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LUND INSTITUTE OF TECHNOLOGY · LUND · SWEDEN
DEPARTMENT OF STRUCTURAL MECHANICS
REPORT NO. 79 - 5

BERTIL FREDLUND

STRUCTURAL DESIGN OF FIRE EXPOSED
RECTANGULAR LAMINATED WOOD BEAMS
WITH RESPECT TO LATERAL BUCKLING

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1. Introduction

This study treats the problem of calculating the ultimate load of laminated beams exposed to fire. Because the section of the beam is reduced by time in a fire and the strength and stiffness of the un-charred material are decreased with increased temperature and moisture content, the ultimate load is decreased by time. The time, that passes until the ultimate load has been decreased to applied load, when failure occurs, is called the time to failure. A method for calculating the time to failure in pure bending of rectangular wood beams is provided in [1]. This reference also calls attention to the increased risk of lateral buckling when the width-height ratio of a beam is decreased by charring. The risk is also raised if intermediate supports fail during the fire exposure.

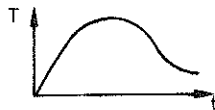
The design method presented below for fire exposed wood beams has been developed according to the method in SBN 1975 [2] regarding lateral buckling of wood beams at normal temperature. The complete design procedure is provided in comments on SBN [3] and briefly divided into 7 steps:

- (1) Determination of the design load at fire
- (2) determination of the design fire load density f , defined as the total heat value of all combustible materials in the fire compartment, divided by the total interior area of the surfaces bounding the compartment, opening areas included, A_{tot}
- (3) determination of the opening factor of the fire compartment $Av\bar{h}/A_{tot}$, where A = total area of window and door openings, h = mean value of the heights of window and door openings, weighed with respect to each individual opening area
- (4) transformation of the design fire load density f and the opening factor $Av\bar{h}/A_{tot}$, with regard to the type of fire compartment, to a fictive fire load density f_{fict} and a fictive opening factor $(Av\bar{h}/A_{tot})_{fict}$ - by that taking into account the real thermal properties of the structures surrounding the fire compartment

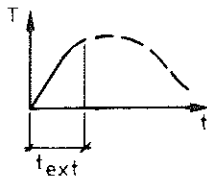
- (5) determination of the average regression rate β of the charred zone with the fictive opening factor as input (fig 1)
- (6) determination of the time to failure \bar{t}
- (7) determination of the corresponding fire duration time t_b - as a temporary solution, t_b is to be taken as the time from the start of the fully developed fire to the time when the gastemperature has been decreased to 300°C .

The design criteria is performed when the time for failure of the load-bearing system \bar{t} according to (6) is greater than the fire duration time t_b according to (7). From the design load at step (1), a safety factor K against failure in pure bending before fire can be identified. The determination of the fire duration time according to step (7) could be divided in three different cases depending on the requirements in the practical application:

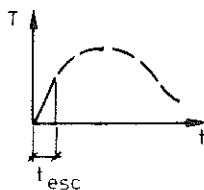
- (a) A complete fire process



- b) a shortened fire process, limited by the time t_{ext} necessary for the fire to be extinguished under the most severe conditions



- c) a shortened fire process, limited by the design evacuation time t_{esc} for the building.



Decisive for which of the cases (a) - (c) the design procedure should be based on is economical considerations from the builder. In this connection the value of the building, the value of contents of the building, costs of dropped production, costs of reconstructing and repairing is a relevant basis for forming an estimation.

If the complete fire process is accounted, i.e. alternative (a), the temporary solution based on (5) and (7) is an available method. There are theoretical models available [4], [5] that give a more accurate information on when the charring is stopped and on magnitude of the average charring rate at step (5) as well. A systematical theoretical calculation of average charring rates for wood beams, exposed to different gastemperature-time curves, is being prepared by the author.

With an assumed average charring rate β and a coefficient μ that accounts for reductions in strength and stiffness of the uncharred material due to the increased temperature and moisture content, the critical load is determined with different safety factors and giving the time until failure \bar{t} . The results are illustrated in the Fig 4a - 4h and 6a - 6h with the ratio $r = \beta\bar{t}/B$ as a function of the width-height ratio B/H for different values of the factor $\eta = \sqrt{\frac{LH}{mB^2}}$ expresses the contribution of geometry and types of support due to slenderness before fire. The time to failure becomes $\bar{t} = r \frac{B}{\beta}$. The calculated design diagrams are applicable to free lateral buckling of beams of rectangular cross section which are prevented from torsion at the supports. If lateral restraints exist and these are kept intact during fire, the instructions in SBN chapter 27:33I are applicable to the reduced section.

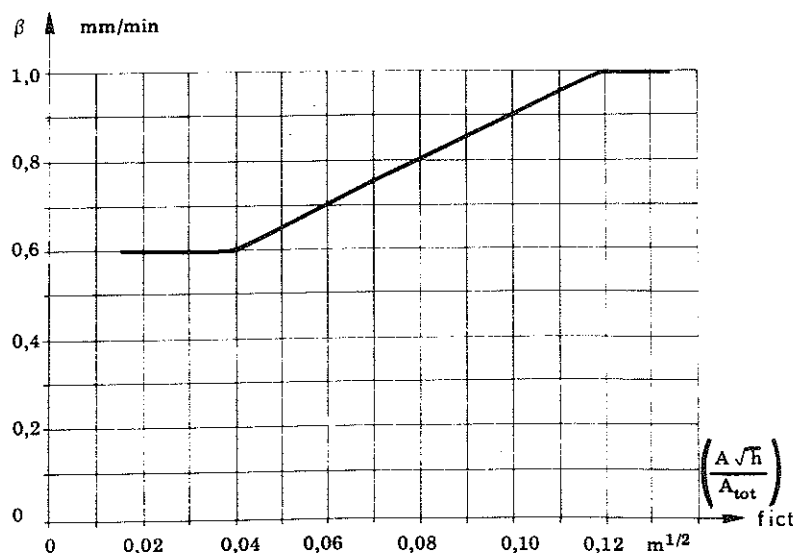


Fig 1. Charring rate β influenced by opening factor $A\sqrt{h}/A_{tot}$ [6].
Temporary relation

2. Allowable stress

The allowable stress for a beam that is loaded at ordinary room temperature by a bending moment in a symmetrical plane must be reduced to account for the risk of lateral buckling. Swedish building regulations [2] specify that the allowable bending stress σ_{ba} be reduced by a coefficient κ_v which is a function of the slenderness α ,

$$\alpha = \sqrt{\frac{\sigma_{ub}}{\sigma_{el}}} \quad (1)$$

where σ_{el} is the maximum bending compression stress for the lateral buckling load according to the theory of elasticity. σ_{ub} is a characteristic value of the ultimate bending stress calculated on the assumption of a linear stress distribution. In SBN [2] σ_{ub} is specified to $3\sigma_{ba}$ where σ_{ba} is the allowable bending stress.

The reduction coefficient κ_v is given by the relations

$$\kappa_v = 1 \quad \text{when } \alpha \leq 0.6 \quad (2a)$$

$$\kappa_v = 1.37 - 0.61\alpha \quad \text{when } 0.6 < \alpha < 1.4 \quad (2b)$$

$$\kappa_v = 1/\alpha^2 \quad \text{when } \alpha \geq 1.4 \quad (2c)$$

The above relations are valid if the initial deformation in the horizontal plane is less than 1/300 of the distance between points restrained from deformations in the horizontal plane.

The reduction coefficient κ_v is represented in Fig 2 as a function of the slenderness α .

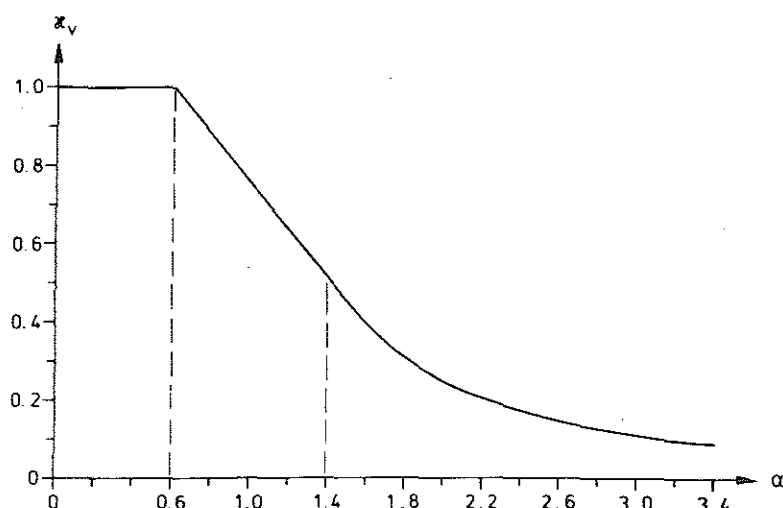


Fig 2. The reduction coefficient κ_v as a function of the slenderness α

If the slenderness is less than 1.4, the beam will buckle non-elastically. When the slenderness is greater than 1.4 the beam will buckle elastically. The curve $\kappa_V(\alpha)$ is drawn so that a desired uniform safety factor with varying α is ensured.

3. The lateral buckling load

Handbooks such as Bygg 1A [7] and comments to Stålbyggnadsnorm 70 [8] provide the solution for the lateral buckling load of a two supported beam with or without intermediate lateral supports and lateral constraints at the ends. For a beam of narrow rectangular section the warping rigidity is negligible and the following equation can be used to the critical load, if the material is considered as elastic:

$$\left. \begin{array}{l} M_{cr} \\ P_{cr} \cdot L \\ q_{cr} \cdot L^2 \end{array} \right\} = m \frac{\sqrt{B_y C}}{L} \quad (3)$$

where M_{cr} , P_{cr} and q_{cr} are the critical moment, concentrated load and distributed load respectively and m is a coefficient depending on the load and support conditions.

The torsional rigidity is given by:

$$C = G K_V \quad (4)$$

and the flexural rigidity in bending about the y-axis by:

$$B_y = EI_y \quad (5)$$

The validity of equation (3) is limited to cross sections with $B_y \ll B_x$ where B_x is the flexural rigidity in bending about the x-axis. For a beam of rectangular cross section equation (3) gives sufficiently accurate results for $B/H < 0.15$ [9]. B is the width and H the height of the cross section. For $B/H > 0.15$ equation (3) must be generalized.

Flint [10] gives the critical value of a concentrated load in a generalized form as

$$P_{cr} \cdot L = \frac{m}{L} \sqrt{\frac{B_y C}{\gamma}} \quad (6)$$

where

$$\gamma = 1 - B_y/B_x \quad (7)$$

Pettersson [9] gives a still more accurate value of the critical moment in pure bending as

$$M_{cr} = \frac{m}{L} \sqrt{\frac{B_y C}{\gamma(1 - \frac{C}{B_x})}} = \frac{m}{L} \sqrt{\frac{B_y C}{(1 - \frac{B_y}{B_x})(1 - \frac{C}{B_x})}} \quad (8)$$

with

$$B_x = EI_x \quad (9)$$

When the section is quadratic the buckling load approaches infinity and is thus no longer the critical load.

