Parametric Identification of Multiple Input, Single Output Linear Dynamic Systems

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PARAMETRIC IDENTIFICATION OF MULTIPLE INPUT, SINGLE OUTPUT LINEAR DYNAMIC SYSTEMS †

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ABSTRACT

In this report a program package for identification by the maximum likelihood method is presented. The program can be used to compute mathematical models of various industrial processes. The application is limited to linear, single-output, time-invariant dynamic systems with normal disturbances having rational spectra.

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TABLE OF CONTENTS

1. INTRODUCTION

In this work a program package for identification of an industrial process by the maximum likelihood method is described. The program produces a mathematical model of the process and its disturbances from given input/output samples. Application is limited to processes that can be described by a linear, single-output, time-invariant dynamic model with normal disturbances having rational spectra. Five input signals can be handled by the program. The program is written in FORTRAN for a CD 3600 computer. Core storage requirement for this version is 17.5 k words. The program package has been tested both for artificially generated data and for data from industrial processes and has produced reasonable models. In this work only a few test examples are presented.

In section 2 a review of the maximum likelihood identification method is given. Computational aspects on the calculation of the gradient and the second partial derivatives of the loss function are given in section 3. Section 4 contains description of the different subroutines in the program package and the use of the programs. Two test examples are given in section 5. Some improvements of the programs are proposed in section 6.

2. STATEMENT OF THE PROBLEM AND ITS SOLUTION, A SUMMARY

The identification problem is:
Given the input/output samples $\{u_i(t), i = 1,2,\ldots,m, y(t); t = 1,2,\ldots,N\}$, where $u_i(t)$, $i = 1,2,\ldots,m$ are the m input signals and $y(t)$ is the output signal, find an estimate of the parameters of the system model

$$A(z^{-1})y(t) = \sum_{i=1}^{m} B_i(z^{-1})u_i(t) + \lambda\, C(z^{-1})\, e(t) \tag{1}$$

$e(t)$ is a sequence of independent normal (0,1) random variables. z denotes the shift operator

$$z\, x(t) = x(t+1) \tag{2}$$

$A(z)$, $B_i(z)$, $i = 1,2,\ldots,m$, and $C(z)$ are polynomials

$$\begin{aligned} A(z) &= 1 + a_1 z + \ldots + a_n z^n \\ B_i(z) &= b_{i0} + b_{i1} z + \ldots + b_{in} z^n \qquad i = 1,2,\ldots,m \\ C(z) &= 1 + c_1 z + \ldots + c_n z^n \end{aligned} \tag{3}$$

We assume that the functions $A(z^{-1})$ and $C(z^{-1})$ have all their zeros inside the unit circle and that there are no factors common to all the polynomials $A(z^{-1})$, $B_i(z^{-1})$, $i = 1,2,\ldots,m$ and $C(z^{-1})$.

A more detailed discussion of this model is to be found in references {10} and {11} where the single input/single output case is considered.

The problem is thus the identification problem of a general, multiple input (m inputs), single output, linear discrete time dynamical system with normal disturbances having rational spectra. This is obvious if we consider the model

$$y(t) = \sum_{i=1}^{m} \frac{B_i^*(z^{-1})}{A_i^*(z^{-1})} u_i(t) + \lambda \frac{C^*(z^{-1})}{D^*(z^{-1})} e(t) \tag{4}$$

where $A_i^*(z)$, $B_i^*(z)$, $i = 1,2,\ldots,m$, $C^*(z)$ and $D^*(z)$ are polynomials. The spectrum of the disturbances of the output is

$$\lambda^2 \frac{C^*(z)\, C^*(z^{-1})}{D^*(z)\, D^*(z^{-1})} \quad (z = e^{i\omega}) \tag{5}$$

where

$C^*(z^{-1})$ and $D^*(z^{-1})$ have no common factors , and the discrete time transfer function from input i to the output is

$$\frac{B_i^*(z^{-1})}{A_i^*(z^{-1})}$$

$A_i^*(z^{-1})$, $i = 1,2,\ldots,m$, and $D^*(z^{-1})$ may have different orders. $A_i^*(z^{-1})$ and $B_i^*(z^{-1})$, $i = 1,2,\ldots,m$ respectively, have no common factors. Writing (4) on common denominators we get a model of the type (1). For the identification we can either choose model (1) or model (4). Model (4) will result in a less number of parameters to estimate. On the other hand this model will result in a more complex algorithm. For the program described in this work the model (1) is used. Descriptions of programs using the other model can be found in ref {2 }.

The problem is a nonlinear estimation problem. An efficient method of obtaining the maximum likelihood estimate has been described in ref {10}. In this work all formulas necessary for the programming of the algorithm are given but the theoretical part of the solution is only briefly described here. More details can be found in ref {10} and {11}.

It follows from (1) that the residuals $\{\varepsilon(t),\ t = 1,2,\ldots,N\}$ defined by

$$C(z^{-1})\ \varepsilon(t) = A(z^{-1})\ y(t) - \sum_{i=1}^{m} B_i(z^{-1})\ u_i(t) \tag{6}$$

are independent and normal $(0,\lambda)$. The logarithm of the likelihood function becomes

$$L = -\frac{1}{2\lambda^2} \sum_{t=1}^{N} \varepsilon^2(t) - N \log \lambda + \text{constant} \tag{7}$$

Maximizing this function is equivalent to minimizing the loss function

$$V(\theta) = \frac{1}{2} \sum_{t=1}^{N} \varepsilon^2(t) \tag{8}$$

where θ is the column vector $(a_1,\ldots,a_n,b_{10},\ldots,b_{1n},b_{20},\ldots,b_{mn},c_1,\ldots,c_n)$. When $\hat{\theta}$, such that $V(\hat{\theta})$ is minimal, has been found, the maximum likelihood estimate of λ will be

$$\hat{\lambda}^2 = \frac{2}{N} V(\hat{\theta}) \tag{9}$$

The maximum likelihood estimate is consistent, asymptotically normal and efficient under mild conditions given in ref {11}.

In order to minimize the loss function an iterative combined Gauss-Newton and Newton-Raphson algorithm is used

$$\theta^{k+1} = \theta^k - \alpha[V_{\theta\theta}(\theta^k)]^{-1} V_\theta(\theta^k) \tag{10}$$

where

V_θ = the gradient vector of V(θ)

$V_{\theta\theta}$ = the matrix of second partial derivatives of V(θ)

α = scaling factor

Differentiating (8) gives

$$\frac{\partial V}{\partial \theta_i} = \sum_{t=1}^{N} \varepsilon(t) \frac{\partial \varepsilon(t)}{\partial \theta_i} \tag{11}$$

$$\frac{\partial^2 V}{\partial \theta_i \, \partial \theta_j} = \sum_{t=1}^{N} \frac{\partial \varepsilon(t)}{\partial \theta_i} \frac{\partial \varepsilon(t)}{\partial \theta_j} + \sum_{t=1}^{N} \varepsilon(t) \frac{\partial^2 \varepsilon(t)}{\partial \theta_i \, \partial \theta_j} \tag{12}$$

If only the first term of (12) is used in the iterative minimizing algorithm we have a Gauss-Newton algorithm. It is used during the first iterations to secure convergence to a minimum. This is not always true {1 }, but in most practical cases convergence occur. Near the minimum a Newton-Raphson procedure is used. This means that we use the whole expression (12) when we compute the second partial derivatives. A rapid convergence near the minimum is thus achieved. The Newton-Raphson algorithm can not be used far from the minimum because it may not converge from a poor approximation. We denote the first term of (12) by the expression "the approximative second derivatives" and analogous the whole expression (12) is called "the exact second derivatives".

Differentiation of (6) gives the formulas necessary for the computation of the first and second order partial derivatives of ε(t) with respect to the parameters. The formulas are given in ref {10} and will be given in the next section.

The minimizing algorithm becomes:

1. Put $\theta^k = \theta^o$ (starting value of θ)
2. Evaluate $V_\theta(\theta^k)$ and $V_{\theta\theta}(\theta^k)$
3. Calculate θ^{k+1} and repeat from 2.

Putting $c_i = 0$, $i = 1,2,\ldots,n$ we obtain in one step the least squares (or Kalman) estimate of $a_i, b_{ki}, k = 1,2,\ldots,m;\ i = 0,1,\ldots,n$. This estimate is then taken as initial value θ^o of the minimizing algorithm. By taking different starting values of c_i, $i = 1,2,\ldots,n$ we investigate whether $V(\theta)$ has several local minima.

For simplification of the program the constant terms, b_{io}, $i = 1,2,\ldots,m$, of the B-polynomials are put equal zero. This will cause no loss of generality, because we can handle this by shifting input and output data one step. Shifting input and output data also handles the case of time lags, that is when the real model should be

$$A(z^{-1})y(t) = \sum_{i=1}^{m} B_i(z^{-1})u_i(t-k_i) + \lambda\ C(z^{-1})e(t) \tag{13}$$

However, the time lag has to be a multiple of the sampling interval.

The parameter accuracy is also given because the information matrix, $\lambda^{-2}\ V_{\theta\theta}$, is estimated. We obtain

$$\sigma_{\theta_i}^2 = \hat{\lambda}^2\ [V_{\theta\theta}(\hat{\theta})]_{ii}^{-1} \tag{14}$$

where σ_{θ_i} is the standard deviation of the parameter θ_i and where $[V_{\theta\theta}(\hat{\theta})]_{ii}^{-1}$ denotes the element in the i-th row and the i-th column of the inverse of the matrix of the second partial derivatives. The standard deviation of the loss function is $\hat{\lambda}^2\sqrt{N/2}$ and that of the parameter λ is approximately $\hat{\lambda}/\sqrt{2N}$.

The maximum likelihood method provides a possibility to test the order of the model. The identification is repeated with increasing order. Let V_n denote the minimal value of the loss function for the n-th order model. It follows from {11} that the parameter estimates for a large number of data are asymptotically normal $(\theta_o, \lambda^2\ V_{\theta\theta}^{-1})$, where θ_o stands for the correct value of θ.

Assuming that asymptotic theory may be applied, we test the hypothesis that the system is of order n, that is the null hypothesis is

$$H_o: a_{n+1}^o = \dots = a_{n+k}^o = b_{1,n+1}^o = \dots = b_{m,n+k}^o =$$
$$= c_{n+1}^o = \dots = c_{n+k}^o = 0$$

(θ_i^o stands for the correct value of θ_i).

Then

$$F_{n+k,n} = \frac{V_n - V_{n+k}}{V_{n+k}} \cdot \frac{N - (m+2)(n+k)}{(m+2)k} \qquad (15)$$

has an $F((m+2)k,\ N - (m+2)(n+k))$ distribution under null hypothesis. When N is large $(m+2)k \cdot F_{n+k,n}$ tends to a χ^2-distribution with (m+2)k degrees of freedom. For instance if we test the model of order (n+1) with one input against the model of order n at a risk level of 5%, the loss function has been significantly reduced, if the test quantity becomes greater than 2.6. Then the order of the model is at least (n+1). The test sometimes seems to give a too high order for the model of a real process than is necessary, ref {4 }, { 5}. Other possibilities to test the order of the model should also be used.

3. COMPUTATIONAL ASPECTS ON THE CALCULATIONS OF THE GRADIENT AND THE SECOND PARTIAL DERIVATIVES OF THE LOSS FUNCTION

From the equations (6), (11) and (12) it is obvious that we must calculate the residuals, $\varepsilon(t)$, the gradient of the residuals, $\frac{\partial\varepsilon(t)}{\partial\theta_i}$, $i = 1,2,\ldots,n(m+2)$, and the second partial derivatives of the residuals, $\frac{\partial^2\varepsilon(t)}{\partial\theta_i\partial\theta_j}$, $i,j = 1,2,\ldots,n(m+2)$. This can be done very economically by choosing appropriate state variables for the computation, and by performing the computations recursively, ref {10}. For large N the computations only increase linearly with the order of the model.

Differentiating (6) gives

$$\begin{cases} C(z^{-1})\,\dfrac{\partial\varepsilon(t)}{\partial a_j} = z^{-j}\,y(t) \\ C(z^{-1})\,\dfrac{\partial\varepsilon(t)}{\partial b_{kj}} = -z^{-j}\,u_k(t) \qquad k = 1,2,\ldots,m \\ C(z^{-1})\,\dfrac{\partial\varepsilon(t)}{\partial c_j} = -z^{-j}\,\varepsilon(t) \end{cases} \tag{16}$$

The last equation of (16) can be differentiated once more:

$$\begin{cases} C(z^{-1})\,\dfrac{\partial^2\varepsilon(t)}{\partial a_i\,\partial c_j} = -z^{-i-j+1}\,\dfrac{\partial\varepsilon(t)}{\partial a_1} \\ C(z^{-1})\,\dfrac{\partial^2\varepsilon(t)}{\partial b_{ki}\,\partial c_j} = -z^{-i-j+1}\,\dfrac{\partial\varepsilon(t)}{\partial b_{k1}} \qquad k = 1,2,\ldots,m \\ C(z^{-1})\,\dfrac{\partial^2\varepsilon(t)}{\partial c_i\,\partial c_j} = -2z^{-i-j+1}\,\dfrac{\partial\varepsilon(t)}{\partial c_1} \end{cases} \tag{17}$$

where the relation

$$\frac{\partial\varepsilon(t)}{\partial a_i} = z^{-i+1}\,\frac{\partial\varepsilon(t)}{\partial a_1} = \frac{\partial\varepsilon(t-i+1)}{\partial a_1} \qquad i \leqslant t+1 \tag{18}$$

and similar formulas for the derivatives with respect to the b- and c-parameters are used. The relation (18) is easily derived from equation (16). Notice that the second partial derivatives of the residuals are zero if no differentiating is made with respect to a c-parameter.

The first problem is the computation of the residuals from equation (6). To carry out these computations we introduce the following state variable representation of equation (6)

$$x(t+1) = \begin{bmatrix} -c_1 & 1 & 0 & \dots & 0 \\ -c_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -c_{n-1} & 0 & 0 & \dots & 1 \\ -c_n & 0 & 0 & \dots & 0 \end{bmatrix} x(t) + \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix} y(t) - \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1,n-1} \\ b_{1n} \end{bmatrix} u_1(t) -$$

$$- \dots - \begin{bmatrix} b_{m1} \\ b_{m2} \\ \vdots \\ b_{m,n-1} \\ b_{m,n} \end{bmatrix} u_m(t) + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} y(t+1) \qquad (19)$$

and

$$\varepsilon(t) = (1,0,\dots,0,0)\ x(t)$$

Now it is very easy to perform the necessary computations. We only need to save the vector x(t) and the last values of $u_i(t)$, $i = 1,2,\dots,m$ and of y(t) in the memory. If we furthermore let the vector x(t) have (n+1) components and put $x_{n+1} = 0$ we can easily write the computation of the residual at time (t+1) as a simple algorithm. In FORTRAN we get

```
   X(N+1) = 0.
   DO  1  I = 1, N
1  X(I)=-C(I)*X(I)+X(I+1)+A(I)*Y1-B(1,I)*U1-...-B(M,I)*UM
   X(1) = X(1) + Y
   E = X(1)
```

where Y1, U1, ..., UM denote the values of y(t), $u_1(t)$, ..., $u_m(t)$ and Y the value of y(t+1). The other notations follow straightforward from (19).

The problem of computing the derivatives remains. We now introduce the following state variable representation of the first equation of (16)

$$x(t+1)=\begin{bmatrix} -c_1 & -c_2 & \cdots & -c_{n-1} & -c_1 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & 1 & 0 \end{bmatrix} x(t)+\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} y(t) \qquad (20)$$

and we get

$$\frac{\partial \varepsilon(t)}{\partial a_1} = x_1(t)$$

$$\frac{\partial \varepsilon(t)}{\partial a_2} = x_1(t-1) = x_2(t) \quad \text{etc.}$$

Put

$$C = \begin{bmatrix} -c_1 & -c_2 & \cdots & -c_{n-1} & -c_n \\ 1 & 0 & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

All the equations of (16) can be represented by state variables in the following way.

$$x(t+1)=\begin{bmatrix} C & & & & \\ & C & & & \\ & & \ddots & & \\ & & & C & \\ & & & & C \end{bmatrix} x(t)+\begin{bmatrix} y(t) \\ 0 \\ \vdots \\ 0 \\ -u_1(t) \\ 0 \\ \vdots \\ 0 \\ \vdots \\ -u_m(t) \\ 0 \\ \vdots \\ 0 \\ -\varepsilon(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (21)$$

C denotes the matrix defined above. All other elements in the matrix of (21) are zeros.

We have

$$x_i(t) = \frac{\partial \varepsilon(t)}{\partial \theta_i} \qquad i = 1,2,\ldots,n(m+2)$$

Notice that this last recursive equation can be iterated by shifts and computation of

$$x_1(t+1) = - \sum_{i=1}^{n} c_i \, x_i(t) + y(t)$$

$$x_{k \cdot n+1}(t+1) = - \sum_{i=1}^{n} c_i \, x_{k \cdot n+i}(t) - u_k(t) \qquad k = 1,2,\ldots,m$$

$$x_{(m+1)n+1}(t+1) = - \sum_{i=1}^{n} c_i \, x_{(m+1)n+i}(t) - \varepsilon(t) \tag{22}$$

because

$$x_2(t) = x_1(t-1) \qquad \text{etc.}$$

Now we have all formulas we need for the computation of the loss function, the gradient and the approximate second derivatives of the loss function. For the computation of the exact second partial derivatives of the loss function we also need the second partial derivatives of the residuals, that is

$$\frac{\partial^2 \varepsilon(t)}{\partial \theta_i \partial \theta_j} \qquad i,j = 1,2,\ldots,n(m+2)$$

From the first equation of (17) we get

$$\frac{\partial^2 \varepsilon(t)}{\partial a_i \partial c_j} = \frac{\partial^2 \varepsilon(t-i-j+2)}{\partial a_1 \partial c_1} \tag{23}$$

and similar formulas also hold for the other equations of (17).

Then this term is identically the same for elements of which the sum of indices is the same. Then it is possible to compute the differences between the elements of the matrix of the approximate second derivatives and those of the matrix of exact second derivatives by computing only (2n-1)(m+2) terms. (The sum of the indices is going from 2 to 2n, that is (2n-1) different values).

The computations of these terms follow the scheme that was given for the gradient of the residuals, because equations (16) and (17) are of the same type.

One problem remains, because we want to compute $V(\theta)$, $V_\theta(\theta)$ and $V_{\theta\theta}(\theta)$ in a way that is as fast as possible. To do this we sum up V, V_θ and $V_{\theta\theta}$ recursively for $t = 1,2,\ldots,N$ according to equations (6), (11) and (12). Notice that N is generally much greater than n and then we can simplify the computation in the following way. V and V_θ are computed directly from (6) and (11), but the computation of $V_{\theta\theta}$ can be simplified.

We have

$$\left(\frac{\partial^2 V}{\partial\theta_i \partial\theta_j}\right)_{appr} = \sum_{t=1}^{N} \frac{\partial\varepsilon(t)}{\partial\theta_i} \frac{\partial\varepsilon(t)}{\partial\theta_j} \tag{24}$$

that is the first term of equation (12).

