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Ziegler-Nichols Auto-Tuners

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Z I E G L E R - N I C H O L S A U T O - T U N E R S

Karl Johan Åström

Abstract:

Methods for automatic tuning of PID-regulators based on Ziegler-Nichols, phase and amplitude margin design criteria are presented. The techniques may be used to provide auto-tuning for simple regulators. The methods are non-parametric and insensitive to modelling errors and disturbances. They will also give the major dynamic characteristics of the process e.g. the dc-gain and the dominant time constants. The techniques may therefore also be used to give initial values for many other types of adaptive regulators.

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1. INTRODUCTION

Many schemes have been proposed for adaptive control. See e.g. the recent survey Åström (1981) where many references are given. Model reference adaptive control (MRAC) and self-tuning regulators (STR) are two common approaches, which are closely related. A self-tuning regulator can be thought of as composed of three parts, an ordinary linear regulator with variable parameters, a parameter estimator and a static system which performs control design calculations based on the estimated parameters. There are many different ways to choose techniques for parameter estimation and control design.

Adaptive techniques may be used in many different ways. In the applications discussed in this paper the purpose is to obtain techniques to tune regulators automatically. The word auto-tuning is used to emphasize this.

In this paper some simple auto-tuners are proposed. The simplicity is obtained by choosing simple design methods and simple estimation techniques. The starting point is the well known Ziegler-Nichols tuning procedure for PID-regulators. This method is based on knowledge of only one point on the Nyquist curve of the open loop system. A very simple estimation procedure which determines this point is proposed. The method has the advantage that a perturbation signal is generated automatically. This perturbation signal is in fact close to optimal for the particular estimation problem.

The Ziegler-Nichols design method normally gives closed loop systems with poor damping. Modifications with better damping are also presented. Other design procedures for PID-regulators which give prescribed phase and amplitude margins are also described.

It is also possible to tune PID-regulators by conventional techniques based on self-tuning regulators or model reference adaptive control, see Wittenmark and Åström (1980) and Landau (1979). The method proposed in this paper has two advantages over these approaches.

Conventional adaptive control based on parameter estimation requires a priori knowledge of the dominating time constant. This is required in order to obtain a prior guess of the sampling period. There are techniques to adjust the sampling period automatically, see Kurz (1979) and Åström and Zhaoying (1981). These techniques will however not work if the initial guess is off by an order of magnitude. The methods proposed in this paper will not require a priori knowledge of the time scale of the process. Therefore they may also be used to initialize more sophisticated

self-tuners.

Conventional approaches to self-tuning PID control result in a microprocessor code of a few kilobytes. The code for the proposed schemes is at least an order of magnitude smaller. The proposed methods may therefore conveniently be incorporated even in very simple regulators.

The major drawback of the methods proposed in this paper compared to conventional adaptive techniques is that they are limited to tuning of simple control laws of the PID type.

The paper is organized as follows: A brief review of the principles of self-tuning control is given in Section 2. The Ziegler-Nichols design method is reviewed in Section 3. Modified design methods which give prescribed phase and amplitude margins, are also given in Section 3. All design procedures require at one point on the Nyquist curve of the open loop system is shown. Methods for estimating of such points are given in Section 4. The methods are based on introduction of nonlinear elements in the closed loop which give rise to a limit cycle oscillation. This gives an automatic generation of a test signal for the estimation. Methods for determining of the period and the amplitude of the oscillation are also discussed in Section 4. Results from simulations and laboratory experiments are given in Section 5. Some concluding remarks are given in Section 6.

2. PRINCIPLES OF SELF-TUNING CONTROL

A schematic diagram of a self-tuning regulator is shown in Fig. 1. The regulator can be thought of as composed of three parts, a regulator with variable parameters, a parameter estimator, and a block which performs design calculations. The control design block is a good starting point for the design of a self-tuner. When the design procedure is known it is easy to see what information about the process that is necessary for the design. This gives guidelines for finding a suitable parameter estimator. Notice that in Fig. 1 the parameter estimation is done in closed loop. To ensure a good tuning in a short time it may be necessary to introduce perturbation signals during the tuning period.

When a self-tuner is used for automatic tuning the blocks which represent parameter estimation and control design in Fig. 1 are only connected when it is desired to tune the regulator. They are disconnected when the tuning is completed. The system then operates as an ordinary constant gain regulator.

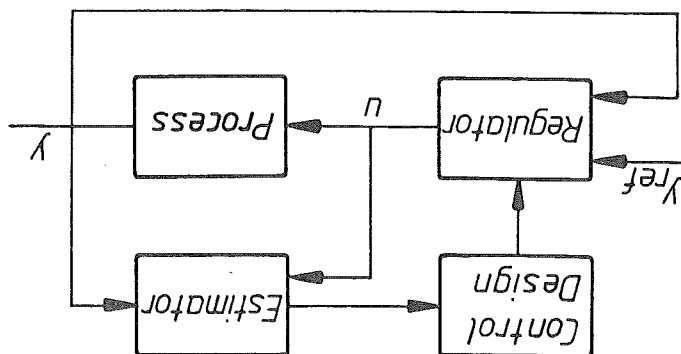
This method is based on a very simple characterization of the process dynamics. The design is based on knowledge of only one point on the Nyquist curve of the process transfer function G , namely the first point where the Nyquist curve intersects the negative real axis. See Fig. 3. For historical reasons this point is characterized by the parameters k and ω_c , which are called the critical gain and the critical frequency. The critical period $t_c = 2\pi/\omega_c$ is sometimes used instead of ω_c . The Ziegler-Nichols design method gives simple formulas for the parameters of the regulator in terms of the critical gain and the critical frequency. See Table 1. The Ziegler-Nichols tuning rules were originally based on results of simulation of many simple systems. They will often give closed loop systems with poor damping. This can be improved by modification of

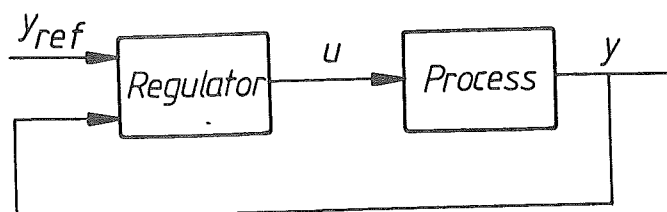
The Ziegler-Nichols method

Following the principles outlined in the previous section the problem of designing a regulator for a process with known characteristics, is first discussed. Consider the closed loop system shown in Fig. 2, which is composed of a process and a simple regulator of the PID-type. It is assumed that the process can be described as a linear time invariant system with the transfer function G . Different methods for finding suitable parameters of a PID-regulator will be discussed.

