section 3 is used to estimate the model parameters for a light dimmer. The model is validated by applying a perturbed voltage to the dimmer, and comparing the current predicted by the model with the measured current. The results show that the method has great potential. There are, however, several sources of error in the process, that have to be dealt with for better performance.

4.1 The Process

A light dimmer is a highly non-linear component and serves as a good test component to validate our method. Ideally, a dimmer works as an open circuit the first part of every half period, and as a short circuit the rest of the half period. This means that the harmonic content of the current is very high.

4.2 Shaping the voltage

A switched voltage converter was used to shape a DC voltage according to a reference signal, which was calculated and output from a PC. The converter switching was at 4 kHz. To get rid of the high frequencies generated by the

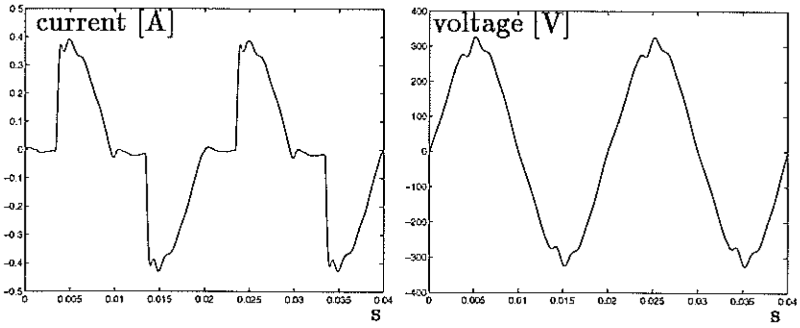

Figure 3 Measured current and voltage from a light dimmer.

switching, a low pass LCL-filter with a bandwidth of 3 kHz was used to smooth the voltage.

The use of a DC generator and the low pass filter resulted in a weak voltage source. The current and the voltage, when the reference voltage is a clean 50 Hz sinusoid, is shown in Figure 3. The plots show that there is a considerable amount of distortion also in the voltage. However, the measurement data show a very good periodicity, and a very low noise level. According to the discussion in Section 3, this means that the voltage source is still suitable.

4.3 Measurement Equipment

The Daqbook Data Acquisition System from IO-tech, [IOt, 1995], was used to measure voltage and current.

The current was measured with a current probe (LEM HEME PR 30). It was filtered through an analog anti-aliasing filter (DBK 18 Filter Module from IOtech, with a bandwidth of 1 kHz). The voltage was connected to a high voltage insulation unit in the Daqbook, to avoid damage of the equipment. The voltage insulation unit low pass filtered the voltage, with approximately the same bandwidth as the current. The frequency response of the two filters were obtained using a Solatron frequency analyzer, see the Bode plots in Figure 4, and the results were used to compensate for the filters in the estimation experiments for the dimmer model.

The Daqbook samples at a maximum frequency of 100 kHz and measures up to 256 analog signals. For the experiments, a sampling rate of 23.810 kHz

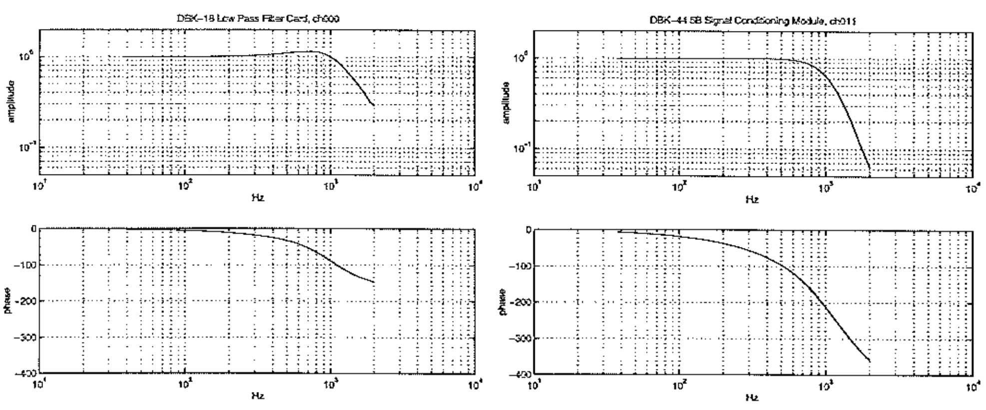

Figure 4 The Bode plots from the anti-aliasing filter (left) and the insulation unit (right). Both are low pass filtering with a bandwidth around 1 kHz.