Now for instance consider the upper left hand part of the symmetric matrix $V_{\theta\theta}$.

We have

$$\begin{bmatrix} \frac{\partial^2 V}{\partial a_1 \partial a_1} & \cdots & \frac{\partial^2 V}{\partial a_1 \partial a_{n-1}} & \frac{\partial^2 V}{\partial a_1 \partial a_n} \\ & & \vdots & \vdots \\ & & \frac{\partial^2 V}{\partial a_{n-2} \partial a_{n-1}} & \frac{\partial^2 V}{\partial a_{n-2} \partial a_n} \\ & & \frac{\partial^2 V}{\partial a_{n-1} \partial a_{n-1}} & \frac{\partial^2 V}{\partial a_{n-1} \partial a_n} \\ & & & \frac{\partial^2 V}{\partial a_n \partial a_n} \end{bmatrix} =$$

$$= \begin{bmatrix} \sum_{t=1}^{N} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t)}{\partial a_1} & \cdots & \sum_{t=1}^{N} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t-n+2)}{\partial a_1} & \sum_{t=1}^{N} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t-n+1)}{\partial a_1} \\ & & \vdots & \vdots \\ & & \sum_{t=1}^{N-n+3} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t-1)}{\partial a_1} & \sum_{t=1}^{N-n+3} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t-2)}{\partial a_1} \\ & & \sum_{t=1}^{N-n+2} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t)}{\partial a_1} & \sum_{t=1}^{N-n+2} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t-1)}{\partial a_1} \\ & & & \sum_{t=1}^{N-n+1} \frac{\partial \varepsilon(t)}{\partial a_1} \frac{\partial \varepsilon(t)}{\partial a_1} \end{bmatrix}$$

From this we can see that if we sum up the n-th column of the matrix from $t = 1$ to $t = N-n+1$, we can easily get the other elements of the matrix. For instance we get the element $\frac{\partial^2 V}{\partial a_{n-1} \partial a_{n-1}}$ from $\frac{\partial^2 V}{\partial a_n \partial a_n}$ by adding one more term etc.

Similar simplifications can be made for the computation of the other parts of the matrix. Furthermore the matrix is symmetric. In appendix C schemes of the computations are given together with an example for a third order model with one input.

4. PROGRAM PACKAGE

The program package has been designed to identify industrial processes. A mathematical model according to (1) is constructed. The program is written in FORTRAN for a CD 3600 computer. The main subroutine in the package is called PROIDE (PROCESS IDENTIFICATION). This subroutine needs the subroutines VV1V2 and GJRV and the integer function NSTABLE. The user has to write a main program containing input of data, data transformations and a sequence of calls of PROIDE. However, in order to simplify the use of the program package, two more subroutines, MISOID and RESTART, have been written. Using these subroutines together with the rest of the package, the user only has to write a main program containing input of data and eventually shifts of input and output sequences. Below we first describe the use of the program package when MISOID and RESTART are not used, and then the use of these subroutines are discussed.

The program accepts data in the form of one dependent variable (the output) and maximum five independent variables (the inputs). It produces the estimates of the parameters of the model (1) together with estimates of their uncertainties. Maximum order of the model is ten. The program accepts sequences of length N, as long as the product N(m+1) is less than 6000, where m is the number of inputs.

In the main program transformations of raw data have to be done. The data for which the identification shall be performed must have zero mean. This is arranged automatically when using MISOID or RESTART but otherwise this transformation of data has to be done. Sometimes there is a drift in the data and this has to be removed by identifying the differences instead. Shift in data is necessary if there is an immediate response to the input i ($b_{io} \neq 0$) or if there are time lags in the process.

Use of program package without MISOID or RESTART

Main program:

This program must contain

1. Program identifier.
2. The same COMMON block as in subroutine PROIDE.
3. Input data in the array DAT in order

   $u_1(1)$, $u_2(1)$, ..., $u_m(1)$, $y(1)$, $u_1(2)$, $u_2(2)$, ..., $u_m(2)$, $y(2)$, ..., $u_1(NP)$, $u_2(NP)$, ..., $u_m(NP)$, $y(NP)$

   Before control is passed over to subroutine PROIDE, the data must be transformed as to have zero mean for each signal. Other necessary transformations of data must also be done here.
4. Possibly starting values for some of the parameters (in the array C). Only necessary if the standard starting point $\theta^o = (0, \ldots, 0)$ is not desired.
5. A call sequence

   CALL PROIDE (.........)

   ⋮

   See next section.
6. FORMAT specifications for input data and requested extra output (information about data etc.)
7. CALL EXIT
8. END

SUBROUTINE PROIDE (NO,NI,NP,L1,L2,IT,ACC,ACC1,IPRINT,IERR):

This subroutine controls the iteration procedure. When called it executes a number of iterations according to (10). Tests of convergence are included. Via the call we can choose between different estimates and different sizes of convergence parameters. The number of iterations, the starting value for the iteration and the amount of printout from the program are also determined by the parameters of the call. Furthermore there is a parameter controlling the convergence of the procedure.

The parameters of the call are defined as follows:

NO	Order of system model (1) (=n)
NI	Number of inputs (=m)
NP	Number of data values (=N)
L1	Parameter determining which starting value θ shall have for the iteration.
L1=-1	An estimation of θ is made starting from a value of θ given in the main program (used when restarting the identification, e.g. if convergence did not occur).
L1=0	An estimation of θ is made starting from the value of θ which was obtained in the preceding iteration.
L1=1	A least squares estimation of $a_1,a_2,\ldots,a_n$, $b_{11},\ldots,b_{mn}$ is made starting from $\theta = (0,0,\ldots,0)$
L1=2	A least squares estimation of $a_1,a_2,\ldots,a_n$, $b_{11},\ldots,b_{mn}$ is made starting from $\theta = (0,0,\ldots,0,c_1,\ldots,c_n)$. The values of the c-coefficients shall be given in the main program.
L2	Parameter determining the number of iterations
L2=0	The iteration procedure continues until the maximum correction of the elements of θ is smaller than a value given by ACC or if the approximate second derivatives are used until this maximum correction is smaller than a value given by ACC1.
L2=N	N iterations shall be made, but if the maximum correction is smaller than ACC1 and approximative second derivatives are used the control is given back to the main program.
IT	Parameter determining whether exact or approximate second derivatives will be used (see (12)).
IT=0	Approximate second derivatives are used.
IT=1	Exact second derivatives are used.

IPRINT	Parameter determining the amount of output from the program.
IPRINT=0	V and θ are printed.
IPRINT=1	V, θ and V_θ are printed.
IPRINT=2	V, θ, V_θ, λ and the standard deviations of the parameters are printed (as well as NS and NH, see below).
IPRINT=3	As IPRINT=2 but $(V_{\theta\theta})^{-1}$ is also printed.
IPRINT=4	As IPRINT=3 but $V_{\theta\theta}$ and the condition number (see below) of this matrix are also printed out.
ACC	Parameter for convergence test. (See the explanation of L2=N and L2=0).
ACC1	Parameter for convergence test (see the explanation of L2=0).
IERR	Parameter determining the convergence of the procedure.
IERR=1	The step length has been halved because no smaller loss function was found. The step length was halved when IT=1 until maximum correction was smaller than ACC.
IERR=2	The maximum correction was smaller than ACC1 when IT=0.
IERR=3	The C-polynomial was not stable in spite of halving the step length so that maximum correction was equal to ACC.
IERR=0	If none of the above conditions are satisfied IERR is zero.

A normal call sequence in the main program is:

```
CALL PROIDE (NO,NI,NP,1,1,0,AC1,AC2,2,IERR)
CALL PROIDE (NO,NI,NP,0,N,0,AC1,AC2,0,IERR)
CALL PROIDE (NO,NI,NP,0,0,1,AC1,AC2,1,IERR)
CALL PROIDE (NO,NI,NP,0,1,1,AC1,AC2,3,IERR)
```

where NO,NI,NP,N have given values. AC1 and AC2 have to be chosen appropriately, e.g. 0,000001 and 0,01 respectively. If IERR is printed out after the last iteration one can determine if the procedure has converged normally.

This call sequence first gives a least squares estimation of the a- and b-parameters with the c-parameters equal zero. Then N iterations using approximate second derivatives starting from this point are performed if the maximum correction of the parameters has not become smaller than AC2. After that the procedure will continue until maximum correction of the parameters is smaller than AC1 and these last iterations use the exact second derivatives. The last call will cause one more iteration with exact second derivatives and will give a printout of the inverse of the matrix of the second derivatives, $V_{\theta\theta}$.

N is chosen as to secure convergence to the minimum point. As a rule N can be chosen a little less than the number of the unknown parameters if data are not too bad. In certain cases N must be 30 or more to secure convergence. This depends on the data, on the order of the system and on the choice of starting value. It may be preferable to use the minimum point for the model of order n as a starting value for the identification of the model of order n+1. If N is chosen too small the exact second derivatives are used before we have reached a point from which convergence to the minimum point will occur. In this case we can get convergence to a maximum point if there exists such a point or extremely slow convergence. In such cases the identification has to be repeated with the obtained parameter values as starting values and using the approximate second derivatives for a few more steps.

The subroutine PRO puts $\alpha = 1$ in the damped Newton-Raphson algorithm

$$\theta^{k+1} = \theta^k - \alpha[V_{\theta\theta}(\theta^k)]^{-1} V_\theta(\theta^k)$$

If the loss function calculated in the new point, $V(\theta^{k+1})$, is larger than $V(\theta^k)$, then the step length α is halved until $V(\theta^{k+1})$ is smaller than $V(\theta^k)$ or until the maximum correction of the coefficients is smaller than ACC. If this last condition is satisfied the iteration is stopped and "NO SMALLER LOSS FUNCTION FOUND EVEN IF STEP EQUAL ACC" is printed. The number of dividings of step length by two is counted (=NH).

The step length is also halved and tested against ACC, if the local quadratic approximation of the loss function is bad. This means that the new value of the loss function, $V(\theta^{k+1})$, is predicted from

$$V(\theta^{k+1}) = V(\theta^k) + V_\theta^T(\theta^k)\Delta\theta + \frac{1}{2}\Delta\theta^T V_{\theta\theta}(\theta^k)\Delta\theta$$

where T stands for transpose and where

$$\Delta\theta = \theta^{k+1} - \theta^k$$

Using (10) we get

$$V_{predicted}(\theta^{k+1}) = V(\theta^k) + \frac{1}{2} V_\theta^T(\theta^k)\Delta\theta$$

Then the value $V(\theta^{k+1})$ is computed and this value is tested against the predicted value. If

$$\left| \frac{V(\theta^{k+1})}{V_{predicted}(\theta^{k+1})} \right| < 0.5$$

the step length is halved and the same procedure is run through again. The value 0.5 is chosen rather arbitrary. It must lie between 0 and 1 and out of experience 0.5 seems to be a good choice.

When θ^{k+1} has been computed there is a test of the direction of the change of the parameters. The direction is downhill if and only if the quantity $V_\theta^T(\theta^k)\Delta\theta$ is negative. If this is not so, the direction is simply reversed.

Another test of the stability is also included. Directly after the corrections of the coefficients are computed, that is when

$$[V_{\theta\theta}(\theta^k)]^{-1} \cdot V_\theta(\theta^k)$$

is computed, a stability test of the C-polynomial

$$C(z) = z^n + c_1 z^{n-1} + \ldots + c_n$$

if performed. The integer function NSTABLE will be zero if all roots of C(z) = 0 lie inside the unit circle. If there is one or more roots outside the unit circle NSTABLE is put equal -1. If the C-polynomial is not stable, the step length is halved until the C-polynomial is stable or until the maximum correction of the coefficients is smaller than ACC. The number of dividings of step length is counted (=NS). If the last condition is satisfied the iteration is stopped and "NO STABILITY OF C-POLYNOMIAL EVEN IF STEP EQUAL ACC" is printed.

If exact second derivatives are used, and L2=0 and maximum absolute value of the elements of

$$[V_{\theta\theta}(\theta^k)]^{-1} V_\theta(\theta^k)$$

is smaller than or equal ACC, the iteration is stopped and "MAX COEFF CORRECTION IS LESS THAN ... " is printed. This is the normal output when the procedure has converged.

In the program a so called condition number, M, is computed for the matrix $V_{\theta\theta}$. It is defined from

$$M = 2\, n \max_{i,k} |a_{ik}| \max_{i,k} |b_{ik}|$$

where $V_{\theta\theta} = (a_{ik})$ and $V_{\theta\theta}^{-1} = (b_{ik})$ and n = the order of the matrix $V_{\theta\theta}$. Large condition number is most often an indication of that numerical difficulties may occur when the matrix is used in the calculations. It is a measure of the singularity of the matrix. It can be used as a test of the order of the model, because a too high order model will give an ill-conditioned second derivative matrix. PROIDE needs the subroutines VV1V2 and

GJRV, which are automatically called from PROIDE. Thus the identification can be performed without any further knowledge of VV1V2 and GJRV.

SUBROUTINE VV1V2 (NO,NI,NP,IT):

This subroutine calculates the loss function, V_θ, the gradient, $V_\theta(\theta)$ and the second partial derivative matrix, $V_{\theta\theta}$, according to formulas given in the preceding sections.

The parameters NO, NI and NP are the same as in subroutine PROIDE. IT is a parameter determining whether exact or approximative second derivatives are computed. These parameters are automatically transferred from the call of PROIDE.

SUBROUTINE GJRV (A,N,EPS,IERR,IA):

This subroutine inverts asymmetric matrices by the method of Gauss-Jordan as proposed by H. Rutishauser (ZAMP 10 (1959), 281-291). It is used for the inversion of the matrix of the second partial derivatives.

The parameters of the call are:

A	the matrix which shall be inverted. After the inversion the inverse is in A, that is the original matrix is destroyed
N	the order of the matrix
EPS	tolerance parameter used for the test of singularity
IERR	parameter indicating if the matrix is considered singular or not
IA	dimension parameter

FUNCTION NSTABLE (A,N):

This function tests the polynomial

$$A(z) = z^n + a_1 z^{n-1} + \ldots + a_n$$

for stability. NSTABLE = -1 if there is one or more roots to A(z) = 0 outside or on the unit circle. NSTABLE=0 if all roots lie inside the unit circle. N is the order of the polynomial. The stability of the polynomial is tested using Ruzicka's algorithm {9 }.

Use of program package together with MISOID or RESTART

Main program

This program must contain

1. Program identifier.
2. The same COMMON block as in subroutine PROIDE.
3. Input data in the array DAT in order
   $u_1(1),u_2(1),\ldots,u_m(1),y(1),u_1(2),u_2(2),\ldots,u_m(2),$
   $y(2),\ldots,u_1(NP),u_2(NP),\ldots,u_m(NP),y(NP)$.
   The only transformation necessary is possibly shift of data.
4. A call of MISOID or RESTART:

   CALL MISOID (NO,NI,NP,IER)

   or

   CALL RESTART (NO,NI,NP,M, KIND,IER).
5. FORMAT specifications for input data and requested extra output (information about data etc.)
6. CALL EXIT
7. END

If RESTART is used with KIND = 0, the starting values of the parameters must be given as data cards. They are read in by RESTART in FORMAT E20.10. This means one value on each card. These data cards must be the last in the data card package. Examples of main programs are given in appendices A and B.

SUBROUTINE MISOID (NO,NI,NP,IER):

The parameters are defined:

NO		Order of system model
NI		Number of inputs
NP		Number of data values
IER	IER=0	The procedure has converged normally
	IER=1	The procedure is stopped, either because no smaller loss function is found or because C-polynomial is not stable, until the maximum correction of the parameters is less than 10^{-6} (that is when IERR=1 or 3 in PROIDE).

In this subroutine we use ACC = 10^{-6} and ACC1 = 0.01. Transformation of data is performed so that they have zero mean. Then a least squares estimation of a- and b-parameters is performed.

After this a number of iterations with approximative second derivatives are done. The number of iterations is chosen as the number of parameters. However, iterations are only performed as long as maximum correction is larger than 0.01. Then two iterations with exact second derivatives are done. Now if the procedure does not converge, 4 more iterations with approximative second derivatives are done, and then we try to use exact second derivatives again. This step is repeated at most three times, and then, or if the procedure converges earlier, iterations with exact second derivatives are performed until maximum correction is less than 10^{-6}. For the different steps the choice of IPRINT has been made, in order to give enough information for the user. Examples of print out from the program are given in appendices A and B.

SUBROUTINE RESTART (NO,NI,NP,M,KIND,IER):

The parameters NO, NI, NP and IER are the same as in MISOID.

M — The number of iterations with approximative second derivatives requested. If the maximum correction is less than 0.001 before M iterations are performed, the procedure directly continues with iterations with exact second derivatives.

KIND

- KIND=0 is used when starting the procedure from a certain value of the parameters. The data is transformed as to have zero mean. The parameter values are read with FORMAT E20.10.
- KIND=1 is used when starting the procedure from a value of the parameters obtained in the same program, that is when MISOID has been used but IER=1. In this case the starting values are available directly in the program.

In the subroutine RESTART we use ACC = 10^{-8} and ACC1 = 0.001. The value are chosen smaller than in MISOID because RESTART shall be used when convergence are not good. The subroutine follows the same scheme as MISOID except that the first step is not a least squares estimation but an ordinary iteration from the given starting value of θ. Furthermore 6 iterations with approximative second derivatives are done instead of 4, if convergence to a minimum does not occur when we try to use exact second derivatives.

Storage and time requirements

The lengths of the subroutines in the program package are given in Table 1.

Subroutine	Length
PROIDE	846
VV1V2	716
GJRV	729
NSTABLE	103
MISOID	354
RESTART	353

Table 1 - Subroutine lengths

Arrays and variables concerning more than one subroutine are declared in a numbered common block. It requires 11408 words for this version of the program. Furthermore about 3000 words are needed for routines required from the CD subroutine library. In this the routines handling the input/output of the computer are included. Totally we need about 17500 words, the main program excluded.

In Table 2 below we show the time that is required to perform one iteration with approximative second derivatives. The computations are performed on a CD 3600 computer. From the table we can see that the time is approximatively linear in both n and N, that is proportional to the product nN. If the step length has to be halved to secure stability or convergence, the time per iteration is prolonged.

n \ N	500	1000
1	1458	2845
2	2122	4138
3	2810	5449

Table 2 - Time in milliseconds for one iteration with approximative second derivatives for different sequence length N and different order of the system n (one input)

Comments on the use of the program package

The program package has been used for identification both of artificial input/output sequences and of measurements from industrial processes. In this section some experiences from these applications are summed up together with general advices for identification.

One problem that is of great importance is the choice of input signal. In general we can say that the input signal has to be such that all modes of the process are excited. This means that the input signal shall contain all frequencies in the frequency bands that are of interest for that particular system. One such appropriate signal, that is often used, is the pseudo random binary signal (PRBS). A rule of thumb is that identification is useless if you cannot see any response of the input signal in the output signal. However, the maximum likelihood method seems to give reasonable estimates even if the noise-signal ratio is rather high.

Another problem is to examine the data for time invariance. This can be tested by dividing the measurement into parts and then identify each part. If the parameters for two different parts differ more than can be expected with regard to the uncertainties of the estimates, the process can be considered not stationary.