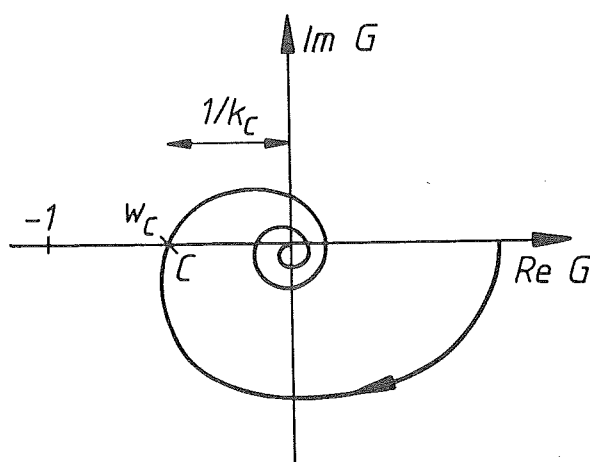
3. METHODS FOR DESIGN OF PID-REGULATORS

Figure 1. Block diagram of a self-tuner.





Figure_2. Block diagram of the closed loop system.



Figure_3. Nyquist curve for the transfer function of the process.

Table_1. - The Ziegler-Nichols design rules.

Regulator	Gain K	I-time	D-time
P	$0.5 k_c$		
PI	$0.4 k_c$	$0.8 t_c$	
PID	$0.6 k_c$	$0.5 t_c$	$0.12 t_c$

the numerical values given in the table.

Phase and amplitude margin design methods

Assuming that one point on the Nyquist curve is known it is possible to use other design methods. With PI or PID control it is for example possible to move the given point on the Nyquist curve to an arbitrary position in the complex plane. This is indicated in Fig. 4. By changing the gain it is possible to move the Nyquist curve in the direction of $G(i\omega)$. The point A may be moved in the orthogonal direction by changing integral or derivative gain. It is thus possible to move a specified point to an arbitrary position. This idea can be used to obtain design methods. By moving A to a point on the unit circle it is possible to obtain systems with a prescribed phase-margin. Analogously it is possible to obtain closed loop systems with a given amplitude margin by moving the point A to the negative real axis. A few examples are given below.

Example 1 - PI control with a specified phase margin.

If the point where the Nyquist curve intersects the negative imaginary axis is known it is possible to obtain a system with a given phase margin by PI-control with positive proportional and integral gains. Let the intersection occur for the frequency ω_d and let the transfer function of the process be G . See Fig. 5. The problem is thus to find a

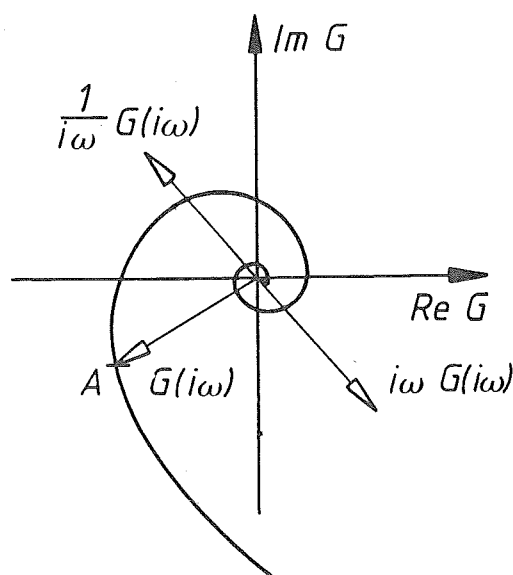


Figure 4. - Shows that a given point on the Nyquist curve may be moved to an arbitrary position in the G -plane by PI, PD or PID control. The point A may be moved in the directions $G(i\omega)$, $G(i\omega)/i\omega$ and $i\omega G(i\omega)$ by changing proportional, integral and derivative gain respectively.

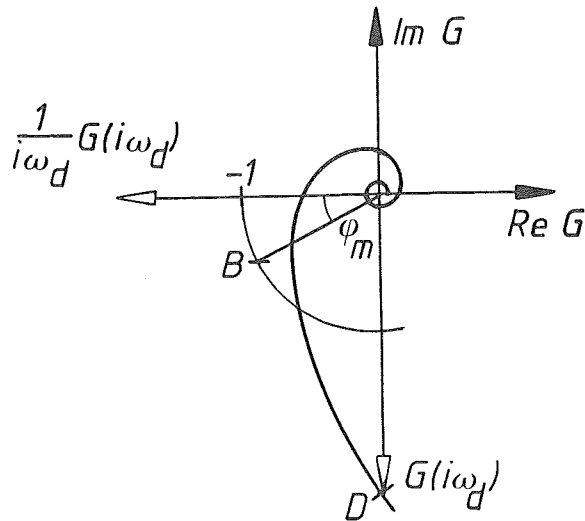


Figure 5. - Illustrates design of a PI-regulator with a given phase-margin.

regulator so that the point D in the figure is moved to B. This may be accomplished by a suitable choice of proportional and integral gains in a PI-regulator.

To see how the parameters of the regulator should be chosen observe that the loop transfer function with PI-control is

$$K \left(1 + \frac{1}{sT_i} \right) G(s).$$

Since the argument of the complex number $G(i\omega_d)$ is $-\pi/2$ the condition

$$\arg\left(1 + \frac{1}{i\omega_d T_i}\right) = \phi_m - \frac{\pi}{2}$$

gives a loop transfer function with the argument $\phi_m - \pi$ at the frequency ω_d . It follows from this condition that the integration time should be chosen as

$$T_i = \frac{1}{\omega_d} \tan \phi_m = \frac{1}{2\pi} t_d \tan \phi_m \quad (1)$$

where $t_d = 2\pi/\omega_d$ is the critical period under pure integral

control. The loop transfer function has unit gain at ω_d if

$$K \sqrt{1 + \left(\frac{1}{\omega_d T_i}\right)^2} |G(i\omega_d)| = 1$$

Combining this expression with equation (1) gives the following formula for the regulator gain

$$K = \frac{\sin \phi}{|G(i\omega_d)|} = k_d \frac{\sin \phi}{m} \quad (2)$$

where k_d is the critical gain under pure integral control.

Notice that the approach is closely related to the Ziegler-Nichols approach in the sense that the regulator parameters are determined simply from one point on the Nyquist curve.

