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Uncoupled Antenna Matching for Performance Optimization in Compact MIMO Systems using Unbalanced Load Impedance

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Abstract—¹Some MIMO applications require antennas to be closely spaced, which result in mutual coupling among antennas and high spatial correlation for signals. In order to compensate for the performance degradation due to correlation and coupling, impedance matching networks may be used. Recently, it was shown that uncoupled matching networks could be optimized against a given performance metric with the constraint of similar matching impedance for all antennas, i.e., balanced matching. In this paper, we investigate the use of uncoupled matching networks with both balanced and unbalanced load impedances, where either the received power or the channel capacity is optimized. For two- and three-element dipole arrays, we show numerically that a significant performance improvement can be achieved by introducing unbalanced matching. Observations suggest that the achieved improvement varies with array geometry and propagation environment. For example, a large capacity gain of up to 23% is realized when matching a linear array to propagation environments that are asymmetrical about the array broadside, whereas the symmetrical environments do not benefit as much from unbalanced matching.

I. INTRODUCTION

MULTIPLE-INPUT-MULTIPLE-OUTPUT (MIMO) wireless communication systems, by employing multiple antennas at both transmitter and receiver sides, offer the potential for linearly increased capacity with the number of antennas in a rich multipath environment [1]–[3]. Such systems perform best when the spatial correlation among signals on different antenna branches is low [3]. Therefore, the implementation of MIMO technology in a compact terminal is very challenging, since it requires multiple antenna elements to be closely spaced, which leads to both high correlation between received signals and high electromagnetic interactions (or mutual coupling) among antenna elements. Mutual coupling is a key concern for multiple antenna systems since strong coupling results in a loss of efficiency of the antenna system [4].

Recent work has shown that, by employing matching networks on the receive side, it is possible to some extent to control the reception characteristics. The optimal performance can be achieved if a coupled matching network is utilized [5]. However, such a network is difficult to construct and typically results in a narrowband operation [6], [7]. As a result, a simple

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uncoupled (or individual port) matching network is recently studied [8]–[10]. Using uncoupled matching, it is possible to optimize the matching network for low correlation, high received power and high channel capacity. A more recent study [6] further concludes that, under certain propagation environments, an uncoupled impedance matching network can be designed to give a comparable performance to that of the optimal termination. It also suggests that the performance of uncoupled matching tends to degrade relative to optimal matching if the propagation environment has reduced directivity. However, [6] only attempts to maximize the received power in a closed form approach, which may result in nonpassive loads. Therefore, a numerical optimization within a constrained search space is still required. Moreover, numerical results are only given for maximized received power of an antenna array with two elements.

In this paper, we extend the study of [6], [8]–[10] in several ways. First, we increase the degrees of freedom for uncoupled matching by introducing unbalanced networks, where different antenna branches are terminated by arbitrary load impedances. Second, we investigate the performance optimization in terms of received power as well as mean capacity with both balanced and unbalanced load impedances, under the scenarios of 2D uniform and 2D Laplacian angular power spectra (APS). We show numerically how much improvement can be achieved by using unbalanced matching instead of balanced matching, which was not covered in [6]. In addition, we study the effects of having different array sizes and geometries by analyzing uniform linear arrays (ULAs) of two and three dipoles, as well as a uniform triangular array (UTA) of three dipoles. To our knowledge, this is the first contribution that considers uncoupled matching for three-dipole arrays and the effects of different array sizes and geometries.

The remainder of the paper is organized as follows. In Section II, we describe the MIMO system model with uncoupled matching using a Z-parameter approach. Then the metrics for system performance are introduced in Section III. In Section IV, we compare numerically the performance optimization for different propagation scenarios between balanced and unbalanced load impedances. Section V concludes the paper.

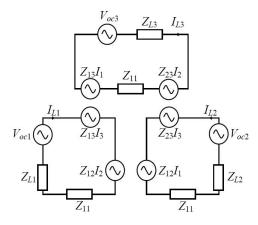


Fig. 1. Equivalent three-dipole array receive circuit

II. SYSTEM MODEL

A MIMO system model consists of M transmit circuits, a multipath propagation channel, and N receive circuits. In this work, we consider half-a-wavelength ($\lambda/2$) electric dipole antennas for the transmit and receive arrays. Our focus is on the receive side, where the array antennas are closely coupled. Using a Z-parameter approach, the m-th transmit circuit consists of the voltage source V_{Sm} , the source impedance Z_{Sm} and a dipole antenna. The antenna array is represented by self and mutual impedances. Since the case of the two-dipole array has been discussed in great detail [8]–[10], only the three-dipole array is analyzed in this paper. The three dipoles are identical so that they have the same self impedance Z_{11} . We assume that transmit antennas are far apart from each others and have negligible mutual coupling, i.e., $Z_{12}=Z_{13}=Z_{23}=0$ and $Z_{12} = Z_{21}, Z_{13} = Z_{31}, Z_{23} = Z_{32}$. The transmit antennas are conjugate matched to the source impedance as follows:

$$Z_{\rm S} = Z_{11}^*$$
 (1)