Identification of a process by this program package ought to be followed by different tests. One of them is to examine the residuals. An assumption was that they were normal and independent and these two things can easily be tested. Single large values of the residuals may indicate errors in the measurements. Another recommendable step is to plot the deterministic part of the output signal from the model, that is

$$y_d(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t)$$

and the difference between the real output $y(t)$ and $y_d(t)$. This difference can be called the error of the deterministic model. From these plots it is possible to see if the error is great compared with $y_d(t)$. In such cases the model does not seem to be good. It is also possible to see if there is a drift or large deviations in the error or if there perhaps is another input signal that we must take into account in order to get a good model of the process. Notice that in this error everything is included that is not caused by manipulations of the input signal, e.g. error in the measurements, variations of the process, disturbances from the surroundings, errors in the estimated model (too low order, unlinearities), other inputs and so on.

We end this section with a recommendation to carefully investigate the model obtained from the program and to try to determine if it is a reasonable model of the process. If not a new measurement probably has to be done.

In ref {2} more aspects on the use of a maximum likelihood identification program package like this one are given.

5. TEST EXAMPLES

Both artificially generated series of data and data from industrial processes have been identified. Below a few examples of identification of artificial input/output sequences are given. For other examples we refer to ref {5},{6},{7} and {8}. In ref {5} the maximum likelihood identification is compared with spectral analysis when identifying a nuclear reactor process. In ref {8} a chemical process, evaporation, is investigated. A distillation column is identified in ref {6}. Ref {7} treats among other things a multivariable process from the paper industry. Identification of further industrial processes will appear in a near future.

Test example 1

The first example is an artificially input/output sequence generated by a second order system with one input. The system was

$$y(t)-1.5y(t-1)+0.7y(t-2)=u(t-1)+u(t-2)+e(t)-e(t-1)+0.2e(t-2)$$

{u(t)} was a PRBS signal with a period of 263 and is a so called quadratic residue code, ref {3 }. The amplitude of the input is ±1. {e(t)} is a sequence of random numbers, obtained from a subroutine RANSS. This subroutine generates random floating point numbers distributed according to normal distribution with a mean of zero and a variance of one. The length of the series was 1000. The first part of the sequences {y(t)} and {u(t)} are plotted in Fig. 1. Both sequences are also listed in appendix A. It contains also the main program for test example 1 and the output from the identification program package. Some of the results are furthermore given in the Tables 3 and 4 below.

Notations: V_n lossfunction for n:th order model
ex exact second derivatives are used

Step	a_1	b_1	c_1	V_1
0	0	0	0	17215
1	-0.88	0.96	0	3381
2	-0.77	0.93	0.58	2903
3	-0.85	0.31	0.45	2516
4	-0.81	0.35	0.60	2515
5 ex	-0.826	0.264	0.551	2483
6 ex	-0.82684	0.26102	0.54119	2482.106
7 ex	-0.82686	0.26107	0.54098	2482.106
8 ex	-0.82686	0.26107	0.54098	
σ_i	0.018	0.064	0.024	
λ	2.228			

Table 3 - Successive estimates of the parameters for the first order model

Step	a_1	a_2	b_1	b_2	c_1	c_2	V_2
0	0	0	0	0	0	0	17215
1	-1.35	0.57	0.95	1.18	0	0	897
2	-1.46	0.66	0.97	1.09	-0.42	-0.18	642
3	-1.498	0.696	0.959	1.035	-0.695	-0.149	544
4	-1.490	0.694	0.962	1.065	-0.920	0.129	488
5	-1.497	0.701	0.955	1.064	-0.992	0.200	484.91
6	-1.4963	0.7005	0.9540	1.0674	-0.9989	0.2072	484.868
7 ex	-1.4963	0.7005	0.9539	1.0675	-0.9998	0.2080	484.867
8 ex	-1.4963	0.7005	0.9539	1.0675	-0.9998	0.2080	484.867
9 ex	-1.4963	0.7005	0.9539	1.0675	-0.9998	0.2080	
σ_i	0.0068	0.0059	0.0310	0.0392	0.0333	0.0330	
λ	0.9848						

Table 4 - Successive estimates of the parameters for the second order model

If we test the order of the model we get

$F_{2,1} = 1365$

$F_{3,2} = 1.2$

with the notations used before. This means that we can accept the second order model, if the test is done on the 5% level. The results of the identification are given in Table 5.

Parameter	Estimated	True
a_1	-1.496 ± 0.007	-1.5
a_2	0.700 ± 0.006	0.7
b_1	0.954 ± 0.031	1.0
b_2	1.068 ± 0.039	1.0
c_1	-1.000 ± 0.033	-1.0
c_2	0.208 ± 0.033	0.2
λ	0.985 ± 0.022	1.0
V	484.9 ± 21.7	

Table 5 - Estimated and true parameter values and the accuracies of the estimated parameters

Notice that the first step of the identification procedure gives the least squares estimate of the parameters, and the significant difference between this estimate and the obtained maximum likelihood estimate. The difference depends on the fact that the least squares estimate is biased when the noise is correlated, ref {12}.

In fig 1 we show the input and output sequences but also the residuals, $\{\varepsilon(t)\}$, the deterministic output $\{y_d(t)\}$ defined by

$$y_d(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t)$$

where $B(z^{-1})/A(z^{-1})$ is the resulting second order model, and the error of the deterministic model, $\{e_d(t)\}$ defined by

$$e_d(t) = y(t) - y_d(t)$$

Only the first part of the series is shown. Notice the different scales. Another step that may be taken to test the goodness of the model is to test the residuals for independence and normality. The results of these tests are shown in fig. 2 and fig. 3. In this example the model is a good approximation of the generating system. All the tests also perform very well.

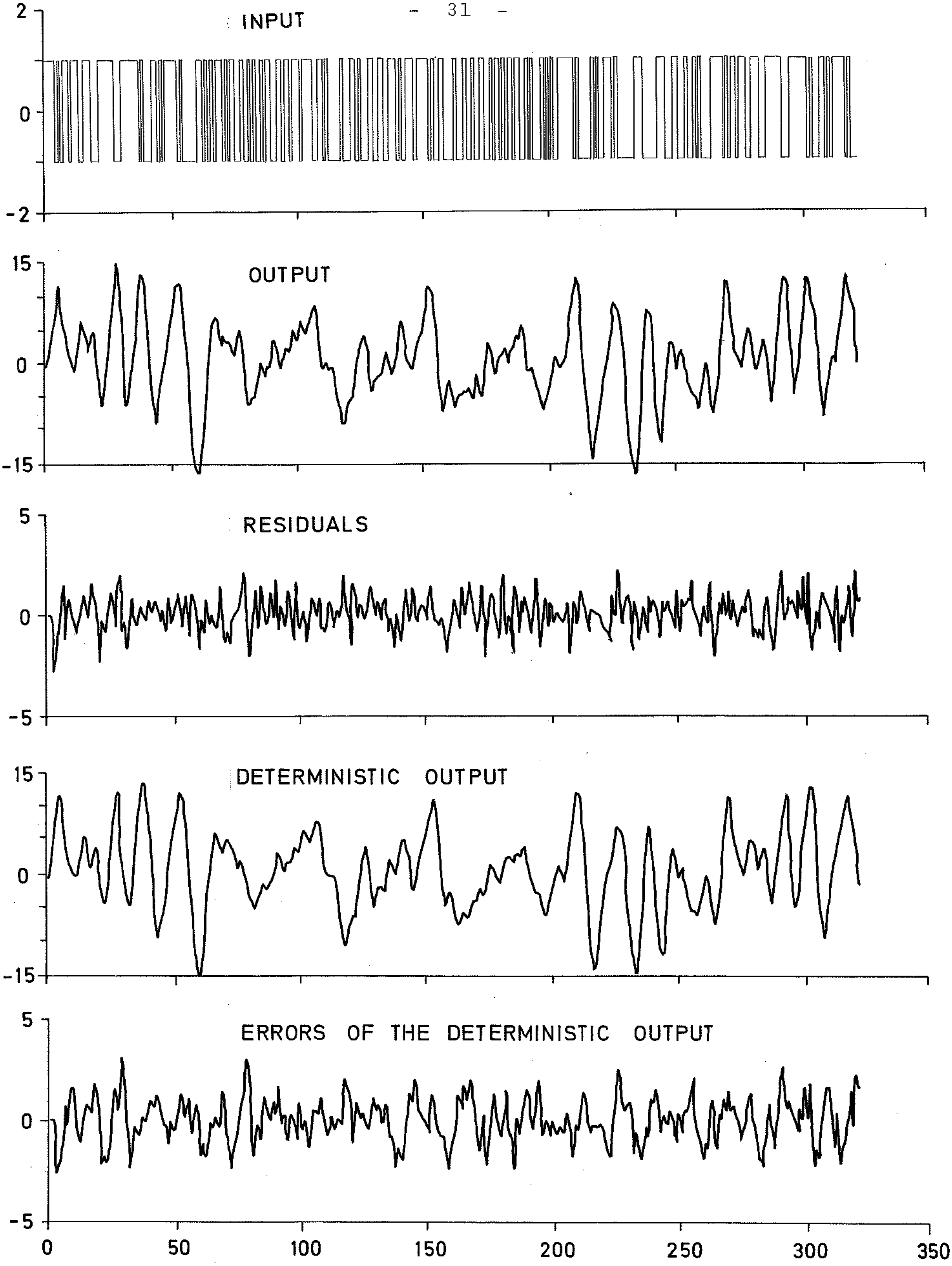

Fig. 1 - Input, output and results from the second order model for test example 1. Only the first part of the sequences is shown. Notice the different scales.

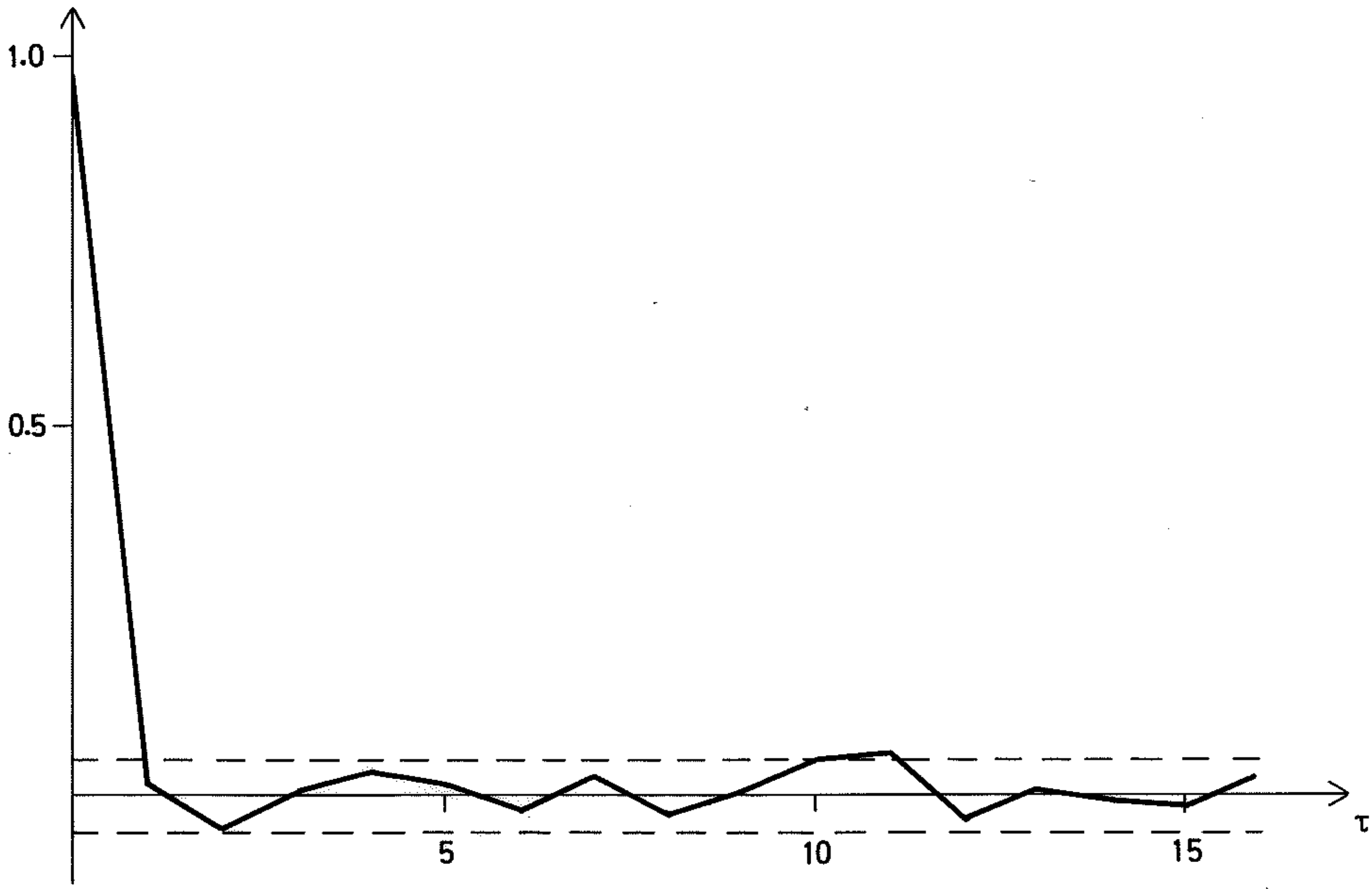

Fig. 2 - Sample covariance function, $r(\tau)$, for the residuals of the second order model for test example 1. The dashed line gives the one sigma limit for $r(\tau)$, $\tau \neq 0$. According to the assumptions $r(\tau)$ should equal zero, $\tau \neq 0$.

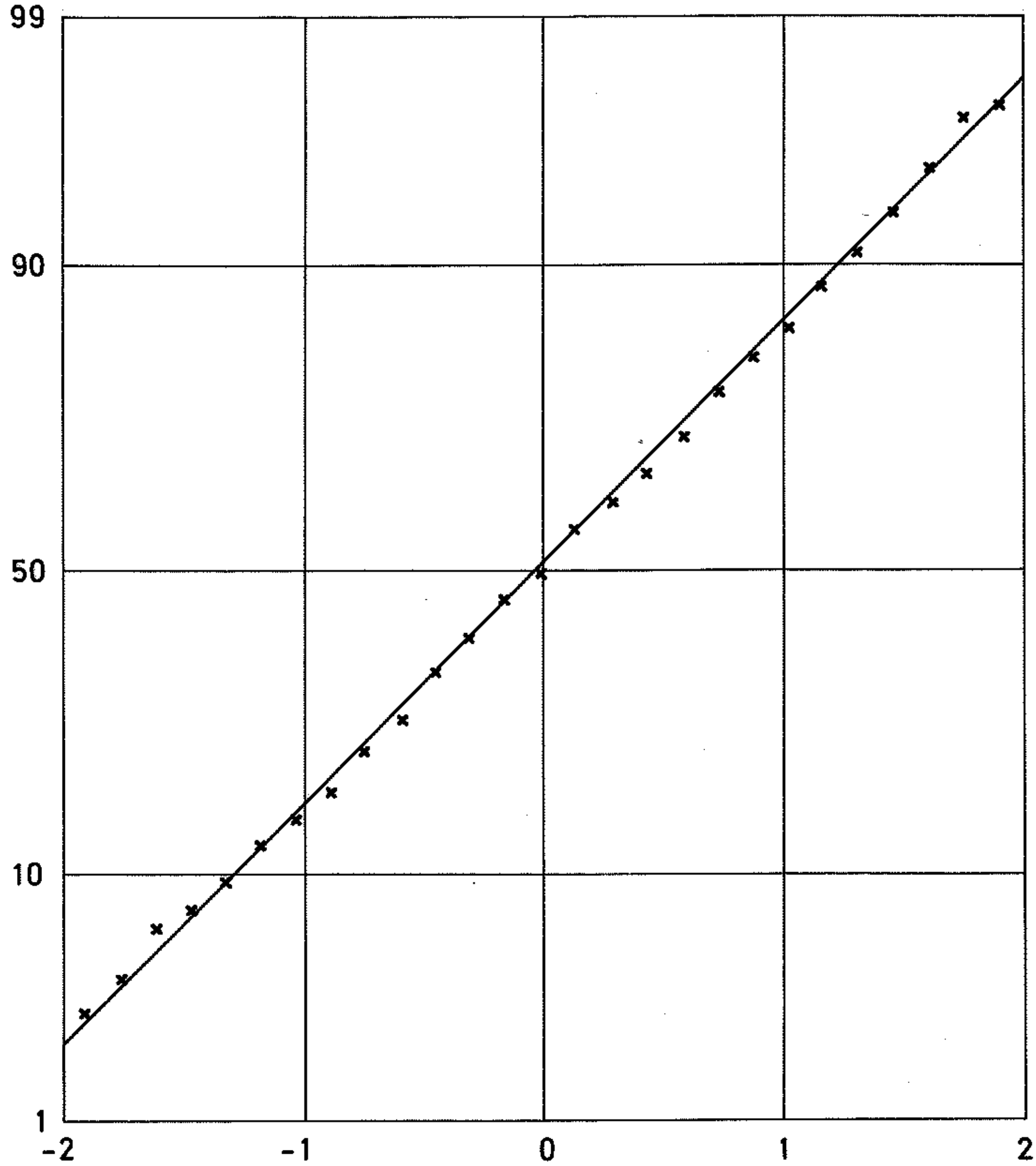

Fig. 3 - Test of normality of the residuals for the model of order 2 for test example 1. The residuals have been divided into class intervals, and in this diagram the cumulative frequencies are plotted with a vertical scale corresponding to a normal distribution, that is a perfect normally distributed variable gives a straight line.

Test example 2

The second example presented in this report is also an artificially generated input/output sequences from a second order system but with two input signals. The system was

$$y(t) - 1.5y(t-1) + 0.7y(t-2) = u_1(t-1) + 0.5u_1(t-2) + 0.7u_2(t-1) - 0.3u_2(t-2) + 1.5(e(t) - 1.0e(t-1) + 0.2e(t-2))$$

In this example all the series $\{u(t)\}$, $\{u_2(t)\}$ and $\{e(t)\}$ are sequences of random normal distributed numbers generated by the subroutine RANSS. The length of the investigated sequence was 500. The input and output signals are plotted in fig. 4. The results from the identification program package are given in Appendix B together with the main program and a table of the input and output data. Tests indicate that the order of the model should be two, and the resulting second order model is given in Table 6.

Parameter	Estimated	True
a_1	-1.517 ± 0.019	-1.5
a_2	0.713 ± 0.015	0.7
b_{11}	0.939 ± 0.068	1.0
b_{12}	0.465 ± 0.087	0.5
b_{21}	0.725 ± 0.069	0.7
b_{22}	-0.336 ± 0.072	-0.3
c_1	-1.043 ± 0.053	-1.0
c_2	0.262 ± 0.051	0.2
λ	1.419 ± 0.045	1.5
V	503.4 ± 31.8	

Table 6 - Estimated and true parameter values and the accuracies of the estimated parameters

As for test example 1 the residuals, the deterministic output of the model and the errors of the deterministic model are shown (fig. 5). The remarks about test example 1 also hold for this example.

