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Åström, Karl Johan

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A TEST EXAMPLE FOR ADAPTIVE CONTROL

K. J. ÅSTRÖM

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Lund Institute of Technology
Division of Automatic Control

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K. J. Åström

Abstract

A simple control system which consists of a DC motor and a PI regulator is described. It is demonstrated that if the regulator is designed for one operating condition, the system performance can change significantly if the moment of inertia is changed moderately. The system is thus a candidate for testing adaptive schemes.

Contents

1. INTRODUCTION

When analysing adaptive control schemes it is desirable to have relevant test examples. To have a system which can be unstable it is necessary to have at least a third-order model. Such a system is analysed in this report.

Consider a DC motor described by

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} = K_1 V$$

where J is the moment of inertia of motor and load, D the damping coefficient, θ the angle of rotation and V the voltage to the amplifier. Let the motor be controlled by a PI regulator with the transfer function

$$G_k(s) = K_2(1 + \frac{1}{sT})$$

The open loop transfer function is then

$$G_o(s) = \frac{K_1 K_2 (1+sT)}{s^2 T(Js+D)}$$

and the closed loop transfer function is

$$G(s) = \frac{G_o(s)}{1 + G_o(s)} = \frac{K(1+sT)}{JTs^3 + DTs^2 + KTs + K}$$

where $K = K_1 K_2$

The root locus of the characteristic equation

$$JTs^3 + DTs^2 + KTs + K = 0$$

with respect to the gain parameter K is shown in Fig. 1.

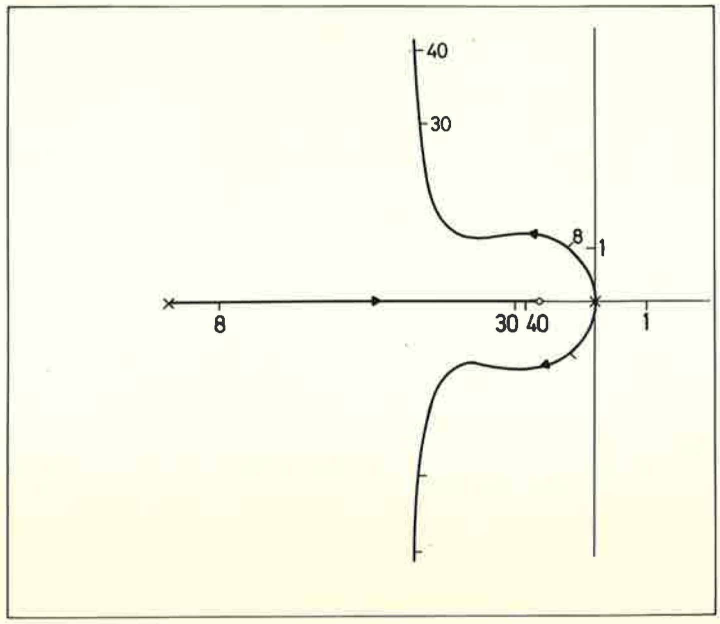

Fig. 1a. Root locus for D/J = 8.

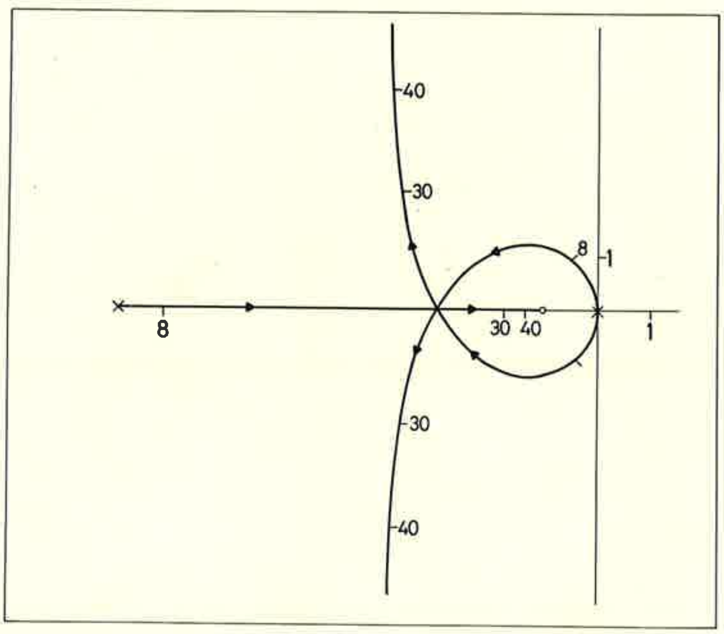

Fig. 1b. Root locus for D/J = 9.