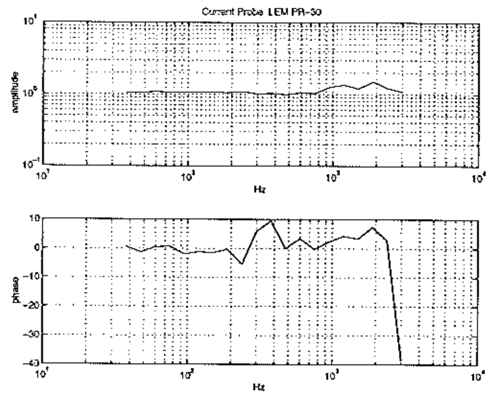

Figure 5 The Bode plot for the current probe.

was used, and the measurement time was 1 s. The reason for the fast sampling rate was to avoid aliasing of the voltage, which according to the documentation of the Daqbook was not low pass filtered. By choosing a sampling frequency that was not a multiple of the fundamental frequency of the voltage, low frequency harmonics were not affected by frequency folding of higher frequency harmonics. Furthermore, only certain fixed frequencies can be selected in the Daqbook. The frequency 23.810 kHz is one possible choice. As the voltage was indeed filtered, the sampling rate was unnecessarily fast.

The Bode plot for the current probe was also obtained using the Solatron frequency analyzer, see Figure 5. The plot shows that the bandwidth is much higher than the frequencies we are considering, and the dynamics of the probe is therefore neglected in our experiments. The phase is not really constant, probably due to resonances. However, the maximum phase shift is less that 10 degrees.

4.4 Spectral Analysis

Matlab's Signal Processing Toolbox [Krauss *et al.*, 1993] was used to analyze the sampled signals. The signals showed a very low noise level, and a good periodicity. The only criterion for the choice of window, is that the spectral leakage between harmonic frequencies shall be minimized, and the peaks detected accurately. In the analysis below, two suitable windows are compared,

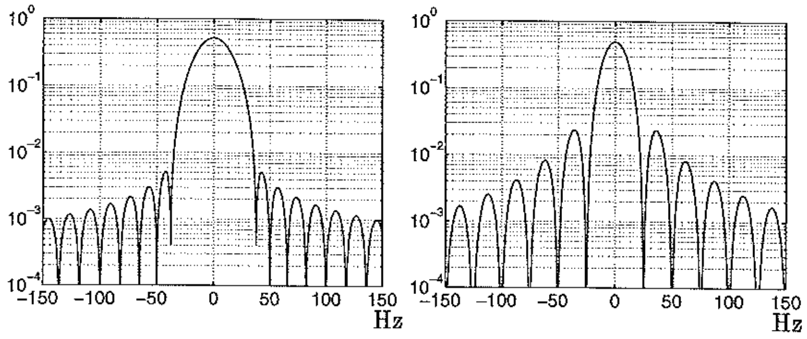

Figure 6 Fourier transforms of two time domain windows. The window to the left is a Kaiser window with $L = 1263$ and $\beta = 5.48$. The right window is a Bartlett window with $L = 1900$. The windows have been chosen to have minimal spectral leakage for periodic signals.

see Figure 6. Both windows show a good performance.

The window to the left is a Kaiser window with $L = 1263$, and $\beta = 5.48$, see Appendix A. It has been chosen so that it has dips for $f = \pm 50$ Hz, and $f = \pm 100$ Hz, to avoid spectral leakage.

The window to the right is a Bartlett window with length $L = 1900$. It has dips for all harmonic frequencies, $f = \pm m f_0$. However, the amplitudes of the side lobes are also much higher than for the Kaiser window, which makes it less suitable in a noisy environment.

4.5 Estimation

As we are only investigating the steady state performance, and assume that the solution is periodic, we only consider harmonic frequencies. If we know that a frequency component is diminishingly small, this frequency does not have to be considered in the estimation experiment. A typical example is that most networks are approximately symmetric, thus only odd harmonics are of importance.

Figure 7 shows that the voltage contains a fair amount of odd harmonics up to order 13. Therefore we chose to work with a model with these frequencies.

To get the $\widehat{V}$-matrix well conditioned, we chose the amplitude of the added harmonic voltage amplitude to be 5% of the nominal voltage. The condition number of the matrix was 6.8, which must be considered acceptable.

Two different models were estimated, one for each of the two windows. In the validation section, the Kaiser window model is referred to as Model 1, whereas Model 2 is the Bartlett window model.

4.6 Validation

To validate the models, current spectra from measurements not used in the estimation process were compared with the current spectra, that our models predicted. Three different measurements for the validation.

1. Measurements from the same time as the estimation experiment.
2. Measurements on the same test rig, but at another time.
3. Using the line voltage from a wall socket as the voltage source.

