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# OPTIMUM BEAMFORMERS FOR UNIFORM CIRCULAR ARRAYS IN A CORRELATED SIGNAL ENVIRONMENT

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## ABSTRACT

By virtue of their geometry, uniform circular arrays (UCAs) are ideally suited to provide 360 degrees of coverage in the azimuthal plane. However, in a correlated signal environment, the well-known technique of spatial smoothing to mitigate the signal cancellation effect as seen in an optimum beamformer will not work since this technique is applicable only to uniform linear arrays. In this paper, we show how the transformation of Davies can be adopted to design optimum beamformers for UCAs in a correlated signal environment. We also introduce derivative constraints to improve the robustness of the optimum beamformers to mismatches between the beamformers' look direction and the actual direction-of-arrival of the desired signal. The effectiveness of our design method is illustrated by a numerical example.

## 1. INTRODUCTION

Typical scenarios for sensor array systems, such as radar, sonar and wireless communications involve all-azimuth-angle (i.e. 360°) coverage [1]. An array geometry that is naturally suited to provide this range of coverage is the uniform circular array (UCA). We consider here the problem of designing beamformers for optimum signal reception for UCAs.

The classical approach to optimum beamformer design is to minimize the beamformer output power subject to a constraint that enforces a fixed response in the direction-of-arrival (DOA) of the desired signal (the *look direction* of the beamformer). If none of the other received signals are correlated with the desired signal, then, as expected, the optimum beamformer will exhibit the target response in the look direction and nulls in the DOAs of the interfering signals. However, if one of the interfering signals is highly correlated with the desired signal, such as that may happen in a multipath environment, then the beamformer will attenuate the desired signal severely.

Many approaches have been proposed to counteract the aforesaid signal cancellation phenomenon (see [2] and the references therein). One general class is *spatial smoothing* [2-6], which exploits the Vandermonde structure of the steering vector of a uniform *linear* array to decorrelate the correlated signals. In this approach, the array is divided

into a number of equal size overlapping subarrays. The decorrelated covariance matrix is obtained by averaging the covariance matrices of the subarrays.

In the case of UCAs, the steering vector does not have a Vandermonde structure. Accordingly, spatial smoothing cannot be applied. In this paper, we follow the approach of Davies [7] (see also [1, 8]) to transform the array sensor outputs to derive the so-called *virtual array* [1]. The key feature of the virtual array is that the resulting steering vector displays the desired Vandermonde structure.

More recently, [9] has generalized the Davies transformation to 2-dimensional arrays of any arbitrary geometry and broadband signals. This method, called *array manifold interpolation* (AMI), is used in [10] to perform optimum beamforming in the presence of correlated wideband signals via frequency averaging. This paper, though close in spirit to the work presented in [10], is nevertheless distinct in that it considers narrowband signals and the averaging is performed over a single virtual array. We also introduce derivative constraints [11] to obtain robustness against directional mismatches.

Finally, we note that the transformation of Davies and AMI both involve an approximation. They thus necessitate some compromises in terms of the size of the virtual array.

## 2. PROPOSED METHOD

### 2.1 Problem Statement

Consider a UCA of  $N$  elements and radius  $r$ . The  $n$ th component of the array response (or steering) vector  $\mathbf{a}(\theta)$ ,  $n \in \{1, 2, \dots, N\}$ , for a narrowband signal of wavelength  $\lambda$  arriving from angle  $\theta$  is given by

$$[\mathbf{a}(\theta)]_n = G_n(\theta) \exp \left[ jkr \cos \left( \theta - \frac{2\pi(n-1)}{N} \right) \right] \quad (1)$$

where  $k = 2\pi/\lambda$ ,  $G_n(\theta)$  is the complex gain pattern of the  $n$ th array element, and  $\theta \in [-\pi, \pi]$ .