□

A drawback with the design method in Example 1 is that the system may be unnecessarily slow because the crossover frequency is chosen as the frequency where the Nyquist curve of the process intersects the negative imaginary axes. With PI control it is, however, not possible to have a crossover frequency which corresponds to the point where the Nyquist curve of the process intersects the negative real axis because the integral time is positive in a PI regulator. With PID control a higher crossover frequency may be obtained. An example of such a design method is given below.

Example 2 - PID control with specified phase margin. Consider a process with the transfer function G . The loop transfer function with PID control is

$$K \left(1 + sT_d + \frac{1}{sT_i}\right) G(s)$$

Assume that the point ω_c where the Nyquist curve of G

intersects the negative real axis is known. The argument of the loop transfer function at ω_c is then

$$\arg \left(1 + i\omega_c T_d + \frac{1}{i\omega_c T_i}\right) - \pi$$

Requiring that the argument is $\phi_m - \pi$ the following condition is obtained

$$\omega T_d - \frac{1}{\omega T_i} = \tan \phi_m \quad (3)$$

There are many T_d and T_i which satisfy this condition. One possibility is to require that there is a constant ratio between T_i and T_d e.g.

$$T_i = 4 T_d \quad (4)$$

Equation (3) then gives a second order equation for T_d which has the solution

$$T_d = \frac{\tan \phi_m + \sqrt{1 + \tan^2 \phi_m}}{2 \omega_c} \quad (5)$$

Furthermore, simple calculations show that the loop transfer function has unit gain at ω_c if the regulator gain is chosen as

$$K = \frac{\cos \phi_m}{|G(i\omega_c)|} = k_c \cos \phi_m \quad (6)$$

where k_c is the critical gain. Compare Fig. 3. There are many other possibilities. The parameter T_i may e.g. be chosen so that $\omega_c T_i$ has a given value.

The design rules are thus given by the equations (3), (4) (5) and (6). Table 2 gives the results for different values of the phase margin. A comparison with Table 1 shows that the design is closely related to the Ziegler-Nichols design method. The gain is slightly higher but the integration time and the derivation times are longer.

□

Table 2 - A phase margin design rule.

ϕ_m	Gain K	I-time	D-time
30	$0.87 k_c$	$0.55 t_c$	$0.14 t_c$
45	$0.71 k_c$	$0.77 t_c$	$0.20 t_c$
60	$0.50 k_c$	$1.29 t_c$	$0.30 t_c$

There are many possible variations of the design methods for PID-regulators given in this section. All methods are closely related because they are based on information about the process to be controlled in terms of one point on the Nyquist curve of the process. The points where the Nyquist curve intersects the real or the imaginary axes are simple choices. Other points on the Nyquist curve may be chosen. See Hägglund (1981). The design methods may also be modified. Other relations between T_i and T_d than those given

by (4) may also be used. Other criteria like amplitude margin or damping may be chosen instead of the phase-margin. It is also possible to have design methods which are based on knowledge of more points on the Nyquist curve.

Specifications on phase and amplitude margins do not guarantee good performance of the closed loop system. The following modification was therefore used. It was required that the Nyquist curve intersects the circle with radius 0.5 at an angle of $\phi_m = 45^\circ$.

4. PARAMETER ESTIMATION

To obtain a self-tuning regulator according to the principles discussed in Section 2, the design principles described in Section 3 should be combined with a parameter estimator. Suitable estimation methods are discussed in this section.

Principles

In the original Ziegler-Nichols scheme the critical gain and the critical frequency are determined in the following way. A proportional regulator is connected to the system. The gain is gradually increased until an oscillation is

obtained. This procedure is not easy to do automatically. Another method for automatic determination of specific points on the Nyquist curve is therefore proposed.

The method is based on the observation that a system with a phase lag of at least π at high frequencies may oscillate with period t_c under relay control. To determine the point C

in Fig. 3 the system is connected in a feedback loop with a relay as is shown in Fig. 6. The error e is then a periodic signal and the parameters k_c and ω_c can be determined from

the first harmonic component of the oscillation.

Let d be the relay amplitude and let a be the amplitude of the first harmonic of the error signal. A simple Fourier series expansion of the relay output then shows that the relay may be described by the equivalent gain

$$k_c = \frac{4d}{\pi a} \quad (7)$$

To obtain the point where the Nyquist curve intersects the negative imaginary axis, an integrator may be connected in the loop after the relay. To obtain other points on the Nyquist curve components with a known phase shift may be introduced into the loop. See Hägglund (1981).

A simple relay control experiment thus gives the information about the process which is needed in order to apply the design methods. Notice in particular that the estimation method will automatically generate an input signal to the process which has a significant frequency content at ω_c .

This ensures that the point A can be determined accurately. See Mannerfelt (1981).

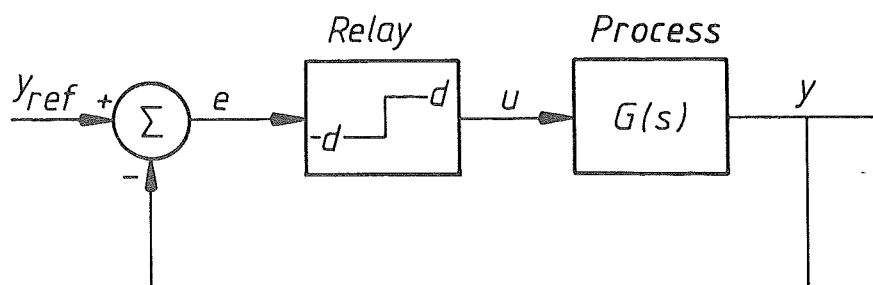


Figure 6. - Relay control of the process.

Difficulties

There are some difficulties associated with the proposed method. If the process has a phase lag less than 180 degrees the system shown in Fig. 6 will not oscillate. An oscillation may be generated by introducing an integrator in the system. If the system still does not oscillate it is strictly positive real. It may then conveniently be controlled by relay feedback. It may also happen that the Nyquist curve intersects the negative real axis at many points. Compare Fig. 7. The closed loop system obtained with relay control may then oscillate with several frequencies.

It may also happen that the system is such that the approximative analysis fails. See Graham and McRuer (1961). This may occur if the error signal has a large content of higher harmonics. In many practical problems the transfer function of the process will however decay rapidly. The signal at the process output is then nearly sinusoidal and describing functions will work very well. It is a good research problem to find precise conditions for this.

Determination of amplitude and period

To complete the description of the estimation method it is also necessary to give methods for automatic determination of the frequency and the amplitude of the oscillation. This may be done in many different ways.