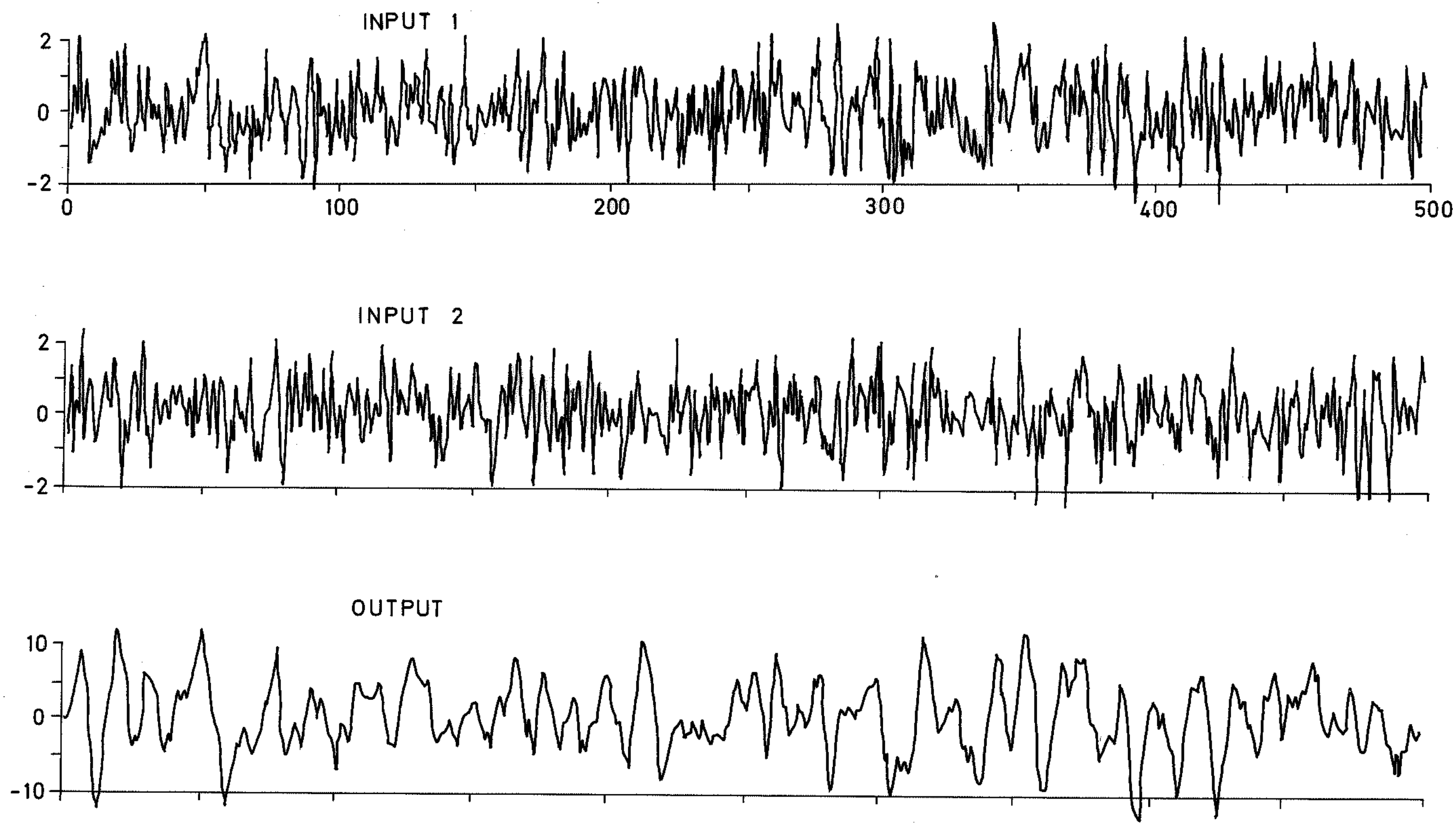

Fig. 4 - Input and output sequences for test example 2

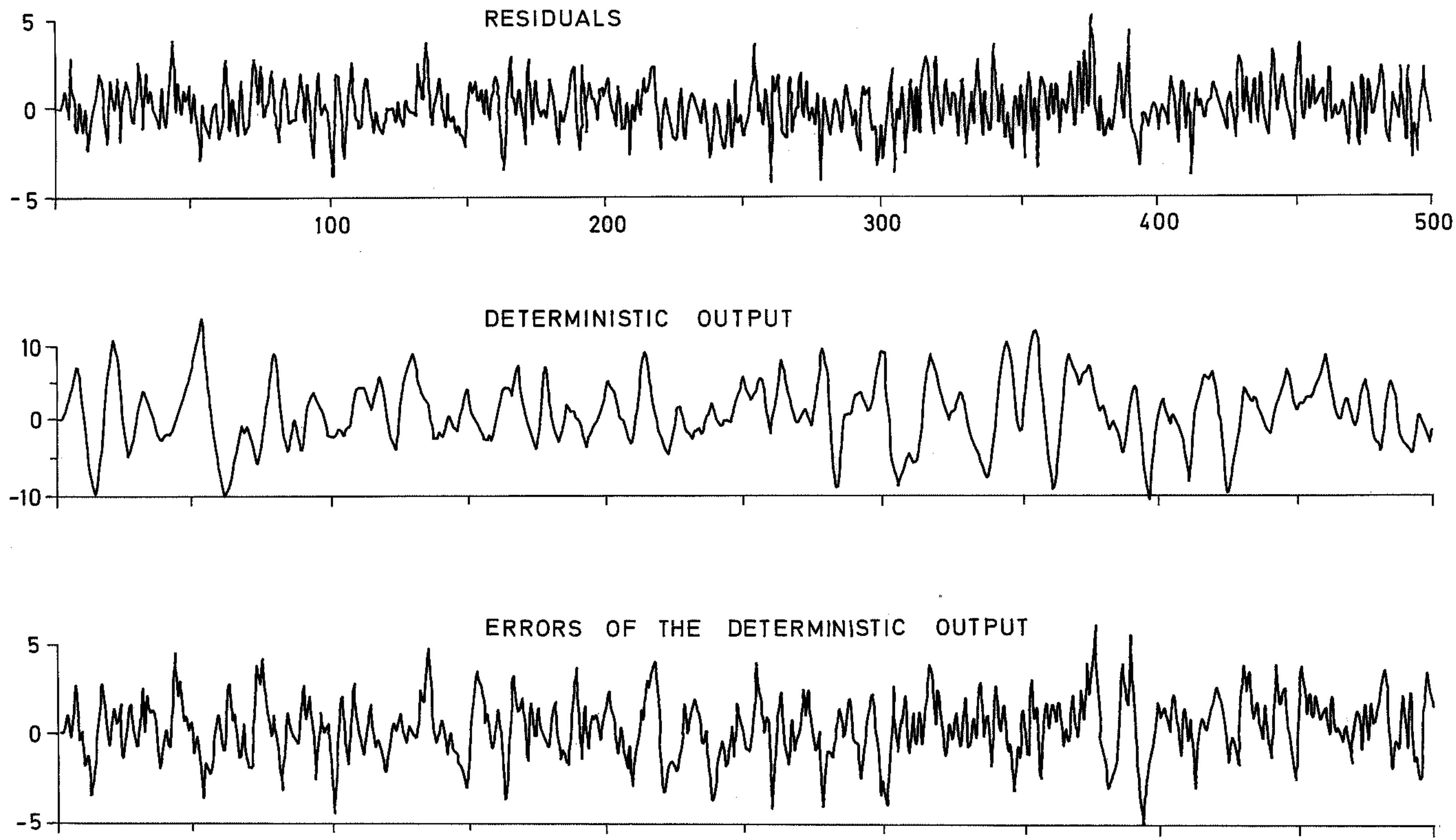

Fig. 5 - Residuals, deterministic output and errors of the deterministic output of the second order model for test example 2. Notice the different scales.

6. EXTENSIONS

This program package has been developed in order to get a working maximum likelihood identification program for analysis of industrial processes. The purpose has been to simplify the use of the program package as much as possible. Therefore many possible improvements are omitted. In this section some extensions are listed (see also ref {2}).

First the model (4) may be preferable because the use of model (1) may give common factors for some of the polynomials. In practice it is difficult to decide whether factors differ significantly from each others or not. Furthermore model (4) gives a less number of parameters.

Another improvement is to determine the order and the delays automatically. This could be done (ref {2}), but it will cause longer computation time. Often the delays can be estimated rather well directly from the data. Somewhere there must be a man "in the loop" to decide whether the model is reasonable or not and the question is where to place him. The same problem arises if there are parameters in the model that do not differ significantly from zero. This can be tested automatically, but it is not done in this version.

Changes in the structure of the model is another valuable possibility to include in the program. A variant of the described program package makes it possible to fix certain parameter values in the model (1).

Other tests that could be done automatically that are not included are for instance test of normality and independence of the residuals and test of time invariance of the process.

There are many strategies to use the exact and approximative second derivatives in the hill-climbing algorithm and the way they are chosen here is not optimal in any sense. Another simple method is to use the exact second derivatives all the time and just reverse the direction if it is not downhill. In this program package there is no possibility to use the pseudoinverse of $V_{\theta\theta}$ instead of the inverse if $V_{\theta\theta}$ is ill-conditioned. Such a possibility would be an improvement.

The hill-climbing routine itself could perhaps be improved by choosing step length in a more "optimal" way or by using another hill-climbing routine, e.g. the algorithm of Fletcher-Powell. However, this is not of essential importance because $V_{\theta\theta}$ is computed rather easily.

We will also emphasize that the available second derivative matrix makes it possible to test the quadratic approximation of the loss function and perhaps to calculate new parameter values for small changes in one parameter without doing all the calculations again.

At last we notice that in the program package many other programs must be included in order to check the obtained model, e.g. programs computing step and impulse responses, Bode diagrams, power density spectrum of the disturbances, minimum variance strategies and test of normality of the residuals. Such programs are available but are not described in this report.

7. REFERENCES

1. Bekey G.A. and McGhee R.B. "Gradient Methods for the Optimization of Dynamic System Parameters by Hybrid Computation", in Computing Methods in Optimization Problems, Balakrishnan and Neustadt, Academic Press, New York 1964.

2. Bohlin T. and Wensmark S. "Process Modelling and Control Design Package", IBM Nordic Laboratory, Report, August 1967.

3. Briggs P.A.N., Hammond P.H., Godfrey K.R. "Estimation of process dynamic characteristics by correlation methods using pseudo random signals", Preprints of the IFAC Symposium in Prague 1967.

4. Gustavsson I. "Parametric Identification of Time Series", Report 6803, March 1968, Lund Institute of Technology, Division of Automatic Control.

5. Gustavsson I. "Maximum likelihood identification of dynamics of the Ågesta reactor and comparison with results of spectral analysis", Report 6903, February 1969, Lund Institute of Technology, Division of Automatic Control.

6. Gustavsson I. "Maximum likelihood identification of dynamics of a distillation column", to appear.

7. Jonson A., Ström L-M "Processidentifiering - Reglerat system och system med flera in- och utsignaler", Thesis for the degree of civ.ing., January 1969, Lund Institute of Technology, Division of Automatic Control.

8. Skarman B. "Experimentell undersökning av processdynamiken hos en indunstare", Thesis for the degree of civ.ing., August 1968, Lund Institute of Technology, Division of Automatic Control.

9. Strejc V. "Synthese von Regelungssystemen mit Prozessrechner" pg 86-93, Akademie-Verlag, Berlin 1967.

10. Åström K.J. and Bohlin T. "Numerical Identification of Linear Dynamic System from Normal Operating Records", Proceedings of the IFAC Conference on Self-Adaptive Control Systems, Teddington (1965).

11. Åström K.J., Bohlin T. and Wensmark S. "Automatic Construction of Linear Stochastic Dynamic Models for Stationary Processes with Random Disturbances Using Operating Records", IBM Nordic Laboratory, Sweden, Report TP 18.150 (1965).

12. Åström K.J. "On the Achievable Accuracy in Identification Problems", IFAC Symposium on Identification in Automatic Control Systems, Prague, June 1967.

```
      PROGRAM TESTEX1
C
C     TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM TEST EXAMPLE 1
C     REFERENCE  GUSTAVSSON I.  PARAMETRIC IDENTIFICATION OF MULTIPLE INPUT
C     SINGLE OUTPUT DYNAMIC SYSTEMS
C     AUTHOR,IVAR GUSTAVSSON 20/2 1969
C
C     SUBROUTINE REQUIRED
C             MISOID
C             PROIDE
C             VV1V2
C             GJRV
C             NSTABLE
C
      COMMON EE,V,Y,U(5),E(10),C(70),EC(70),V1(70),VCC(140),ECC(140),
     FV2(70,70),DAT(6000)
      DIMENSION UA(1000),YA(1000)
C
      READ 100,NI,NP
  100 FORMAT(2I5)
      NT=NP*(NI+1)
      READ 101,(DAT(I),I=1,NT)
  101 FORMAT(10F8.3)
      PRINT 102
  102 FORMAT(/5X*TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM
     FTEST EXAMPLE 1*/)
      DO 1 I=1,NP
      I1=2*I-1
      I2=2*I
      UA(I)=DAT(I1)
    1 YA(I)=DAT(I2)
      PRINT 103
  103 FORMAT(/5X*INPUT*/)
      PRINT 107,(UA(I),I=1,NP)
      PRINT 104
  104 FORMAT(/5X*OUTPUT*/)
      PRINT 107,(YA(I),I=1,NP)
  107 FORMAT(10F10.3)
C
      DO 2 I=1,3
      CALL MISOID(I,NI,NP,IER)
      PRINT 105,IER
  105 FORMAT(//5X*IER=*I5)
      IF(IER-1)  6,4,6
    4 CALL RESTART(I,NI,NP,10,1,IER)
    6 IF(I-1)  3,5,3
C     TEST QUANTITY COMPUTED (F-TEST)
    3 TQ=(VOLD-V)*(NP-(NI+2)*I)/(V*(NI+2))
      PRINT 106,TQ
  106 FORMAT(//5X*TEST QUANTITY=*,F12.5)
    5 VOLD=V
    2 CONTINUE
C
      CALL EXIT
      END
```

TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM TEST EXAMPLE 1

INPUT

1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000
1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000
-1.000	1.000	1.000	1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000
1.000	1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000
-1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	1.000
1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	1.000
-1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000	-1.000
1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000
-1.000	1.000	1.000	-1.000	1.000	1.000	-1.000	1.000	1.000	1.000
-1.000	1.000	1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000
1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	-1.000
1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000
-1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	1.000
-1.000	-1.000	1.000	1.000	1.000	-1.000	1.000	1.000	1.000	1.000
1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	-1.000
-1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000
-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000
1.000	-1.000	1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000
-1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000
-1.000	-1.000	1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000
-1.000	-1.000	-1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000
1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000
-1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000
-1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000
-1.000	-1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000
1.000	1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000	1.000	1.000
1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000	1.000	1.000	1.000

-1.000	-1.000	-1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
-1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000
1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000
-1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000
1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000
1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000
1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000	1.000	-1.000
1.000	1.000	1.000	-1.000	1.000	1.000	1.000	1.000	-1.000	-1.000
1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000
-1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000
1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	1.000
1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	1.000
1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000
-1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000
-1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000
1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000	1.000	1.000
-1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000
1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	1.000	1.000
1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000	-1.000
1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000
-1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000
1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000
1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000	1.000	1.000
-1.000	1.000	-1.000	1.000	1.000	-1.000	1.000	1.000	1.000	-1.000
-1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000
1.000	1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000	1.000
1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000
-1.000	1.000	-1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000
-1.000	1.000	-1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	1.000
1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000
-1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000
1.000	1.000	-1.000	1.000	1.000	1.000	-1.000	1.000	1.000	1.000
1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000

-1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000
-1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000
-1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000
1.000	-1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000
-1.000	1.000	1.000	-1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000
-1.000	1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000
-1.000	1.000	-1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000
-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000
-1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000
1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000
-1.000	1.000	-1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000
1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000
1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000	1.000
-1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	-1.000	1.000	-1.000
-1.000	1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	1.000	1.000
1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000	1.000
1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000
1.000	1.000	1.000	1.000	1.000	1.000	-1.000	-1.000	-1.000	1.000
1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000	-1.000
-1.000	1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	1.000	1.000
1.000	-1.000	1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	1.000
1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	1.000	-1.000
1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000
-1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000
1.000	1.000	-1.000	1.000	1.000	-1.000	1.000	1.000	1.000	-1.000
1.000	1.000	1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	1.000
-1.000	-1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000
1.000	-1.000	1.000	-1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000
1.000	1.000	-1.000	-1.000	1.000	1.000	1.000	-1.000	1.000	-1.000
-1.000	1.000	1.000	1.000	-1.000	1.000	1.000	1.000	1.000	1.000
-1.000	1.000	-1.000	-1.000	1.000	1.000	-1.000	-1.000	-1.000	-1.000
1.000	-1.000	-1.000	-1.000	1.000	-1.000	-1.000	1.000	-1.000	-1.000
1.000	1.000	-1.000	-1.000	1.000	-1.000	1.000	1.000	-1.000	1.000
-1.000	1.000	1.000	-1.000	1.000	1.000	-1.000	-1.000	1.000	-1.000
1.000	-1.000	-1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000
-1.000	1.000	1.000	1.000	1.000	1.000	1.000	-1.000	1.000	-1.000

OUTPUT

-0.591	2.109	2.933	6.655	9.427	11.877	8.119	5.896	3.148	2.145
0.014	-1.142	0.437	4.069	6.467	3.831	1.649	3.181	4.495	4.825
-0.384	-3.916	-6.746	-4.893	0.663	6.411	9.666	14.461	15.276	8.351
0.548	-6.578	-6.211	-2.502	1.522	6.178	10.815	13.304	12.636	10.359
6.153	0.314	-5.367	-9.006	-7.914	-4.798	-2.875	0.887	1.794	4.832
8.214	11.622	12.043	11.199	8.784	3.446	-0.759	-5.904	-10.882	-15.820
-16.555	-14.764	-6.899	-1.185	3.359	5.547	5.730	6.786	4.197	2.871
3.795	2.806	2.881	2.110	1.206	2.589	4.901	2.716	-0.371	-3.988
-6.099	-5.142	-5.255	-2.084	-0.006	-1.598	-2.074	-0.603	-0.688	3.355
3.270	1.987	-0.474	0.187	2.148	1.660	2.169	4.964	3.282	3.961
6.322	4.971	4.366	5.703	6.781	8.153	8.629	3.457	-0.767	-0.788
-0.135	-0.991	-0.533	-2.035	-5.048	-5.969	-8.808	-8.656	-8.676	-5.809
-5.320	-4.840	-0.539	2.179	3.370	4.201	2.905	-1.264	-4.304	-2.866
-2.036	-1.947	-1.504	-0.335	1.546	-1.207	-1.337	-0.837	1.877	4.092
6.194	4.833	1.244	-0.224	-0.874	1.735	3.182	3.926	4.748	8.027
10.939	11.409	9.804	6.272	2.972	-2.204	-7.127	-5.699	-3.353	-2.387
-3.580	-6.709	-5.307	-4.876	-4.339	-4.259	-4.027	-3.918	-5.302	-4.427
-1.469	-4.771	-4.831	-0.238	2.954	2.132	0.293	-1.446	0.555	1.055
1.459	2.074	0.250	2.075	2.200	3.786	3.397	5.490	5.243	0.753
-1.101	-0.896	-0.694	-3.287	-4.150	-5.658	-7.066	-5.089	-4.017	-1.977
0.062	0.958	-0.427	-0.650	0.076	1.455	6.647	9.283	12.115	12.672
9.611	5.275	-1.345	-6.688	-11.414	-14.405	-13.587	-10.550	-6.824	-2.672
-2.019	0.812	3.325	8.934	8.353	7.203	5.300	1.153	-3.636	-10.762
-13.003	-15.585	-16.715	-13.585	-5.898	1.187	8.051	8.062	5.371	-0.077
-7.253	-10.791	-12.017	-7.903	-1.179	2.896	3.008	2.317	-0.963	0.835
1.413	0.053	-2.824	-3.235	-4.228	-5.966	-6.936	-5.790	-3.009	-1.348
0.245	-2.378	-7.540	-7.011	-4.136	1.748	6.544	12.042	12.056	10.857
7.221	2.410	1.466	0.730	-0.346	2.183	5.309	4.774	2.095	-1.083
-0.785	0.901	3.495	2.861	-2.210	-5.823	-3.620	1.051	7.594	10.315
12.862	13.021	8.085	1.279	-4.653	-4.152	-2.211	3.782	6.834	12.552

