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WATER QUALITY MAINTENANCE AND CONTROL THEORY

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ABSTRACT.

River pollution control, or water quality control to use
less emotive language, really needs no introduction to
the chemists and biologists who have nurtured the subject
since its emergence with the Industrial Revolution. In
contrast, the control engineer has begun only recently
to contemplate the application of his particular exper-
tise and knowledge to this sphere of research. It is in-
tended, therefore, that this report should review the com-
ponent features of water quality control and locate those
areas where control and systems theory may provide useful
results; in this respect it is furthermore advantageous
to show where coordinated research is required between
the control engineer and members of other sciences such
as biology, ecology, hydrology and economics. Indeed,
water quality control is a field which denies the strict
classical separation of scientific research into well-
defined disciplines and faculties.

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1. INTRODUCTION.

It is probably true of all countries that national re-
quirements for potable and municipal water supply are
increasing; and, similarly, with expanding population
and industries the production of waste material from
society is also increasing. The combination of these
two events places a great strain on the maintenance of
river water quality and requires more sophisticated
techniques for the management of water resources in ge-
neral. With specific reference to the United Kingdom,
one might define the problem as not so much a case of
the traditional diminishing resource of finite quanti-
ty, but rather one of finite retrieval of fresh-water
in terms of reservoirs and impoundments; rising prices
and increasing competition for land use have generated
considerable interest in an alternative to fresh-water
reservoirs, namely the use and re-use of water courses
for the purification of effluents and the abstraction
of potable water supplies.

Thus, given a situation where the problem of planning
and control of river water quality is too complex to be
considered outside the realms of relatively advanced
mathematical and engineering methods, control and sys-
tems theory may offer the techniques to solve many of
the current problems. Nevertheless, it should not be
overlooked that the control engineer is naturally depen-
dent upon many other disciplines of science in order to
approach the subject with any hope of success in prac-
tical terms. As will become evident in the present dis-
cussion, control theory has advanced far beyond the
current needs and available technology of waste water
treatment; thus it is unfortunate that published mate-
rial suggests that the control theoreticians have ten-
ded to assume too high a level of technology in water

pollution studies, which belies a requisite minimum intimacy with the system to which they have chosen to apply their knowledge.

2. GENERAL ASPECTS OF RIVER WATER QUALITY CONTROL.

2.1. The Degradation of Water Quality.

Principally, polluting materials enter a water course through effluent discharges, although water quality can be significantly impaired by agricultural chemicals in surface water run-off and land drainage and by power station cooling water. The effects of the latter two inputs are mainly indirect in the sense that they provide nutrients and higher temperature conditions, respectively, which can increase the growth rate of certain components of the aquatic ecology, e.g. algae, thus causing an undesirable imbalance in the system.

The numbers and types of pollutant substances are many and varied, but first let us make a distinction between conservative and non-conservative substances by the following definitions: a non-conservative substance can be degraded or altered by biological and chemical processes in the stream, whereas a conservative substance is unaffected by such action. To the control engineer the interaction of soluble non-conservative matter with the aquatic environment is the more challenging topic since the underlying physico-chemical and biochemical mechanisms are complex and dynamic variations are quite pronounced.

2.2. Waste Water Treatment Technology.

Ultimately, the maintenance of river water quality is
dependent to a large extent on the technology and design
of sewage treatment plants. Currently available tech-
niques of waste water treatment conform to a comparative-
ly standardized pattern and are oriented towards the re-
moval of two indices of pollution level: (i) suspended
solids; (ii) biochemical oxygen demand (BOD), an aggre-
gate or macro-measure of the degradable organic waste
content of the effluent. Usually the quality of the final
effluent is specified as a maximum permissible level of
suspended solids and BOD related to a given effluent
/stream dilution ratio.

