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EXPERIMENTS WITH COMPUTER CONTROL OF
ROOM AIR TEMPERATURE

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ABSTRACT

Various regulators for controlling room air temperature from radiator effect are discussed and tested experimentally on a full scale test room. The regulators are implemented on a process computer. A mathematical model of the heating dynamics is derived and used to determine optimal feedback regulators. These regulators are shown to have better performance on the test room than conventional control.

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1 INTRODUCTION

The heating of a room by an electric radiator is usually not a difficult control problem. A simple thermostat keeps the desired temperature within $\pm 1^{\circ}\text{C}$ under normal conditions. However, this may not be sufficient in some applications.

In this report various regulators for room air temperature control are discussed and tested experimentally. The regulators are implemented on a process computer. Regulators that are based on modern control theory and derived from a model of the heating process are compared with conventional ones.

The control experiments have been carried out on a special test room which is further described in section 2. In this section also the control equipment is discussed. In section 3 conventional control and the possible need for more elaborate regulators are discussed. A model of the system to be controlled is a central concept in modern control theory. In section 4 a discrete time model of the dynamics relating room air temperature to radiator effect is derived. For the identification the Maximum Likelihood method is used. The validity of the model for control design purposes is investigated in section 5. Here also optimal feedback regulators are derived and tested experimentally. Finally, in section 6, the different regulators are evaluated and compared.

2 EXPERIMENT EQUIPMENT

The control experiments have been carried out on a special full scale experiment room. The room is heated by electrical radiators, which are controlled by a process computer (PDP-15). In section 2.1 the experiment room is described and in section 2.2 its control equipment is discussed. Computer hardware and software are described in section 2.3.

2.1 Experiment room

The experiment room has been developed by the division of Building Science, Lund Institute of Technology, Adamson (1969). A schematic picture of the room is given in fig. 2.1.

The room has a length of 4.5 m, a width of 3.6 m and a height of 3.0 m. It is connected to outdoor air through a window and a front wall. The other three walls of the room divide the room from another room. All the walls, the ceiling and the floor are built with only one half of the normal thickness. This is done so that the masses connected to the room air should be the same as if the room was surrounded by similar rooms under the same circumstances. More details are given in table 2.1. The room is heated by two electrical radiators situated under the window. The power used in the radiators is governed by a thyristor device from 0 W to 2000 W. The average heating power was approximately 1000 W for all experiments. The room is further equipped with a fan which leads outdoor air into the room. The capacity of the fan allows the room air to be changed 10-11 times per hour. The fan and an extra 500 W-radiator are controlled by relays from the computer.

2.2 Measurement and control equipment

Thermistors were used to measure the room air temperature and wall temperatures. The measurement interval is 15 to 35 degrees centigrade and with an accuracy of ± 0.06 degrees centigrade. The room air temperature is measured in the middle of the room.

Table 2.1 Summary of building data from the testroom at the division of Building Science.

Surface	Materials	Thickness m	Heatcond. W/°Cm	Heatcap. Wh/°Cm ³
Floor 16.2 m ²	Reinforced concrete	0.125	1.57	560
	Mineral wool (Gullifiber S200, 200 kg/m ³)	0.20	0.0406	48.8
Ceiling 16.2 m ²	Reinforced concrete	0.125	1.51	560
	Mineral wool (Gullifiber 3004, 16 kg/m ³)	0.20	0.0406	3.90
Partition walls 2 · 13.4 m ²	Light concrete	0.075	0.151	125
	Mineral wool 16 kg/m ³	0.10	0.0406	3.90
Corridor wall 8.7 m ²	Light concrete	0.075	0.151	125
	Mineral wool 16 kg/m ³	0.10	0.0406	3.90
Door to corridor 1.5 m ²	Masonite	0.004	0.13	465
	Air	0.04	0.0256	0.30
	Masonite	0.004	0.13	465
Door window 0.5 m ²	Machine glass P3	0.002	0.81	605
	Light concrete	0.25	0.151	125
Window 7.6 m ²	Machine glass	0.003	0.81	605
	Air	0.009	0.0256	0.30
	Machine glass	0.003	0.81	605

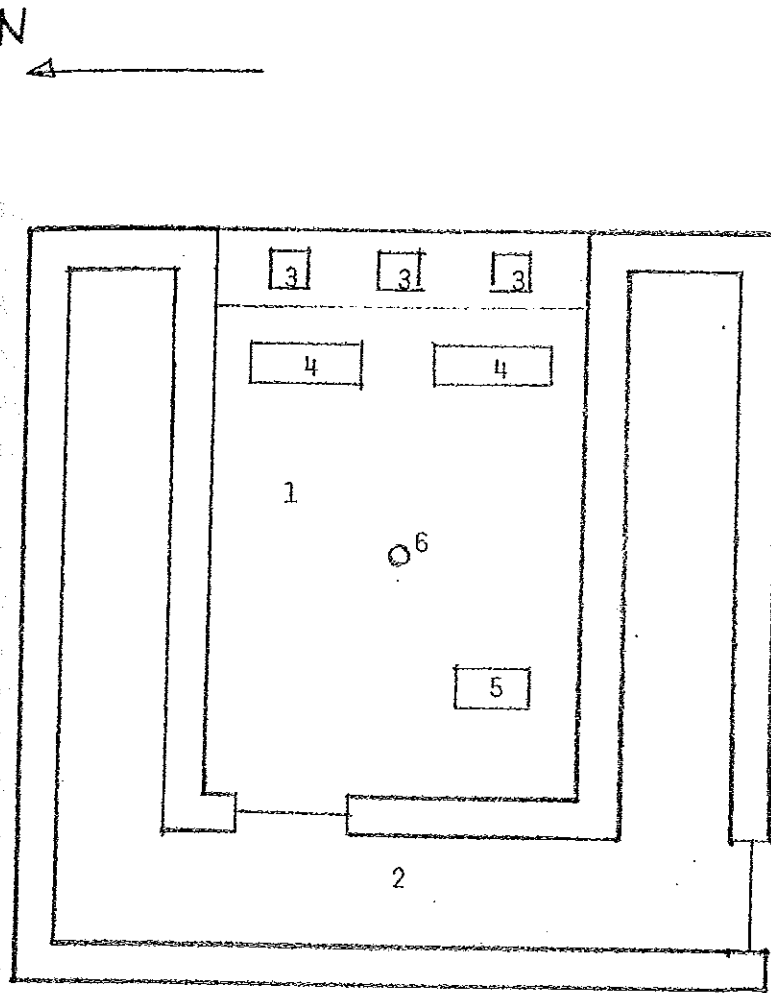


Figure 2.1 A schematic picture of the test room seen from above. Testroom (1), surrounding room (2), air inlet (fan) (3), electrical radiator 1 kW (4), electrical radiator 0.5 kW (5) and room air temperature sensor (6).

The heating power is not measured directly, since it is proportional to an analog control signal to the thyristor device. At the room the measurements and the control are carried out by a remote computer device, called coupler/controller (HP 2570A), Jensen (1973 a). The coupler/controller is connected to a PDP-15 computer 400 m away.

The coupler/controller measures analog signals from the thermistors with a digital voltmeter and sets out analog and logical outputs with a specially built interface. The time to measure one analog input is 2.6 seconds and to set out one analog output is 0.7 seconds. The low speed is due to the fact that the communication between the coupler/controller and the computer goes with 110 baud (normal teletypespeed). Further details are given in Jensen (1973 a).

2.3 Computer programs

The process computer used for the control is a PDP-15 (16k core memory, 256k disk and three Dectape units). The programs have been run under the time-sharing executive RSX. The core memory is divided into partitions, in which different programs are run on various priority levels. A program, which once has entered the core memory, will always run to completion before it leaves its partition.

To run the control experiment three programs are used. One program measures analog inputs and sets out analog and logical outputs with the coupler/controller. Between the input part and the output part a second and high priority program computes the new inputs from the latest inputs and outputs.

The third program updates regulator parameters and other experiment variables at every sampling-interval. Thus the control law parameters can be changed automatically to pre-

determined values. This is a useful property since the experiments have to be made over 10-12 hours during the night. Usually three different control laws were tested during one night.

One program logs the experiment and stores data on disk and on Dectape so that offline studies can be made. Another help program performs online plotting on a x-y-display of different experiment variables.

3 CONVENTIONAL CONTROL OF ROOM AIR TEMPERATURE

Room temperature is usually controlled by thermostats. For larger installations PID-controllers or variants thereof can be used. In many cases these regulators give reasonable constant room air temperature. In this section the performance of these regulators is shown when controlling the temperature of the experiment room. To be able to compare different regulators a test disturbance cycle is designed. Most "natural" disturbances are covered by the test disturbances. In section 3.1 natural disturbances are discussed and in section 3.2 the test cycle is described. The result of conventional control is shown in section 3.3.

