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# On Event-Based Sampling for LQG-Optimal Control

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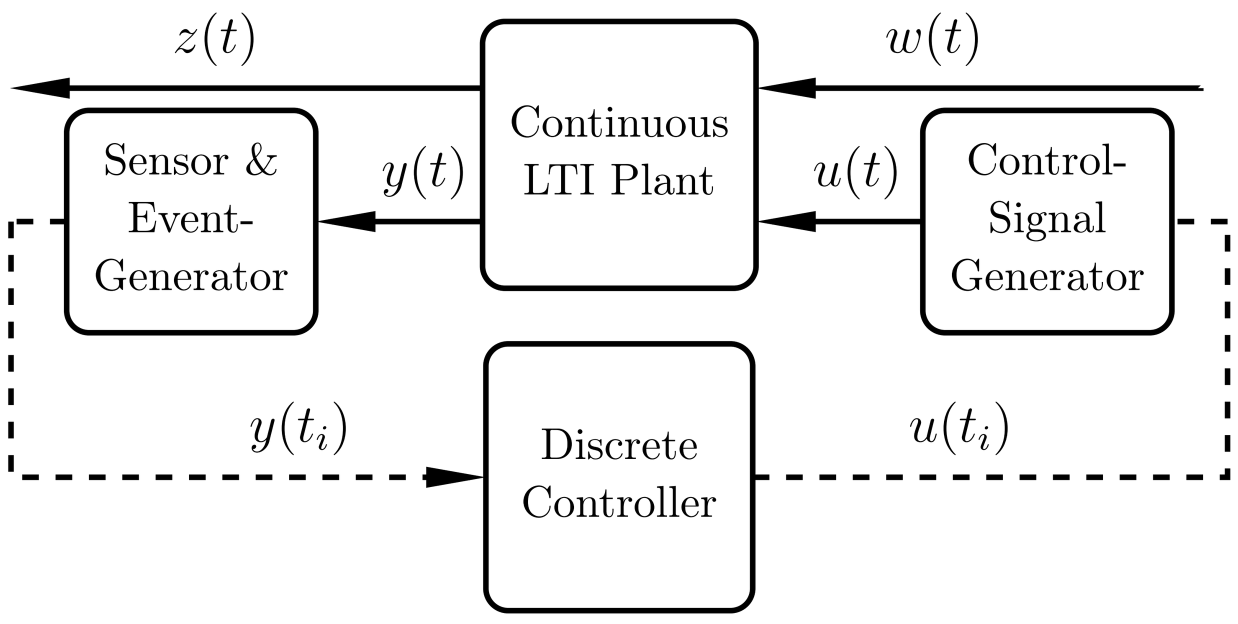
## Introduction

**Event-based control** is the concept of sampling and actuating based on system signals rather than a periodic timer, with the goal of more resource-efficient control. We consider the classic LQG formulation, with an added penalty $\rho$ on the average sampling rate $f$:

$$\text{Minimize } J = \lim_{T\to\infty} \frac{1}{T}\mathbb{E}\Big[\int_0^T z(t)^\intercal z(t)dt\Big] + \rho f.$$

For this objective, the optimal controller structure is available [1], and we consider the remaining problem of optimizing the sampling policy [2].