In the first two measurements, most of the voltage distortion is in the 7th harmonic, but similar results are obtained from distortions of other harmonics. For the third measurement, the stiff line voltage is used. This has a very

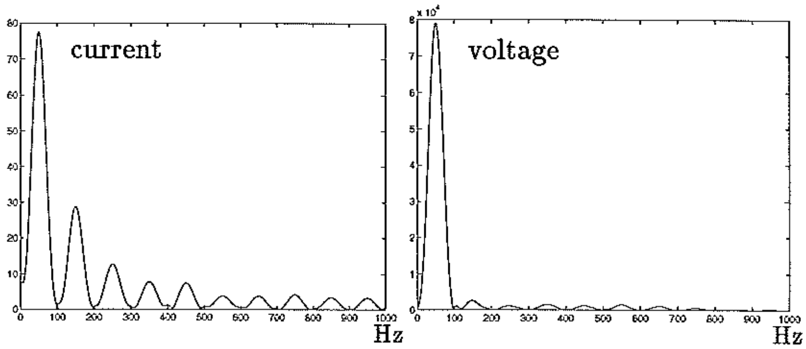

Figure 7 The current and voltage spectra from measurements on the dimmer. The spectra are obtained using a Kaiser window.

low distortion level. The deviation from the "nominal" voltage used in the estimation process is still large, as the "nominal" voltage was very distorted.

The results from the validation of the two models are shown in a number of figures below. In all figures there are three plots. The upper plot shows a reconstruction of the current using the Fourier components of the odd harmonics up to order 13. The lower left plot shows the amplitude of the Fourier coefficients of the deviation from the nominal current. The lower right plot shows the phase of the Fourier coefficients. The values estimated with the models are marked with a ring, (o), and the measured values are marked with a plus, (+).

Figures 8–10 show the validation of the model obtained with the Kaiser window. Figure 8 shows an almost perfect match between the estimated and measured current. The small errors in amplitude were expected, because of the linearization of the model. The distortion in the 7th harmonic of the voltage is about 7% of the nominal amplitude. Figure 9 shows a little larger deviations, but still the result is very good. The difference probably comes from the power electronics used to shape the voltage. It is thus an experimental error and not a method error. The plots in Figure 10 show a good result from measurements using another voltage source. The phase is perfect, and the error in amplitude can be explained by the large voltage distortion of 8% in the fundamental frequency.

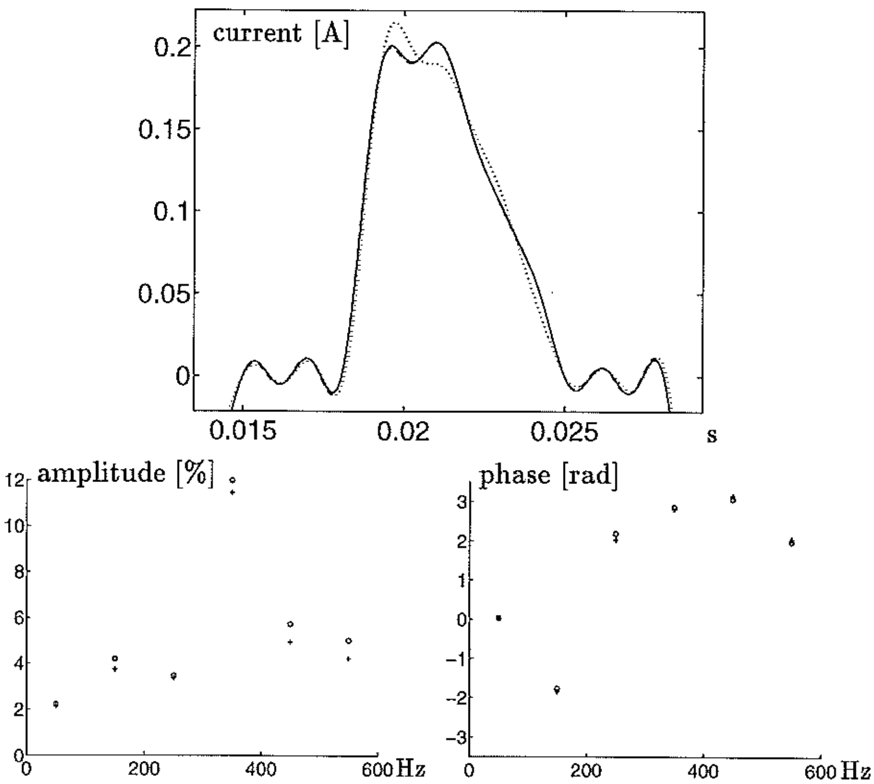

Figure 8 Validation of Kaiser window model using measurement 1. The upper plot shows a reproduction of the current using Fourier coefficients of odd harmonics up to order 13. The estimated current is solid, the measured current is dashed, and the nominal current is dotted. The lower plots show amplitude and phase of the Fourier coefficients of the current. Estimated values are marked with rings, (o), and measured values with a plus, (+). There is an almost perfect match between estimated and measured current.