3.1 Disturbances

The purpose of the control is to maintain a constant room air temperature under various disturbances. Some of the most important disturbances may be the following:

- Outdoor air temperature
- Sun radiation
- Wind
- Persons
- Illumination
- Machines
- Change of room air
- Random air circulations

Some of the disturbances appear more or less like step disturbances, while the heating process has a slow response. Since feedforward terms are not feasible, the control cannot be able to eliminate the disturbances without some overshoot.

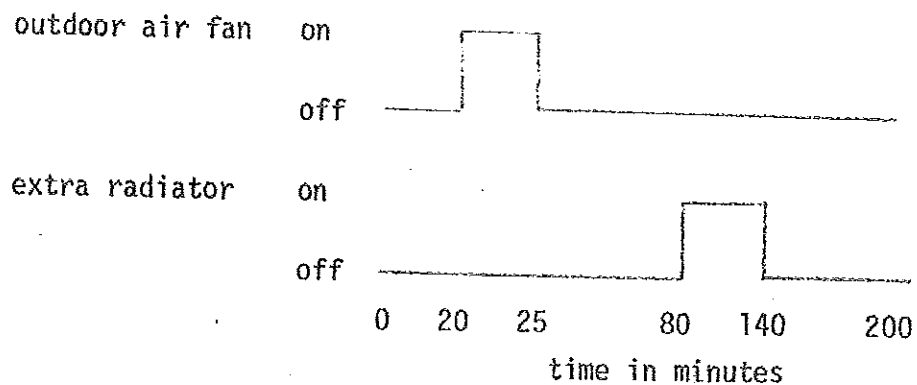
If a window is opened for a short while the temperature usually changes quickly. But the heat capacity in walls, ceiling

and floor together is in many cases 100 times larger the heat capacity of the room air. Therefore the air temperature resumes its old value even without control, due to heat exchange with the walls, ceiling and floor. Consequently the control law should react differently on this kind of disturbance than on a more long term disturbances like change of outdoor temperature. The temperature noise due to random aircirculations must also be regarded when a control is designed.

3.2 Imposed test disturbances

To test a control it would be convenient to artificially generate some of the earlier discussed disturbances in a full scale experiment.

To simulate that a door or a window is opened, outdoor air is fanned into the room during 5 minutes. To simulate steplike disturbances an extra 500 W-radiator is switched on for 60 minutes. A standard disturbance cycle, which is used in all experiments, consisted of both above mentioned disturbances in the following way:



In addition, some of the other disturbances listed above are present, which makes the experiments not exactly identical. The open loop response to the standard disturbance cycle is given in fig. 6.1.

3.3 Conventional control

In many cases it is sufficient with proportional control (P-control). At low gains the deviation of the output from the desired value can be large. At higher gains the deviation decreases but usually the system starts to oscillate. If the oscillations do not become too large the gain may be infinite and an on/off control results. The well known thermostat is of this type. In fig. 6.2 is shown how this regulator behaves.

Another way of eliminating the mean deviation and the oscillations is to use PI or PID control. One drawback with the PID regulator is that three parameters must be tuned, which may be time consuming. In fig. 6.3 is shown the result for a PID regulator tuned by the rule of Ziegler-Nichols, Harriot (1964). A main feature of the above controls is that none of them uses any information of old inputs.

The results of fig. 6.2 and 6.3 may for many applications be quite satisfactory. However, in some cases the requirements on constant temperature may be more restrictive. Notice also that the results are given for quite a small room. Usually the oscillations in temperature increase with the size of the application. It is therefore of interest to investigate if better control than shown in fig. 6.2. and 6.3 can be obtained. Modern control theory will be used to design regulators. For this purpose a model of the system is required.

4 DETERMINATION OF MODELS

The process under consideration is quite slow. It takes several hours to evaluate the performance of a regulator. Thus even the tuning of a straightforward PID regulator will be very time consuming. Consequently, simulation of different control laws on a good model of the process saves a lot of time. Also, more complex regulators may have to be used. This is often the case when there are time delays in the system. To determine such regulators a model is very valuable. Optimal control laws can be synthesized from the model, applying e.g. Linear Quadratic control theory.

Several different methods to obtain mathematical models of a system exist. In this section we will derive a model by fitting parameters to input output data from special experiments.

4.1 Identification method

A suitable process model for computer control purposes is

$$\begin{aligned}
 y(t) + a_1 y(t-1) + \dots + a_n y(t-n) = \\
 = b_1 u(t-k-1) + b_2 u(t-k-2) + \dots + b_n u(t-k-n) + \\
 + \lambda (e(t) + c_1 e(t-1) + \dots + c_n e(t-n))
 \end{aligned}
 \tag{4.1}$$

where $(u(t), y(t), t=1, 2, \dots, N)$ is in the input output sequence. The sequence $(e(t), t=1, 2, \dots, N)$ is a sequence of independent normal (0.1) random variables, which lumps together all kinds of random disturbances acting on the process. In the present case y is the room air temperature in $^{\circ}\text{C}$ (relative to \bar{y}) and u is the effect to the electrical radiator in kW (relative to \bar{u}). The temperature \bar{y} is the steady state temperature when $u(t) \equiv \bar{u}$. Usually \bar{u} is chosen as 1 kW which gives different \bar{y}

depending on outdoor temperature.

Naturally, disturbances like change of room air cannot be modelled by the $e(t)$ -variables. Therefore such disturbances must be avoided during the identification experiment. The $e(t)$ -variables will then mainly model random air circulation.

The parameters a_i , b_i , c_i , λ , k and n can be determined by the Maximum Likelihood (ML) method, Aström, Bohlin (1965), Gustavsson (1969). In this method \hat{a}_i , \hat{b}_i , \hat{c}_i and \hat{k} are chosen for a given input output sequence so that

$$V(\hat{a}_i, \hat{b}_i, \hat{c}_i) = \sum_{t=1}^N \varepsilon^2(t)$$

is minimized for fixed \hat{n} .

$\varepsilon(t)$ is calculated as

$$\begin{aligned} \varepsilon(t) + \hat{c}_1 \varepsilon(t-1) + \dots + \hat{c}_{\hat{n}} \varepsilon(t-\hat{n}) &= \\ = y(t) + \hat{a}_1 y(t-1) + \dots + \hat{a}_{\hat{n}} y(t-\hat{n}) & \\ - \hat{b}_1 u(t-\hat{k}-1) - \hat{b}_2 u(t-\hat{k}-2) - \dots - \hat{b}_{\hat{n}} u(t-\hat{k}-\hat{n}) & \end{aligned}$$

The number \hat{n} is increased until no more significant reduction in the minimal value of V occurs, Aström-Bohlin (1965).

4.2 Identification experiment and result

To achieve a suitable input output sequence $(u(t), y(t), t=1, 2, \dots, N)$ to perform the parameter estimation the input sequence must be chosen with some care.

Experiments on the test room have been performed with a PRBS (Pseudo Random Binary Signal) as input, Gustavsson (1971). The sampling interval was chosen as 1 minute, which will give suitable

models for control. The experiments and the results are thoroughly discussed in Jensen (1973 b). The minimizing parameter estimates for $0 \leq \hat{k} \leq 3$ and $1 \leq \hat{n} \leq 3$ are given in table 4.1. It turns out that suitable values of the parameters are

$$\begin{array}{llll} n=2 & a_1 = -1.664 & b_1 = 0.0488 & c_1 = -0.091 \\ k=3 & a_2 = 0.683 & b_2 = 0.0042 & c_2 = 0.1403 \\ \lambda = 0.0151 & & & \end{array}$$

The λ -value can be interpreted as the standard deviation of the prediction error, i.e. knowing old inputs and outputs the next output can be predicted with an accuracy of $\pm 0.015^\circ\text{C}$. It should be noticed that this figure refers to the identification situation when 8 temperature sensors have been used. As mentioned before, only one sensor is used in the control experiments. The prediction error then increases by a factor of ca 3. The input output sequence of the experiment together with model output (\hat{y}), model error ($=y-\hat{y}$) and residuals ($=\epsilon(t)$) is given for the above model in figure 4.1. The model output is defined as

$$\begin{aligned} \hat{y}(t) = & -\hat{a}_1 \hat{y}(t-1) - \hat{a}_2 \hat{y}(t-2) - \dots - \hat{a}_n \hat{y}(t-n) + \\ & + \hat{b}_1 u(t-k-1) + \hat{b}_2 u(t-k-2) + \dots + \hat{b}_n u(t-k-n) \end{aligned}$$

Table 4.1.

Result from Maximum Likelihood identification.

Number of data points 270.