The excitation current is the source of radiation and it is given by

$$\begin{bmatrix} I_{S1} \\ I_{S2} \\ I_{S3} \end{bmatrix} = \frac{1}{2 \operatorname{Re} Z_{11}} \begin{bmatrix} V_{S1} \\ V_{S2} \\ V_{S3} \end{bmatrix}.$$
 (2)

The receive circuit for the three-dipole array is given in Figure 1, where $\{V_{\text{oc1}}, V_{\text{oc2}}, V_{\text{oc3}}\}$ denote open-circuit voltages, Z_{11} is the self impedance, $\{Z_{12}, Z_{13}, Z_{23}\}$ are mutual impedances between different pairs of elements. $\{Z_{\text{L1}}, Z_{\text{L2}}, Z_{\text{L3}}\}$ are the equivalent impedance loads (i.e., matching network and the rest of the receive circuit) with which the antennas are terminated. The open-circuit voltages depend on the scattering environment and they are correlated due to the finite spacing among the receive antennas. The complex open-circuit correlation coefficient α_{ij} depends on the antenna spacing and the angular spread, and is given by

$$\alpha_{ij} = \frac{\mathrm{E}\{V_{\mathrm{oc}i}V_{\mathrm{oc}j}^*\}}{\mathrm{E}\{V_{\mathrm{oc}i}V_{\mathrm{oc}i}^*\}}.$$
 (3)

It is known that the currents and thus the voltages over the loads can be partly decorrelated by choosing different load impedances [8]. For the three-dipole array, the antenna impedance \mathbf{Z}_{A} and the termination load impedance \mathbf{Z}_{L} are given by

$$\mathbf{Z}_{A} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12} & Z_{11} & Z_{23} \\ Z_{13} & Z_{23} & Z_{11} \end{bmatrix}, \tag{4}$$

$$\mathbf{Z}_{L} = \begin{bmatrix} Z_{L1} & 0 & 0 \\ 0 & Z_{L2} & 0 \\ 0 & 0 & Z_{L3} \end{bmatrix}.$$
 (5)

Note that we have the freedom for choosing $\mathbf{Z}_{\rm L}$ with arbitrary $\{Z_{\rm L1}, Z_{\rm L2}, Z_{\rm L3}\}$. Finally, the voltages over the loads are

$$\begin{bmatrix} V_{\mathrm{L1}} \\ V_{\mathrm{L2}} \\ V_{\mathrm{L3}} \end{bmatrix} = \mathbf{Z}_{\mathrm{L}} \begin{bmatrix} I_{\mathrm{L1}} \\ I_{\mathrm{L2}} \\ I_{\mathrm{L3}} \end{bmatrix} = \mathbf{Z}_{\mathrm{L}} \left(\mathbf{Z}_{\mathrm{L}} + \mathbf{Z}_{\mathrm{A}} \right)^{-1} \begin{bmatrix} V_{\mathrm{oc1}} \\ V_{\mathrm{oc2}} \\ V_{\mathrm{oc3}} \end{bmatrix}.$$
(6)

For the multipath propagation environment, we use the Kronecker model in which the channel matrix can be described as

$$\mathbf{H}_{ch} = \mathbf{\Psi}_{R}^{1/2} \mathbf{H}_{iid} (\mathbf{\Psi}_{T}^{1/2})^{\mathrm{T}}, \tag{7}$$

where the transmit correlation matrix $\Psi_T = \mathbf{I}_3$, \mathbf{I}_m is the identity matrix of dimension $m \times m$, and the receive correlation matrix Ψ_R is represented as

$$\Psi_{\mathbf{R}} = \begin{bmatrix}
1 & \alpha_{12} & \alpha_{13} \\
\alpha_{12}^* & 1 & \alpha_{23} \\
\alpha_{13}^* & \alpha_{23}^* & 1
\end{bmatrix}.$$
(8)

The entries of $\mathbf{H}_{\mathrm{iid}}$ are complex Gaussian random variables of zero mean and variance of 1.