The formula for calculating the torsion constant for a rectangular cross section can be put in the form

$$K_V = \frac{1}{3} \left(1 - \frac{192}{\pi^5} \frac{B}{H} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^5} \tanh \frac{n\pi H}{2B} \right) B^3 H = \frac{1}{3} f \left(\frac{B}{H} \right) B^3 H \quad (10)$$

Substitution of Eqs. (4), (5), (9), (10) and expressions for I_x and I_y into Eq. (8) yields

$$M_{cr} = \frac{mB^3 H}{6L} \sqrt{\frac{f(B/H)}{1 - (B/H)^2} \cdot \frac{EG}{1 - 4\frac{G}{E} f(B/H) \left(\frac{B}{H}\right)^2}} \quad (11)$$

4. The critical stress

The critical value of the moment corresponds to a critical stress which, if linear stress distribution is assumed, can be represented in the form

$$\sigma_{cr} = \frac{mB^2}{LH} \sqrt{\frac{f(B/H)}{1 - (B/H)^2} \cdot \frac{EG}{1 - 4\frac{G}{E} f(B/H) \left(\frac{B}{H}\right)^2}} \quad (12)$$

Substitution of the critical value σ_{cr} for σ_{e1} in Eq. (1) where it is assumed that $E/G = 20$ is a suitable value for wood, yields a new expression for the slenderness

$$\alpha = \frac{\sqrt{\sigma_{ub}}}{4\sqrt{G \cdot E}} \sqrt{\frac{LH}{mB^2}} \sqrt{\frac{[1 - (B/H)^2][1 - 1/5 f(\frac{B}{H})(B/H)^2]}{f(\frac{B}{H})}} \quad (13)$$

Numerical values applicable to wood according to SBN 1975 yield

$$\frac{\sqrt{\sigma_{ub}}}{4\sqrt{GE}} = 0.13$$

5. Fire exposure on four sides

In Fig 3 the section of a beam before exposure to fire is defined as having height H and width B . The beam is exposed to fire from four sides. After the time \bar{t} , the section has been reduced by charring to height h and width b . The notation β is used for the average charring rate. In a standard fire, if the charred zone $\beta \bar{t}$ is less than or equal to $B/4$ or $\beta \bar{t}/B$ is less than or equal to $1/4$ when B is less than or equal to H , then the charring rate is approximately constant [11], [12], [13].

Before fire exposure the bending moment in the decisive cross section is M_1 , where the magnitude of M_1 is $1/K$ of the ultimate moment, i.e. if linear stress distribution is assumed

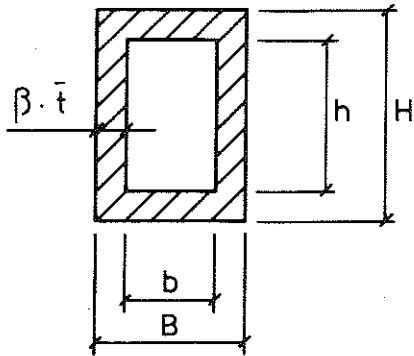


Fig 3. Section of a beam exposed to fire from four sides. Illustrated before fire and after the time \bar{t}

$$M_1 = \frac{1}{K} \sigma_1 \frac{BH^2}{6} \quad (14)$$

where σ_1 is the ultimate bending stress at normal temperature.

The ultimate bending moment in a fire M_2 follows the equation

$$M_2 = \sigma_2 \frac{bh^2}{6} \quad (15)$$

and the ultimate stress in a fire is

$$\sigma_2 = \mu \sigma_1 \kappa_V(\alpha_2) \quad (16)$$

where μ is a reduction coefficient that accounts for the decrease in strength and stiffness due to the increased temperature and moisture content in the non-charred portion of the beam. A value of 0.8 is normally assumed for μ which is supposed to be on the safe side. The function $\kappa_V(\alpha_2)$ gives the reduction coefficient according to Eq. (2) with the slenderness α_2 of the beam with charred cross section.

Failure of the fire exposed beam will occur at the moment M_1 if

$$M_1 = M_2 \quad (17)$$

Substitution of Eqs. (14), (15) and (16) yields:

$$BH^2 = \mu K \kappa_V(\alpha_2) bh^2 \quad (18)$$

With an average charring rate β for the time \bar{t}

$$B-b = H-h = 2\beta\bar{t} \quad (19)$$

and, if b is eliminated from Eqs. (18) and (19)

$$b = B-H+h$$

$$BH^2 = \mu K \kappa_V(\alpha_2) (B-H+h)h^2$$

The last equation can be expressed in the form

$$\frac{B}{H} = \mu K \kappa_V(\alpha_2) \left(\frac{B}{H} - 1 + \frac{h}{H} \right) \left(\frac{h}{H} \right)^2 \quad (20)$$

With known μ , K , α_2 and B/H Eq. (20) can be solved for the critical value of the ratio h/H . After the value of the critical ratio h/H has been determined, the corresponding time of failure \bar{t} can be obtained from Eq. (19), rewritten as

$$\frac{\beta\bar{t}}{B} = \frac{1 - h/H}{2 B/H} \quad (21)$$

The slenderness ratio α_2 that enters into Eq. (20) is determined from Eq. (13) expressed in the form

$$\alpha_2 = 0.13 \sqrt{\frac{LH}{mB^2}} \frac{B}{b} \sqrt{\frac{h}{H}} \sqrt[4]{\frac{[1 - (b/h)^2][1 - 1/5 f(\frac{b}{h})(b/h)^2]}{f(\frac{b}{h})}} \quad (22)$$

When solving Eq. (20), it is suitable to express B/b and b/h as functions of h/H . From Eq. (19)

$$\frac{b}{h} = \frac{B}{h} - \frac{H}{h} + 1 \quad (23)$$

With $B/h = \frac{B/H}{h/H}$ substituted into Eq. (23)

$$\frac{b}{h} = 1 - \frac{1 - B/H}{h/H} \quad (24)$$

Analogously

$$\frac{B}{b} = 1 / (1 - \frac{1 - h/H}{B/H}) \quad (25)$$

6. Numerical solution

In combination with Eqs. (2), (22), (24) and (25), Eq. (20) can now be solved for varying B/H , safety factor K and some parameter, expressing the susceptibility of the non-exposed beam to lateral buckling. Eqs. (13) and (22) indicate as natural to choose this parameter as

$$\eta = \sqrt{\frac{LH}{mB^2}} \quad (26)$$

Eq. (20) is of the third degree in h/H and can be solved by the Newton-Raphson method. A suitable start value for h/H is 1.0 and $\kappa_V(\alpha_2) = 1.0$. The equation then gives a new value of h/H . After the new value of h/H has been determined, the corresponding $\kappa_V(\alpha_2) \leq 1.0$ is obtained from Eq. (22) and (2). The procedure is repeated until the error in h/H is less than some stipulated value (in this calculation $(\frac{h}{H})_{i-1} - (\frac{h}{H})_i < 0.002 \frac{B}{H}$). After the value of h/H has been determined, the corresponding value $\beta \bar{t}/B$ is obtained from Eq. (21)

The solution of Eqs. (20) and (21) is illustrated in Fig 4a to 4h for several values of the safety factor K . The curves give the ratio $r = \beta \bar{t}/B$ as a function of B/H for different values of the parameter η . The time of failure is then

$$\bar{t} = r B / \beta \quad (27)$$

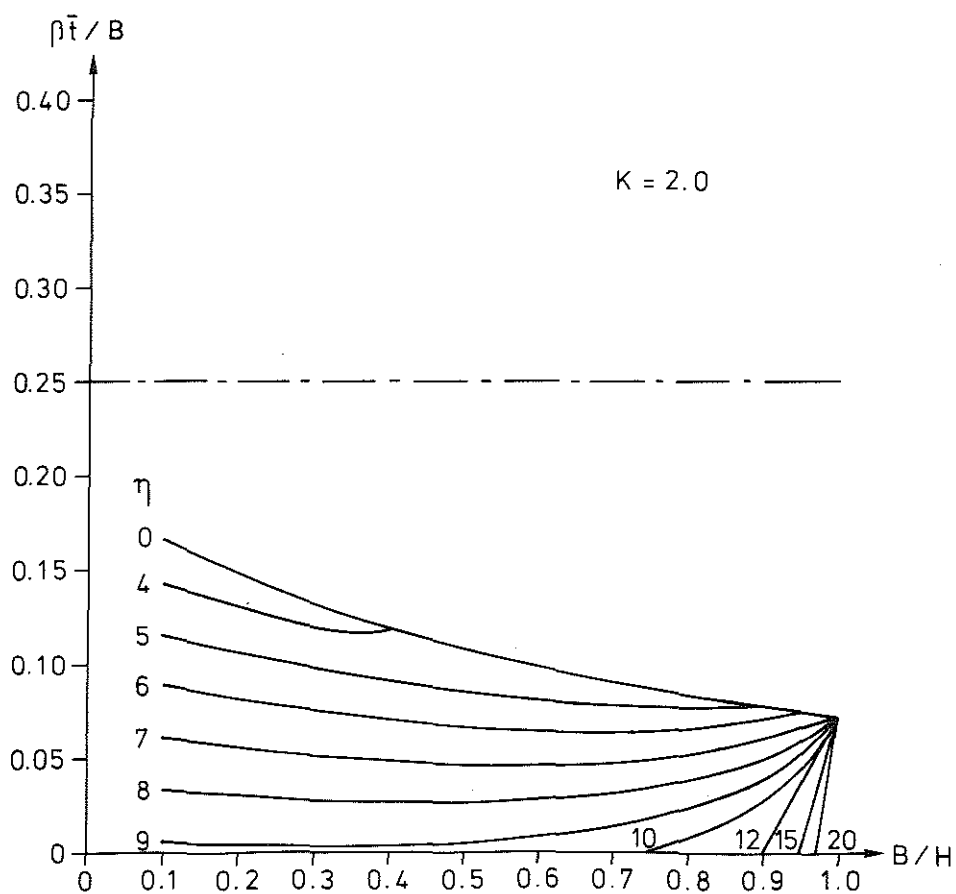
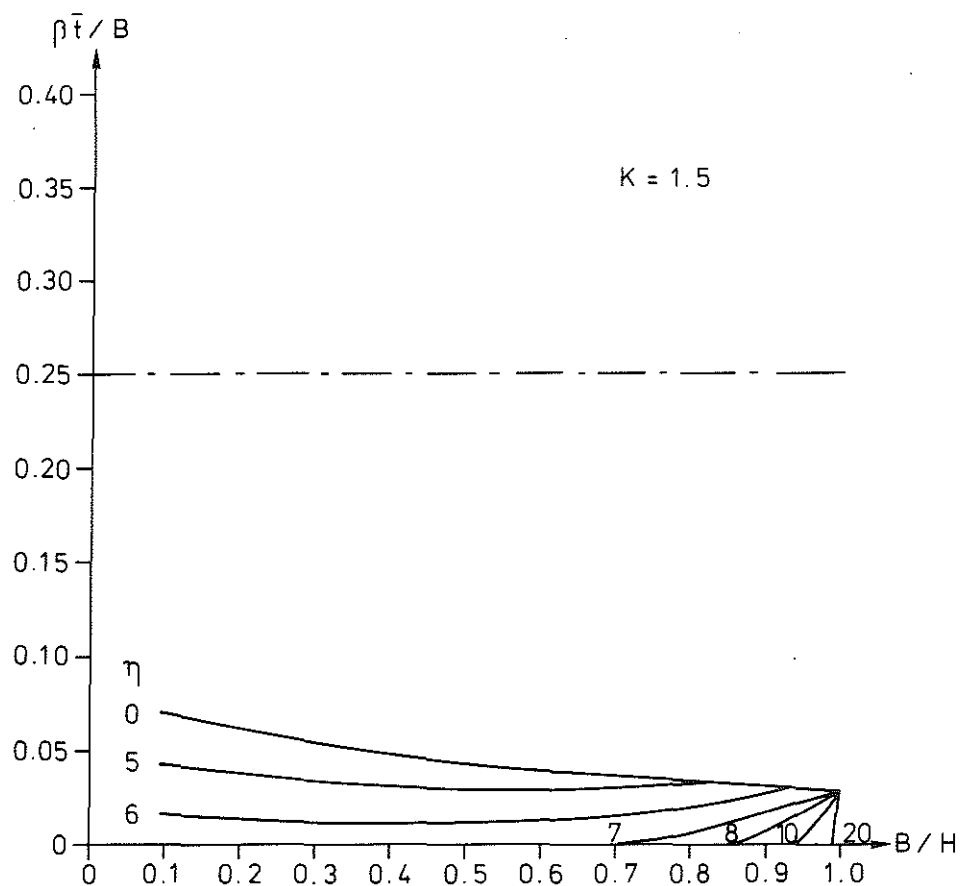


