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Åström, Karl Johan

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

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THREE LECTURES ON
MODELING, IDENTIFICATION AND ADAPTIVE CONTROL

KARL JOHAN ÅSTRÖM

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
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LUND INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AUTOMATIC CONTROL
Box 725
S 220 07 Lund 7 Sweden

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Karl Johan Åström

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Three Lectures on Modeling, Identification and Adaptive Control

Abstract

This report consists of the slides for three lectures given at a workshop on control theory at Mathematisches Forschungsinstitut, Oberwolfach in March 1980.

The first lecture gives an overview of the modeling problem including modeling from physics and from process experiments. Two particular problems are discussed in detail, namely the sensitivity of a control design to modeling errors and modeling of large systems. It is also emphasized that modeling is largely a craft and not a science. The lecture on identification starts with the problem formulation. Selection of experimental conditions, criteria and model classes are then discussed. Identification of estimation of parameters of models of dynamical systems is discussed in detail. The main elements of the relevant estimation theory is given and the role of interactive computing is discussed. In the lecture on adaptive control some design principles are first given. A simple adaptive controller based on minimum variance control and recursive least squares estimation is then discussed. The properties of the closed loop obtained in a simple case is analysed. Results in more general cases are then quoted.

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MODELING IDENTIFICATION AND ADAPTIVE CONTROL

LECTURE 1 MODELING

LECTURE 2 IDENTIFICATION

LECTURE 3 ADAPTIVE CONTROL

MODELING

1. INTRODUCTION
2. PHYSICS & PROCESS EXPERIMENTS
3. LARGE SYSTEMS
4. MODELING IS A CRAFT

THOUGHTS ON APPLIED WORK

- PURPOSE

  PRIMARILY TO GET SOMETHING TO WORK NOT TO WRITE A PAPER

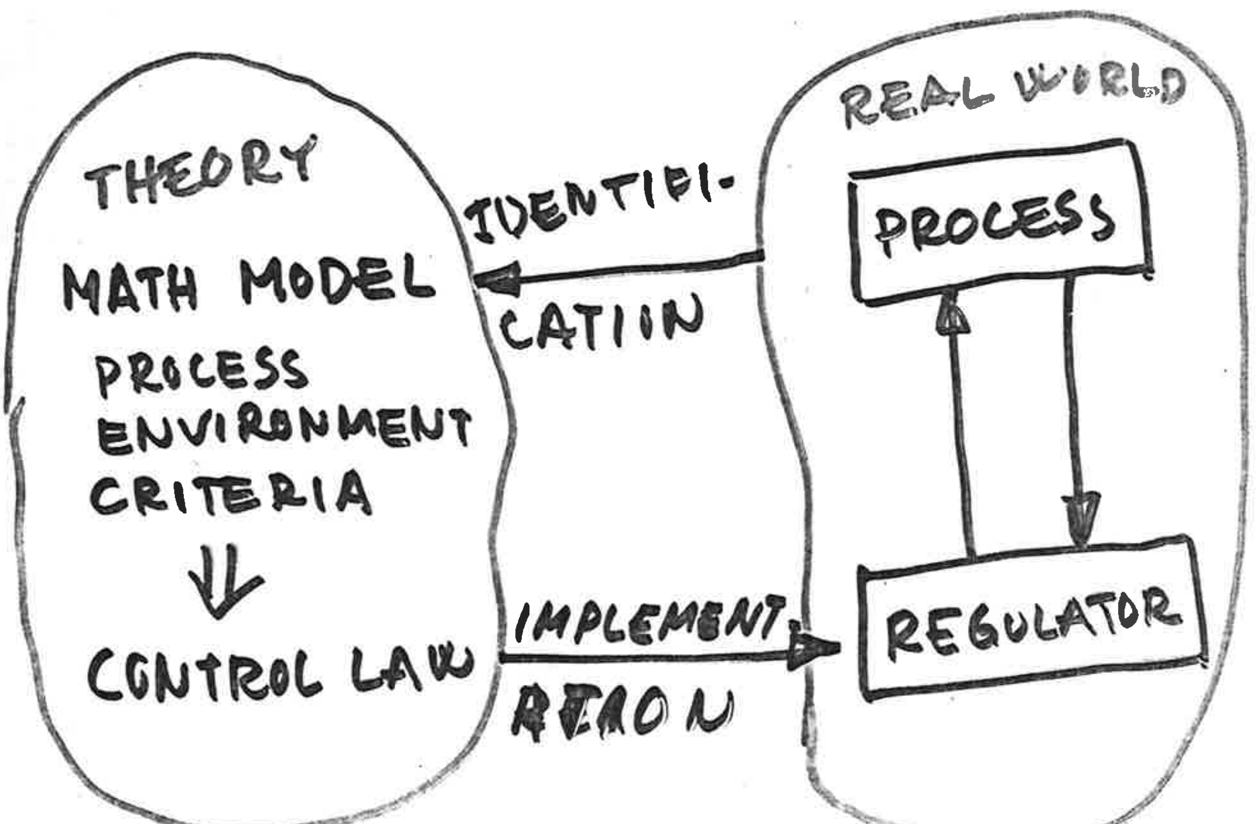
- SIMPLICITY

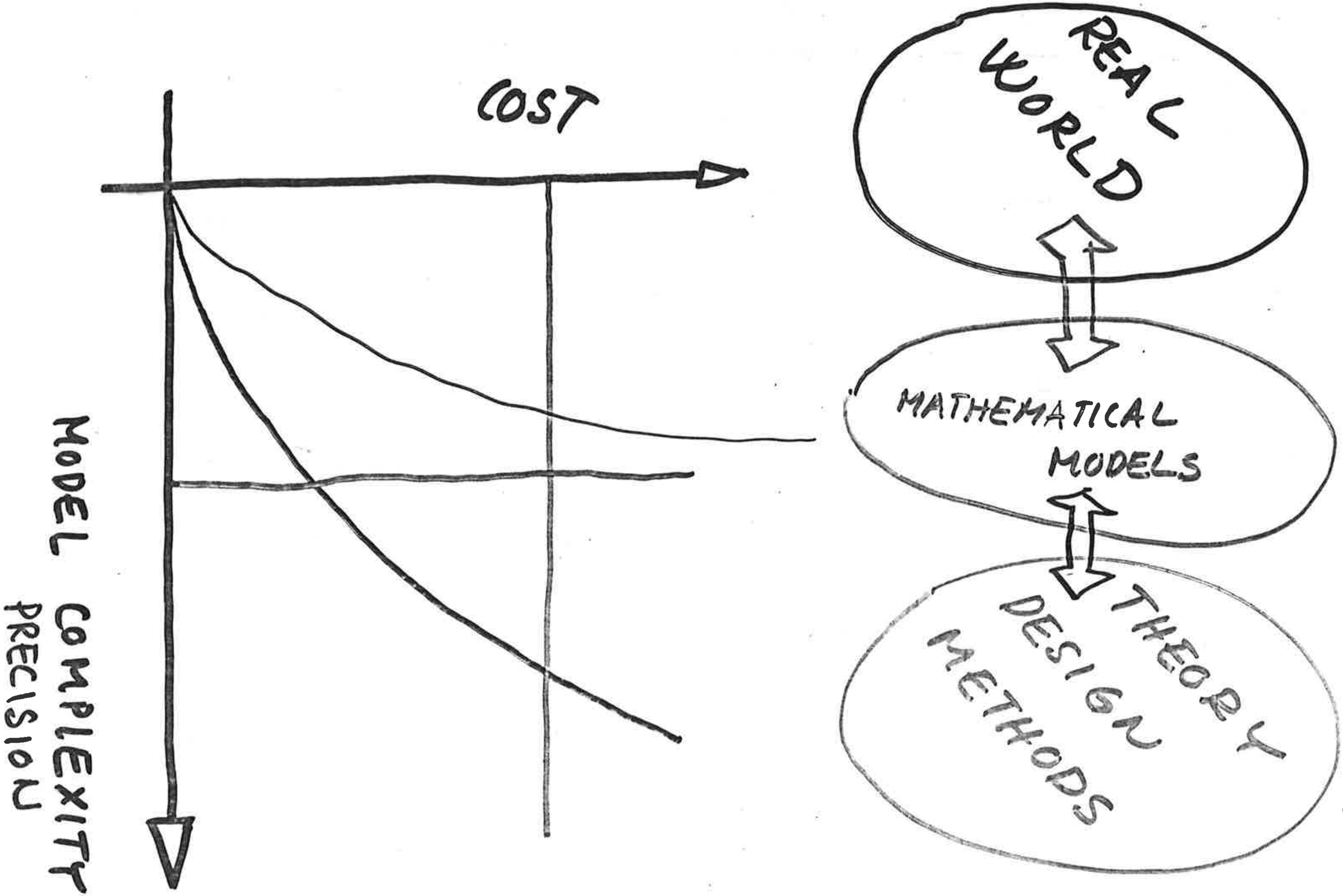
  YOU MUST BE ABLE TO EXPLAIN WHAT YOU ARE DOING

- SCIENTIFIC APPROACH

MOTIVATION & BACKGROUND

- WHY DID WE DO THIS




COST
MODEL COMPLEXITY
PRECISION
REAL WORLD
MATHEMATICAL MODELS
THEORY
DESIGN
METHODS

MODELING

WHY USE A MODEL?

