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Identification and Modelling of Ship Dynamics

Åström, Karl Johan; Källström, Claes

1972

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Åström, K. J., & Källström, C. (1972). *Identification and Modelling of Ship Dynamics*. (Technical Reports TFRT-7015). Department of Automatic Control, Lund Institute of Technology (LTH).

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

IDENTIFICATION AND MODELLING OF SHIP DYNAMICS

K. J. ÅSTRÖM
C. KÄLLSTRÖM

REPORT 7202 MARS 1972
LUND INSTITUTE OF TECHNOLOGY
DIVISON OF AUTOMATIC CONTROL

IDENTIFICATION AND MODELLING OF SHIP DYNAMICS

K.J. Åström

C. Källström

ABSTRACT.

This report describes the application of identification and modelling techniques to the determination of ship dynamics. The analysis is based on data obtained from an experiment on MS Atlantic Song in collaboration with Wallenius Lines. The results indicate that identification techniques can be profitably exploited to determine hydrodynamic derivatives. The analysis also illustrate several features of the maximum likelihood identification method.

This work has been supported by the Swedish Board for Technical Development under Contract 71-50/U33.

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1. INTRODUCTION.

This report applies modelling and identification techniques to the problem of determining ship dynamics. A summary of physical models is given in Section 2 where it is found that ship dynamics can be characterized as a third order system where the rudder is the input and the heading the output. The state variables are arbitrarily chosen as the component of ship velocity orthogonal to the ship's center line, v, the heading angle, φ, and its derivative. It is also shown in Section 2 that it is not possible to determine the hydrodynamic derivatives from measurements of input (rudder angle) and output (heading) alone. However, if the velocity component v is also measured e.g. by a doppler radar it is shown that the hydrodynamic derivatives can in fact be determined from an experiment where the rudder is perturbed in an arbitrary fashion and the heading φ and the across ship velocity component are measured. Hence a new technique to determine hydrodynamic derivatives! An experiment aimed at analysing the feasibility of the proposed scheme is discussed in Section 3. The experiment was performed by two students, Mr. Ulf Ekwall and Mr. Anders Edvardsson, as part of their MS thesis. The experiment was done on MS Atlantic Song in collaboration with the Wallenius Line of Stockholm, Sweden. The experiment lasted for about 30 minutes and was done in bad sea conditions. In the experiment the input signal was generated manually and the outputs were also read manually.

Section 4 presents an application of the maximum likelihood identification technique to the data obtained in the experiment. The analysis shows that the data is not perfect and that part of it has to be discarded. An analysis of a data set of 1500 seconds indicates that a third order model is compatible with the data.

Accuracy estimates also indicate that the parameters are reasonably accurate. Typical examples are

$$a_1 = - 2.18 \pm 0.04$$

$$b_1 = 0.14 \pm 0.01$$

In Section 5 the parameters of a physical model are estimated from the input-output data. The results are compatible with those of Section 4.

2. SHIP DYNAMICS.

The equations describing ship dynamics are well-known. See e.g. Abkowitz [1], Goclowski and Gelb [6] and Zuidweg [7]. They are obtained directly from Newton's laws expressing conservation of linear and angular momentum. The essential difficulty in obtaining the equations is to describe the hydrodynamic forces acting on the hull. These forces are in general complicated functions of the ship's motion (velocity and angular velocity) and the rudder position. The equations can, however, be linearized around a straight line motion with constant velocity. In that case the hydrodynamic forces can be characterized by a linear function of the velocities, accelerations, angular velocities and angular accelerations. The coefficients of the linear function are called the hydrodynamic derivatives which are usually determined by model tests in tanks. The equations of motion can be somewhat simplified if they are expressed in a reference frame fixed to the ship. See Fig. 2.1.

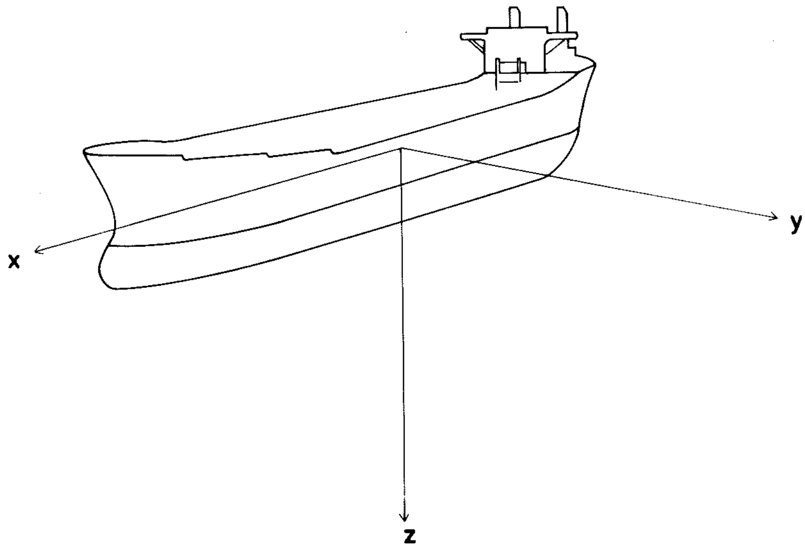

Fig. 2.1 - Definition of coordinates fixed to the ship. Rotation around the x, y and z axes are called roll, pitch and yaw respectively.

The symmetry of the hull implies that some of the hydrodynamical forces will vanish. In particular it turns out that for a ship with symmetric port and starboard sides the yaw motion can be separated from the roll and pitch motions. See e.g. Abkowitz [1, p. 21].

State Equations.

To give the equations for the yaw motion let v be the velocity component in the y-direction and ω the component of the angular velocity on the z-axis. Abkowitz [1] has shown that the equations for the yaw motion linearized about a straight line motion with constant velocity is given by the equation

$$\begin{bmatrix} m - Y_{\dot{v}} & mx_g - Y_{\dot{\omega}} \\ mx_g - N_{\dot{v}} & I - N_{\dot{\omega}} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} =$$

$$= \begin{bmatrix} Y_v & Y_\omega - m \\ N_v & N_\omega - mx_g \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \end{bmatrix} \delta \qquad (2.1)$$

where m is the mass of the ship, I its moment of inertia about the z-axis, Y the component of the hydrodynamic force on the y-axis, N the z-component of the torque due to the hydrodynamic forces, x_g the x-coordinate of the center of mass, and δ the rudder deflection. The hydrodynamic force Y is a complicated function of the motion, i.e.

$$Y = Y(v, \omega, \delta, \dot{v}, \dot{\omega}) \qquad (2.2)$$

and analogous for N.

The quantities Y_v, $Y_{\dot{v}}$, N_v, $N_{\dot{v}}$, N_ω, $N_{\dot{\omega}}$ and N_δ denote the hydrodynamic derivatives. The derivative Y_x is defined by

$$Y_x = \frac{\partial Y}{\partial x} \tag{2.3}$$

and evaluated at the nominal motion. The other derivatives are defined analogously. The linearized equations for the yaw motion can be simplified further if equation (2.1) is solved for $\dot{v}$ and $\dot{\omega}$. This gives

$$\frac{d}{dt}\begin{bmatrix} v \\ \omega \\ \varphi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \\ \varphi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \delta \tag{2.4}$$

where the heading φ defined by $\varphi = \omega$ has also been introduced as an extra state variable. See Fig. 2.2.

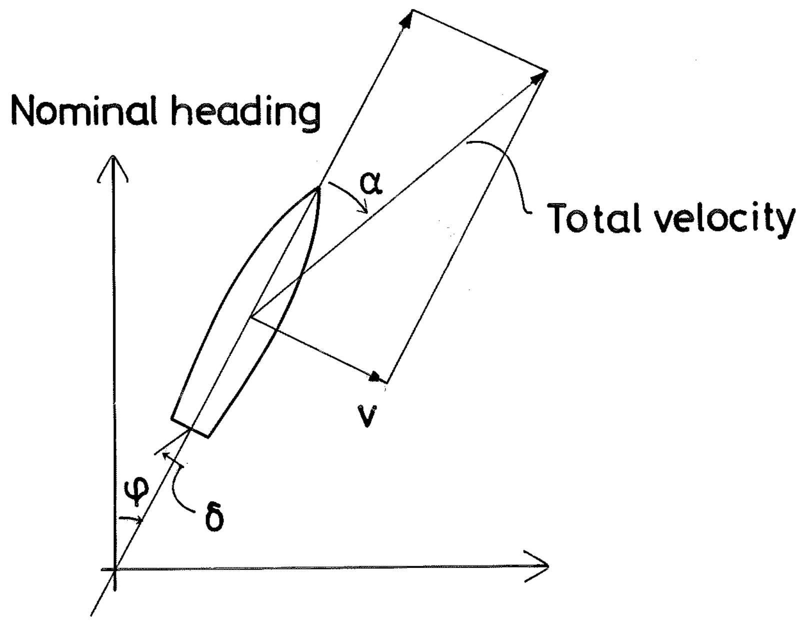

Fig. 2.2 - Quantities used to describe the linearized yaw motion of a ship.

The linearized yaw motion of a ship can thus be described as a third order dynamical system where the state variables can be chosen as

v the component of the ship velocity on the y-axis of the ship fixed coordinate system,

φ the deviation in heading,

ω the ship's angular velocity about the z-axis.

Other state variables are sometimes chosen. The angle of attack i.e. α in Fig 2.2 is for example sometimes used instead of the velocity component v.

The model contains six parameters a_{11}, a_{12}, a_{21}, a_{22}, b_1 and b_2 which are functions of the hydrodynamic derivatives, the ship's mass and its moment of inertia.