Fig 4a-b. The ratio $r = \beta \bar{t} / B$ as a function of the width-height ratio B/H for different values of the factor $\eta = \sqrt{LH/mB^2}$ and safety factor K . Fire exposure on four sides. The time to failure is $\bar{t} = r \frac{B}{\beta}$ where β is the charring rate

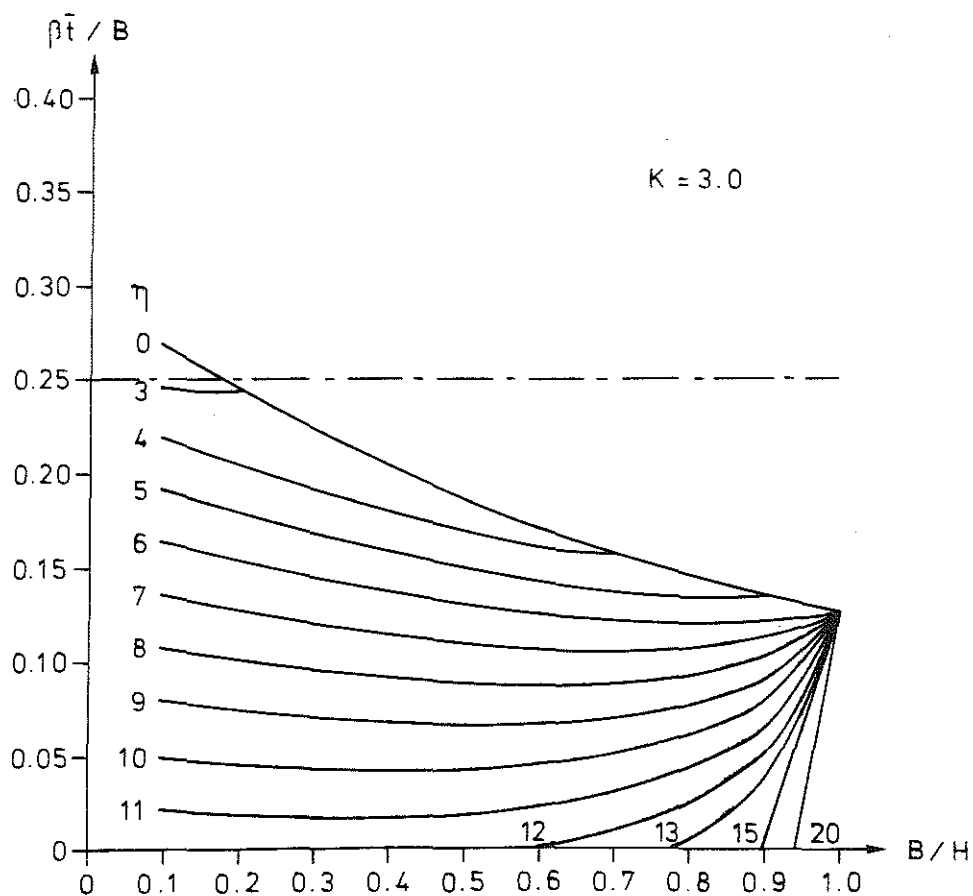
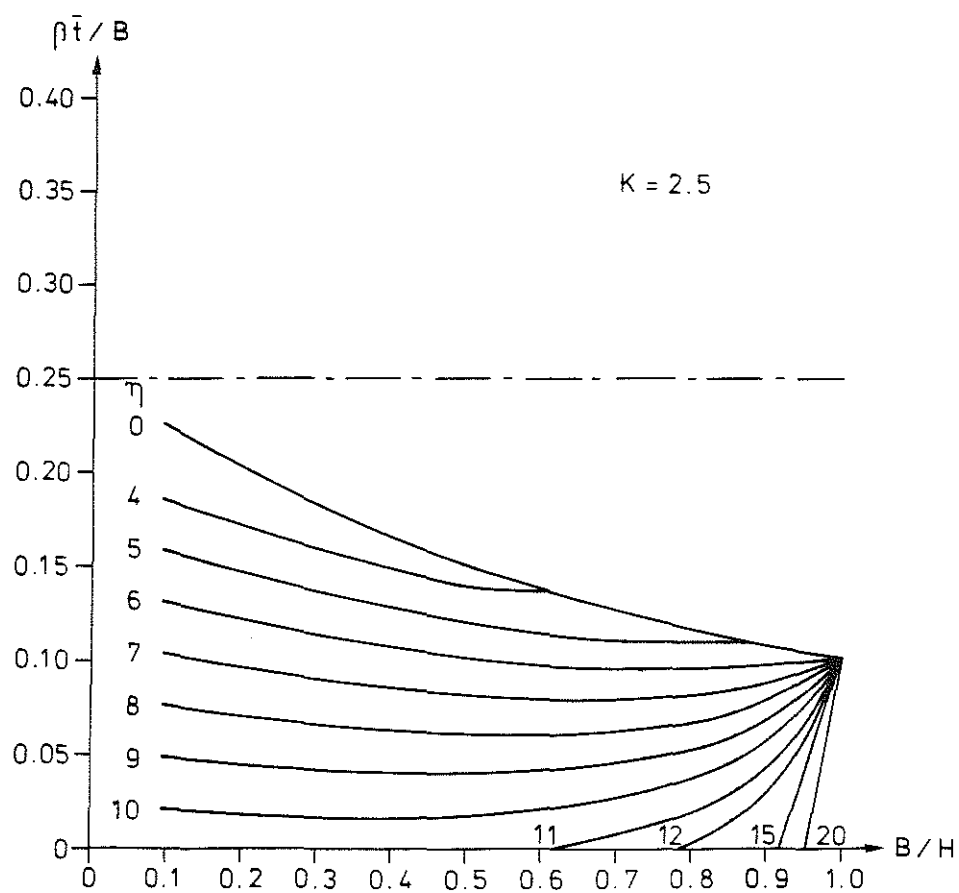