- O COMPACT SUMMARY OF KNOW-LEDGE (NEWTON & KEPPLES VS TYCHO BRAHE)
  - COMMUNICATION
  - EDUCATION
- O EASIER TO WORK WITH MODELS THAN REAL LIFE
  - DESIGN
  - OPTIMIZATION
- O SOMETIMES A NECESSITY
  NO ALTERNATIVE AVAILABLE

CAUTION!

"WHEN MAP DISAGREES WITH NATURE TRUST NATURE"
SWEDISH ARMY MANUAL

EMPIRICS

MODELING IS OFTEN MORE TIMECONSUMING THAN INITIALLY ANTICIPATED

EXAMPLE OF SYSTEMIDENTIFICATIN

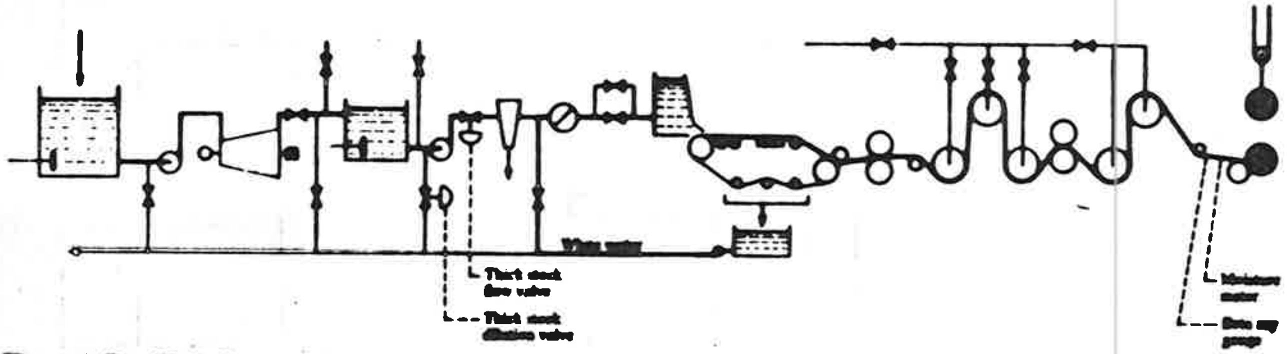
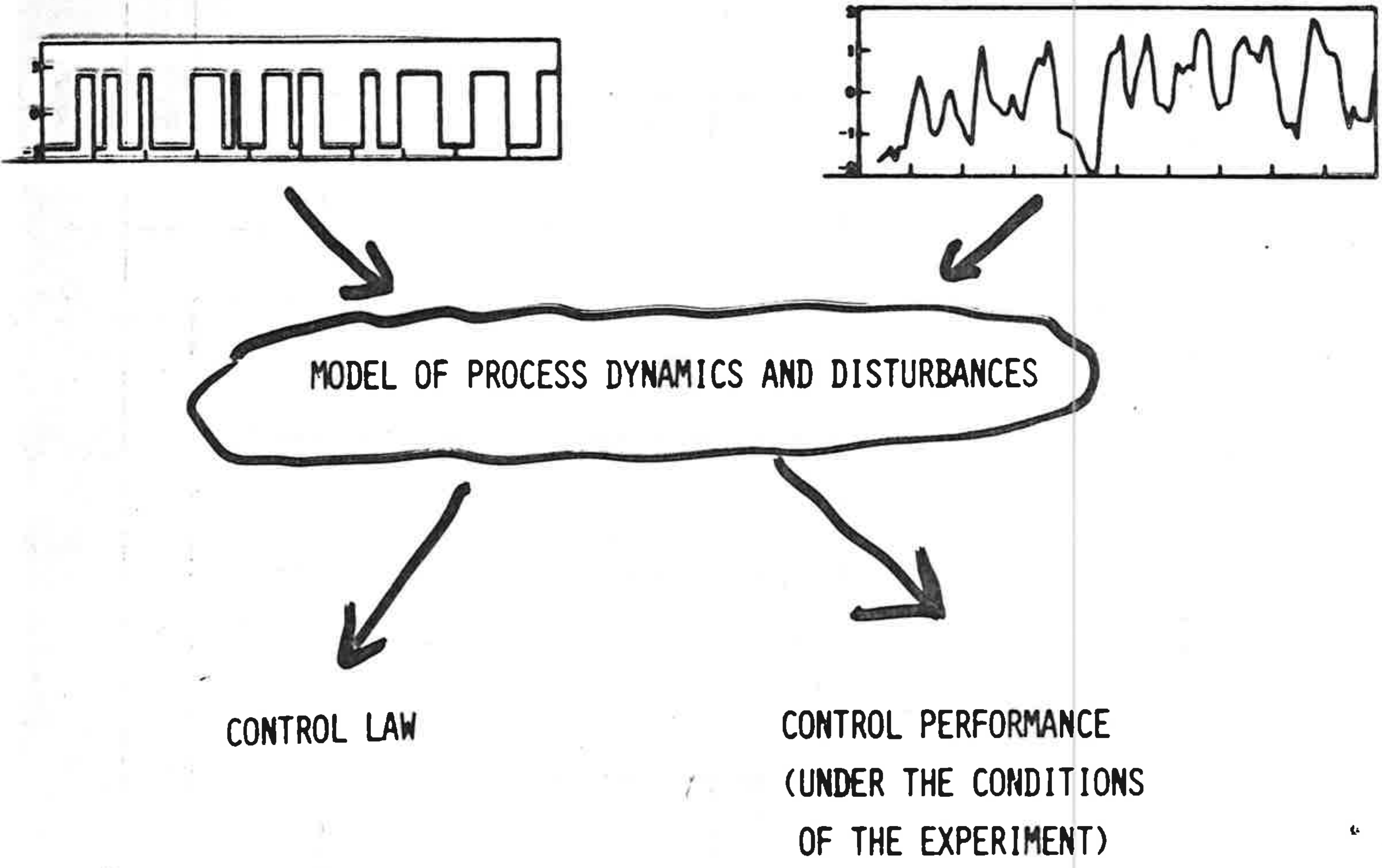
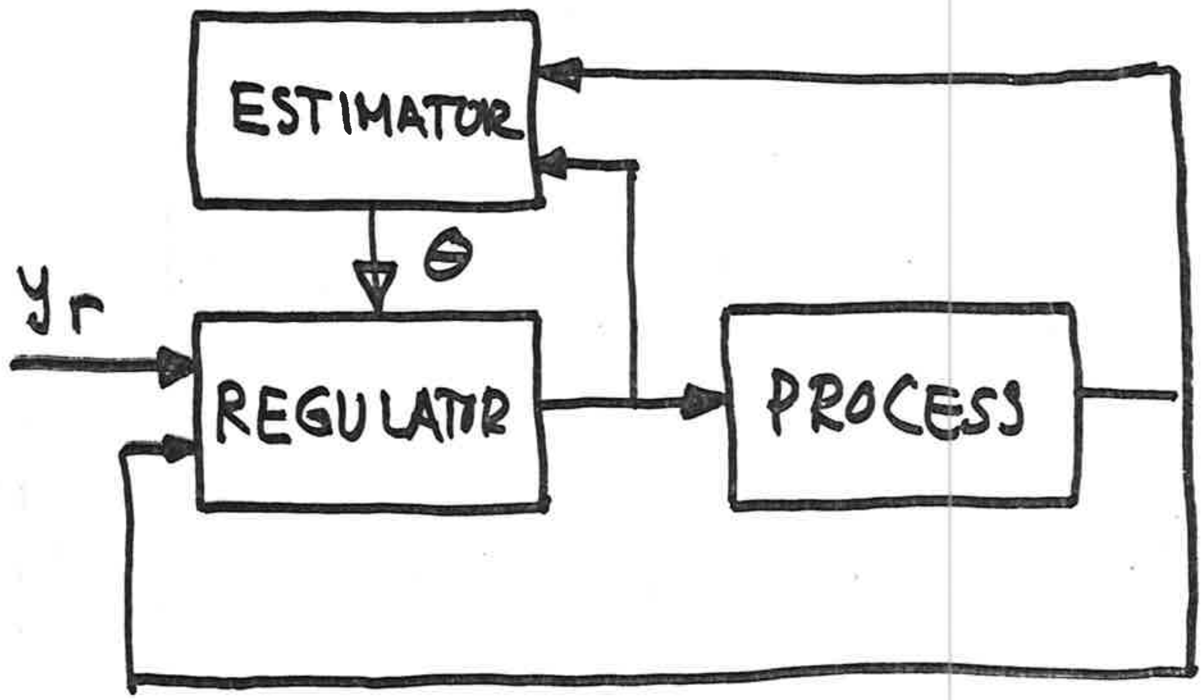