12.153	10.218	7.926	3.558	-0.310	-5.333	-7.817	-5.971	-3.599	-2.828
1.347	0.309	3.679	6.499	9.711	13.392	10.504	10.051	5.920	0.390
-5.054	-11.101	-15.025	-15.596	-14.187	-7.500	-1.678	2.808	5.401	5.574
5.716	4.500	4.958	6.165	5.749	4.050	1.849	0.111	1.545	1.410
1.055	-2.447	-2.758	-3.991	-4.549	-4.827	-4.527	-2.887	-2.750	-2.995
0.610	0.693	2.294	2.433	0.671	-0.130	-1.608	0.646	-0.031	2.058
4.165	4.092	6.253	7.969	7.637	5.176	5.160	3.306	6.524	5.969
4.190	2.070	1.714	2.532	2.376	0.732	-1.979	-5.701	-10.504	-12.161
-13.301	-8.902	-6.932	-6.983	-4.266	-1.486	3.692	5.983	5.643	2.194
-3.406	-6.326	-6.070	-3.715	-2.294	-2.382	1.851	3.057	2.896	1.079
1.774	3.621	4.176	4.321	4.067	-0.247	-2.381	-2.543	0.665	1.721
4.307	6.940	8.799	9.834	11.927	11.304	8.689	4.632	-0.782	-5.599
-4.977	-4.808	-5.149	-7.768	-7.689	-6.383	-6.083	-5.043	-3.713	-2.912
-2.920	-3.949	-3.445	-1.945	-3.379	-5.155	-1.988	0.046	0.227	-0.635
-0.999	-1.495	0.273	2.032	2.921	3.776	0.919	3.585	2.765	2.729
4.403	4.460	2.581	-1.287	-3.693	-3.805	-3.605	-4.086	-4.408	-5.825
-5.650	-3.288	-1.485	0.352	2.133	-0.135	0.051	-0.252	3.659	5.163
9.268	12.104	11.629	10.520	4.643	-3.049	-8.948	-11.709	-14.770	-15.684
-9.574	-4.165	0.914	1.688	1.364	3.152	3.514	4.841	6.666	5.131
0.508	-3.998	-8.099	-12.236	-13.826	-14.336	-12.543	-5.888	3.304	8.397
6.985	3.054	-2.790	-7.444	-11.969	-12.585	-8.418	-2.076	3.672	4.235
1.536	0.728	0.406	1.014	-2.271	-3.794	-5.039	-3.255	-4.393	-4.532
-4.620	-2.048	-1.598	-1.373	-1.018	-5.522	-7.989	-4.353	3.288	7.806
9.804	9.133	7.033	4.225	1.158	0.522	0.106	-1.262	0.864	4.462
7.342	3.004	2.164	1.572	2.274	3.738	2.687	-1.838	-4.969	-1.682
-1.038	4.525	10.021	13.446	12.639	6.801	-0.906	-4.544	-5.484	-1.220
2.235	5.549	9.692	13.251	13.505	11.056	6.001	-0.943	-6.797	-9.453
-8.075	-4.943	-2.832	-0.221	1.957	4.629	8.341	9.697	10.007	9.296
8.669	5.406	0.270	-6.255	-10.251	-14.185	-15.393	-13.521	-7.304	-1.850
0.989	4.794	4.968	5.644	4.692	5.236	6.711	5.407	3.526	0.551
0.274	2.573	2.510	0.605	-1.806	-1.203	-4.794	-3.791	-2.953	-1.976
-1.924	-2.769	-3.356	-1.684	-1.273	-0.152	2.303	1.231	0.164	3.006
3.474	4.488	5.486	4.923	3.301	4.060	4.395	6.200	6.911	5.505
4.654	7.923	8.505	4.609	1.238	0.064	-0.988	-1.442	-0.892	-1.671
-6.449	-10.058	-12.437	-11.717	-8.492	-6.501	-4.671	-4.384	0.251	4.310

4.568	4.633	1.680	-3.629	-8.124	-5.165	-4.381	-4.343	-1.182	1.511
3.855	3.679	0.537	0.970	2.732	2.050	2.486	2.782	1.327	-1.884
-1.166	0.695	1.218	2.667	3.870	8.414	11.489	12.380	9.407	6.950
3.798	-1.795	-4.823	-2.561	-1.087	-2.725	-3.977	-7.618	-8.864	-5.623
-6.643	-8.281	-6.623	-2.054	-2.194	-0.911	-3.171	-3.794	-3.476	-2.547
1.606	-0.953	-1.223	-1.899	-1.590	1.576	1.422	0.929	1.703	3.010
2.745	3.194	2.751	4.028	3.961	2.272	-0.453	-2.645	-4.295	-2.926
-3.983	-7.374	-7.492	-6.987	-3.637	-0.298	1.992	2.884	1.795	-1.220
1.362	3.403	7.649	10.561	12.272	11.158	6.205	3.934	-1.675	-6.343
-10.402	-13.408	-13.371	-8.274	-3.884	-0.416	1.094	4.102	4.623	5.925
7.438	4.711	1.262	0.492	-5.791	-9.818	-12.891	-13.842	-13.255	-11.736
-5.917	2.476	6.764	7.612	4.826	-1.181	-5.217	-11.139	-12.147	-7.473
-0.117	4.637	4.596	2.099	-0.008	-0.475	0.495	-1.978	-4.689	-4.889
-5.274	-5.536	-5.051	-3.200	-3.113	0.622	0.408	-3.055	-6.226	-7.942
-6.817	-0.473	6.822	7.966	9.458	8.737	8.621	2.604	2.718	1.178
-0.441	-0.101	5.317	6.786	2.460	-0.196	1.731	1.205	2.175	-0.321
-3.286	-5.784	-3.691	0.045	4.524	9.559	13.937	12.750	6.507	-1.005
-4.213	-4.745	-1.534	1.369	6.097	10.067	12.687	9.789	7.488	6.014
-0.101	-6.917	-9.213	-7.931	-6.841	-3.447	-0.506	-0.207	1.290	6.935
10.855	12.864	10.994	7.871	4.219	0.057	-5.689	-11.000	-14.802	-15.491
-11.360	-5.815	-2.120	2.694	4.868	4.035	4.650	3.915	4.472	4.455
6.250	3.572	0.778	0.087	2.109	3.650	-0.065	-0.948	-1.999	-4.163
-4.230	-4.400	-3.996	-1.434	-2.474	-0.406	-0.520	-0.569	-0.048	2.077
1.273	-0.924	-1.029	0.256	0.727	3.964	2.471	4.035	4.389	6.282
5.093	3.779	3.652	5.081	10.531	8.528	6.174	1.217	0.572	0.684
-1.383	-1.562	-2.098	-3.897	-6.087	-9.675	-10.516	-9.084	-7.372	-5.636
-6.751	-0.906	2.850	6.587	5.513	1.909	-1.767	-3.294	-4.263	-3.451
-2.820	-0.949	1.760	3.021	0.044	-0.011	2.000	3.352	4.583	5.671
2.486	-1.710	-3.124	-3.049	-0.969	1.297	2.107	4.534	6.688	10.322
9.531	9.215	6.269	2.345	-0.823	-3.858	-3.863	-0.741	-0.139	-4.158
-7.655	-9.463	-6.341	-5.736	-8.277	-5.870	-6.270	-3.996	-2.273	-2.107
-0.731	0.111	1.364	2.285	1.201	-0.776	-2.367	-2.161	-0.880	1.753
3.348	4.082	3.937	5.542	4.893	3.703	3.897	3.172	0.029	-1.897
-0.313	-0.896	-1.100	-3.025	-4.068	-5.734	-4.648	-4.449	-1.819	0.705
0.282	-0.703	-0.371	1.957	2.713	5.698	10.916	12.542	12.508	8.678

```
IDENTIFICATION WITH MODEL OF ORDER   1


LEAST SQUARES ESTIMATION

LOSS FUNCTION     1.7214598234+004

LAMBDA=  5.8676398+000

GRADIENT OF V
3.0316854+004 -9.1968473+002  0.0000000+000

NEW ESTIMATION OF THE COEFFICIENTS
-8.8352715-001  9.5773849-001  0.0000000+000

STANDARD DEVIATIONS OF THE COEFFICIENTS
3.1656119-002  1.8568950-001  5.8676398+000


ITERATION NUMBER   1

LOSS FUNCTION     3.3813078636+003

LAMBDA=  2.6005030+000

GRADIENT OF V
-1.4463440-006  5.9098238-007 -3.1596729+003

NEW ESTIMATION OF THE COEFFICIENTS
-7.6801235-001  9.3321237-001  5.8430925-001

STANDARD DEVIATIONS OF THE COEFFICIENTS
1.5675217-002  8.2309867-002  3.5363698-002


ITERATION NUMBER   2

LOSS FUNCTION     2.9026656934+003

NEW ESTIMATION OF THE COEFFICIENTS
-8.5476443-001  3.1120537-001  4.5018800-001


ITERATION NUMBER   3

LOSS FUNCTION     2.5164756983+003

NEW ESTIMATION OF THE COEFFICIENTS
-8.1485299-001  3.4767152-001  6.0123488-001


TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER   4

LOSS FUNCTION     2.5153801061+003

NEW ESTIMATION OF THE COEFFICIENTS
-8.2612978-001  2.6399488-001  5.5084899-001
```

```
ITERATION NUMBER  5

   LOSS FUNCTION    2.4826559687+003

   NEW ESTIMATION OF THE COEFFICIENTS
-8.2684398-001  2.6101568-001  5.4119211-001


   EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

   LOSS FUNCTION    2.4821063987+003

   LAMBDA=  2.2280513+000

   GRADIENT OF V
-3.8019317-001  2.0590625-001  2.1006605+000

   INVERSE OF SECOND DERIVATIVE MATRIX
 6.8743602-005 -3.0296526-005  2.4524584-005
-3.0296526-005  8.1246981-004 -1.1286519-004
 2.4524584-005 -1.1286519-004  1.1603850-004

   NEW ESTIMATION OF THE COEFFICIENTS
-8.2686313-001  2.6107396-001  5.4098092-001

   STANDARD DEVIATIONS OF THE COEFFICIENTS
 1.8473166-002  6.3508055-002  2.4000829-002

   LOSS FUNCTION    2.4821061866+003

   LAMBDA=  2.2280513+000

   GRADIENT OF V
-1.8432969-004  3.6083045-005  8.6209225-004

   INVERSE OF SECOND DERIVATIVE MATRIX
 6.8725723-005 -3.0301985-005  2.4528904-005
-3.0301985-005  8.1278370-004 -1.1295639-004
 2.4528904-005 -1.1295639-004  1.1613533-004

   NEW ESTIMATION OF THE COEFFICIENTS
-8.2686313-001  2.6107402-001  5.4098082-001

   STANDARD DEVIATIONS OF THE COEFFICIENTS
 1.8470763-002  6.3520319-002  2.4010840-002

   MAX COEFF CORRECTION IS LESS THAN  1.0000000-006


   IER=    0
```

```
IDENTIFICATION WITH MODEL OF ORDER  2


LEAST SQUARES ESTIMATION

LOSS FUNCTION    1.7214598234+004

LAMBDA=  5.8676398+000

GRADIENT OF V
3.0316854+004  2.1279435+004 -9.1968473+002 -2.4266965+003  0.0000000+000  0.0000000+000

NEW ESTIMATION OF THE COEFFICIENTS
-1.3516873+000  5.6800730-001  9.5468379-001  1.1809049+000  0.0000000+000  0.0000000+000

STANDARD DEVIATIONS OF THE COEFFICIENTS
7.1417395-002  7.0708235-002  1.8573339-001  1.9808659-001  5.8676398+000  5.8676398+000


ITERATION NUMBER  1

LOSS FUNCTION    8.9672978562+002

LAMBDA=  1.3392011+000

GRADIENT OF V
-1.1151191-005 -5.3106342-006  6.6802022-007  3.8597500-007  8.1693328+002 -9.4557894+001

NEW ESTIMATION OF THE COEFFICIENTS
-1.4637335+000  6.5778095-001  9.7160232-001  1.0857645+000 -4.2050650-001 -1.7596087-001

C-POLYNOMIAL WAS NOT STABLE UNTIL THE STEP HAS BEEN HALVED    1 TIMES

STANDARD DEVIATIONS OF THE COEFFICIENTS
2.0120651-002  1.8693123-002  4.2444481-002  4.6310780-002  4.4228042-002  3.8993664-002
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ITERATION NUMBER  2

LOSS FUNCTION    6.4191219239+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4977030+000  6.9615451-001  9.5850761-001  1.0348963+000 -6.9463758-001 -1.4903209-001

ITERATION NUMBER  3

LOSS FUNCTION    5.4423678609+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4896602+000  6.9426403-001  9.6201631-001  1.0652546+000 -9.1963374-001  1.2909817-001

ITERATION NUMBER  4

LOSS FUNCTION    4.8800541875+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4967036+000  7.0072844-001  9.5539561-001  1.0638826+000 -9.9208931-001  1.9952454-001

ITERATION NUMBER  5

LOSS FUNCTION    4.8490675578+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4962792+000  7.0045165-001  9.5402613-001  1.0673670+000 -9.9885036-001  2.0722740-001

TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER  6

LOSS FUNCTION    4.8486785444+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4962949+000  7.0047128-001  9.5386922-001  1.0675442+000 -9.9982653-001  2.0804804-001
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ITERATION NUMBER  7

LOSS FUNCTION    4.8486741442+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.4962948+000  7.0047125-001  9.5386917-001  1.0675443+000 -9.9982596-001  2.0804771-001


EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

LOSS FUNCTION    4.8486741435+002

LAMBDA=  9.8475115-001

GRADIENT OF V
 5.6284480-006  5.0599920-006 -3.3821561-007 -4.1397288-007 -6.2653271-008 -3.8548023-008

INVERSE OF SECOND DERIVATIVE MATRIX
 4.8150294-005 -3.9422238-005 -1.9038993-005  1.4904450-004  5.2609204-005 -3.7326676-005
-3.9422238-005  3.5933666-005  9.5286327-006 -9.8230365-005 -4.4313710-005  3.6605896-005
-1.9038993-005  9.5286327-006  9.9045402-004 -8.9203950-004  1.2062580-005  2.8802150-005
 1.4904450-004 -9.8230364-005 -8.9203950-004  1.5832043-003  1.0393999-004 -3.0242908-005
 5.2609204-005 -4.4313710-005  1.2062580-005  1.0393999-004  1.1448668-003 -9.7413847-004
-3.7326676-005  3.6605896-005  2.8802150-005 -3.0242908-005 -9.7413847-004  1.1246135-003

NEW ESTIMATION OF THE COEFFICIENTS
-1.4962948+000  7.0047125-001  9.5386917-001  1.0675443+000 -9.9982596-001  2.0804771-001

STANDARD DEVIATIONS OF THE COEFFICIENTS
 6.8332289-003  5.9030609-003  3.0991576-002  3.9182756-002  3.3319922-002  3.3023883-002

MAX COEFF CORRECTION IS LESS THAN  1.00000000-006


IER=    0


TEST QUANTITY=  1364.80976
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IDENTIFICATION WITH MODEL OF ORDER  3


LEAST SQUARES ESTIMATION

LOSS FUNCTION     1.7214598234+004

LAMBDA=  5.8676398+000

GRADIENT OF V
3.0316854+004  2.1279435+004  1.0541295+004 -9.1968473+002 -2.4266965+003 -3.0053129+003  0.0000000+000  0.0000000+000
0.0000000+000

NEW ESTIMATION OF THE COEFFICIENTS
-8.9485222-001 -1.2903287-001  3.5547169-001  9.9043471-001  1.6168773+000  7.2579111-001  0.0000000+000  0.0000000+000
0.0000000+000

STANDARD DEVIATIONS OF THE COEFFICIENTS
1.3875526-001  2.0079963-001  1.0591726-001  1.8596799-001  2.2836971-001  2.5710870-001  5.8676398+000  5.8676398+000
5.8676398+000


ITERATION NUMBER  1

LOSS FUNCTION     6.4284519915+002

LAMBDA=  1.1338829+000

GRADIENT OF V
5.0132629-005  5.5208919-005  4.4919667-005  7.3528645-007  3.6826532-007 -1.5100886-006  2.6056824+002  4.0332484+002
-9.5608132+001

NEW ESTIMATION OF THE COEFFICIENTS
-1.1116511+000  1.4186494-001  2.4710156-001  9.7903338-001  1.4062533+000  4.5441017-001 -3.8647720-001 -1.4100065-001
-1.2798836-001

C-POLYNOMIAL WAS NOT STABLE UNTIL THE STEP HAS BEEN HALVED    1 TIMES

STANDARD DEVIATIONS OF THE COEFFICIENTS
1.4252358-001  2.0840930-001  9.7171929-002  3.6016824-002  1.4653793-001  1.5659346-001  1.4614263-001  8.0826365-002
5.6804383-002


ITERATION NUMBER  2

LOSS FUNCTION     5.3112944459+002

NEW ESTIMATION OF THE COEFFICIENTS
-7.7097667-001 -3.8162534-001  5.0537014-001  9.5885417-001  1.7435836+000  7.6850389-001 -2.4291387-001 -5.0884470-001
9.8531467-002


ITERATION NUMBER  3

LOSS FUNCTION     4.8746017020+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.1388296+000  1.6656704-001  2.4976587-001  9.5678188-001  1.3997391+000  3.9471011-001 -6.3172339-001 -1.8144177-001
1.0475267-001
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    ITERATION NUMBER  4

    LOSS FUNCTION     4.8361365583+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.8527438+000  1.2361942+000 -2.5102189-001  9.5479442-001  7.1175998-001 -3.4963126-001 -1.3442309+000  5.0794413-001
-1.7057626-002

    ITERATION NUMBER  5

    LOSS FUNCTION     4.8321464352+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4111735+000  5.7581689-001  5.7942059-002  9.5582213-001  1.1317708+000  1.2200234-001 -9.0286593-001  7.2039320-002
 6.8964453-002

    ITERATION NUMBER  6

    LOSS FUNCTION     4.8315775656+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.5056812+000  7.1725017-001 -8.2794311-003  9.5587321-001  1.0415187+000  2.1800434-002 -9.9733788-001  1.0652966-001
 4.8868362-002

    ITERATION NUMBER  7

    LOSS FUNCTION     4.8315724555+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4095905+000  5.7341679-001  5.9070121-002  9.5580215-001  1.1332844+000  1.2412167-001 -9.0127863-001  7.0796219-002
 6.8468119-002

    ITERATION NUMBER  8

    LOSS FUNCTION     4.8314993583+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4592064+000  6.4768920-001  2.4290874-002  9.5586164-001  1.0858088+000  7.1351536-002 -9.5089814-001  1.2046365-001
 5.8298646-002

    TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER  9

    LOSS FUNCTION     4.8314993382+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4583622+000  6.4642587-001  2.4882337-002  9.5586103-001  1.0866747+000  7.2251202-002 -9.5005416-001  1.1952024-001
 5.8470974-002

    ITERATION NUMBER 10

    LOSS FUNCTION     4.8314993154+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4583624+000  5.4642610-001  2.4882232-002  9.5586103-001  1.0866746+000  7.2250987-002 -9.5005433-001  1.1952044-001
 5.8470897-002
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    EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

    LOSS FUNCTION    4.8314993154+002

    LAMBDA=  9.8300553-001

    GRADIENT OF V
-5.9301965-007 -2.6565976-007 -2.9453076-008  8.1476173-008  1.0902295-007  7.7649020-008  3.0039058-008  4.4012205-008
 2.4479959-008

    INVERSE OF SECOND DERIVATIVE MATRIX
 1.5449526-001 -2.3109722-001  1.0815823-001 -2.0442239-004  1.4783140-001  1.6487087-001  1.5359683-001 -1.5279453-001
 3.1414759-002
-2.3109722-001  3.4572952-001 -1.6182596-001  3.0321078-004 -2.2114483-001 -2.4646238-001 -2.2975098-001  2.2860993-001
-4.7031812-002
 1.0815823-001 -1.6182596-001  7.5755914-002 -1.4303627-004  1.0350084-001  1.1531799-001  1.0752730-001 -1.0701397-001
 2.2030739-002
-2.0442243-004  3.0321084-004 -1.4303630-004  1.0144043-003 -1.1393960-003 -7.6334291-005 -1.9149951-004  2.8398107-004
-7.8506331-005
 1.4783140-001 -2.2114483-001  1.0350084-001 -1.1393959-003  1.4333920-001  1.5073605-001  1.4696263-001 -1.4628829-001
 3.0154947-002
 1.6487087-001 -2.4646238-001  1.1531799-001 -7.6334246-005  1.5073605-001  1.7757214-001  1.6391569-001 -1.6287538-001
 3.3421706-002
 1.5359683-001 -2.2975098-001  1.0752730-001 -1.9149947-004  1.4696263-001  1.6391569-001  1.5373291-001 -1.5288959-001
 3.1412115-002
-1.5279453-001  2.2860993-001 -1.0701397-001  2.8398163-004 -1.4628829-001 -1.6287538-001 -1.5288959-001  1.5310520-001
-3.2068743-002
 3.1414759-002 -4.7031812-002  2.2030739-002 -7.8506323-005  3.0154947-002  3.3421706-002  3.1412115-002 -3.2068743-002
 7.3946949-003

    NEW ESTIMATION OF THE COEFFICIENTS
-1.4583623+000  6.4642609-001  2.4882236-002  9.5586103-001  1.0866746+000  7.2250993-002 -9.5005432-001  1.1962043-001
 5.8470898-002

    STANDARD DEVIATIONS OF THE COEFFICIENTS
 3.8637905-001  5.7799515-001  2.7056040-001  3.1308445-002  3.7216750-001  4.1423174-001  3.8542456-001  3.8464073-001
 8.4531016-002

    MAX COEFF CORRECTION IS LESS THAN  1.00000000-006

    IER=    0

    TEST QUANTITY=     1.17426
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```
      PROGRAM TESTEX2
C
C     TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM TEST EXAMPLE 2
C     REFERENCE  GUSTAVSSON I. PARAMETRIC IDENTIFICATION OF MULTIPLE INPUT
C     SINGLE OUTPUT DYNAMIC SYSTEMS
C     AUTHOR,IVAR GUSTAVSSON 20/2 1969
C
C     SUBROUTINE REQUIRED
C              MISOID
C              PROIDE
C              VV1V2
C              GJRV
C              NSTABLE
C
      COMMON EE,V,Y,U(5),E(10),C(70),EC(70),V1(70),VCC(140),ECC(140),
     FV2(70,70),DAT(6000)
      DIMENSION UA1(1000),UA2(1000),YA(1000)
C
      READ 100,NI,NP
  100 FORMAT(2I5)
      NT=NP*(NI+1)
      READ 101,(DAT(I),I=1,NT)
  101 FORMAT(10F8.3)
      PRINT 102
  102 FORMAT(/5X*TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM
     FTEST EXAMPLE 2*/)
      DO 1 I=1,NP
      I1=3*I-2
      I2=3*I-1
      I3=3*I
      UA1(I)=DAT(I1)
      UA2(I)=DAT(I2)
    1 YA(I)=DAT(I3)
      PRINT 103
  103 FORMAT(/5X*INPUT 1*/)
      PRINT 107,(UA1(I),I=1,NP)
      PRINT 108
  108 FORMAT(/5X*INPUT 2*/)
      PRINT 107,(UA2(I),I=1,NP)
      PRINT 104
  104 FORMAT(/5X*OUTPUT*/)
      PRINT 107,(YA(I),I=1,NP)
  107 FORMAT(10F10.3)
C
      DO 2 I=1,3
      CALL MISOID(I,NI,NP,IER)
      PRINT 105,IER
  105 FORMAT(//5X*IER=*I5)
      IF(IER-1)  6,4,6
    4 CALL RESTART(I,NI,NP,10,1,IER)
    6 IF(I-1)  3,5,3
C     TEST QUANTITY COMPUTED (F-TEST)
    3 TQ=(VOLD-V)*(NP-(NI+2)*I)/(V*(NI+2))
      PRINT 106,TQ
  106 FORMAT(//5X*TEST QUANTITY=*,F12.5)
    5 VOLD=V
    2 CONTINUE
C
      CALL EXIT
      END
```