Fig 4c-d. Continued

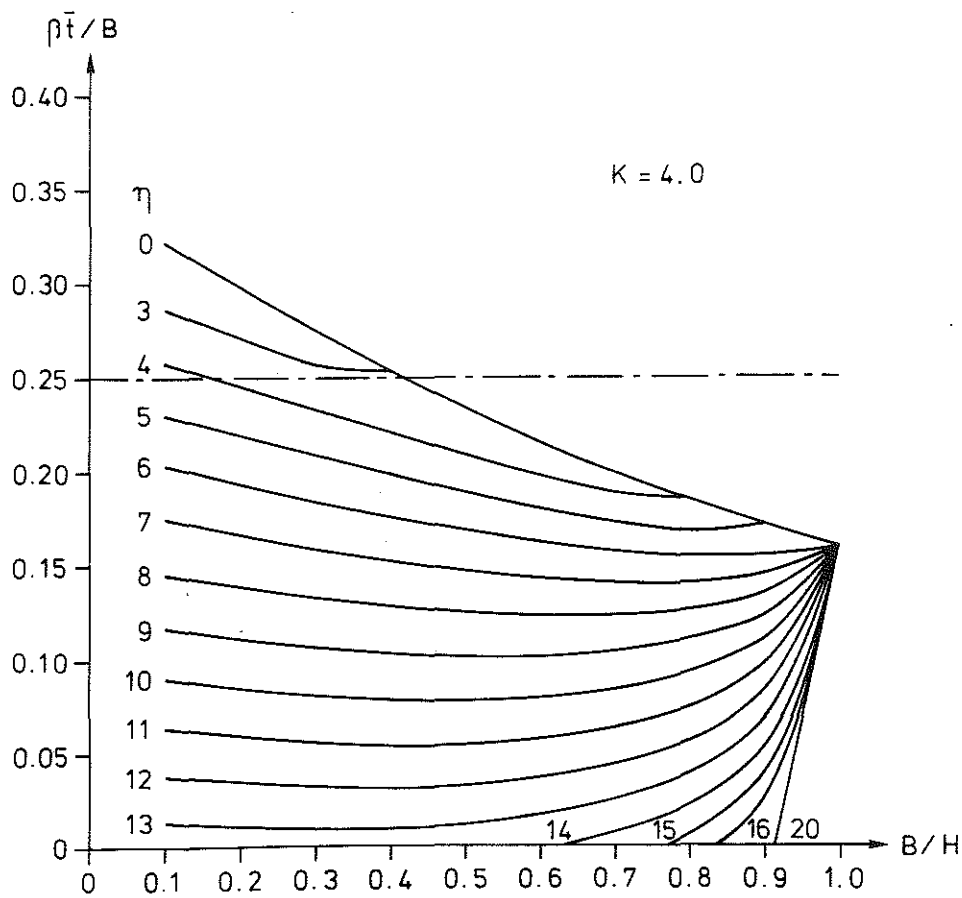
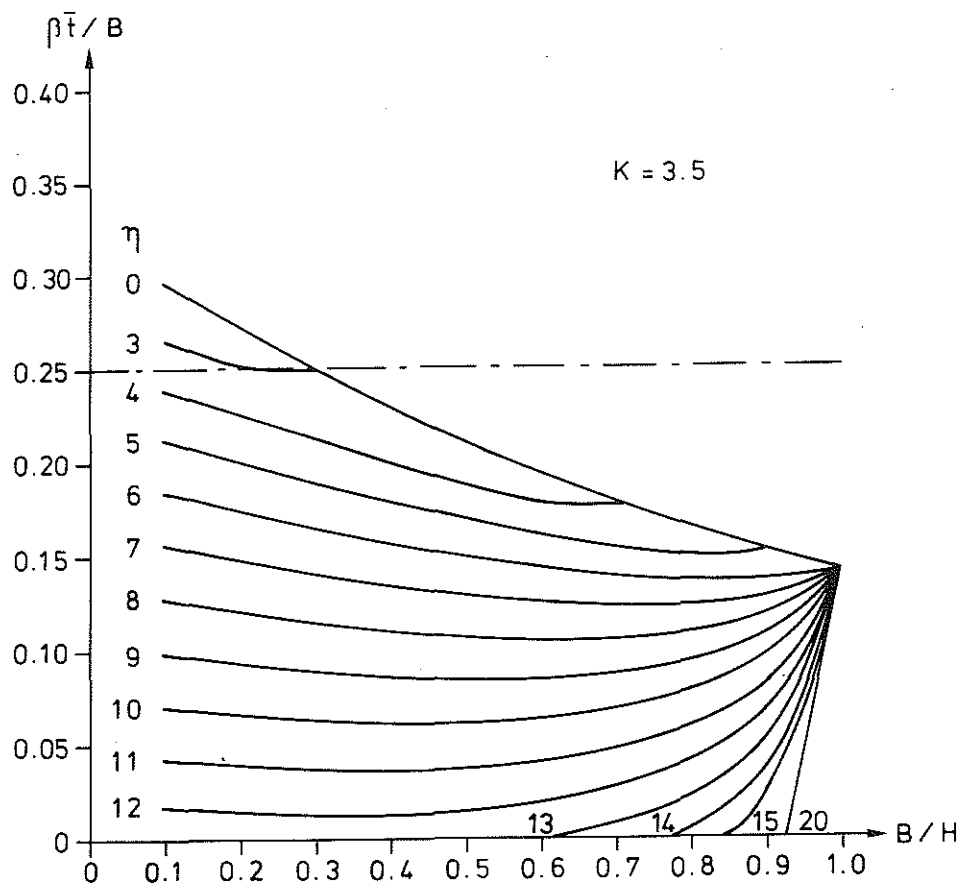


Fig 4e-f. Continued

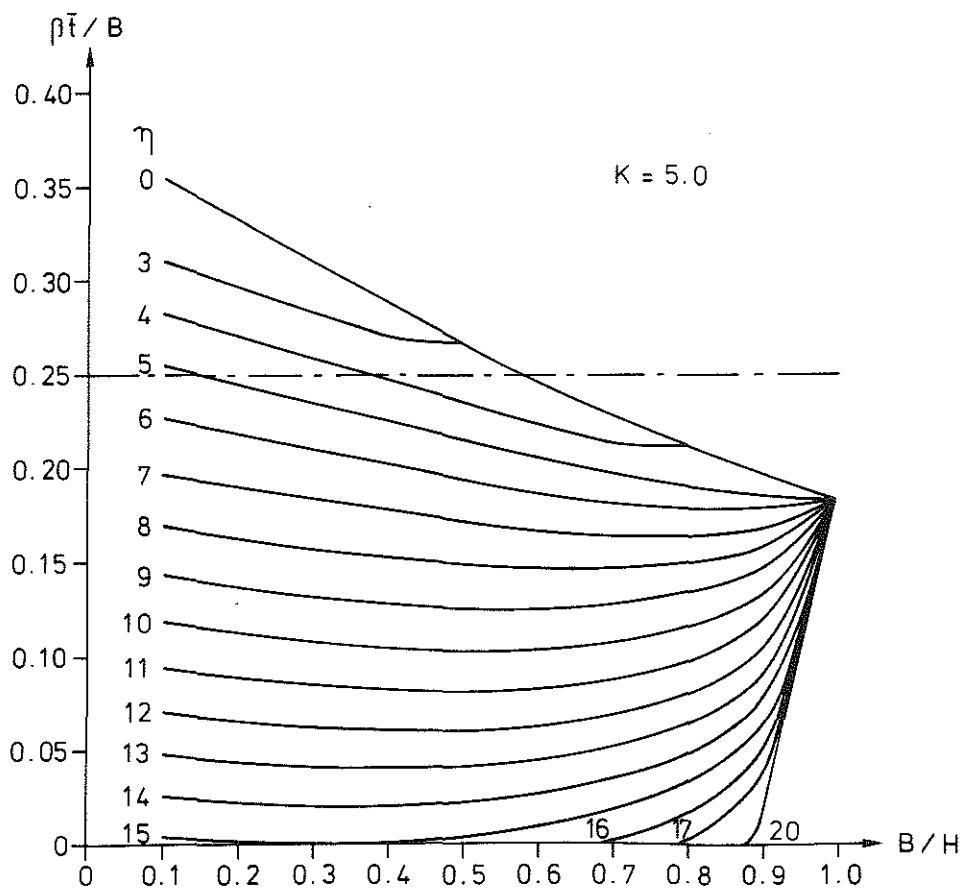
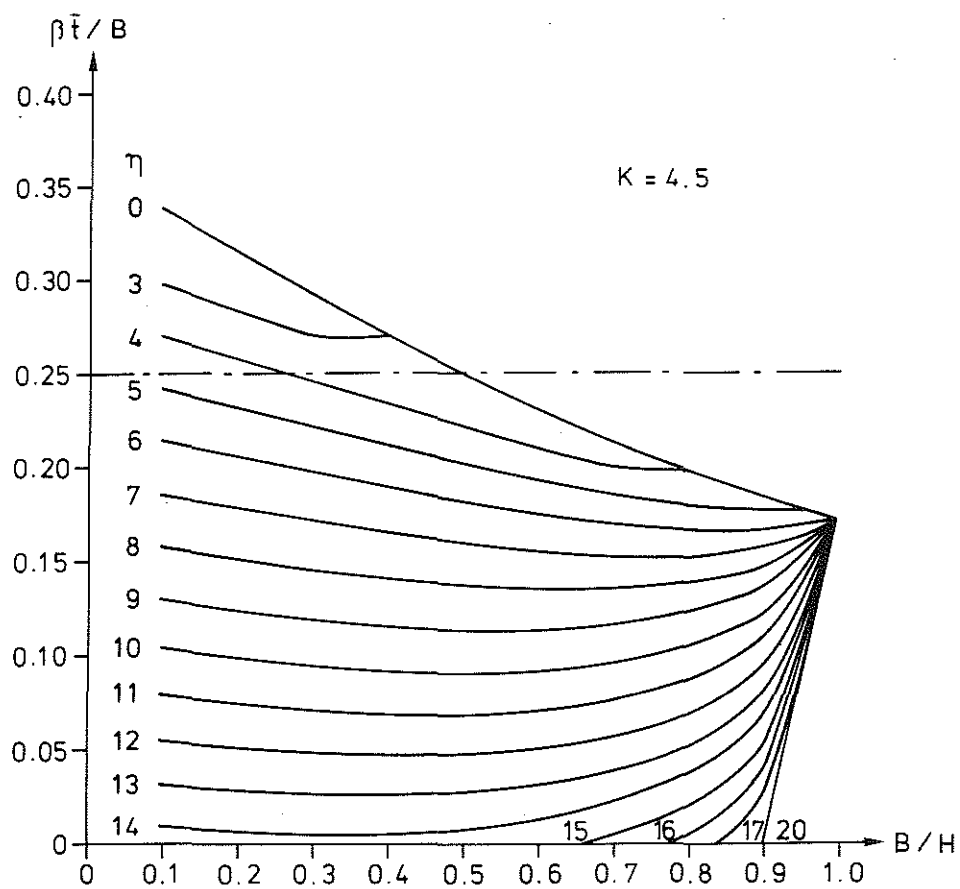


Fig 4g-h. Continued

The special case $\eta = 0$ corresponds to the slenderness $\alpha = 0$, i.e. the failure occurs in pure bending.

7. Fire exposure on three sides

A similar calculation can be performed for a beam exposed to fire on three sides, as illustrated in Fig 5.

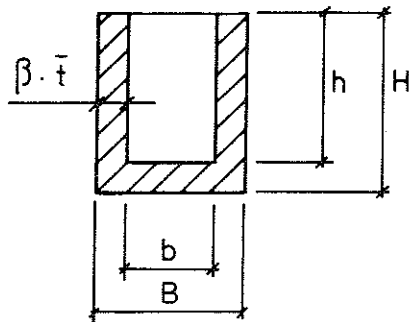


Fig 5. Section of a beam exposed to fire from three sides. Illustrated before fire and after the time \bar{t}

Equation (20) then becomes

$$\frac{B}{H} = \mu K \kappa_V(\alpha_2) \left(\frac{B}{H} - 2 + 2 \frac{h}{H} \right) \left(\frac{h}{H} \right)^2 \quad (28)$$

and (21)

$$\frac{\beta \bar{t}}{B} = \frac{1 - \frac{h}{H}}{\frac{B}{H}} \quad (29)$$

The solution of Eqs. (28) and (29) is illustrated in Fig 6a to 6h.

8. Different loading cases

Although the curves in Fig 4a to 4h and 6a to 6h are directly valid when the applied load is a constant bending moment, they can be used approximately also for other loading cases, giving results on the safe side.