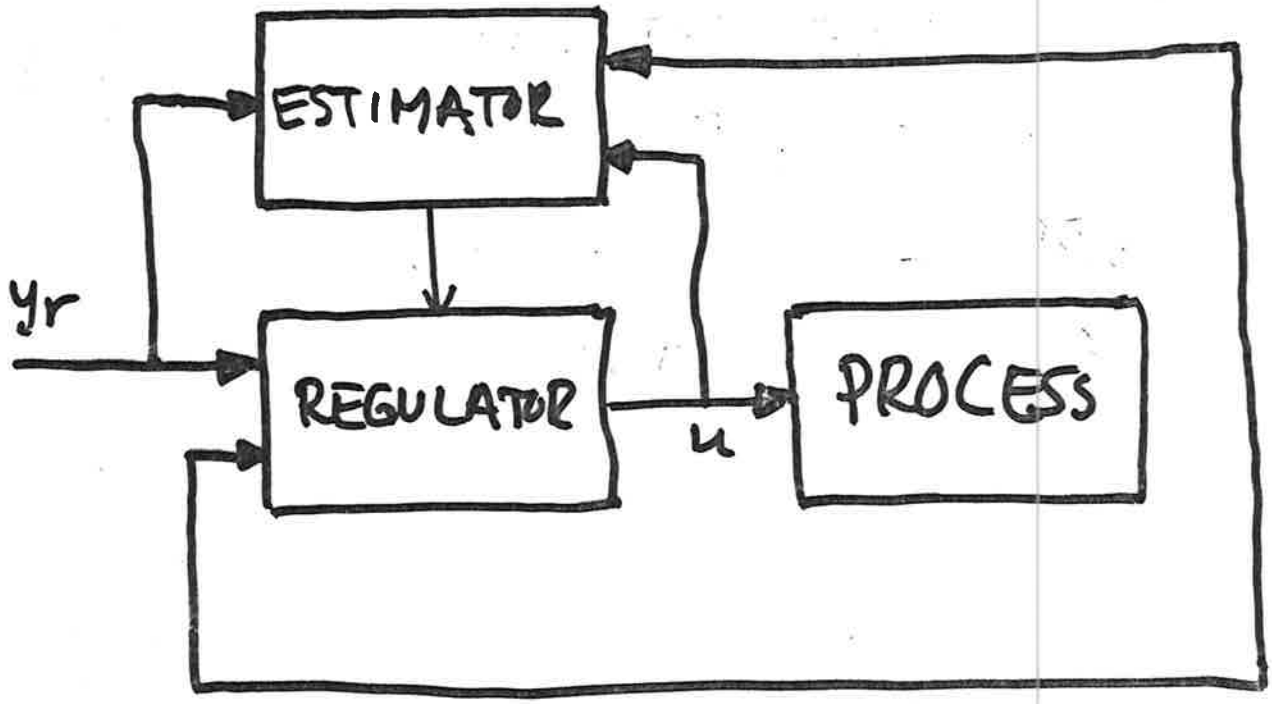
Figure 1 Simplified diagram of a kraft paper machine.



BASIC STRUCTURES FOR IMPLICIT METHODS



A) OBSERVER POLYNOMIAL GIVEN
(ADAPTS TO PROCESS DYNAMICS)



B) OBSERVER POLYNOMIAL ESTIMATED
(ADAPTS TO DISTURBANCES & COMMAND)

MODELING IN AUTOMATIC CONTROL

GOAL CLEAR?

EXAMPLES

- PROCESS DESIGN
- REGULATOR STRUCTURE
  SENSORS & ACTUATORS
- REGULATOR DESIGN
- TROUBLE SHOOTING
- PERFORMANCE EVALUATION
- ASSESSMENT OF POSSIBLE CONTROL PERFORMANCE

  CAN FLUCTUATIONS BE REDUCED?
  HOW MUCH?
  HOW COMPLEX A REGULATOR IS NEEDED?

  DOES THIS LOOP REQUIRE TUNING?

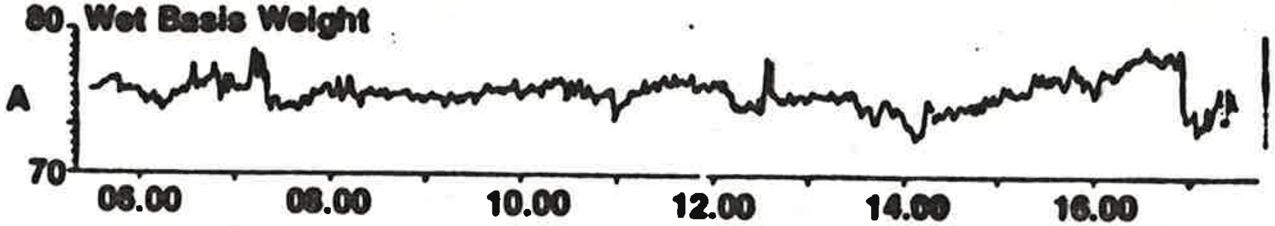
- KNOWLEDGE ABOUT PROCESS DYNAMICS MAY GIVE A POSSIBILITY TO RESOLVE A DIFFICULT DESIGN COMPROMISE (EX AIRPLANE FBW)

- MOVE CONTROL DESIGN CLOSER TO PROCESS DESIGN

- KNOWLEDGE ABOUT CONTROL & DYNAMICS MAY GIVE A POSSIBILITY TO AVOID PROBLEMS BY GOOD PROCESS DESIGN

ASSESSMENT OF BENEFITS OF CONTROL

DATALOGGING:

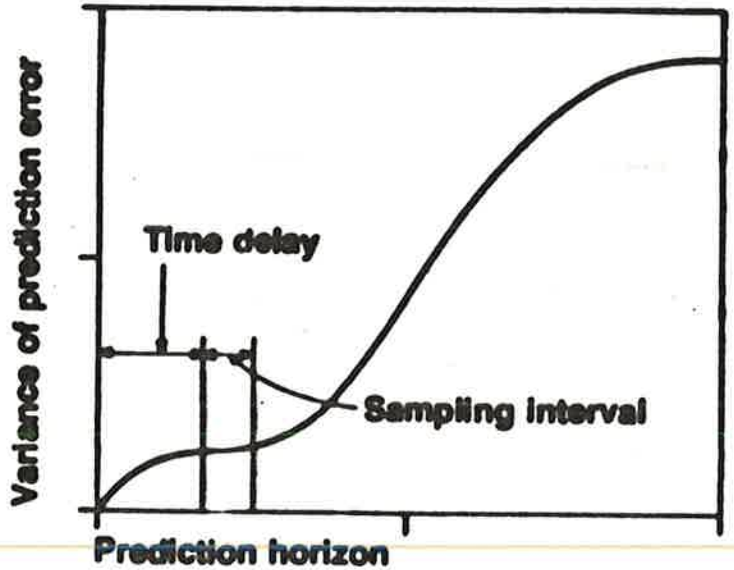


PROCESS IDENTIFICATION: PROCESS MODEL

PREDICTION

ERROR ANALYSIS

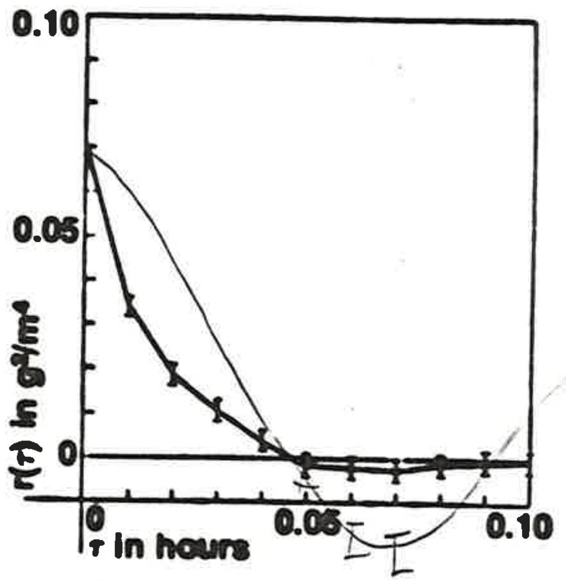
HEDGE



ASSESSMENT OF NEED FOR RETUNING OF MINIMUM VARIANCE CONTROL

LOG CONTROLLED OUTPUT DURING NORMAL OPERATION

CALCULATE COVARIANCE FUNCTION OF OUTPUT (COV Y)



USE KNOWLEDGE OF PROCESS DEAD TIME AND SAMPLING PERIOD TO TEST IF COVARIANCE FUNCTION SATISFIES MINIMUM VARIANCE CONDITIONS

THE ROLES OF MODELS IN CONTROL SYSTEM DESIGN

CLASSICAL (EXTERNAL MODELS)

PROCEDURE: FIX REGULATOR COMPLEXITY (PI, LEAD LAG, ETC). INVESTIGATE IF A VARIETY OF SPECIFICATIONS CAN BE SATISFIED. IF NOT, INCREASE REGULATOR COMPLEXITY.

DESIGN PARAMETERS: REGULATOR COMPLEXITY AND PARAMETERS.

MODEL: RESULTS ARE BETTER IF MODEL MORE ACCURATE. LITTLE PENALTY ON MODEL COMPLEXITY.

MODERN (INTERNAL DESCRIPTIONS)

PROCEDURE: CHOOSE MODEL AND CRITERIA. APPLY DESIGN PROCEDURE. CHECK SPECIFICATIONS WHICH ARE NOT DIRECTLY GIVEN BY CRITERIA. ALTER MODEL AND CRITERIA.

DESIGN PARAMETERS: CRITERIA AND MODEL.

MODEL: THE REGULATOR COMPLEXITY IS UNIQUELY GIVEN BY MODEL COMPLEXITY. HENCE LARGE PENALTY ON COMPLEX MODEL.

COMMENT

1. JET ENGINE MULTIVARIABLE DESIGN COMPETITION.
2. OFTEN QUOTED CRITICISM AGAINST LQG: "A KALMAN FILTER FOLLOWED BY A STATE FEEDBACK $u = -L\hat{x}$ CARRIES WITH IT, HOWEVER, THE PENALTY OF MAKING THE COMPENSATOR AT LEAST EQUAL IN ORDER TO THE PROCESS MODEL, WHICH WILL NOT BE ATTRACTIVE FOR MOST INDUSTRIAL APPLICATIONS."

