Maximum Likelihood Identification of Dynamics of the Ågesta Reactor and Comparison with results of Spectral Analysis

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MAXIMUM LIKELIHOOD IDENTIFICATION OF DYNAMICS OF THE ÅGESTA REACTOR AND COMPARISON WITH RESULTS OF SPECTRAL ANALYSIS †

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ABSTRACT

The transfer function from reactivity to nuclear power has been determined by identification of dynamic measurements of a nuclear reactor. Two different methods have been used. A parametric model of the process has been developed by the maximum likelihood method. This model is compared with the results of spectral analysis of the same measurements. Differences in the results show that the maximum likelihood method may be preferable if the noise to signal ratio is high. An example with simulated data indicates that the differences between the results from the two methods may be remarkable, and that it may be difficult to get a reasonable parametric model from the results of the spectral analysis.

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1. INTRODUCTION

This work is a part of a systematic investigation of various identification methods with respect to practical applications to industrial processes. The purpose of this paper is to compare the properties of the maximum likelihood estimate with the results of spectral analysis, when identifying data from dynamic measurements of a nuclear reactor.

Some sort of identification must be performed if complex control systems have to be designed or if a theoretically developed model shall be compared with the real process. The choice of identification method depends on what information of the process that is needed. If a parametric model is required, for instance, there are two ways to get it. One method is to estimate the transfer function and to approximate a parametric model from the Bode diagram. Another method is to use an identification procedure, such as the least squares estimate or the maximum likelihood estimate, which directly gives a parametric model. A parametric discrete time model is appropriate for determining control strategies and preferable if a digital computer is to be used to implement the control law. For such applications the direct methods seem to have many advantages. If on the other hand only a rough estimate of the system is required or if we only want to compare a model of a system with the real system, perhaps spectral analysis is an appropriate identification method. For such applications the Bode diagram may be sufficient.

In this work measurements from a nuclear reactor have been analysed. The transfer function from control rod position (approximately proportional to the reactivity) to nuclear power has been determined. The maximum likelihood estimate of the parameters of a linear time discrete model with a disturbance with rational spectrum, was computed. The spectrum of the disturbance as well as the transfer function were obtained. This transfer function is then compared with the results from the spectral analysis of the same data. It turns out that the two methods give comparable results, but there are differences in the amplitude estimates of about 0.5 dB and in the phase estimates of about 5°. These differences are most probably due to the uncertainty of the estimates

obtained from the spectral analysis. An example is given where it is shown that the difference between the estimates from the two methods may differ remarkably. This seems to be the case when the noise to signal ratio is high. The maximum likelihood method may give good estimates even if the disturbances are not small compared with the input signal, while spectral analysis seems to be more sensitive in this respect. However, the evaluation of the coherence function will give a possibility to check the reliability of the spectral analysis estimate.

The conclusion is that the choice of identification method is primarily based on what is the main purpose of the investigation. Other factors one must have in mind are available computer time and memory and requested accuracy of the results. And all these factors ought to be remembered already when the measurements are planned together with the usual problems of choosing input signal, sampling interval etc.

Furthermore pseudo random binary signals and spectral analysis are discussed shortly in this report.

2. PHYSICS EXPERIMENTS AT ÅGESTA NUCLEAR POWER STATION

The data, for which the identification is performed, have been received from AB Atomenergi, Studsvik Sweden. The measurements are described and some results of the analysis of these data at AB Atomenergi are given in {1}. The data are from dynamic measurements performed at the Ågesta reactor, the first power reactor built in Sweden. A simplified flow diagram is outlined in figure 1 (from {1}).

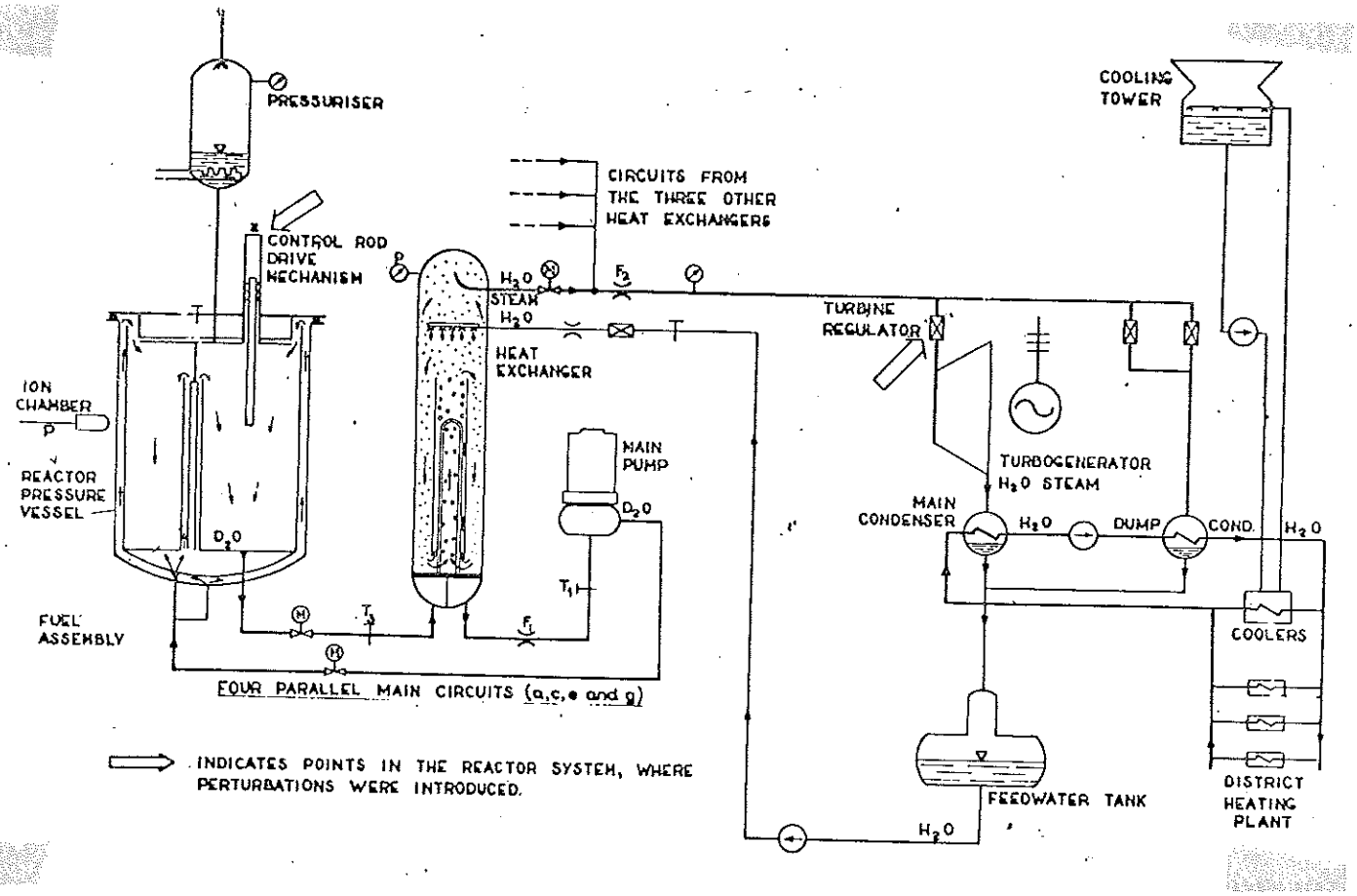


Fig. 1 - Simplified flow diagram of the Ågesta Power Reactor.

The measurements were performed at different power levels with the automatic control system switched off. The purpose of the whole experiment was to measure the dynamic characteristics of the reactor in order to check dynamic models derived from physical equations and to give information on the system, which could be of value for operation of the Ågesta reactor and for design of future reactors etc. The system was disturbed by different perturbation functions in the reactivity and in the load. The responses of nuclear power, coolant inlet and outlet temperature and control rod position were recorded.

The two series analyzed here, denoted AR60 and AR61, are both such that the input signal is the control rod position and the output signal is the nuclear power. It ought to be mentioned that these two series are only two of many measurements performed in order to get information on the reactor dynamics. The input signal was in these two cases a pseudo random binary signal denoted BDG in {1}. The two input-output sequences are plotted in figures 2-3. The results from the analysis at AB Atomenergi are presented as transfer functions (Bode diagrams) from reactivity to power, figure 4 (from {1}).