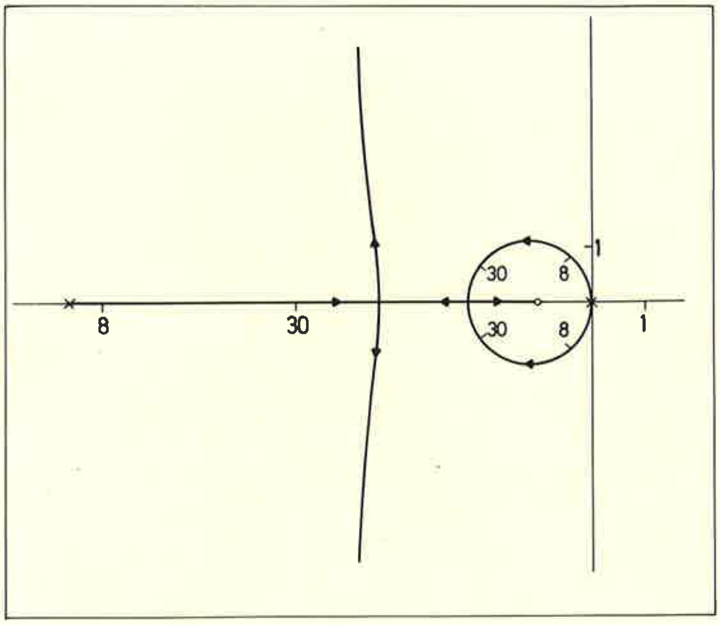

Fig. 1c. Root locus for D/J = 10.

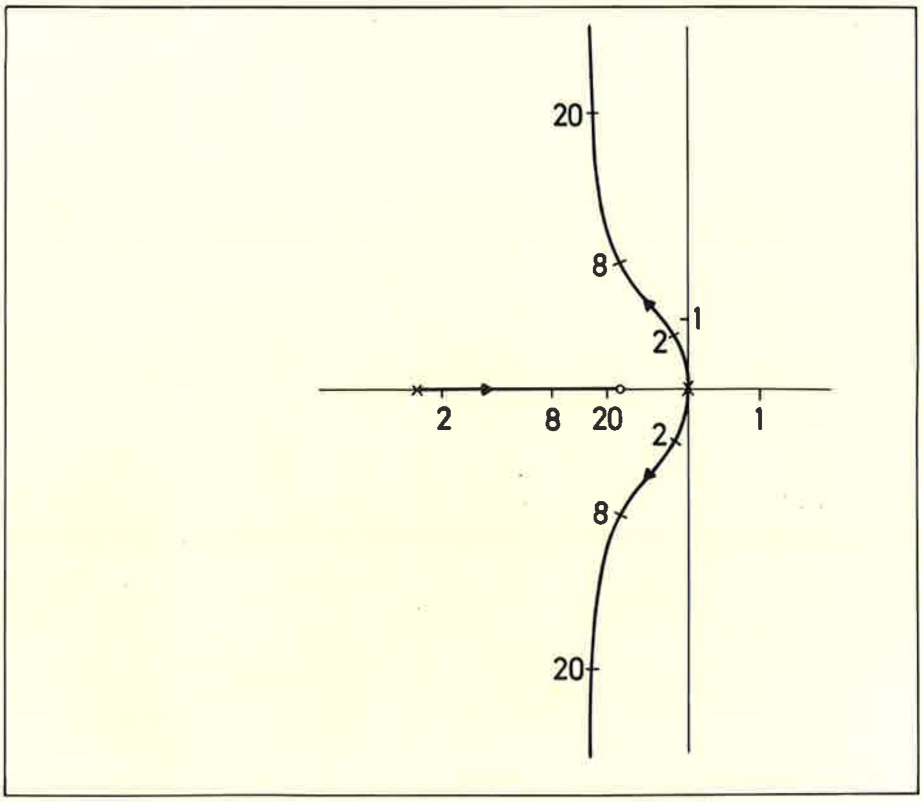

Fig. 2. Root locus of the system with D/J = 4.