Suppose the array receives  $L$  signals,  $s_1(t), \dots, s_L(t)$ , each arriving from a distinct direction  $\theta_1, \dots, \theta_L$ . The array output vector is given by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

where  $\mathbf{A} = [\mathbf{a}(\theta_1) \ \dots \ \mathbf{a}(\theta_L)]$ ,  $\mathbf{s}(t) = [s_1(t) \ \dots \ s_L(t)]^T$ ,

$\mathbf{n}(t) = [n_1(t) \cdots n_N(t)]^T$ ,  $n_n(t)$  is the noise output of the  $n$ th sensor, and  $\mathbf{n}(t)$  and  $\mathbf{s}(t)$  are assumed to be stationary, zero mean, and uncorrelated with each other. The covariance matrix is given by

$$\mathbf{R}_x = \mathbf{E}[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma_n^2\mathbf{\Sigma}_n \quad (3)$$

where  $\mathbf{R}_s$  is the signal covariance matrix, and  $\mathbf{\Sigma}_n$  is the normalized noise covariance matrix.

Let  $s_1(t)$  be the desired signal. If none of the other signals is correlated with  $s_1(t)$ , then the optimum beamformer defined by the following problem:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_l) = 1, \quad (P1)$$

where  $\theta_l$  is the look direction of the beamformer, will exhibit the expected characteristic of unity gain in the desired signal direction if  $\theta_l = \theta_1$ , and nulls in the other signal directions. The solution to (P1) is given by

$$\mathbf{w}_{\text{opt}} = \mathbf{R}_x^{-1} \mathbf{a}(\theta_l) [\mathbf{a}^H(\theta_l) \mathbf{R}_x^{-1} \mathbf{a}(\theta_l)]^{-1}. \quad (4)$$

We shall refer to the beamformer as defined by (P1) the *conventional optimum beamformer*.

The difficulty with the above design method is that if one of the interfering signals is highly correlated with the desired (or look direction) signal, then the desired signal can be cancelled [4]. A remedy is to perform spatial smoothing on  $\mathbf{R}_x$  but this requires the steering vector to be in Vandermonde form which is not the case with UCAs.

## 2.2 Modal Transformation

To simplify the following presentation, we shall assume that  $G_n(\theta) = 1$ ,  $n = 1, \dots, N$ . Consider the matrices  $\mathbf{J}$  and  $\mathbf{F}$  defined as follows where  $p = 1, \dots, M$ ,  $n = 1, \dots, N$ , and  $M = 2h + 1$ :

$$\mathbf{J} = \text{diag} \left[ \left( \sqrt{N} j^{p-1-h} J_{p-1-h}(kr) \right)^{-1} \right] \quad (5)$$

$$[\mathbf{F}]_{pn} = \frac{1}{\sqrt{N}} e^{j2\pi(p-1-h)(n-1)/N}, \quad (6)$$

where  $J_p(\cdot)$  denotes a  $p$ th order Bessel function of the first kind.

In [1], it is shown that if the sensor outputs are transformed by  $\mathbf{J}$  and  $\mathbf{F}$  as illustrated in Fig. 1, then the array response vector of the resultant  $M$ -element virtual array will have, approximately, the Vandermonde form

$$\mathbf{a}_v(\theta) = \mathbf{J}\mathbf{F}\mathbf{a}(\theta) \approx \left[ e^{-jh\theta} \cdots 1 \cdots e^{jh\theta} \right]^T. \quad (7)$$

An appropriate choice for  $h$  is given by

$$\max \left\{ h \mid h \leq \frac{N-1}{2} \text{ and } \frac{|J_{h-N}(kr)|}{|J_h(kr)|} < \varepsilon \right\} \quad (8)$$

where  $\varepsilon$  determines the accuracy of the approximation.