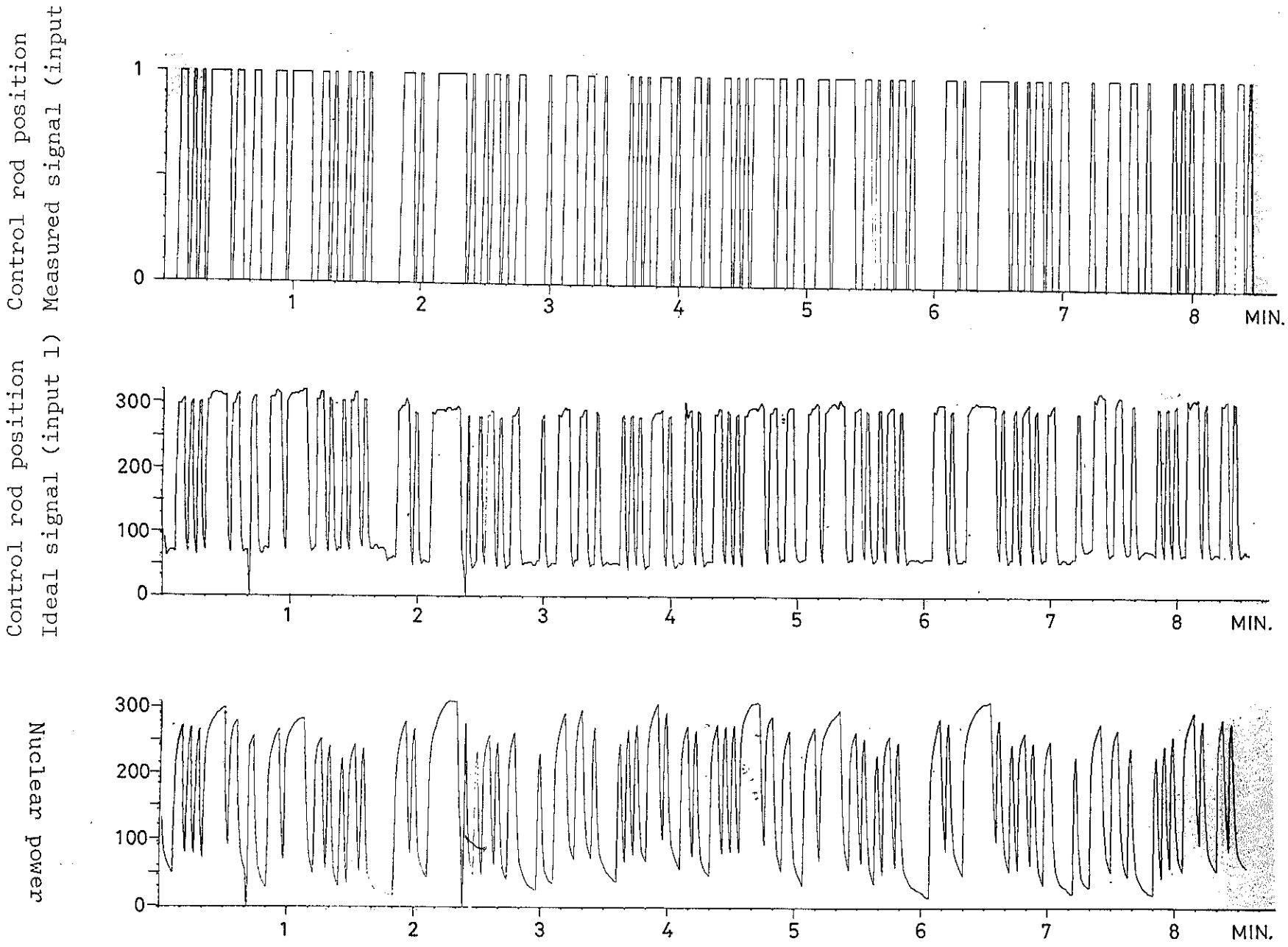


Fig. 2 - Results of an experiment for determination of process dynamics showing response of nuclear power to perturbation of control rod position. The sequence is denoted AR60 A. (Digital units on vertical scale).

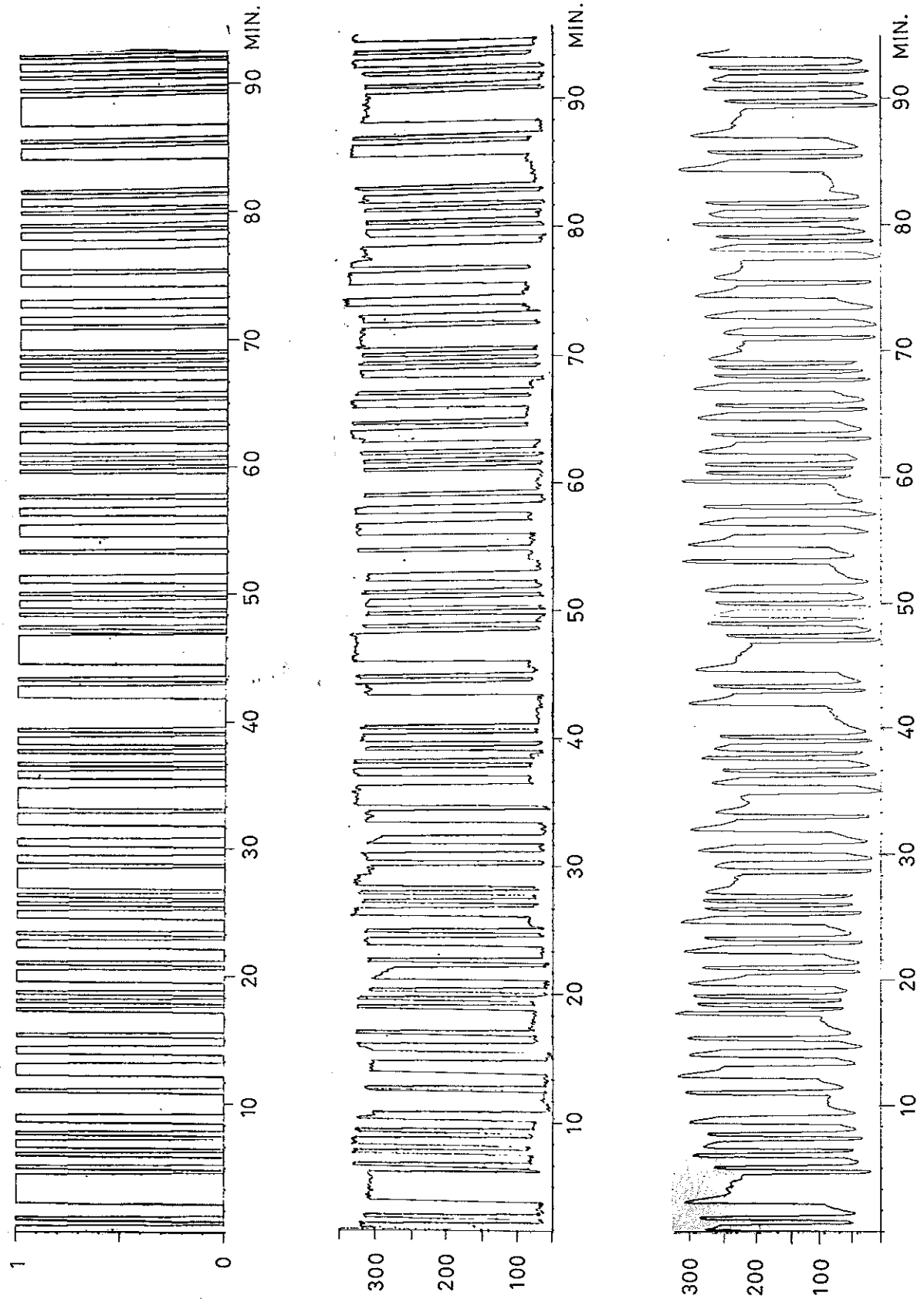


Fig. 3 - Results of an experiment for determination of process dynamics showing response of nuclear power to perturbation of control rod position. The sequence is denoted AR61. (Digital units on vertical scale).