The design of a waste water treatment plant for effici-
ent operation depends essentially upon the regulation
of a constant volumetric flow through the biological
oxidation process of a percolating filter or activated
sludge stage. At present, therefore, the automation of
the plant is usually a function of routing the flow
according to pre-specified schedules based on the de-
sign flow. Possibly because of its hitherto relatively
insignificant economic importance, the waste water treat-
ment industry has not been subjected to a wealth of
technological innovation; the application of control
theory to the unit process involved, e.g. activated
sludge treatment [8] and chemical precipitation, is mere-
ly in its infancy. Generally, only a few local control-
lers are currently used for operation and the introduc-
tion of further on-line control is hampered by the con-
siderable effort required to develop suitable and reli-
able instrumentation [23] as we shall see later.

Thus, a truly variable control of waste water treatment
is not available at present and it is simply not possible

to regulate either the effluent flow or quality in a
manner which can maintain prescribed in-stream condi-
tions on a day-to-day operational basis.

2.3. Water Quality Monitoring and Instrumentation.

In contrast to the process industries, the monitoring
of water quality is severely encumbered by the accessi-
bility problems associated with rivers and the multi-
tude of variables to be observed; it is often difficult
to place a monitor in a suitable position owing to the
nature of the river bank or bed and even then the re-
trieval of data can be delayed by the long distances
over which it must be transmitted to the processing
centre. The technology of the instrumentation is limited
and integrated monitoring systems with reliable, robust
equipment are few and far between; clearly, it is not
feasible to produce an automatic system capable of mo-
nitoring all the physical, chemical and biological va-
riables that might at some time be significant [7].

However, sensors have been developed for the determina-
tion of temperature, dissolved oxygen (DO) concentra-
tion, pH value, conductivity and turbidity; more recent-
ly, ion-selective electrodes for the measurement of nit-
rate, sulphide, ammonia, and cyanide, for example, have
been made available, but are only at the evaluation
stage [7]. Suffice it to say that, in the majority of
cases, the observation of water quality variables is a
matter of manual or automatic sampling followed by la-
boratory analysis; indeed, experience shows that it is
an extremely tedious affair.

With respect to control applications the difficulty in

obtaining suitable data cannot be emphasised too strongly. Many locations for monitors prevent the use of a mains power-supply and the alternative is beset by recurrent failure of batteries; in addition the sensory equipment requires continual maintenance to prevent fouling and the loss of calibration. Such problems in conjunction with the laboratory analysis of samples make the data collection procedure a labour-intensive operation which can only be accomplished in an arduous manner and with a certain dedication to duty. The net result is a debilitating lack of data in the published studies.

2.4. Water Quality Standards.

It should be understood that in the course of most water quality studies one encounters the vexed question of specifying standards for acceptable stream conditions; as yet, no-one has solved the problem of establishing suitable and socially-fair levels of quality to be maintained and nor is there an effective characterisation of the benefits to accrue from the amenity value of water courses. In the case where stream quality holds the balance between the competing demands of effluent disposal and potable water abstraction it is more a mathematical problem to formulate the cost-benefit analysis; however, amenity values are less easy to cost objectively and the problem formulation tends to stray into the less tangible sphere of politics.

2.5. The Control Problem.

The water quality control problem can best be defined by reference to Figure 1; briefly, it divides into three separate categories:

(i)   On-line automatic control for waste water treat-ment plants (WT);

(ii)   On-line automatic control for potable water abst-raction plants (PWA);

(iii)   On-line control of water quality in reaches $j = 1$, ...,m to be consistent with pre-specified stan-dards for the stream.

As will become evident in Section 3 of the report, the majority of control theory applications have examined category (iii), either for the single reach case or mul-ti-reach case where $j = 1,...,m$; it is, in general, true that control (iii) will be dependent upon (i), which is invariably assumed to be available in order to obtain solutions for (iii). Other areas of active research have been confined mainly to the control of one of the unit processes in waste water treatment (i), namely the acti-vated sludge stage [8]. However, the synthesis of a control system for the combined operation of (i) + (iii) or (i), (ii) + (iii) has not been attempted to the best of the author's knowledge.