Parameters	k=0	k=1	k=2	k=3
a_1	-0.9867	-0.9818	-0.9749	-0.9655
b_1	0.0681	0.0852	0.1084	0.1200
c_1	0.7904	0.7128	0.6293	0.4459
λ	0.0386	0.0362	0.0317	0.0287
V	0.820	0.709	0.543	0.444
a_1	-1.8069	-1.7448	-1.6981	-1.6636
a_2	0.8196	0.7601	0.7155	0.6833
b_1	0.0007	0.0124	0.0306	0.0488
b_2	0.0367	0.0339	0.0145	0.0042
c_1	-0.2691	-0.3546	-0.3129	-0.2988
c_2	0.1164	0.1888	0.1861	0.1403
λ	0.0142	0.0125	0.0125	0.0151
V	0.111	0.084	0.085	0.124
a_1	-2.2583	-2.4359	-2.6157	-2.6382
a_2	1.6942	2.0041	2.2976	2.3328
a_3	-0.4272	-0.5623	-0.6796	-0.6930
b_1	0.0038	0.0139	0.0379	0.0478
b_2	0.0091	0.0129	-0.0248	-0.0478
b_3	0.0153	-0.0074	-0.0058	0.0415
c_1	-0.9168	-1.1186	-1.2916	-1.1150
c_2	0.4241	0.5041	0.5078	0.3078
c_3	0.05406	-0.0144	-0.0383	0.0011
λ	0.0116	0.0115	0.0119	0.0145
V	0.074	0.071	0.076	0.114

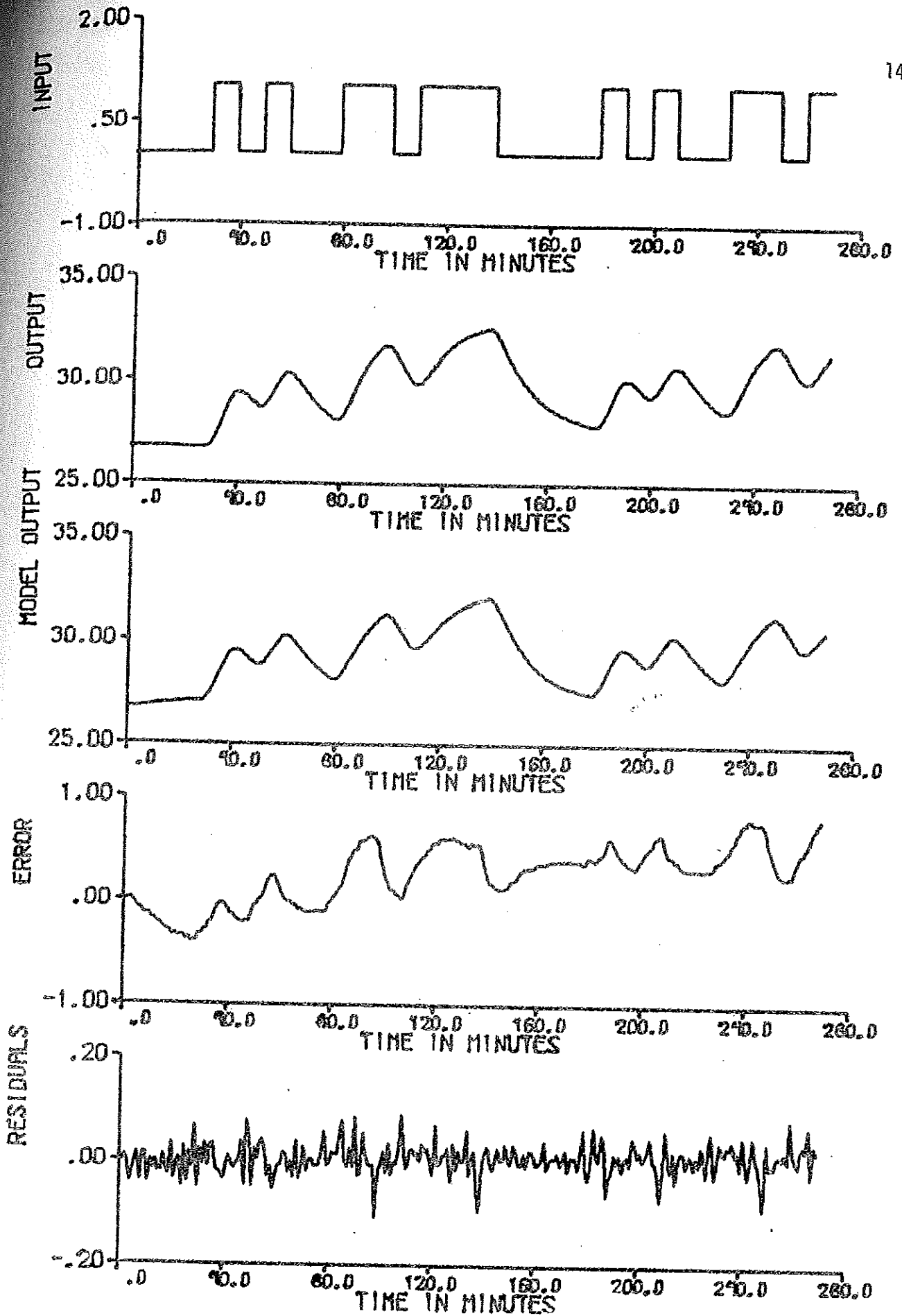


FIG. 4.1 Input, output, model output, model error and residuals from a model with $n = 2$ and $k = 3$.

5 DESIGN OF REGULATORS

In this section is described how a model can be used to design regulators. One possibility is to tune regulator parameters by simulation. Optimal regulators can also be derived directly from the model.

Both these aspects are considered. In section 5.1 the properties of the model of section 4 are compared with the real process. It is found that the model is well suited for simulation and regulator synthesis. In section 5.2 is described how the model has been used to tune the parameters of a PID regulator and in section 5.3 optimal control laws are derived.

5.1 Evaluation of the model

In section 4.2 it was found that the model

$$\begin{aligned}
 y(t+1) = & 1.6636 y(t) - 0.6833 y(t-1) + 0.0488 u(t-3) \\
 & + 0.0042 u(t-4)
 \end{aligned}
 \tag{5.1}$$

well describes the behaviour of the system under the experiment conditions. u is here the radiator effect in kW and y is the temperature in $^{\circ}\text{C}$ relative to the steady state temperature corresponding to $u=0$. The time unit is one minute.

The fact that the model adequately describes the behaviour of the system under the identification experiment conditions does not necessarily imply that it is also valid for control design. Therefore the model was also tested with feedback regulators. Simulated closed loop behaviour was compared with experiment results for various regulators. The comparisons are shown in fig. 5.1 and 5.2. In the simulations digitally generated noise was used to simulated random disturbances. A 500 W regulator was switched on during 60 minutes. The regulators used are the ones calculated

in section 5.3.

It is seen that the model accurately describes the actual system. Simulation is thus an appropriate tool for testing various control laws.

5.2 Parameter tuning using simulation

The parameters of a digital version of a PID controller

$$u(t) = a_0 z(t) + a_1 y(t) + a_2 y(t-1) \quad (5.3)$$

where

$$z(t) = \sum_{k=1}^t y(k) \quad (5.4)$$

were tuned. The parameters were changed using a simple trial and error scheme, until the simulated behaviour was satisfactory. The same disturbances as described in section 5.1 were used. Suitable values were found to be

$$\begin{aligned} a_0 &= -0.092 \\ a_1 &= -3.30 \\ a_2 &= +2.20 \end{aligned}$$

To decrease the influence of the noise y in (5.3) was replaced by a filtered value

$$y_f(t) = 0.5y(t) + 0.33y(t-1) + 0.16y(t-2) \quad (5.5)$$

This improves the performance of the regulator.

Control of a climat room is quite slow. If a large disturbance enters it will be compensated only after a relatively long time. The integral term $z(t)$ would then increase considerably. To avoid this, terms $y(k)$ in (5.4) with an absolute value greater than a certain value (usually 0.6°C) are not included in the sum.