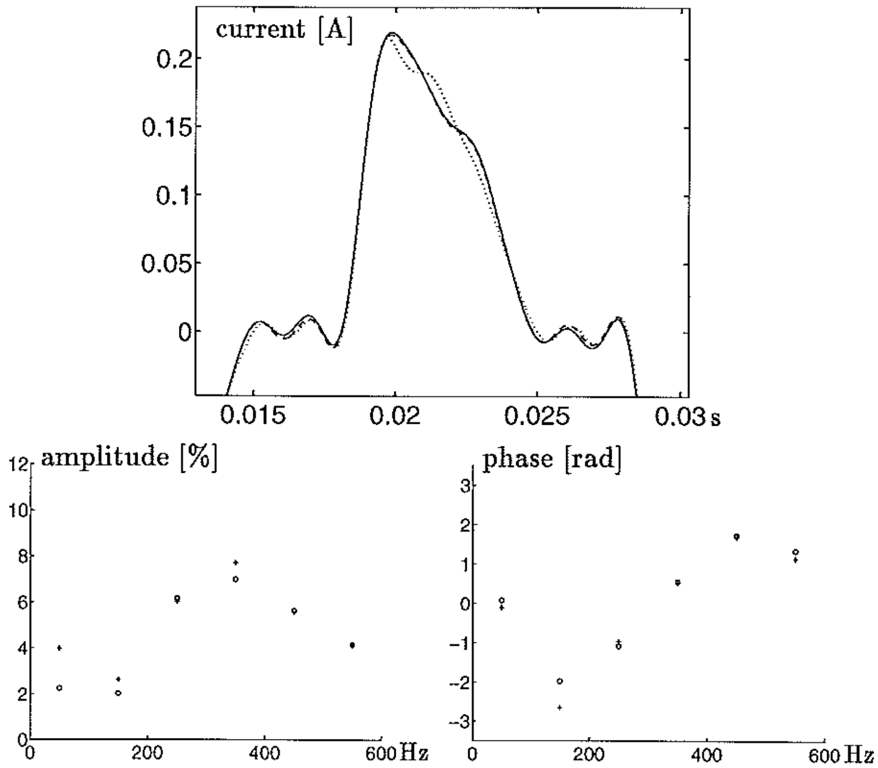

Figure 9 Validation of Kaiser window model using measurement 2.

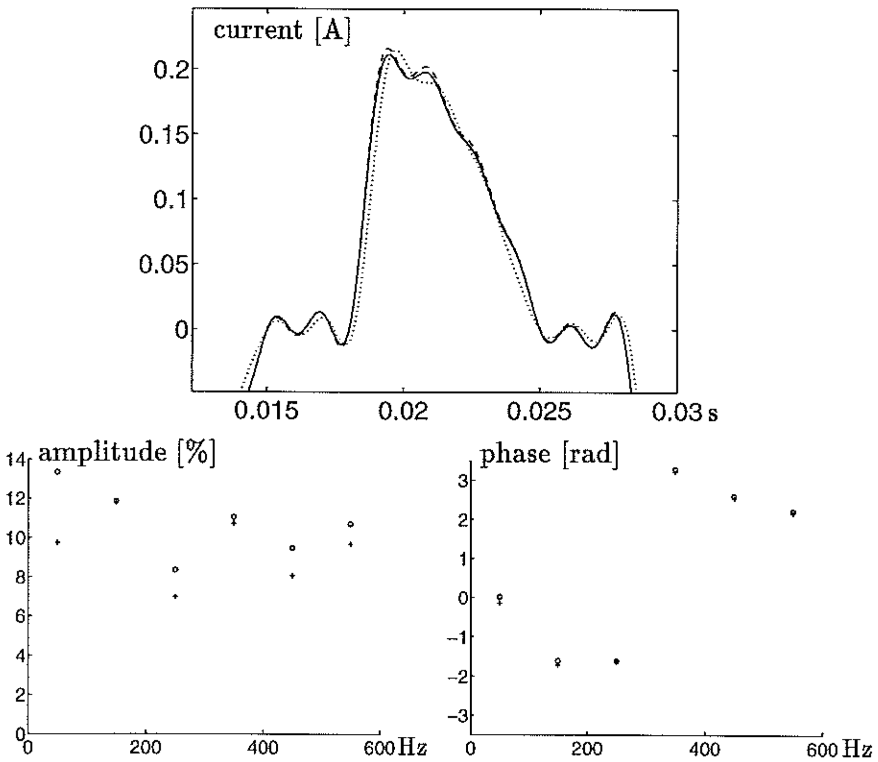

Figure 10 Validation of Kaiser window model using measurement 3. There is a voltage distortion of 8% in the amplitude of the fundamental frequency. Such a large deviation results in a small error when using linearized models.