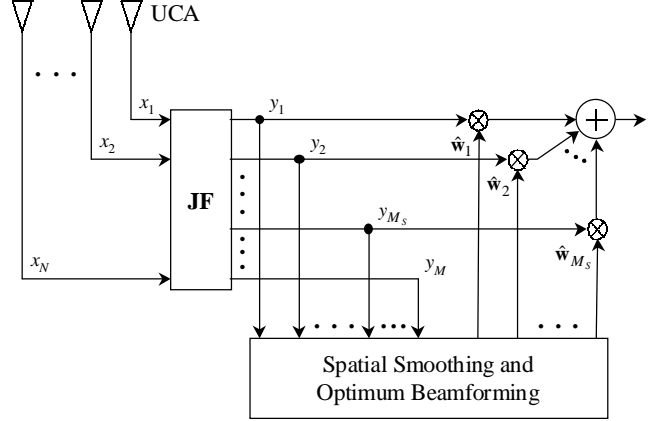


Fig. 1. Modal transformation and optimum beamforming for uniform circular arrays.

## 2.3 Spatial Smoothing

In Fig. 1, the output vector of the virtual array is given by

$$\mathbf{y}(t) = \mathbf{J}\mathbf{F}\mathbf{x}(t), \quad (9)$$

and the corresponding covariance matrix is given by

$$\mathbf{R}_y = \mathbf{E}[\mathbf{y}(t)\mathbf{y}^H(t)] = \mathbf{A}_v \mathbf{R}_s \mathbf{A}_v^H + \sigma_n^2 \mathbf{J}\mathbf{F}\mathbf{\Sigma}_n \mathbf{F}^H \mathbf{J}^H \quad (10)$$

where the steering matrix of the virtual array,  $\mathbf{A}_v = \mathbf{J}\mathbf{F}\mathbf{A}$ , is Vandermonde.

To perform forward/backward spatial smoothing [5], we divide the virtual array into overlapping subarrays of  $M_S$  elements each. The  $p$ th forward subarray covariance matrix  $\mathbf{R}_p^{(f)}$  is then the  $p$ th  $M_S \times M_S$  principal submatrix of  $\mathbf{R}_y$ , where  $p = 1, \dots, \hat{M}$  and  $\hat{M} = M - M_S + 1$  is the total number of subarrays; and the  $p$ th backward subarray covariance matrix is given by

$$\mathbf{R}_p^{(b)} = \tilde{\mathbf{I}} \left( \mathbf{R}_p^{(f)} \right)^* \tilde{\mathbf{I}} \quad (11)$$

where  $\tilde{\mathbf{I}}$  is the reverse permutation matrix. The spatially smoothed covariance matrix is now found as

$$\mathbf{R}_{av} = \frac{1}{2\hat{M}} \sum_{p=1}^{\hat{M}} \left[ \mathbf{R}_p^{(f)} + \tilde{\mathbf{I}} \left( \mathbf{R}_p^{(f)} \right)^* \tilde{\mathbf{I}} \right] = \hat{\mathbf{A}}_v \bar{\mathbf{R}}_s \hat{\mathbf{A}}_v^H + \bar{\mathbf{R}}_n \quad (12)$$

where

$$\bar{\mathbf{R}}_s = \frac{1}{2\hat{M}} \sum_{p=1}^{\hat{M}} \mathbf{B}^{p-1} \mathbf{R}_s \left( \mathbf{B}^{p-1} \right)^H + \mathbf{B}^{p-M} \mathbf{R}_s^* \left( \mathbf{B}^{p-M} \right)^H, \quad (13)$$

$$\bar{\mathbf{R}}_n = \frac{1}{2\hat{M}} \sum_{p=1}^{\hat{M}} \left( \mathbf{J}\mathbf{F}\mathbf{\Sigma}_n \mathbf{F}^H \mathbf{J}^H \right)_p, \quad (14)$$

$$\mathbf{B} = \text{diag} \left( e^{-jh\theta}, e^{-j(h-1)\theta}, \dots, e^{-j(h-M_S+1)\theta} \right), \quad (15)$$

$$\hat{\mathbf{A}}_v = [\hat{\mathbf{a}}_v(\theta_1) \cdots \hat{\mathbf{a}}_v(\theta_L)], \quad (16)$$

$\hat{\mathbf{a}}_v(\theta)$  consists of the first  $M_S$  elements of  $\mathbf{a}_v(\theta)$ , and

$(\mathbf{J}\mathbf{F}\Sigma_n\mathbf{F}^H\mathbf{J}^H)_p$  is the  $p$ th  $M_S \times M_S$  principal sub-matrix of  $\mathbf{J}\mathbf{F}\Sigma_n\mathbf{F}^H\mathbf{J}^H$ .

