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PREFACE

The work presented in this report has been carried out at the Divison of Building Materials at the Lund Institute of Technology in Sweden. The author would like to express his thanks especially to Ragnar Hedström, who carried out most of the tests, Britt Andersson, who produced the drawings and Anni-Britt Nilsson, who typed the report.

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Per-Erik Petersson

CONTENTS

SUMMARY

In /1/ it is shown that the method of determining the fracture energy (G_F) by means of stable three-point bend tests on notched beams seems suitable for concrete and similar materials. It is also shown that G_F, for most practical applications, can be considered as a material property, i.e. G_F is reasonably independent of the specimen dimensions.

This report can be regarded as being complementary to /1/. Among other things the testing method has been further analysed and discussed and tests have been carried out in order to study the effect on the test results of loading velocity, relative notch depth, distance between the supports, notch tip geometry and curing conditions during the last day before testing. According to the test results it seems as though none of the studied parameters has any marked effect on the determined G_F-values but due to the wide scatter of the test results this conclusion is a little uncertain.

Finally the effect of the scatter on the necessary number of specimens in a test series in order to obtain a test result within a defined confidence interval is analysed. The necessary number of specimens increases with the decreasing area of the uncracked ligament (A_{lig}) and the increasing size of the largest aggregate particles (d_{aggr}^{max}). The relationship between the necessary number of specimens in a test series and $\sqrt{A_{lig}}/d_{aggr}^{max}$ is presented.

1 INTRODUCTION

The fracture energy (G_F) is defined as the amount of energy necessary to create one unit of area of a crack. In /1/ methods of determining G_F on concrete by means of three-point bend tests on notched beams are discussed and analysed. In this report these methods are further analysed and the aim of the work presented here has been to obtain a basis for a standard test method suitable for determining G_F for mortar and concrete /2/. Among other things the effect on the test results of the notch geometry, the curing conditions and the loading velocity has been investigated. Another thing that has been analysed is the effect of the size of the largest aggregate particles on the scatter of the test results and a relation showing the necessary number of specimens in a test series in order to obtain relevant results of G_F is presented.

2 PRINCIPLES OF DETERMINING G_F ON A NOTCHED BEAM

The most accurate method of determining G_F by means of a three-point bend test on a notched beam is presented in Figure 1. The beam is twice as long as the distance between the supports and consequently the energy supplied by the weight (between the supports) of the beam during the test is compensated for. This means that the area under the stable load-deflection curve (F-δ curve) equals the total amount of energy consumed during the test. In /1/ it is shown that the energy consumption outside the fracture zone is negligible, at least when the relative notch depth, i.e. the relation between the notch depth and the beam depth, is not less than 0.5 (the fracture zone is a narrow, micro-cracked band which develops in front of the crack tip during the crack propagation). Thus the area under the F-δ curve represents the energy consumption in the fracture zone when the crack propagates through the beam. G_F is then obtained as the total energy consumption (W_{TOT}) divided by the cross sectional area of the ligament over the notch (A_{lig}), i.e.:

$$G_F = W_{TOT}/A_{lig} \qquad (2:1)$$

Another method of determining G_F is presented in Fig 2. In this case the energy supplied by the weight of the beam is only partly compensated for. The beam length is $\ell + 2\alpha\ell/2$ where α is a figure between 0 and 1. In order to determine G_F it must then be possible to determine not only the area under the F-δ curve but also the areas representing W_1, W_2 and W_3 in Fig 2. F_0 in the figure represents a point load producing a moment at the center of the beam equalling the moment due to the weight of the beam

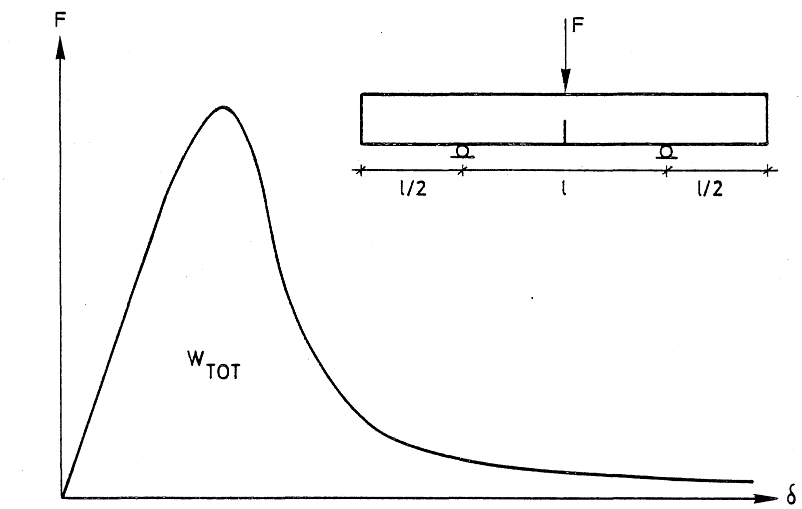

Fig 1. Load-deflection curve when the energy supplied by the weight of the beam is compensated for.

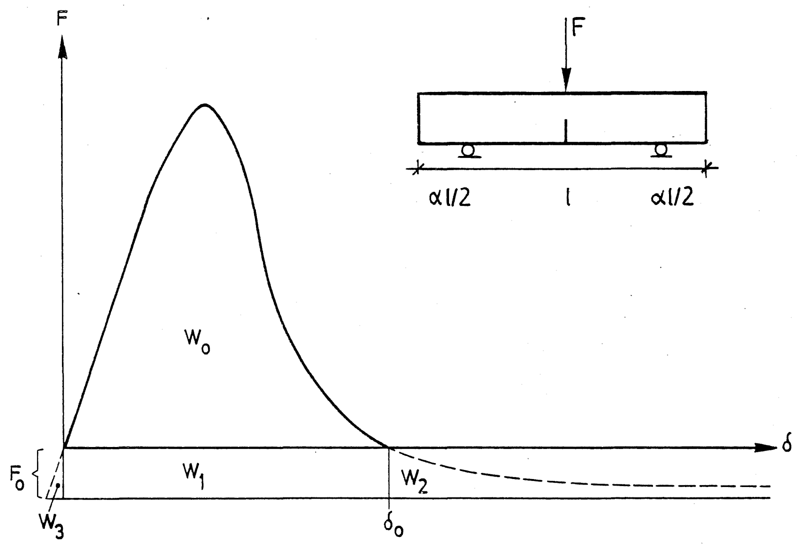

Fig 2. Load-deflection curve when the energy supplied by the weight of the beam is only partly compensated for (unless $\alpha=1$).

and δ_o represents the deflection at the final fracture of the beam.

It can easily be shown that $F_o = Mg(1-\alpha^2)/2$ (M=weight of the beam between the supports and g=9.81 m/s^2) and consequently $W_1=F_o\delta_o=Mg\delta_o(1-\alpha^2)/2$. Theoretical estimations in /1/ indicate that $W_1 \cong W_2$, at least for small values of F_o, and that W_3 is normally so small that it is negligible. This means that G_F can be calculated as:

$$G_F = \left[W_o + Mg\delta_o(1-\alpha^2)\right] /A_{lig} \qquad (2:2)$$

Eq (2:2) is always an approximation unless α=1 in which case eq (2:1) and eq (2:2) are identical. In order to determine the difference, for different values of F_o, between the correct G_F-value according to eq (2:1) and the approximate value according to eq (2:2), the following experiments were carried out. Stable load-deflection curves were determined for four notched concrete beams according to the method presented in Fig 1. The cross sectional areas were 50 x 50 mm^2 for two of the beams and 200 x 50 mm^2 for the others (beam depth = 200 mm) and for all the beams the relative notch depth was 0.5. The water-cement ratio of the concrete quality used was 0.6 and the size of the largest aggregate particles was 8 mm. The beams were kept at 100 % RH until the time for testing when the beams were 28 days old.