The Bartlett window model is validated in Figures 11–13. The Bartlett window has a lower spectral leakage for periodic frequencies. Thus it ought to give a better result. In Figure 11, it is shown that this window performs better that the Kaiser window. Figures 12, and 13 show a worse results, however. This is probably because the signals are not completely periodic. It is then desired to split the measurement data in many segments, to get down the variance of the estimated parameters. The window should thus be short. For non-periodic signals, it is also important that the window has low spectral leakage not only for periodic frequencies. The Kaiser window would be the better choice in this situation. In Figure 13 there seems to be a systematic error in the amplitude.

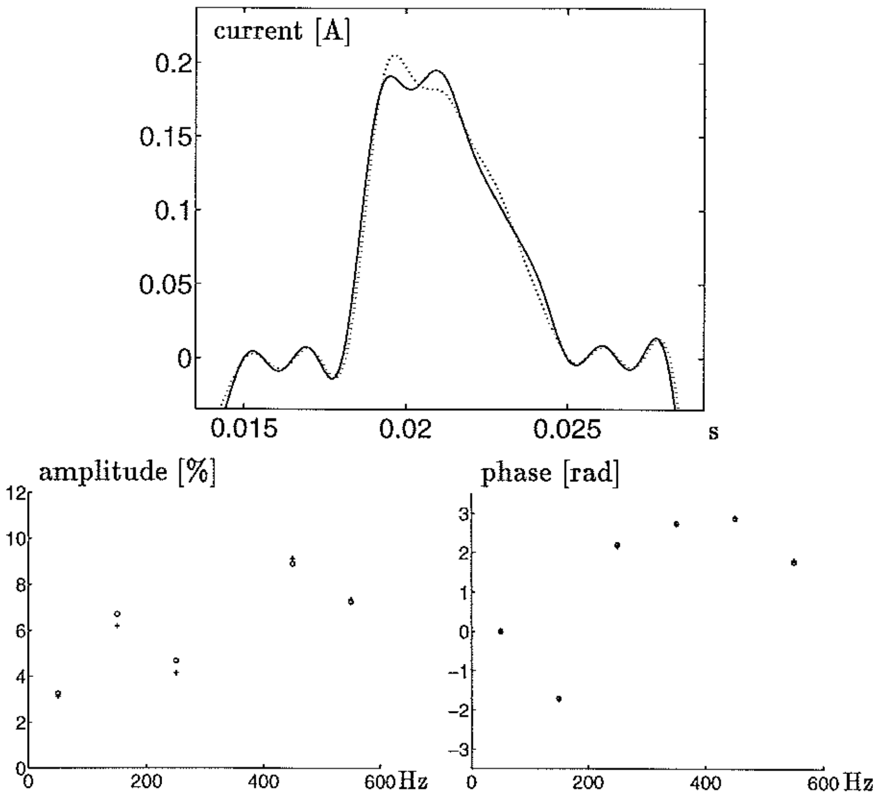

Figure 11 Validation of Bartlett window model using measurement 1. The result is even better than for the Kaiser window model.

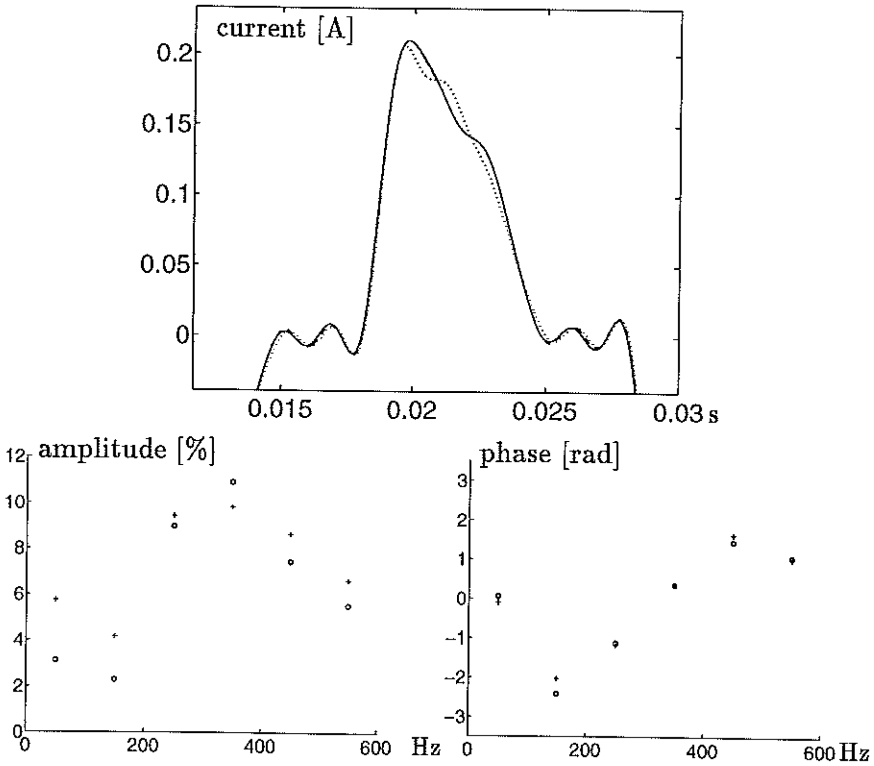

Figure 12 Validation of Bartlett window model using measurement 2. This shows a worse behavior. Probably due to non-periodicity in the signals.