It is customary to normalize the equations by expressing all quantities in nondimensional units. We have

$$\begin{aligned} a_{11} &= (V/\ell)\alpha_{11} & a_{12} &= V\alpha_{12} \\ a_{21} &= (V/\ell^2)\alpha_{21} & a_{22} &= (V/\ell)\alpha_{22} \\ b_1 &= (V^2/\ell)\beta_1 & b_2 &= (V/\ell)^2\beta_2 \end{aligned} \qquad (2.5)$$

Choosing the length ℓ of the ship as the length unit and the time unit as ℓ/V where V is the ship velocity the linearized equation of motion (2.4) can then be written as

$$\frac{d}{dt'}\begin{bmatrix} v' \\ \omega' \\ \varphi \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v' \\ \omega' \\ \varphi \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix}\delta \qquad (2.6)$$

where all parameters and variables are dimension free.

The dimension free parameters α_{ij}, β_i, i,j = 1, 2 are remarkably similar for different ships as is seen from Table 2.1.

Ship	Minesweeper	Freighter	Tanker
Length (m)	60	160	310
Speed V (m/sec)	4	7.8	8.5
α_{11}	-0.863	-0.89	-0.466
α_{12}	-0.482	-0.286	-0.226
α_{21}	-5.25	-4.39	-3.08
α_{22}	-2.45	-2.72	-1.66
β_1	+0.175	+0.108	+0.139
β_2	-1.38	-0.93	-1.00
Ref.	[1]	[7]	[4]

Table 2.1 - Examples of dimensionless parameters for different ships.

Input-Output Relations. Identifiability.

Consider the rudder angle δ to be the input and the heading angle φ the output. The input-output relation of the dynamical system (2.6) can then be represented by the following transfer function

$$G(s) = \frac{b_1 s + b_2}{s(s^2+a_1 s+a_2)} \tag{2.7}$$

where

$$a_1 = -\alpha_{11} - \alpha_{22}$$

$$a_2 = \alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21} \tag{2.8}$$

$$b_1 = \beta_2$$

$$b_2 = -\alpha_{11}\beta_2 + \alpha_{21}\beta_1$$

In an experiment where the rudder angle is perturbed and the heading observed we thus find that the parameter β_2 is identifiable but the parameters α_{11}, α_{12}, α_{21}, α_{22} and β_1 are not identifiable. We can thus immediately conclude that in order to determine all six parameters of the model (2.6) it is necessary to measure not only the heading angle but also some other system variable.

Assume, however, that the velocity component v is also measured. The transfer function from rudder angle to v is given by

$$G_2(s) = \frac{c_1 s + c_2}{s^2 + a_1 s + a_2} \tag{2.9}$$

where

$$c_1 = \beta_1 \tag{2.10}$$

$$c_2 = -\beta_1\alpha_{22} + \beta_2\alpha_{12}$$

Hence if both v and φ are considered as outputs the parameters β_1 and β_2 can be determined directly from an input-output experiment. The coefficients α_{11}, α_{12},

α_{21} and α_{22} are related to the parameters of the input-output relation through nonlinear equations. The possibilities to determine these will, of course, depend on the actual parameter values. For the values of Table 2.1 the nonlinear equations will have a solution and we can thus conclude that the system (2.6) or (2.4) is identifiable if both heading angle φ and crosstrack velocity v are measured. In practice it would be more realistic to assume that the crosstrack deviation is measured instead of v.

3. THE EXPERIMENT.

The experiment was performed on Sunday, Dec. 21, 1969, with the MS Atlantic Song of the Wallenius Lines. Atlantic Song is a freighter of 15000 tons, 197 m length with a maximum speed of 21 knots. The measurements were done east of the coast of Denmark. They were started at Lat. N 54°17' Long 4°51', course 217°. See Fig. 3.1. The experiment lasted for about half an hour. The wind was about 8 Beaufort (17-20 m/sec, fresh gale!). The wave height was estimated to 3.5 - 4 meters. The sight was poor due to heavy snow-fall. During the experiment both wind and waves were on the port. The ship has a luffing tendency which means that a wind gust will cause a port yaw. The impact of the waves on the bow induced sudden and violent starboard yaws.

In the beginning of the experiment the speed was 18.5 knots. Towards the end of the experiment the speed was, however, reduced to 18 knots due to the large rudder angles.

The experiment was carried out by two students, Mr. Ekwall and Mr. Edvardsson. The experimental arrangement is shown in Fig. 3.2. Mr. Ekwall was on the bridge together with the captain, Mr. Tärnsjö, the second mate, Mr. Håkansson, and the helmsman, Mr. Brand. Mr. Edvardsson was at the rudder servo. Mr. Ekwall who acted as a coordinator was coordinating the experiment. He ordered the rudder angle to be read by Mr. Edvardsson every 15 seconds. He read the heading angle from the gyro-compass simultaneously. Edvardsson also ordered the changes in the rudder angle to be performed by the helmsman. After a command the rudder was found to settle in less than 2.5 seconds.

The rudder angle was measured using the arrangement shown in Fig. 3.3. The length of the scale was about

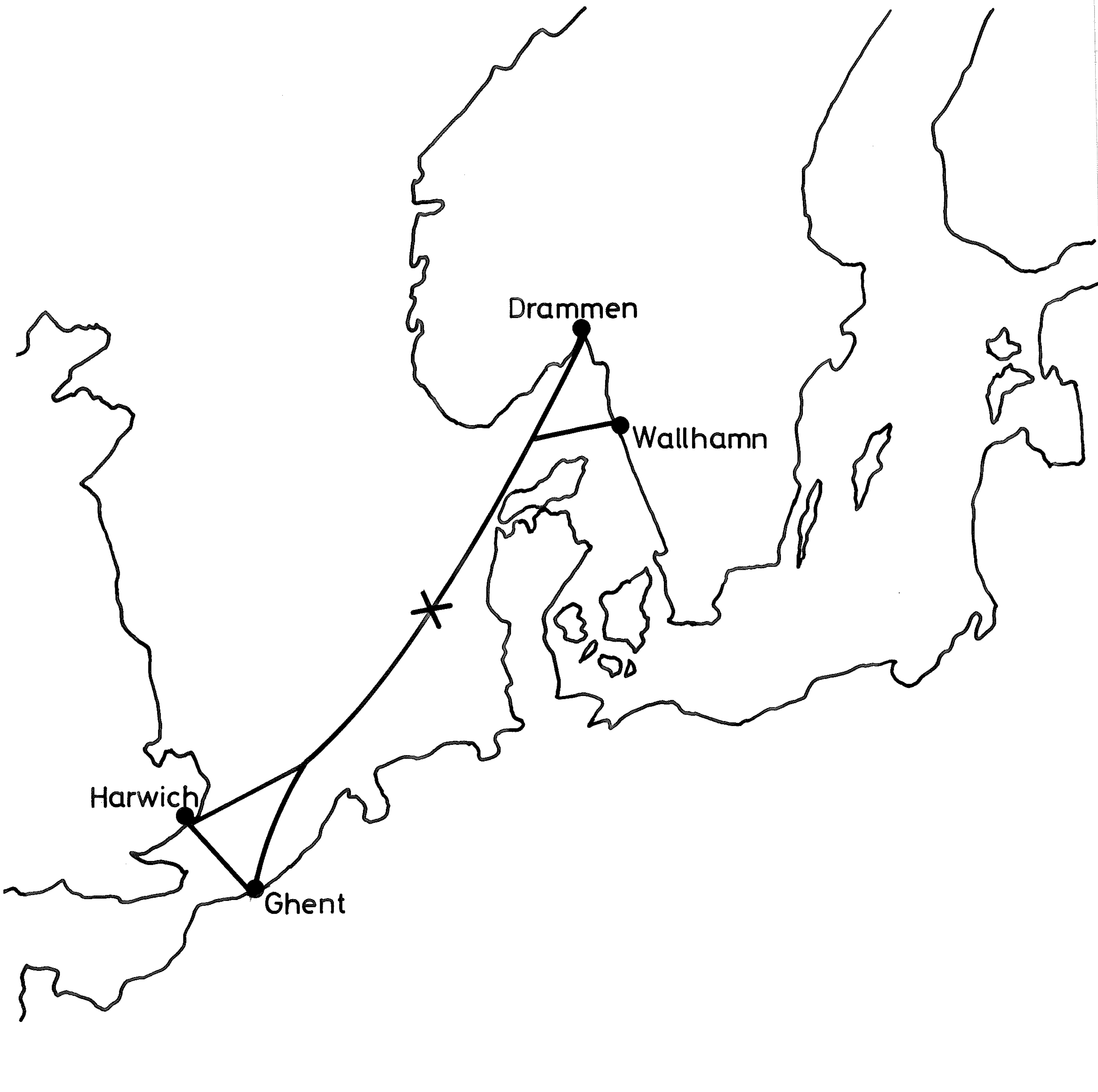

Fig. 3.1 - The route of the ship. The experiments were performed at the position marked X.