Detection of peaks and zero crossings

The period of an oscillation can easily be determined by measuring the times between zero-crossings. The amplitude

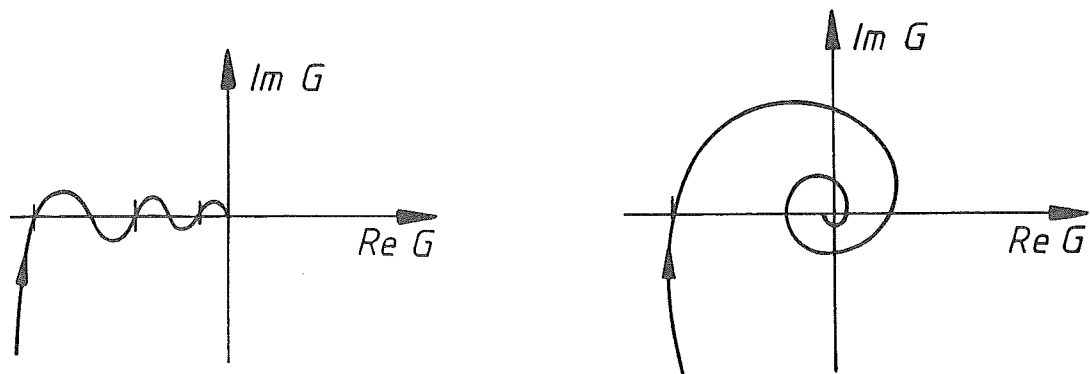


Figure 7. - Nyquist curves with several intersections of the negative real axis. Oscillations may occur at the frequencies indicated by dots in the figure.

may be determined by measuring the peak-to-peak values. These estimation methods are very easy to implement because they are based on counting and comparisons only.

Since the describing function analysis is based on the first harmonic of the oscillation the simple estimation techniques require that the first harmonic dominates. If this is not the case it may be necessary to filter the signal before measuring. See e.g. Åström (1975).

Least squares estimation

More elaborate estimation schemes may also be used to determine the amplitude and the period of the oscillation.

The period may be estimated based on the observation that a sinusoidal function with period T satisfies the linear difference equation

$$y(t) - \theta y(t-h) + y(t-2h) = 0, \quad (8)$$

where h is the sampling period and

$$\theta = 2 \cos(2\pi h/T).$$

The period can thus be obtained by estimating the parameter θ in (9) by least squares and computing an estimate of T from

$$\hat{T} = 2\pi h / \cos^{-1}(\hat{\theta}/2). \quad (9)$$

Since the signal may have a non-zero average it should be high-pass filtered before estimation. Alternatively, the parameters θ_1 and θ_2 of the model

$$y(t) - \theta_1 y(t-h) + y(t-2h) + \theta_2 = 0$$

could be estimated.

When the period T is obtained the amplitude may be obtained from the solution to the following least squares problem

$$\min_{\theta} \sum_{k=1}^N [y(kh) - \theta_1 \sin \omega kh - \theta_2 \cos \omega kh - \theta_3]^2$$

where

$$\omega = 2\pi/\hat{T}.$$

An estimate of the amplitude is then given by

$$\hat{a} = \sqrt{\theta_1^2 + \theta_2^2}$$

Extended Kalman filtering

The period and damping of an oscillation may also be determined by extended Kalman filtering. This is based on the observation that a signal which is the sum of a sinusoid and a constant may be represented by the following differential equations

$$\frac{dx_1}{dt} = 2\pi x_2 / x_3$$

$$\frac{dx_2}{dt} = -2\pi x_1 / x_3$$

$$\frac{dx_3}{dt} = 0$$

$$\frac{dx_4}{dt} = 0$$

$$y = x_1 + x_4$$

The states of this model may be estimated by an extended Kalman filter. See Jazwinski (1970) and Balchen et al (1976). The period and the amplitude of the oscillation are then given by

$$\hat{a} = \sqrt{x_1^2 + x_2^2}$$

$$\hat{T} = x_3$$

Combinations of different estimation schemes

There are many possibilities to combine different estimation schemes. A crude estimate of the period may e.g. be obtained by determining peaks and zero crossings. The estimate obtained could then be refined by least squares or by extended Kalman filtering. When discussing this it should also be kept in mind that the design rules are based on fairly crude characterizations of the desired performance. Thus it makes little sense to obtain very accurate estimates.

5. SIMULATIONS AND EXPERIMENTS

A number of simulations and experiments have been performed in order to find out if a useful auto-tuner may be designed based on the ideas described in the previous sections. Consequences of using estimators having different complexity have also been explored. The results are summarized in this section. Some representative examples are also presented.

Practical aspects

There are several practical problems which must be solved in order to implement an auto-tuner based on the ideas described previously. It is necessary to account for measurement noise, level adjustment, saturation of actuators and automatic adjustment of the amplitude of the oscillation.

Measurement noise may give errors in detection of peaks and zero crossings. A dead-zone in the relay in Fig. 6 is a simple way to reduce the influence of measurement noise. Filtering is another possibility. The estimation schemes based on least squares and extended Kalman filtering can be made less sensitive to noise.

When the regulator is switched on it may happen that the process output is far from the desired equilibrium condition. It would be desirable to have the regulator reach the equilibrium automatically. For a process which has finite low frequency gain there is no guarantee that the desired steady state will be achieved with relay unless the relay amplitude is sufficiently large. To guarantee that the output can actually reach the reference value it may be necessary to introduce manual or automatic reset.

It is highly desirable that the relay amplitude is adjusted automatically. A reasonable approach is to make sure that the oscillation is a given percentage of the admissible swing in the output signal.

Regulator complexity

The consequences of using estimation schemes of different complexity have been explored by simulation. In these experiments processes having different dynamics have been regulated with different types of auto-tuners. The effects of measurement noise and load disturbances have been investigated. Although work still remains to be done the experiments have shown that the simple estimation method based on zero-crossing and peak detection works very well. The experiments also indicate that simpleminded level adjustment methods often are satisfactory. A representative simulation of a simple auto-tuner is given below.

Simulation of a simple PI auto-tuner

A simple PI auto-tuner based on the design method given in Example 1 has been simulated. The design procedure in Example 1 was used and it was required that the Nyquist curve intersects the circle with radius 0.5 at an angle of 45° . A small dead-zone was also added to the relay to make the system less sensitive to measurement noise. An integrator was introduced into the process in order to be able to determine the intersection of the Nyquist curve with the negative imaginary axis. Compare Fig. 5. This integrator will also automatically ensure that the output will oscillate around the set point during the experiment.