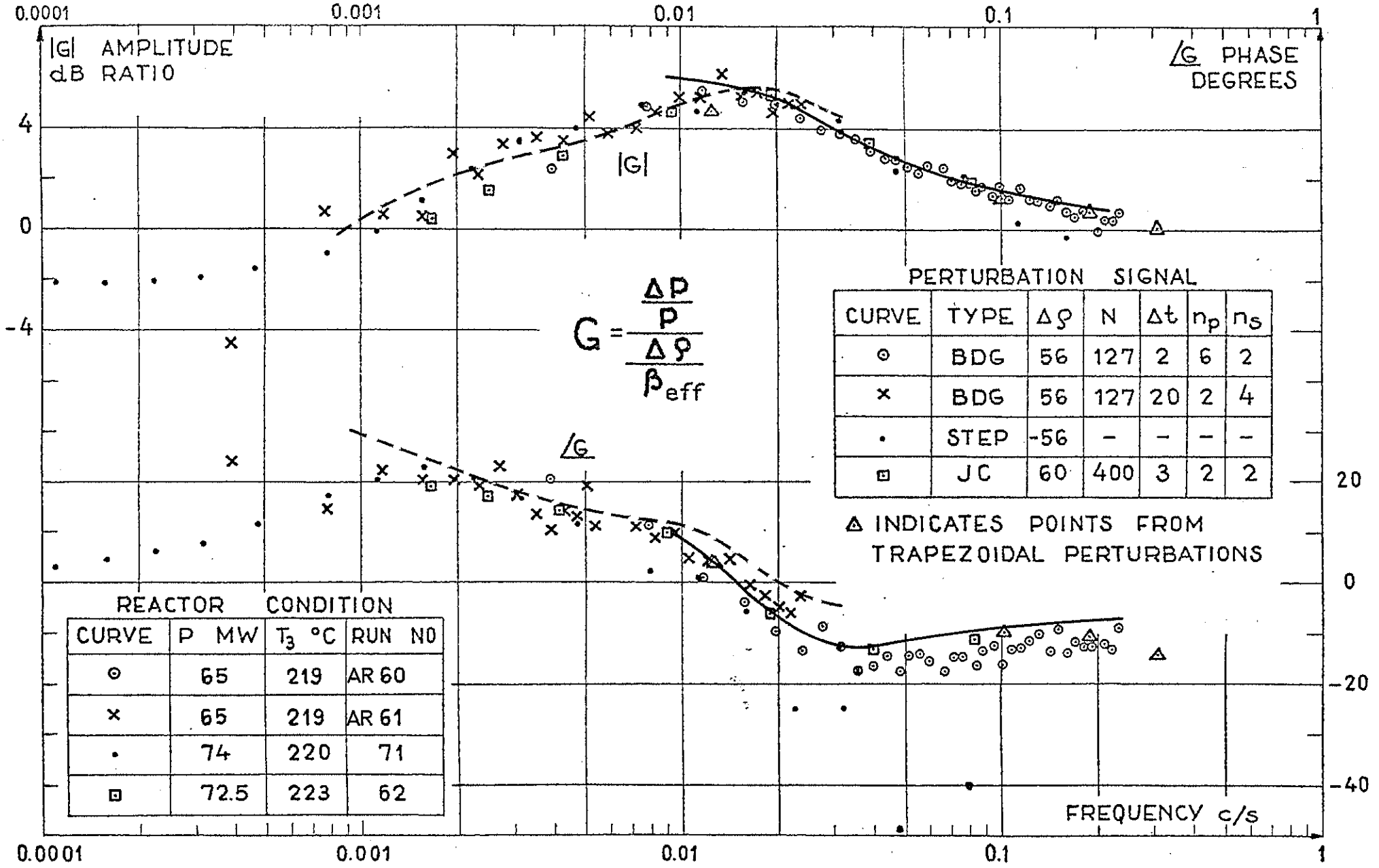


Fig. 4 - Comparison between different perturbation methods. The curves show the results of maximum likelihood identification (perturbation signal BDG; run AR60 and run AR61 (dashed line))

3. OUTLINE OF THE MAXIMUM LIKELIHOOD ESTIMATION METHOD

The problem is to determine an appropriate model of a process from which we have input-output samples. Assuming the process to be linear of n-th order and to be subject to disturbances that are stationary random processes with rational power spectra, we choose the model

$$A(z^{-1})\, y(t) = B(z^{-1})\, u(t) + \lambda\, C(z^{-1})\, e(t) \qquad (1)$$

where $\{u(t), y(t), t = 1,2,\ldots,N\}$ is the input-output sequence and where $\{e(t), t = 1,2,\ldots,N\}$ is a sequence of independent normal (0,1) random variables. z denotes the shift operator

$$z\, x(t) = x(t+1) \qquad (2)$$

and A(z), B(z) and C(z) are polynomials

$$A(z) = 1 + a_1 z + \ldots + a_n z^n$$

$$B(z) = b_1 z + \ldots + b_n z^n$$

$$C(z) = 1 + c_1 z + \ldots + c_n z^n \qquad (3)$$

Since the identification is described elsewhere {10}, {8}, {9}, the details are not given here. However, a short summary is presented.

The problem is solved by determining the maximum likelihood estimate of the parameters $\theta = (a_1,\ldots,a_n,b_1,\ldots,b_n,c_1,\ldots,c_n)$. The maximum likelihood estimate is consistent, asymptotically normal and efficient under mild conditions given in {11}. Maximizing the likelihood function is equivalent to minimizing the loss function

$$V(\theta) = \frac{1}{2} \sum_{t=1}^{N} \varepsilon^2(t) \qquad (4)$$

where the residuals $\varepsilon(t)$ are obtained from

$$C(z^{-1})\, \varepsilon(t) = A(z^{-1})\, y(t) - B(z^{-1})\, u(t) \qquad (5)$$

The identification problem is then reduced on a problem of minimizing a function of several variables.

has an F(3k, N - 3(n+k)) distribution under null hypothesis. When N is large $3k \cdot F_{n+k,n}$ tends to a χ^2-distribution with 3k degrees of freedom. Most often the test is used with k = 1, that is we test the model of order (n+1) against the model of order n. It is used at a risk level of 5%, that is if the test quantity is greater than 2.6 (N is supposed to be larger than 100), the loss function has been reduced significantly and the order of the model is at least (n+1).

FORTRAN programs for the identification procedure are available. The programs are described in {9}. They can handle the multiple-input, single output case and are in this sense a bit more general than what has been described in this section. But principally there is no difference. Notice that the input and output signals should be transformed before the identification procedure as to have zero mean. There is no loss in generality for the identification procedure by assuming the constant term of the B-polynomial, b_o, to be zero or by assuming the time delay k in the model

$$A(z^{-1})\, y(t) = B(z^{-1})\, u(t-k) + \lambda\, C(z^{-1})\, e(t) \qquad (8)$$

to be zero. These cases can be handled with by shifting input/output signals appropriately. On this matter we only point out that for one run of the identification procedure with reactor data the convergence was extremely slow when starting from $\theta = (0, \ldots, 0)$ using the model (1), but when a slightly different model, that is $a_n = c_n = 0$, was used no such convergence difficulty appeared. Even in the bad case fast convergence to a minimum was obtained when choosing another starting point. However, for certain cases it may occur that the uncertainty of the parameters can be remarkably reduced by choosing another structure.

We solve this problem by a recursive technique, which uses both the gradient with respect to the parameters, V_θ, and the matrix of the second partial derivatives, $V_{\theta\theta}$. Other minimization methods do not use the second derivatives but this extra work is not of essential importance because the computations of $V_{\theta\theta}$ are done very economically and increase only linearly with the order of the model for large N. Furthermore this method directly gives the accuracy of the parameters, because an estimate of the inverse of the information matrix is available ($\lambda^2\{V_{\theta\theta}\}^{-1}$). The parameter λ is determined from

$$\hat{\lambda} = \frac{2}{N} V(\hat{\theta}) \qquad (6)$$

where $\hat{\theta}$ is such that $V(\hat{\theta})$ is minimal.

To obtain a starting value for the minimizing algorithm we put $c_i = 0$, $i = 1, \ldots, n$. The loss function $V(\theta)$ is quadratic in a_i and b_i and the algorithm converges in one step to the least squares estimate of the a- and b-parameters. This estimate is then taken as the starting point for the gradient routine. By taking different starting values of c_i, $i = 1, \ldots, n$ we investigate whether $V(\theta)$ has several local minima.

This method of identification also gives a possibility to test the order of the model, when it is unknown. The identification is repeated for increasing order of the model. Now let V_n denote the minimal value of the loss function for the n-th order model. It follows from {10} that the parameter estimates for large N are asymptotically normal (θ_o, $\lambda^2 V_{\theta\theta}^{-1}$), where θ_o stands for the correct value of θ. Assuming that asymptotic theory may be applied we test the hypothesis that the system is of order n, that is the null hypothesis is

$$H_o: a_{n+1}^o = \ldots = a_{n+k}^o = b_{n+1}^o = \ldots = b_{n+k}^o =$$

$$= c_{n+1}^o = \ldots = c_{n+k}^o = 0$$

(θ_i^o stands for the correct value of θ_i).

Then

$$F_{n+k,n} = \frac{V_n - V_{n+k}}{V_{n+k}} \cdot \frac{N - 3(n+k)}{3 \cdot k} \qquad (7)$$

4. THE PERTURBATION SIGNAL

For the two measurements AR60 and AR61 a pseudo-random binary sequence (PRBS) was chosen as the perturbation signal. Such sequences may be generated with different methods, generating m-sequences, quadratic residue codes, Hall-sequences etc. {4}. Those sequences have the same autocovariance function

$$r_{xx}(\tau) = \frac{1}{N} \sum_{t=1}^{N} x(t)\, x(t+\tau) = \begin{cases} 1 \text{ if } \tau = 0, \text{ modulo } N \\ -\frac{1}{N} \quad \text{elsewhere} \end{cases}$$

$r_{xx}(\tau)$ = the autocovariance function

$x(t)$ = the value of the sequence at time t (+1 or -1)

N = the period of the sequence

The autocovariance function is thus approximately an impulse function. This fact makes correlation analysis easy, and for instance it is possible to obtain the impulse response function out of the measured crosscorrelation function, if the period, N, and the pulse length, Δt, have been chosen appropriately for the investigated system. Rules of thumb for this choice are available {4}. Unfortunately you have to compromise because those conditions contradict the conditions for high reliability of the estimates of the weighting coefficients of a system with noise.

An investigation of the spectrum of the PRBS shows that the spectrum is a line spectrum with a lowest frequency of $1/N \cdot \Delta t$ c/s and that an upper frequency limit of $1/4\Delta t$ c/s may be defined (the 3 dB point is approximately $0.22/\Delta t$ c/s).

From the rules of thumb it follows for this case, where the shortest time constant is about 2-3 seconds and the longest one 50-150 seconds, that N should be 500-1000 and Δt 1-2 seconds. If you instead of this say that the interesting part of the spectrum is between 0.001 c/s and 0.25 c/s, you get Δt = 1 second and N = 1000.

For these experiments one has chosen to investigate the system in two steps, first in the frequency band 0.0004-0.02 c/s and then in the band 0.004-0.2 c/s. The values of N and Δt are for AR60 127 and 2 and for AR61 127 and 20, respectively. The frequency bands were chosen to overlap each others, because a good estimate was desirable around 0.01 c/s.

This division of the investigation will give shorter measurement time, but different identification methods may give the model quite different structures. If only a frequency diagram is desired nothing is changed, but if you want a model of the system for designing control laws it will be more difficult. For the maximum likelihood estimate you will have one model for each frequency band and it will be rather difficult to get a model describing the total system. It may be solved by using different control loops for each frequency domain especially when the time constants of the system differ very much.