2. NUMERICAL EXAMPLES

To fix the ideas the following parameters are assumed

$J = 1$
$D = 4$
$T = 1$

The closed loop system transfer function then becomes

$$G(s) = \frac{K(s+1)}{s^3 + 4s^2 + Ks + K}$$

The root locus in this particular case is shown in Fig. 2. It is seen from this figure that $K = 8$ is a reasonable value. The characteristic equation then becomes

$$s^3 + 4s^2 + 8s + 8 = (s+2)(s^2+2s+4) = 0$$

and the poles are thus

$$s_1 = -2, \quad s_{2,3} = -1 \pm i\sqrt{3}$$

The stepresponses are shown in Fig. 3. It is clear from this figure that the responses are poorly damped. For $K = 8$ the stepresponse is e.g.

$$g(t) = 1 + e^{-2t} - 2e^{-t} \cos\sqrt{3}t$$

which has an overshoot of 43%.

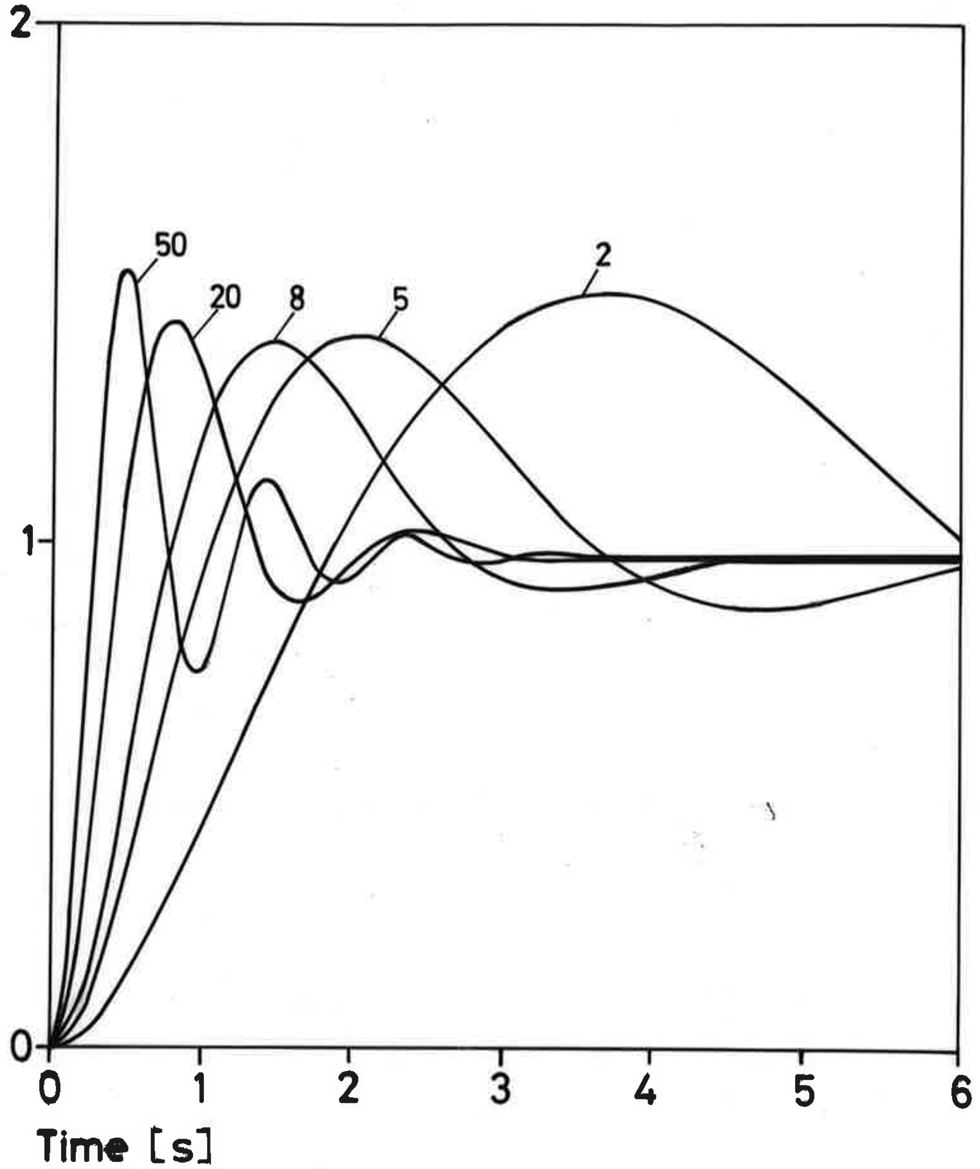

Fig. 3. Stepresponses for the system with D/J = 4 and different values of the gain.

3. IMPROVED STEPRESPONSES

To reduce the overshoot in the stepresponse, to command inputs the system can be modified as shown in Fig. 4. This way to compensate in order to obtain desired response to commands is discussed in depth in Bengtsson (1974).

The transfer function from command input θ_r to is θ

$$G(s) = \frac{K(1+sT)}{JTs^3 + DTs + KTs + K} \qquad \text{Case A}$$

$$G(s) = \frac{K}{JTs^3 + DTs + KTs + K} \qquad \text{Case B}$$

$$G(s) = \frac{K(1+aTs)}{JTs^3 + DTs + KTs + K} \qquad \text{Case C}$$

The corresponding stepresponses are shown in Fig. 5. We find that case B gives a reasonable response. Since B is also simpler than C we choose B.