The period was estimated from the zero crossings and the amplitude by peak detection. The amplitude of the oscillation was adjusted automatically in the following way. A prior value was given. The amplitude was estimated by detecting the peak during the first full period of the oscillation, starting at the first zero crossing. The amplitude is then recalculated to give the desired error amplitude which was chosen as 0.1. The period and the amplitude are then determined by analysing three full periods.

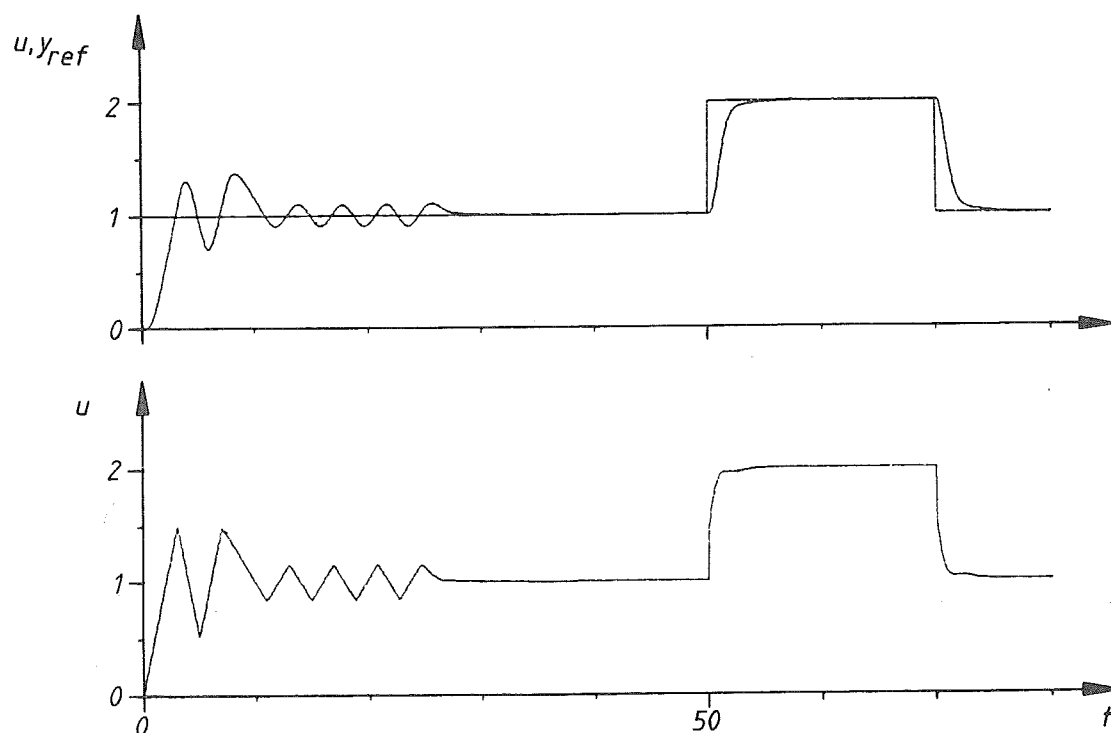


Figure 8. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = 1/(1+0.25s)^4$.

regulator is applied to processes with a wide variation in the process gain. The gain is changed by a factor of 10 up and down in comparison with the reference case. The initial value of the relay amplitude is 0.1 in all cases. The amplitude settles to 3, 0.3 and 0.03 respectively. The tuning algorithm gives an integration time of 0.64 in all cases. The gain is 4.5, 0.46 and 0.044 respectively. The experiment verifies that the algorithm behaves in the desired way.

Variations in process time constant

Similarly Fig. 10 shows corresponding results for variations in the process time constant. The initial value of the relay amplitude is 0.1 in all cases. The amplitude adjusts to 0.015, 0.079 and 0.79. The regulator gain tunes to 0.65 in all cases. The integration time tunes to 25, 2.6 and 0.25 respectively.

Effects of load disturbances and measurement noise

It is of considerable practical interest to know how the system reacts to load disturbances and measurement noise.

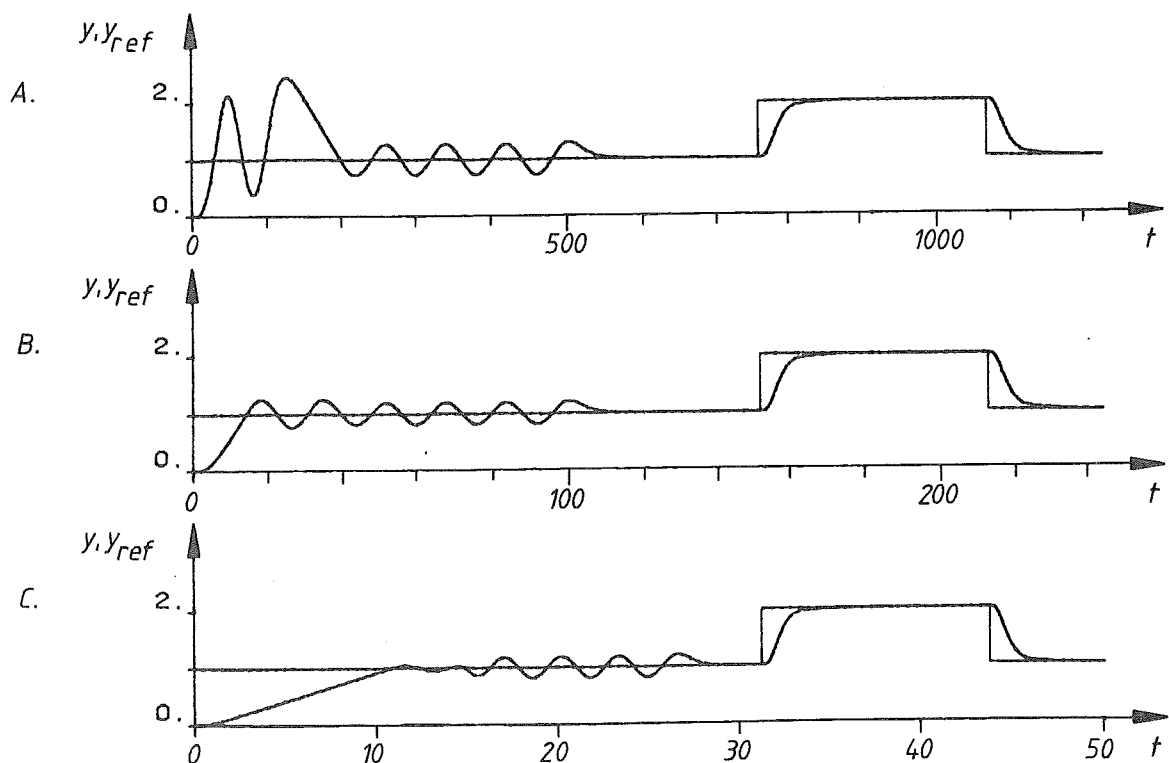


Figure 10. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = 1/(1+sT)^4$. The process time constant T is 5, 1 and 0.2 in A, B, and C respectively.