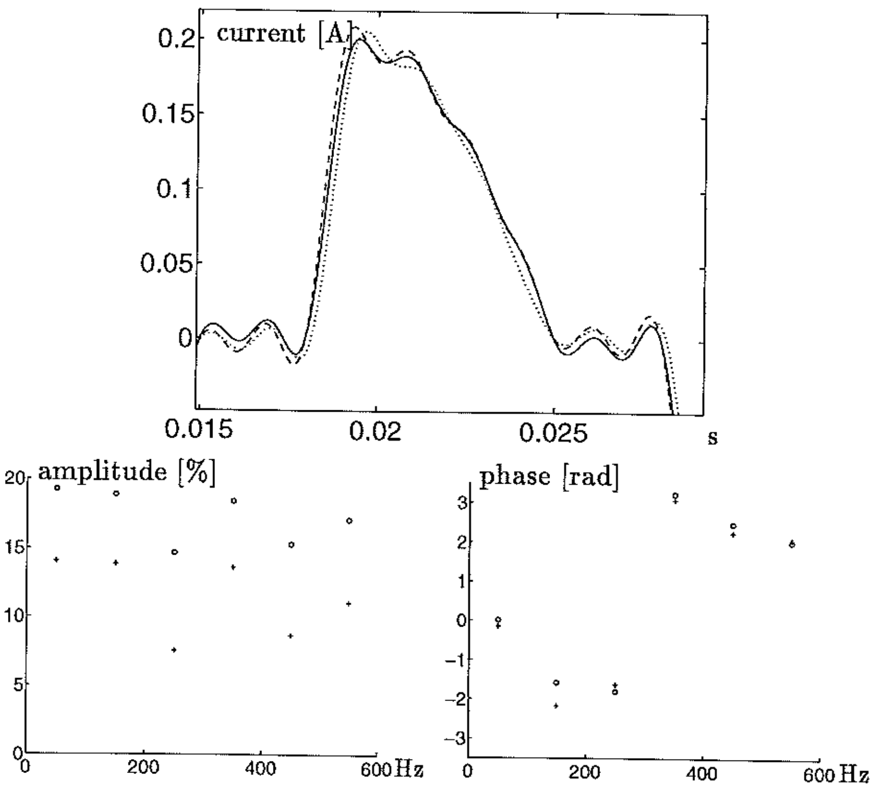

Figure 13 Validation of Bartlett window model using measurement 3. There seem to be a systematic error in the amplitude.

4.7 Discussion

The experiments were made in a lab environment, with little noise and almost periodic signals. In real applications, the results are probably dependent on proper signals processing. To decrease the variance, the sampled signals are split into smaller segments. The length of each segment is equal to the length of the window. Each segment is Fourier transformed, and the mean value is taken as the final result. The more segments there are, the smaller is the variation. This means that it is desired to have as short windows as possible. Short windows, however result in wide main lobes. The final choice of window must be a compromise.

The worst source of error is probably the voltage source used in the estimation process. The power electronic devices used to convert direct voltage to the desired shape is very temperature dependent. The voltage is also weak, which results in a considerable degree of distortion even for high harmonics.

A better way to shape the voltage in the estimation process would probably be to use the stiff line voltage, and modulate it with, for instance, a power amplifier. This way, the voltage would be more clean, and the $\widehat{V}$-matrix better conditioned, with smaller amplitudes for the harmonics that are not considered, and thus increased accuracy.

A close look at the current plot in Figure 3 shows that the current is not symmetric. When the dimmer is turned off, and the current is approximately zero, the shape of the curve is not exactly the same for positive and negative voltage. This results in harmonics of even order, which have not been considered in the model. There are also harmonics of order higher than 13.

The characteristics of a dimmer can be temperature dependent. In the experiment, the voltage had to be turned off between the different measurements. This means that the temperature, and thus the behavior of the dimmer might have been different for different measurements. It would be preferable to have the voltage on all the time, and just alter the distortion between measurements.

5. Conclusions and Future Work

We have presented a new method to model nonlinear loads for steady state analysis of distribution networks. In this report, we have shown that the parameters of the models can be obtained through real measurements, with good accuracy.