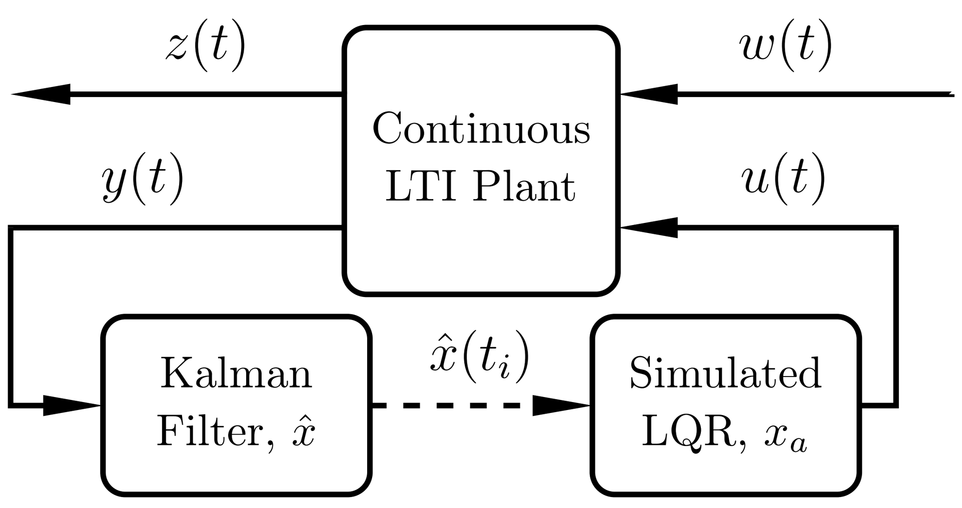



**Our Contributions**:

- *Identifying an equivalent free boundary PDE* formulation of the optimal sampling problem
- *Deriving a numerical scheme* to approximately compute the optimal sampling policy for general LTI systems, and observing that sampling thresholds can be non-convex
- *Deriving tight bounds* on the improvement over periodic sampling in the special case of multidimensional integrators ($A = 0$), see [2].

## Optimal Controller Structure

The optimal controller structure from [1] can intuitively be represented as:




When sampling we assign $x_a(t_i) = \hat{x}(t_i)$, and the error $\tilde{x} = \hat{x} - x_a$ is reset to zero. The error dynamics are:

$$\text{Dynamics: } \begin{cases} d\tilde{x} = A\tilde{x}dt + d\epsilon, \\ \tilde{x}(t_i) = 0, \end{cases} \qquad \text{Noise: } \begin{cases} \mathbb{E}[d\epsilon] = 0, \\ \mathbb{E}[d\epsilon d\epsilon^\intercal] = R\,dt \succ 0. \end{cases}$$

This fundamental reset system determines the closed-loop cost

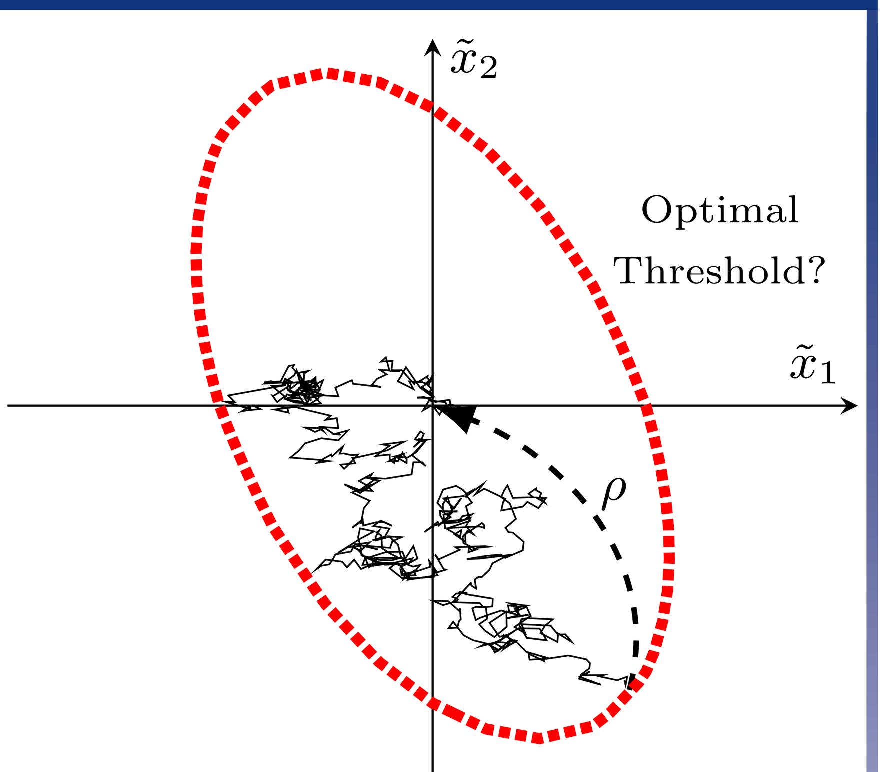
$$J = \gamma_0 + \lim_{T\to\infty}\frac{1}{T}\mathbb{E}\Big[\int_0^T \tilde{x}(t)^\intercal Q\tilde{x}(t)dt\Big] + \rho f,$$

where $\gamma_0$ is the optimal continuous-time LQG-cost.