For different theoretical values of F_o the areas under the stable curves were split into the smaller areas W_o, W_1 and W_2 according to Fig 2. For each value of F_o the fracture energy was calculated as:

$$G_F^{CALC} = (W_o + 2F_o\delta_o)/A_{lig} \qquad (2:3)$$

G_F^{CALC} was then, for different values of F_o, compared with the real G_F-value calculated according to eq (2:1). The curves in Fig 3 show the ratio G_F^{CALC}/G_F as function of $W_o/(W_o+2F_o\delta_o)=W_o/(W_o+2W_1)$ for the four beams tested. As can be seen in the Figure, the ratio G_F^{CALC}/G_F decreases with decreasing value of $W_o/(W_o+2W_1)$. The difference between the values of G_F^{CALC} and G_F seems however to be negligible when the amount of energy represented by the measured area under the F-δ curve, i.e. W_o, is larger than 50 % of the calculated energy consumption during the test, i.e. W_o+2W_1. Not even when W_o is as small as 30-40 % of W_o+2W does the value of G_F^{CALC} differ from the real G_F-value by more than about 10 %. According to test results in /1/ and to experience gained in connection with the tests presented in Chapter 4, the ratio $W_o/(W_o+2W_1)$, for the actual specimen dimensions, in most cases seems to be greater than 0.3-0.4, even if α=0, and then eq (2:2) ought to be useful as an approximate method of determining G_F.

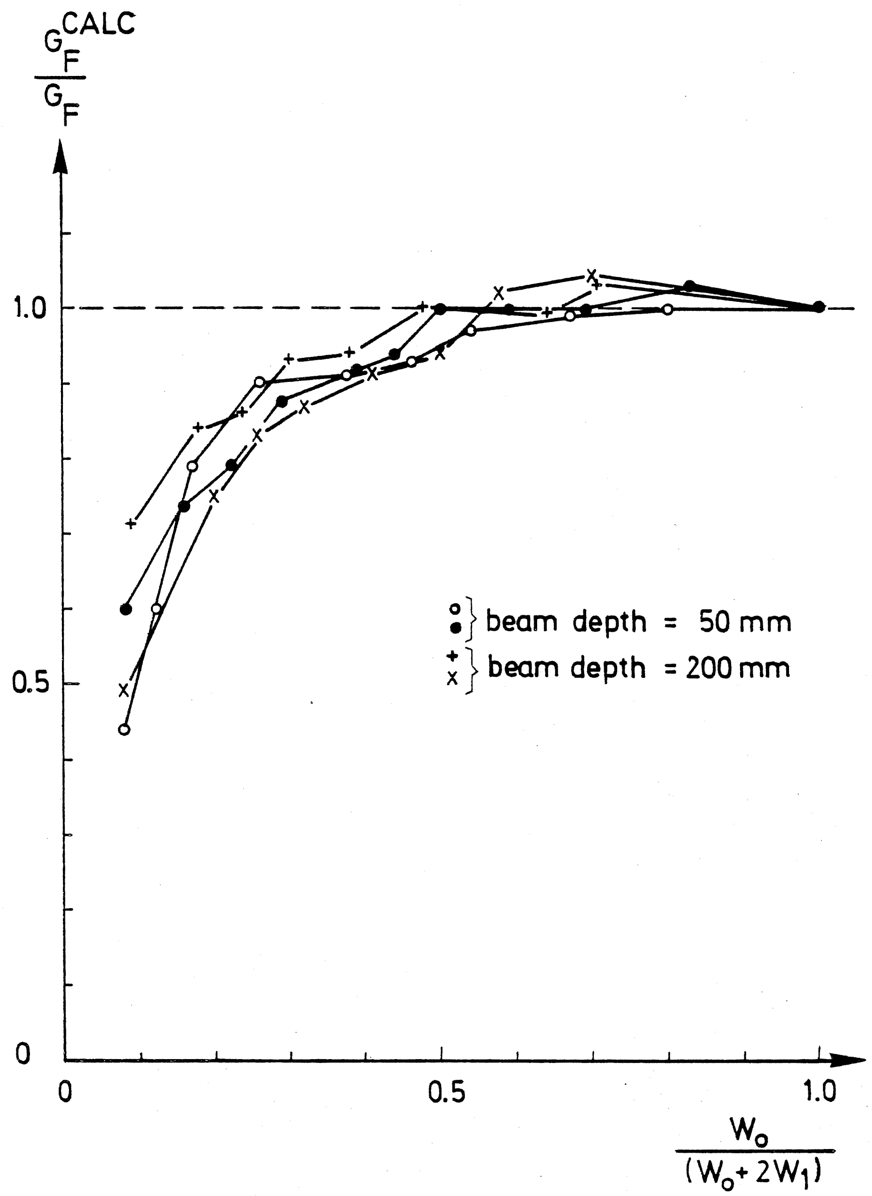

Fig 3. G_F^{CALC} as function of $W_0/(W_0+2W_1)$ for 4 concrete beams. G_F^{CALC}, G_F and $W_0/(W_0+2W_1)$ are defined by eq (2:3), eq (2:1) and Fig 2 respectively.

However, in some cases the ratio $W_0/(W_0+2W_1)$ may be less than 0.3-0.4, for example for deeply notched beams, for long and slender beams and for low strength concrete qualities and in these cases the restrictions given in Fig 3 concerning the evaluation method according to eq (2:2) must always be considered.

The test results presented in Fig 3 are in aggreement with calculation results in /1/, which imply that eq (2:2) can be used as an approximation even if the measured area under the F-δ curve is only 30-35 % of the total energy consumption according to eq (2:1).

As the difference between the two evaluation methods according to eq (2:1) and eq (2:2) in most cases seems to be very small, even if $\alpha=0$, both of the methods ought to be suitable for determining G_F. However, there are some advantages of the test method according to eq (2:2). These advantages are ($\alpha<1$):

a) smaller specimens can be used which considerably facilitate the performance of the test.

b) the test method according to Figure 1 produces a long "tail" of the F-δ curve which can cause problems when measuring the area under the curve. Consequently it is easier to evaluate the test results according to Figure 2 and eq (2:2), especially when $\alpha=0$. A small fault in the balance of the loading system in Figure 1 may also give rise to substantial errors due to the long tail.

When determining the area under the F-δ curve it is not necessary to measure the real deflection of the beam but the movement of the cross-head of the testing machine can be measured as well. This is possible as the testing machine and the supports are supposed to act in an elastic way, which means that the amount of elastic energy stored in the testing machine and the supports at the maximum load will be completely released during the unloading. When measuring the deformation as the movement of the cross-head of the testing machine it must be checked that no irreversible deformations take place at the supports or at the loading point.

All the tests reported below are evaluated by means of eq (2:2) and $\alpha=0$. This method has been used primarily for two reasons; first of all it seems to produce acceptable results in most cases and secondly it is the most convenient way of determining G_F.

3 SUITABLE DIMENSIONS FOR STANDARD TEST SPECIMENS

When determining the fracture energy by means of a three-point bend test, the following points concerning suitable dimensions for standard test specimens have to be considered:

a) it must be easy to handle the specimen during the test

b) it must be possible to obtain a stable fracture by means of ordinary available testing machines.

c) the beam has to be representative of the material and the dimensions of the specimen will then be determined by the dimensions of the largest irregularities in the material.

c) the dimensions must allow the evaluation method concerned to be used.

When using a specimen with the dimensions 100 x 100 x 840 mm^3 (800 mm between the supports) and with a notch depth of 50 mm, the demands above seem to have been met (these dimensions have been used for all the tests presented in Chapter 4). When using these dimensions it means that the weight of the beam becomes about 17-20 kg and consequently the beam can then be handled by one man.

For practical reasons the beam length has to exceed the distance between the supports by a few centimeters and in this case $\alpha=0$ should not be used. In the case in question $\alpha=20/400=0.05$ and consequently the correction factor $1-\alpha^2$ in eq (2:2) becomes $0.9975 \cong 1$. As can be seen, $\alpha=0$ can be used without producing any fault of practical importance.

Stability conditions according to /1/ for a three-point bend test on a notched beam are shown in Fig 4. The curves are relevant for a concrete beam with $d/\ell_{ch}=1$ (d=beam depth, $\ell_{ch}=G_F E/f_t^2$, E=Young's modulus, f_t=tensile strength). In the figure k is the stiffness of the testing machine and b is the beam width. For the actual beam depth, 100 mm, the curves in Fig 4 consequently correspond to a ℓ_{ch}-value of 100 mm, which is a very low value for normal concrete qualities and a low value for mortar. However, a higher value of ℓ_{ch} produces a more stable fracture and therefore $d/\ell_{ch}=1$ ought to be suitable when estimating a value of the stiffness of the testing machine which is on the safe side.