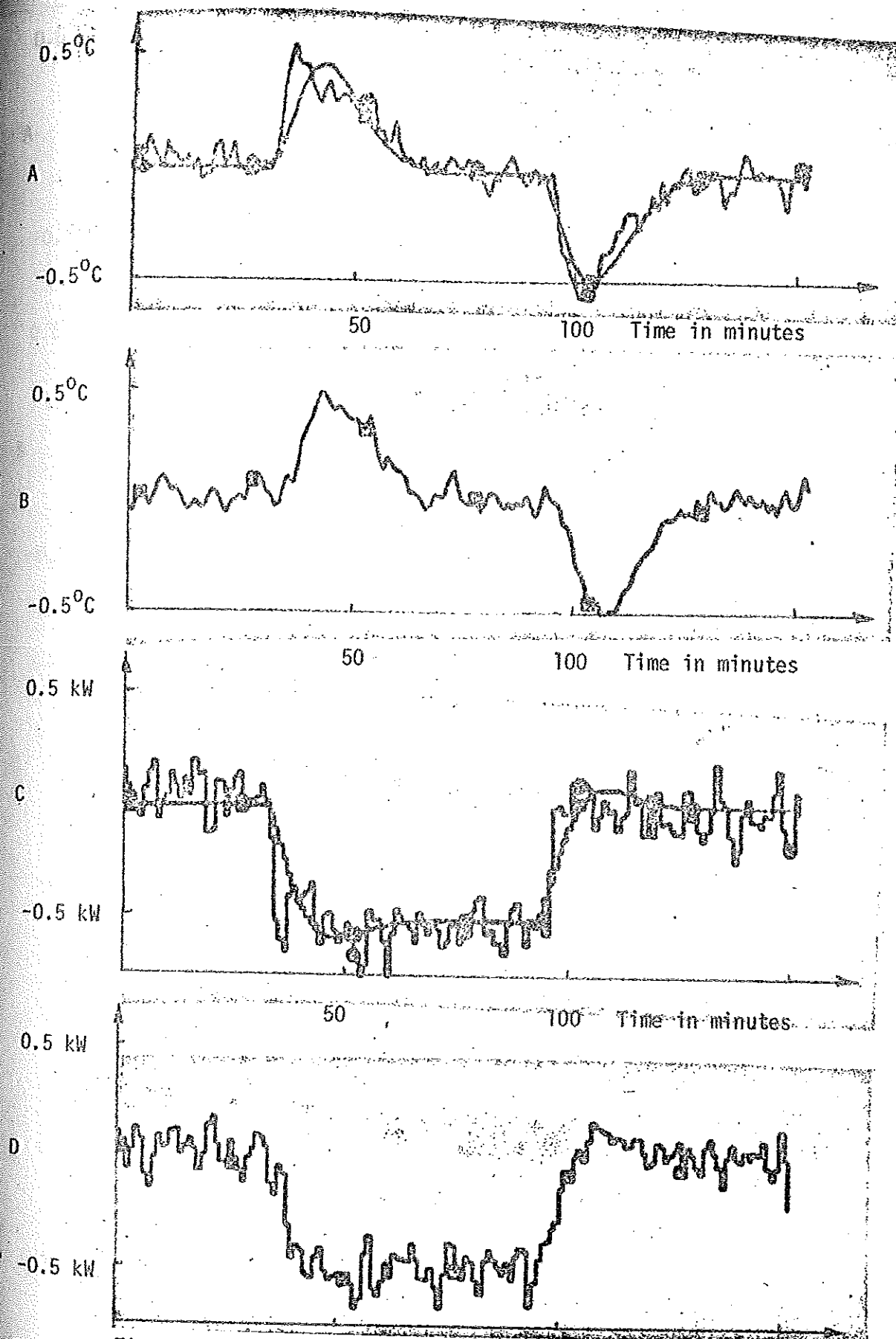


Fig. 5.1 Comparison between simulation and experimental results for the regulator
 $u(t) = -0.093z(t) - 1.546y(t) - 0.0197y(t-1) + 0.159y(t-2) + 0.337y(t-3) - 0.141u(t-1) - 0.149u(t-2) - 0.157u(t-3) - 0.0125u(t-4)$
 A: Smooth line: Simulated noise free temperature. Rough line: Experimental result. B: Simulated temperature. C: Smooth line: Simulated noise free input signal. Rough line: Experimental result. D: Simulated input signal

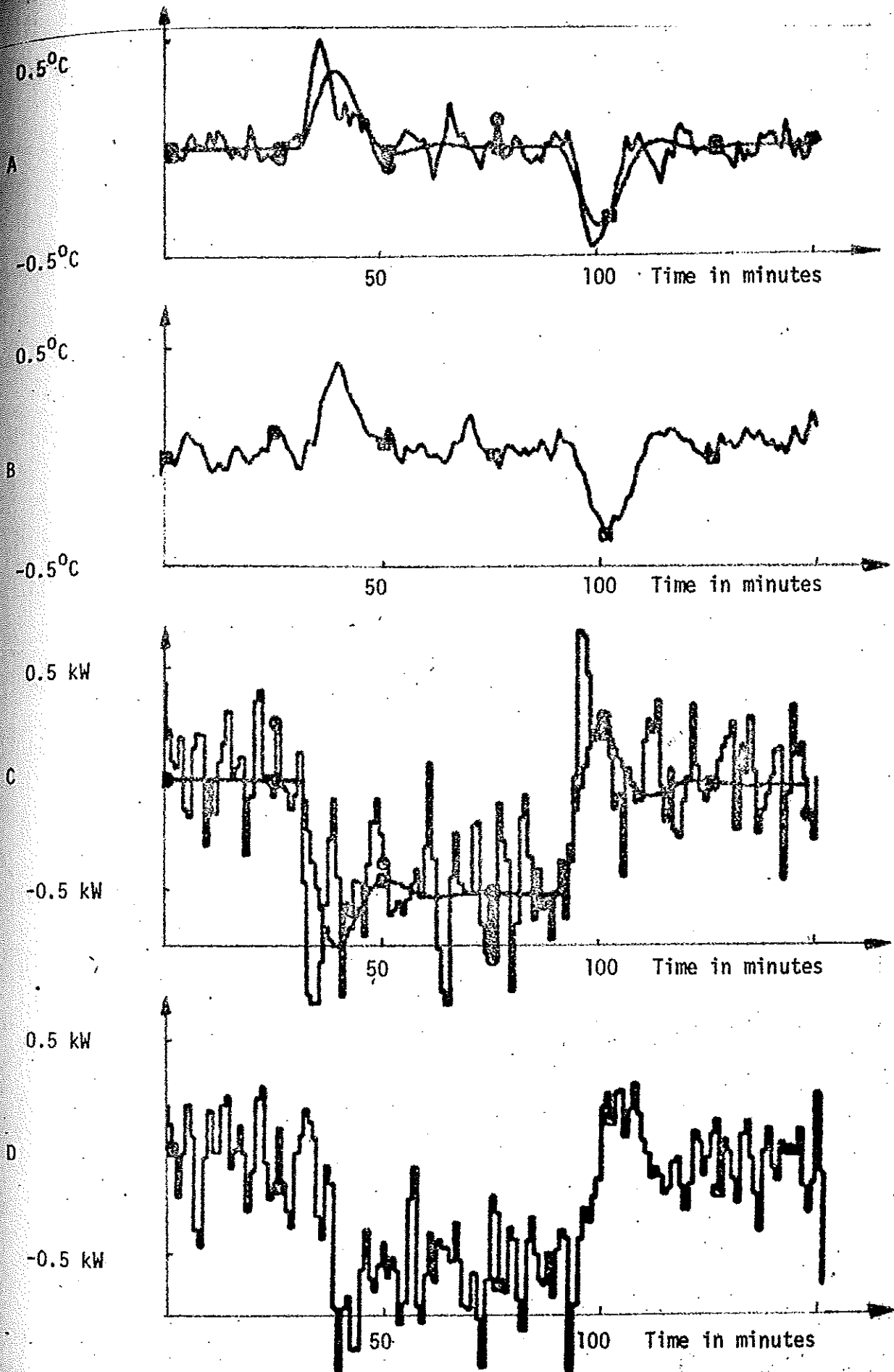


Fig. 5.2 Comparison between simulation and experimental results for the regulator.

$$u(t) = -0.273z(t) - 3.63y(t) - 0.122y(t-1) + 0.322y(t-2) + 0.767y(t-3) - 0.291u(t-1) - 0.324u(t-2) - 0.355u(t-3) - 0.028u(t-4)$$

Experimental results for the resulting regulator are shown in fig. 6.4.

5.3 Synthesis of optimal regulators

Several possibilities to determine control laws from a model exist. Minimum variance and dead beat control laws are sometimes used in similar contexts, Aström (1970). They are, however, not applicable here, since the input u is limited to the interval $(-1,+1)$, which is too small. A feasible way to handle this difficulty is to use Linear quadratic control theory to determine the regulator.

For the present application it was decided not to include the noise structure in the model. This is since the C-parameters in the model (4.1) seem to depend on the experiment conditions. Thus, optimal Kalman filtering of the outputs is replaced by the simple filter (5.5).

To put the deterministic model on a state space form the following state vector is introduced:

$$x(t) = (z(t) \ y(t) \ y(t-1) \ u(t-1) \ u(t-2) \ u(t-3) \ u(t-4))^T$$

Then

$$x(t+1) = Ax(t) + Bu(t)$$

with

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.6636 & -0.6833 & 0 & 0 & 0.0488 & 0.004217 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

and

$$B^T = (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0)$$

The criterion to be minimized is

$$Q = \sum_1^{\infty} q_1 z^2(t) + q_2 y^2(t) + q_3 u^2(t)$$

The optimal feedback law then is

$$u(t) = -Lx(t)$$

where the matrix L is given in table 5.1 for two different criteria.