## Optimal Sampling Problem

**Problem:** Find the optimal trigger threshold in the $\tilde{x}$-space, from which we reset $\tilde{x}$ to zero and pay $\rho$. This threshold can be expressed in the *relative value function* $V(\tilde{x})$ of the optimization problem, where it is optimal to sample when

$$V(\tilde{x}) = 0.$$




$V$ satisfies a free boundary PDE:

$$\tilde{x}^\intercal Q\tilde{x} + \tilde{x}^\intercal A^\intercal \nabla V + \frac{1}{2}\mathrm{Tr}(R\nabla^2 V) - J = 0, \qquad V(\tilde{x}) \le 0, \quad \forall \tilde{x}$$

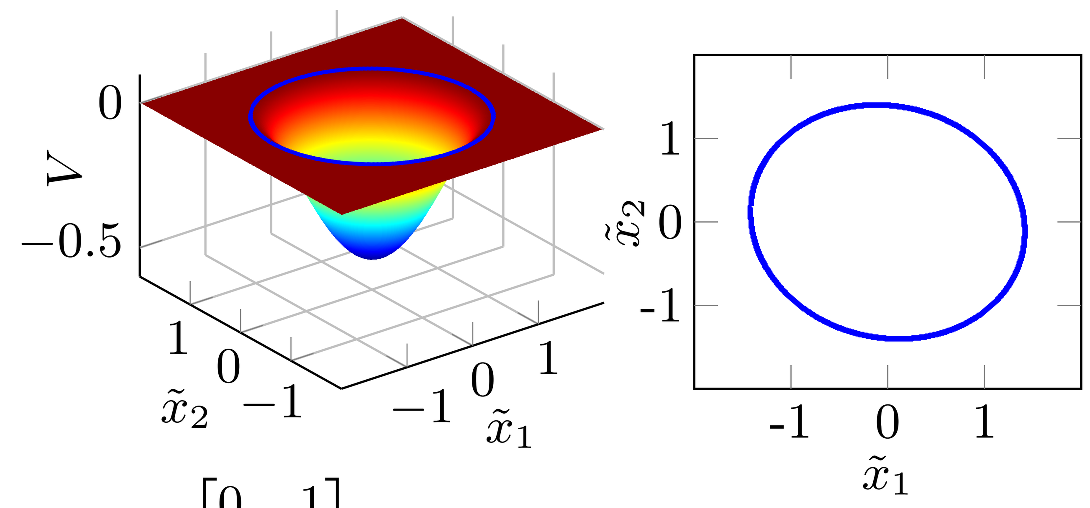
**Conditions on the Free Boundary:**

$$\text{Dirichlet: } V(\tilde{x}) = 0, \quad \text{Neumann: } \nabla V = 0.$$

## Numerical Solution

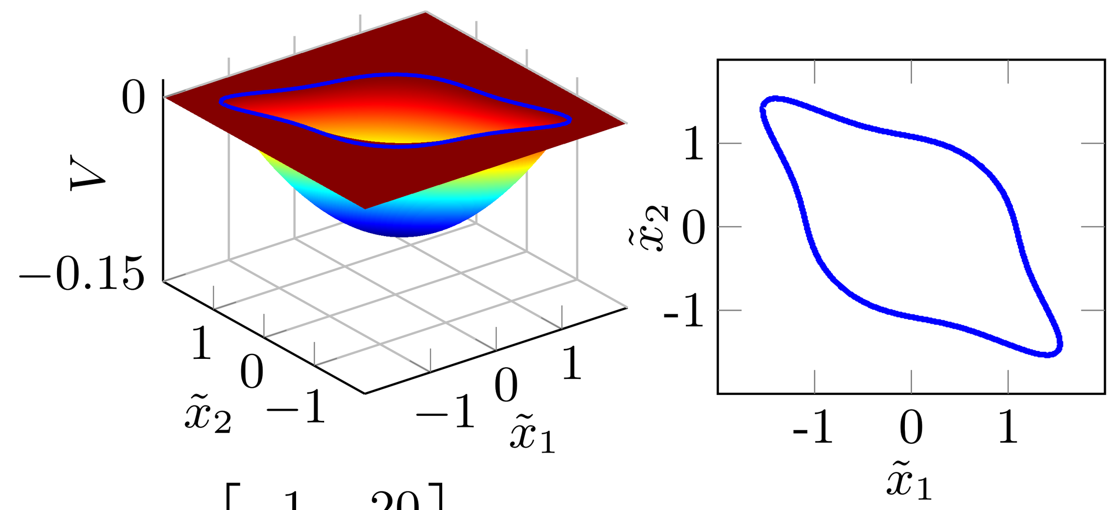
**Numerical scheme:**

- Finite-difference approximation
- Simulate dynamic version of PDE
- Enforce $V(\tilde{x}) \le 0$
- Run until stationarity



$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; Q = R = I.$$

**Note:**
The optimal thresholds are not necessarily convex. Typically, this is the case for systems with strong cross-coupling between the states.