TEST OF IDENTIFICATION PROGRAM PACKAGE USING DATA FROM TEST EXAMPLE 2

INPUT 1

-0.486	0.757	-0.085	2.131	-0.271	0.216	0.912	-1.456	-0.835	-1.040
-0.860	-0.481	-0.660	0.161	-0.466	1.471	0.380	1.703	-0.421	-0.139
1.891	-0.346	-1.162	-0.412	-0.503	1.250	-0.183	-0.598	1.287	-0.215
0.170	-0.305	0.110	-0.279	-1.236	0.773	-0.450	0.237	-0.860	-0.610
0.074	0.424	-0.878	0.871	0.761	0.150	1.209	0.926	1.901	1.796
2.182	-1.355	-0.202	-0.143	0.902	-0.912	-1.103	-1.731	-0.830	0.301
-1.208	-0.853	-0.001	-0.625	-0.580	0.189	-1.911	0.243	-0.666	-0.132
-1.126	-0.180	1.758	-0.828	0.340	0.996	0.740	-0.138	-0.046	-1.222
-0.671	0.151	0.721	0.211	0.054	-1.838	-1.801	0.127	1.461	1.456
-2.183	1.063	-0.159	0.186	-0.713	0.274	-1.242	0.007	0.933	-0.745
0.391	-0.444	-1.174	1.147	-1.398	0.847	1.508	0.341	-0.277	0.608
-0.321	-0.197	0.904	1.545	-0.029	0.677	-1.223	-0.656	-0.164	-0.312
-1.026	-0.002	1.402	1.269	-0.316	0.751	0.211	1.104	0.302	-0.376
-0.052	1.718	-0.295	-0.253	-0.555	0.352	0.578	0.699	-1.256	0.220
0.754	-1.495	-0.901	0.118	0.146	2.094	-0.487	-0.379	-0.990	-0.479
0.197	-0.052	-0.322	-0.460	-0.023	0.498	-0.366	0.421	0.621	-0.285
1.034	-0.515	-0.088	0.287	1.186	1.754	-1.422	-0.385	1.204	-1.691
0.287	0.211	-0.049	0.927	2.126	0.169	-1.633	-0.354	-1.072	0.956
-0.531	1.737	0.595	-0.376	-1.177	0.526	-0.813	0.064	-0.728	-0.355
-0.769	0.018	-0.330	0.902	-1.327	0.602	0.950	0.827	0.136	-0.618
0.927	0.186	-1.210	0.763	1.224	-2.071	-0.322	1.268	0.343	0.711
1.261	0.797	0.599	-0.489	-1.121	0.893	0.320	-0.059	-1.390	0.414
-0.091	-0.267	-0.049	0.727	-1.613	-0.006	-1.590	0.513	-0.555	0.876
-0.631	0.099	-0.671	0.714	0.761	-1.143	0.689	-2.249	0.863	-0.468
1.325	-1.194	0.028	-0.409	1.116	1.082	-0.176	0.738	-0.582	-0.038
0.427	1.127	-0.605	1.999	-1.102	0.556	-1.502	0.669	2.167	0.467
0.581	1.483	0.189	-0.384	-0.548	-0.527	0.586	0.135	0.524	-0.025
0.065	-1.127	0.028	1.307	0.875	2.115	-0.266	-0.030	-0.453	-0.395
-1.703	-1.563	2.485	1.784	-0.091	-1.771	-0.018	-0.048	0.450	-0.090
0.864	-1.689	0.473	0.496	1.309	0.976	0.012	2.107	-0.424	-0.151

-1.421	-1.883	2.061	-1.997	-1.371	0.865	-1.816	-0.712	-1.368	-0.781
-1.505	1.405	0.866	1.011	0.035	1.100	-0.702	0.128	-0.679	1.093
-0.507	-0.103	1.010	0.555	-0.431	0.959	0.579	-0.570	-1.142	-0.465
-0.089	-0.689	-1.236	-0.446	-1.363	-0.898	-1.558	1.392	-0.670	-1.514
2.585	1.874	-0.287	0.522	0.646	-0.532	-0.415	0.386	0.607	1.214
1.690	0.963	1.269	1.942	-0.412	0.716	-0.818	-1.011	-0.690	-0.245
-0.967	-0.002	0.304	0.700	0.659	0.401	1.498	-0.847	0.778	-0.130
1.297	1.022	-0.083	0.587	0.183	-1.731	1.545	0.317	1.583	-0.775
-1.780	1.934	-0.258	-0.859	-0.702	-2.170	1.374	0.974	-0.243	1.094
-0.319	-1.067	-2.479	-1.252	-0.529	-0.735	1.248	0.239	-1.034	-0.533
0.023	-0.658	-0.667	0.412	0.600	-1.598	0.351	-1.443	-1.185	-2.056
2.144	0.660	0.379	-0.730	-0.015	0.404	-0.778	1.865	0.752	-1.690
0.876	-0.939	-1.325	-2.589	1.696	-0.307	-0.410	-0.216	-0.757	0.381
0.149	-1.240	0.954	0.257	-0.051	0.570	-0.982	0.097	-0.202	0.099
1.563	-0.278	0.849	-0.058	0.602	1.463	-0.597	-0.863	0.620	0.637
0.768	-0.765	0.387	1.131	0.203	0.832	0.746	-0.183	1.972	0.972
-0.577	0.407	-0.571	-0.791	1.417	0.462	0.992	-0.659	-0.917	0.055
-0.163	0.746	1.574	-0.830	0.743	-1.617	-0.665	0.300	0.174	0.116
-0.389	0.655	1.001	-1.869	0.468	-0.044	-0.486	-0.752	-0.552	-0.433
-0.728	-0.191	1.367	-0.968	-1.879	0.396	-0.737	-1.328	1.177	0.638

INPUT 2

-0.591	1.404	-1.121	0.329	0.052	2.380	-0.727	0.827	0.960	0.346
-0.847	-0.577	-0.248	0.482	1.200	0.083	0.273	1.644	0.468	0.449
-2.120	-0.173	-0.889	-0.370	1.089	1.153	-0.550	1.669	2.129	-0.644
-0.356	-1.781	0.330	0.896	-0.243	-0.278	0.383	-0.155	0.019	0.768
0.223	0.406	0.831	0.014	-0.315	0.451	-0.704	1.047	-0.361	0.192
0.485	1.131	-0.607	0.437	0.974	-1.004	1.022	0.869	0.107	-1.696
-0.160	-0.826	0.864	-0.142	-0.007	-0.300	-0.537	1.596	-0.267	-1.261
-0.781	-1.406	0.079	0.113	0.154	0.390	2.220	1.319	-0.139	-1.933
-0.282	1.319	-0.436	1.500	1.028	-0.319	-0.206	1.247	0.053	1.769
-0.486	0.591	-0.478	-0.307	1.325	-0.045	-1.127	1.754	-0.666	-0.504
0.307	-1.332	-0.057	0.842	0.136	-0.059	1.061	-0.708	-0.832	0.092
0.769	-0.590	0.115	0.304	-0.087	2.030	0.660	0.631	-1.358	1.661
1.253	-0.005	0.741	0.342	-0.082	0.520	1.500	0.714	0.040	0.603
-0.155	0.825	0.847	-0.759	0.066	-1.542	0.001	-1.368	-1.171	-0.207
1.397	-0.155	-0.104	1.220	-0.427	0.219	0.272	0.596	-0.372	1.162

1.458	-0.155	-0.099	-0.515	-0.069	-0.237	-1.964	-0.468	0.131	0.878
0.505	-0.680	1.469	-0.006	0.960	1.797	0.929	-0.289	-0.718	0.123
1.726	-1.964	-1.013	1.049	1.181	0.506	-0.568	-1.061	1.981	-0.596
-0.728	0.015	-1.678	1.470	-0.932	0.713	0.158	1.059	0.413	-1.066
0.290	1.786	1.608	-1.623	0.350	0.941	-0.613	0.748	-0.459	-0.122
0.184	-0.309	-0.167	0.556	-0.659	-1.883	0.765	-0.526	0.164	1.267
0.317	-0.207	-0.800	0.264	0.100	0.081	0.093	-0.177	-0.705	-0.491
-1.138	0.931	-0.148	2.180	-0.510	-0.019	0.426	0.674	0.933	-1.702
0.706	-0.568	-1.148	-0.457	0.552	-0.832	1.200	0.182	0.856	0.329
-1.220	0.749	0.085	0.505	0.751	-0.218	-0.529	1.347	-0.881	0.751
0.415	0.782	0.785	1.667	0.157	-0.065	-1.042	-0.592	0.438	-0.331
1.742	0.119	-2.031	0.580	0.088	1.017	0.026	1.272	-0.159	0.790
0.195	-0.784	0.083	0.457	0.026	1.147	0.935	-0.956	-0.493	-1.186
-0.780	-1.224	0.527	1.021	-1.140	-1.849	-0.054	0.721	2.215	-0.269
-0.007	0.995	0.553	0.621	-0.402	1.197	0.037	1.990	-0.405	2.146
-1.664	-1.318	0.120	-0.795	1.083	0.864	0.613	0.462	0.225	-1.477
1.548	-1.848	0.001	-0.432	0.105	1.566	-1.241	2.116	0.560	0.682
1.076	-0.308	-0.435	0.798	-0.426	0.278	0.004	0.023	0.038	-0.529
0.599	0.532	0.622	0.394	0.241	-0.095	-0.345	-0.154	0.589	-0.211
1.693	-1.307	0.005	0.700	0.205	-0.731	-1.244	-0.573	0.088	-0.688
2.471	0.290	-0.523	-1.104	-0.371	0.416	-2.454	0.430	-1.204	-0.734
0.564	-0.005	0.913	0.368	0.246	-0.529	0.164	-2.541	0.603	-0.391
1.291	1.333	0.617	1.760	1.414	0.003	0.306	0.085	-1.314	0.273
-1.875	0.606	0.091	-0.845	0.492	-1.314	1.522	1.190	0.138	-0.182
-0.956	-0.601	-1.375	-0.293	1.011	-0.472	1.145	-0.148	1.229	0.134
0.070	-0.241	-0.269	0.371	0.934	-0.464	0.187	-0.866	-0.090	-0.973
1.236	1.114	0.271	-0.458	0.822	1.212	1.129	0.399	0.524	-0.738
0.031	-1.084	-0.510	-1.704	0.757	0.811	-1.231	0.220	2.060	1.143
-0.420	-0.256	0.263	0.772	-0.154	-1.755	-0.346	-0.576	0.260	-0.568
-0.508	-0.835	-0.917	-0.174	0.073	0.925	-1.790	0.875	-0.737	0.179
0.572	0.302	1.161	-0.937	-1.124	-0.104	-0.361	0.318	1.587	-0.549
-0.866	0.355	0.019	0.226	0.788	-0.345	1.244	-1.110	0.714	-1.566
0.376	0.505	0.391	1.839	-1.236	-2.253	-1.130	0.899	-0.345	-2.250
1.432	1.099	1.272	0.454	-0.329	-0.132	-2.325	-0.523	1.807	0.432
-0.452	0.294	0.637	-0.307	0.424	0.323	-0.479	0.348	1.799	1.049

OUTPUT

0.000	0.000	1.907	2.552	3.633	7.870	9.311	6.404	5.019	0.004
-3.614	-10.117	-12.441	-11.985	-8.740	-2.893	1.105	4.883	7.922	12.101
11.698	8.396	7.433	-0.098	-2.788	-4.147	-2.866	-3.182	-2.027	2.773
6.357	4.235	4.170	2.199	1.726	-1.183	-4.103	-4.962	-2.769	-2.252
-3.040	-0.543	3.772	2.433	3.740	2.905	4.484	4.496	7.146	7.375
10.901	12.538	9.969	7.361	2.488	-0.610	-3.150	-4.685	-6.637	-10.466
-11.958	-7.636	-6.727	-5.862	-3.173	-3.700	-2.329	-1.375	-2.800	-3.970
-5.149	-3.301	-2.723	-1.631	2.392	1.911	3.772	6.843	10.168	7.588
1.088	-4.920	-4.571	-3.534	-2.526	-0.472	-1.181	-4.200	-3.204	-0.789
2.235	4.203	3.742	0.222	1.417	2.774	0.614	-2.216	-2.401	-4.287
-6.952	-1.952	-0.325	-0.567	-2.636	-3.132	-0.960	4.796	4.991	4.310