According to Fig 4 the highest value of Eb/k producing a stable fracture is about 200 when $\ell/d=8$ and $a/d=0.5$. This means that, in order to obtain a stable fracture, k has to exceed Eb/200, i.e. in this case E/2 N/mm. Noramally E is less than 40,000 MPa and then the fracture process ought to be stable when k exceeds 20,000 N/mm. It must however be observed that k includes the weaknesses of the total testing arrangement, except the specimen, i.e. k includes the weaknesses of the supports, of the load cell etc, etc.

The actual specimen dimensions seem to be usable when the size of the largest aggregate particles is less than 30-35 mm, see below. For larger

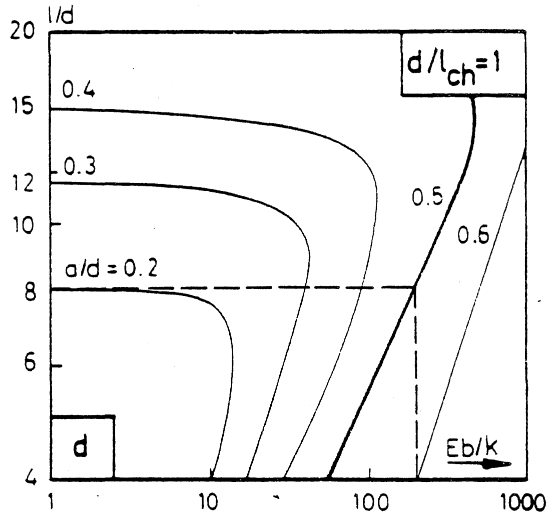

Fig 4. Conditions of stability for a three-point bend test on a notched beam. The areas of stability are limited by the curves and the axis. ℓ=beam length, d=beam depth, a=notch depth, E=Young's modulus, b=beam width, k=stiffness of the testing machine, $\ell_{ch}=EG_F/f_t^2$, f_t=tensile strength, G_F=fracture energy.

aggregate particles the dimensions of the beam must be increased and then the stability conditions have to be analysed for each test, for example by means of the stability conditions for three-point bend test on notched beams presented in /1, 3/. When larger beams are used, the value of G_F will probably be slightly overestimated. Test results in /1/ indicate that G_F increases about 15-25 % when the beam depth increases four times.

According to experience derived from the tests presented below, the ratio $W_0/(W_0+2W_1)$, see Fig 3, normally seems to exceed 0.45 for the specimen dimensions concerned and ordinary concrete qualities. This means that the evaluation method according to eq (2:2) and Fig 2 can be used in most cases but the restrictions according ti Fig 3 must always be kept in mind.

4 THE EFFECT OF SOME PARAMETERS ON THE EXPERIMENTALLY DETERMINED VALUE OF G_F

4.1 Introduction

According to test results in /1/, G_F can be considered to be a material parameter, i.e. G_F is very slightly affected by the specimen dimensions. In this chapter the effect of some other parameters on the experimentally determined value of G_F is investigated. These parameters are loading velocity, relative notch depth, distance between the supports, type of notch tip and curing conditions. In order to study the effect of these parameters on the test results, a basic test method and a standard concrete quality were defined and then the five parameters were varied one by one.

4.2 Standard concrete quality and basic test method

The composition of the standard concrete quality is presented in Table 4:1.

Table 4:1. The composition of the standard concrete quality.

Cement (ordinary Portland)	341 kg/m^3
Water	204 kg/m^3
Natural gravel 0-8 mm	896 kg/m^3
Crushed quartzite 8-16 mm	896 kg/m^3

Specimens with dimensions according to Chapter 3 (100 x 100 x 840 mm^3) were cast in plywood moulds. The specimens were kept under wet sackcloth during the first 24 hours after casting and then in lime-saturated water (+20^{o}C) until the time for testing. A day before the testing a 5 mm wide and 50 mm deep rectangular notch was sawn at the middle of the specimen with the use of a diamond saw.

The testing arrangement for the basic test method is shown in Fig 5. The loading velocity was chosen so that the maximum load was reached about 30 seconds after the start of the test. The fracture area was measured by means of vernier callipers and the area under the F-δ curve was measured with a planimeter. The G_F-value was calculated by using eq (2:2). Each test series consisted of 6 specimens which were cast on two different occasions, 3 specimens on each occasion.

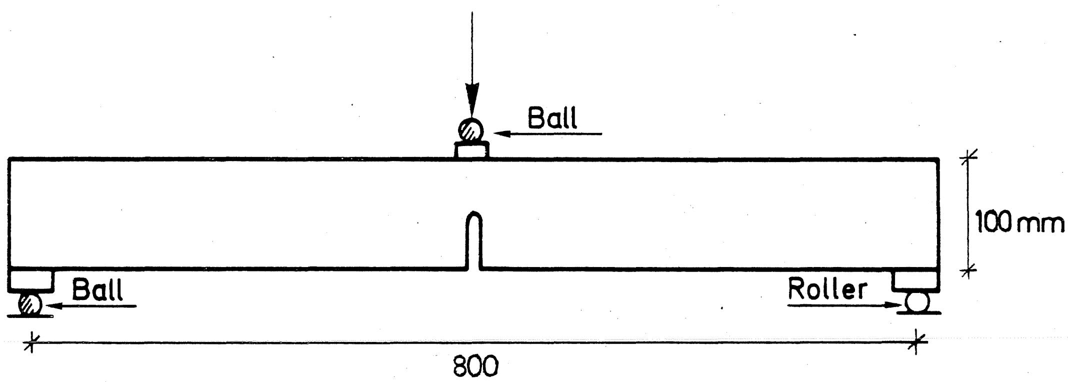

Fig 5. Testing arrangement for determining G_F (basic test method).

In Fig 6 results representing 4 different test series are presented. The tests were carried out according to the basic test method described above and the standard concrete quality according to Table 4.1 was used. Each circle represents the result of a single specimen and the dashed lines represent arithmetic mean values.

As can be seen in the figure there is a wide scatter in the results; the individual values vary between 79 and 185 N/m and the arithmetic mean values between 94 and 127 N/m. This wide scatter, which is natural for the testing method, of course makes it difficult to analyse the test results and this must be remembered when the test results below are discussed. The effect of the scatter on the choice of testing procedure is further discussed in Chapter 5.

4.3 Loading velocity

The loading velocity is not only dependent on the velocity of the cross head of the testing machine but also on the stiffness of the machine; a weak testing machine makes the loading velocity slow during the loading but fast during the unloading after the peak load is reached. The velocity of the cross head only defines the mean loading velocity during the test. However, for a specific testing machine it is suitable to use the time until the peak load is reached for defining the loading velocity and this definition is used here.

In Fig 7 test results are presented showing G_F as a function of the loading velocity. The arithmetic mean values decrease slightly as the loading velocity decreases but due to the wide scatter this effect is far from being significant. The only conclusion that can be be drawn from the results is that the effect, if there is any, of the loading velocity on the fracture energy is very small within the range of velocities investigated here. However, no conclusions regarding the effects of very high loading velocities can be drawn from the results in Fig 7, dynamic loading may result in quite different G_F-values.

A consequence of the results in Fig 7 is that the time until the peak load is reached ought to, independent of the stiffness of the testing machine, be sufficient for defining the loading velocity when determining G_F experimentally.