The performance of the auto-tuner is illustrated in Fig. 8 where control of a process whose dynamics is described by four cascaded first order lags is shown. The figure shows clearly that the auto-tuning algorithm performs well. The amplitude adjustment works well during the estimation period. The relay amplitude is initially too large (0.5) but the algorithm adjusts it to 0.16 which gives an error amplitude of 0.098 which is close to the desired value 0.10. The period is estimated to $t_d = 3.91$ which is close to the

correct value 3.79. An estimate of the critical gain under pure integral control can be calculated from (7). This gives 2.12 which is also close to the correct value 1.92. These numbers are in good agreement with experiences from other applications of describing function analysis. Accuracies of period and amplitude of the order of a few percent are e.g. reported in Graham and McRuer (1961).

Fig. 8 also shows that the system responds well to set point changes. Notice in particular that the modified design procedure gives much better damping than the normal Ziegler-Nichols rule.

Variations in process gain

Fig. 9 shows how the closed loop system behaves when the

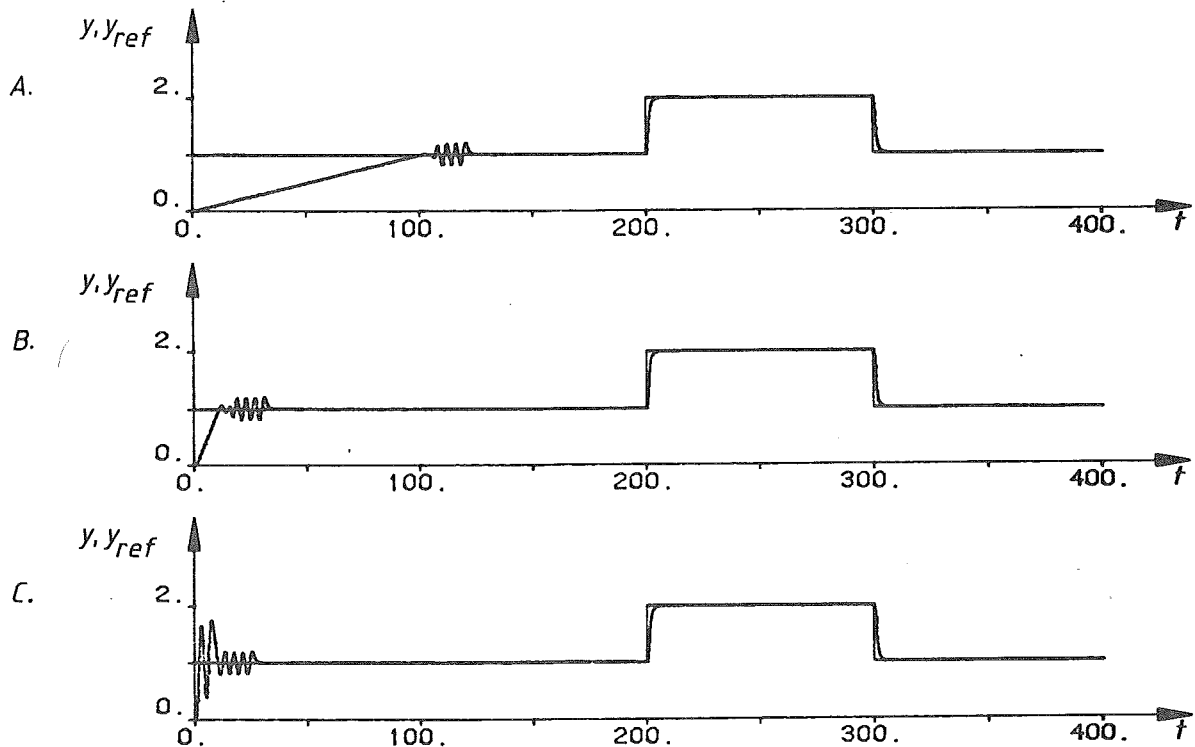


Figure 9. - Simulation of a PI auto-tuner applied to a process with the transfer function $G(s) = k/(1+0.25s)^4$. The process gain k is 0.1, 1 and 10 in A, B, and C respectively.

Fig. 11 illustrates the behaviour of the system under such conditions. The measurement noise was simulated as white noise added to the process output. The figure shows that the performance does not deteriorate appreciable when disturbances are added. The simple algorithm is in fact very robust with respect to measurement noise. A peak detection overestimates the amplitude in the presence of measurement noise. A consequence of this is that the gain will decrease with increasing noise level. Using the zero crossing method the period is underestimated when there is high frequency measurement noise. The regulator gain will thus decrease with increasing noise level and the integral action will increase. The effects of these desirable properties are illustrated in Fig. 12. The tuning algorithm gives the gain 0.2 an 0.013 and the integration times 0.4 and 0.05 in A and B respectively.

Laboratory experiments

Several of the algorithms have been applied to control of laboratory processes. See Elfgrén (1981). In these experiments the algorithms have been coded in Pascal on a DEC LSI 11/03. Different processes have been simulated on analog computers. Small laboratory processes have also been controlled. The general results found in the simulations,