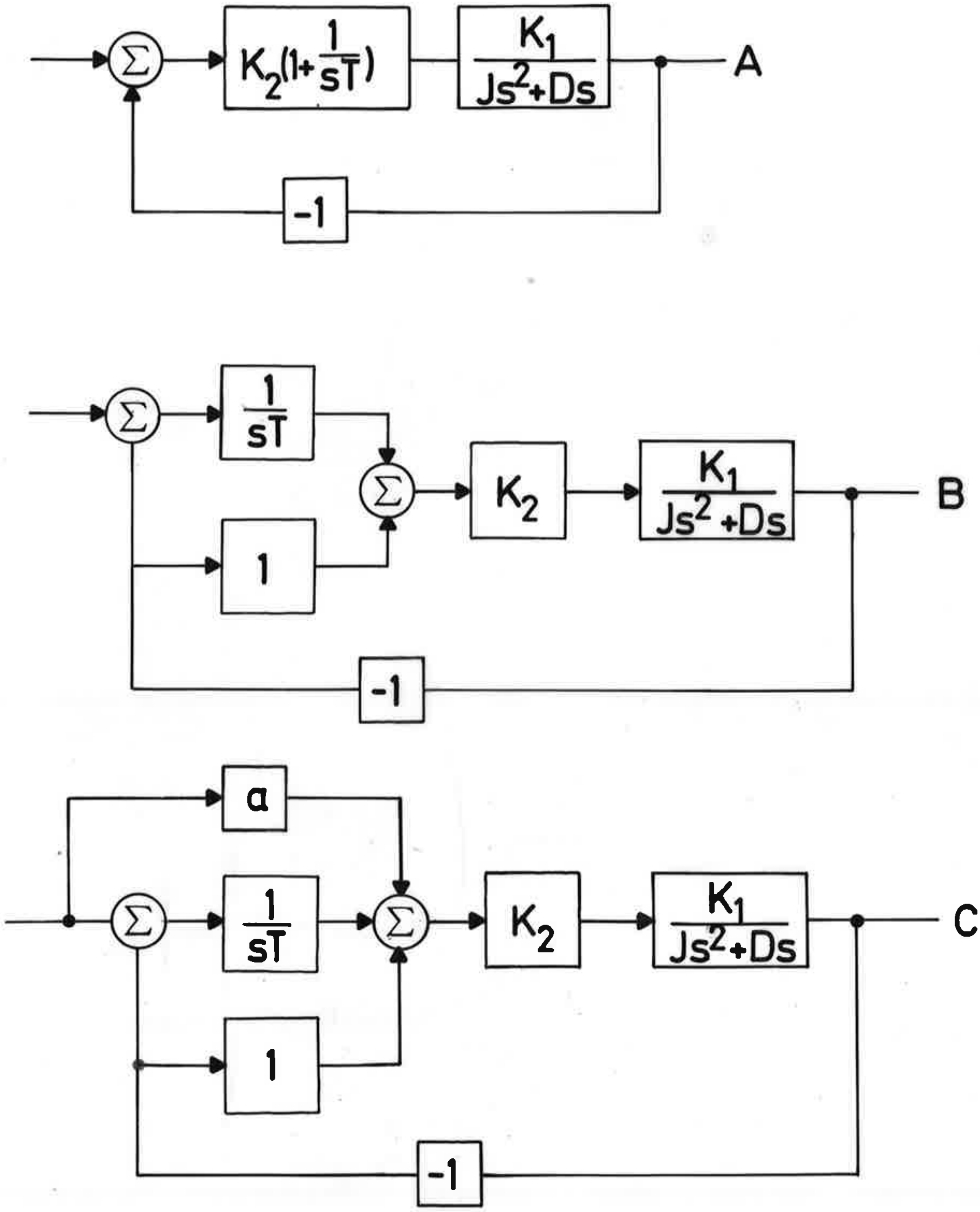

Fig. 4. Block diagrams for the original system A and different modifications B and C to achieve an improved response to command inputs.