3.126	2.971	2.808	2.769	3.268	2.591	5.003	3.436	0.403	-3.451
-3.382	-3.785	-4.050	-2.505	2.193	3.934	5.244	7.919	8.446	8.000
6.042	5.772	4.557	4.520	5.409	2.395	-2.699	-3.331	-2.325	-1.785
-2.244	-1.166	0.130	-2.282	-2.540	-3.738	-1.289	0.642	0.680	1.179
1.972	2.257	2.374	0.754	-0.503	-2.628	-1.710	-4.067	-2.676	0.752
3.255	3.160	0.101	2.679	3.344	6.974	8.621	8.154	6.089	3.035
-2.306	-0.277	-2.640	-5.027	-1.071	3.159	6.529	5.976	2.352	2.007
-0.038	-2.624	-4.035	-2.115	-0.110	-0.104	0.775	1.979	3.407	-0.342
-4.528	-2.643	-4.284	-1.012	-0.764	-0.091	-0.305	1.938	5.013	5.909
6.259	4.483	2.994	-0.559	-0.176	-1.679	-4.571	-4.653	-6.532	-2.451
1.018	6.447	8.475	11.082	9.942	6.905	4.788	3.214	-1.587	-6.823
-8.389	-7.485	-5.775	-4.545	-1.238	-0.786	-0.214	0.157	-2.415	-1.690
-1.639	-0.168	-0.422	-1.626	-2.651	0.137	-1.454	-2.214	-3.453	-2.147
-1.974	-1.948	-2.497	-2.732	-0.547	-0.831	4.067	4.049	5.725	3.211
2.066	1.059	3.072	6.650	6.810	6.472	3.614	1.003	-1.842	-5.210
0.842	5.159	9.522	7.292	5.351	1.912	1.855	-2.171	-1.267	-0.825
2.471	1.751	1.719	-1.119	-0.309	0.517	5.985	4.785	6.619	4.361
-0.013	-5.480	-9.687	-8.583	-3.492	-0.364	1.749	1.946	0.930	1.833
1.930	0.115	1.485	1.579	2.127	3.499	4.460	4.693	5.081	5.916
4.211	0.375	-4.976	-4.310	-10.266	-9.590	-8.124	-4.900	-6.895	-6.579
-5.735	-7.360	-5.332	-3.769	1.256	6.452	11.808	10.488	7.443	7.359
3.739	0.784	-1.769	-0.826	-0.490	0.792	2.219	1.774	3.131	2.894

-2.838	-3.686	-3.386	-5.667	-3.365	-4.472	-7.875	-8.047	-8.682	-6.471
-1.267	1.478	4.991	9.619	8.663	8.921	4.489	-1.350	-2.427	-4.062
-0.986	1.187	7.534	12.279	11.824	12.054	6.647	4.310	0.532	-4.640
-9.331	-9.027	-9.419	-4.941	-2.948	2.338	4.682	8.038	8.560	5.862
3.303	6.042	5.744	9.063	8.012	8.468	8.833	3.163	-0.262	-0.052
-1.857	-5.601	-4.957	-3.750	-2.712	-1.545	-2.127	-3.235	-0.855	5.697
4.443	2.740	-0.780	-8.377	-11.816	-13.043	-13.662	-9.371	-3.439	1.091
1.702	2.924	2.064	-1.007	0.208	1.309	-0.677	-3.673	-3.721	-5.109
-10.333	-7.495	-6.736	0.591	2.799	4.864	5.239	4.730	5.463	6.629
5.277	2.993	-0.496	-4.900	-11.546	-13.163	-9.013	-7.312	-5.854	1.422
5.615	5.080	5.116	2.046	2.659	2.919	-0.479	0.052	-1.395	-3.158
-4.739	0.797	3.692	3.013	4.001	5.746	7.171	5.016	2.072	-2.163
0.070	4.200	3.231	2.391	3.836	2.441	4.433	4.061	4.424	6.964
8.622	5.973	6.782	2.416	-0.633	-1.949	-1.665	1.589	1.529	-0.985
-0.780	-1.297	-0.505	-0.144	5.290	3.072	2.027	-2.902	-4.180	-3.632
-3.678	-0.121	2.091	3.529	1.942	2.061	0.871	-2.729	-2.523	-4.473
-6.333	-3.859	-7.086	-3.836	-3.668	-3.465	-0.363	0.193	-2.313	-1.732

IDENTIFICATION WITH MODEL OF ORDER 1

LEAST SQUARES ESTIMATION

LOSS FUNCTION 6.0075927839+003

LAMBDA= 4.9020782+000

GRADIENT OF V

1.0280835+004	-6.0946632+002	-3.2372053+002	0.0000000+000

NEW ESTIMATION OF THE COEFFICIENTS

-8.5133550-001	8.1808319-001	7.4323858-001	0.0000000+000

STANDARD DEVIATIONS OF THE COEFFICIENTS

4.4922301-002	2.3307330-001	2.4568370-001	4.9020782+000

ITERATION NUMBER 1

LOSS FUNCTION 1.2617751737+003

LAMBDA= 2.2465753+000

GRADIENT OF V

1.4263205-006	-3.1548552-008	5.2445102-008	-5.7324896+002

NEW ESTIMATION OF THE COEFFICIENTS

-7.8974947-001	7.6202468-001	6.8058948-001	2.9467172-001

STANDARD DEVIATIONS OF THE COEFFICIENTS

2.3176889-002	1.0725387-001	1.1311404-001	5.0935266-002

ITERATION NUMBER 2

LOSS FUNCTION 1.2087788173+003

NEW ESTIMATION OF THE COEFFICIENTS

-8.3239910-001	5.0452737-001	6.4157206-001	2.0613092-001

ITERATION NUMBER 3

LOSS FUNCTION 1.1886629622+003

NEW ESTIMATION OF THE COEFFICIENTS

-8.0499703-001	5.4963189-001	6.3829696-001	3.0207837-001

ITERATION NUMBER 4

LOSS FUNCTION 1.1855628959+003

NEW ESTIMATION OF THE COEFFICIENTS

-8.2166225-001	4.8210590-001	6.2962984-001	2.4949847-001

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     TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER  5

  LOSS FUNCTION     1.1834917318+003

  NEW ESTIMATION OF THE COEFFICIENTS
-8.1506686-001  4.9985323-001  6.3099342-001  2.7184062-001


  ITERATION NUMBER  6

  LOSS FUNCTION     1.1823963213+003

  NEW ESTIMATION OF THE COEFFICIENTS
-8.1491345-001  4.9973087-001  6.3099546-001  2.7167101-001

  EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

  LOSS FUNCTION     1.1823961203+003

  LAMBDA=  2.1747608+000

  GRADIENT OF V
 2.8270349-004  2.7891656-006 -1.9747822-006 -5.6159217-005

  INVERSE OF SECOND DERIVATIVE MATRIX
 1.4357019-004 -6.4807663-005 -7.2968833-005  8.4338108-005
-6.4807663-005  2.5489331-003 -3.8833980-004 -4.4584156-004
-7.2968833-005 -3.8833980-004  2.3346888-003 -1.5715614-004
 8.4338108-005 -4.4584156-004 -1.5715614-004  4.2391134-004

  NEW ESTIMATION OF THE COEFFICIENTS
-8.1491348-001  4.9973086-001  6.3099548-001  2.7167101-001

  STANDARD DEVIATIONS OF THE COEFFICIENTS
 2.6058153-002  1.0979706-001  1.0508143-001  4.4776383-002

  MAX COEFF CORRECTION IS LESS THAN  1.00000000-006


  IER=    0
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IDENTIFICATION WITH MODEL OF ORDER  2

LEAST SQUARES ESTIMATION

LOSS FUNCTION     6.0075927839+003

LAMBDA=  4.9020782+000

GRADIENT OF V
1.0280835+004  7.2183096+003 -6.0946632+002 -1.0342251+003 -3.2372053+002 -5.1510788+002  0.0000000+000  0.0000000+000

NEW ESTIMATION OF THE COEFFICIENTS
-1.0821559+000  3.2558207-001  7.4104023-001  9.2142693-001  7.3203450-001  6.2237085-002  0.0000000+000  0.0000000+000

STANDARD DEVIATIONS OF THE COEFFICIENTS
9.8726749-002  9.5520851-002  2.3433995-001  2.4737674-001  2.4707568-001  2.5746982-001  4.9020782+000  4.9020782+000

ITERATION NUMBER  1

LOSS FUNCTION     7.8311830901+002

LAMBDA=  1.7698794+000

GRADIENT OF V
1.7239363-006 -1.6664139-006  1.8058927-008  4.2804459-008  5.7072612-008  8.8836714-008  4.1996798+002 -8.8811948+001

NEW ESTIMATION OF THE COEFFICIENTS
-1.5821695+000  7.3001038-001  8.2714578-001  5.4916831-001  7.2651693-001 -2.9702659-001 -8.1989918-001 -1.6530535-001

STANDARD DEVIATIONS OF THE COEFFICIENTS
6.0604548-002  5.2426380-002  8.5025240-002  9.6692531-002  8.9231682-002  9.9364837-002  8.0657209-002  5.3780184-002

ITERATION NUMBER  2

LOSS FUNCTION     6.3764453694+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.5841147+000  7.6896746-001  8.9050189-001  4.9249165-001  7.0896004-001 -3.6636149-001 -9.6302159-001  1.8012551-002

ITERATION NUMBER  3

LOSS FUNCTION     5.5071779127+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.5128028+000  7.1269590-001  9.1413416-001  5.2084395-001  7.0904906-001 -3.2265721-001 -9.7392040-001  1.7641382-001

ITERATION NUMBER  4

LOSS FUNCTION     5.0709442325+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.5161867+000  7.1245087-001  9.3167001-001  4.7349897-001  7.2069752-001 -3.3140731-001 -1.0279261+000  2.4884406-001
```

```
    ITERATION NUMBER  5

    LOSS FUNCTION     5.0349326793+002

    NEW ESTIMATION OF THE COEFFICIENTS
 -1.5167257+000  7.1264078-001  9.3751349-001  4.6753845-001  7.2422108-001 -3.3487233-001 -1.0400087+000  2.5950333-001

    ITERATION NUMBER  6

    LOSS FUNCTION     5.0340380660+002

    NEW ESTIMATION OF THE COEFFICIENTS
 -1.5170656+000  7.1290388-001  9.3842169-001  4.6563619-001  7.2465226-001 -3.3562414-001 -1.0419960+000  2.6179346-001

    TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER  7

    LOSS FUNCTION     5.0340075733+002

    NEW ESTIMATION OF THE COEFFICIENTS
 -1.5171295+000  7.1294160-001  9.3869288-001  4.6521814-001  7.2476144-001 -3.3578484-001 -1.0425125+000  2.6224680-001

    ITERATION NUMBER  8

    LOSS FUNCTION     5.0340064813+002

    NEW ESTIMATION OF THE COEFFICIENTS
 -1.5171295+000  7.1294158-001  9.3869288-001  4.6521818-001  7.2476141-001 -3.3578478-001 -1.0425123+000  2.6224666-001

    EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

    LOSS FUNCTION DOES NOT DECREASE UNTIL THE STEP HAS BEEN HALVED    1 TIMES

    LOSS FUNCTION     5.0340064820+002

    LAMBDA=  1.4190147+000

    GRADIENT OF V
  8.5867941-007  6.5448694-007 -8.0704922-008 -8.4633939-008  7.3923729-009 -1.3753038-008 -2.8681825-008 -1.5978003-008

    INVERSE OF SECOND DERIVATIVE MATRIX
  1.7259031-004 -1.3626361-004 -8.6903471-005  4.5250456-004 -5.3060066-006  1.8938776-004  2.2293754-004 -1.3508814-004
 -1.3626361-004  1.1859107-004  4.9515972-005 -2.9782945-004 -1.0179874-005 -1.3522419-004 -1.7578859-004  1.1827989-004
 -8.6903471-005  4.9515972-005  2.3157638-003 -2.2086460-003 -5.5302905-004  3.7245623-004 -4.7396952-004  3.1992450-004
  4.5250456-004 -2.9782945-004 -2.2086460-003  3.7488406-003  4.1961822-004 -3.0684101-006  9.2866652-004 -6.5282694-004
 -5.3060066-006 -1.0179874-005 -5.5302905-004  4.1961822-004  2.3741413-003 -1.8789845-003 -8.3539480-005  1.2284797-004
  1.8938776-004 -1.3522419-004  3.7245623-004 -3.0684103-006 -1.8789845-003  2.5671804-003  2.8404554-004 -1.9935265-004
  2.2293754-004 -1.7578859-004 -4.7396952-004  9.2866652-004 -8.3539480-005  2.8404554-004  1.3950186-003 -1.1435317-003
 -1.3508814-004  1.1527989-004  3.1992450-004 -6.5282694-004  1.2284797-004 -1.9935265-004 -1.1435317-003  1.2707181-003

    NEW ESTIMATION OF THE COEFFICIENTS
 -1.5171295+000  7.1294159-001  9.3869288-001  4.6521816-001  7.2476142-001 -3.3578481-001 -1.0425124+000  2.6224673-001

    STANDARD DEVIATIONS OF THE COEFFICIENTS
  1.8642111-002  1.5453002-002  6.8286368-002  8.6883112-002  6.9141724-002  7.1897713-002  5.3000124-002  5.0583504-002

    MAX COEFF CORRECTION IS LESS THAN  1.00000000-006

    IER=    0

    TEST QUANTITY=   165.90452
```

```
    IDENTIFICATION WITH MODEL OF ORDER  3


    LEAST SQUARES ESTIMATION

    LOSS FUNCTION    6.0075927839+003

    LAMBDA=  4.9020782+000

    GRADIENT OF V
 1.0280835+004  7.2183096+003  3.6343964+003 -6.0946632+002 -1.0342251+003 -1.1420544+003 -3.2372053+002 -5.1510786+002
-5.2961876+002  0.0000000+000  0.0000000+000  0.0000000+000

    NEW ESTIMATION OF THE COEFFICIENTS
-8.0776210-001 -1.1251362-001  2.7223583-001  8.1707589-001  1.0837597+000  5.9943187-001  7.2020569-001  2.3696364-001
-5.6166515-002  0.0000000+000  0.0000000+000  0.0000000+000

    STANDARD DEVIATIONS OF THE COEFFICIENTS
 1.2562212-001  1.6812279-001  1.0398664-001  2.3545843-001  2.5278513-001  2.7294253-001  2.4710056-001  2.6431758-001
 2.5792154-001  4.9020782+000  4.9020782+000  4.9020782+000

    ITERATION NUMBER  1

    LOSS FUNCTION    6.2954492078+002

    LAMBDA=  1.5868773+000

    GRADIENT OF V
 8.5943611-008 -2.5457448-006 -2.9411531-006 -6.8699592-008  3.5375706-008  6.9077942-008  9.0221874-010 -2.8202250-008
 4.6900823-008  1.8250999+002  2.8110026+002 -9.8356015+001

    NEW ESTIMATION OF THE COEFFICIENTS
-9.5364731-001 -1.6874149-001  4.0300360-001  8.8384790-001  1.0379219+000  2.9513531-001  7.2248415-001  1.3147851-001
-2.2070779-001 -3.7122295-001 -4.4427498-001 -3.7621881-002

    STANDARD DEVIATIONS OF THE COEF ICIENTS
 2.2059178-001  3.0397723-001  1.3683440-001  7.6762802-002  1.9302958-001  1.6326786-001  8.0098886-002  1.8075722-001
 9.5639690-002  2.2205588-001  1.3236811-001  7.6368935-002

    ITERATION NUMBER  2

    LOSS FUNCTION    5.7249044218+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.1080418+000  1.0059338-001  2.8139517-001  9.2274291-001  8.8936939-001  1.6874011-001  7.1396790-001 -1.0904069-002
-1.4310969-001 -5.9948843-001 -2.5813033-001  1.0575064-001

    ITERATION NUMBER  3

    LOSS FUNCTION    5.0732323264+002

    NEW ESTIMATION OF THE COEFFICIENTS
-2.1704296+000  1.6740189+000 -4.3938881-001  9.2301151-001 -9.2323964-002 -4.2039517-001  7.1387592-001 -7.5883950-001
 1.4126468-001 -1.6741774+000  7.9951073-001  4.7158827-003
```

```
    ITERATION NUMBER  4

    LOSS FUNCTION     5.0023044116+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.5321900+000  7.1783425-001  2.5381162-003  9.2243722-001  5.1876947-001 -1.1739534-001  7.1723966-001 -3.8900010-001
-4.8422414-002 -1.0374626+000  1.8521895-001  7.2172689-002

    ITERATION NUMBER  5

    LOSS FUNCTION     4.9936159223+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.7364977+000  1.0292729+000 -1.4406256-001  9.2237155-001  3.2309539-001 -2.0351343-001  7.1648960-001 -4.5392368-001
 1.8430135-002 -1.2420813+000  3.9730694-001  2.4363389-002

    ITERATION NUMBER  6

    LOSS FUNCTION     4.9931431942+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.6218439+000  8.5323686-001 -6.1071024-002  9.2108038-001  4.4160075-001 -1.6727413-001  7.1521503-001 -3.7723448-001
-1.6242513-002 -1.1280572+000  2.8055965-001  4.9012289-002

    ITERATION NUMBER  7

    LOSS FUNCTION     4.9926729079+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.7232568+000  1.0084998+000 -1.3421354-001  9.2176340-001  3.3959482-001 -2.0341562-001  7.1731546-001 -4.4713689-001
 1.5897745-002 -1.2294040+000  3.8555828-001  2.4885358-002

    ITERATION NUMBER  8

    LOSS FUNCTION     4.9925367100+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.6544285+000  9.0294081-001 -8.4460860-002  9.2112472-001  4.1000854-001 -1.8055958-001  7.1818651-001 -4.0060377-001
-5.2590821-003 -1.1607887+000  3.1482105-001  4.0531825-002

    ITERATION NUMBER  9

    LOSS FUNCTION     4.9924220112+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.7086967+000  9.8610604-001 -1.2365457-001  9.2157327-001  3.5494380-001 -1.9922149-001  7.1761840-001 -4.3770717-001
 1.1737043-002 -1.2149795+000  3.7090385-001  2.7823555-002

    ITERATION NUMBER 10

    LOSS FUNCTION     4.9923769722+002

    NEW ESTIMATION OF THE COEFFICIENTS
-1.6686946+000  9.2478598-001 -9.4755431-002  9.2123167-001  3.9567913-001 -1.8566933-001  7.1808196-001 -4.1032515-001
-6.4873638-004 -1.1750663+000  3.2969469-001  3.7043105-002
```

ITERATION NUMBER 11

LOSS FUNCTION 4.9923439454+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.6991724+000 9.7150369-001 -1.1677372-001 9.2149355-001 3.6467858-001 -1.9604533-001 7.1774786-001 -4.3130285-001
8.8496781-003 -1.2054906+000 3.6115807-001 2.9950889-002

ITERATION NUMBER 12

LOSS FUNCTION 4.9923286268+0 2

NEW ESTIMATION OF THE COEFFICIENTS
-1.6761122+000 9.3615930-001 -1.0011693-001 9.2130111-001 3.8812905-001 -1.8818608-001 7.1800662-001 -4.1500567-001
1.6904542-003 -1.1824756+000 3.3738249-001 3.5290717-002

TRIAL TO USE EXACT SECOND DERIVATIVES-ITERATION NUMBER 13

LOSS FUNCTION 4.9923184402+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.6859710+000 9.5127660-001 -1.0724299-001 9.2138035-001 3.7809979-001 -1.9152080-001 7.1790150-001 -4.2232390-001
4.7591547-003 -1.1923197+000 3.4756508-001 3.3005388-002

ITERATION NUMBER 14

LOSS FUNCTION 4.9923057930+002

NEW ESTIMATION OF THE COEFFICIENTS
-1.6859091+000 9.5118198-001 -1.0719828-001 9.2139190-001 3.7813531-001 -1.9148065-001 7.1789692-001 -4.2227624-001
4.7422966-003 -1.1922538+000 3.4749964-001 3.3013285-002