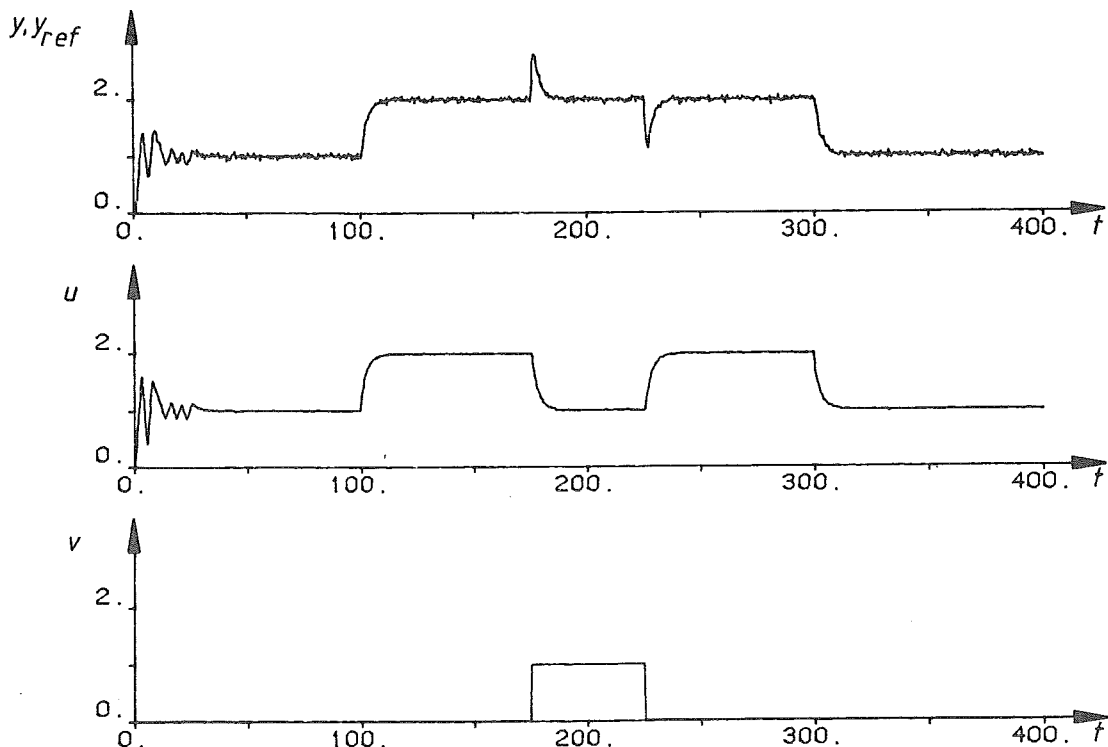


Figure 11. - Simulation of the system with measurement noise and load disturbances v . The standard deviation of the measurement noise is 0.03.

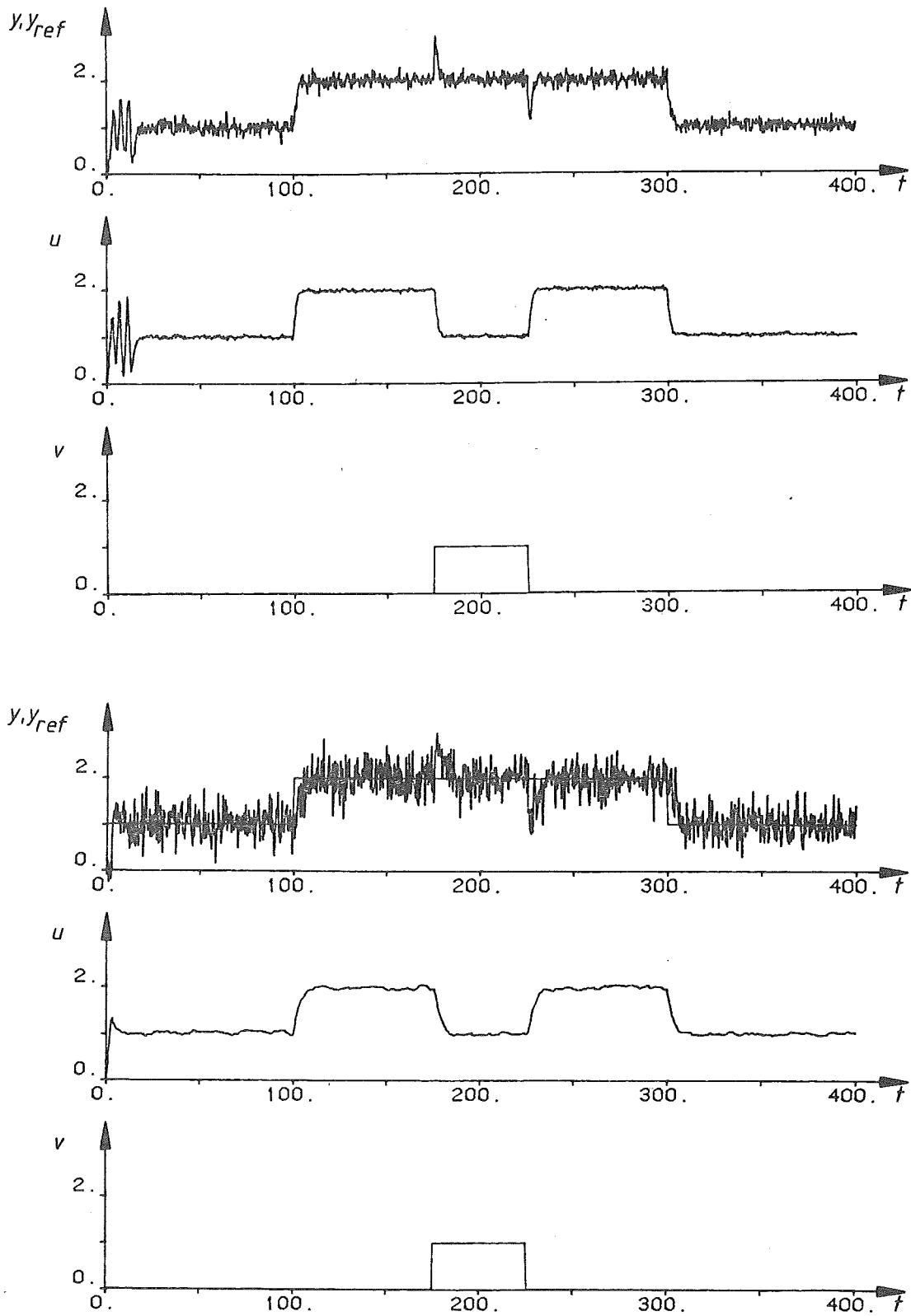


Figure 12. - Simulation of the system with variable noise level. The standard deviation of the measurement noise is 0.1 and 0.3 in A and B respectively.

that the simple algorithms work well, are also supported by experiences from the laboratory experiments. A representative experiment is described below.

Temperature control

Derivative action is often useful in temperature control. A few simple experiments were designed in order to investigate problems where PID-control may be beneficial. A lamp with a carbon filament was controlled. The surface temperature of the lamp bulb was measured using a thermocouple. The current through the lamp was controlled using a conventional thyristor system. A picture of the process is shown in Fig. 13.

The algorithm used in this case was an auto-tuner based on the Ziegler-Nichols design method. The critical gain and the critical period were determined by detecting peaks and zero crossings. The amplitude of the oscillations during the estimation phase was controlled automatically. A simple regulator with very low gain and integral action was introduced in order to provide an automatic start.