3. CONTROL AND SYSTEMS THEORY APPLICATION TO PROBLEMS OF WATER QUALITY CONTROL.

A cursory review is given here of those aspects of water quality maintenance which have proved amenable to the application of control and systems theory. The discussion is not exhaustive, since it is largely restricted to considerations of dissolved oxygen as a specific index of water quality, and should be read as a collection of example problems and their modes of solution.

3.1. System Identification and Estimation of Parameters. In-stream water quality modelling.

What is really required of a water quality model is that it be able to predict the effects of a multitude of pollutant materials on, and their interactions with, the aquatic environment. However, for the purposes of the present discussion, the problem is simplified by the conservative nature of many pollutants, where independent component mass balances and time-series analysis of the associated system inputs and outputs at various fixed spatial locations can be applied to obtain models.

On the other hand, the dynamic variations of non-conservative pollutants are more difficult to characterise since they interact with the aquatic life system in a complex biochemical or biological manner. Historically one particular index of water quality, DO [*], has received

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[*] Appropriately so, since DO level reflects the general health and balance of the aquatic life; BOD is the principal non-conservative substance interacting with DO and hence the tendency to refer to BOD-DO models in the literature.

much attention [9], [13], [26], although the major re-
search effort has been directed towards the development
of theoretical models for use in a steady-state, deter-
ministic parameter estimation context. Unfortunately,
steady state is the exception rather than the rule for
river systems and the more realistic problem of identi-
fying a dynamic BOD-DO model in the presence of system
and observation noise has been tackled using the extended
Kalman filter [3]; it was first necessary, however, to
identify the system approximately by deterministic simu-
lation, where it was found that the complexity of the
model should be extended to include explicit representa-
tion of the effects of algae [2].

The emergence of algal growth and decay as important
constituents of the BOD-DO balance raises the intriguing
issue of the control engineer's ability to model ecolo-
gical systems in general. Although preliminary steps have
been taken already in this direction [1], it would be
reasonable to suggest that data collection for verifica-
tion purposes could prove to be quite restrictive.

Stream flow modelling.

The synthesis of stream flow data plays an integral part
in the prediction of water quality variations, since the
flowrate of a river affects the dispersive and retention
time properties of each reach. An example of recursive
time-series analysis (see e.g. Young [33]) applied to
this kind of problem is the rainfall-flow modelling work
carried out recently in connection with a systems analy-
sis study of water quality in the Bedford-Ouse river sys-
tem [4], [35]. The flow model of a 55 km stretch of the
river between the site of the new city of Milton Keynes

and Bedford uses a combination of lumped-parameter dif-
ferential-difference equations to characterise the de-
terministic variations in flow, and a time-series model
to account for stochastic variations resulting from rain-
fall and run-off effects. These latter time-series mo-
dels have been developed using an Instrumental Variable -
- Approximate Maximum Likelihood (IV-AML) approach [5],
[6] and seasonally adjusting the rainfall data to account
for varying rates of evaporation [30]. Similar studies
besides that of the Bedford-Ouse system have also used
recursive time-series analysis techniques to model rain-
fall-run off relationships [18].

3.2. Control System Synthesis: In-Stream Water Quality Control.

The general approach to the in-stream water quality con-
trol problem has been to assume largely unverified dyna-
mic models of BOD-DO interaction and the availability of
a method of on-line waste water treatment plant control.

Single-reach control.

The single-reach case may be thought of as a subset of
the multireach problem discussed below; however, the
concern of hierarchical strategies, in particular, with
the optimisation of the global system and their assump-
tion of sub-optimal or optimal singe-reach controllers,
shows a surprising, and almost total, disregard in the
literature for more simple dynamic control investiga-
tions.