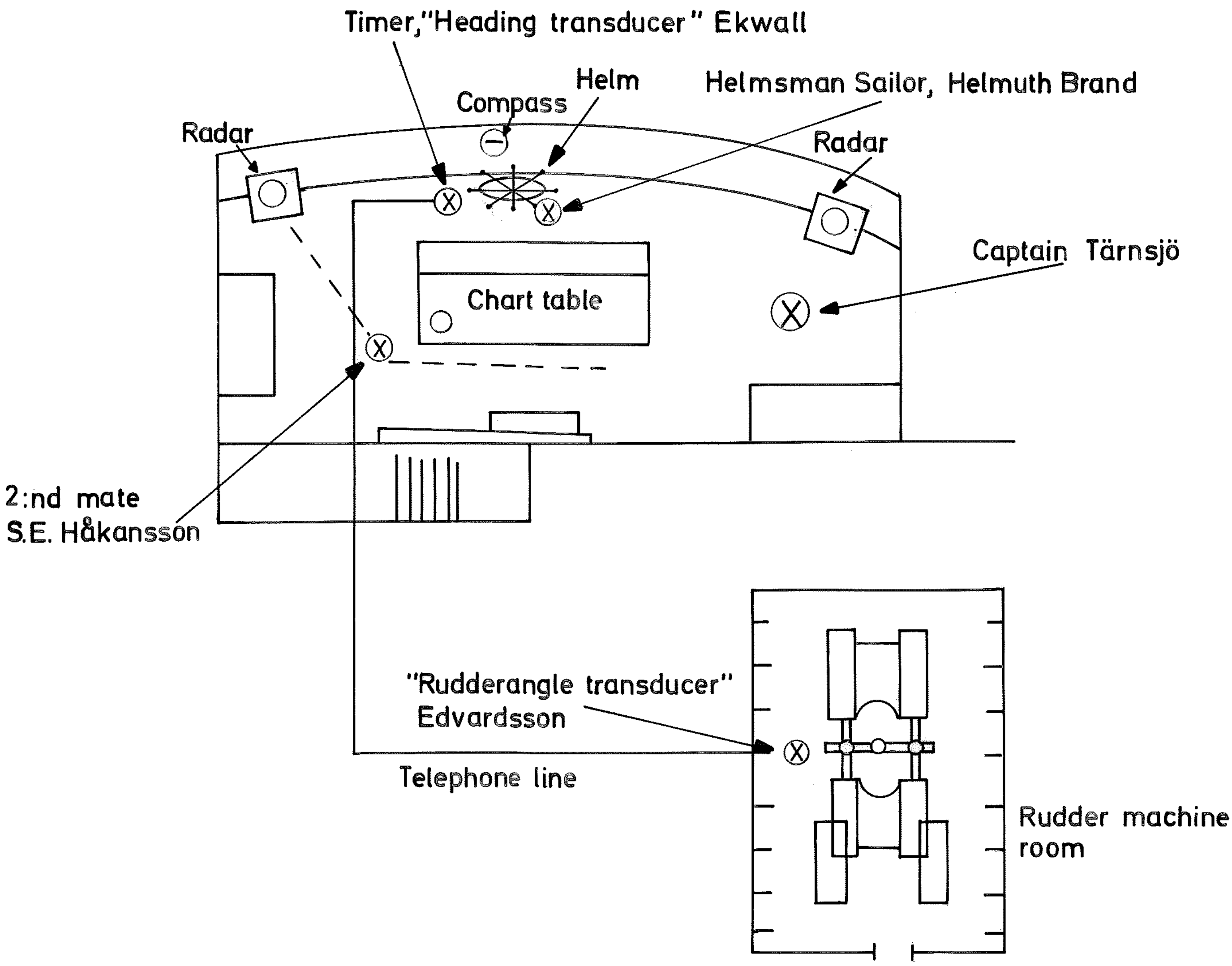

Fig. 3.2 - Outline of experimental arrangement.

30 cm corresponding to a rudder angle of 20°. The rudder angle could conveniently be read with an error less than 0.1°.

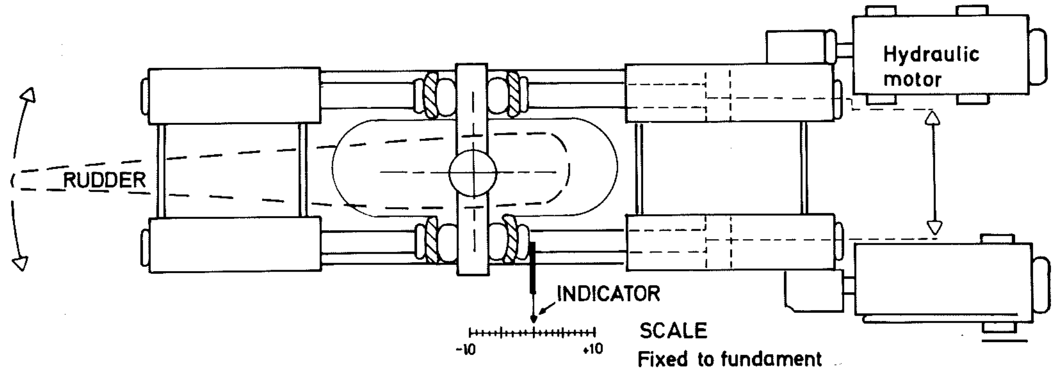

Fig. 3.3 - Arrangement for measuring rudder angle.

The input signal was chosen as two periods of a PRBS signal with a period of N = 64. Mr. Ekwall had the signal recorded on a table. The amplitude changed somewhat during the experiment. The peak-to-peak amplitude was between 10° and 15°. Before the experiment was started the "null position", i.e. the rudder position which gave a straight line course, was determined. This was done by steering the ship for constant heading observations. A copy of the heading recording obtained before and during the experiment is shown in Fig. 3.4.

A list of the input-output data obtained in the experiment is shown in Table 3.1. A few readings of the heading and rudder angles were somewhat uncertain as is always the case when manual recording is used. These readings are underlined in Table 3.1. The input-output data obtained in the experiment is graphed in Fig. 3.5.

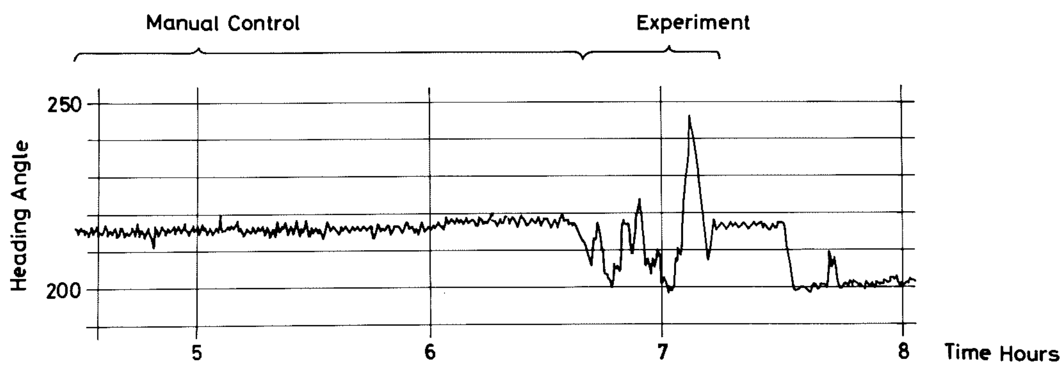

Fig. 3.4 - Recorded heading before, during and after the experiment.

T	U(T)	Y(T)	T	U(T)	Y(T)	T	U(T)	Y(T)	T	U(T)	Y(T)
0	-1.8	217.3	465	-6.6	197.7	930	-6.0	210.0	1395	5.8	193.7
15	-3.7	216.0	480	-6.4	200.2	945	6.7	204.0	1410	-8.6	198.0
30	-6.3	214.7	495	5.7	199.9	960	6.7	200.4	1425	-8.4	202.1
45	.1	211.5	510	-6.6	198.7	975	-8.4	200.6	1440	11.4	202.1
60	-8.4	206.0	525	-6.3	199.7	990	3.9	201.3	1455	-12.0	202.7
75	5.9	206.0	540	5.8	198.4	1005	-8.2	200.2	1470	-12.0	206.3
90	-6.3	205.0	555	5.8	198.0	1020	6.7	200.0	1485	12.5	204.0
105	5.8	201.6	570	5.8	202.3	1035	-10.2	198.0	1500	12.5	201.6
120	5.8	199.8	585	-6.7	207.9	1050	3.8	198.5	1515	12.5	207.0
135	5.8	202.0	600	2.0	211.4	1065	3.8	196.8	1530	-13.0	216.0
150	-6.7	206.2	615	2.1	213.6	1080	3.8	198.5	1545	11.3	214.0
165	-6.5	210.0	630	-6.5	216.0	1095	-8.4	201.0	1560	11.4	226.0
180	6.7	210.0	645	.0	218.2	1110	-8.2	203.8	1575	-13.5	233.0
195	6.7	209.7	660	.1	217.2	1125	5.8	201.0	1590	-13.5	238.7
210	-6.7	212.1	675	.1	216.7	1140	5.8	200.5	1605	8.2	241.4
225	-6.5	216.6	690	.2	216.6	1155	-8.7	203.5	1620	8.2	244.1
240	-6.4	216.6	705	-6.5	217.1	1170	-8.7	206.3	1635	8.2	249.0
255	-6.3	213.6	720	-6.3	215.9	1185	-8.4	204.0	1650	-12.5	257.0
270	5.8	210.0	735	-6.3	212.0	1200	-8.3	201.7	1665	-12.0	262.0
285	-6.5	206.6	750	5.7	208.0	1215	5.7	206.5	1680	-12.0	261.0
300	-6.3	206.9	765	5.8	205.1	1230	1.8	193.0	1695	-1.8	255.7
315	-6.2	203.5	780	5.8	206.0	1245	-8.7	194.5	1710	-1.8	250.0
330	5.8	200.0	795	5.8	209.0	1260	-8.5	195.6	1725	-1.7	246.2
345	-6.4	196.5	810	5.9	214.0	1275	5.8	193.0	1740	-1.7	243.6
360	5.8	196.5	825	-6.6	220.0	1290	-8.4	191.0	1755	-1.6	242.5
375	-6.5	195.5	840	-1.8	225.0	1305	5.8	191.5	1770	-10.2	240.5
390	-6.3	195.6	855	-6.5	226.1	1320	-6.5	190.0	1785	.0	237.0
405	6.7	193.4	870	-6.3	226.4	1335	-6.4	191.0	1800	-6.4	231.5
420	-6.6	192.0	885	-6.2	223.7	1350	5.8	190.1	1815	-6.3	228.0
435	5.8	192.7	900	-6.2	220.0	1365	-6.6	190.4	1830	-6.2	222.5
450	5.9	193.2	915	-6.1	215.1	1380	5.8	192.5	1845	-6.2	216.5
									1860	-6.2	211.1

Table 3.1 - Input-output data obtained in the experiment. The uncertain readings are underlined. (The original data set)