5. RESULTS OF IDENTIFICATION BY THE MAXIMUM LIKELIHOOD METHOD

As mentioned in a previous section identification of two series of data, AR60 and AR61, has been performed. AR60 contains 1704 data points and is separated into three parts, A,B, and C. Series AR60 A contains the first 514 data, B the next 512 data and C the rest. AR61 contains 1126 data. The sampling interval is 1 and 5 seconds respectively. Two input signals are available, input 1, the ideal input, and input 2, the measured input. First we give an example showing the results of identification of a certain input-output sequence for increasing order of the model. The input- and output sequences have been shifted so that the model we use (1) also includes a direct input-output connection. As an example we choose series AR60, input 1 (table 1).

	n = 1	2	3	4
a_1	-0.345±0.010	-1.078±0.036	-2.062±0.028	-1.811±0.348
a_2		0.200±0.017	1.253±0.042	0.796±0.720
a_3			-0.188±0.015	0.030±0.442
a_4				-0.010±0.069
b_1	165.4±2.0	168.5±1.6	168.4±1.5	166.5±1.8
b_2		-131.0±5.3	-297.6±4.3	-248.4±59.8
b_3			129.6±3.9	53.5±104.7
b_4				28.9±45.8
c_1	0.039±0.039	-0.924±0.057	-2.009±0.050	-1.758±0.348
c_2		0.267±0.043	1.216±0.089	0.784±0.694
c_3			-0.196±0.045	-0.006±0.417
c_4				-0.004±0.075
λ	20.01	17.27	16.23	16.14
V	102657	76508	67554	66797

Table 1 - Results from maximum likelihood identification of series AR60 A, input 1, for increasing order of the model. In the table the estimated values of the parameters are given together with the estimated standard deviation of the parameters. Furthermore the estimated value of λ and the minimal loss function, V, for each model are given. Tests indicate a third order model.

If we test the order with a statistical F-test, we get the following test quantities (notation according to (7))

$F_{2,1} = 57.8$

$F_{3,2} = 22.3$

$F_{4,3} = 2.4$

From this we conclude that the system is of third order. Notice that the large increase of uncertainty of the parameters, when going from n = 3 to n = 4, also indicates that there are redundant parameters in the fourth order model.

For this example we also give the least squares estimates of the coefficients for the third and fourth order model (table 2)

	n = 3	n = 4	n = 3
a_1	-0.296	-0.218	-2.062
a_2	-0.259	-0.233	1.253
a_3	0.004	-0.135	-0.188
a_4		0.006	
b_1	166.0	167.8	168.4
b_2	6.6	15.8	-297.6
b_3	-44.7	-30.7	129.6
b_4		-27.8	
V	81642	77903	67554

Table 2 - Least squares estimates (series AR60, input 1). The estimated parameter values are given for n = 3 and n = 4 together with the values of the loss function. In the table the maximum likelihood estimates for n = 3 are given in the last column.

The least squares estimates seem to be very poor but on the other hand a test of the reduction of the loss function shows that the model should be of higher order. The reason why a high order model is needed to get a good least squares estimate, is that the assumption of uncorrelated noise does not hold for this example. A discussion of this matter can be found in {12}.

The F-test worked very well for this example and there is no indication that the system should be of order less than three. But in other cases the F-test and the examination of the uncertainty of the parameters do not give enough information of the order of the system. Other methods of testing the order have to be considered.

For instance take another example, the identification of AR61, input 1. The F-test gives results which are shown in table 3.

n \ k	2	3	4	5
1	1986	1505	1167	883
2		163	123	84
3			54	29
4				4

Table 3 - Test quantities $F_{k,n}$ for example AR61, input 1. The model of order k has been tested against the model of order n.

We can see that the system seems to be of at least 5:th order. The uncertainty of the parameters does not indicate a lower order system either, because they are of the same order for the fourth and fifth order models. But examining the polynomials A, B and C we find that for the fifth order model these polynomials have a factor in common (table 4).

n	A	B	C
2	0.157 0.905	0.953	-0.237 0.194
3	-0.002 0.578 0.761	0.343 0.897	-0.372 0.364±0.358 i
4	0.054 0.961 0.714±0.288 i	0.986 0.681±0.226 i	-0.255 0.461 0.643 0.739
5	0.050 0.965 0.709±0.286 i -1.001	0.984 0.678±0.219 i -1.002	-0.293 0.781 0.538±0.087 i -1.001

Table 4 - Roots of the polynomials A, B and C respectively for the models of order 2, 3, 4 and 5 for the sequence AR61, input 1. The system seems to be of 4:th order.

Further comments on the problem of testing the order can be found in {8}.

In order to present the results of the identification in a clear way and to get a basis for comparison with the results of the spectral analysis, the phase and amplitude functions (Bode diagrams) for the reactivity to power transfer function are given for the obtained models.

First we give the results obtained when identifying series AR60 A, input 2 (fig. 5). The obtained model is

$$(1 - 2.02z^{-1} + 1.15z^{-2} - 0.13z^{-3})\, y(t) = (0.76 - 1.37z^{-1} + 0.61z^{-2})\, u(t) + 6.55(1 - 1.59z^{-1} + 0.70z^{-2} - 0.07z^{-3})\, e(t)$$

If we identify AR60, input 2, that is a longer measurement with the same conditions, we also find a third order model and the phase curve for this is given in fig. 6. The difference between the models for AR60 and AR60 A is small and this also holds for the amplitude curves. The advantage of a longer series is that

the uncertainty of the parameters will be smaller, but on the other hand the series must not be so long that it is no longer stationary. In this case the system seems to be stationary.

For the series AR61, input 2, we give the phase and amplitude curves for the fourth order model. (Fig. 7). Different tests of the order show that the system probably is of fourth order. The model has the following parameter values

$$(1 - 2.44z^{-1} + 2.08z^{-2} - 0.67z^{-3} + 0.03z^{-4})\ y(t) =$$

$$= (0.77 - 1.81z^{-1} + 1.43z^{-2} - 0.39z^{-3})\ u(t) +$$

$$+ 4.94(1 - 1.65z^{-1} + 0.80z^{-2} - 0.02z^{-3} - 0.06z^{-4})\ e(t)$$

To check if any important change will occur if we instead of the real input signal, input 2, use the ideal input signal, input 1, the series have been identified also for this input signal. The phase curves for AR61, input 1 and 2, n = 4, are shown in fig. 8. No significant difference can be seen.

Notice that the parameter values given above are from a model relating output signal to input signal measured in digital units. If we want this relation in physical units an amplitude factor must be introduced. This has been done when transferring the results to the amplitude diagrams in order to achieve comparability with the results presented in reference {1}. The results of the identification for AR60 A, input 2 and for AR61, input 2, are also given in fig. 4, in order to get a comparison with the results obtained by other identification methods.

The series have also been identified with a slightly different model. The data have been shifted as before, but a_n and c_n have been put equal zero to get the same order of the A, B and C-polynomials (compare the models given above). The differences between the obtained models are rather small, see fig. 9 where an example is given.

In figure 10 the results of the maximum likelihood identification of series AR60 A are shown for the first part of the series. In this figure we show

1. input 2
2. output y(t)
3. the deterministic output $y_d(t)$ defined by

$$y_d(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t)$$

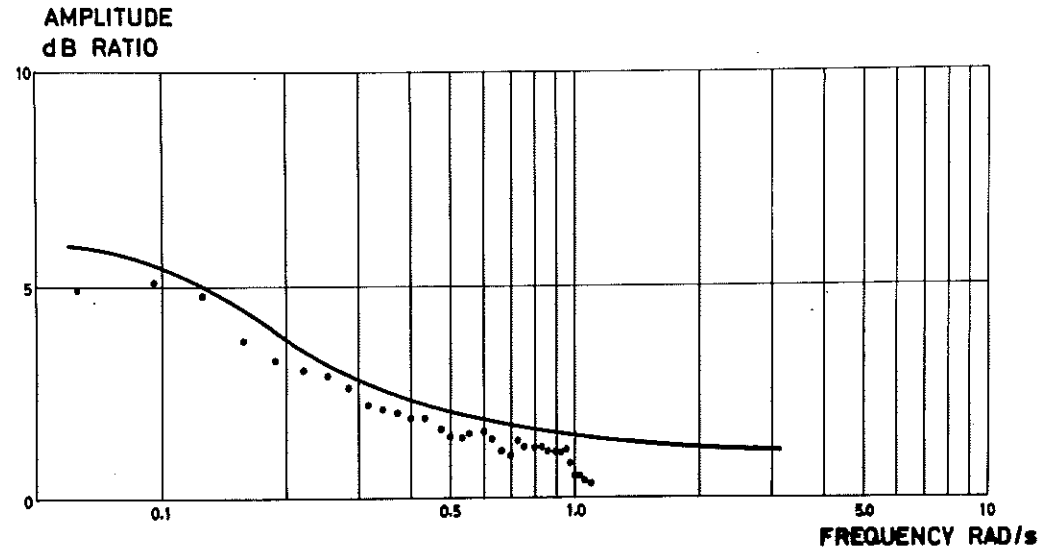
where $B(z^{-1})/A(z^{-1})$ is the resulting third order model.

4. the error of the deterministic model

$$e_d(t) = y(t) - y_d(t)$$

Notice the different scales.

The error of the deterministic model is rather small. This fact indicates that the model is good. Large value occur only when there are sudden jumps in the input signal. Both these values of the input and output signals may depend on failures in the recording unit, because they have no correspondence anywhere else in the measurements.