WHY DO SIMPLE MODELS WORK SO WELL FOR CONTROL SYSTEM DESIGN ?

AN UNEXPLOITED BUT INTERESTING PROBLEM AREA

- REQUIRES SYSTEMATIC APPROACH TO DESIGN
- RELATED TO SINGULAR PERTURBATIONS
- STATE SPACE NOT NECESSARILY THE RIGHT FRAMEWORK

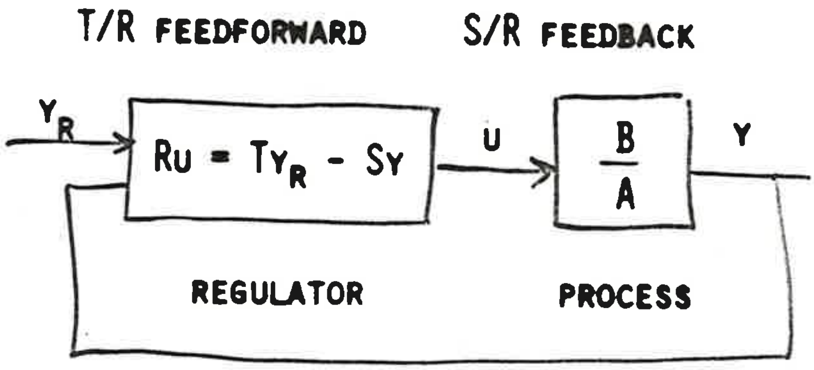
AN EXAMPLE

POLE-PLACEMENT DESIGN

PROCESS: $Y = \frac{B}{A} U$

DESIRED: $Y = \frac{Q}{P} Y_R$

REGULATOR STRUCTURE:



THEOREM:

CONSIDER A REGULATOR OBTAINED BY APPLYING POLE-PLACEMENT DESIGN TO THE STABLE MODEL $G = B/A$ WITH THE SPECIFICATION THAT THE CLOSED LOOP TRANSFER FUNCTION SHOULD BE $G_D = Q/P$. LET THE REGULATOR CONTROL A STABLE SYSTEM WITH THE PULSE TRANSFER FUNCTION $G_0 = B_0/A_0$. THE CLOSED LOOP SYSTEM IS THEN STABLE IF

$$|G - G_0| < \left|\frac{BPT}{AQS}\right| = \left|\frac{G}{G_D}\right| \cdot \left|\frac{G_{FF}}{G_{FB}}\right| \qquad G_{FF} = \frac{T}{R}, \; G_{FB} = \frac{S}{R}.$$

ON THE UNIT CIRCLE AND AT $Z = \infty$.

MODELING BASED ON PHYSICAL PRINCIPLES

- SPECIFY PURPOSE OF MODEL

  DEFINE SYSTEM BOUNDARIES

  INPUTS, OUTPUTS, DISTURBANCES

  ⇒ QUALITATIVE MODEL

- WRITE BALANCE EQUATIONS

  MASS
  MOMENTUM
  ENERGY

  VARIABLES REQUIRED TO ~~BE~~ DESCRIBE STORAGE OF THESE ARE CALLED STATE VARIABLES.

- WRITE CONSTITUTIVE EQUATIONS

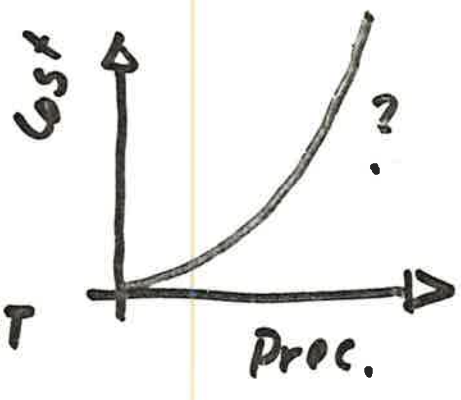
  HOOKES
  ARRHENIUS
  THERMODYNAMICAL STATE EQ

  ⇒ QUANTITATIVE MODEL

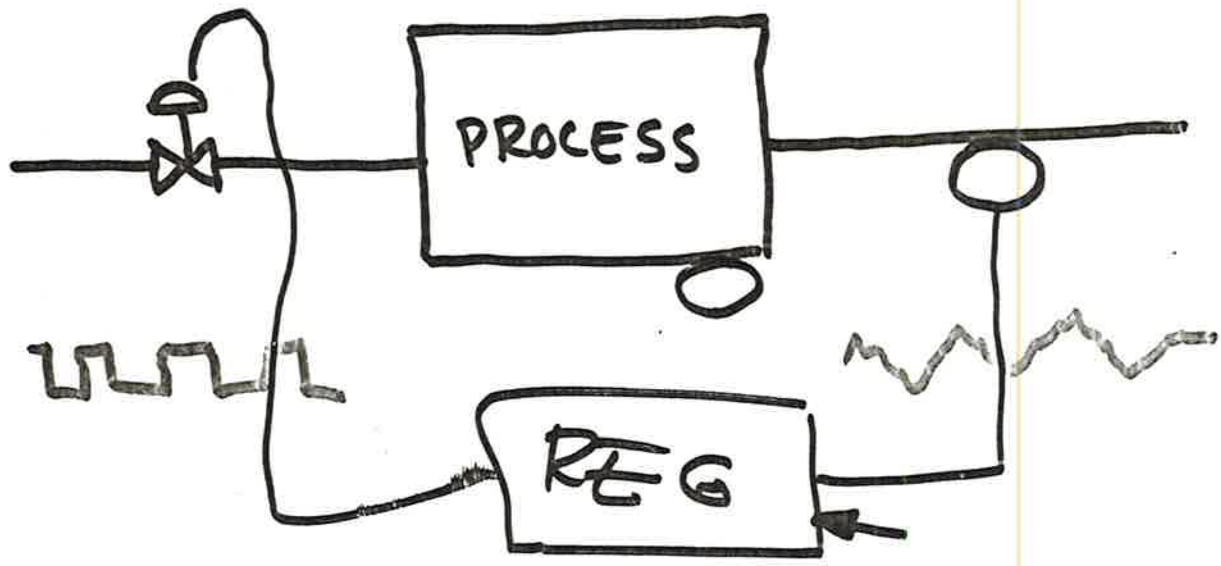
- VALIDATE !

  DIFFICULTY

  APPROX, TIME, COST



MODELING BY PROCESS EXPERIMENTS



- EXPERIMENTAL PLANNING
- CHOOSE MODEL STRUCTURE
- PARAMETER ESTIMATION
- VALIDATION

EXAMPLES

FREQUENCY RESPONSE
TRANSIENT RESPONSE
CORRELATION METHODS

CLASSICAL NON PARAMETRIC

LEAST SQUARES
MAXIMUM LIKELIHOOD

PARAMETRIC ESTIMATION

MODELING OF LARGE SYSTEMS

DESIRABLE FEATURES

- MODEL SHOULD BE EASY TO WRITE, CHECK, AND MODIFY.
- MODEL MANIPULATIONS SHOULD BE AUTOMATED.
- PROPERTIES OF MODEL SHOULD BE EASY TO FIND
  (SIMULATION, ANALYSIS, LINEARIZATION, ...)

PROCEDURE

- CUT SYSTEM INTO SUBSYSTEMS.
- WRITE BALANCE EQUATIONS (MASS, MOMENTUM, ENERGY) AND CONSTITUTIVE EQUATIONS.
- DESCRIBE INTERCONNECTIONS HIERARCHICALLY.
- LET THE COMPUTER DO THE REST (COMPUTE STEADY STATE, GENERATE CODE FOR SIMULATION, LINEARIZATION ETC).