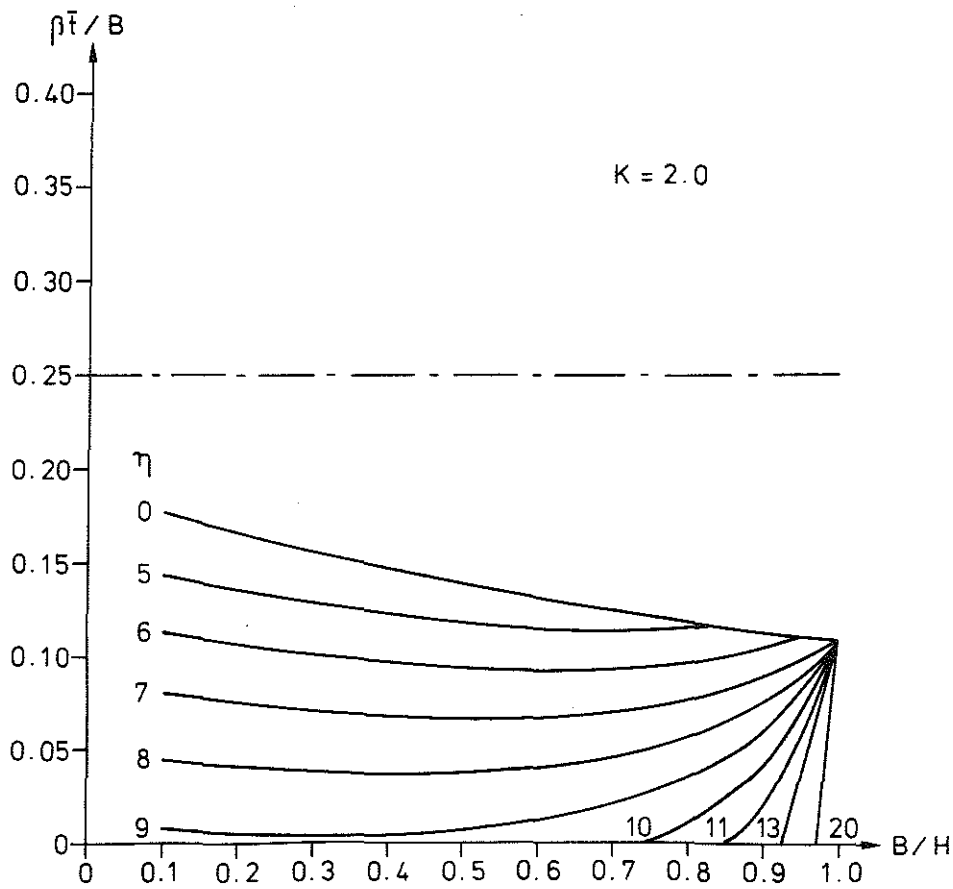
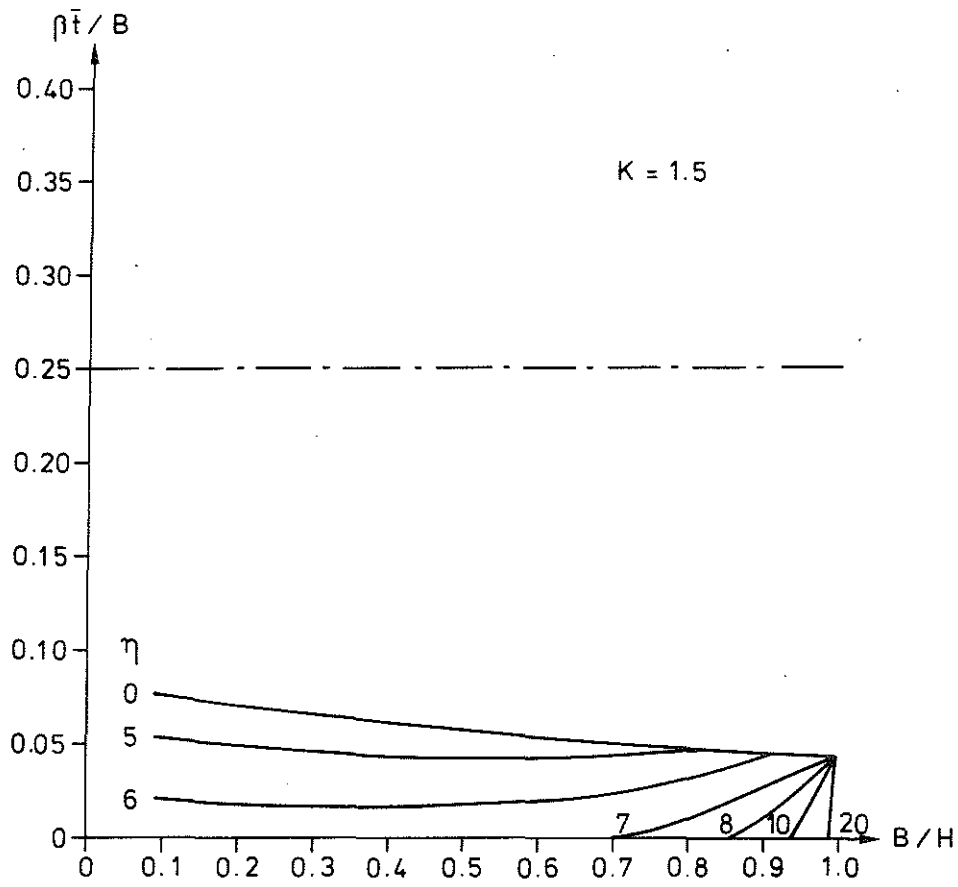


Fig 6a-b. The ratio $r = \beta \bar{t} / B$ as a function of the width-height ratio B/H for different values of the factor $\eta = \sqrt{LH/mB^2}$ and safety factor K . Fire exposure on three sides. The time to failure is $\bar{t} = r \frac{B}{\beta}$ where β is the charring rate

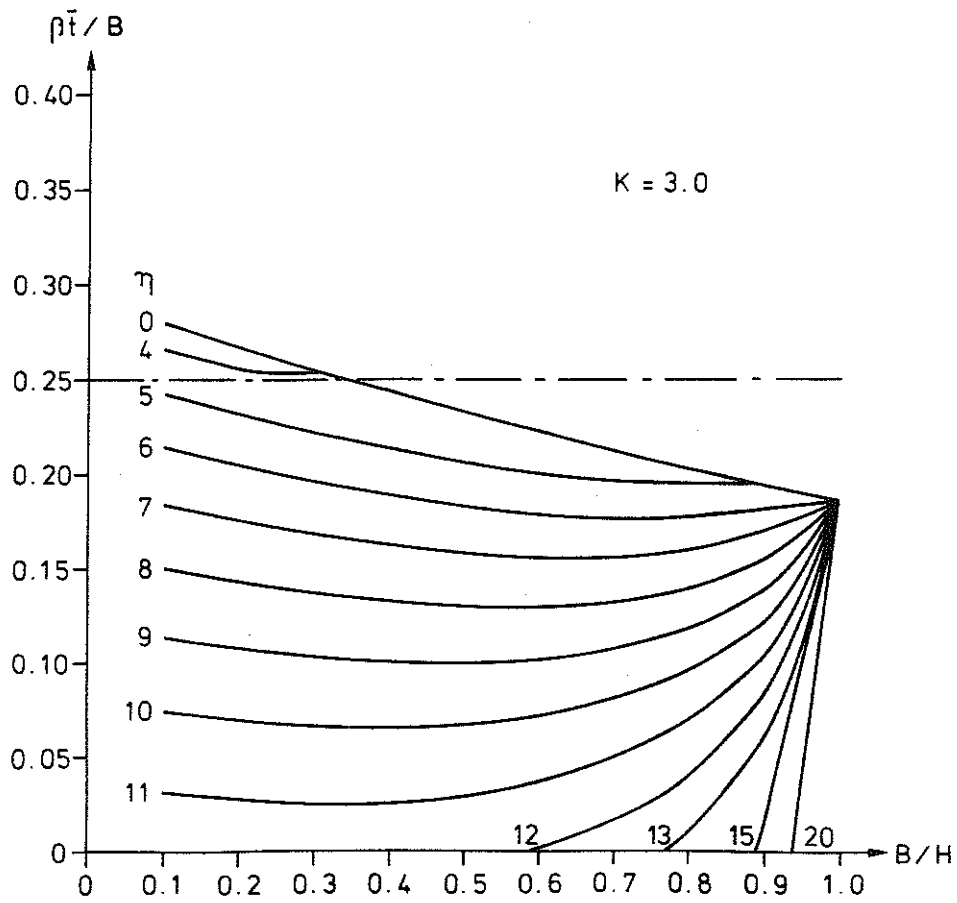
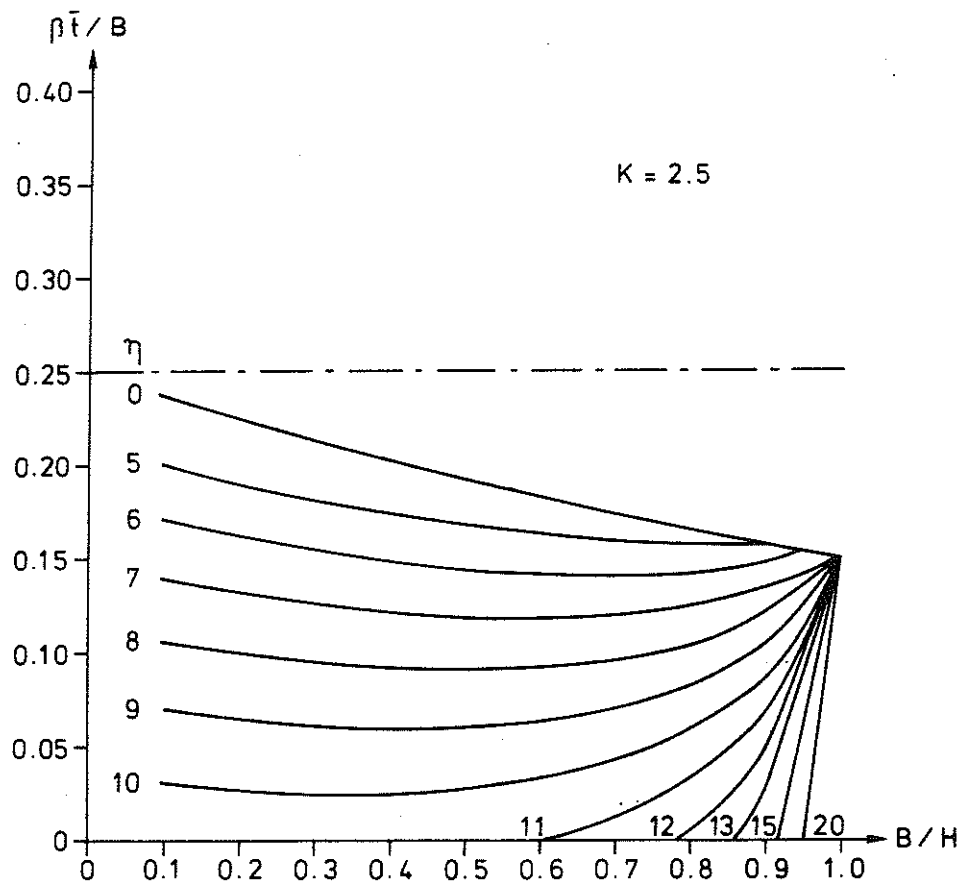