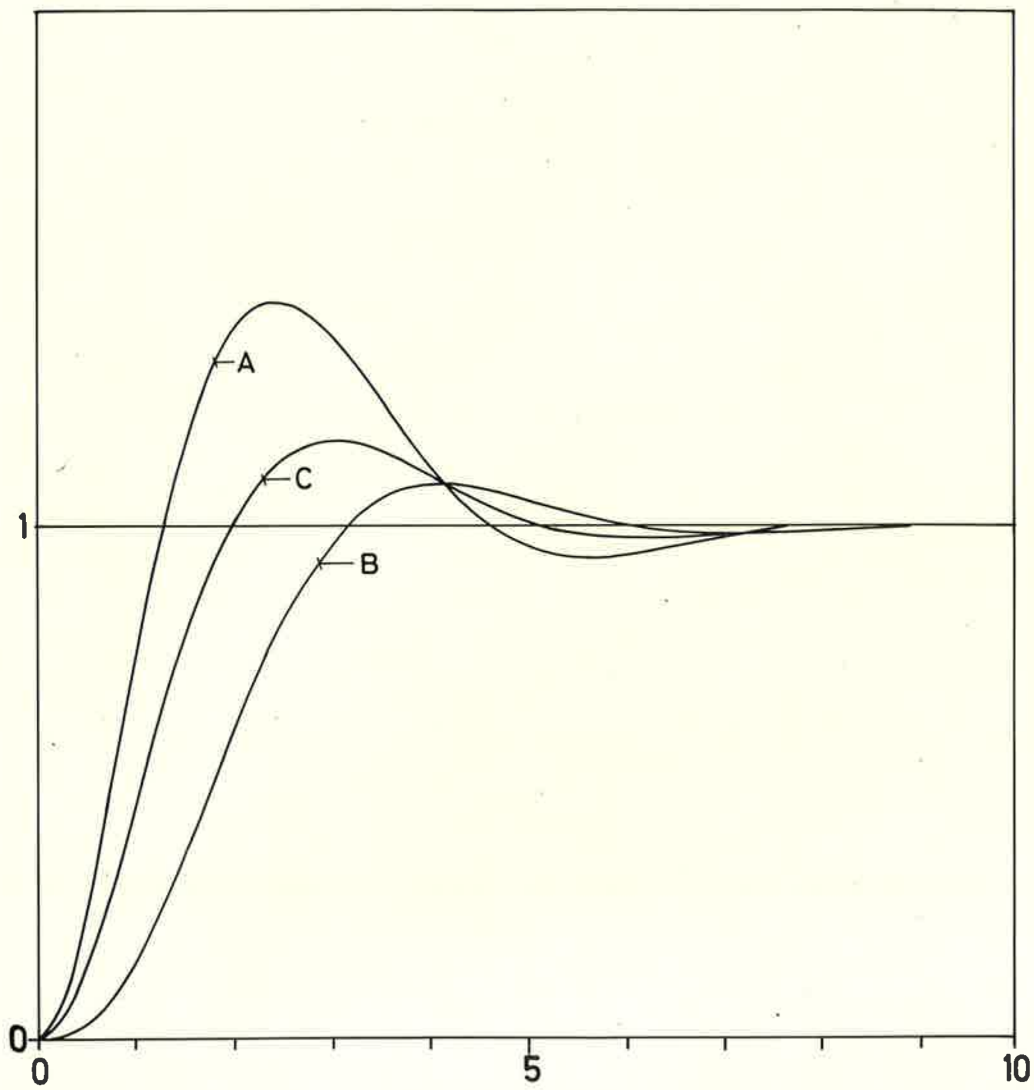

Fig. 5. Stepresponses of the systems A, B and C.

4. VARIATION IN THE MOMENT OF INERTIA

It has been found that for the case $J = 1$ and $D = 4$ a reasonable regulator is obtained if we select $K = 8$ and $T = 1$ and use the scheme B of Fig. 4. We will now analyse what happens to the system if the moment of inertia changes. The characteristic