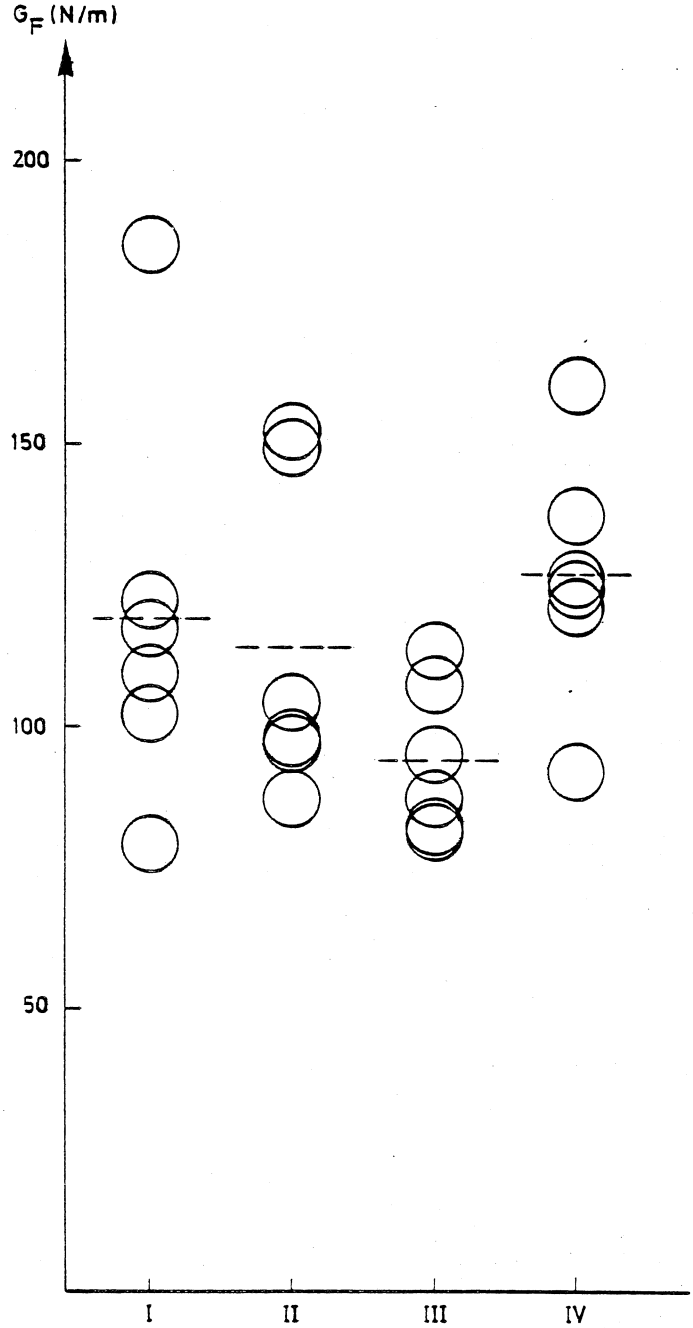

Fig 6. G_F-values representing 4 different test series. A concrete quality according to Table 4.1 was used. The circles represent individual test results and the dashed lines represent arithmetic mean values.

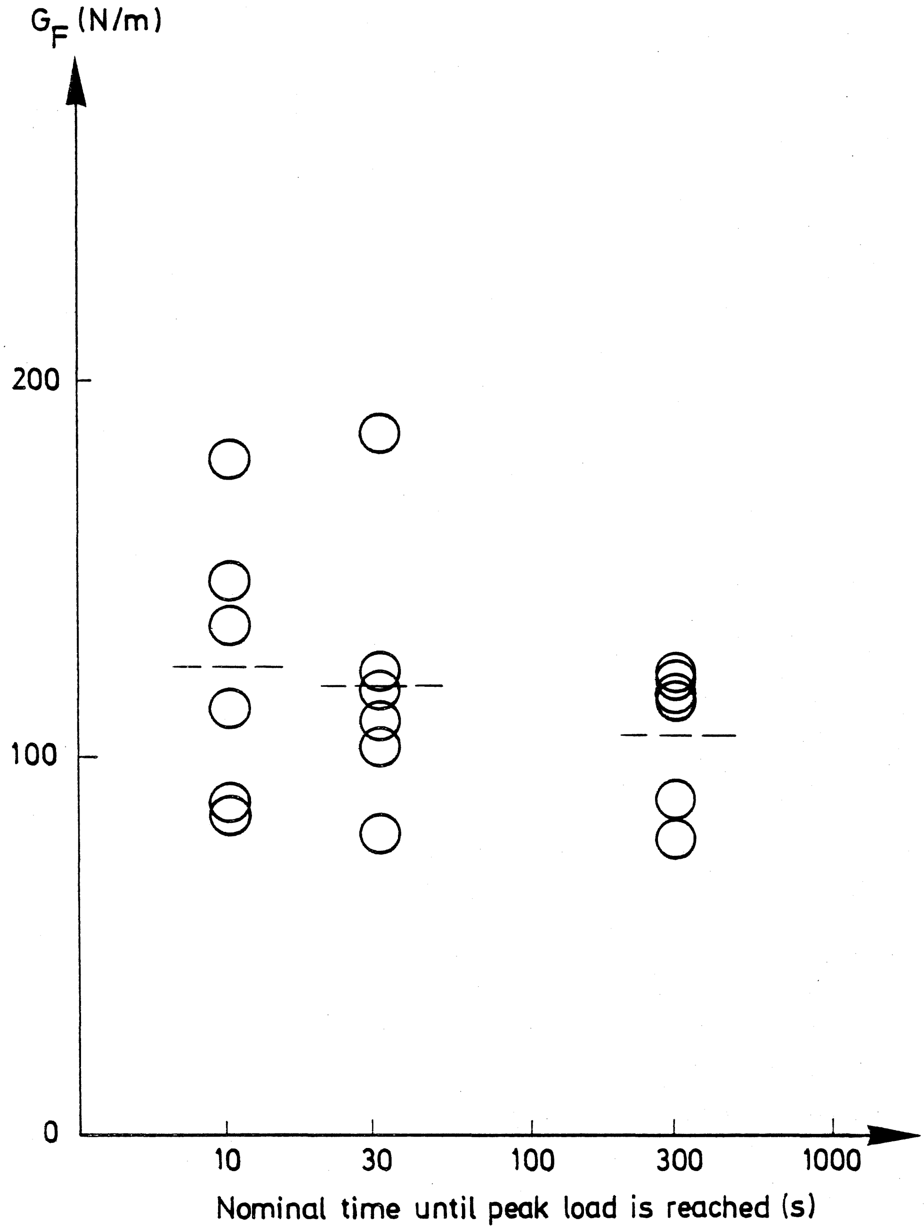

Fig 7. G_F as a function of the loading velocity.

4.4 Notch depth

The highest possible fracture moment of a notched beam subjected to pure bending is obtained for a stress distribution according to Dugdale along the fracture zone (no strain-hardening), see Fig 8. The highest possible fracture moment can be expressed as:

$$M_{max} = f_t(d-a)^2b/2 \qquad (4:1)$$

where f_t=tensile strength, d=beam depth and a=notch depth.

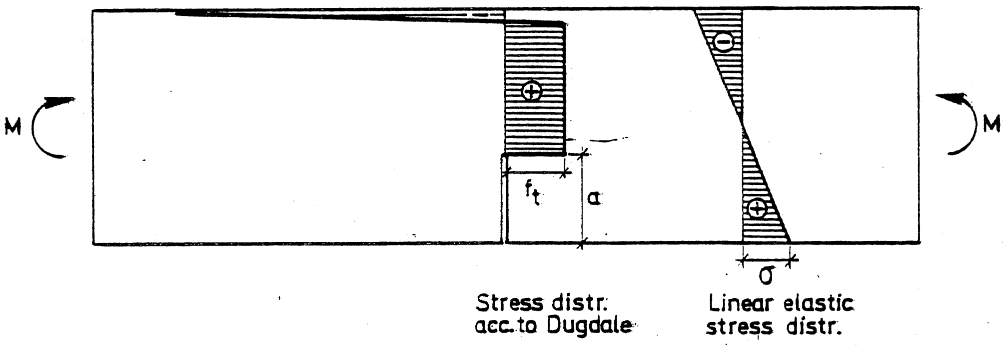

Fig 8. Stress distributions in a beam subjected to pure bending.

A second fracture zone will develop if the maximum stress σ according to the linear elastic stress distribution, see Fig 8, reaches the tensile strength, i.e.:

$$\sigma = 6M/(d^2b) = f_t \Rightarrow M = f_td^2b/6 \qquad (4:2)$$

By equating eq (4:1) and eq (4:2), the following expression is obtained:

$$\frac{f_td^2b}{6} = \frac{f_t(d-a)^2b}{2} \Rightarrow d^2 = 3(d-a)^2 \Rightarrow a = 0.423d \qquad (4:3)$$

Eq (4:3) implies that only a single fracture zone can develop if $a \gtrsim 0.423d$. This is of course a figure which is on the safe side and normally only a single fracture zone develops even if the notch depth is much smaller than 0.423d. However, to be on the safe side, which is necessary, for example, in a standard test method, it is suitable to use notch depths exceeding the critical value discussed above.