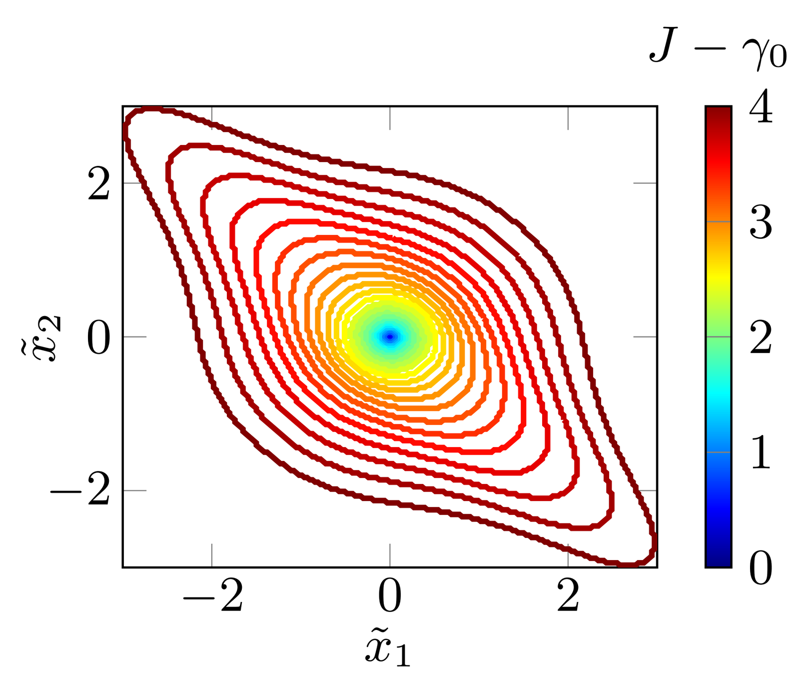


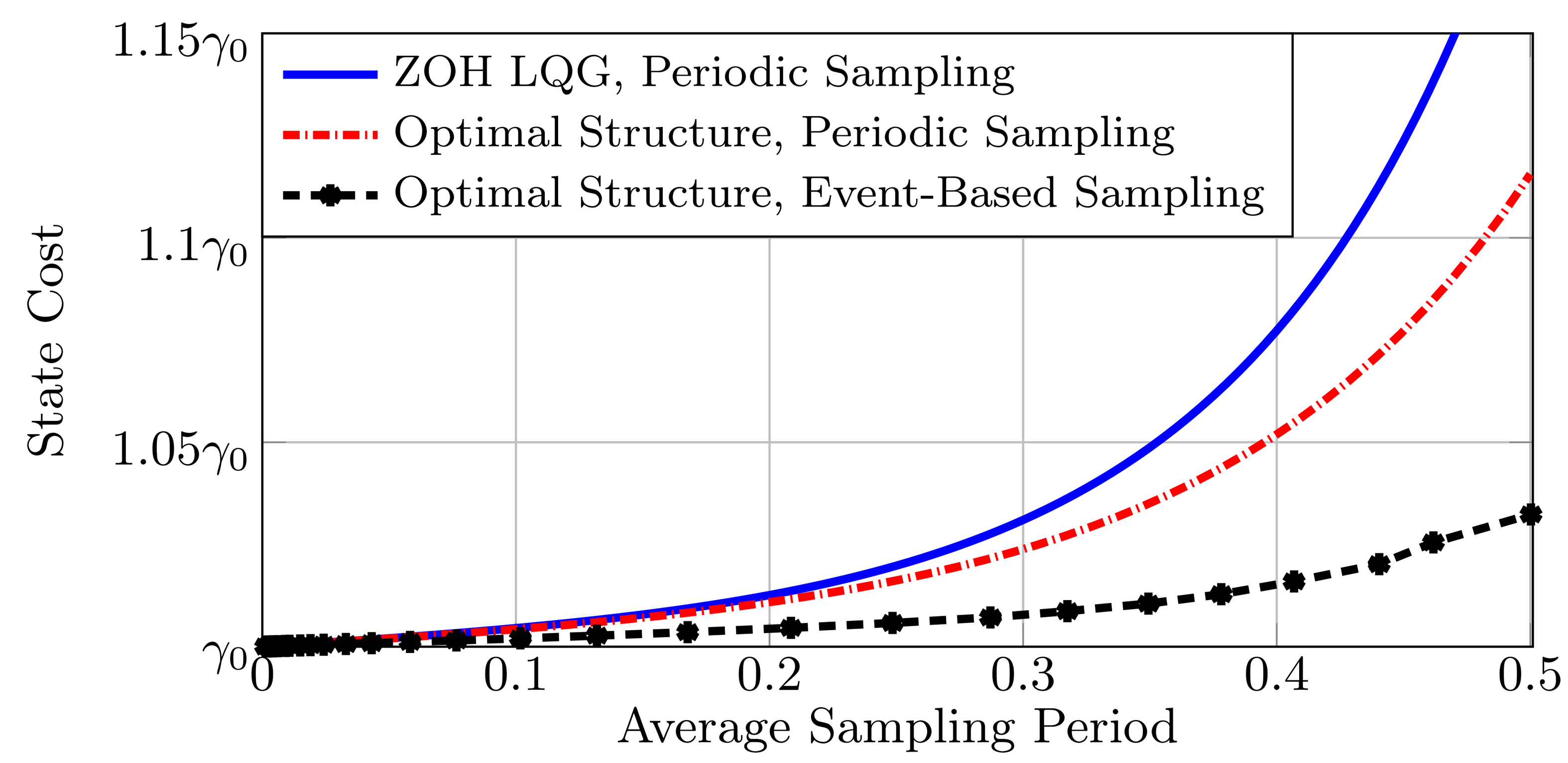
$$A = \begin{bmatrix} -1 & 20 \\ 20 & -1 \end{bmatrix}, \; Q = R = I.$$

## Comparison to Periodic Sampling

By numerically computing the optimal sampling policy, we can compare the closed-loop performance using periodic- and event-based sampling for different average sampling periods. Example:

$$A = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}, \quad R = Q = I.$$




## References

[1] L. Mirkin. "Intermittent Redesign of Analog Controllers via the Youla Parameter". *IEEE Trans. Automatic Control*, vol 62, no. 4, 2017.

[2] M. Thelander Andrén, B. Bernhardsson, A. Cervin, K. Soltesz. "On Event-Based Sampling for LQG-Optimal Control". In: *56th IEEE Conference on Decision and Control*, 2017.