Table 5.1

Optimal feedback matrix

Loss fcn Q1: $q_1=1$ $q_2=10$ $q_3=1$

$$L = (0.093 \ 3.10 \ -2.02 \ 0.141 \ 0.149 \ 0.156 \ 0.012)$$

Loss fcn Q2: $q_1=1$ $q_2=10$ $q_3=0.1$

$$L = (0.273 \ 7.27 \ -4.60 \ 0.291 \ 0.323 \ 0.355 \ 0.028)$$

These control laws, modified by data filtering (5.4) and limiting the integral term as described in section 5.2, were tested on the climat room. The results are shown in fig. 6.5 and 6.6.

The sensitivity of the optimal feedback matrix L with respect to the model parameters was considered. In section 4.2 it was found that the model

$$y(t+1) = 1.70y(t) - 0.72y(t-1) + 0.0375u(t-2) + 0.014u(t-3) \quad (5.6)$$

also fits the experimental data well. The optimal L for this model with loss function Q1 is

$$L=(0.093 \quad 3.16 \quad -2.17 \quad 0.144 \quad 0.153 \quad 0.042 \quad 0.0)$$

A comparison with table 5.1 shows that the feedback parameters have changed little indeed, although even the number of time delays in the models differs. The closed loop behaviour differs only slightly.

Instead of using Linear quadratic control theory to determine control laws it is possible to modify the dead beat control law in another way. For the dead beat controller the input at time t is chosen to give zero output at time $t+k$, where k is the number of time delays in the model. Instead, it is possible to determine $u(t)$ so as to give zero output at time $t+k+N$, where N is a suitable number. With $N=10$ the control law is

$$\begin{aligned} u(t) = & -0.14z(t) - 4.10y(t) + 2.74y(t-1) - 0.201u(t-1) \\ & - 0.217u(t-2) - 0.082u(t-3) \end{aligned} \quad (5.7)$$

Further details are given in Jensen (1973 c). The behaviour of this regulator is shown in fig. 6.7.

6 CONCLUSIONS

Seven control experiments have been reported in the previous sections. In this section they will be summarized and evaluated. General experiences and comments are given in section 6.2.

6.1 Summary of control experiments

The reported control experiments are:

- (1) The open loop system. $u(t)=0$ See fig. 6.1.
- (2) The system controlled with a proportional regulator with very large gain.

$$u(t) = \begin{cases} +1 & \text{if } y(t) < 0 \\ -1 & \text{if } y(t) > 0 \end{cases} \quad \text{See fig. 6.2.}$$

- (3) A PID regulator tuned using the Ziegler Nichols rule

$$u(t) = -0.0111z(t) - 1.725y(t) + 0.225y(t-1)$$

See fig. 6.3.

- (4) A PID regulator tuned by simulation

$$u(t) = -0.092z(t) - 3.30y(t) + 2.20y(t-1)$$

See fig. 6.4.

- (5) The optimal regulator for the loss function $Q1$ (cf section 5.3)

$$u(t) = -0.093z(t) - 3.10y(t) + 2.02y(t-1) - 0.141u(t-1) - 0.156u(t-2) - 0.012u(t-3)$$

See fig. 6.5.

(6) The optimal regulator for the loss function Q2 (cf section 5.3)

$$u(t) = -0.273z(t) - 7.27y(t) + 4.60y(t-1) - 0.241u(t-1)$$

$$-0.323u(t-2) - 0.012u(t-3)$$

See fig. 6.6.

(7) Modified dead beat regulator (cf section 5.3)

$$u(t) = -0.14z(t) - 4.10y(t) + 2.74y(t-1) - 0.201u(t-1)$$

$$-0.217u(t-2) - 0.082u(t-3)$$

See fig. 6.7.

In all these cases the control signal is limited to the interval $(-1, +1)$. The variable $z(t)$ is the integral term:

$$z(t) = \sum_{k=0}^t y(k)$$

As before $y(t)$ is in $^{\circ}C$ relative to the setpoint and $u(t)$ is in kW relative to 1 kW. For regulators 4-6 the y-term has been filtered according to (5.5). For regulators 4-6 the integral term is updated only if $|y(k)| < 0.6$. Regulator 7 has been implemented as

$$u(t) = -4.24y(t) + 6.84y(t-1) - 2.74y(t-2) + 0.798u(t-1)$$

$$-0.016u(t-2) + 0.135u(t-3) + 0.082u(t-4)$$

which for unconstrained input is equivalent to the regulator given above (5.7). The limits on $u(t)$ in (6.1) imply limits on the integral term, that have an effect similar to deleting terms from the sum $z(t)$.

Filtering of measured data gives improved behaviour. A sampling period for the control of one minute is sufficient in the present application. It would however be desirable to sample the temperature faster to obtain better filtered values. In the present application the input to the process is an analog

Regulators that use old input values when computing the new one have better performance than conventional regulators. Because of the time delay in the system the effect of the input is not seen until after ca 3 minutes. It is therefore valuable to know the inputs during this period.

The noise structure was not used for the control synthesis. The signal to noise ratio was quite high in the identification experiment. Therefore probably also models from least squares identification would have been suitable.

The experiences of the experiments can be summarized as follows: It is very valuable to have a model of the system. It can be used for tuning control parameters by simulation as well as for control law synthesis. It was found that the models from ML-identification are quite appropriate for this purpose.

6.2 Conclusions

The experimental results for the regulators are shown in fig. 6.1-6.7. At point A (20 min) the outdoor air inlet fan is switched on and at B (25 min) it is switched off. A 500 W radiator is switched on between point C (80 min) and D (140 min). In table 6.1 some statistical evaluation of the regulators is given. It should be noticed, however, that the experiment conditions have not been exactly identical due to external disturbances (wind, outdoor temperature, etc.)

signal in the range $(-1, +1)$. This signal is by a thyristor bridge converted to on-off pulses which are the real input to the radiator. It would be desirable to directly design a control law that gives the logical input "on or off". This, however, cannot be done straightforwardly. If the gain of the discussed regulators is increased, on-off control laws with bad performance are obtained.

It is possible to get much better control than a conventional thermostat control. The feasibility to use a process computer to control the temperature naturally depends on the size of the application. However, digital regulators that perform control of the type $u(t) = a_1 y(t) + \dots + a_n y(t-n+1) + b_1 u(t-1) + \dots + b_n u(t-n)$ will probably soon be available at low cost.

FIG. 6.1 The open loop system.