Fig 6c-d. Continued

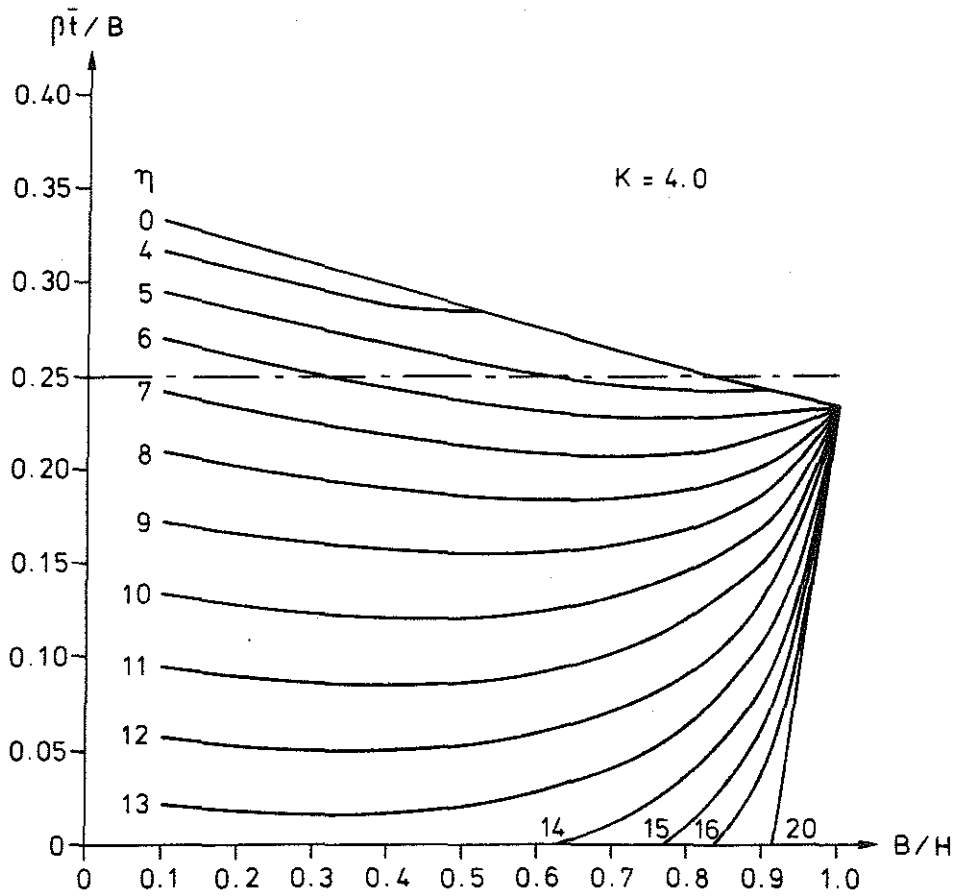
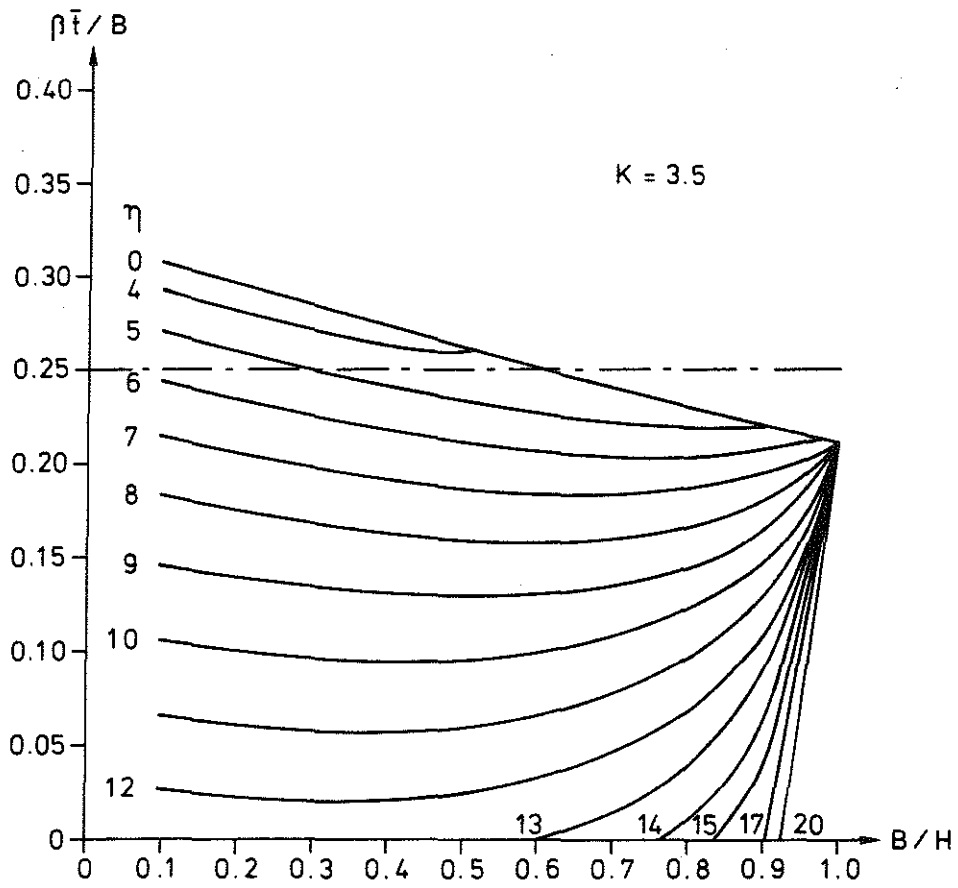


Fig 6e-f. Continued

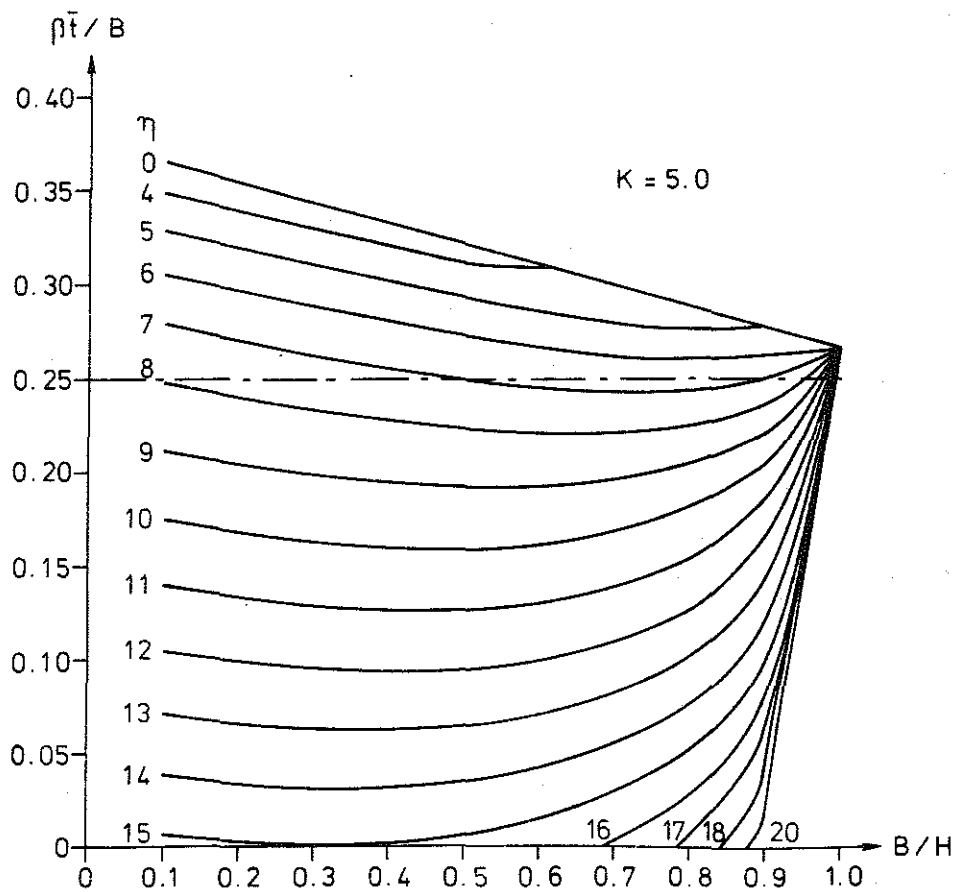
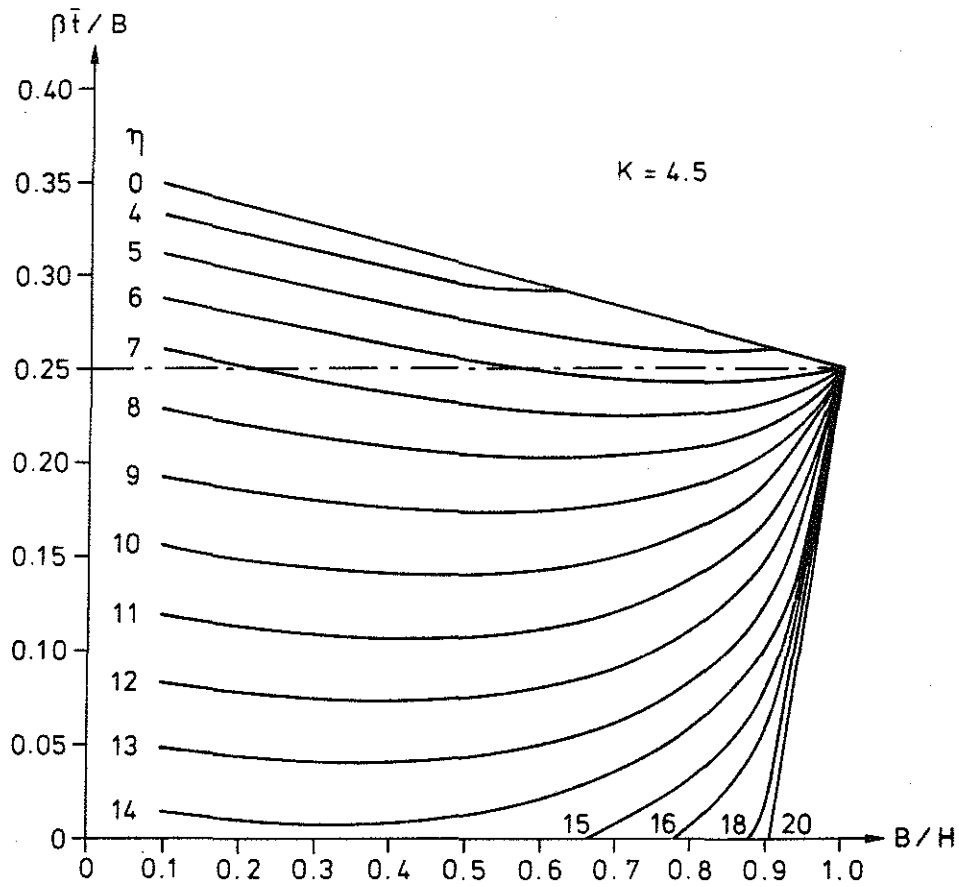