In order to study if the relative notch depth has any effect on the experimentally determined G_F-value, tests were carried out on beams with varying relative notch depths, ranging from 0.4 to 0.7. The results are presented in Fig 9. According to the results in the figure there seems to be no significant effect of the relative notch depth on the G_F-values. The small decrease of the mean values with increasing relative notch depths can mostly be explained by the fact that the ratio $W_0/(W_0+2W_1)$, see Fig 3, decreases with increasing relative notch depth. For the relative notch depth 0.7, $W_0/(W_0+2W_2)$ is about 0.35, which, according to Fig 3, means that the G_F-value is underestimated by about 5-10 %.

4.5 Distance between the supports

In Fig 10 G_F-values as a function of the distance between the supports are presented. For the two shorter distances between the supports the G_F-values are in good agreement with other test results, compare Fig 6, but for the longer distance, 1200 mm, the G_F-values seem to be significantly smaller. The explanation can be found in Fig 3. The value of $W_0/(W_0+2W_1)$ is about 0.65, 0.5 and 0.3 for the 600, 800 and 1200 mm respectively long distances between the supports. According to the results in Fig 3 this means that the G_F-values are correct for the two shorter distances but underestimated by about 10-15 % for the longer distance between the supports, which explains, at least to some extent, the low G_F-values determined on these 1200 mm beams. Consequently this long beam is unsuitable for determining G_F by the test method in question.

4.6 Type of notch

When a cast notch is used, the concrete closest to the notch tip will be disturbed during the casting. This is not the case when sawn notches are used and to study how the notch type affects the test results, the three types of notches presented in Fig 11 were tested. The results are presented in Fig 12.

As can be seen in Fig 12 the G_F-values seem to be very little affected by the choice of notch type, at least where the actual size of the specimen is concerned. According to the test results it seems as if a notch width of at least 15 mm can be used, which in the case in question is almost the same as the size of the largest aggregate particles (16 mm).

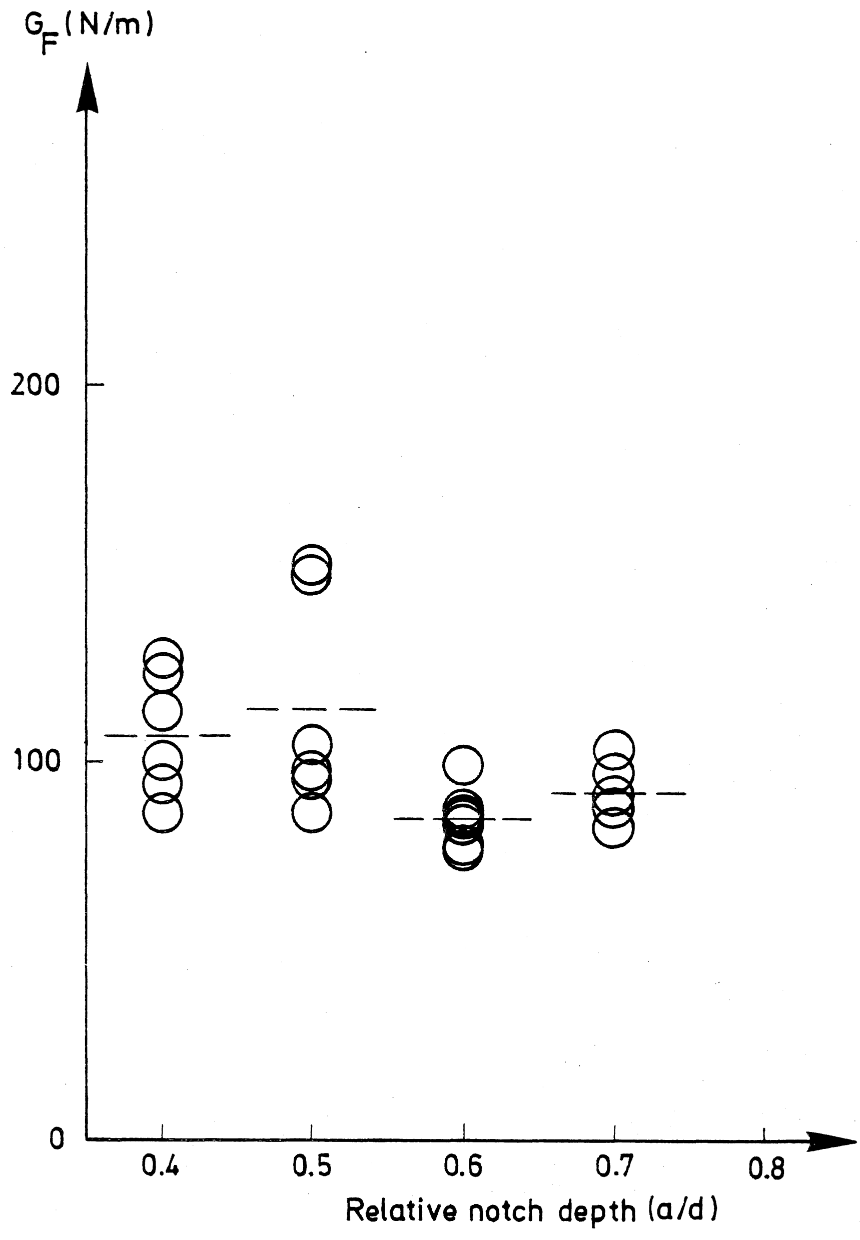

Fig 9. G_F as a function of the relative notch depth.

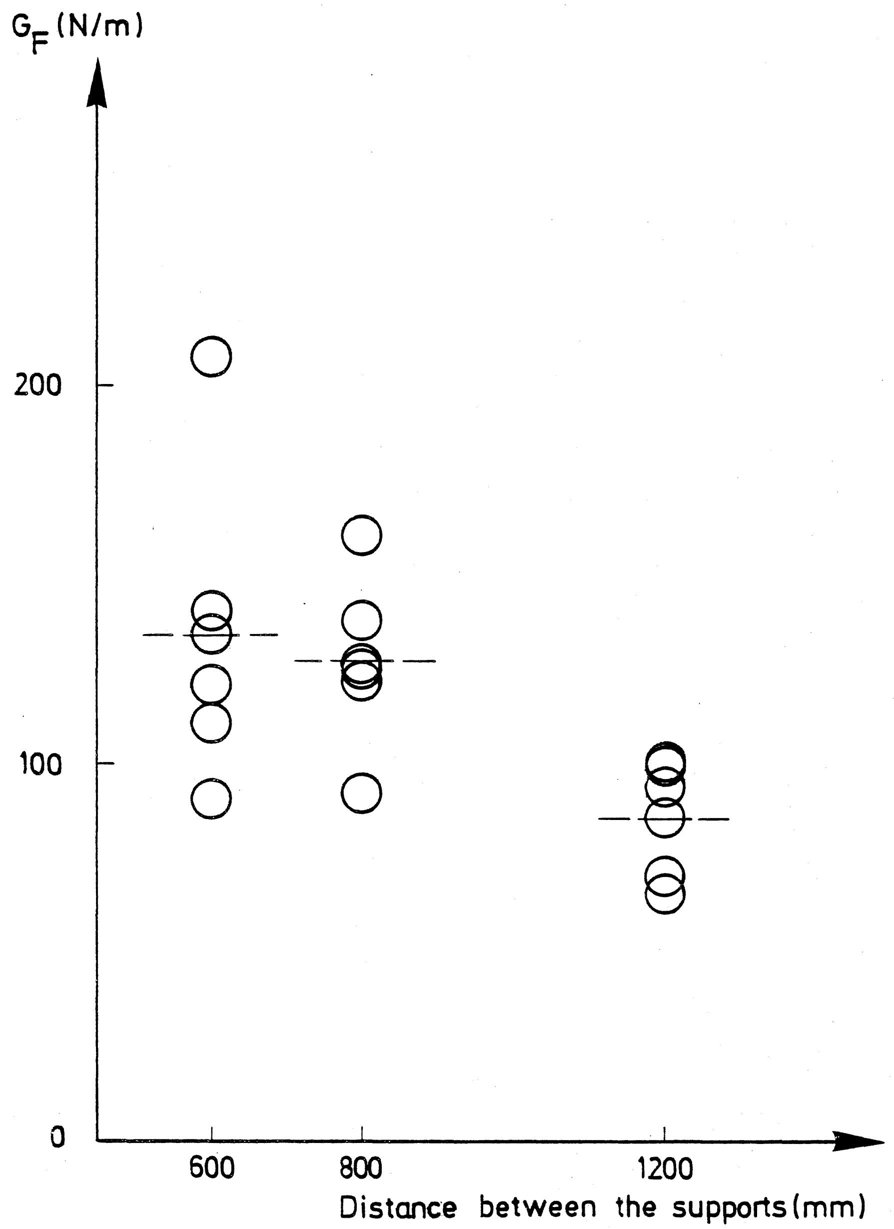

Fig 10. G_F as a function of the distance between the supports.