As long as the number of signals in the largest subgroup of highly correlated signals is not more than  $2M/3$ , the above procedure ensures that  $\bar{\mathbf{R}}_s$  is well-conditioned [5]. This means that the spatially smoothed  $\bar{\mathbf{R}}_s$  has reduced signal correlations compared to  $\mathbf{R}_s$  and optimum beamforming will now work [4, 6].

Summarizing, the optimum beamformer for a UCA in a correlated signal environment is found as follows

$$\min_{\hat{\mathbf{w}}} \hat{\mathbf{w}}^H \mathbf{R}_{av} \hat{\mathbf{w}} \quad \text{subject to} \quad \hat{\mathbf{w}}^H \hat{\mathbf{a}}_v(\theta_l) = 1 \quad (P2)$$

where  $\hat{\mathbf{w}}$  is as defined in Fig. 1. The resulting optimal weight vector is given by

$$\hat{\mathbf{w}}_{\text{opt}} = \mathbf{R}_{av}^{-1} \hat{\mathbf{a}}_v(\theta_l) \left[ \hat{\mathbf{a}}_v^H(\theta_l) \mathbf{R}_{av}^{-1} \hat{\mathbf{a}}_v(\theta_l) \right]^{-1}. \quad (17)$$

We shall refer to the beamformer as defined by (P2) the *spatially smoothed optimum beamformer*.

Note that only one subarray of the virtual array is used in the spatially smoothed optimum beamformer. This suggests a compromise in the effective aperture of the beamformer when using this approach to deal with highly correlated signals.

Note also that for a signal  $s(t)$  arriving from the look direction, the output of the virtual array is given by

$$\mathbf{y}(t) = \mathbf{J}\mathbf{F}\mathbf{x}(t) = \mathbf{J}\mathbf{F}\mathbf{a}(\theta_l)s(t) = \mathbf{a}_v(\theta_l)s(t). \quad (18)$$

The output of the spatially smoothed optimum beamformer is given, accordingly, by

$$\hat{\mathbf{w}}^H \hat{\mathbf{a}}_v(\theta_l) s(t) = s(t) \quad (19)$$

which follows from the look direction constraint in (P2). Thus, the beamformer is able to receive the desired signal with no distortion, although strictly speaking, there will be some distortion due to the approximation in (7).

## 2.4 Derivative Constraints

In an uncorrelated signal environment, the conventional optimum beamformer is known to be highly sensitive to mismatches between the beamformer's look direction  $\theta_l$  and the actual DOA of the desired signal  $\theta_1$ . In [11], it is shown that this sensitivity can be reduced by appending *derivative constraints* to the optimization problem (P1). In the case of the spatially smoothed optimum beamformer, we expect a similar problem will arise. We derive, accordingly, derivative constraints for this beamformer.

Following [11], the derivative constraints are found by setting to zero the partial derivatives (wrt  $\theta$ ) of the power response of the virtual array, i.e.,

$$\left. \frac{\partial^n}{\partial \theta^n} \left| \hat{\mathbf{w}}^H \hat{\mathbf{a}}_v(\theta) \right|^2 \right|_{\theta=\theta_l} = 0, \quad n = 1, 2, \dots \quad (20)$$