There are two basic alternative approaches to the local
(i.e. single-reach) control of, for instance, in-stream
dissolved oxygen level:

(i)   artificial aeration of the stream;

(ii) regulation of an effluent discharge to the stream.

Tarrasov et al [28] have provided optimal control sur-
faces for the solution of (i) by operation of a series
of submerged pipes and diffusers attached to the river
bed, but it is (ii) which would appear to be the more
immediately practicable of the two methods. Thus, in an
attempt to place a lesser demand on the possible techno-
logical advances in this field, a preliminary examina-
tion employing simple algebraic techniques of state-space
control system synthesis has demonstrated the initial
feasibility of approach (ii) [34]; this study was ori-
ented specifically towards the use of field data prima-
rily collected for identification purposes [2], [3]. Ade-
quate control can be achieved by either of two state-
variable feedback laws: (i) assuming a variable treat-
ment process the BOD concentration of the effluent is
controlled in a prescribed manner; (ii) for a fixed treat-
ment process the effluent flow is detained in storage la-
goons or tanks and subsequently discharged to the river
in accordance with the control law.

Multi-reach control.

Several investigators have tackled the optimal or near-optimal control of a multi-reach system and adjacent treatment plants; by multi-reach it is assumed that it is possible to decompose the global river system into a series of sequential, discrete reaches (usually defined in terms of the physical placement of treatment plants, tributaries and weirs) as in Figure 1. Recent work has shown that methods of hierarchical-multilevel control are appropriate for the solution of the operational problem [12], [24], [25], [27], although other suboptimal procedures have been suggested [19]. The structure of the hierarchical strategy requires a "first-level" local controller, which computes the optimal control trajectories for each reach, while a "second (or higher) level" controller co-ordinates this information to be consistent with an optimal or near-optimal control law for the global system.

3.3. Long-Term Planning and Investment Analysis.

For a long time capital investment in the necessary facilities has dominated the economic evaluations of water resources management; indeed, operational costs are so traditionally thought of as negligible that for this reason, and, in addition, the lack of a clearly defined "final product" specification, the automatic control of waste water treatment has remained somewhat of a backwater in the stream of technological advance.

However, in the broadest setting, investment policies need to consider a multitude of objectives for supply and demand of water; in the United Kingdom the precursor

for this type of large-scale study has been the Trent Research Programme *) [14], [15]. A steady-state model describing many indeces of water quality is a fundamental part of this analysis, which examined the complex problem of assessing the different ways in which water from the Trent and its tributaries can be used for domestic, industrial, agricultural and amenity purposes. The problem is constructed in a manner whereby the costs and benefits of each facility can be evaluated to determine the most efficient solution using a dynamic programming routine. Similar approaches have been used by Dysart [10] and Futagami [11], for example, to solve the less ambitious problem of optimising future investment needs for a series of sewage treatment plants.

The more general principles of water resources management have been formulated by several people in the USA; in particular, Thomann [29] has provided a review of systems analysis application and Kneese (see e.g. [20]) has contributed much to the advance of economic theory in this area.

The omissions of this review section are a reflection of the general absence of control and systems theory application to the water quality maintenance problem. However, we have not considered the identification and control system synthesis for waste water treatment plants, since the review by Olsson et al [23] illustrates the many problems to be addressed in that respect; neither is men-

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*) It is worth mentioning in passing that the Trent Research Programme is a comprehensive survey which has hypothesised several interesting methods of obtaining potable supplies from moderately polluted river water, e.g. sandstone aquifer recharging, and naturally filtered water from alluvial gravels.

tion made of the technological progress required for
the on-line control of potable abstraction plants, where
it can be assumed that latent control problems exist,
although this area is outside the previous experience
of the author. And finally, since the discussion has
been limited to water quality as opposed to water re-
sources in the wide sense, it is interesting to observe
that reservoir systems, for example, are not immune to
simple control analysis [16], [17], [31].