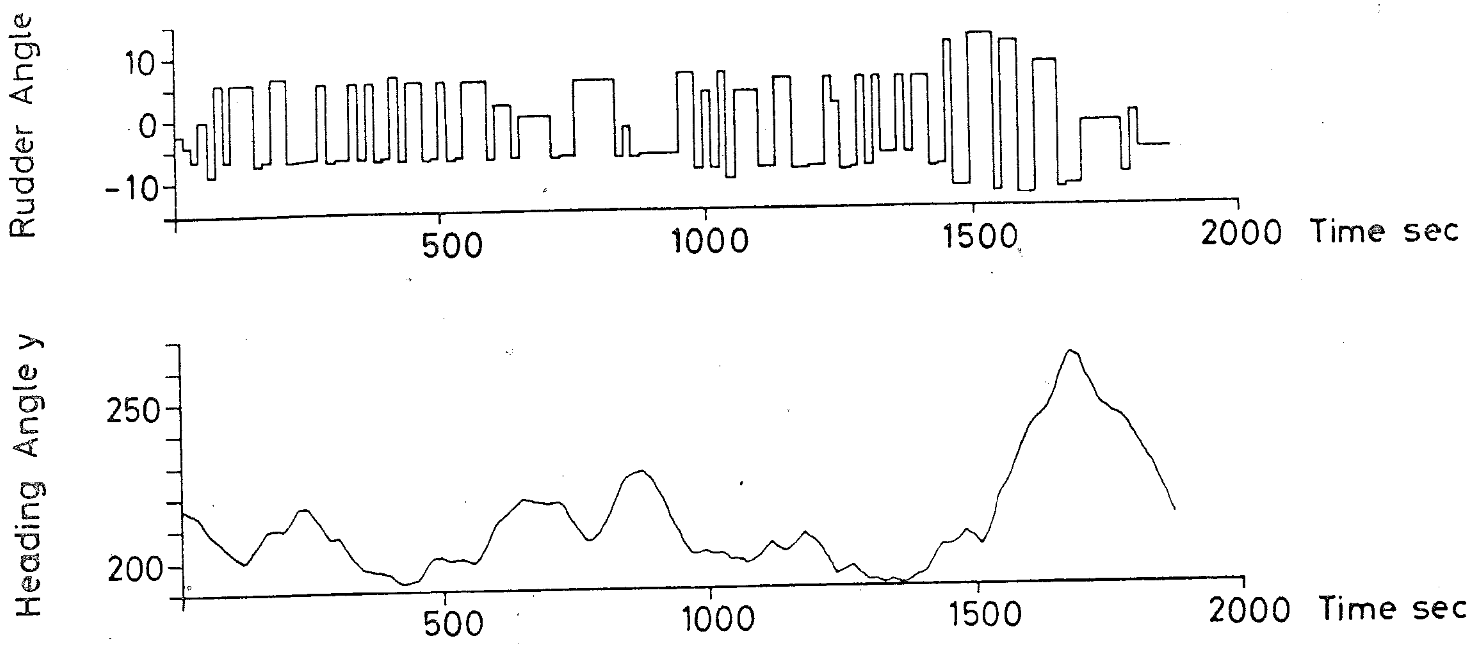

Fig. 3.5 - Input (rudder angle) and output (heading angle) signals obtained in the experiment.

4. MAXIMUM LIKELIHOOD IDENTIFICATION.

In a first attempt to obtain a parametric model we will neglect all physical knowledge about the system and simply determine a sampled input-output model given by

$$y(t) + a_1 y(t-1) + \dots + a_n y(t-n) =$$

$$= b_1 u(t-1) + \dots + b_n u(t-n) +$$

$$+ \lambda[e(t) + c_1 e(t-1) + \dots + c_n e(t-n)] \qquad (4.1)$$

from the measured input-output sequences using the method of maximum likelihood [2], [5].

A Straightforward Approach.

In order to estimate the order n of the system (4.1) the identification is repeated for different values of n. Table 4.1 gives the minimal value of the loss function versus n.

n	V_n	$F_{n/n-1}$
1	677.82	
2	232.83	76
3	226.28	1.1
4	208.31	3.2

Table 4.1 - Minimal value of loss function V_n and test quantities for models of different orders based on original data without fitting initial conditions.

In the table is also given the statistics defined by

$$F_{n/m} = \frac{V_m - V_n}{V_n} \cdot \frac{N - k_n}{k_n - k_m}, \qquad n > m \qquad (4.2)$$

where V_n is the minimal loss function for a model of order n, N the number of input-output pairs, k_n the number of parameters in a model of N:th order.

Under the assumption that the input-output data was actually generated by a model of type (4.1) where $\{e(t)\}$ is a sequence of normal random variables and some other regularity conditions it can be shown that $F_{n/m}$ for large N has an $F(N-k_n, k_n-k_m)$ distribution. At a risk level of 5% we have F(100,3) = 3.

If the assumption is satisfied we can then conclude from Table 4.1 that the reduction in the loss function is significant when n is increased from 1 to 2. The reduction obtained when increasing the order from 2 to 3 does not give a significant reduction in the loss function. Similarly we find $F_{4/2} = 2.21$. Hence a reduction of the system order from 2 to 4 is not significant either.

An analysis of the test quantities thus indicates that a second order model is consistent with the data if the assumption required for the tests are fulfilled.

The successive iterates of the estimates for the second order model are given in Table 4.2. This gives an indication of the convergence of the maximum likelihood estimate. The least squares estimate is taken as the initial estimate.

N	a_1	a_2	b_1	b_2
0	-1.372	0.378	0.142	0.256
1	-1.783	0.777	0.182	0.215
2	-1.745	0.750	0.180	0.198
3	-1.737	0.743	0.187	0.182
4	-1.730	0.735	0.191	0.179
5	-1.7296	0.73518	0.19108	0.17862
6	-1.729574	0.735193	0.191070	0.178442
7	-1.729580	0.735199	0.191072	0.178440

N	c_1	c_2	λ
0	0.000	0.000	2.50
1	-0.785	-0.126	
2	-0.849	0.160	
3	-1.004	0.400	
4	-1.005	0.393	
5	-1.00498	0.39303	
6	-1.006647	0.398059	
7	-1.006679	0.398070	

Table 4.2 - Successive iterates of the maximum likelihood estimate for the second order model.

A straightforward application of the maximum likelihood method thus leads to the following model:

$$
\begin{aligned}
a_1 &= -1.730 \pm 0.033 & a_2 &= 0.735 \pm 0.034 \\
b_1 &= 0.191 \pm 0.024 & b_2 &= 0.178 \pm 0.029 \\
c_1 &= 1.007 \pm 0.090 & c_2 &= 0.398 \pm 0.080
\end{aligned}
\qquad (4.3)
$$

The estimated parameter accuracies are obtained from the estimate of the covariances given by the second derivatives of the loss function.

Since the data-set is fairly short N = 125 the initial conditions may influence the results. To investigate this the initial conditions are introduced as parameters and fitted to the data. The values of the loss function obtained in this case are shown in Table 4.3.

n	V_n	$F_{n/n-1}$
1	651.04	
2	201.45	65.5
3	192.08	1.4
4	170.92	3.4

Table 4.3 - Minimal values of the loss function V_n and test quantities for models of different orders based on original data with initial conditions fitted.

It is also found that $F_{4/2} = 2.4$. The coefficients of the second order model are

$$
\begin{aligned}
a_1 &= -1.729 \pm 0.031 & a_2 &= 0.735 \pm 0.032 \\
b_1 &= 0.190 \pm 0.022 & b_2 &= 0.176 \pm 0.027 \\
c_1 &= -1.043 \pm 0.086 & c_2 &= 0.456 \pm 0.076
\end{aligned}
\qquad (4.4)
$$

It is found that the coefficients do not deviate significantly from those obtained when initial conditions are put equal to zero.

The results of the identification of the second order model are illustrated in Fig. 4.1. In this figure we