It can be shown, similarly to [11], that the first order

( $n=1$ ) derivative constraint is linear in  $\tilde{\mathbf{w}}$  where

$$\tilde{\mathbf{w}}^T = \begin{bmatrix} \text{Re}\{\hat{\mathbf{w}}\}^T & \text{Im}\{\hat{\mathbf{w}}\}^T \end{bmatrix}, \quad (21)$$

whereas the higher order derivative constraints are quadratic in  $\tilde{\mathbf{w}}$ . We consider, therefore, only the first order constraint. The *robust optimum beamformer* is defined, accordingly, by

$$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H \tilde{\mathbf{R}} \tilde{\mathbf{w}} \quad \text{subject to} \quad \tilde{\mathbf{C}}^T(\theta_l) \tilde{\mathbf{w}} = \tilde{\mathbf{f}} \quad (P3)$$

where

$$\tilde{\mathbf{R}} = \begin{bmatrix} \text{Re}\{\mathbf{R}_{av}\} & -\text{Im}\{\mathbf{R}_{av}\} \\ \text{Im}\{\mathbf{R}_{av}\} & \text{Re}\{\mathbf{R}_{av}\} \end{bmatrix}, \quad (22)$$

$$\tilde{\mathbf{C}}(\theta_l) = [\mathbf{c}_1(\theta_l) \quad \mathbf{c}_2(\theta_l) \quad \mathbf{c}_3(\theta_l)], \quad (23)$$

$$\tilde{\mathbf{f}} = [1 \quad 0 \quad 0], \quad (24)$$

$$\mathbf{c}_1^T(\theta_l) = \begin{bmatrix} \text{Re}\{\hat{\mathbf{a}}_v(\theta_l)\}^T & \text{Im}\{\hat{\mathbf{a}}_v(\theta_l)\}^T \end{bmatrix}, \quad (25)$$

$$\mathbf{c}_2^T(\theta_l) = \begin{bmatrix} -\text{Im}\{\hat{\mathbf{a}}_v(\theta_l)\}^T & \text{Re}\{\hat{\mathbf{a}}_v(\theta_l)\}^T \end{bmatrix}, \quad (26)$$

$$\mathbf{c}_3^T(\theta_l) = \begin{bmatrix} \text{Re}\{\hat{\mathbf{a}}'_v(\theta_l)\}^T & \text{Im}\{\hat{\mathbf{a}}'_v(\theta_l)\}^T \end{bmatrix}, \quad (27)$$

and where  $\hat{\mathbf{a}}'_v(\theta_l)$  is the first derivative of  $\hat{\mathbf{a}}_v(\theta)$  wrt  $\theta$ , evaluated at  $\theta = \theta_l$ ,

$$\hat{\mathbf{a}}'_v(\theta_l) = -j \cdot \begin{bmatrix} h e^{-jh\theta_l} & \dots & (h - M_S + 1) e^{-j(h - M_S + 1)\theta_l} \end{bmatrix}^T. \quad (28)$$

The solution to (P3) is given by

$$\tilde{\mathbf{w}}_{\text{opt}} = \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}(\theta_l) \left[ \tilde{\mathbf{C}}(\theta_l)^T \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{C}}(\theta_l) \right]^{-1} \tilde{\mathbf{f}}. \quad (29)$$

Note that if  $\tilde{\mathbf{C}}(\theta_l)$  is not full rank, then the redundant constraint(s) will have to be located and removed.

## 3. NUMERICAL EXAMPLE

In the following example, we consider a UCA of 15 elements with  $d/\lambda = 0.3$ . We choose  $\varepsilon = 0.05$  which by (8), results in a 13-element virtual array. The subarray length was set to 9 for 5 subarrays. The signal scenario consists of uncorrelated sensor noise of 0 dB, and three equal power (10 dB) signals arriving from  $-120^\circ$ ,  $0^\circ$  and  $90^\circ$ . The signals arriving from  $-120^\circ$  and  $90^\circ$  are fully correlated while the signal arriving from  $0^\circ$  is not correlated with either of the other two signals.