EXAMPLE DYMOLA

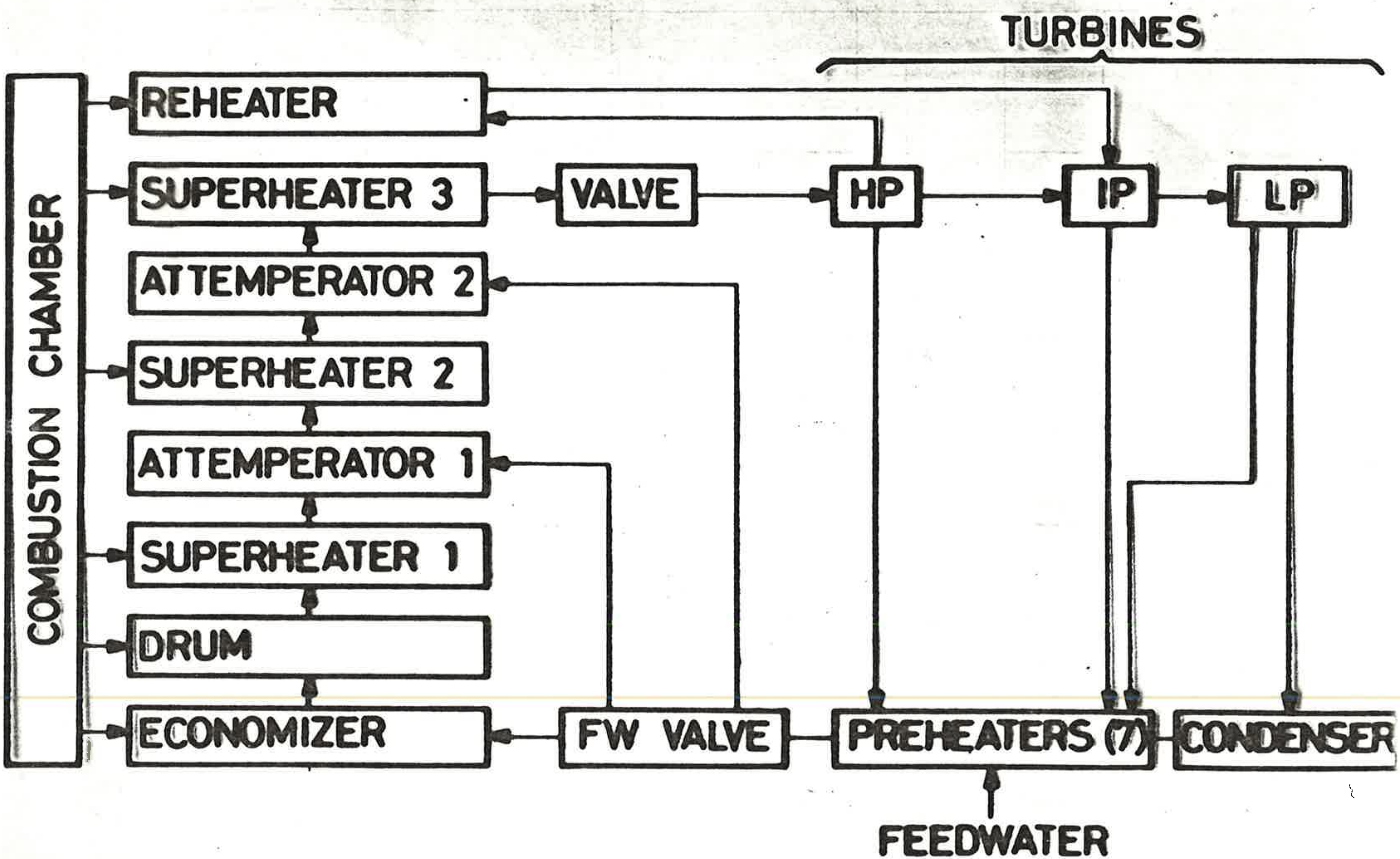
H. ELMQVIST: A STRUCTURED MODEL LANGUAGE FOR LARGE CONTINUOUS SYSTEMS.

PhD DISSERTATION, LUND, MAY 1978.

LANGUAGE TRANSLATOR FOR OPERATING ON THE MODEL.
SOLVE FOR STEADY STATE OR DX/DT, FORMULA MANIPULATION ETC.

EXAMPLE: MODEL OF A DRUMBOILER TURBINE

ORIGINAL DOCUMENTATION IS A 60 PAGE REPORT + STEAM TABLES.
DYMOLA DESCRIPTION REQUIRES 9 PAGES OF CODE + STEAM TABLES.



TURBINES
COMBUSTION CHAMBER
REHEATER
SUPERHEATER 3
VALVE
HP
IP
LP
ATTEMPERATOR 2
SUPERHEATER 2
ATTEMPERATOR 1
SUPERHEATER 1
DRUM
ECONOMIZER
FW VALVE
PREHEATERS (7)
CONDENSER
FEEDWATER

```
MODEL POWERSTATION

SUBMODEL DRUMSYST
SUBMODEL (SUPERHEATER) SUPERH1, SUPERH2, SUPERH3
SUBMODEL CONTROLVALVE
SUBMODEL LPTURB
.
.
.
CONNECT (HEAT) COMBCHAMBER TO (ECONOMIZER, DRUMSYST::RISERS,
    SUPERH1, SUPERH2, SUPERH3, REHEATER)
CONNECT (STEAM) DRUMSYST::DRUM TO SUPERH1 TO ATTEMP1 →
    TO SUPERH2 TO ATTEMP2 TO SUPERH3 TO →
    CONTROLVALVE TO HPTURB TO REHEATER TO IPTURB →
    TO  LPTURB TO CONDENSOR
.
.
.
END
```

```
model powerstation

submodel drumsyst
submodel (superheater) superh1, superh2, superh3
submodel (attemperator) attemp1, attemp2
submodel reheater
submodel controlvalve
submodel (turbsection) HPturb
submodel IPturb
submodel LPturb
submodel condensor
submodel (preheater) preh1, preh2, preh3, preh4, preh5,
         preh6, preh7
submodel splitsteam
submodel dearator
submodel feedwaterpump
submodel feedwatervalve
submodel combchamber
submodel economizer

connect (heat) combchamber to (economizer,
  drumsyst::risers, superh1, superh2, superh3, reheater)

connect (steam) drumsyst::drum to superh1 to attemp1  ->
  to superh2 to attemp2 to superh3 to  ->
  controlvalve to HPturb to reheater to IPturb  ->
  to LPturb to condensor

connect (extractsteam) HPturb to preh7,
  IPturb to (preh6, preh5, preh4,
    splitsteam to (dearator, preh3) ),
  LPturb to (preh2, preh1)

connect (feedwater) condensor to preh1 to preh2 to  ->
  preh3 to dearator to feedwaterpump to preh4  ->
  to preh5 to preh6 to preh7 to  ->
  feedwatervalve to  ->
  (economizer to drumsyst::drum, attemp1, attemp2)

connect (condensate) preh7 to preh6 to preh5  ->
   to preh4 to dearator,
  preh3 to preh2 to preh1 to condensor

connect (power) HPturb to IPturb to LPturb

HPturb.N1 = 0
LPturb::LP3.Wp = 0

end
```

```
MODEL TYPE SUPERHEATER

CUT INSTEAM (W, H1, P1)
CUT OUTSTEAM (W, H2, P2)
PATH STEAM   INSTEAM - OUTSTEAM
CUT HEAT (Q)
PARAMETER Cm, m, K, Vs, F
LOCAL Tm, TmH, T2, T2H, R2

P1**2 - P2**2 = F*W**2

  { ENERGY BALANCE }
  { DER(m*Cm*Tm + Vs*R2*H2) = }
(m*Cm*TmH + Vs*R2)*DER(H2) = Q - W*(H2 - H1)

Tm = T2 + K*W*(H2 - H1)
TmH = T2H + K*W

R2 = RHP(H2, P2)
T2 = THP(H2, P2)
T2H = THPH(H2, P2)

END
```

NO MODEL IS EVER A PERFECT FIT TO REALITY

- DON'T BELIEVE THE 33-RD ORDER CONSEQUENCES OF A FIRST ORDER MODEL
- DON'T EXTRAPOLATE BEYOND THE REGION OF FIT
(DON'T GO OFF THE DEEP END)
- DON'T APPLY A MODEL UNTIL YOU UNDERSTAND THE SIMPLIFYING ASSUMPTIONS ON WHICH IT IS BASED AND CAN TEST THEIR APPLICABILITY
(USE ONLY AS DIRECTED)

DISTINGUISH AT ALL TIMES BETWEEN THE MODEL AND THE REAL WORLD

- DON'T BELIEVE THAT THE MODEL IS REALITY
  (DON'T EAT THE MENU)
- DON'T DISTORT REALITY TO FIT THE MODEL
  (THE PROCRUSTES METHOD)
- MORE THAN ONE MODEL MAY BE USEFUL FOR UNDERSTANDING DIFFERENT ASPECTS OF THE SAME PHENOMENON