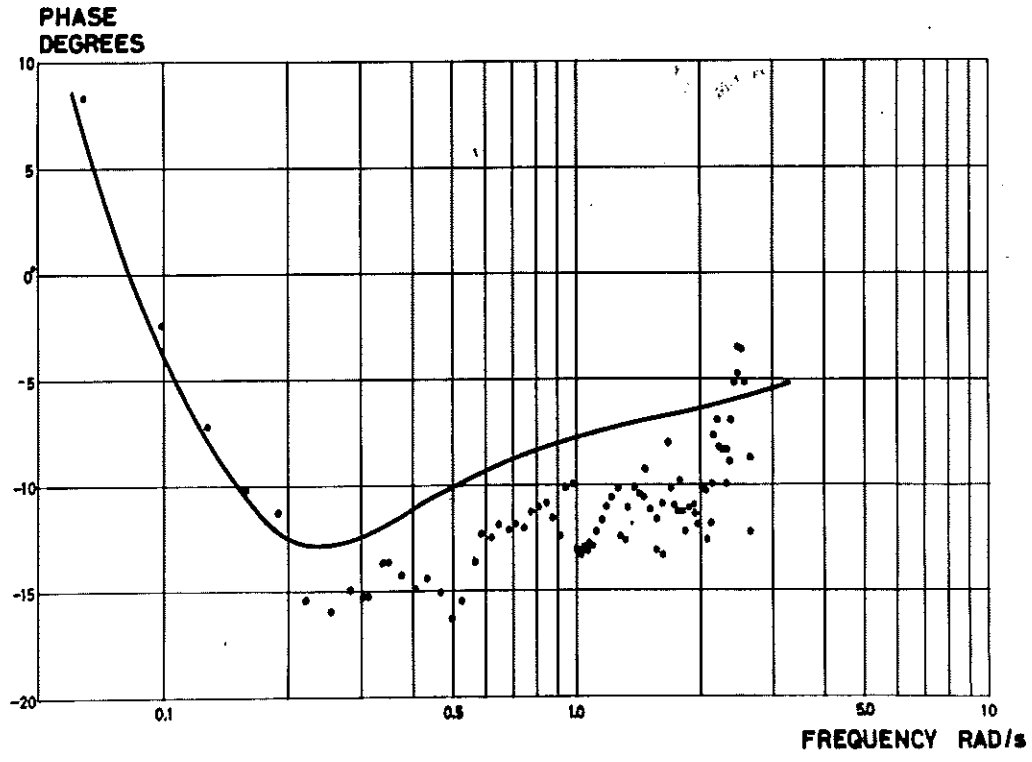


Fig. 5 - Reactivity to power transfer function resulting from maximum likelihood identification of series AR60 A, input 2 (3rd order model), the continuous line, and from spectral analysis of the same series, the dots. For simplicity the amplitude estimates from spectral analysis are omitted for frequencies above 1.1 rad/s.

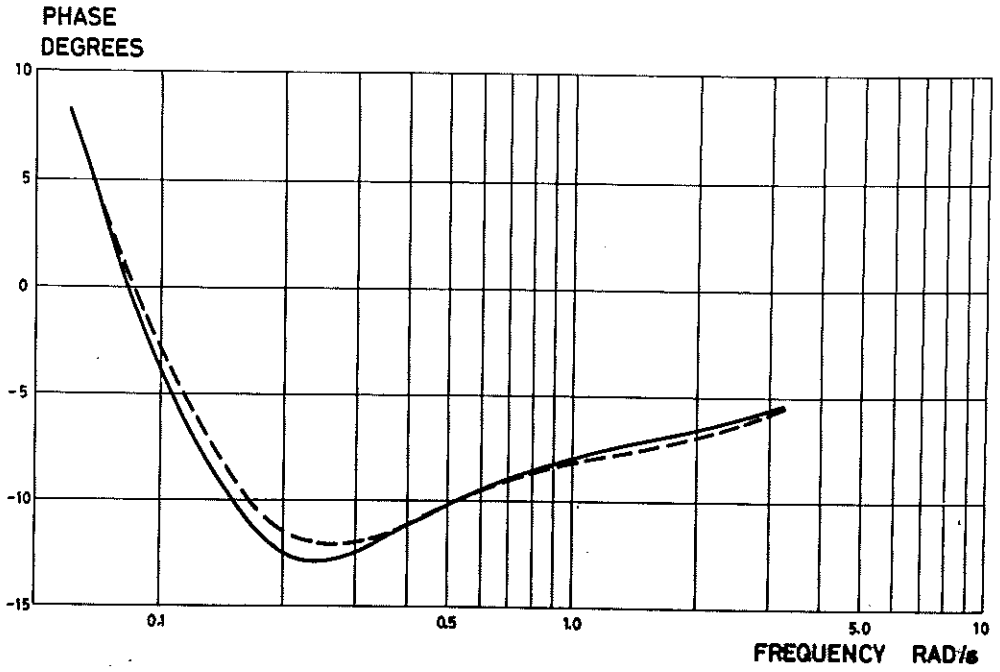


Fig. 6 - Comparison between the phase curves for the estimated models of the series AR60 A (the continuous line) and the series AR60 (the dashed line).

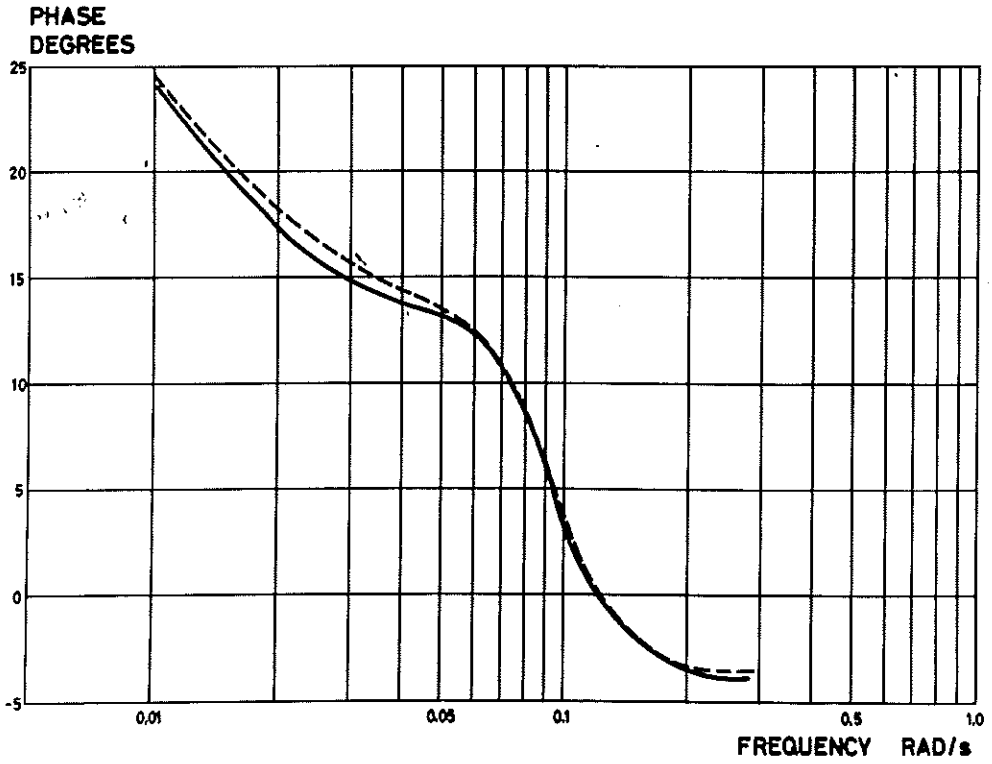


Fig. 8 - Comparison between the phase curves for the models obtained by the maximum likelihood method when identifying series AR61 using input 1, the dashed line, and input 2, the continuous line.