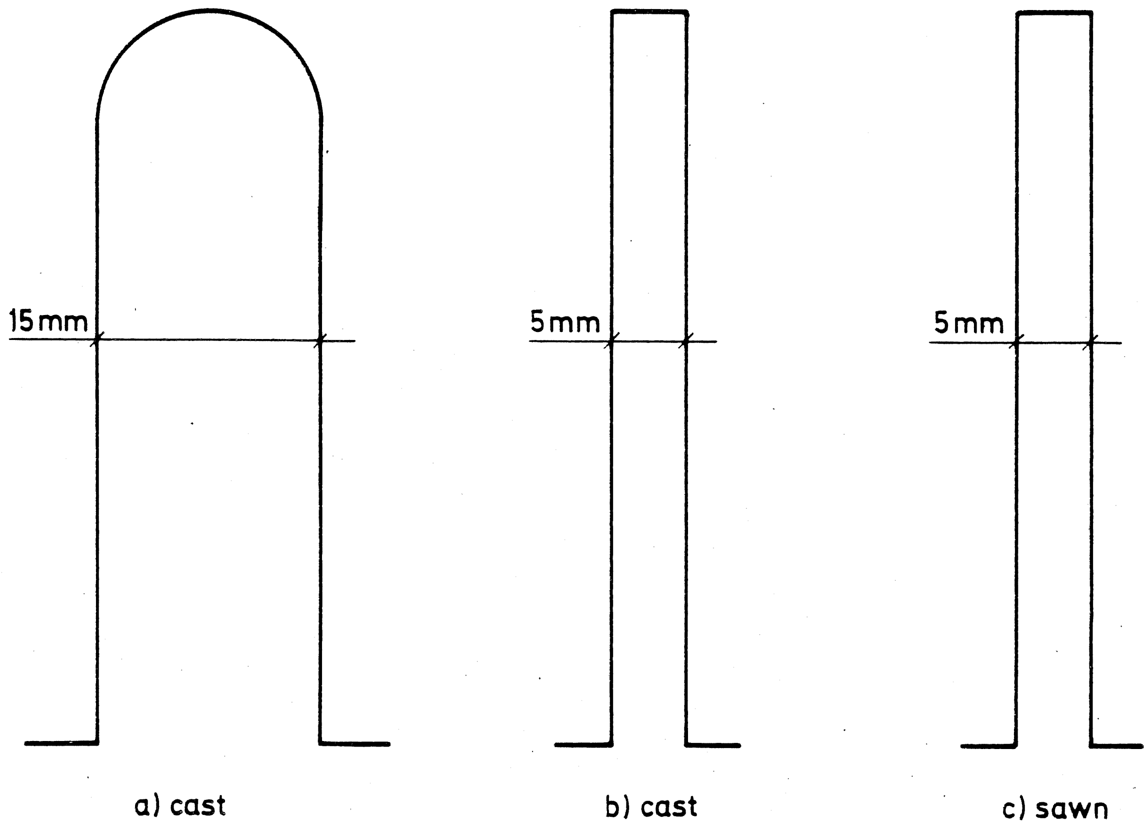

Fig 11. The three different notch types used during the test.

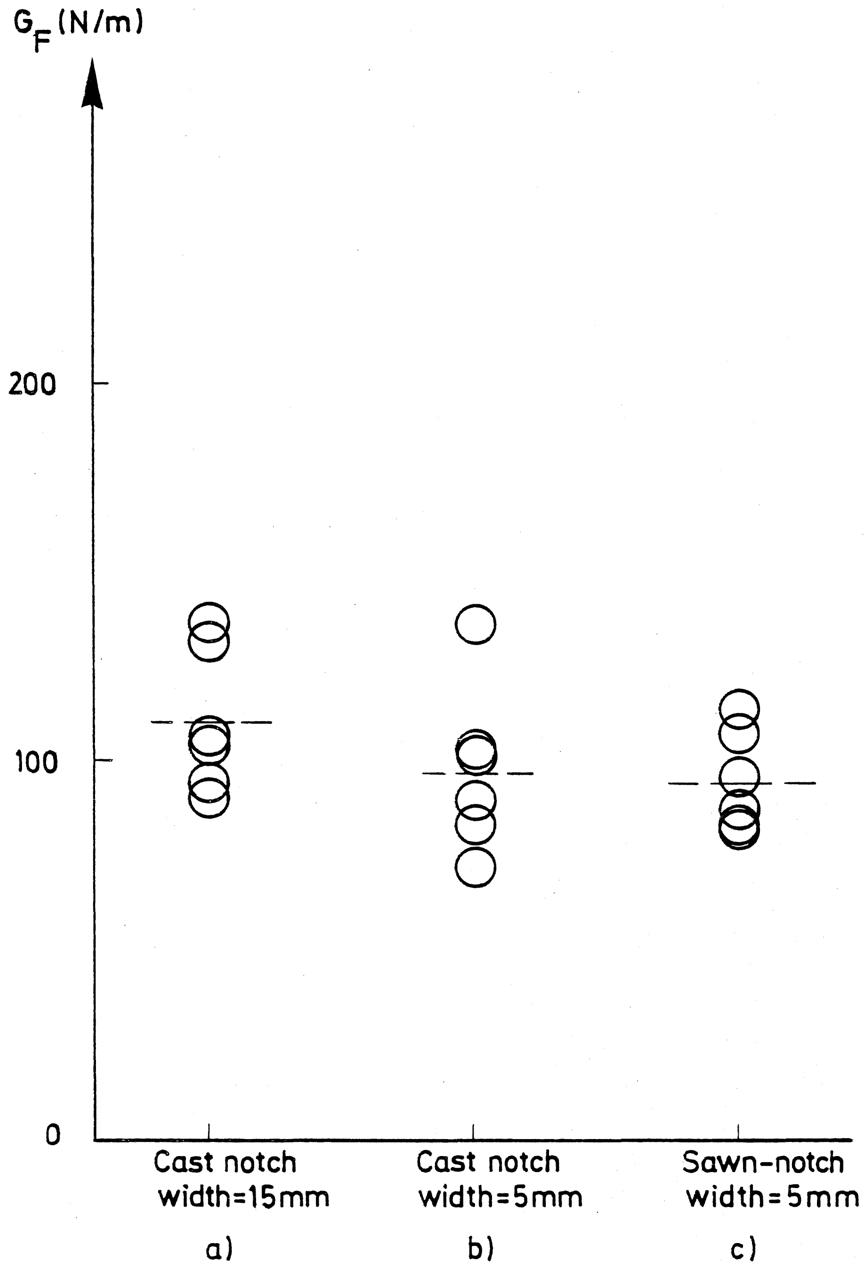

Fig 12. G_F-values for different types of notches. The notch types denoted a), b) and c) are described in Fig 11.