4. A CRITICAL APPRAISAL OF SOME ASPECTS OF CONTROL
APPLICATIONS.

There are several general criticisms to be made of the
current tenor of control applications in water quality
systems. Firstly, by virtue of their distributed-para-
meter nature they are an example problem for the formu-
lation of estimation techniques with partial differen-
tial equations [21], [22]; however, these studies, ei-
ther merely assembling the optimal state estimation
equations [21] or "verifying" a stochastic approximation
technique with synthetic data [22], while they may indi-
cate the initial viability of an approach, have tended
to overlook practical considerations in order to obtain
theoretical completeness. The very limiting constraints
of field data collection, where the most intensive
sampling programmes so far published in the literature
are a maximum of some eighty daily samples from the
author's experimental work, and, more recently, a further
block of seventy daily samples from the Bedford-Ouse stu-
dy, seem to indicate a certain naïvety in discussing un-
biased parameter convergence over several hundred itera-
tions of daily samples [22]. And in any case, it is not
necessarily true that partial differential equation mo-
dels are the best representation of quality and flow dy-
namics in a river; certainly, the lumped-parameter dif-
ferential-difference equations used for deterministic si-
mulation of BOD-DO interaction [2] and rainfall-flow pre-
diction [30] adequately account for the majority of vari-
ations observed.

Secondly, since a river system can be decomposed into a
sufficiently complex sequential, discrete, multi-reach
problem it has become popular to apply techniques of
hierarchical or multi-level control in this area, again,
largely as a convenient example formulation. Experience

shows that even a simple PI controller on the DO level
in a river is difficult to justify unequivocally in an
economic context; thus, hierarchical, multi-reach con-
trol solutions tend to meet with implacable refusal, on
behalf of the practising profession, to admit the feasi-
bility of on-line control in any form whatsoever. It
seems, therefore, that some subtlety is required of the
control engineer and he should proffer more immediately
practicable solutions to the problems of water quality
control. At the very least he should be able to demonst-
rate that operational costs are not negligible or that
the introduction of on-line control could reduce capital
investment costs.

Yet, in making such criticisms of hierarchical control
applications in this field it is not denied that such
studies can provide useful references against which the
performance of less sophisticated control systems can
be judged; moreover, they may be of significance in the
possible extensions to the simple local control schemes
when emergency situations require co-ordinating action
between individual reaches and their controllers. How-
ever, it should not be forgotten that we are, after all,
dealing with the solution of essentially practical prob-
lems and in this case it should be a matter of the prob-
lem specifying an applicable technique rather than a
piece of theory searching for justification in an app-
lications sense.

5. FUTURE PROBLEMS FOR IDENTIFICATION AND CONTROL.

Some of the points which have been reviewed are developed
further here since they give an insight to the control
engineer of the type of complexities involved and they
also define the peripheral areas where he may be required
to make compromise solutions in view of the intricate bio-
logical and biochemical nature of water quality systems.

Let us take as our example the BOD-DO interaction dis-
cussed previously and although we shall consider in-
stream aspects of the relationships they may well apply
to similar processes occurring during the treatment of
effluent before discharge to the stream. Superficially
it would appear that we are concerned with chemical con-
stituents interacting in a manner which can be described
largely by the rate kinetics of purely chemical and phy-
sical phenomena. However, without giving too much detail,
which may confuse the issue at present, the very defini-
tion of BOD and its "decay" properties, i.e. the oxida-
tion of degradable material, are a crude, but neverthe-
less viable, approximation of complex biological proces-
ses by first-order chemical reaction kinetics. Thus,
from the outset the most basic mathematical descriptions
of BOD-DO interaction are considerable simplifications
of the true dynamics of the system.