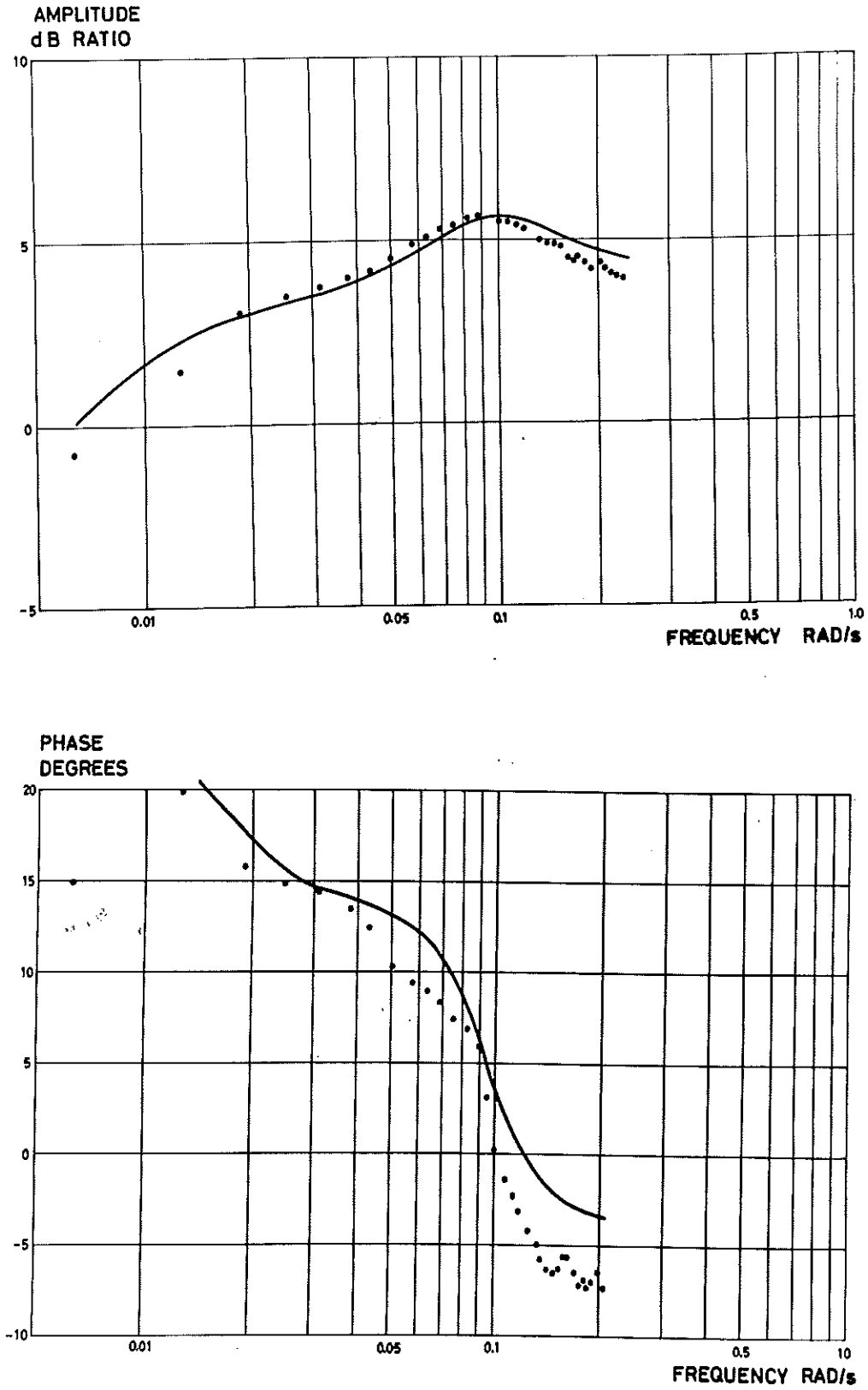
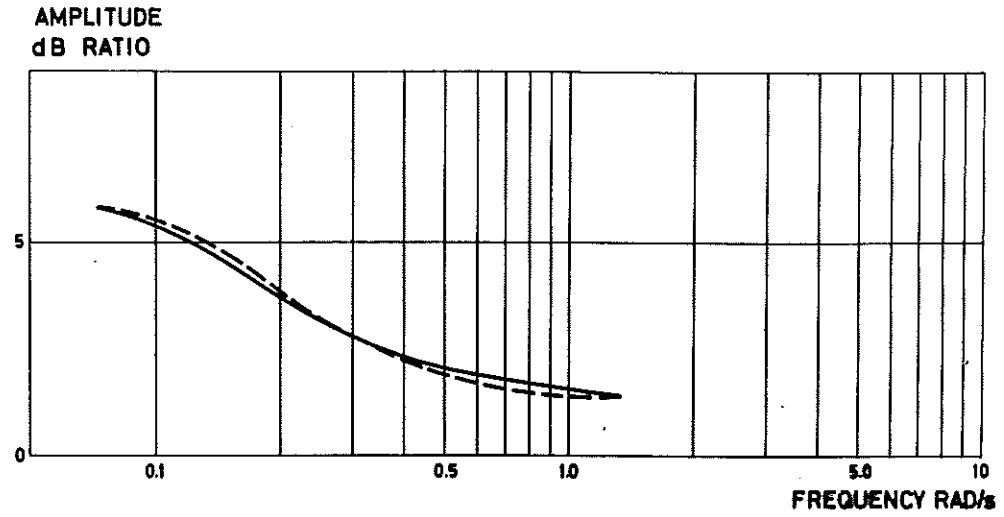


Fig. 7 - Reactivity to power transfer function resulting from maximum likelihood identification of series AR61, input 2 (4th order model), the line, and from spectral analysis of the same series, the dots.



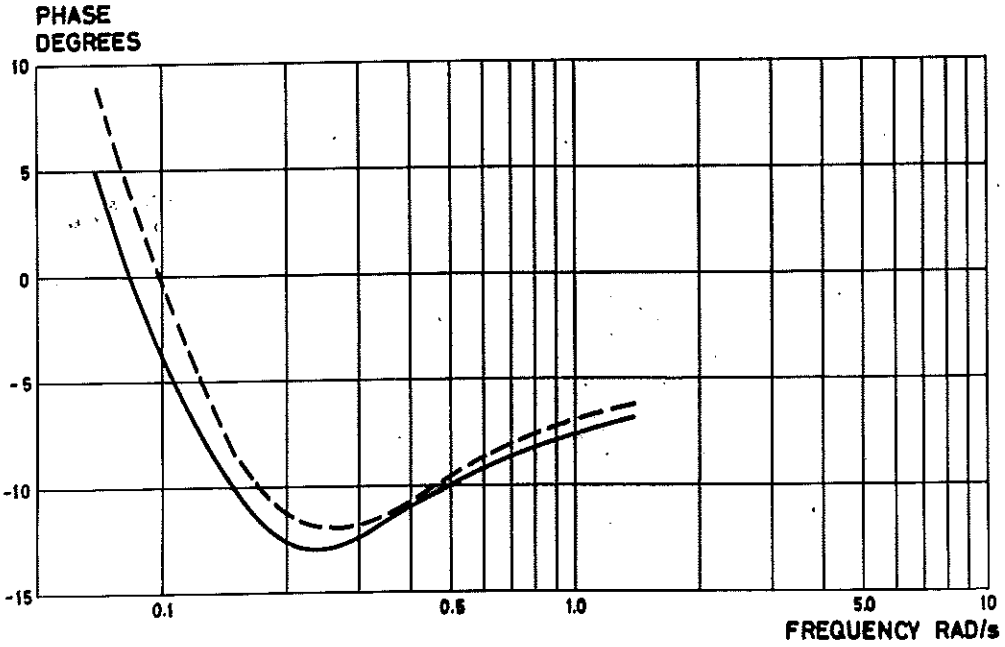


Fig. 9 - Comparison between phase and amplitude curves from models obtained at the maximum likelihood identification of series AR60 A, using different model structures (the dashed line represents the model with $a_n = c_n = 0$)

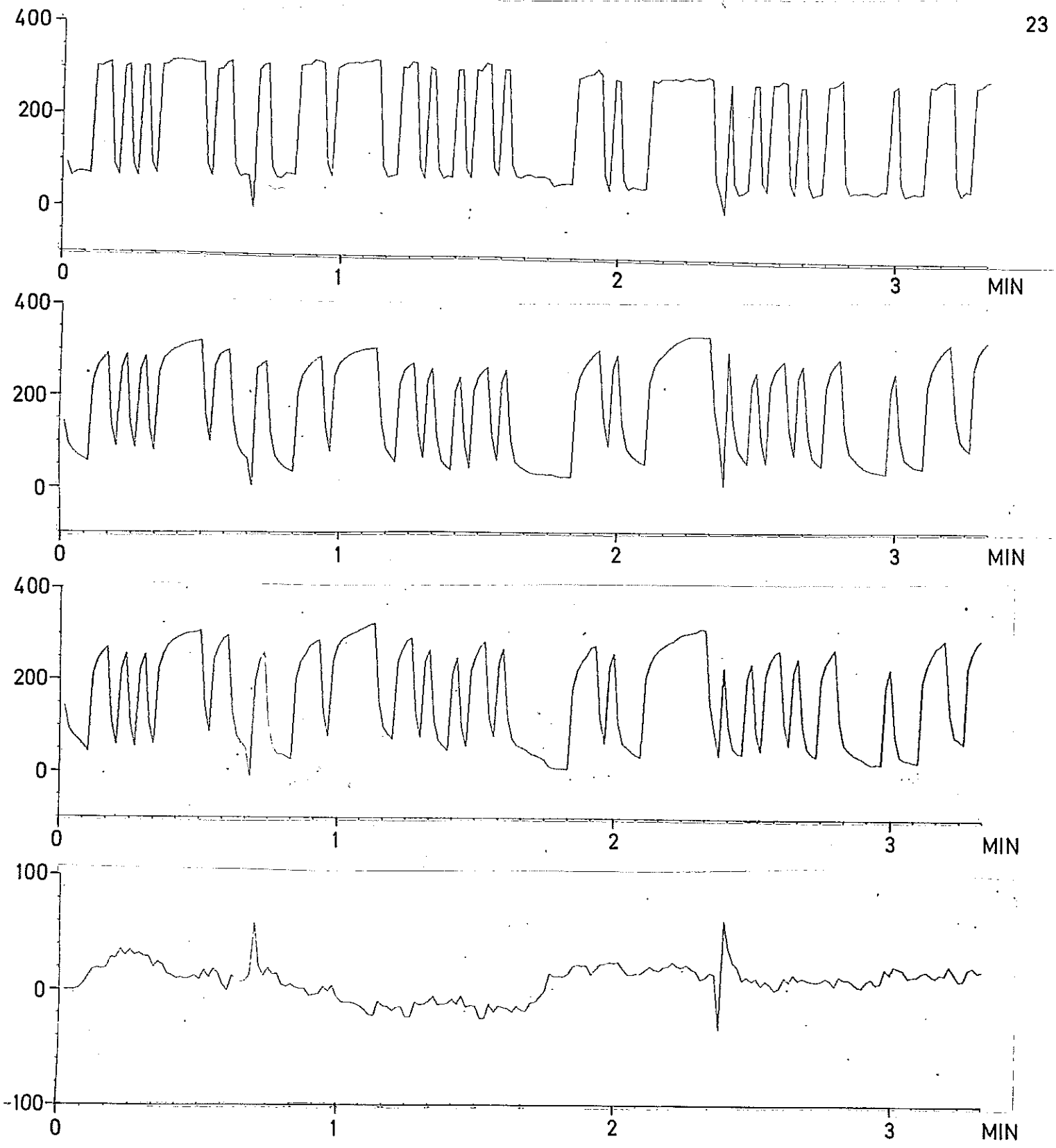


Fig. 10 - Results of maximum likelihood identification of series AR60A (the first part is shown):

1. input 2
2. output y(t)
3. the deterministic output $y_d(t)$
4. the error of the deterministic output $e_d(t)$

Notice the different scales.