Fig. 14 shows a typical experimental result. Notice that a fairly high gain is needed. The control signal is noisy. This is due to the high gain and to the fact that the Ziegler-Nichols design gives a closed loop system with a poor damping. The effects of the noise can be reduced significantly by a slight modification of the Ziegler-Nichols rule.

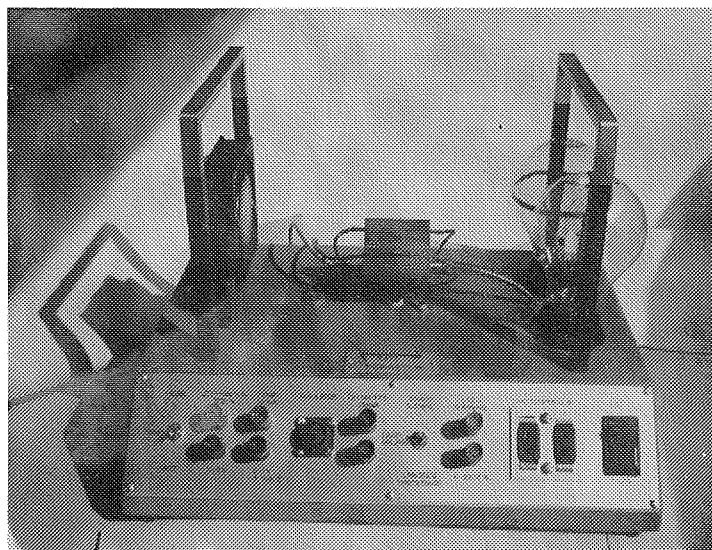


Figure 13. - System used for temperature control.

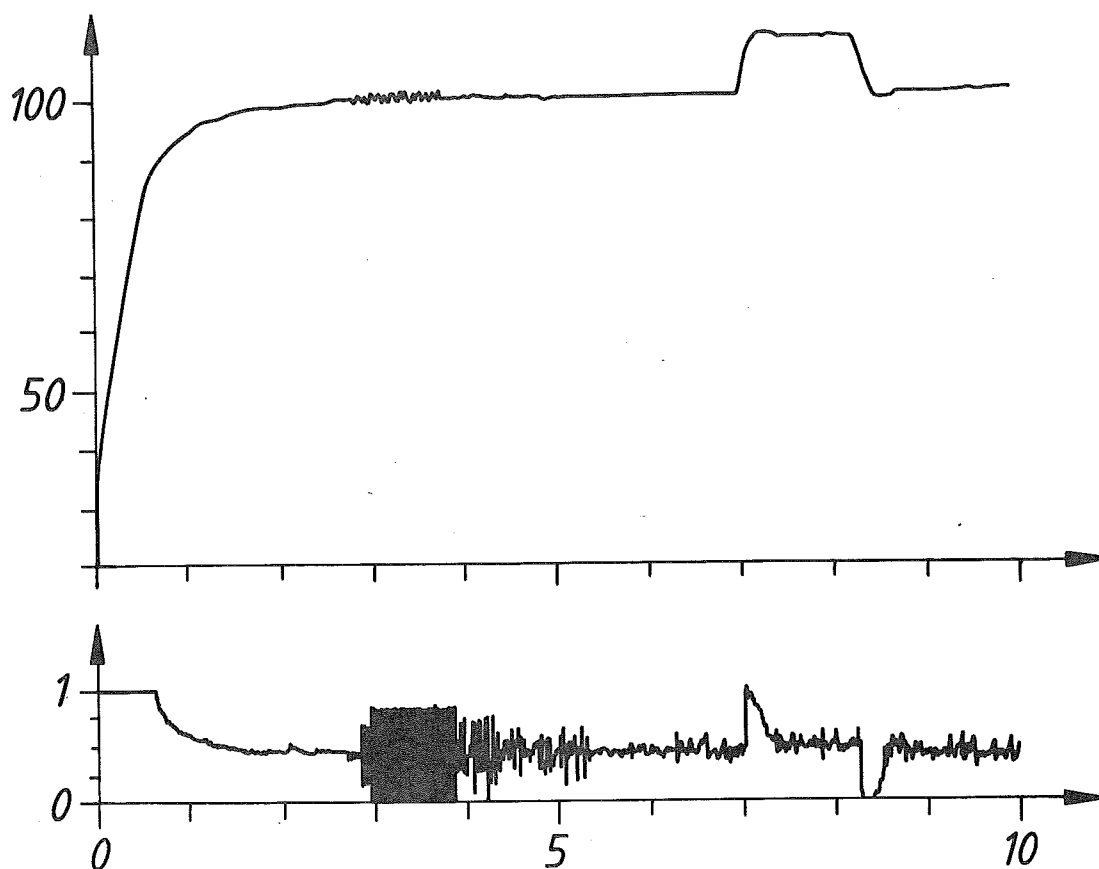


Figure 14. - Result from experiments with temperature control using a PID-regulator with auto-tuning.

6. CONCLUSIONS

There are many possibilities to introduce auto-tuning in regulators of the PID type. Self-tuning regulators based on minimum variance control or pole placement design may be configured in such a way that the regulators obtained correspond to PID control. Such approaches have been considered by Wittenmark et al (1980) and Wittenmark and Åström (1980). These types of regulators have the disadvantage that some information about the time scale of the process must be provided a priori. This is necessary in order to obtain a reasonable estimate of the sampling period in the regulator. There are some possibilities to tune the sampling period automatically. Different schemes have been proposed by Kurz (1980) and Åström and Zhaoying (1981). These methods will, however, only work well for moderate changes in the process time constants.

The method proposed in this paper does not suffer from this disadvantage. It may be applied to processes having widely different time scales. The test signal which is generated automatically by the algorithm will have a considerable energy at the crossover frequency of the process.

Conventional self-tuning regulators based on recursive estimation of a parametric model requires a computer code of a few kilobytes. The algorithms proposed in this paper which are based on determination of zero crossings and peak detection may be programmed in a few hundred bytes. It is thus possible to use these methods also in very simple regulators.

The methods proposed will of course inherit the limitations of the PID algorithms. They will not work well for problems where more complicated regulators are required.

The experiences reported also indicate that the simple versions of the algorithms work very well and that they are robust. It thus appears worthwhile to explore these algorithms further.

The algorithms discussed in this paper may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide the apriori information which is required by more sophisticated adaptive algorithms. When combined with a bandwidth self-tuner like the one discussed in Aström (1980) it is possible to obtain an adaptive regulator which may set a suitable closed loop bandwidth automatically.

7. ACKNOWLEDGEMENTS

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