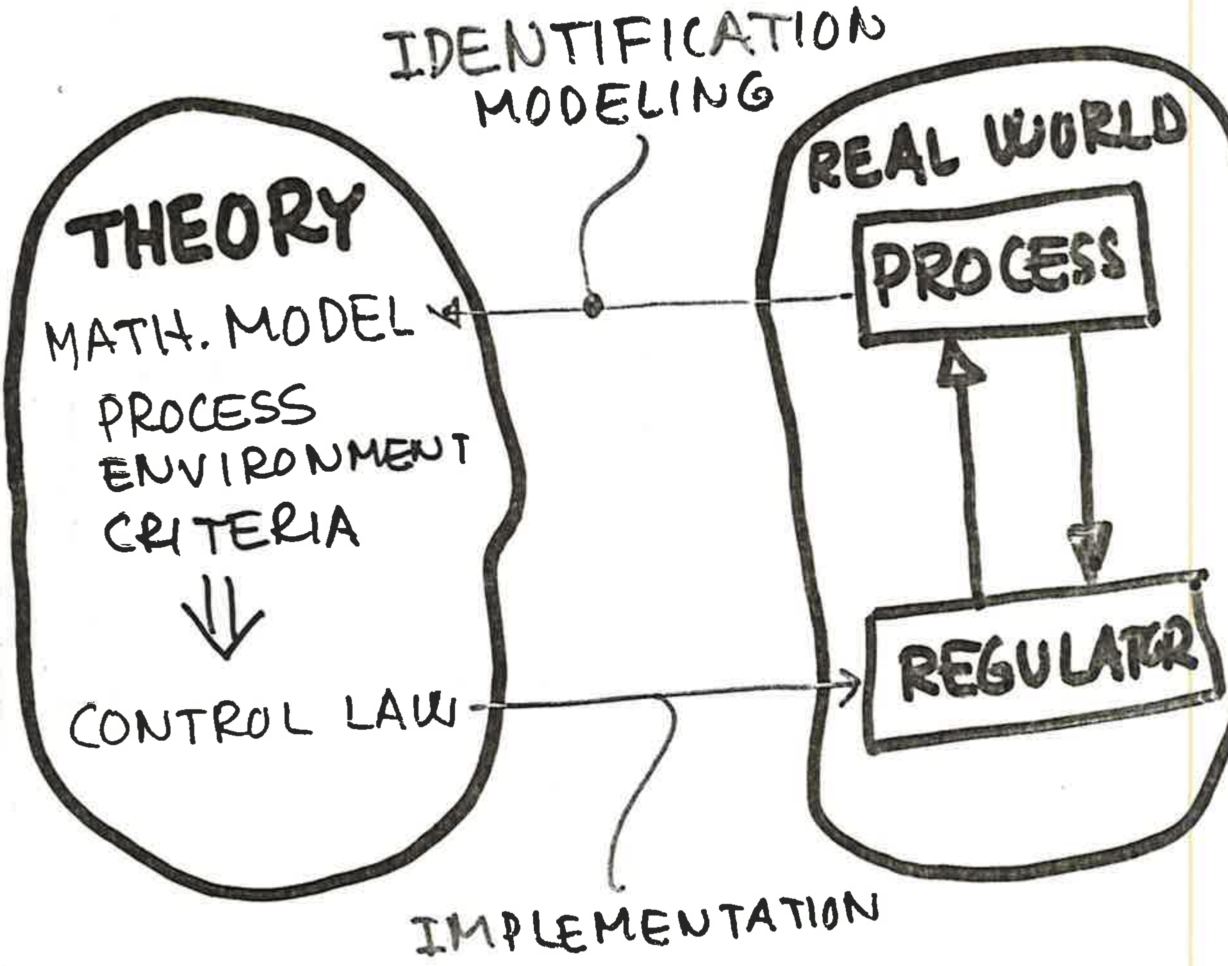
A USEFUL MODEL MUST SERVE PRACTICAL ENDS NOT PEDANTRY

- DON'T APPLY THE TERMINOLOGY OF "SUBJECT A" TO THE PROBLEMS OF "SUBJECT B"
  (NEW NAMES FOR OLD)
- DON'T EXPECT THAT BY HAVING NAMED A DEMON YOU HAVE DESTROYED HIM
- THE PURPOSE OF NOTATION AND TERMINOLOGY SHOULD BE TO ENHANCE INSIGHT AND FACILITATE COMPUTATION - NOT TO IMPRESS OR CONFUSE THE UNINITIATE
  (GOBBLEDYGOOK)

A MODEL MUST BE PERMITTED TO EVOLVE AS CONDITIONS CHANGE OR AS ADDITIONAL DATA BECOME AVAILABLE

- DON'T RETAIN A DISCREDITED MODEL
- DON'T FALL IN LOVE WITH YOUR MODEL
- DON'T REJECT DATA IN CONFLICT WITH THE MODEL. USE SUCH DATA TO REFUTE, MODIFY OR IMPROVE THE MODEL.

(PEARL HARBOUR)



CLASSICAL:

TRANSFER FUNCTION
IMPULSE & FREQUENCY RESPONSE

"MODERN":

PARAMETRIC STATE SPACE MODELS
⋮
LEAST SQUARES
MAXIMUM LIKELIHOOD
⋮

TRADE EXPERIMENTAL SIMPLICITY FOR COMPUTATIONS!

IDENTIFICATION

1. INTRODUCTION
2. CRITERIA
3. ESTIMATING PARAMETERS IN DYNAMICAL SYSTEMS
4. MODEL STRUCTURES
5. ESTIMATION THEORY
6. INTERACTIVE COMPUTING
7. CONCLUSIONS

MOTIVATION

PROCESS MODELLING

DESIGN OF CONTROL LAWS

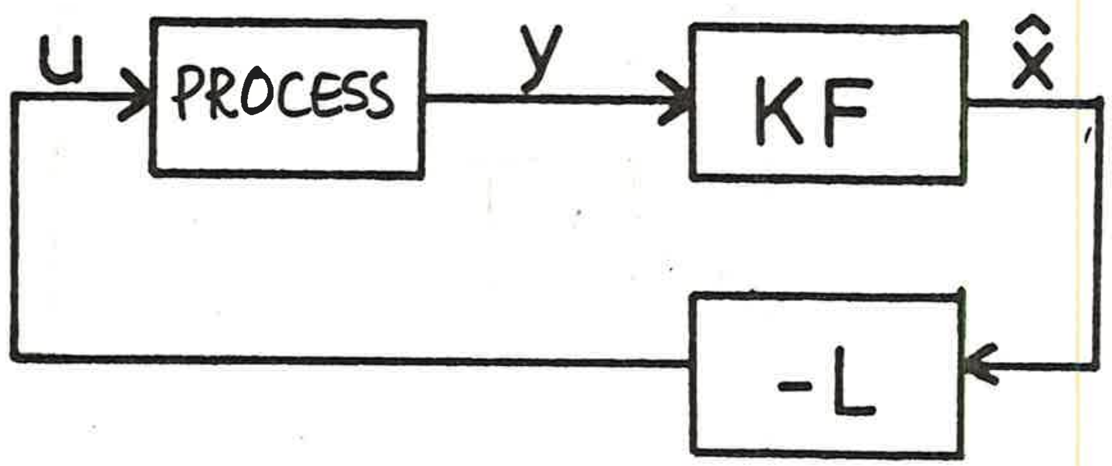
Ex: Given the system

$$x(t+1) = Ax(t) + Bu(t) + V(t)$$

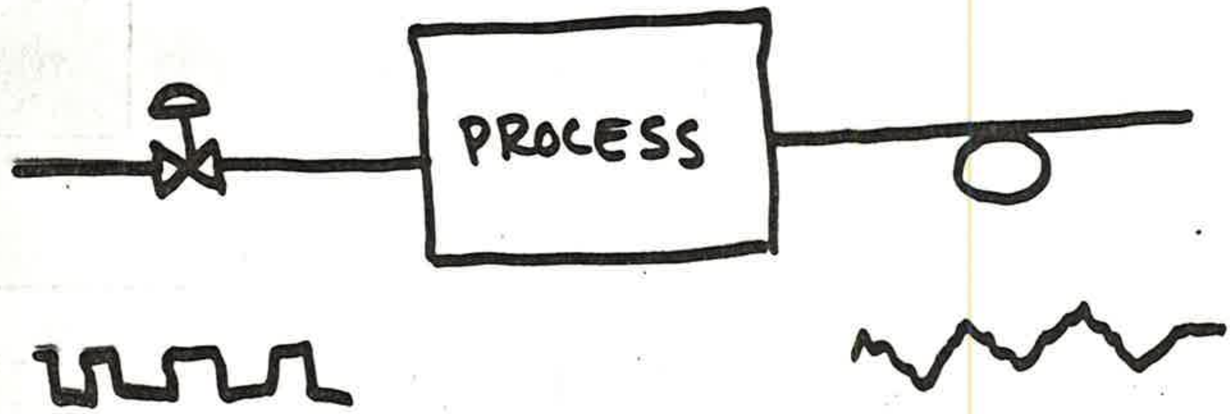
$$y(t) = Cx(t) + e(t)$$

Find control which minimizes

$$E \sum_{t=1}^{N} x^T(t) Q_1 x(t) + u^T(t) Q_2 u(t) \}$$



MODELING BY PROCESS EXPERIMENTS



- EXPERIMENTAL PLANNING
- CHOOSE MODEL STRUCTURE
- PARAMETER ESTIMATION
- VALIDATION

PARAMETER ESTIMATION

GIVEN

- INPUT-OUTPUT DATA $\mathcal{D}$ $\{u(t), y(t),\ 0 \le t \le T\}$ FROM AN EXPERIMENT

- A CLASS OF MODELS $\mathcal{M}(\theta)$

- A CRITERION $\mathcal{C}$

FIND A MODEL IN THE CLASS WHICH FITS THE DATA BEST ACCORDING TO $\mathcal{C}$.

PROBLEMS

- HOW TO CHOOSE THE EXPERIMENT, $\mathcal{M}$ AND $\mathcal{C}$

- HOW TO FIND THE BEST FIT (OPTIMIZATION)

The probability of the errors

$$\Omega = h^{\mu}\pi^{-\frac{1}{2}\mu}\, e^{-hh(vv+v'v'+v''v''+\ldots)}$$

must become a minimum.

"Therefore, that will be the most probable system of values of the unknown quantities p, q, r, s, etc., in which the sum of the squares of the differences between the observed and computed values of the functions V, V', V'', etc. is a minimum, ..."

PRINCIPLE OF LEAST SQUARES

"In conclusion, the principle that the sum of the squares of the differences between the observed and computed quantities must be a minimum may, in the following manner, be considered independently of the calculus of probabilities."

"Denoting the differences between observation and calculation by Δ, Δ', Δ'', etc., the first condition will be satisfied not only if $\Delta\Delta + \Delta'\Delta' + \Delta''\Delta'' +$ etc., is a minimum (which is our principle), but also if $\Delta^4 + \Delta'^4 + \Delta''^4 +$ etc., or $\Delta^6 + \Delta'^6 + \Delta''^6 +$ etc., or in general, if the sum of any of the powers with an even exponent becomes a minimum. But of all these principles ours is the most simple; by the others we should be led into the most complicated calculations."