The next step is to find applications for the model. It would be interesting to see how the method can be of help in designing harmonic filters, that possibly can be adaptive. It would also be interesting to use the method to analyze large system. This could be combined with methods that guarantee stability and periodicity.

The method will constitute the basis of a licentiate thesis by Erik Möllerstedt. The thesis will be available early 1998.

6. References

FRIMAN, E. (1994): "Proposals for limits and responsibility sharing with regards to power quality." Technical Report. Swedish Transmission Research

Institute.

GILMORE, R. J. and M. B. STEER (1991): "Nonlinear circuit analysis using the method of harmonic balance – A review of the art." *Int J. Microwave and Millimeter-Wave Computer-Aided Eng.*, **1:1**, pp. 22–37.

IOTECH, INC. (1995): *Daqbook/Daqboard/Daq – User's Manual.*

KRAUSS, T. P., L. SHURE, and J. N. LITTLE (1993): *Signal Processing Toolbox.* MathWorks, Inc.

KUNDERT, K. S. and A. SANGIOVANNI-VINCENTELLI (1986): "Simulation of nonlinear circuits in the frequency domain." *IEEE Trans. on Computer-Aided Design*, **5:4**, pp. 521–535.

MÖLLERSTEDT, E., B. BERNHARDSSON, and S. E. MATTSSON (1997a): "A simple model for harmonics in electrical distribution networks." In *Proceedings of the 36th IEEE Conference on Decision and Control.* San Diego, California, USA.

MÖLLERSTEDT, E., S. E. MATTSSON, and B. BERNHARDSSON (1997b): "Modeling of electricity distribution network and components." report ISRN LUTFD2/TFRT--7557--SE. Department of Automatic Control, Lund Institute of Technology, Lund, Sweden.

MÖLLERSTEDT, E., S. E. MATTSSON, and B. BERNHARDSSON (1997c): "A new approach to steady-state analysis of power distribution networks." In SYDOW, Ed., *15th IMACS World Congress*, vol. 5, pp. 677–682. W& T Verlag, Berlin, Germany.

A. Discrete Fourier Transformation

To work with signals in the frequency domain, it is necessary to transform sampled time domain signals to the frequency domain. This is often done by discrete Fourier transformation, (DFT). The discrete Fourier transform, $X(k)$, of a sampled signal, $x(n)$, is defined as

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1)e^{-j\frac{2\pi kn}{N}}$$

$$x(n+1) = \frac{1}{N}\sum_{k=0}^{N-1} X(n+1)e^{j\frac{2\pi kn}{N}}.$$

This appendix describes how the DFT is used for Fourier analysis of time domain signals.

A.1 Sampling of signals

When analyzing a continuous time signal using the DFT, we first have to sample the signal. To avoid aliasing problems, the continuous time signal must be filtered through an anti-alias filter before sampling to make sure that all frequencies above the Nyquist frequency are attenuated. The Nyquist frequency is half the sampling frequency. The filters affect the phase and amplitude of the Fourier coefficients. This has to be compensated for, as we are considering nonlinear loads.

A.2 The Effect of Windowing

The DFT requires a finite sampled data series. This means that we have to pick out a small part of the often very long, or infinite signal. This is often referred to as windowing. The effect of windowing is reduced resolution and spectral leakage. The reduced resolution results in that peaks in the spectrum are smeared out. It makes it hard to separate peaks that are close to each other. With spectral leakage is meant the fact that a signal of a certain frequency affects other frequencies in the Fourier spectrum. This is shown in Figure 14. The frequency spectrum of constant signal has a discrete peak for zero frequency. Since the signal is of finite length, the peak is no longer discrete, but smeared out. There are also side lobes, which means that the constant signal affect the spectrum for frequencies far from zero. The resolution is determined by the width of the main lobe, whereas the spectral leakage is determined by the relative amplitude of the main lobe and the side lobes.

The window in Figure 14 is rectangular, that is all samples in the time domain signal are weighted equally. A rectangular window results in considerable side lobes, which gives severe spectral leakage. This is called Gibbs' phenomenon. By choosing another shape of the window the amplitude of the side lobes can be reduced. This reduces the problem with spectral leakage.