Furthermore, when it was found necessary to include the
effects of algal growth and decay in the model [2], [3]
the identification problem moved into the more mathema-
tically nebulous area of describing the interplay bet-
ween components of an ecological system. Algal popula-
tion kinetics are dependent upon sunlight intensity, and
the concentrations of nitrogen- and phosphorus-bearing
chemicals; their particular importance lies in their abi-
lity to expand their numbers suddenly under so-called

"bloom" conditions, thus completely disrupting the ba-
lance of the aquatic life structure. Indeed, algae have
been the subject of much research effort in connection
with the eutrophication *) of lakes and reservoirs (see
e.g. [32]), which, as is indicated below, may be of im-
portance in waste water treatment control system synthe-
sis. However, for the purposes of a mathematical BOD-DO
model there exists a large gap between the macroscopic
needs of a control engineer and the microscopic details
which can be provided by a biologist; one may compare
the requirements of the situation to those of a bilate-
ral international meeting without an interpreter!

Ultimately, in view of limited data, the compromise so-
lution was to predict algal growth and decay by a pseu-
do-empirical relationship which low-pass filtered the
hours of sunlight incident on the system each day. Quite
obviously, it would be preferable to augment the BOD-DO
equations with properly identified state-space equations
for algal population dynamics, since this would be more
theoretically satisfying and it may provide a more easi-
ly manipulable control variable than sunlight intensity,
e.g. phosphate or nitrate concentration. However, it is
now appropriate that we make mention of the other prin-
ciple non-conservative substances in a stream, namely
those bearing nitrogen in their structure; in fact, the
nitrogen cycle interacts with the BOD-DO balance by the
processes of nitrification and denitrification **), while
the corresponding fluctuations in nutrient concentration

**) Nitrification is the oxidation of ammonia products,
for example, through nitrite to nitrate compounds;
denitrification is the reverse process.

*) i.e. nutrient-rich; nitrates and phosphates etc. are
nutrients in this sense.

influence the growth rates of algae. It is hoped that a combined BOD-DO, algae, and nitrogen-cycle model can be identified from data currently being analysed at the Control Division, University of Cambridge Engineering Department, in connection with the Bedford-Ouse River study.

Thus, there is much to be accomplished in the modelling of water quality dynamics, but the problem will probably be constrained by the availability of suitable data and the interpretation of complex biological information into a tractable mathematical form. On the other hand, for control system synthesis it may be adequate to make several approximations, although it seems reasonable to assume here that nitrate and phosphate removal at the treatment stage may eventually be closely linked to that of BOD. In particular, the feasibility of lagoon detention schemes for effluent control (see Section 3.2) rests upon the suppression of eutrophication and the subsequent possible transmission of large, active algal populations to the stream.

6. CONCLUSIONS.

The problems of water quality maintenance are no diffe-
rent in nature from those in many other systems; rather,
it might be concluded that they are more emphasised.
For instance, data are particularly difficult to obtain,
the technology of the plant is currently not amenable to
control application, cost-benefit analyses are difficult
to formulate and there is a lack of clearly-defined "fi-
nal product" specifications for stream quality control
and potable water abstraction.

All the possible control applications cannot be reviewed,
since many of them are only in a nascent form and others
will emerge as further research is undertaken. But the
necessary investigations should not assume the appearance
of running before they can walk, so to speak, and even
the most elementary control theory may be genuinely use-
ful before the implementation of more sophisticated techniques.

7. Footnote.

This report is based on research *) carried out by the
author in the Control Division of the University Engi-
neering Department, Cambridge between October, 1970, and
October, 1973. During the final year, from October, 1972,
a three-year project was commenced on the modelling and
control of water quality in the Bedford-Ouse river; part
of this study is the responsibility of Dr. Peter C. Young
and Paul Whitehead of the Control Division at Cambridge
and to whom the author is indebted for many useful dis-
cussions.