THE LIKELIHOOD FUNCTION

INTRODUCE

$$y_{t_k}^T = \left[y^T(t_0)\; y^T(t_1) \ldots\; y^T(t_k) \right]$$

USING BAYES RULE THE LIKELIHOOD FUNCTION BECOMES

$$L = p(y_{t_k}) = p(y(t_k) | y_{t_{k-1}})\; p(y_{t_{k-1}})$$

$$p(y(t_k) | y_{t_{k-1}})\; p(y(t_{k-1}) | y_{t_{k-2}}) \ldots p(y(t_1) | y(t_0))\; p(y(t_0))$$

FOR LINEAR GAUSSIAN PROBLEMS

$$p(y(t_k) | y_{t_{k-1}}) = N(\hat{y}(t_k | t_{k-1}),\; R(t_k))$$

$$= (1/2)(2\pi)^{-m/2} (\det R(t_k))^{-1/2} \exp - (1/2)\, \varepsilon^T(t_k) R^{-1}(t_k) \varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k)$$

THE LIKELIHOOD FUNCTION CAN THEN BE WRITTEN AS

$$-\log L = (1/2) \left[\Sigma \log \det R(t_k) + \Sigma \varepsilon^T(t_k) R^{-1}(t_k) \varepsilon(t_k) \right] + \text{const.}$$

NOTICE RELATIONS TO FILTERING THEORY !

INTERPRETATION FOR NON GAUSSIAN PROCESSES

PREDICTION ERROR INTERPRETATION

Notice that the ML-criterion gives a loss function N of the form

$$V(\theta) = \sum_{t=1}^{N} q(\varepsilon(t_k))$$

where

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k | t_{k-1})$$

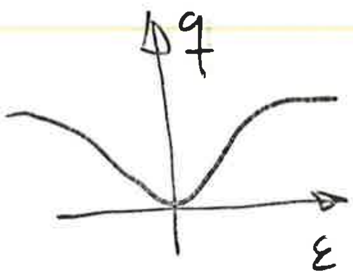
is the prediction error.

Alternative: Postulate prediction model and error criterion

Compare Gauss

Consequences for modeling

Dynamical systems



NOTICE q QUADRATIC FOR GAUSSIAN DISTURBANCES

ROBUSTNESS

THE MAXIMUM LIKELIHOOD PRINCIPLE Fisher 1912

RULE

Let Y be a random variable with probability density $p(y, \theta)$. To estimate θ from an observation y choose $\hat{\theta}$ such that

$$L(y, \hat{\theta}) \geq L(y, \theta) \qquad \forall\ \theta$$

where L is the likelihood function defined by $L(y, \theta) = p(y, \theta)$.

INDEPENDENT SAMPLES

$$L(y_1, y_2, \ldots y_n, \theta) = p(y_1, \theta)\, p(y_2, \theta) \ldots p(y_n, \theta)$$

PROPERTIES

Consistency
Asymptotic normality
Efficiency

OTHER PREDICTION ERROR CRITERIA

ML:

fen cte

$$V(\theta) = -\log L = \frac{1}{2}\sum_{k=1}^{N} \log\det R(t_k) + \frac{1}{2} m_y N \log 2\pi + \frac{1}{2}\sum_{k=1}^{N} \varepsilon^T(t_k) R^{-1}(t_k)\varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$

MORE GENERAL

$$V(\theta) = g(G(\theta))$$

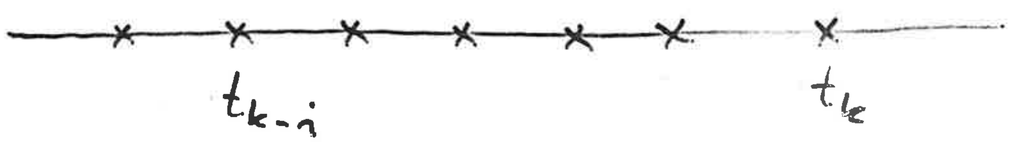
$$G(\theta) = \sum_{k=1}^{N} F(\varepsilon(t_k), \theta, t_k)$$

LONGER PREDICTION HORIZON

$$V(\theta) = g(G_1(\theta), G_2(\theta), \dots, G_s(\theta))$$

$$G_i(\theta) = \sum_{k=1}^{N} F_i(\varepsilon(t_k|t_{k-i}), \theta, t_k)$$

$$\varepsilon(t_k|t_{k-i}) = y(t_k) - \hat{y}(t_k|t_{k-i})$$



ESTIMATING PARAMETERS OF DYNAMICAL SYSTEMS

Example

$$\dot{x} = Ax + Bu + \overset{R_1}{v}$$

$$y(t_k) = Cx(t_k) + \overset{R_2}{e(t_k)}$$

How to obtain the likelihood function

Computational aspects

The minimization problem

Properties of the ML-estimate

THE LIKELIHOOD FUNCTION

INTRODUCE

$$y_{t_k}^T = \left[y^T(t_0)\; y^T(t_1) \ldots y^T(t_k) \right]$$

USING BAYES RULE THE LIKELIHOOD FUNCTION BECOMES

$$L = p(y_{t_k}) = p(y(t_k) \mid y_{t_{k-1}})\, p(y_{t_{k-1}})$$

$$p(y(t_k) \mid y_{t_{k-1}})\, p(y(t_{k-1}) \mid y_{t_{k-2}}) \ldots p(y(t_1) \mid y(t_0))\, p(y(t_0))$$

FOR LINEAR GAUSSIAN PROBLEMS

$$p(y(t_k) \mid y_{t_{k-1}}) = N(\hat{y}(t_k \mid t_{k-1}),\; R(t_k))$$

$$= (1/2)(2\pi)^{-m/2} (\det R(t_k))^{-1/2} \exp - (1/2)\, \varepsilon^T(t_k) R^{-1}(t_k) \varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k)$$

THE LIKELIHOOD FUNCTION CAN THEN BE WRITTEN AS

$$-\log L = (1/2) \left[\Sigma \log \det R(t_k) + \Sigma\, \varepsilon^T(t_k) R^{-1}(t_k) \varepsilon(t_k) \right] + \text{const.}$$

NOTICE RELATIONS TO FILTERING THEORY !

INTERPRETATION FOR NON GAUSSIAN PROCESSES

EXAMPLE

$$\dot{x} = Ax + Bu + v$$

$$y(t_k) = Cx(t_k) + e(t_k)$$

THE KALMAN BUCY THEORY GIVES:

$$\hat{y}(t_k|t_{k-1}) = C\hat{x}(t_k|t_{k-1})$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$

$$R(t_k) = R_2 + C\,P(t_k|t_{k-1})\,C^T$$

$$\hat{x}(t_k|t_k) = \hat{x}(t_k|t_{k-1}) + K(t_k)\cdot\varepsilon(t_k)$$

$$K(t_k) = P(t_k|t_{k-1})\,C^T R^{-1}(t_k)$$

$$P(t_k|t_k) = P(t_k|t_{k-1}) - K(t_k)CP(t_k|t_{k-1})$$

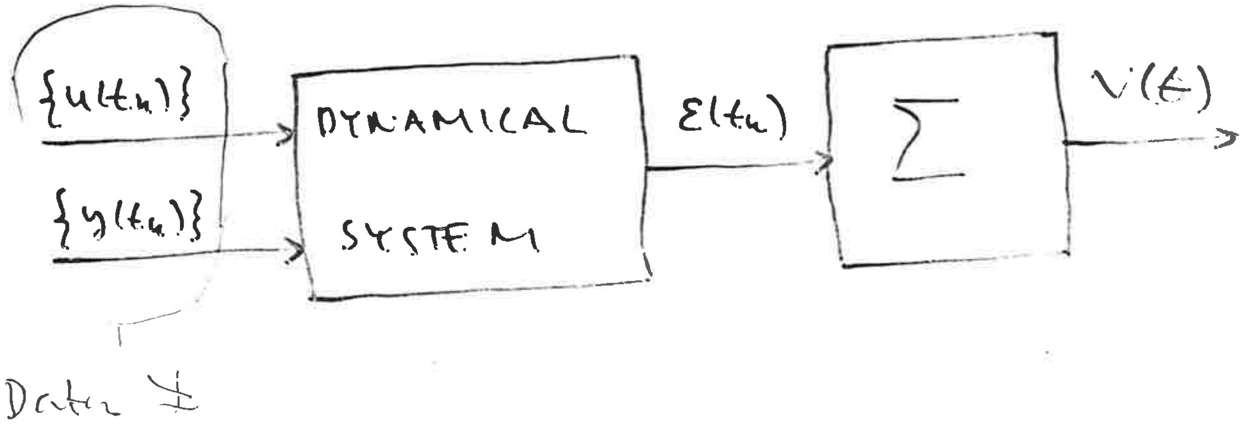
$$\frac{d\hat{x}(t|t_k)}{dt} = A\hat{x}(t|t_k) + Bu(t) \qquad t_k \leq t \leq t_{k+1}$$

$$\frac{dP(t|t_k)}{dt} = AP(t|t_n) + P(t|t_k)A^T + R_1 \qquad t_k \leq t \leq t_{k+1}$$

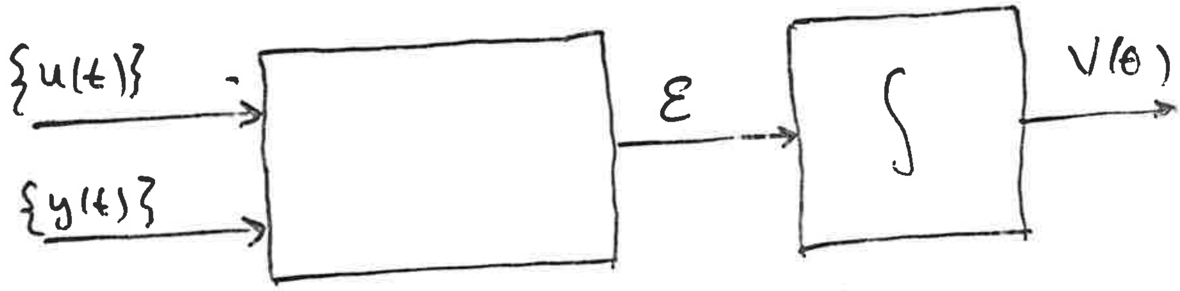
THE LIKELIHOOD FUNCTION

$$(-2\log L)_{t_k} = (-2\log L)_{t_{k-1}} + \varepsilon^T(t_k)R^{-1}(t_k)e(t_k) + \log\det R(t_k)$$

NOTICE THE STRUCTURE OF THE LIKELIHOOD FUNCTION



CONTINUOUS TIME DATA



$$\frac{dz}{dt} = F(z, u(t), y(t), t) = G(z, t)$$

$$\varepsilon = H(z, t)$$

$$V(\theta) = \frac{1}{2}\int_0^T K(\varepsilon, t, \theta)\,dt$$

COMPUTATIONAL ASPECTS

What must be done?