Fig 6g-h. Continued

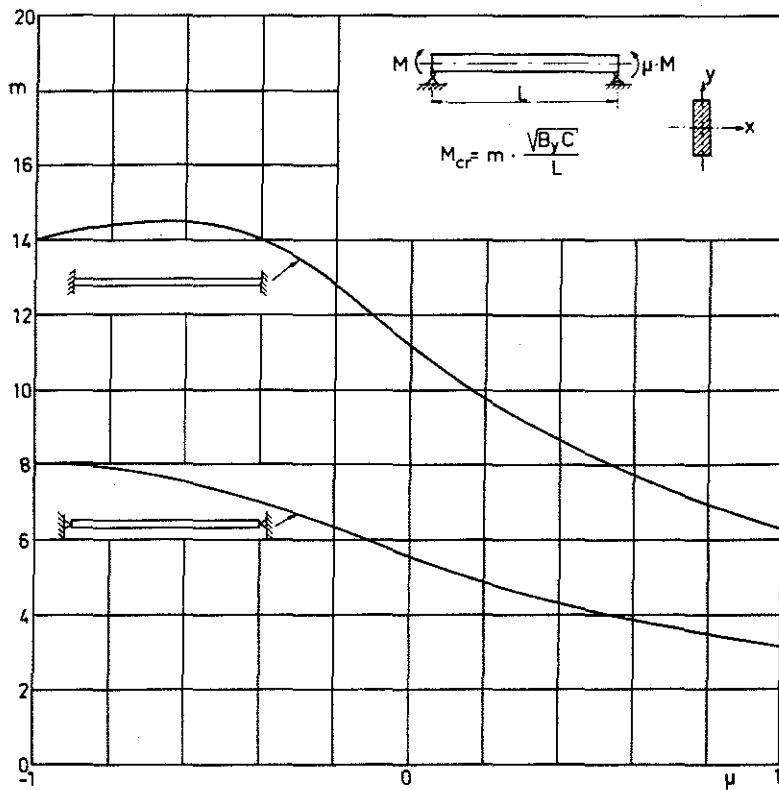


Fig 7a. Critical load of a simply supported beam with negligible warping rigidity and bisymmetrical cross section, at the supports submitted to bending moments of different magnitude. Lateral restraints according to Fig 7f case 1 or 3

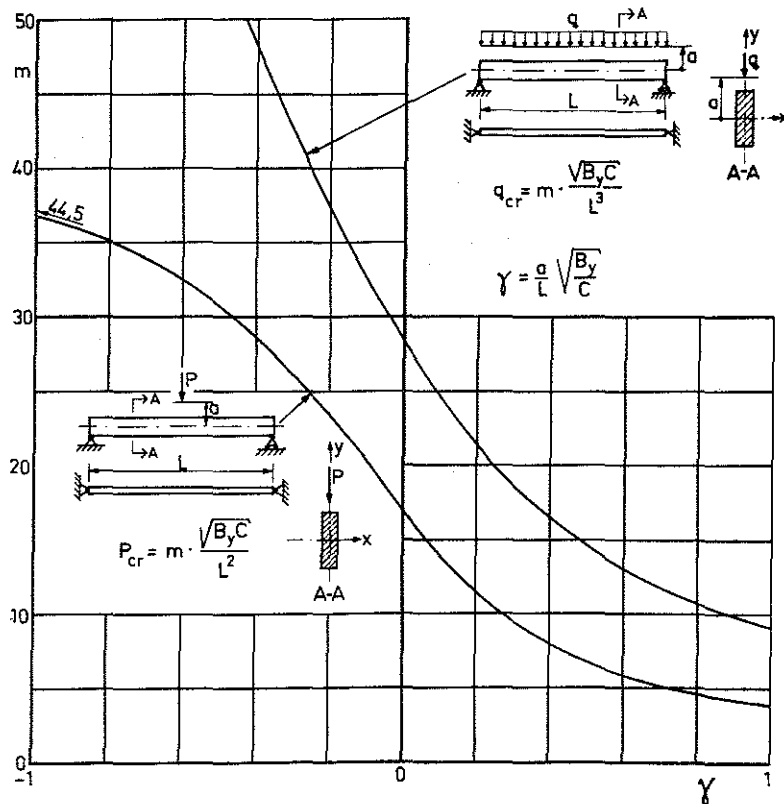


Fig 7b. Critical load of a simply supported beam with negligible warping rigidity and bisymmetrical cross section submitted to a concentrated load P at the center or a uniformly distributed load q . Lateral restraints according to Fig 7f case 1

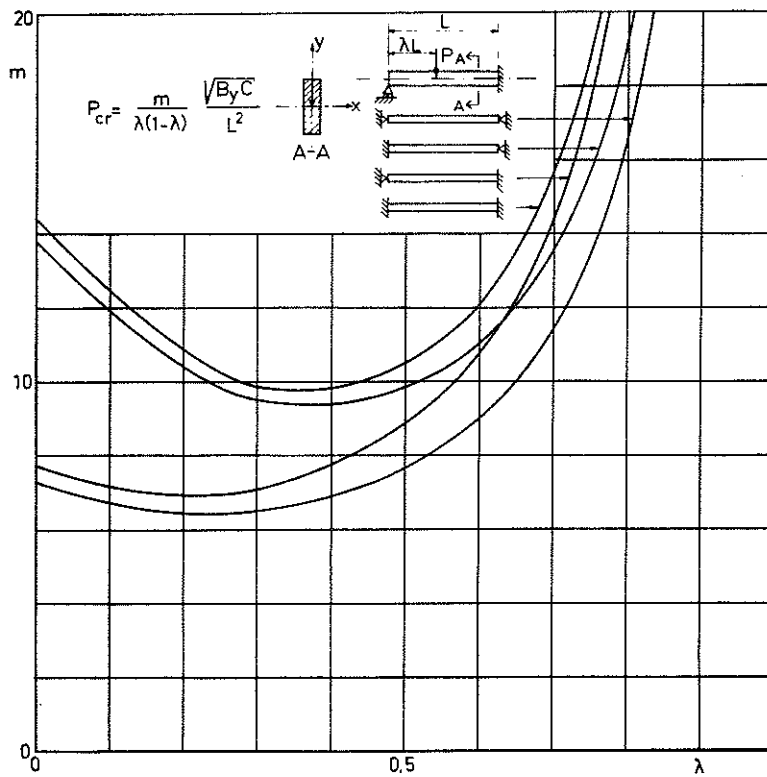


Fig 7c. Critical load of a beam with negligible warping rigidity and bisymmetrical cross section submitted to a concentrated load applied at the centre of shear equal the centre of gravity λL from a support. Clamped at one support in the rigid direction. Lateral restraints according to Fig 7f

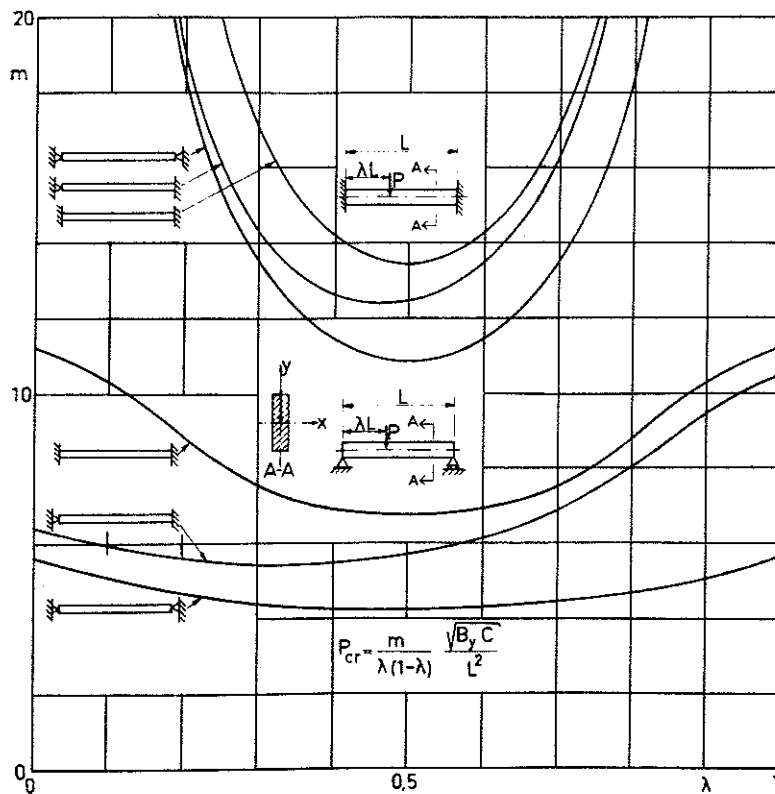


Fig 7d. Critical load of a beam with negligible warping rigidity and bisymmetrical cross section submitted to a concentrated load applied at the centre of shear equal the centre of gravity λL from a support. Simply supported or clamped at two supports in the rigid direction. Lateral restraints according to Fig 7f

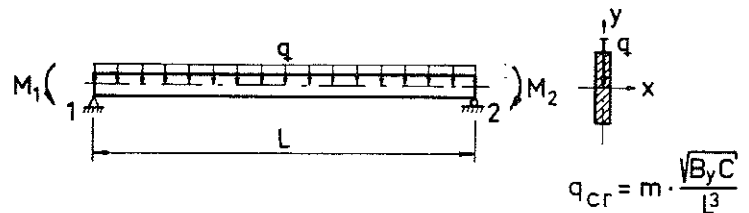
			
Supporting moment	$M_1 = M_2 = 0$	$M_1 = qL^2/8 \quad M_2 = 0$	$M_1 = M_2 = qL^2/12$
Supportcase acc to fig. 7f	1 2 3	1 2 ₁ 2 ₂ 3	1 2 3
m	28 33 48	56 65 80 87	98 107 130

Fig 7e. Critical load of a beam with negligible warping rigidity and bisymmetrical cross section submitted to a uniformly distributed load applied at the centre of shear equal the centre of gravity. Simply supported, clamped at one support or clamped at two supports in the rigid direction. Lateral restraints according to Fig 7f

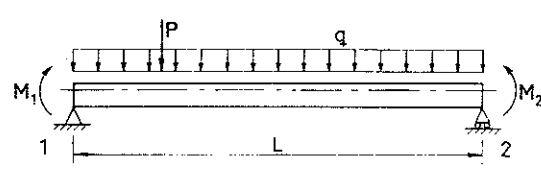
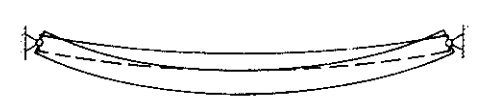