$$Js^3 + 4s^2 + 8s + 8 = 0$$

The Routh - Hurwitz stability criterion gives

$$8J < 32 \Rightarrow J < 4$$

The system is thus stable if $J < 4$ and unstable if $J \geq 4$.

The root locus of this equation with respect to J is shown in Fig. 6.

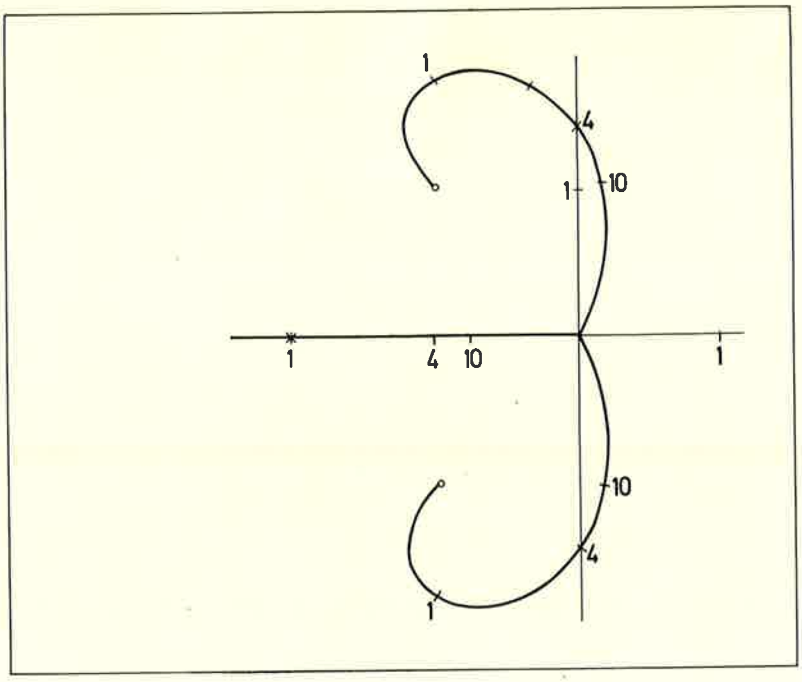

Fig. 6. Root locus of the system with respect to the moment of inertia J.