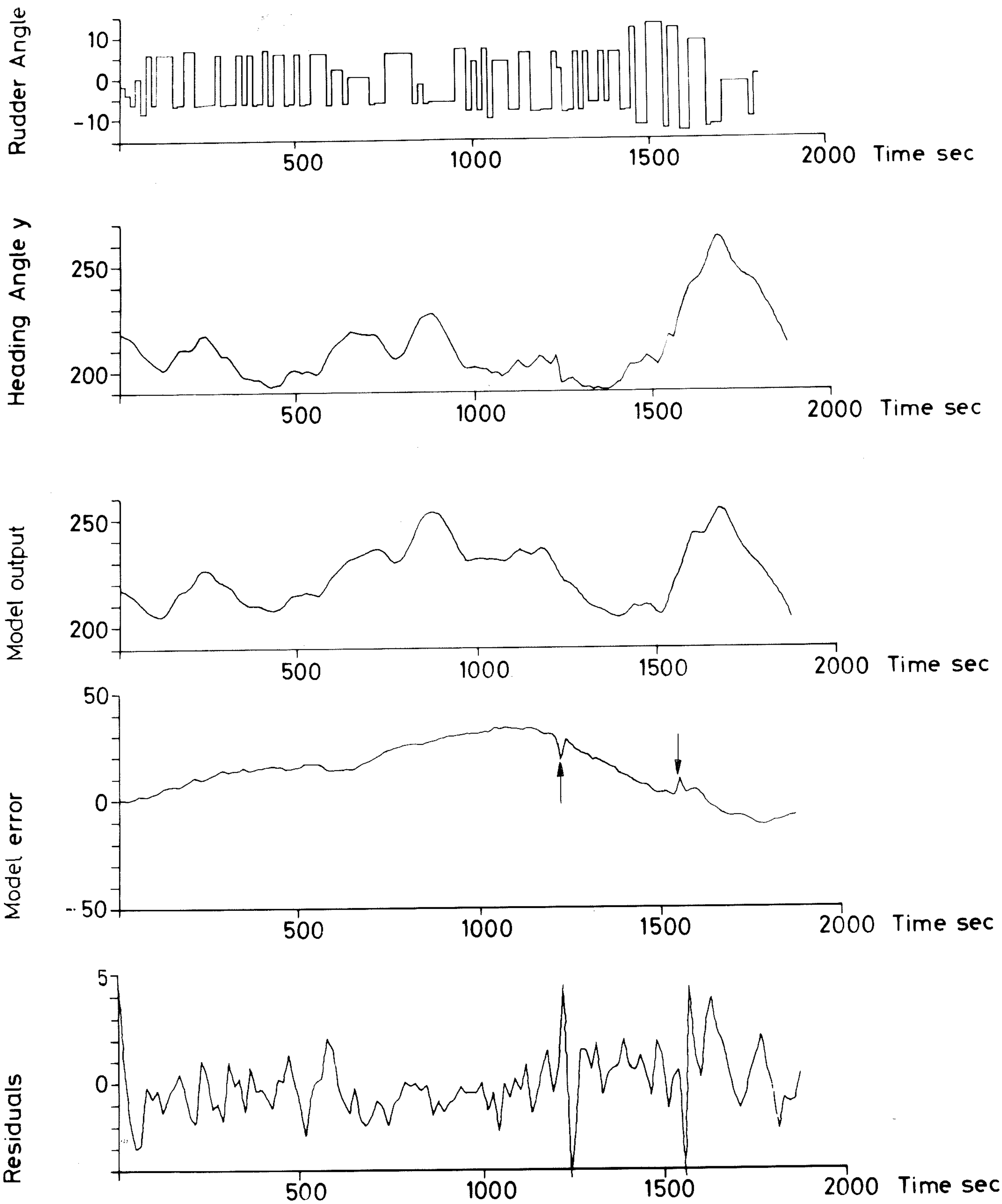

Fig. 4.1 - Illustration of the results of the identification of a second order model when the initial conditions are not fitted.

T	U(T)	Y(T)	*	T	U(T)	Y(T)	*	T	U(T)	Y(T)	*	T	U(T)	Y(T)
0	-1.8	217.3	*	465	-6.6	197.7	*	930	-6.0	210.0	*	1395	5.8	193.7
15	-3.7	216.0	*	480	-6.4	200.2	*	945	6.7	204.0	*	1410	-8.6	198.0
30	-6.3	214.7	*	495	5.7	199.9	*	960	6.7	200.4	*	1425	-8.4	202.1
45	.1	211.5	*	510	-6.6	198.7	*	975	-8.4	200.6	*	1440	11.4	202.1
60	-8.4	208.0	*	525	-6.3	199.7	*	990	3.9	201.3	*	1455	-12.0	202.7
75	5.9	206.0	*	540	5.8	198.4	*	1005	-8.2	200.2	*	1470	-12.0	206.3
90	-6.3	203.0	*	555	5.8	198.0	*	1020	6.7	200.0	*	1485	12.5	204.0
105	5.8	201.6	*	570	5.8	202.3	*	1035	-10.2	198.0	*	1500	12.5	201.6
120	5.8	199.8	*	585	-6.7	207.9	*	1050	3.8	198.5	*	1515	12.5	207.0
135	5.8	202.0	*	600	2.0	211.4	*	1065	3.8	196.8	*	1530	-13.0	216.0
150	-6.7	206.2	*	615	2.1	213.6	*	1080	3.8	198.5	*	1545	11.3	221.0
165	-6.5	210.0	*	630	-6.5	216.0	*	1095	-8.4	201.0	*	1560	11.4	226.0
180	6.7	210.0	*	645	.0	218.2	*	1110	-8.2	203.8	*	1575	-13.5	233.0
195	6.7	209.7	*	660	.1	217.2	*	1125	5.8	201.0	*	1590	-13.5	238.7
210	-6.7	212.1	*	675	.1	216.7	*	1140	5.8	200.5	*	1605	8.2	241.4
225	-6.5	216.6	*	690	.2	216.6	*	1155	-8.7	203.5	*	1620	8.2	244.1
240	-6.4	216.6	*	705	-6.5	217.1	*	1170	-8.7	206.3	*	1635	8.2	249.0
255	-6.3	213.6	*	720	-6.3	215.9	*	1185	-8.4	204.0	*	1650	-12.5	257.0
270	5.8	210.0	*	735	-6.3	212.0	*	1200	-8.3	201.7	*	1665	-12.0	262.0
285	-6.5	206.6	*	750	5.7	208.0	*	1215	5.7	197.4	*	1680	-12.0	261.0
300	-6.3	206.9	*	765	5.8	205.1	*	1230	1.8	193.0	*	1695	-1.8	255.7
315	-6.2	203.5	*	780	5.8	206.0	*	1245	-8.7	194.5	*	1710	-1.8	250.0
330	5.8	200.0	*	795	5.8	209.0	*	1260	-8.5	195.6	*	1725	-1.7	246.2
345	-6.4	196.5	*	810	5.9	214.0	*	1275	5.8	193.0	*	1740	-1.7	243.6
360	5.8	196.5	*	825	-6.6	220.0	*	1290	-8.4	191.0	*	1755	-1.6	242.5
375	-6.5	195.5	*	840	-1.8	225.0	*	1305	5.8	191.5	*	1770	-10.2	240.5
390	-6.3	195.6	*	855	-6.5	226.1	*	1320	-6.5	190.0	*	1785	.0	237.0
405	6.7	193.4	*	870	-6.3	226.4	*	1335	-6.4	191.0	*	1800	-6.4	231.5
420	-6.6	192.0	*	885	-6.2	223.7	*	1350	5.8	190.1	*	1815	-6.3	228.0
435	5.8	192.7	*	900	-6.2	220.0	*	1365	-6.6	190.4	*	1830	-6.2	222.5
450	5.9	193.2	*	915	-6.1	215.1	*	1380	5.8	192.5	*	1845	-6.2	216.5
			*				*				*	1860	-6.2	211.1

Table 4.4 - Input-output data for the adjusted data set. The changed data are underlined.
(The adjusted data set.)

T	U(T)	Y(T)	*	T	U(T)	Y(T)	*	T	U(T)	Y(T)	*	T	U(T)	Y(T)
0	-1.8	217.3	*	465	-6.6	197.7	*	930	-6.0	210.0	*	1395	5.8	193.7
15	-3.7	216.0	*	480	-6.4	200.2	*	945	6.7	204.0	*	1410	-8.6	198.0
30	-6.3	214.7	*	495	5.7	199.9	*	960	6.7	200.4	*	1425	-8.4	202.1
45	.1	211.5	*	510	-6.6	198.7	*	975	-8.4	200.6	*	1440	11.4	202.1
60	-8.4	208.0	*	525	-6.3	199.7	*	990	3.9	201.3	*	1455	-12.0	202.7
75	5.9	206.0	*	540	5.8	198.4	*	1005	-8.2	200.2	*	1470	-12.0	206.3
90	-6.3	203.0	*	555	5.8	198.0	*	1020	6.7	200.0	*	1485	12.5	204.0
105	5.8	201.6	*	570	5.8	202.3	*	1035	-10.2	198.0	*	1500	12.5	201.6
120	5.8	199.8	*	585	-6.7	207.9	*	1050	3.8	198.5	*	1515	12.5	207.0
135	5.8	202.0	*	600	2.0	211.4	*	1065	3.8	196.8	*	1530	-13.0	216.0
150	-6.7	206.2	*	615	2.1	213.6	*	1080	3.8	198.5	*	1545	11.3	221.0
165	-6.5	210.0	*	630	-6.5	216.0	*	1095	-8.4	201.0	*	1560	11.4	226.0
180	6.7	210.0	*	645	.0	218.2	*	1110	-8.2	203.8	*	1575	-13.5	233.0
195	6.7	209.7	*	660	.1	217.2	*	1125	5.8	201.0	*	1590	-13.5	238.7
210	-6.7	212.1	*	675	.1	216.7	*	1140	5.8	200.5	*	1605	8.2	241.4
225	-6.5	216.6	*	690	.2	216.6	*	1155	-8.7	203.5	*	1620	8.2	244.1
240	-6.4	216.6	*	705	-6.5	217.1	*	1170	-8.7	206.3	*	1635	8.2	249.0
255	-6.3	213.6	*	720	-6.3	215.9	*	1185	-8.4	204.0	*	1650	-12.5	257.0
270	5.8	210.0	*	735	-6.3	212.0	*	1200	-8.3	201.7	*	1665	-12.0	262.0
285	-6.5	206.6	*	750	5.7	208.0	*	1215	5.7	197.4	*	1680	-12.0	261.0
300	-6.3	206.9	*	765	5.8	205.1	*	1230	1.8	193.0	*	1695	-1.8	255.7
315	-6.2	203.5	*	780	5.8	206.0	*	1245	-8.7	194.5	*	1710	-1.8	250.0
330	5.8	200.0	*	795	5.8	209.0	*	1260	-8.5	195.6	*	1725	-1.7	246.2
345	-6.4	196.5	*	810	5.9	214.0	*	1275	5.8	193.0	*	1740	-1.7	243.6
360	5.8	196.5	*	825	-6.6	220.0	*	1290	-8.4	191.0	*	1755	-1.6	242.5
375	-6.5	195.5	*	840	-1.8	225.0	*	1305	5.8	191.5	*	1770	-10.2	240.5
390	-6.3	193.6	*	855	-6.5	226.1	*	1320	-6.5	190.0	*	1785	.0	237.0
405	6.7	193.4	*	870	-6.3	226.4	*	1335	-6.4	191.0	*	1800	-6.4	231.5
420	-6.6	192.0	*	885	-6.2	223.7	*	1350	5.8	190.1	*	1815	-6.3	228.0
435	5.8	192.7	*	900	-6.2	220.0	*	1365	-6.6	190.4	*	1830	-6.2	222.5
450	5.9	193.2	*	915	-6.1	215.1	*	1380	5.8	192.5	*	1845	-6.2	216.5
			*				*				*	1860	-6.2	211.1

Table 4.4 - Input-output data for the adjusted data set. The changed data are underlined. (The adjusted data set.)

n	V_n	$F_{n/n-1}$
1	362.59	
2	55.11	163
3	47.91	4.25
4	46.96	0.55

Table 4.5 - Minimal values of the loss function V_n and the test quantities $F_{n/n-1}$ for models of different order based on the adjusted data set. The initial state is also fitted.

the loss functions and the test quantities are sensitive.