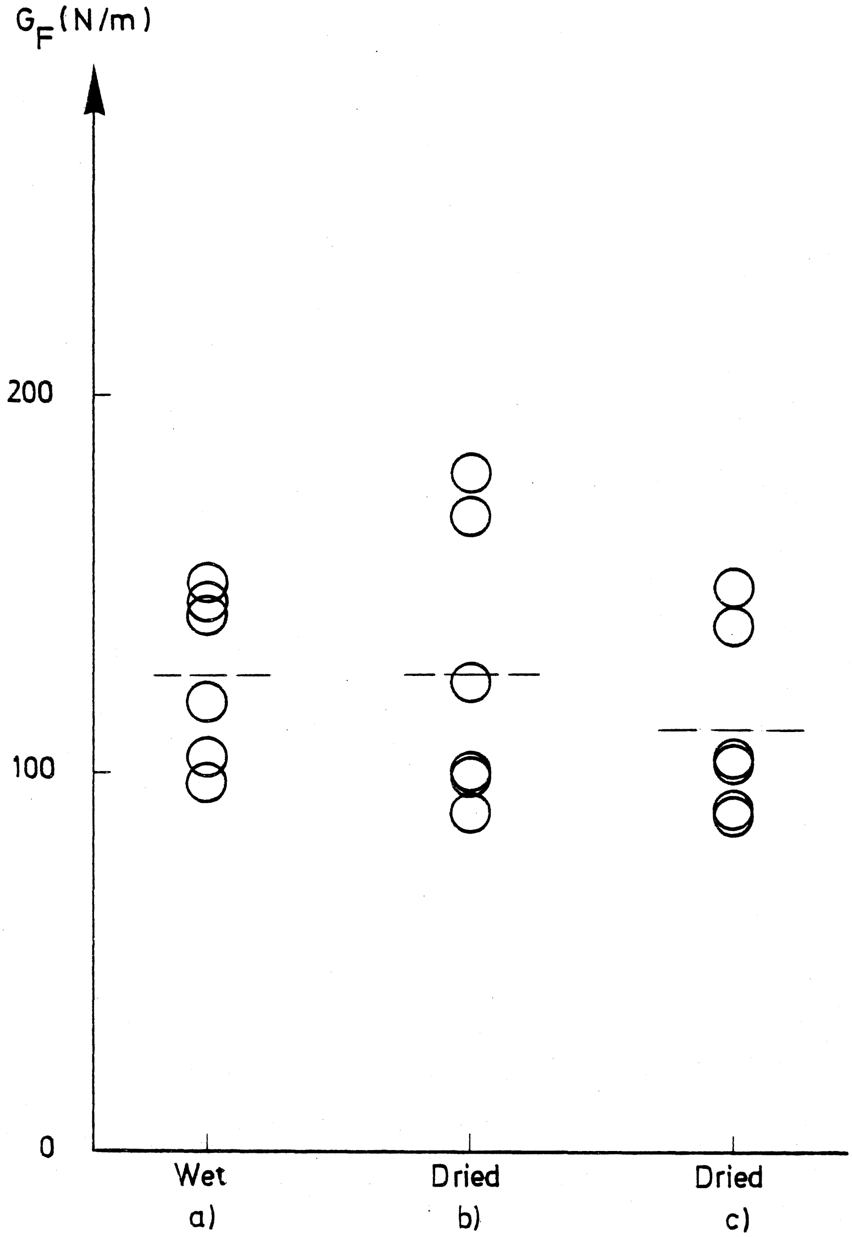

Fig 13. G_F-values representing different curing conditions. The curing conditions denoted a), b) and c) are defined in the text.

4.7 Curing conditions

It is desirable if the specimens can be stored in water until the time for testing as this is the most well defined curing condition. However. it is easier to perform the test if the specimen is allowed to dry in the air for a few minutes while it is adapted to the testing machine. In order to investigate how a short drying period affects the test results, the following curing conditions were analysed:

a) 28 days wet, the notch sawn on the 27th day
b) 27 days wet, 1 day in the air, the notch sawn on the 27th day
c) 27 days wet, 1 day in the air, the notch sawn on the 28th day

The tests were performed 28 days after casting as the different curing conditions during the 28th day then have a lesser effect on the hydration process than when the concrete is only 7 days old. The possible differences between the results should then mainly be dependent on the effects of drying (or shrinkage).

The results are presented in Fig 13. As can be seen, no significant differences were obtained between the results for the specimens stored in different curing conditions. The results,however, give no information about possible effects caused by different curing conditions during longer times.

5 THE EFFECT OF THE SCATTER ON THE NECESSARY NUMBER OF SPECIMENS IN A TEST SERIES

When determining G_F of concrete by the use of stable fracture tests the scatter becomes wide, which is clearly indicated by the results presented above. The scatter influences the accuracy of the test results and in order to improve the accuracy it is necessary to increase the number of specimens used in a test serie.

For G_F-tests on concrete the scatter ought to increase with decreasing value of the ratio $\sqrt{A_{lig}}/d_{aggr}^{max}$, where A_{lig} is the area of the fracture area and d_{aggr}^{max} is the size of the largest aggregate particles. For a beam with a square cross sectional area and the relative notch depth 0.5, $\sqrt{A_{lig}}/d_{aggr}^{max}$ can be expressed as $\sqrt{2} \times d/d_{aggr}^{max}$, where d=beam depth.

In order to study the effect of $\sqrt{A_{lig}}/d_{aggr}^{max}$ on the scatter and thereby on the necessary number of specimens in a test series, beams with different cross sectional areas and different sizes of the largest aggregate particles were analysed. The three concrete mixtures used are presented in Table 5.1. The concrete beams were cast and cured according to the description in 4.2.

24 specimens (cross sectional area = 100 x 100 mm^2, relative notch depth = 0.5) of mixture 2 were tested. These specimens are in fact identical to those used for the tests presented in 4.2 and Fig 6.

Table 5:1. The three concrete mixtures used in the tests

		Mix 1	Mix 2	Mix 3
Cement (ordinary Portland)	kg/m^3	442	341	341
Water	"	265	204	204
Natural gravel (0-4 mm)	"	1567	448	448
Natural gravel (0-4 mm)	"	-	448	448
Crushed quartzite (8-12 mm)	"	-	448	298
Crushed quartzite (12-16 mm)	"	-	448	298
Crushed quartzite (16-25 mm)	"	-	-	298

Table 5:2. Test results and results found in the literature of the coefficient of variation (v=s/m, s=standard deviation, m=mean value) for G_F-determinations on concrete by use of three-point bend tests on notched beams.

References	d_{aggr}^{max} (mm)	A_{lig} (mm^2)	$\sqrt{A_{lig}}/d_{aggr}^{max}$	Number of specimens	v=s/m×100 %	Notations in Fig 14
Test presented in this chapter	4 4 16 25 25	5000 1250 5000 5000 1250	17.7 8.8 4.4 2.8 1.4	12 12 24 12 12	5.2 11.6 24.0 26.0 44.2	●
/3/	8 12 16	1250 1250 1250	4.4 2.9 2.2	12 12 12	12.3 18.2 23.7	+
/1/	8 8 8 8	5000 5000 1250 1250	8.8 8.8 4.4 4.4	6 6 10 11	16.1 9.5 15.0 11.3	▽
/4/	4 32	1250 5000	8.8 2.2	12 11	11.0 22.5	▼

Each of the mixtures 1 and 3 were tested by using 12 specimens with the cross sectional area 100 x 100 mm^2 and 12 specimens with the cross sectional area 50 x 50 mm^2. For all the specimens the relative notch depth was 0.5 and the distance between the supports was 8 times the beam depth.

The results, i.e. the scatter, for the 5 test series are presented in Table 5.2. The scatter is defined by using the coefficient of variation (v), i.e. the standard deviation over the mean value (s/m). In addition to the results from the 5 test series a few test results found in the literature are also presented in the Table.