6. SPECTRAL ANALYSIS

An estimate of the transfer function, $G(i\omega)$, for a system with input $u(t)$ and output $y(t)$ may be obtained by spectral analysis if appropriate assumptions hold for the input and for the dynamical system, $G(s)$. For infinite length of $u(t)$ and $y(t)$ we get

$$G(i\omega) = \frac{\phi_{uy}(\omega)}{\phi_u(\omega)} = \frac{Y(i\omega)}{U(i\omega)} \qquad (9)$$

in the continuous case where

$\phi_{uy}(\omega)$ = the Fourier transform of the crosscovariance function, $r_{uy}(\tau)$

$\phi_u(\omega)$ = the Fourier transform of the autocovariance function $r_{uu}(\tau)$

$Y(i\omega)$ = the Fourier transform of $y(t)$

$U(i\omega)$ = the Fourier transform of $u(t)$

e.g.

$$Y(i\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} y(t)\, e^{-i\omega t}\, dt$$

The equalities (9) indicate that there are two different approaches to the problem of estimating $G(i\omega)$. The first method, the indirect one, is to calculate the auto- and crosscovariances and then to Fourier analyse them. The other method, the direct one, is to Fourier analyse $x(t)$ and $y(t)$ directly and to relate corresponding frequencies to each others. If $u(t)$ and $y(t)$ are defined zero outside the measure interval, $(0,T)$, and if the covariances are computed for lags from 0 to T, using the formula

$$r_{uy}(\tau) = \frac{1}{T-\tau} \int_{0}^{T-\tau} u(t)\, y(t+\tau)\, dt \qquad (10)$$

the two methods are equivalent. But if the covariances are computed for a maximum lag that is shorter than T, e.g. about 10 per cent of the total time T, which is generally done, it is no longer obvious that the methods give comparable results. The direct approach seems to have received less attention, perhaps

because it has been considered less efficient. However, there is now available a fast algorithm for machine calculation of Fourier series, the Fast Fourier Transform (or the Cooley-Tukey algorithm). In {6} it is pointed out that crosscorelation of input and output filters out extraneous noise to the same extent as direct Fourier analysis and no more. This is said under the assumption that the input signal is a pseudorandom binary sequence. Numerical examples are also given. Unfortunately the maximum lag used is not published.

In general properties of transfer function estimates are difficult to analyse but in certain special cases accuracy of the estimate has been studied {9} {11}. As far as known by the author no general study of this problem is available. On the other hand the calculation of $\phi_u(\omega)$ has been thoroughly investigated with respect to accuracy etc., e.g. in Ref. {7} . The calculation of $\phi_{uy}(\omega)$ is treated in an analogous way. The estimate of $G(i\omega)$ should then be good, if good estimates of $\phi_u(\omega)$ and ϕ_{uy} are available.

However, the described way of computing $\phi_u(\omega)$ from $r_{uu}(\tau)$ is not very good, because the estimates of the power density function (the periodogram) are inefficient. The variability of these estimates does not decrease with increased record length. A way out of this is some form of weighting of the estimates of the power density function. One method is to perform this weighting in the covariance function before the Fourier transformation and different methods have been proposed,"hanning", "hamming" etc. see Ref {3}. In the sampled case this weighting is easily applied after the Fourier analysis. The power spectrum density function will be smoothed by this procedure. Another method is to compute the periodogram directly by the Cooley-Tukey algorithm and then smooth it by averaging over an appropriate frequency interval. By this method it is also possible to use a rectangular spectral window, which is desirable for obtaining good estimates even if there are peaks of reasonable magnitude. The method seems to be preferable especially if covariance function are not required.

Other problems, that arise when trying to perform spectral analysis, are to determine appropriate values of the sampling interval, Δt, the length of the measurements, $N \cdot \Delta t$, and the maximum lag, m, used for the calculation of the covariance functions. These properties must be chosen so that we get no aliasing, required frequency resolution and required accuracy. The sampling interval Δt must be chosen so that $\Delta t < 1/2f_c$ where f_c is the cut-off frequency above which the power spectrum (almost) vanishes As a rule of thumb m should be less than 10% of the sample size, N, to avoid instabilities in the estimates of the covariance function. The maximum lag m determines the frequency resolution for the power spectrum density function in the frequency interval $(0,f_c)$. The frequency resolution will be

$$\Delta f = \frac{1}{m\Delta t}$$

Furthermore we have the uncertainty relation

$$\varepsilon^2 = \frac{m}{N}$$

where ε = normalized standard error for the spectral calculation,
$\varepsilon^2 = \mathrm{Var}\{\hat{\phi}_u\}/\phi_u^2$.
$\hat{\phi}_u$ = Estimated value
ϕ_u = True value

ε can be reduced by increasing N or for given N by sacrificing frequency resolution.

From knowledge of f_c one can choose m in order to receive a desired frequency resolution Δf. For small uncertainty in the estimates of the power spectrum density function one should choose $m << N$. On the other hand high resolution will result if m is large. Thus a compromise choice for m is necessary. Notice that the estimates of the power spectrum density function, that we get, are in some sense an average over a certain frequency band (of length $1/m \cdot \Delta t$ or more). This means that the resolution must be chosen so that the power spectrum density function not changes very much in such an interval.

Notice that we in this section have assumed that u(t) and y(t) are aperiodic functions. The knowledge that they are periodic in a special case may be explored and may in such cases improve the estimates.

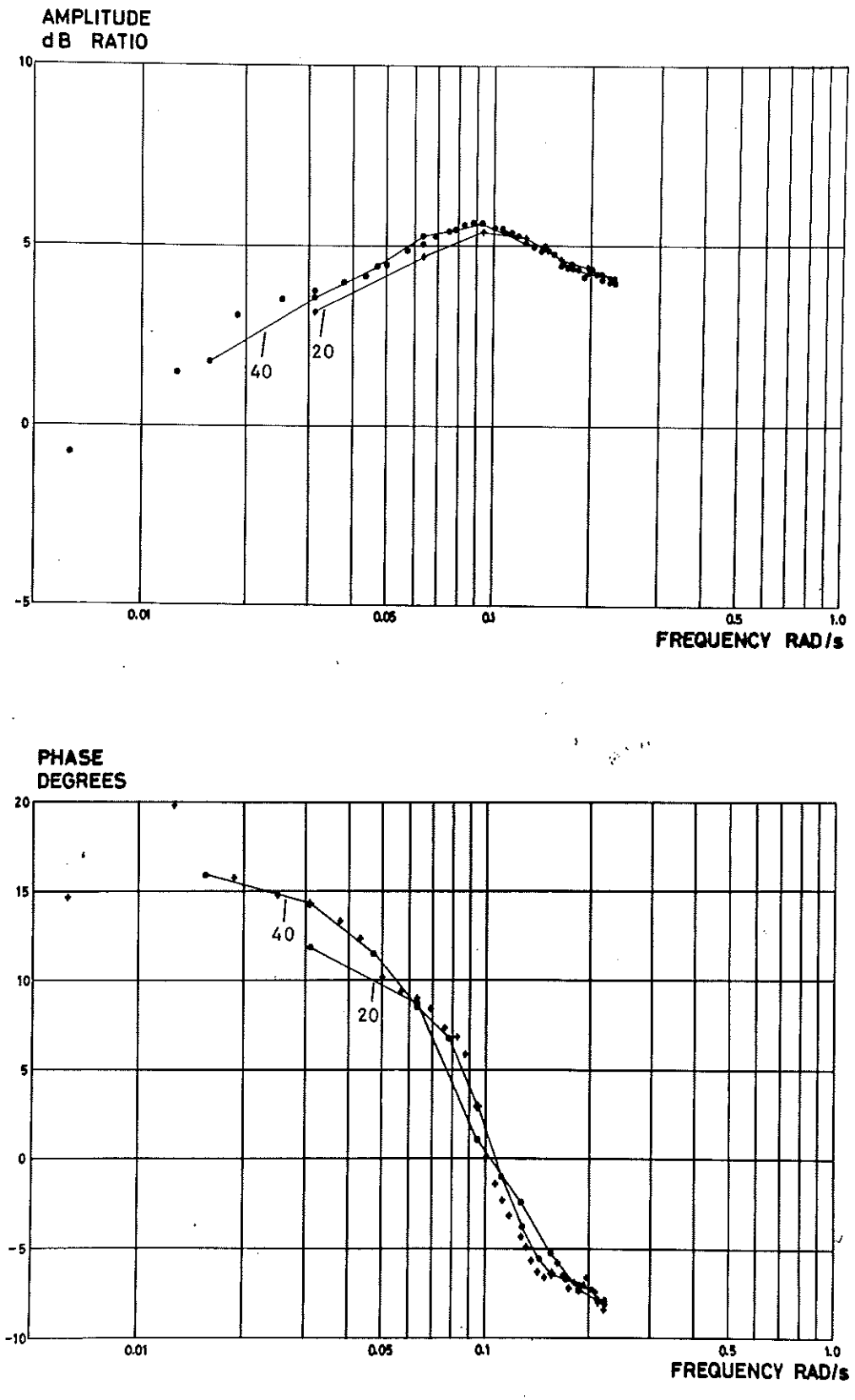


Fig. 11 - Comparison of results of spectral analysis of series AR61 using different maximum lag (20, 40 and 100 respectively). Notice that a greater maximum lag gives a better frequency resolution.