III. PERFORMANCE METRICS

In this paper, we investigate the performance metrics in terms of received power as well as channel capacity. The mean received power at the n-th receive antenna element is given by

$$P_n = \frac{\mathrm{E}\{|I_n|^2 \operatorname{Re} Z_{\operatorname{L}n}\}}{\mathrm{E}\{|I_0|^2 \operatorname{Re} Z_{11}\}},\tag{9}$$

where I_0 is the current induced by open-circuit voltage $V_{\rm oc}$ over a single conjugate matched receive antenna. In this way, the received power is normalized by the power of an optimally matched single antenna. The closed form for the received power can be found by using the network expressions (3) through (6), and for the two-dipole array case it has been given in [8]–[10].

Another important criterion is the channel capacity which represents the theoretical limit for error-free transmission data rate. In this paper, we only consider the case of no channel state information (CSI) at the transmitter, *i.e.*, equal power over transmit antennas. The mean capacity can be obtained from either independent realizations of the channel in (7), or the closed form expression of [11], which is a function of the mean received power and output correlations. For our purpose,

both methods show good agreement and we adopt the latter approach in this paper.

The performance optimization is performed by searching for the \mathbf{Z}_{L} that maximizes either the total mean received power or mean capacity. Previous studies in [8]-[10] show that, using uncoupled matching with similar load impedances, there exist two maxima for the received power, whereas a single maximum is observed for the mean capacity. Therefore, we note that local optima may occur and the global solution cannot be guaranteed through our use of Matlab's simple optimization routines. However, a large number of random initial points are utilized to minimize the risk of obtaining local solutions. Besides, in order to get passive networks, we also constrain the search space within the rectangular region defined by the diagonally opposite corner points between $0 - i500 \Omega$ and $500 + i500 \Omega$. We find that the above strategy suffices for our purpose of determining the performance gain that can be achieved by the use of unbalanced matching in comparison to balanced matching.

IV. NUMERICAL STUDIES AND DISCUSSIONS

In this section, we numerically evaluate the matching performance of thin half-a-wavelength ($\lambda/2$) dipoles that are closely spaced in the receive antenna array. We use the induced EMF method to calculate the self and mutual impedances of the dipoles. The antenna self impedance is $Z_{11} = 73 + i42 \Omega$. The mutual impedance is however related to the array size and structure and it will be given separately for each of the different array types we study. Our investigation concerns different propagation environments: uniform 2D APS and Laplacian 2D APS defined by the mean and standard deviation of the distribution. We study the matching performance in terms of maximized total mean received power and mean capacity, making comparison between balanced and unbalanced matching networks. The array structures for the analysis include a ULA of two dipoles, a ULA of three dipoles and a UTA of three dipoles. A reference SNR of 20 dB is assumed.

A. Two-dipole ULA

Two identical dipole antennas are placed next to each other. At the separation distance $d=0.1\lambda$, the mutual impedance is $Z_{12}=67+i7.6\,\Omega$. In uniform 2D APS, the open-circuit correlation is 0.9. Three Laplacian 2D APS are considered having different means of $\mu=\{0^\circ,45^\circ,90^\circ\}$ but with the same standard deviation of $\sigma=\{30^\circ\}$. The number of degrees of freedom of the optimization search space is 4, *i.e.*, $\{\operatorname{Re} Z_{\mathrm{L}i}, \operatorname{Im} Z_{\mathrm{L}i}, i=1,2\}$. The obtained maximized power and capacity for the two-dipole ULA is given in Table I for uniform APS and Table II for Laplacian APS.

For comparison, the theoretical mean capacity of uncorrelated Rayleigh channels for a 2×2 MIMO system with no CSI at the transmitter and SNR of $20\,\mathrm{dB}$ is $11.3\,\mathrm{bits/s/Hz}$. The results show that it is possible to improve the received power in all propagation environments relative to that of a single dipole ($0\,\mathrm{dB}$ corresponds to the power received by a single conjugate matched dipole). For the uniform APS and the

TABLE I $\label{eq:continuous} \text{Optimized matching performance for two-dipole ULA under }$ uniform 2D APS

Uniform 2D APS	Balanced	Un-Balanced
Total mean received power (dB)	1.41	1.41
Mean capacity (bits/s/Hz)	9.98	9.98