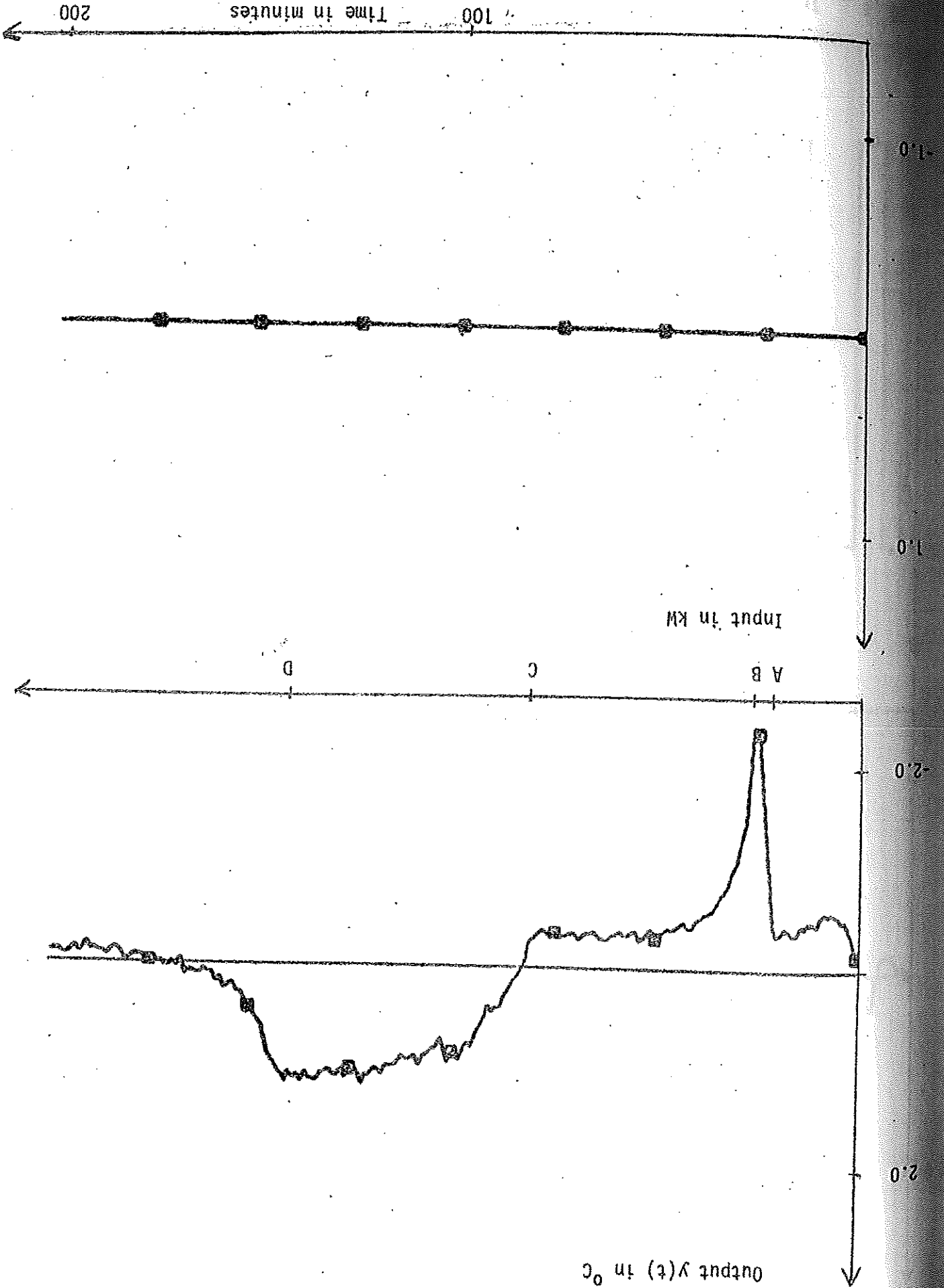


FIG. 6.2 The system controlled with a proportional regulator with a very large gain.

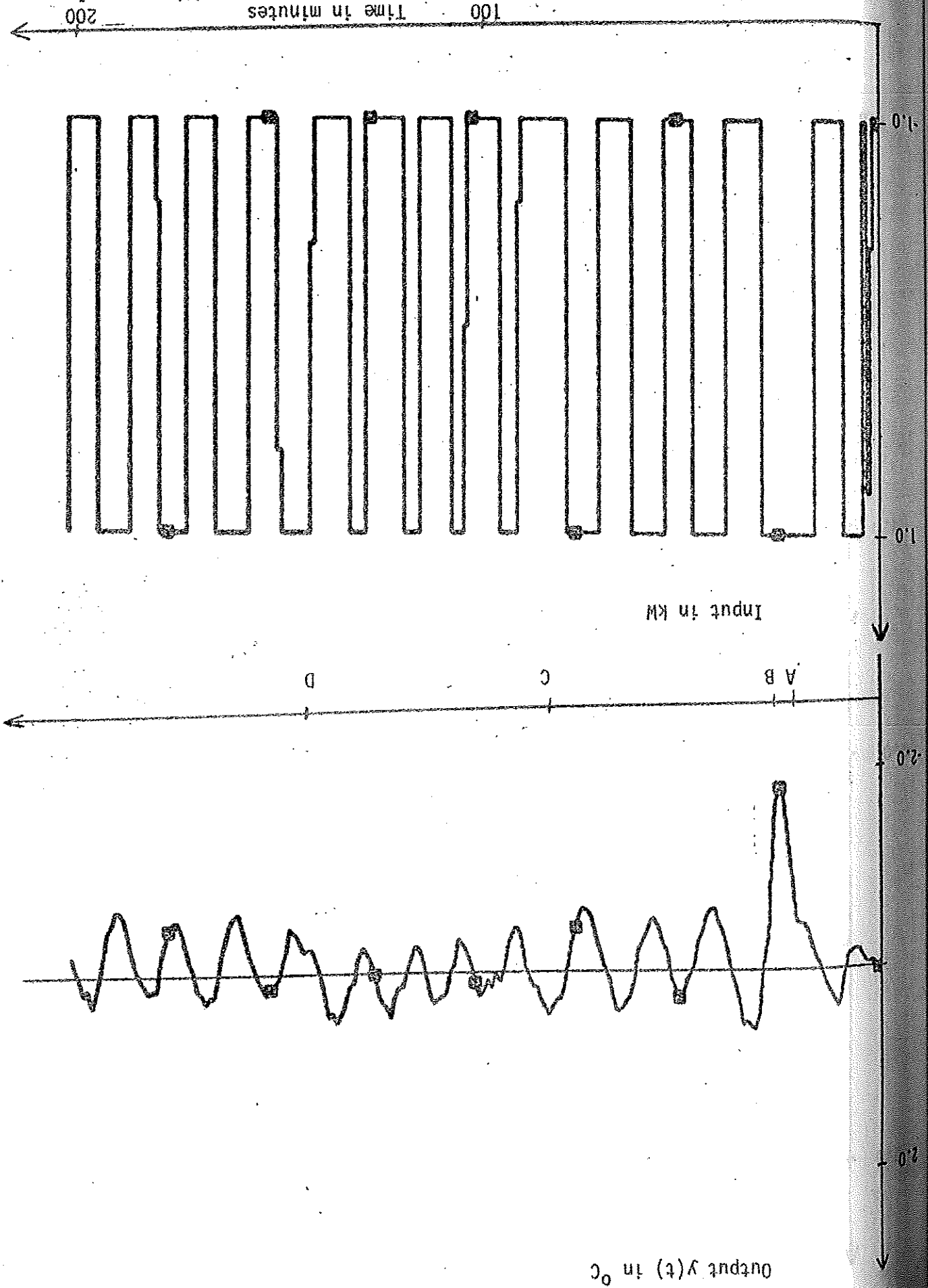


Fig. 6.3 A PID regulator tuned using the Ziegler Nichols rule.

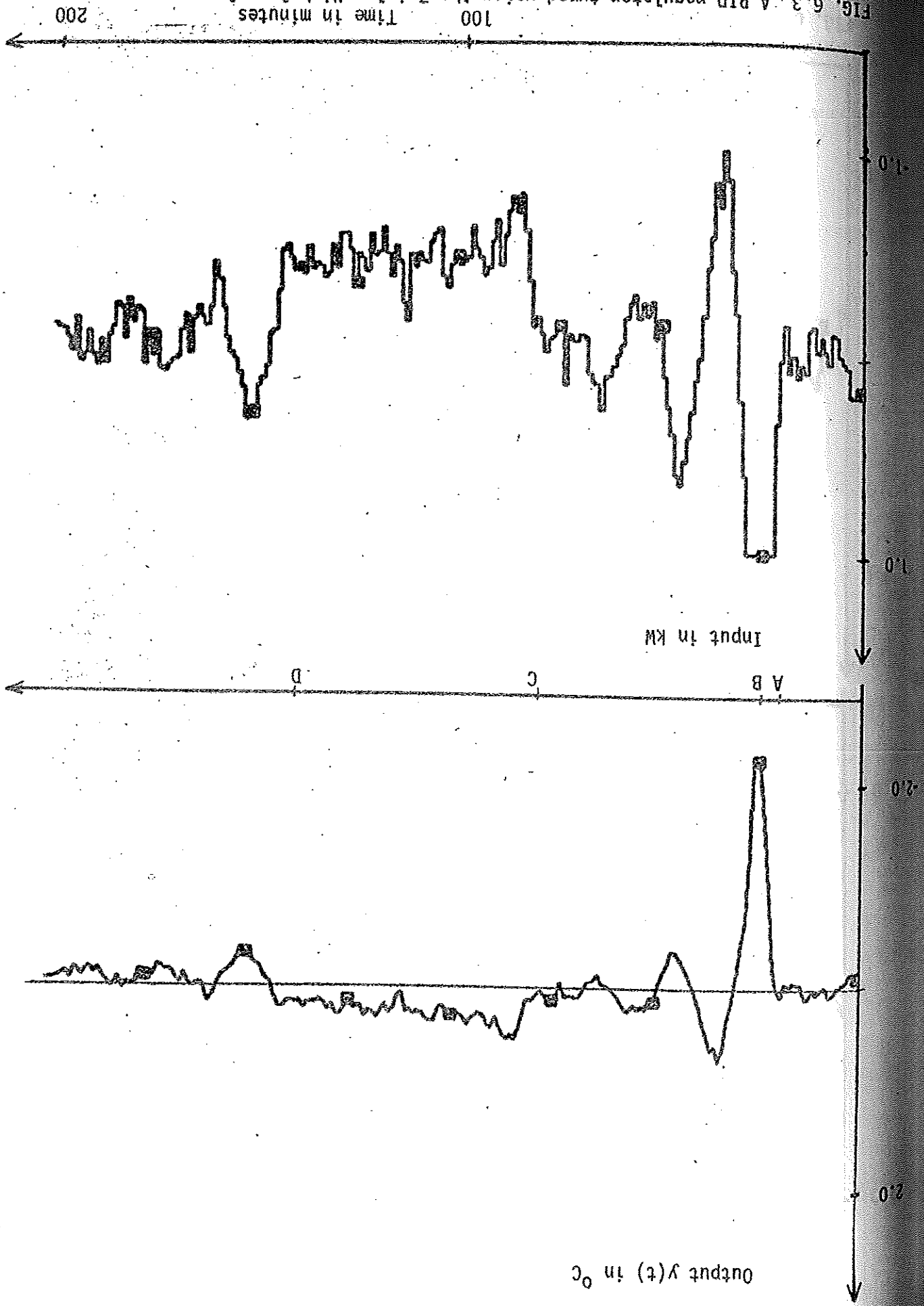


FIG. 6.4 A PID regulator tuned by simulation.