The coefficients of the third order model are given below:

$a_1 = - 2.28 \pm 0.10$

$a_2 = 1.65 \pm 0.18$

$a_3 = - 0.36 \pm 0.07$

$b_1 = 0.153 \pm 0.012$

$b_2 = 0.156 \pm 0.024$

$b_3 = - 0.195 \pm 0.024$

$c_1 = - 0.99 \pm 0.15$

$c_2 = 0.29 \pm 0.16$

$c_3 = 0.01 \pm 0.10$

The results of the identification of a third order model are illustrated in Fig. 4.2. A comparison between the Figures 4.1 and 4.2 shows a significant improvement in the residuals.

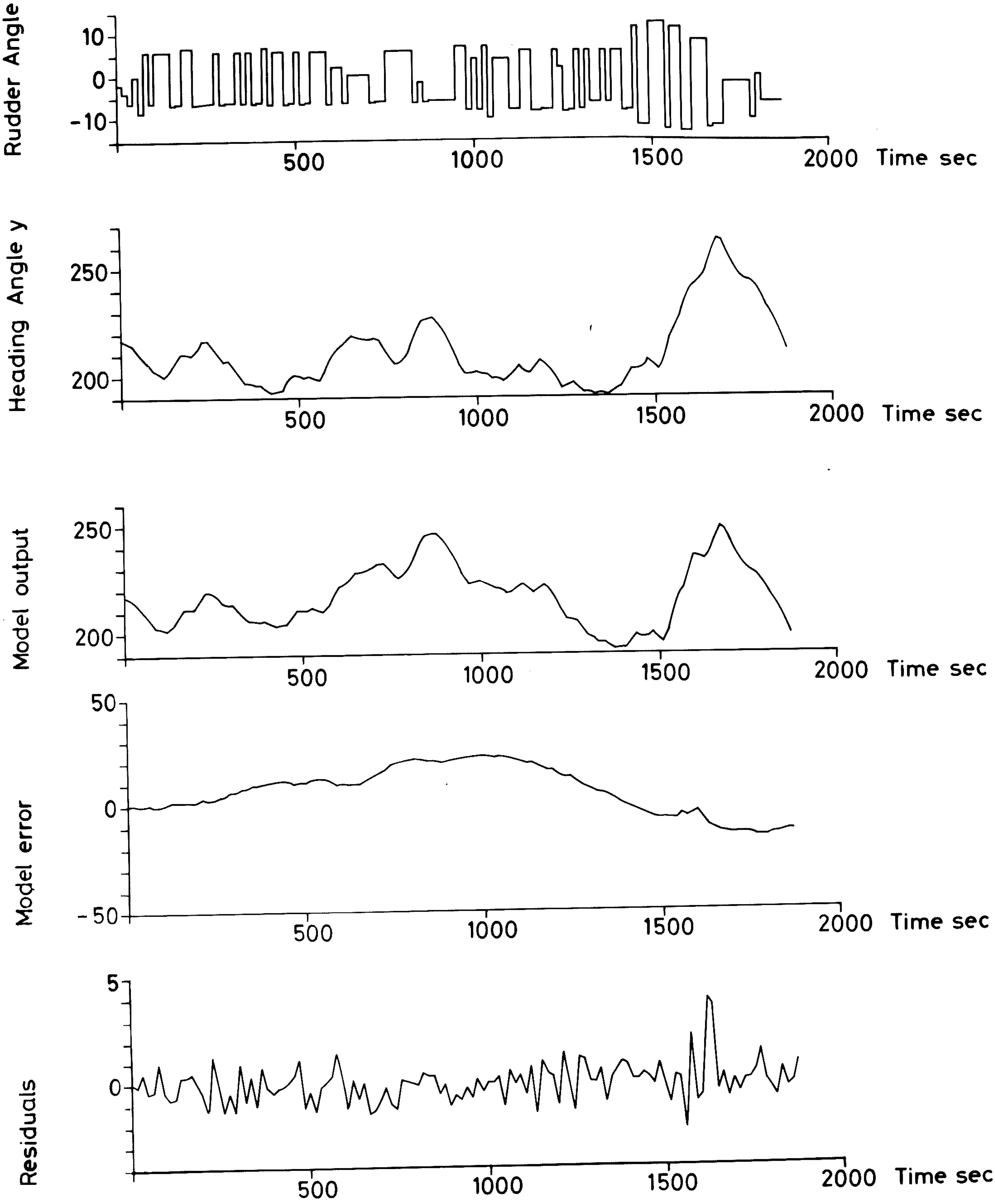

Fig. 4.2 - Results of identification of a third order model from the adjusted input-output data. The initial state of the model is also estimated.

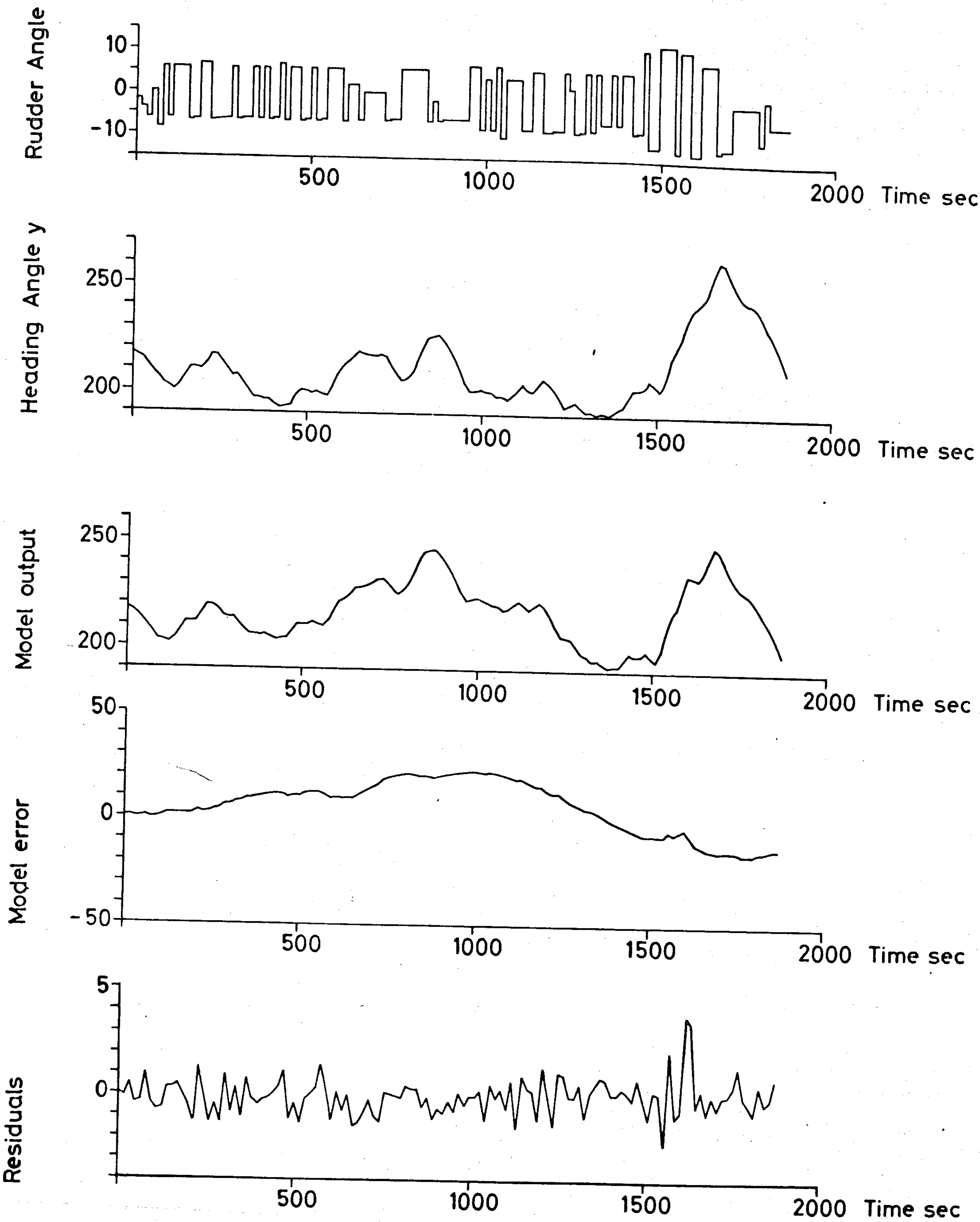

Fig. 4.2 - Results of identification of a third order model from the adjusted input-output data. The initial state of the model is also estimated.

Computations Based on 100 Data Points.

Notice that in Fig. 4.2 the residuals at time 1500 - - 1700 are still somewhat large. One explanation may be that the speed was reduced to 18 knots at time 1485. For this purpose we will also carry out the identification for a data set that is truncated. Four cases labelled A, B, C and D will be considered:

A. Original data. Initial state set equal zero.

B. Original data. Initial state is estimated.

C. Adjusted data. Initial state set equal zero.

D. Adjusted data. Initial state is estimated.

The loss functions and the test quantities are shown in Table 4.6. It is clear from this table that there is a significant difference in the loss functions when the initial state is estimated and when it is not. This is not surprising since the system contains an integrator.

n	Case A		Case B	
	V_n	$F_{n/n-1}$	V_n	$F_{n/n-1}$
1	462.59		371.40	
2	190.17	45	102.36	61
3	179.14	1.8	91.02	2.7
4	175.61	0.6	88.06	0.7
	Case C		Case D	
1	336.01		207.79	
2	112.01	63	24.98	180
3	101.78	3.0	18.01	8.6
4	101.10	0.2	16.49	1.9

Table 4.6 - Loss functions and test quantities for models of different orders in the cases A, B, C and D.