TABLE II

OPTIMIZED MATCHING PERFORMANCE FOR TWO-DIPOLE ULA UNDER

LAPLACIAN 2D APS

Power (dB)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	0.39	0.87	3.31
Un-Balanced	0.39	3.21	4.64
Capacity (bits/s/Hz)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	8.71	8.42	8.00
Un-Balanced	8.71	8.49	8.21

broadside ($\mu=0^\circ$) Laplacian APS, unbalanced matching does not perform better than balanced matching. It is mainly due to the fact that the incident field is symmetrical about the broadside of the two-dipole ULA. Therefore, the performance is already optimum with balanced matching. However, for other Laplacian APS that is asymmetrical about the array broadside, the unbalanced matching network facilitates better "tuning" to the asymmetrical APS. We observe that the optimized received power and capacity is improved compared to the balanced network. A significant gain of up to $2.3\,\mathrm{dB}$ is observed for the total mean received power whereas a modest of $0.2\,\mathrm{bits/s/Hz}$ is achieved for the mean capacity. We also observe that the super directivity characteristics [9] of the two closely coupled dipoles enabling more than $3\,\mathrm{dB}$ of power gain against the reference single dipole.

B. Three-dipole ULA

We extend our investigation to study the performance of three-dipole arrays. A three-dipole ULA is first arranged as to the left in Figure 2. The separation between adjacent elements is $d=0.1\lambda$. The mutual impedances are calculated to be

$$\begin{bmatrix} Z_{12} \\ Z_{13} \\ Z_{23} \end{bmatrix} = \begin{bmatrix} 67 + i7.6 \\ 51 - i19 \\ 67 + i7.6 \end{bmatrix} \Omega.$$
 (10)

In this case, the number of degrees of freedom of the optimization search space is increased to 6, *i.e.*, $\{\operatorname{Re} Z_{\operatorname{L}i}, \operatorname{Im} Z_{\operatorname{L}i}, i=1,2,3\}$. The optimization results are given in Tables III and IV.

For comparison, the theoretical mean capacity of uncorrelated Rayleigh channels for a 3×3 MIMO system with no CSI at the transmitter and SNR of 20 dB is $16.7 \, \mathrm{bits/s/Hz}$. First, as was observed for the two-dipole ULA in [9], we find that a single maximum is obtained for the mean capacity of the three-dipole ULA. An example is shown in Figure 3, where the array is matched to the Laplacian $(90^{\circ}, 30^{\circ})$ APS with balanced matching. Second, by using unbalanced load impedances, the

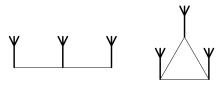


Fig. 2. (a) three-dipole ULA and (b) three-dipole UTA

TABLE III

Optimized matching performance for three-dipole ULA under uniform 2D APS

Uniform 2D APS	Balanced	Un-Balanced
Total mean received power (dB)	2.24	3.38
Mean capacity (bits/s/Hz)	12.19	12.67

optimization performance can be improved for all scenarios, including the symmetrical uniform APS and broadside Laplacian APS. In this linear array structure, the center element suffers from the strongest mutual coupling, such that one cannot adequately compensate for it with uncoupled matching. This phenomenon is better illustrated with the antenna patterns. Figure 4 shows the antenna patterns of the three-dipole ULA matched for maximized receive power under uniform APS for both balanced and unbalanced matching. The antenna patterns are normalized against the omnidirectional azimuth pattern of a single conjugate matched dipole (i.e., the reference pattern is at 0 dB). We observe that the patterns for elements 1 and 3 are symmetrical about the array broadside and have reasonable gain values of 0 dB and 4 dB for balanced and unbalanced matching, respectively, whereas the center element has very low gains of $-4 \, dB$ and $-5 \, dB$. With unbalanced matching, we can partially decouple the patterns of elements 1 and 3, which leads to a larger maximum receive power than with balanced matching. Numerically, we can improve the maximized total mean received power by up to $1.6\,\mathrm{dB}$ and the maximized channel capacity by up to 2.2 bits/s/Hz (23%). The results also suggest that the performance can be improved much more significantly under propagation environments that are asymmetrical about the array broadside.