The qualitative features of the system are easily obtained from Fig. 6. If the regulator is designed to make the system behave well for J = 1, we find that the system will become less and less damped if J is increased. For J = 4 the system becomes unstable. See the stepresponses in Fig 7.

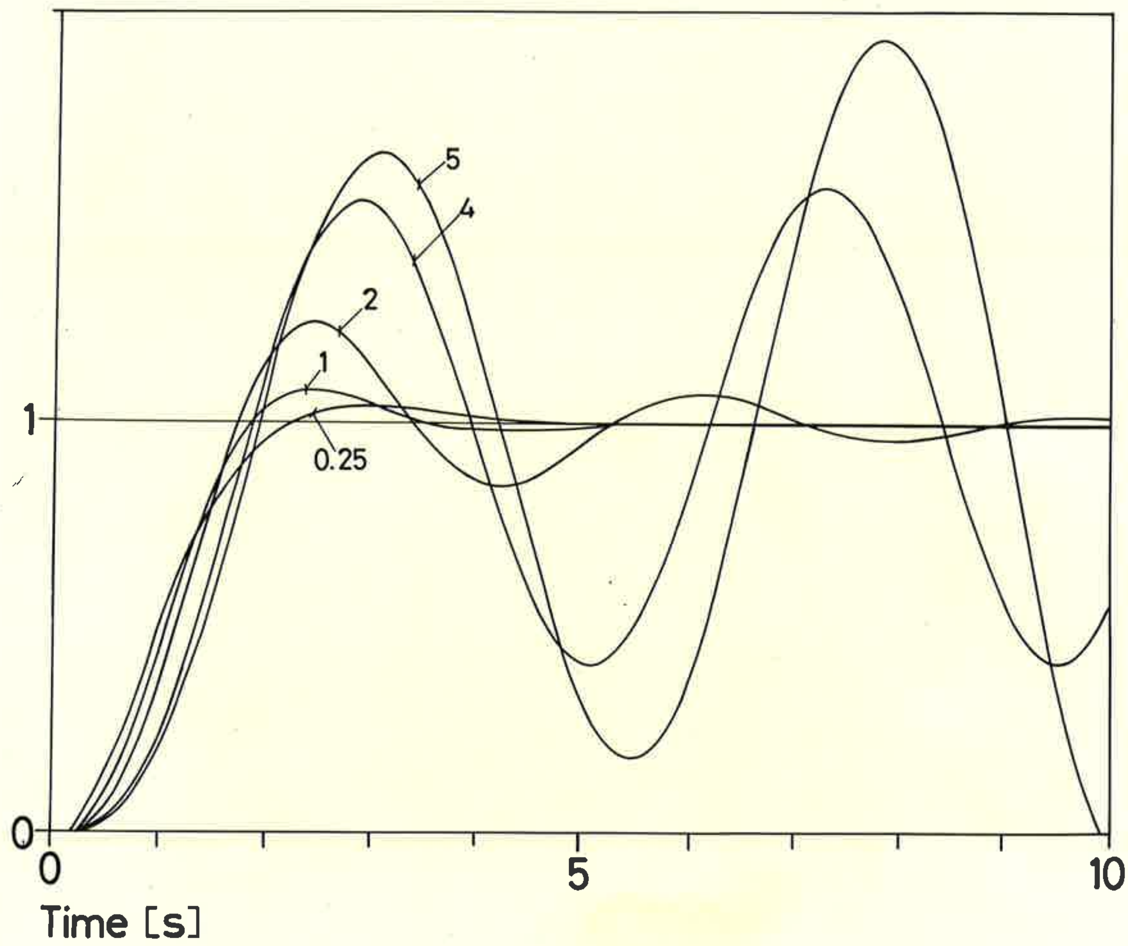

Fig. 7. Stepresponses of the system for different values of the moment of inertia.

REFERENCES

Bengtsson, G.: A Theory for Control of Linear Multivariable Systems.