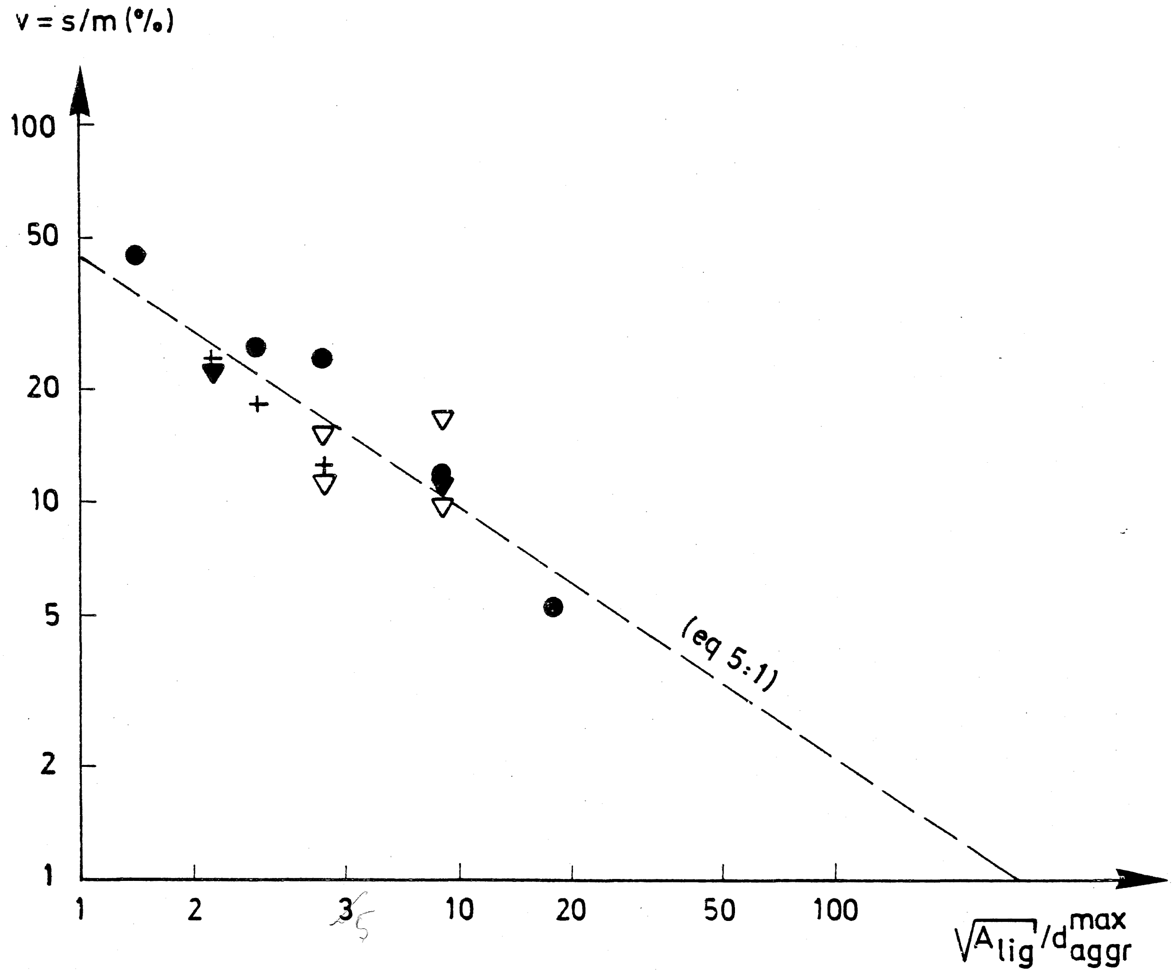

Fig 14. Coefficient of variation as the function of $\sqrt{A_{lig}}/d_{aggr}^{max}$. The definition of the dots is given in Table 5.2 and the straight line is defined in eq (5:1).

In Fig 14 the coefficients of variation according to Table 5:2 are shown as the function of $\sqrt{A_{lig}}/d_{aggr}^{max}$. The position of the straight line is calculated by means of linear regression. The coefficient of correlation is -0.89, which means that the line fits the data fairly well. The equation for the straight line is:

$$v = 44\left(\frac{\sqrt{A_{lig}}}{d_{aggr}^{max}}\right)^{-0.77} \qquad (5:1)$$

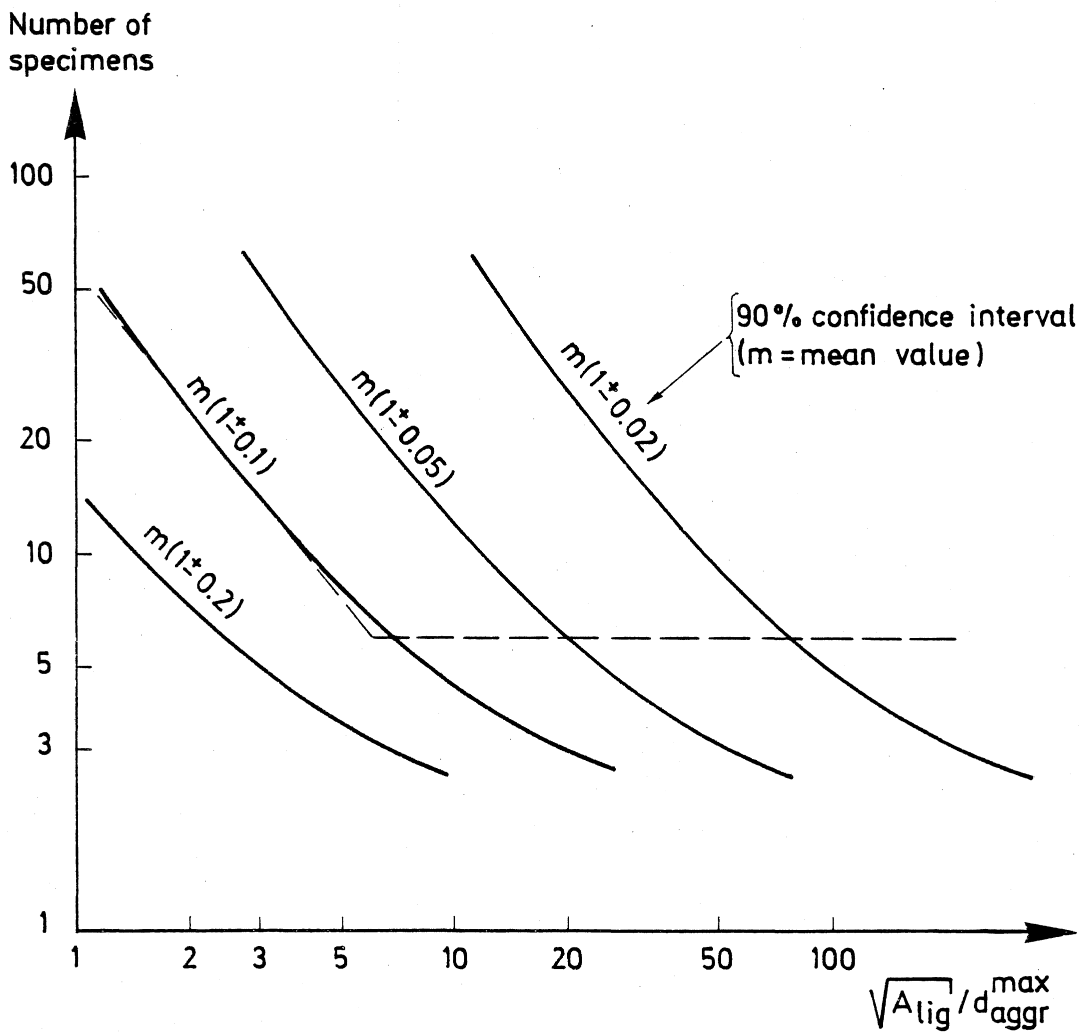

Fig 15. The number of specimens necessary to use in a test series, in order to obtain a mean value within a defined confidence interval, as functions of $\sqrt{A_{lig}}/d_{aggr}^{max}$. The continuous curves represent different widths of the 90 % confidence intervals and the dashed line represents a curve recommended in /2/.

Eq (5:1) makes it possible, by using ordinary statistical methods, to calculate the number of specimens necessary to use in a test series in order to obtain a mean value within a defined confidence interval. These calculation results are presented in Fig 15. The curves are relevant for 90 % confidence intervals, which ought to be satisfactory in most cases where concrete is concerned and the curves represent different widths of the confidence intervals ranging from ±0.02 m to ±0.2 m (m=mean value).

As can be seen in Fig 15 it is necessary to accept a confidence interval of about ±0.1 m as the number of specimens in a test series would otherwise be too large for ordinary testing conditions ($\sqrt{A_{lig}}/d_{aggr}^{max}$=3-30). The number of specimens in a test series should always exceed at least 6 and therefore the dashed line in Fig 15 is recommended for standard tests /2/.

6 REFERENCES

/1/ Petersson, P-E. "Crack growth and development of fracture zones in plain concrete and similar materials." Report TVBM-1006, Division of Building Materials, Lund Institute of Technology, Lund, Sweden,

/2/ Proposed RILEM recommendation, 29th January, 1982, revised version June 1982. "Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams." Division of Building Materials, Lund Institute of Technology, Lund, Sweden, 1982.

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