C. Three-dipole UTA

In order to investigate the impact of having different array geometries, we also study a three-dipole UTA. The geometry

TABLE IV $\label{eq:continuous} \text{Optimized matching performance for three-dipole ULA under } \\ \text{Laplacian 2D APS}$

Power (dB)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	0.94	2.26	3.72
Un-Balanced	1.92	3.42	5.30
Capacity (bits/s/Hz)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	10.46	9.89	8.89
Un-Balanced	11.16	12.13	11.78

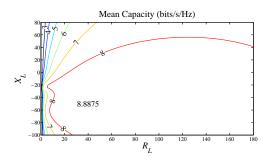


Fig. 3. Mean capacity of three-dipole ULA under Laplacian $(90^\circ, 30^\circ)$ APS with balanced matching

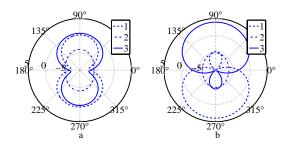


Fig. 4. Antenna patterns of the three-dipole ULA under uniform APS with (a) balanced (b) unbalanced matching for maximized receive power.

is given as the right in Figure 2, where $d=0.1\lambda$. In this case, the antenna mutual impedances are identical between different pairs of antenna elements. The optimization results are given in Tables V and VI.

First, the results show that the array has similar maximized receive power when using balanced matching to all different propagation scenarios. However, we note that the received power on individual antenna branches are different for the different scenarios, even though they share the same total power (which is the objective function of the optimization). Second, no improvement is achieved using unbalanced matching under uniform APS. This is because, as illustrated in Figure 5, that three antenna patterns are already evenly displaced from one another in the azimuth plane with balanced matching, which fits well with the distribution of the uniform APS. Furthermore, comparisons between balanced and unbalanced matching in Tables V and VI reveal that the performance benefit of using unbalanced matching is noticeably smaller than the previous case of the three-dipole ULA. Numerically, we obtain up to 0.85 dB improvement for the maximized total

TABLE V $\label{eq:continuous} \text{Optimized matching performance for three-dipole UTA under }$ uniform 2D APS

Uniform 2D APS	Balanced	Un-Balanced
Total mean received power (dB)	4.04	4.04
Mean capacity (bits/s/Hz)	14.89	14.89

TABLE VI
OPTIMIZED MATCHING PERFORMANCE FOR THREE-DIPOLE UTA UNDER
LAPLACIAN 2D APS

Power (dB)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	4.04	4.04	4.04
Un-Balanced	4.89	4.84	4.85
Capacity (bits/s/Hz)	$(0^{\circ}, 30^{\circ})$	$(45^{\circ}, 30^{\circ})$	$(90^{\circ}, 30^{\circ})$
Balanced	12.52	12.69	12.75
Un-Balanced	12.75	12.82	12.75

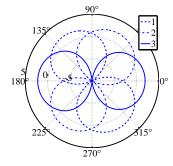


Fig. 5. Antenna patterns of the three-dipole UTA under uniform APS with matching for maximized receive power.

mean received power and up to 2% improvement for the maximized mean capacity. The modest improvement can be explained by noting that, relative to the 1D three-dipole ULA, the 2D three-dipole UTA demonstrates a more uniform effective aperture as seen from different azimuth angles. This results in increased symmetry between the array and the propagation environment. The improvement introduced by the unbalanced matching network increases with reducing symmetry between the array geometry and the propagation environment.

It is mentioned in [6] that the matching network can synthesize the antenna patterns such that they form beams towards the incident field. However, no example or illustration was provided. In our study, we confirm that the uncoupled matching network with unbalanced load impedances adapts (or beamform) the antenna patterns to the given propagation environment. This is demonstrated in Figure 6, in which the three-dipole UTA is matched to maximize the received power under various environments, using unbalanced load impedances.

V. CONCLUSIONS

In this paper, we show that significant improvements can be realized with unbalanced matching network when employing uncoupled antenna matching for performance optimization in compact MIMO systems. The achieved improvement varies with array geometry and propagation environment. In particular, we conclude that unbalanced matching is especially effective for arrays with effective aperture that can vary significantly with respect to the propagation environment, such as ULAs. Finally, the antenna patterns of the three-dipole

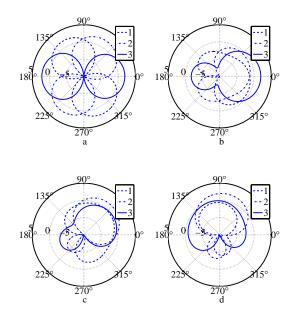


Fig. 6. Antenna patterns of the three-dipole UTA with unbalanced matching for maximized receive power under various environments. (a) uniform APS; (b) Laplacian $(0^{\circ}, 30^{\circ})$ APS; (c) Laplacian $(45^{\circ}, 30^{\circ})$ APS; (d) Laplacian $(90^{\circ}, 30^{\circ})$ APS.

UTA under different propagation environments are shown to illustrate the beamforming ability of the matching networks.

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