7. COMPARISON

After this discussion we will show some results obtained when using spectral analysis on the series AR60 and AR61. As mentioned before these series have been analyzed with a type of spectral analysis {1}. Some of the results can be seen in figure 4. Notice that these results have been obtained by a method that uses the fact that the input signal is periodic.

To obtain more insight in these problems the same data has been processed once more with a standard program for spectral analysis, BMD02T, from BMD Biomedical Computer Programs, ed. W.J. Divon, Health Sciences Computing Facility, UCLA. The program was available at Uppsala Computing Centre. Unfortunately the program has certain constraints, so that only 1000 data in each series can be used. This fact must be remembered when comparing the results of series AR61, because only the first 1000 data of this series have been used for the spectral analysis.

Roughly this program computes the auto- and crosscovariances and then calculates the raw estimates of the power spectrum. These estimates are then smoothed by "hamming". In figures 5 and 7 the results of the spectral analysis are shown. The maximum lag used is 100 for both series. In figure 11 a comparison for different maximum lag is done. Less maximum lag will result in averaging the power spectrum in some sense (see previous section). Notice in figure 5 the dispersion of the points. The estimates for points near each other may differ very much.

From the figures we can see that the maximum likelihood estimate and the results of spectral analysis may differ remarkably (e.g. figure 5). To test why this happens both methods were applied to 2 simulated input-output sequences. The input was a sequence of random numbers obtained from a subroutine, generating random floating point numbers distributed according to normal distribution with a mean of 0 and a variance of 1. The model used was the following

$$(1 - 1.073\, z^{-1} + 0.149 z^{-2})\, y(t) = (0.090 z^{-1} - 0.116 z^{-2})\, u(t) + \\ + \lambda (1 - 0.4 z^{-1} + 0.21 z^{-2})\, e(t)$$

Another sequence of random numbers was used as e(t). The lengths of the series were 1000 points. λ was chosen 0.03 and 0.2 respectively. Notice that the noise-signal ratio is higher for this example than for the models of the reactor process, which were given in section 5. However, a change of λ to a comparable level only changes the results of the identification of the example with λ equals 0.03 very little.

The input-output sequences were identified and analyzed by the BMD 02T-program. The models obtained from the identification by the maximum likelihood method were

$$(1-1.076z^{-1} + 0.149z^{-2})y(t) = (0.091z^{-1} - 0.116z^{-2})u(t) + \\ + 0.03(1 - 0.428z^{-1} - 0.157z^{-2})e(t)$$

and

$$(1 - 1.114z^{-1} + 0.182z^{-2})y(t) = (0.094z^{-1} - 0.117z^{-2})u(t) + \\ + 0.20(1 - 0.476z^{-1} + 0.148z^{-2})e(t)$$

respectively.

The Bode diagrams are shown in figure 12.

From the results we can see the difficulties to obtain a good estimate of the transfer function from the spectral analysis. If a parametric model is required it is difficult to find the parameters of the model starting from the results of the spectral analysis. Whatever method we use, we can see directly from figure 12 that the model we get cannot be as good as the one we get by the maximum likelihood identification. Notice that this holds particularly when the noise level is high.

In this case we get an indication at the spectral analysis that the estimates may be bad. According to ref. {5} reduced statistical confidence must be placed on the results when the coherence function, $\gamma_{xy}^2(\omega)$, defined by

$$\gamma_{xy}^2(\omega) = \frac{|\phi_{xy}(\omega)|^2}{\phi_x(\omega) \cdot \phi_y(\omega)} \qquad (11)$$

becomes less than unity. For this example the coherence function is 0.2-0.4 when λ is 0.2, and 0.90-0.95 when λ is 0.03. The value

of the coherence function at the analysis of the reactor measurements was about 0.99. The coherence function is less than unity for the example depending on the disturbances. To get better estimates the sequence must be much longer.

A comparison between the two methods of identification shows that the spectral analysis is faster than the maximum likelihood method especially when the order of the system is unknown. On the computer we use, a CD 3600, the computing time for spectral analysis of a series of 500 data was about 30 seconds, but a simpler program may reduce this time to about 20 seconds. The identification of a third order model with the same amount of data with the maximum likelihood method will take 25-50 seconds, depending on the convergence rate. But on the other hand no information about the system order is given when spectral analysis is used. The choice between the methods must be based upon what is desired of the results. If control laws have to be designed the maximum likelihood method would be the best one, but if only a very approximate estimation of the properties of the system is desired, spectral analysis would be preferable because it is faster. The example above shows, however, that the error of this estimation may be very large, compared with the results of the maximum likelihood method. The conclusion is that possibilities of error sources at identification must always be thoroughly investigated.

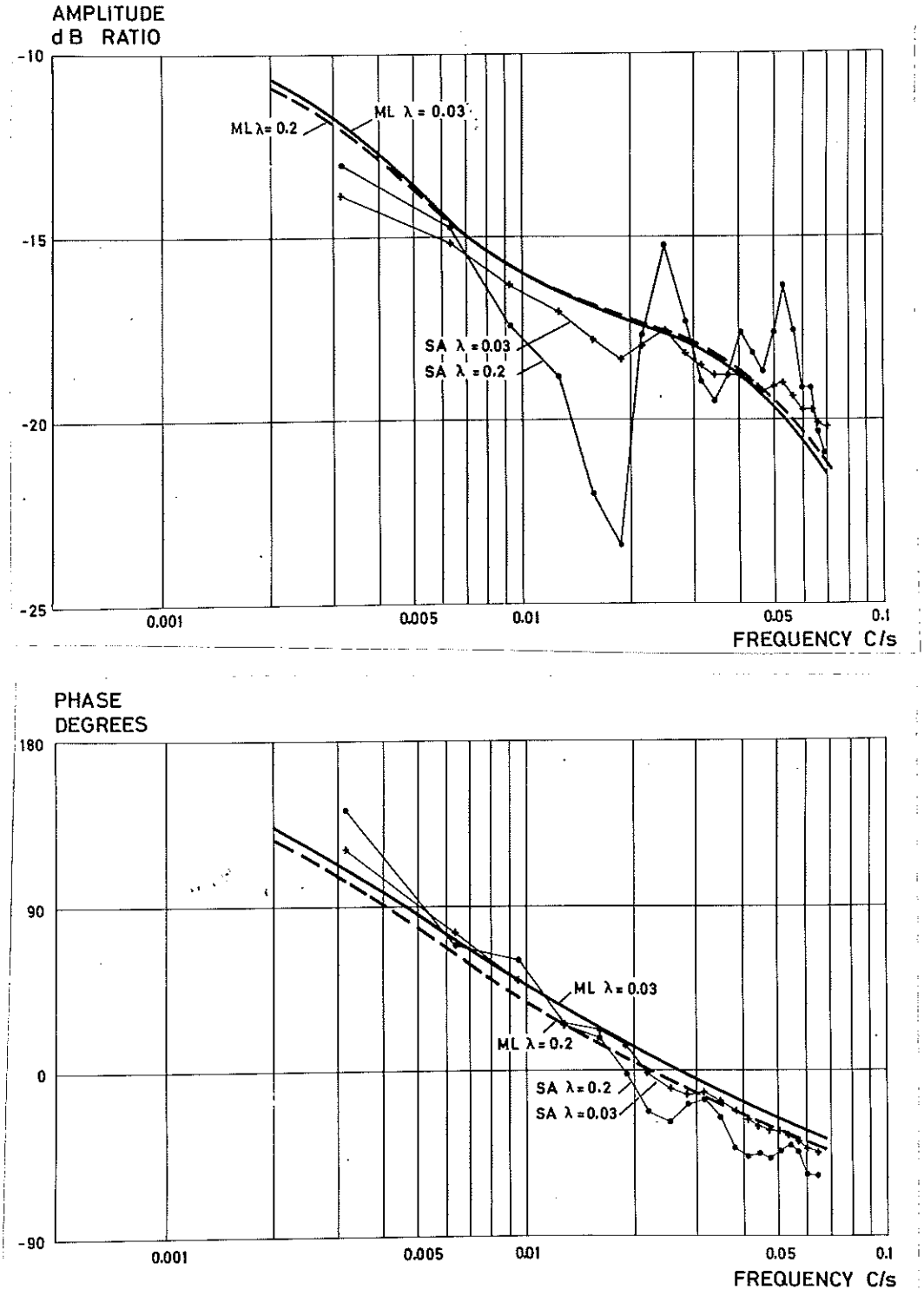


Fig. 12 - Comparison between maximum likelihood identification (ML) and spectral analysis (SA) of two artificial input - output sequences with different noise level, $\lambda = 0.03$ and $\lambda = 0.2$. The amplitude and phase curves for the generating system and for the model from the maximum likelihood identification of the system with $\lambda = 0.03$ cannot be separated in the diagram.

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