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of the advice and comments on this report given by Pro-
fessor G. Olsson of the Lund Institute of Technology,
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WASTEWATER TREATMENT PLANTS (WT)
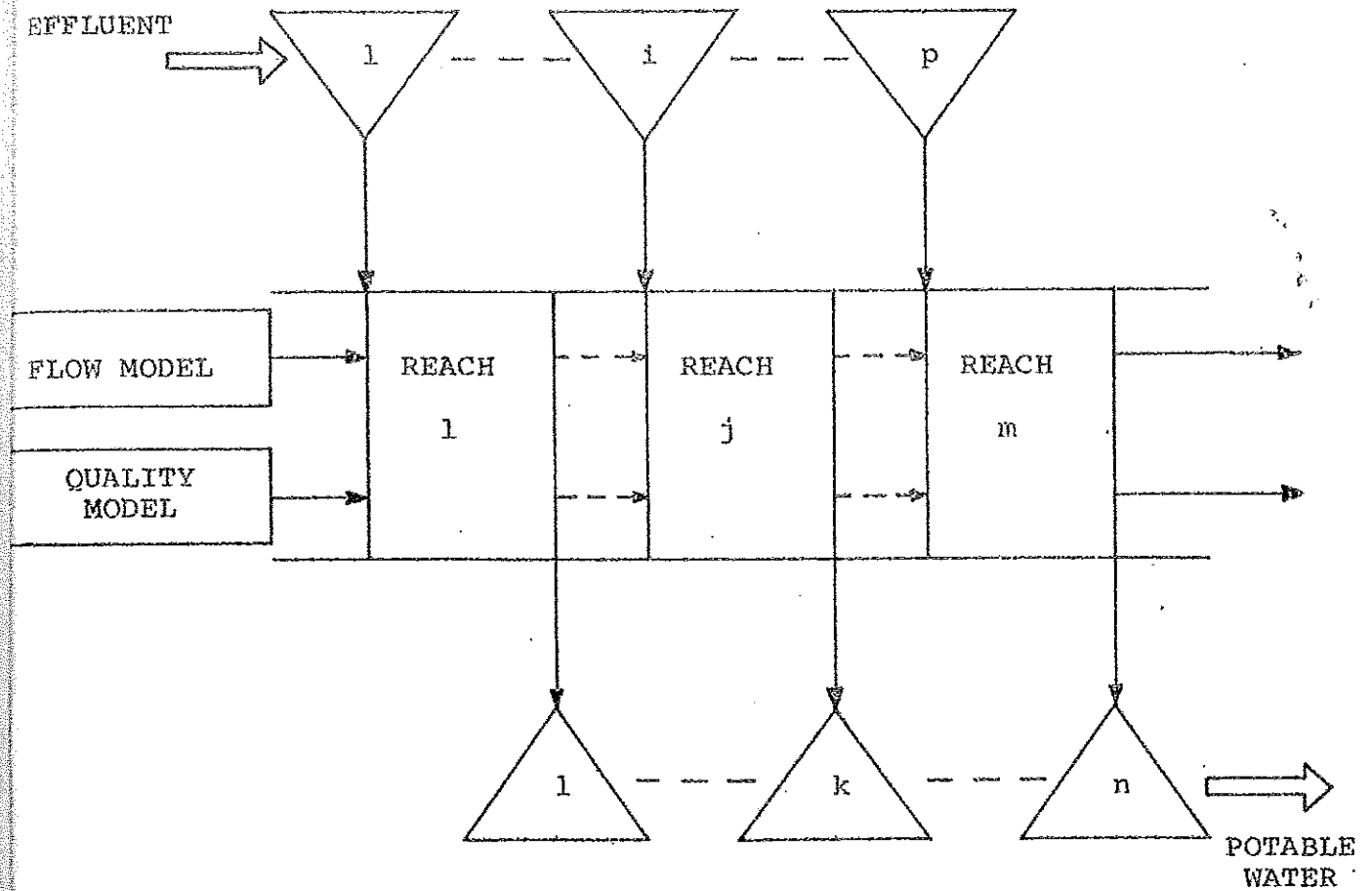
Figure 1. A Schematic Diagram of a River System and Adjacent
Treatment and Abstraction Facilities.