Support-case	Supportcondition		
	1	2	
1	$u = 0$ $u'' = 0$ $\phi = 0$ $\phi'' = 0$	$u = 0$ $u'' = 0$ $\phi = 0$ $\phi'' = 0$	
2	$u = 0$ $u' = 0$ $\phi = 0$ $\phi' = 0$	$u = 0$ $u'' = 0$ $\phi = 0$ $\phi'' = 0$	
3	$u = 0$ $u' = 0$ $\phi = 0$ $\phi' = 0$	$u = 0$ $u' = 0$ $\phi = 0$ $\phi' = 0$	

Fig 7f. Supportcase and supportcondition of two supported beams

In Figs. 7a to 7f the critical load for several loading cases is given [7]. From these figures we get the coefficient m . If the greatest critical moment M_{\max} for these loading cases is calculated and compared to critical moment in Eq. (11), a new coefficient m_1 can be derived. Hence

$$m_1 = m M_{\max} / qL^2 \quad \text{distributed load } q \quad (30a)$$

$$m_1 = m M_{\max} / PL \quad \text{concentrated load } P \text{ applied at the center} \quad (30b)$$

$$m_1 = m M_{\max} / PL\lambda(1-\lambda) \quad \text{concentrated load } P \text{ applied } \lambda L \text{ from a support} \quad (30c)$$

With a known m_1 value, η_1 can be calculated

$$\eta_1 = \sqrt{\frac{LH}{m_1 B^2}} \quad (31)$$

and used as an input value in Fig 4a - 4h and 6a - 6h.

Since the method described above is based on the section of the beam with the greatest bending moment, an extra margin of safety is ensured when the method is used for loading cases with an unsymmetrical distribution of moment. Methods are available [14] that can be used to determine the position of a beam section which will provide the same safety factor as for a symmetrical moment distribution.

9. Example

Calculate the time to failure of a simply supported laminated wood beam, quality L40, with length 12 m, width 0.15 m, and height 0.50 m according to Fig. 8. The beam is carrying the uniformly distributed load $q = 3.8$ kN/m, the dead weight of the beam included, and is prevented from torsion of the supports. The fictitious opening factor of the fire compartment is $(A\sqrt{h}/A_{\text{tot}})_{\text{fict}} = 0.08 \text{ m}^{1/2}$. Divide the calculation into the three cases a) fire exposure on three sides, b) fire exposure on four sides, and c) fire exposure on three sides with lateral restraints of secondary wood beams with centre distances = 2 m attached to the primary beams in such a way, that they prevent lateral deflection and torsion of the beam in the restraint sections.

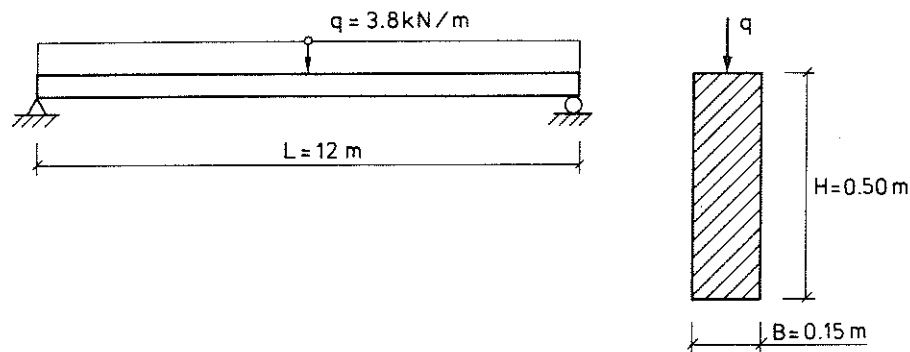


Fig 8. Simply supported beam prevented from torsion at the supports, quality L40, acted upon by a uniformly distributed load $q = 3.8 \text{ kN/m}$ in a fire compartment with $(A\sqrt{h}/A_{\text{tot}})_{\text{fict}} = 0.08 \text{ m}^{1/2}$

The maximum bending moment M_{max} and bending stress σ are

$$M_{\text{max}} = \frac{qL^2}{8} = \frac{3.8 \cdot 10^{-3} \cdot 12^2}{8} = 0.0684 \text{ MNm}$$

$$\sigma = \frac{M}{W} = \frac{0.0684 \cdot 6}{0.15 \cdot 0.5^2} = 10.94 \text{ MPa}$$

According to SBN [2] and with L40, the characteristic value of the ultimate bending stress, determined on the assumption of a linear bending stress distribution, is $3 \times 13 = 39 \text{ MPa}$.

The safety factor against failure in pure bending before fire

$$K = \frac{39}{10.94} = 3.56$$

Calculate lateral buckling coefficient m_1 according to equation (30a)

$$m_1 = m \frac{M_{\text{max}}}{qL^2} = \frac{m}{8}$$

where m is obtained from [8] or Fig 7b.

Input value in Fig 7b is

$$\gamma = \frac{a}{L} \sqrt{\frac{B_y}{C}} = \frac{a}{L} \sqrt{\frac{EI_y}{GK_V}}$$

with

$$E = 11\,000 \text{ MPa}$$

$$G = 550 \text{ MPa}$$

$$I_y = \frac{1}{12} B^3 H = 1.406 \cdot 10^{-4} \text{ m}^4$$

$$K_V = \frac{1}{3} (1 - 0.63 \frac{B}{H}) B^3 H = 4.56 \cdot 10^{-4} \text{ m}^4$$

$a = 0.25 \text{ m}$ (the complete load q approximated to be applied at the top of the beam).

Hence

$$\gamma = 0.052$$

From Fig 7b

$$m = 27$$

giving

$$m_1 = 3.4$$

From equation (31)

$$\eta_1 = \sqrt{\frac{LH}{m_1 B^2}} = \sqrt{\frac{12 \cdot 0.50}{3.4 \cdot 0.15^2}} = 8.86$$

$$\left. \begin{array}{l} \text{a) Fire exposure on three sides} \\ B/H = 0.30 \\ \eta_1 = 8.86 \\ K = 3.56 \Rightarrow \text{Fig 6e} \end{array} \right\}$$

$$\text{interpolated } \frac{\beta \bar{t}}{B} = 0.140$$

i.e.

$$\bar{t} = \frac{0.140 \cdot 0.15}{\beta} = \frac{0.021}{\beta}$$

In Fig 1, the average charring rate corresponding to $(A\sqrt{h}/A_{\text{tot}})_{\text{fict}} = 0.08 \text{ m}^{1/2}$ is 0.8 mm/min . Substituted into the expression above yields

$$\bar{t} \approx 26 \text{ min}$$

If failure occurs in pure bending η_1 becomes zero and from Fig 6e $\frac{\beta \bar{t}}{B} = 0.285$ (corresponding to 53 min). But $\frac{\beta \bar{t}}{B}$ is maximated to 0.25, because the charring rate is not constant when more than the half width is charred [11], [12], [13].

Hence

$$\bar{t} = \frac{0.25 \cdot 0.15}{0.8 \cdot 10^{-3}} \approx 47 \text{ min}$$

$$\left. \begin{array}{l} \text{b) Fire exposure on four sides} \\ B/H = 0.30 \\ \eta_1 = 8.86 \\ K = 3.56 \Rightarrow \text{Fig 4e} \end{array} \right\} \text{interpolated } \frac{\beta \bar{t}}{B} = 0.095$$

i.e.

$$\bar{t} = \frac{0.095 \cdot 0.15}{\beta} = \frac{0.0143}{\beta}$$

and with $\beta = 0.08 \text{ mm/min}$ substituted

$$\bar{t} \approx 18 \text{ min}$$

If failure occurs in pure bending $\eta_1 = 0$ and Fig 4e gives $\frac{\beta \bar{t}}{B} = 0.25$
i.e.

$$\bar{t} \approx 47 \text{ min}$$

c) Fire exposure on three sides and with lateral restraints with centre distances = 2 m.

According to the instructions in SBN 1975, chapter 27.331 [2], L in equation (31) and in Fig 7b is inserted as the centre distance 2 m, i.e.

$$\gamma = \frac{a}{L} \sqrt{\frac{By}{C}} = \frac{0.25}{2} \sqrt{\frac{11000 \cdot 1.406 \cdot 10^{-4}}{550 \cdot 4.56 \cdot 10^{-4}}} = 0.310$$

$$m = 18; \quad m_1 = \frac{m}{8} = 2.25 \text{ (Fig 7b)}$$

$$\eta_1 = \sqrt{\frac{LH}{m_1 B^2}} = \sqrt{\frac{2 \cdot 0.5}{2.25 \cdot 0.15^2}} = 4.44$$

$$B/H = 0.30$$

$$\eta_1 = 4.44$$

$$K = 3.56 \Rightarrow \text{Fig 6e}$$

$$\text{interpolated } \frac{\beta \bar{t}}{B} = 0.265$$

i.e.

$$\bar{t} = \frac{0.265 \cdot 0.15}{\beta} = \frac{0.0398}{\beta}$$

and with $\beta = 0.8$ mm/min substituted

$$\bar{t} \approx 50 \text{ min}$$

If failure occurs in pure bending

$$\bar{t} \approx 53 \text{ min}$$

according to case a).

But the maximum allowable ratio of $\frac{\beta \bar{t}}{B}$ is 0.25, and with $\beta = 0.8$ mm/min substituted

$$\bar{t} \approx 47 \text{ min}$$

This is, because the maximum allowable ratio $\frac{\beta \bar{t}}{B} = 0.25$, equal the time to failure obtained if failure occurs in pure bending.

In this case it is important to note that this time to failure requires that the lateral restraints are kept intact during the fire in such a way that they prevent lateral deflection and torsion of the beam in the restraint sections. The fulfilment of this requirement can be checked by the same method described above regarding the design of the primary beams.

10. Conclusion

The described method makes it possible in a simple way to determine the risk of lateral buckling of a beam exposed to fire. The values of

the coefficient m can easily be found in handbooks, such as [7], [8]. The coefficient m_1 can then be determined from expressions similar to Eq. (30) for a great number of loading cases.

Figs. 4a-4h and 6a-6h illustrate explicitly the necessity of determining the risk of lateral buckling already when $\frac{B}{H}$ is slightly < 1 . It is probably always the buckling load that is the determining factor for a fire exposed beam, if not the phenomena adjacent to supports, joints, mountings etc are decisive.

Further knowledge is needed, especially of the charring rate in real fires and the material properties when the temperature and the moisture contents are increased in the uncharred part of the cross section. In [5] a model is presented which makes it possible to calculate the charring rate, the temperature profile and the moisture at an arbitrary fire exposure.

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