It is also clear from Table 4.6 that the loss function is decreased significantly when the data is adjusted. Compare cases A and B with C and D respectively. Assuming that the order test can be applied, it then follows that a second order model would be selected in all cases except case D where a third order model would be preferred. The coefficients of the second order model are summarized in Table 4.7 and those of the third order models in Table 4.8. Analysing the numbers in Table 4.7 and Table 4.8 we find that even if there are significant differences in the loss functions (V) in the different cases the model parameters a_i and b_i do not change significantly. The parameters c_i do, however, differ significantly. This is quite natural since these parameters are used to describe the disturbances. In Fig. 4.3 we illustrate the results of the identification in case D. An analysis of the covariance of the residuals in this case shows that the largest normalized covariance is 0.12 which is well inside the 5% confidence band (-0.20, -0.20).

	Case A	Case B	Case C	Case D
a_1	-1.681±0.044	-1.671±0.028	-1.632±0.049	-1.586±0.025
a_2	0.710±0.044	0.699±0.028	0.661±0.050	0.610±0.026
b_1	0.100±0.034	0.118±0.025	0.131±0.026	0.141±0.013
b_2	0.260±0.041	0.248±0.029	0.270±0.031	0.284±0.013
c_1	-0.942±0.117	-1.127±0.106	-0.607±0.124	-0.316±0.128
c_2	0.175±0.113	0.450±0.109	0.027±0.109	0.159±0.112
λ	1.95	1.43	1.43	0.707
V	190.2	102.4	112.0	25.0

Table 4.7 - Coefficients of the second order models obtained in the cases A, B, C and D.

	Case A	Case B	Case C	Case D
a_1	-2.104±0.475	-2.258±0.067	-2.162±0.242	-2.184±0.041
a_2	1.365±0.821	1.632±0.120	1.459±0.417	1.493±0.071
a_3	-0.244±0.361	-0.361±0.055	-0.281±0.183	-0.294±0.032
b_1	0.099±0.032	0.113±0.024	0.133±0.024	0.142±0.011
b_2	0.258±0.078	0.224±0.042	0.218±0.050	0.214±0.016
b_3	-0.189±0.105	-0.213±0.028	-0.204±0.057	-0.212±0.015
c_1	-1.387±0.501	-1.775±0.134	-1.192±0.265	-1.080±0.111
c_2	0.523±0.540	1.104±0.229	0.311±0.238	0.288±0.147
c_3	-0.049±0.150	-0.248±0.120	-0.022±0.101	0.062±0.114
λ	1.89	1.35	1.43	0.60
V	179.1	91.0	101.8	18.0

Table 4.8 - Coefficients of the third order models obtained in the cases A, B, C and D.

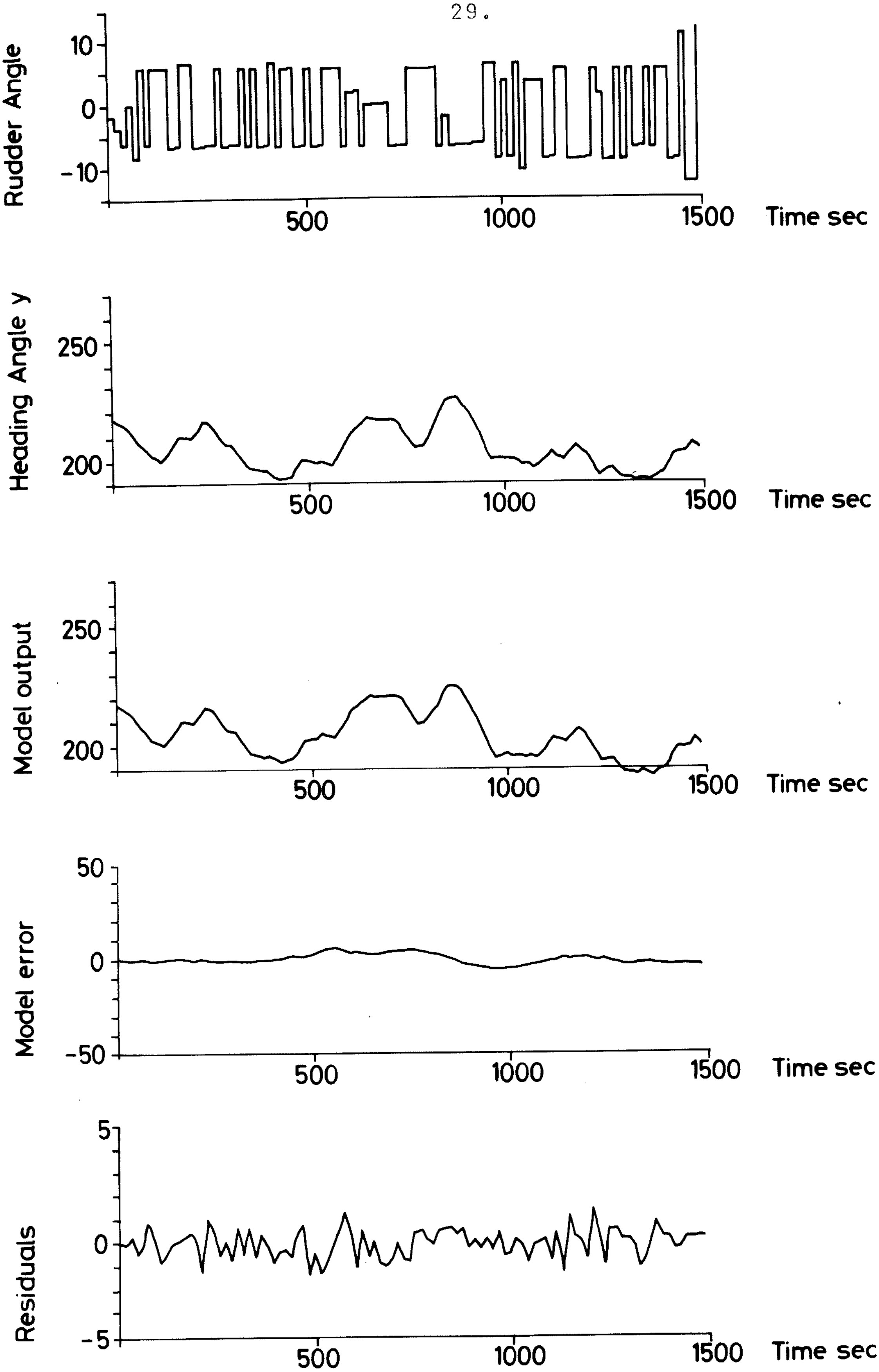

Fig. 4.3 - Illustrates the results of the identification of a third order model based on the truncated (0 - 1500) part of the adjusted data set.

5. EXPLOITING PHYSICAL KNOWLEDGE.

A direct application of the maximum likelihood method indicates that a model of second or third order is consistent with the data. Notice, however, that the technique used in Chapter 4 is a "black-box" method in the sense that only inputs and outputs are considered and that no physical knowledge about the system is exploited. Comparing with the physical models discussed in Chapter 2 we find that they are of third order. The physical model contains a pure integrator. An analysis of the models obtained by the maximum likelihood method shows that the corresponding pulse transfer functions have poles close to one. Hence there are at least some similarities between the physical models and the models obtained from the identification procedure.

It seems feasible to try to combine physical models and identification techniques. One way to do this is to try to fit a model of the form (2.4) to the observed input-output data. As was shown in Section 2 it is, however, not possible to determine all the parameters of the model from input-output data. We will therefore first change the coordinates in state space so that we get a model such that all parameters can be determined from input-output data. Introduce the transformation

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & -a_{11} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \\ \varphi \end{bmatrix} \tag{5.1}$$

The equation (2.4) is then transformed to

$$\frac{d}{dt}\begin{bmatrix}\omega\\ x\\ \varphi\end{bmatrix} = \begin{bmatrix}-a_1' & 1 & 0\\ -a_2' & 0 & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\omega\\ x\\ \varphi\end{bmatrix} + \begin{bmatrix}b_1'\\ b_2'\\ 0\end{bmatrix}\delta \qquad (5.2)$$

where

$x = a_{21}v - a_{11}\omega$

$a_1' = -(a_{11}+a_{22})$

$a_2' = a_{11}a_{22} - a_{12}a_{21}$

$b_1' = b_2$

$b_2' = a_{21}b_1 - a_{11}b_2$

If we consider the rudder angle δ to be the input and the heading angle φ the output, the input-output relation of the dynamical system (5.2) can be represented by the transfer function

$$G(s) = \frac{b_1's + b_2'}{s(s^2+a_1's+a_2')} \qquad (5.3)$$

and we conclude that all the parameters a_1', a_2', b_1' and b_2' of the system (5.2) are identifiable.

The Structural Identification Computer Program.

Consider the time invariant, linear, stochastic state model

$$\begin{cases} dx = Axdt + Budt + Kde \\ dy^* = Cxdt + Dudt + de \end{cases} \qquad (5.4)$$

with initial state $x(t_0)$, where u is an m-vector, x an n-vector, y^* an r-vector and $\{e(t)\}$ an r-dimensional Wiener-process. Suppose that we measure the output vector $y(t) = dy^*/dt$ only at the times $t = t_0$, $t_0 + \tau$, $t_0 + 2\tau$, ..., where τ is the sampling interval, and suppose that the control variables are constant over the sampling intervals, then the system (5.4) can be described at the sampling instants by the following stochastic difference equations

$$\begin{cases} x(t+\tau) = \tilde{A}x(t) + \tilde{B}u(t) + \tilde{K}\tilde{e}(t) \\ y(t) = Cx(t) + Du(t) + \tilde{e}(t) \end{cases} \tag{5.5}$$

where $\{\tilde{e}(t), t = t_0, t_0 + \tau, \ldots\}$ is a sequence of independent Gaussian vectors with zero mean value.

If the system (5.5) is assumed to contain no state variable noise, then put the matrix K equal to zero.