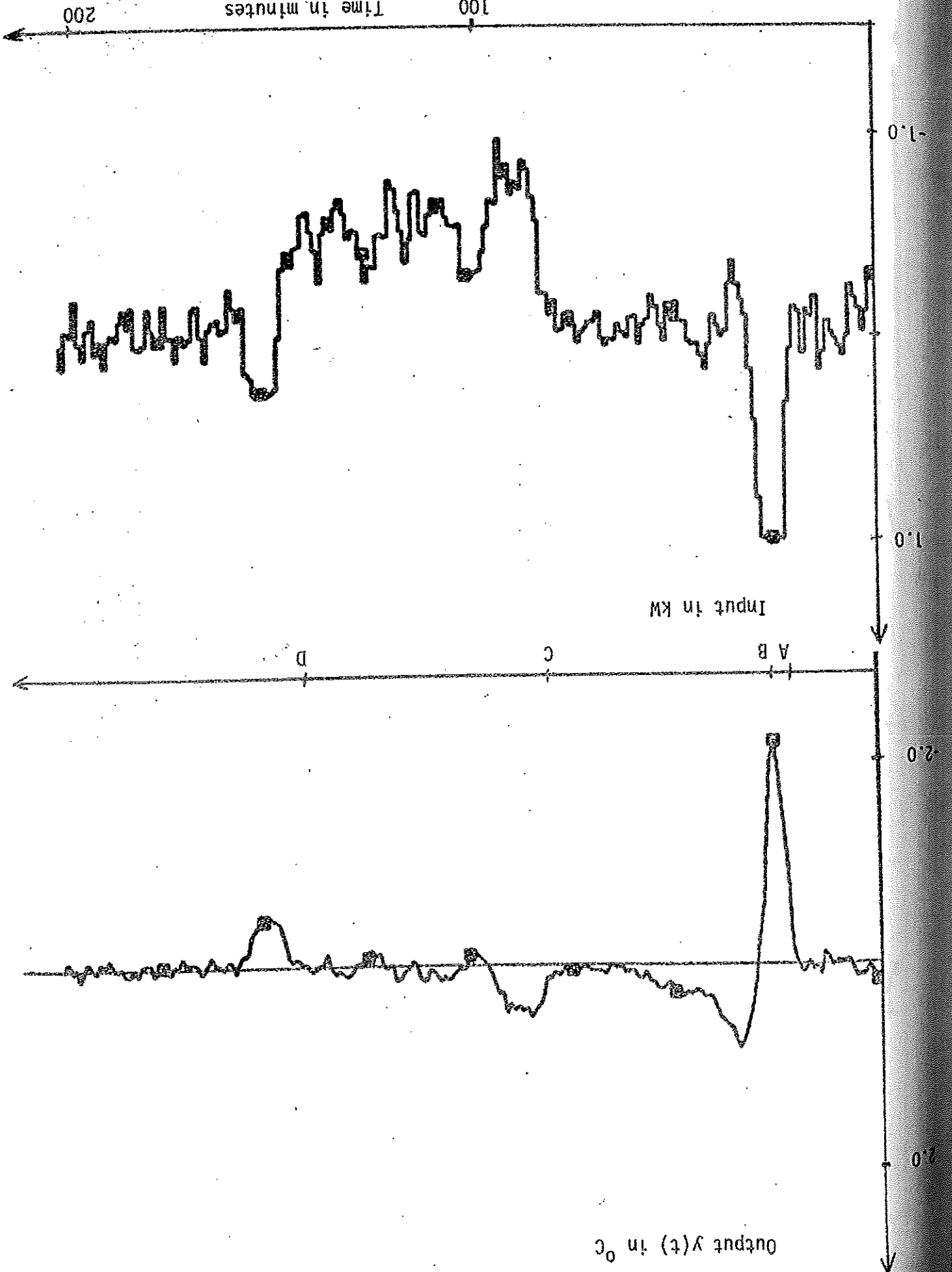


FIG. 6.5 The optimal regulator for the loss function Q1.

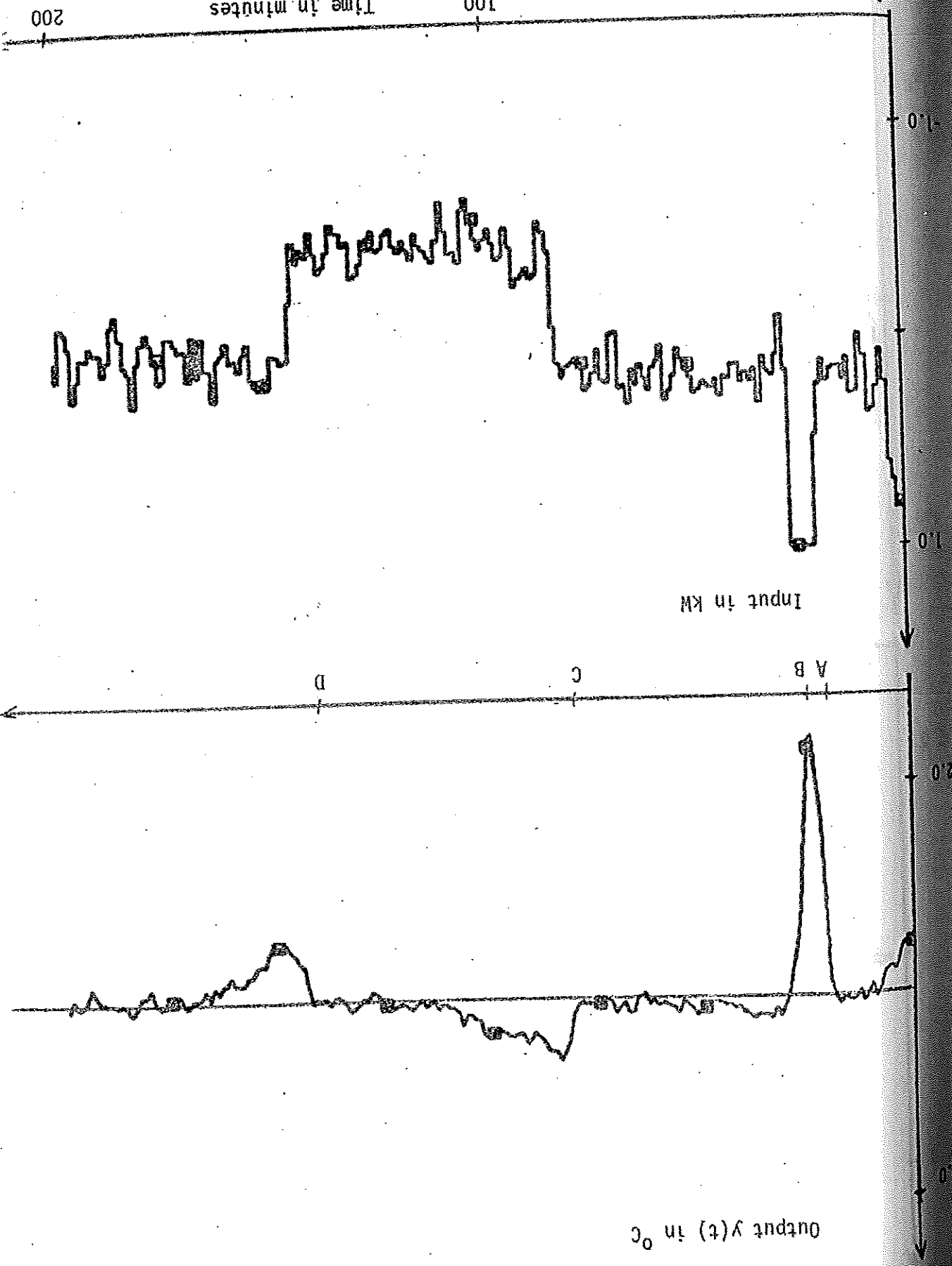


FIG. 6.6 The optimal regulator for the loss function Q2.