In Fig. 2, we plot the power response of the conventional, spatially smoothed, and robust optimum beamformers, as the look direction is swept across the  $360^\circ$  azimuthal plane. As can be seen, in contrast to the spatially smoothed and robust optimum beamformers, the conventional optimum beamformer was not able to receive the two correlated signals arriving from  $-120^\circ$  and  $90^\circ$ . Also, the peaks of the robust optimum beamformer are broader than the peaks of the spatially smoothed optimum

beamformer because of the derivative constraints.

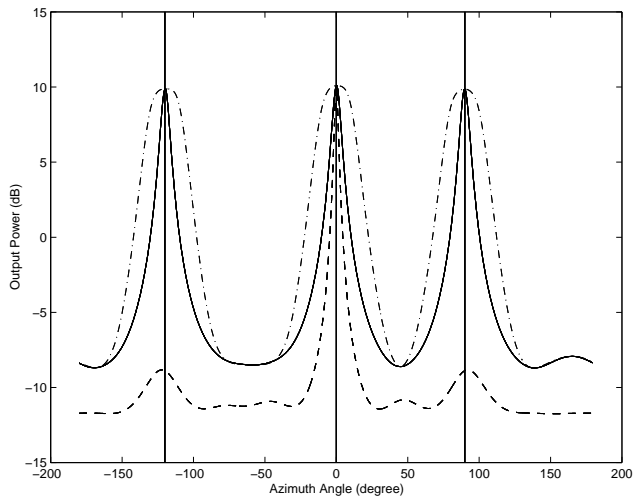


Fig. 2. Output power against look direction. Dashed line: conventional optimum beamformer. Solid line: spatially smoothed optimum beamformer. Dashed-dot line: robust optimum beamformer.

In Fig. 3, we compare the array patterns of the three beamformers when they are steered towards the look direction of  $-120^\circ$ . As can be seen, they all achieved the target response (0 dB gain) in the look direction, and placed nulls in the DOA of the uncorrelated interferer at  $0^\circ$ . However, only the spatially smoothed and robust optimum beamformers were able to place a “null” in the DOA of the correlated interferer at  $90^\circ$ . The “null” at  $90^\circ$  can be made deeper by increasing the number of subarrays used but this will reduce the subarray size and hence the number of degrees of freedom [6].

#### 4. CONCLUSIONS

The transformation technique as presented in this paper provides a solution to the design of optimum beamformers for UCAs in the presence of coherent signals. However, the technique is limited by the approximation involved in the modal transformation and may necessitate a compromise in the size of the virtual array. We also showed how derivative constraints can be easily incorporated into the design, and showed that they are effective in increasing the robustness of the beamformer to directional mismatches.

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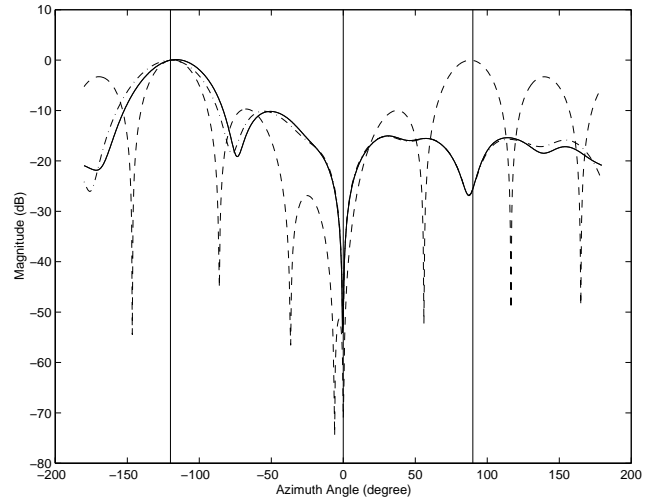


Fig. 3. Array pattern for look direction of  $-120^\circ$ . Dashed line: conventional optimum beamformer. Solid line: spatially smoothed optimum beamformer. Dashed-dot line: robust optimum beamformer.

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