```
EXACT SECOND DERIVATIVES ARE USED UNTIL MAX COEFF CORR IS LESS THAN ACC

LOSS FUNCTION     4.9923057903+002

LAMBDA=  1.4131250+000

 GRADIENT OF V
-1.1434313-006 -3.3755787-006 -5.5809505-006  6.4529013-007  5.3650001-007  6.2829349-007 -6.4993219-007 -1.0211201-006
-1.2493765-006  8.0865721-007  2.9134681-006  4.7152862-006

 INVERSE OF SECOND DERIVATIVE MATRIX
 1.3249762-001 -2.0046567-001  9.4155008-002  1.7754139-003  1.1828647-001  6.8251559-002 -1.0385839-003  1.0017645-001
-4.7774553-002  1.3312813-001 -1.4105084-001  3.8895920-002
-2.0046567-001  3.0341948-001 -1.4254687-001 -2.6902624-003 -1.7903627-001 -1.0290741-001  1.5607508-003 -1.5156420-001
 7.2418409-002 -2.0141288-001  2.1355262-001 -5.8939558-002
 9.4155008-002 -1.4254687-001  6.6987532-002  1.2635567-003  8.4085752-002  4.8281645-002 -7.3708412-004  7.1180543-002
-3.4041938-002  9.4599404-002 -1.0034677-001  2.7720114-002
 1.7754138-003 -2.6902622-003  1.2635566-003  2.3696537-003 -1.2110805-003  1.7579212-003 -5.9703440-004  1.9671619-003
-7.3905650-004  1.4817479-003 -1.6063496-003  4.3668289-004
 1.1828647-001 -1.7903627-001  8.4085752-002 -1.2110804-003  1.1108275-001  5.7741867-002 -1.4245297-004  8.8105588-002
-4.2156090-002  1.1916771-001 -1.2656478-001  3.5015114-002
 6.8251559-002 -1.0290741-001  4.8281645-002  1.7579213-003  5.7741867-002  3.8951775-002 -8.6840744-004  5.2313340-002
-2.4700926-002  6.8531335-002 -7.1931373-002  1.9560427-002
-1.0385839-003  1.5607506-003 -7.37 8406-004 -5.9703440-004 -1.4245290-004 -8.6840740-004  2.5516680-003 -3.7719630-003
 1.3827048-003 -1.1065646-003  1.2136934-003 -3.1063101-004
 1.0017645-001 -1.5156420-001  7.1180543-002  1.9671619-003  8.8105588-002  5.2313340-002 -3.7719630-003  8.1395990-002
-3.9125598-002  1.0075727-001 -1.0685776-001  2.9490261-002
-4.7774553-002  7.2418409-002 -3.4041938-002 -7.3905655-004 -4.2156090-002 -2.4700926-002  1.3827049-003 -3.9125598-002
 1.9940230-002 -4.8064203-002  5.1105333-002 -1.4123568-002
 1.3312813-001 -2.0141288-001  9.4599404-002  1.4817481-003  1.1916771-001  6.8531335-002 -1.1065647-003  1.0075727-001
-4.8064203-002  1.3483231-001 -1.4302951-001  3.9527265-002
-1.4105084-001  2.1355262-001 -1.0034677-001 -1.6063497-003 -1.2656478-001 -7.1931373-002  1.2136935-003 -1.0685776-001
 5.1105333-002 -1.4302951-001  1.5280737-001 -4.2773131-002
 3.8895919-002 -5.8939558-002  2.7720114-002  4.3668294-004  3.5015114-002  1.9560427-002 -3.1063103-004  2.9490261-002
-1.4123568-002  3.9527265-002 -4.2773131-002  1.2496675-002

 NEW ESTIMATION OF THE COEFFICIENTS
-1.6859091+000  9.5118190-001 -1.0719824-001  9.2139190-001  3.7813536-001 -1.9148062-001  7.1789692-001 -4.2227620-001
 4.7422775-003 -1.1922538+000  3.4749958-001  3.3013300-002

 STANDARD DEVIATIONS OF THE COEFFICIENTS
 5.1438065-001  7.7839909-001  3.6574431-001  6.8789638-002  4.7098155-001  2.7889724-001  7.1382650-002  4.0316433-001
 1.9954722-001  5.1889271-001  5.5239880-001  1.5797116-001

MAX COEFF CORRECTION IS LESS THAN  1.00000000-006


IER=    0


TEST QUANTITY=      1.01907
```

ON THE COMPUTATION OF THE MATRIX OF THE PARTIAL DERIVATIVES

Below a scheme of the computations of the matrix of the second partial derivatives is shown, fig 6 and 7. In fig 8-10 an example for a third order system with one input is given.

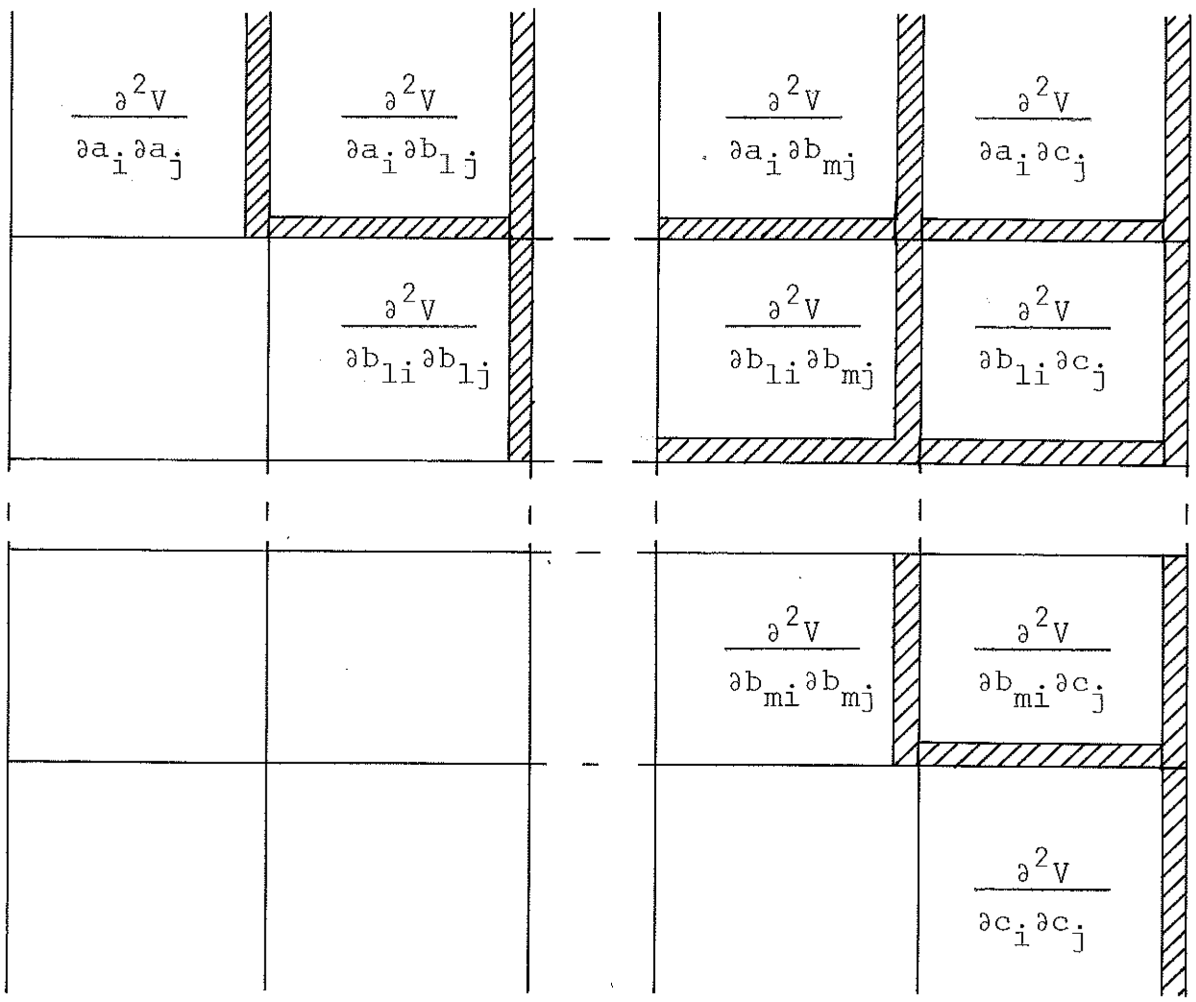

Fig. 6 - Scheme of the computations of $V_{\theta\theta}$ for $t = 1,2,\ldots, N-n+1$. Only vectors marked in the figure have to be computed. Notice that only the diagonal parts of the matrix are symmetric. For instance the upper righthand part $\frac{\partial^2 V}{\partial a_i \partial c_j}$ is not symmetric.

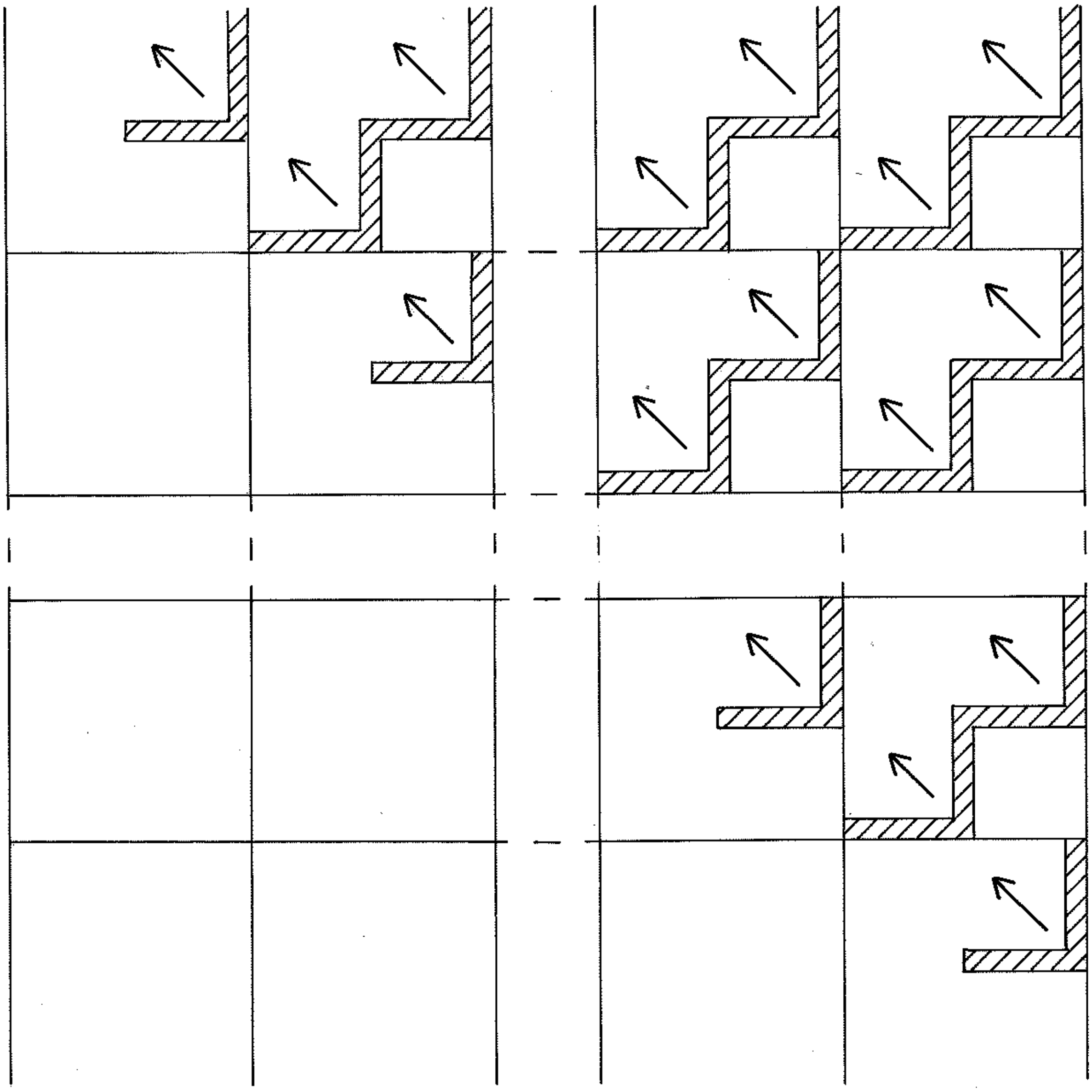

Fig. 7 - Scheme of the computations of $V_{\theta\theta}$ for $N-n+1 < t \leq N$. The matrix is partitioned as in fig 6. The elements of the matrix are computed successively in the direction of the arrows. The elements that are changed for a certain t in the interval $N > t > N - n + 1$ are marked in the figure.

$$
\left[\begin{array}{ccc|ccc|ccc}
 & & \frac{\partial^2 V}{\partial a_1 \partial a_3} & & & \frac{\partial^2 V}{\partial a_1 \partial b_3} & & & \frac{\partial^2 V}{\partial a_1 \partial c_3} \\
 & & \frac{\partial^2 V}{\partial a_2 \partial a_3} & & & \frac{\partial^2 V}{\partial a_2 \partial b_3} & & & \frac{\partial^2 V}{\partial a_2 \partial c_3} \\
 & & \frac{\partial^2 V}{\partial a_3 \partial a_3} & \frac{\partial^2 V}{\partial a_3 \partial b_1} & \frac{\partial^2 V}{\partial a_3 \partial b_2} & \frac{\partial^2 V}{\partial a_3 \partial b_3} & \frac{\partial^2 V}{\partial a_3 \partial c_1} & \frac{\partial^2 V}{\partial a_3 \partial c_2} & \frac{\partial^2 V}{\partial a_3 \partial c_3} \\
\hline
 & & & & & \frac{\partial^2 V}{\partial b_1 \partial b_3} & & & \frac{\partial^2 V}{\partial b_1 \partial c_3} \\
 & & & & & \frac{\partial^2 V}{\partial b_2 \partial b_3} & & & \frac{\partial^2 V}{\partial b_2 \partial c_3} \\
 & & & & & \frac{\partial^2 V}{\partial b_3 \partial b_3} & \frac{\partial^2 V}{\partial b_3 \partial c_1} & \frac{\partial^2 V}{\partial b_3 \partial c_2} & \frac{\partial^2 V}{\partial b_3 \partial c_3} \\
\hline
 & & & & & & & & \frac{\partial^2 V}{\partial c_1 \partial c_3} \\
 & & & & & & & & \frac{\partial^2 V}{\partial c_2 \partial c_3} \\
 & & & & & & & & \frac{\partial^2 V}{\partial c_3 \partial c_3}
\end{array}\right]
$$

Fig. 8 - Scheme of the computations of $V_{\theta\theta}$ for $t = 1,2,\ldots,N-2$. Only elements marked in the figure have to be computed.

$$
\left[\begin{array}{ccc|ccc|ccc}
 & & \frac{\partial^2 V}{\partial a_1 \partial a_3} & & & \frac{\partial^2 V}{\partial a_1 \partial b_3} & & & \frac{\partial^2 V}{\partial a_1 \partial c_3} \\
 & \frac{\partial^2 V}{\partial a_2 \partial a_2} & \frac{\partial^2 V}{\partial a_2 \partial a_3} & \frac{\partial^2 V}{\partial a_2 \partial b_1} & \frac{\partial^2 V}{\partial a_2 \partial b_2} & \frac{\partial^2 V}{\partial a_3 \partial b_3} & \frac{\partial^2 V}{\partial a_2 \partial c_1} & \frac{\partial^2 V}{\partial a_2 \partial c_2} & \frac{\partial^2 V}{\partial a_2 \partial c_3} \\
 & & & & & & & & \\
\hline
 & & & & & \frac{\partial^2 V}{\partial b_1 \partial b_3} & & & \frac{\partial^2 V}{\partial b_1 \partial c_3} \\
 & & & & \frac{\partial^2 V}{\partial b_2 \partial b_2} & \frac{\partial^2 V}{\partial b_2 \partial b_3} & \frac{\partial^2 V}{\partial b_2 \partial c_1} & \frac{\partial^2 V}{\partial b_2 \partial c_2} & \frac{\partial^2 V}{\partial b_2 \partial c_3} \\
 & & & & & & & & \\
\hline
 & & & & & & & & \frac{\partial^2 V}{\partial c_1 \partial c_3} \\
 & & & & & & & \frac{\partial^2 V}{\partial c_2 \partial c_2} & \frac{\partial^2 V}{\partial c_2 \partial c_3} \\
 & & & & & & & &
\end{array}\right]
$$

Fig. 9 - Scheme of the computations of $V_{\theta\theta}$ for $t = N-1$. Only elements marked in the figure have to be computed.

$$
\left[\begin{array}{ccc|ccc|ccc}
\frac{\partial^2 V}{\partial a_1 \partial a_1} & \frac{\partial^2 V}{\partial a_1 \partial a_2} & \frac{\partial^2 V}{\partial a_1 \partial a_3} & \frac{\partial^2 V}{\partial a_1 \partial b_1} & \frac{\partial^2 V}{\partial a_1 \partial b_2} & \frac{\partial^2 V}{\partial a_1 \partial b_3} & \frac{\partial^2 V}{\partial a_1 \partial c_1} & \frac{\partial^2 V}{\partial a_1 \partial c_2} & \frac{\partial^2 V}{\partial a_1 \partial c_3} \\
 & & & & & & & & \\
 & & & & & & & & \\
\hline
 & & & \frac{\partial^2 V}{\partial b_1 \partial b_1} & \frac{\partial^2 V}{\partial b_1 \partial b_2} & \frac{\partial^2 V}{\partial b_1 \partial b_3} & \frac{\partial^2 V}{\partial b_1 \partial c_1} & \frac{\partial^2 V}{\partial b_1 \partial c_2} & \frac{\partial^2 V}{\partial b_1 \partial c_3} \\
 & & & & & & & & \\
 & & & & & & & & \\
\hline
 & & & & & & \frac{\partial^2 V}{\partial c_1 \partial c_1} & \frac{\partial^2 V}{\partial c_1 \partial c_2} & \frac{\partial^2 V}{\partial c_1 \partial c_3} \\
 & & & & & & & & \\
 & & & & & & & &
\end{array}\right]
$$

Fig. 10 - Scheme of the computations of $V_{\theta\theta}$ for $t = N$. Only elements marked in the figure have to be computed.