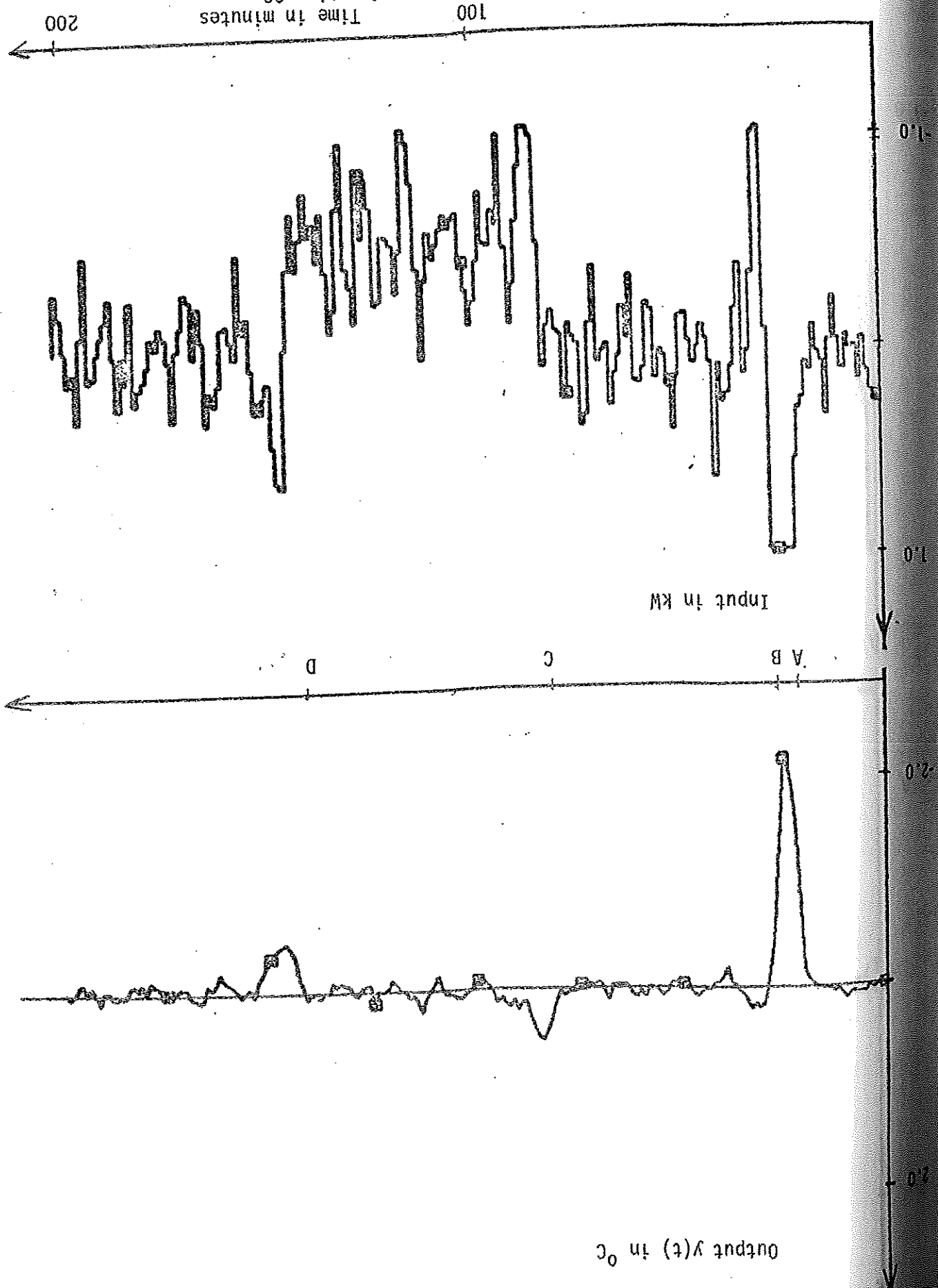


FIG. 6.7 Modified dead beat regulator.

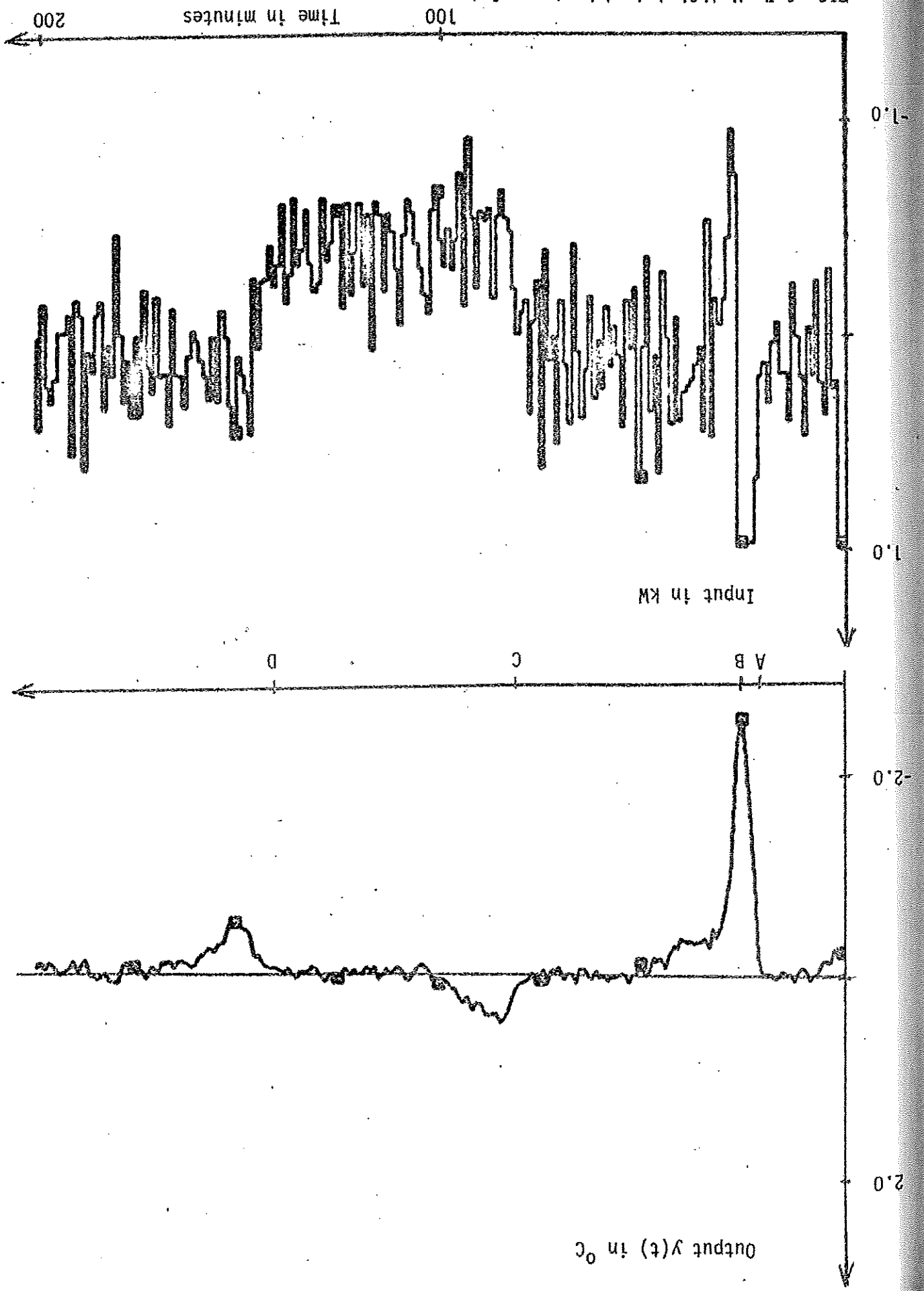


Table 6.1 Evaluation of regulators

Period 1: from the 31st to the 60th minute.
 Period 2: from the 61st to the 175th minute.

A: root mean square of output (in °C)
 B: standard deviation of input (in km)
 C: maximum of absolute value of output (in °C)

Experiment number	Period 1			Period 2		
	A	B	C	A	B	C
1	0.516	0.000	0.707	1.192	0.586	0.510
2	0.367	9.981	0.288	0.492	0.554	0.492
3	0.301	4.157	0.222	0.492	0.554	0.492
4	0.387	1.158	0.183	0.492	0.554	0.492
5	0.098	0.654	0.227	0.492	0.554	0.492
6	0.112	2.148	0.151	0.492	0.554	0.492
7	0.245	3.000	0.180	0.492	0.554	0.492