Now, let the elements of the matrices A, B, C, D, K and the initial state vector $x(t_0)$ depend on parameters α_i, $i = 1, \ldots, \ell$ and define a loss function (See ref. [3]).

$$V(\alpha_1, \ldots, \alpha_\ell) = \det\left\{\sum_{i=0}^{N-1} \tilde{e}(t_0+i\tau)\tilde{e}^T(t_0+i\tau)\right\} \tag{5.6}$$

N is the number of sample events and the residuals $\tilde{e}(t)$ are defined as

$$\tilde{e}(t) = y(t) - Cx(t) - Du(t) \tag{5.7}$$

where y(t) is the recorded output vector.

The computer program now tries to fit the parameters α_i, $i = 1, \ldots, \ell$ in such a way that the loss function V becomes as small as possible.

Result of Identification.

The computer program is provided with the following model (cf. (5.2)):

$$\begin{bmatrix} d\omega \\ dx \\ d\varphi \end{bmatrix} = \begin{bmatrix} -\frac{V}{\ell}\,\alpha_1 & 1 & 0 \\ -\left(\frac{V}{\ell}\right)^2 \alpha_2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ x \\ \varphi \end{bmatrix} dt + \begin{bmatrix} \left(\frac{V}{\ell}\right)^2 \alpha_3 \\ \left(\frac{V}{\ell}\right)^3 \alpha_4 \\ 0 \end{bmatrix} \delta dt + \begin{bmatrix} \alpha_5 \\ \alpha_6 \\ \alpha_7 \end{bmatrix} de$$

$$dy^* = [0 \quad 0 \quad 1] \begin{bmatrix} \omega \\ x \\ \varphi \end{bmatrix} dt + de \qquad (5.8)$$

$$\begin{bmatrix} \omega(t_0) \\ x(t_0) \\ \varphi(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 217.3 \end{bmatrix}$$

ℓ = length of the ship = 197 m

V = the ship velocity = 9.26 m/s

The sampling interval τ is 15 s. The normalizing factors are chosen in such a way that the parameters α_1, α_2, α_3 and α_4 are dimensionfree (cf. (2.5)). The value of $\varphi(t_0)$ is picked up so the residual at the time t_0, $\tilde{e}(t_0)$, is equal to zero. Only the adjusted data set is used.

The structural identification leads to the following model

$$\alpha_1 = 0.4434 \qquad \alpha_2 = 0.0365$$
$$\alpha_3 = 0.8944 \qquad \alpha_4 = -0.0269 \qquad (5.9)$$

$$\tilde{K} = \begin{bmatrix} 163.0607 \\ -2.8145 \\ 1.0522 \end{bmatrix}$$

with minimum value of the loss function

$$V_{min} = 60.65$$

The value $V_{min} = 60.65$ is comparable with the value $V_3 = 47.91$ of Table 4.5. The difference is due to the fact that V_{min} is obtained from a system with less freedom degrees caused by the special structure of the system.

In Fig. 5.1 the results of the identification are illustrated. In this figure we show the recorded input (rudder angle), the recorded output (heading angle), the model output y_d, the model error $y-y_d$ and the residuals $\tilde{e}(t)$. The model output is the response of the system

$$\frac{d}{dt}\begin{bmatrix} \omega \\ x \\ \varphi \end{bmatrix} = \begin{bmatrix} -\frac{V}{\ell}\alpha_1 & 1 & 0 \\ -\left(\frac{V}{\ell}\right)^2\alpha_2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \omega \\ x \\ \varphi \end{bmatrix} + \begin{bmatrix} \left(\frac{V}{\ell}\right)^2\alpha_3 \\ \left(\frac{V}{\ell}\right)^3\alpha_4 \\ 0 \end{bmatrix}\delta$$

$$y = [0 \quad 0 \quad 1]\begin{bmatrix} \omega \\ x \\ \varphi \end{bmatrix} \qquad (5.10)$$

$$\begin{bmatrix} \omega(t_0) \\ x(t_0) \\ \varphi(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 217.3 \end{bmatrix}$$

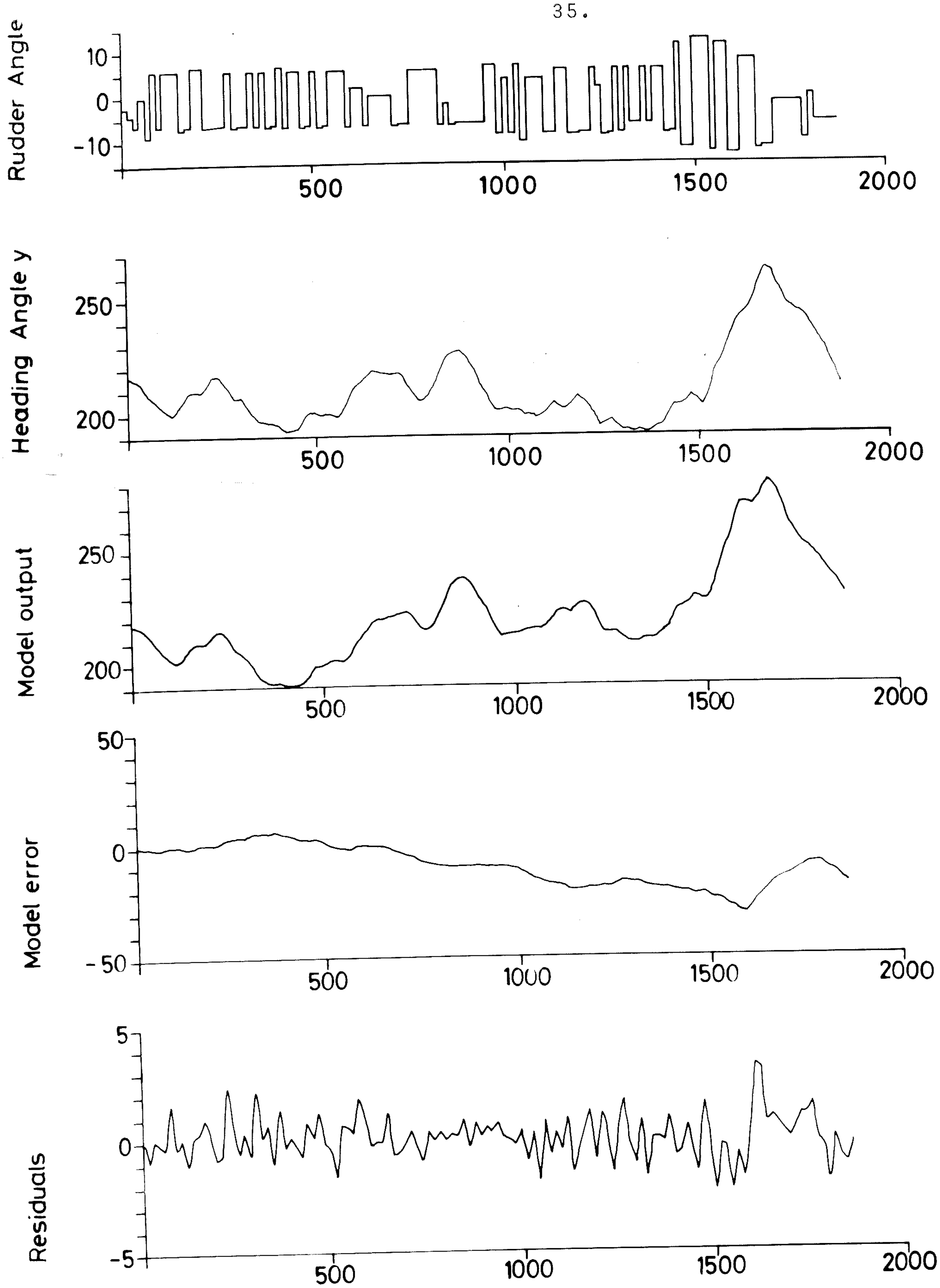

Fig. 5.1 - Results of fitting a model (5.8) to the adjusted input-output set.

with the parameter values (5.9) to the rudder angle input.

An estimate of the variance of the residuals is obtained by dividing V_{min} with the number of sample events, i.e. 125:

$$\mathrm{var}\{\tilde{e}(t)\} = 0.97$$

The transfer function of the system (5.10) is obtained quite analogous to (5.2):

$$G(s) = \frac{\left(\frac{V}{\ell}\right)^2 \alpha_3 s + \left(\frac{V}{\ell}\right)^3 \alpha_4}{s\left(s^2 + \frac{V}{\ell}\alpha_1 s + \left(\frac{V}{\ell}\right)^2 \alpha_2\right)} \qquad (5.11)$$

The value of G(s) is zero, when

$$s = -\left(\frac{V}{\ell}\right)\frac{\alpha_4}{\alpha_3} = 0.0014$$

The fact that G(s) has a zero in the right half plane indicates that the system (5.10) is not a minimum phase system.

The eigenvalues of the system (5.10) are obtained as the roots of the equation

$$s\left(s^2 + \frac{V}{\ell}\alpha_1 s + \left(\frac{V}{\ell}\right)^2 \alpha_2\right) = 0 \qquad (5.12)$$

We get the eigenvalues

$$s_1 = -\left(\frac{V}{\ell}\right)0.334 = -0.115$$

$$s_2 = -\left(\frac{V}{\ell}\right)0.109 = -0.00517$$

$$s_3 = 0$$

and the two time constants

$$t_1 = -1/s_1 = 63.7 \text{ s}$$

$$t_2 = -1/s_2 = 194.8 \text{ s}$$

6. ACKNOWLEDGEMENTS.

We would like to express our gratitude to the Wallenius Lines for their positive attitude towards research of this type and their willingness to allow experiments to be performed on their ship. We are also grateful to Mr. Ulf Ekwall and Mr. Anders Edvardsson who made the experiment.

The programs used for the MLE identification were written by Gustavsson [5].

The report has been expertly typed by Mrs. G. Christensen and the figures are drawn by Miss M. Steinertz.

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