A popular window is the Kaiser window

$$w[n] = \begin{cases} \frac{I_0(\beta(1-((n-\alpha)/\alpha)^2)^{1/2})}{I_0(\beta)}, & 0 \leq n \leq M, \\ 0, & \text{otherwise,} \end{cases}$$

where $\alpha = M/2$, and I_0 is the zeroth-order modified Bessel function of the first

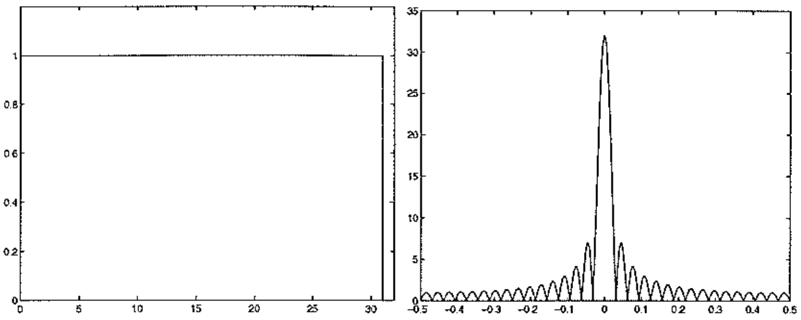

Figure 14 A finite data series gives rise to reduced resolution and spectral leakage. A rectangular time window gives very high side lobes. This is often referred to as Gibbs' phenomenon.

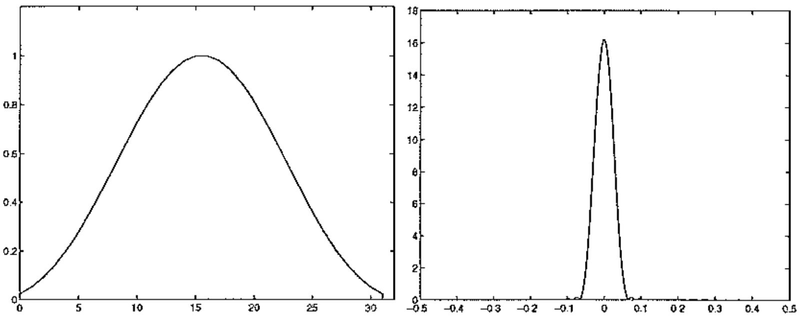

Figure 15 A Kaiser window reduces the side lobes, and thus the spectral leakage considerable.

kind. The window has two design parameters, the length, $(M+1)$, and a shape parameter β. With M you decide the width of the main lobe, and thus the resolution. The amplitudes of the side lobes are determined by β. Since these two design criteria are, almost, separated, it is very simple to design a Kaiser window that suits your needs. In Figure 15 it is shown that the amplitude of the side lobes can be reduced considerably, and thus the spectral leakage, by choosing a Kaiser window.

Much been written about how to choose windows for frequency analysis. Most of the time, the objective is to get the power spectrum for a signal. The power spectrum is the squared amplitude of the Fourier spectrum. This means that the phase of the spectrum is not investigated. As our method is very phase sensitive, this is a subject that requires more consideration.

A.3 Frequency Analysis of Periodic Signals

For periodic signals, we know that we only have the nominal frequency and harmonic overtones. This means that we do not need a high resolution, as the frequencies of the peaks in the spectrum are separated by at least the nominal frequency. It is, however, of major importance to correctly estimate the maximum of the peaks.

As shown in the definition, the discrete Fourier transform results in a discrete spectrum. The number of frequency samples equals the number of point in the sampled signal, x. The frequency samples are equally spaced up to the Nyquist frequency. The more samples there is in the sampled signal, the denser the spectrum is.

A problem with a large frequency interval between the samples in the spec-

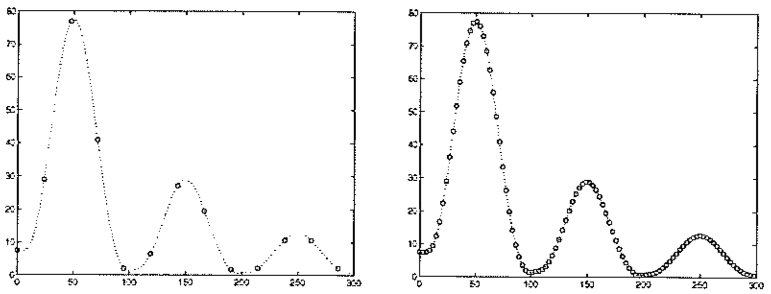

Figure 16 Zero padding the data file results in a denser spectrum, which makes the peaks easier to estimate. The resolution is, however, unchanged.

trum is that it is hard to estimate the maximum of the peaks, see Figure 16. To give the spectrum a denser spacing, the data series can be zero padded. This means that the size of the time domain series is increased by adding zeros at the end. The effect of zero padding is shown in Figure 16. Notice that the peaks are detected more accurately. The accuracy is, however, not increased.

An interesting aspect of the DFT is that the variance of the Fourier coefficients, $X(k)$, does not decrease if the data file is longer. This only gives a higher resolution. One way to decrease the variance is to split the data series into smaller, non-overlapping parts and take the mean value of the Fourier coefficients. This split reduces the resolution, as less data points are used for each Fourier transform, but as mentioned previously, resolution is not of importance for our application.