Minimization algorithms → FUNCTION EVALUATION
GRADIENT —"—
HESSIAN —"—

Simplifications

constant sampling rate

special model structures

USING ADJOINT VARIABLES TO CALCULATE GRADIENTS

$$\frac{dx}{dt} = f(x,\theta,t)$$

$$V(\theta) = \int_0^T g(x,s)\,ds$$

$$V_\theta(\theta) = \int_0^T g_x x_\theta \,ds = -\int_0^T p^T(s) f_\theta \,ds$$

$$\begin{cases} \dfrac{dp}{dt} = -\left(\dfrac{\partial f}{\partial x}\right)^T p + g_x^T \\ p(T) = 0 \end{cases}$$

PROOF:

$$\frac{dx_\theta}{dt} = f_x x_\theta + f_\theta$$

$$V_\theta = \int_0^T [g_x x_\theta + p^T \dot{x}_\theta - p^T f_x x_\theta - p^T f_\theta]\,ds$$

$$= p^T x_\theta \Big]_0^T + \int_0^T [g_x x_\theta - \dot{p}^T x_\theta - p^T f_x x_\theta - p^T f_\theta]\,ds$$

$$= p^T x_\theta \Big]_0^T - \int_0^T [g_x - p^T f_x - \dot{p}^T] x_\theta - \int_0^T p^T f_\theta \,ds$$

EXAMPLE $\quad t_{k+1} - t_k = 1$

$$x(t+1) = A\,x(t) + B\,u(t) + K\,\varepsilon(t)$$

$$y(t) = C\,x(t) + \varepsilon(t)$$

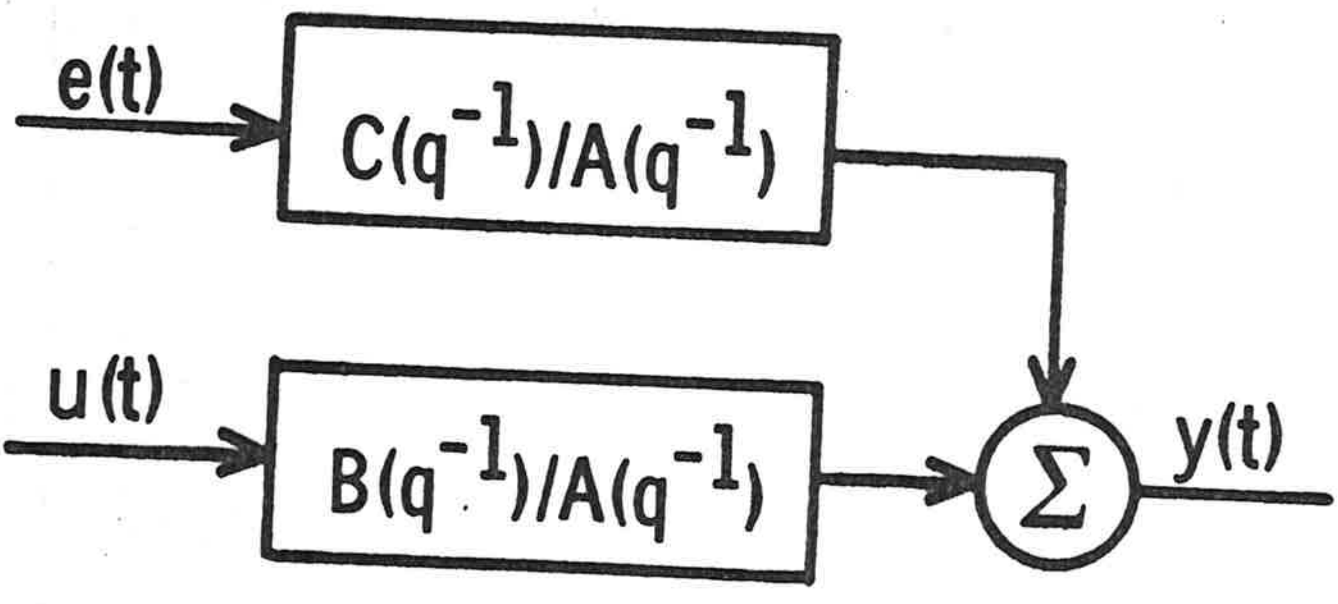
$$-2 \log L = \sum_{1}^{N} \varepsilon^T(t)\, R^{-1}\, \varepsilon(t) + N \log \det R + c$$

MINIMIZE $\quad$ W.R.T. $\quad$ R !

$$-2 \log L = N \log \det \frac{1}{N} \sum_{1}^{N} \varepsilon^T(t)\, \varepsilon^T(t) + r\,N + \text{const.}$$

EXAMPLE (ARMAX MODEL)

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) =$$
$$= b_1 u(t-1) + \ldots + b_n u(t-n) +$$
$$+ \lambda \left(e(t) + c_1 e(t-1) + \ldots + c_n e(t-n) \right)$$



$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t) + C(q^{-1})\, e(t)$$
$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$$
$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_n q^{-n}$$
$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_n q^{-n}$$

THE ARMAX MODEL

CANONICAL FORM FOR LINEAR TIMEINVARIANT SYSTEM WHOSE DYNAMIS IS RATIONAL TRANSFER FUNCTION + TIME DELAY

DISTURBANCES ARE STATIONARY WITH RATIONAL SPECTRAL DENSITY

CAN BE EXTENDED TO MISO?

$$Ay = B_1 u_1 + B_2 u_2 + \dots + B_r u_r + Ce$$

MINIMIZATION

$$-\log L = \frac{1}{\lambda^2} V(\theta) + \frac{N}{2} \log \lambda + \text{const}$$

$$V(\theta) = \frac{1}{2} \sum_{t=1}^{N} \varepsilon^2(t)$$

$$C(q^{-1})\varepsilon(t) = A(q^{-1})y(t) - B(q^{-1})u(t)$$

$$\theta^{k+1} = \theta^k - \left[V_{\theta\theta}(\theta^k)\right]^{-1} V_\theta(\theta^k)$$

$$V_\theta = \sum_{t=1}^{N} \varepsilon(t)\, \varepsilon_\theta(t)$$

$$V_{\theta\theta} = \sum_{t=1}^{N} \varepsilon_\theta(t)\varepsilon_\theta(t) + \sum_{t=1}^{N} \varepsilon(t)\varepsilon_{\theta\theta}(t)$$

$$C(q^{-1})\frac{\partial \varepsilon(t)}{\partial a_i} = y(t-i)$$

$$C(q^{-1})\frac{\partial \varepsilon(t)}{\partial b_i} = -u(t-1)$$

$$C(q^{-1})\frac{\partial \varepsilon(t)}{\partial c_i} = -\varepsilon(t-i)$$

MODEL STRUCTURES

$$dx = Ax\,dt + Bu\,dt + dw \leftarrow R_1$$

$$dy = Cx\,dt + de \leftarrow R_2$$

$$y(t) + A_1 y(t-1) + \ldots + A_n y(t-n) =$$

$$= B_1 u(t) + \ldots + B_n u(t-n) + e(t) + C_1 e(t-1) + \ldots + C_n e(t-n)$$

$$y(t) = H(s)\,u(t) + G(s)\,e(t)$$

NONLINEAR MODELS

$$\frac{d\hat{x}(t|t_k)}{dt} = f(\hat{x}(t|t_k), u(t))$$

$$\hat{y}(t_k|t_{k-1}) = g(\hat{x}(t_k|t_{k-1})) +$$

$$\hat{x}(t_k|t_k) = h(\hat{x}(t_k|t_{k-1}), \varepsilon(t_k))$$

ESTIMATION THEORY

HOW WILL THE METHODS WORK
UNDER IDEAL CIRCUMSTANCES

HOW ARE THE RESULTS INFLUENCED
BY DIFFERENT CHOICES OF THE
PROBLEM ELEMENTS $\mathcal{D}$, $\mathcal{M}$, $\mathcal{E}$

CLASSICAL STATISTICS

- CONSISTENCY
- ASYMPTOTIC DISTRIBUTIONS
- EFFICIENCY

GENERAL COMMENT ON RESULTS

- LARGE SAMPLE PROPERTIES $N \to \infty$
- CHARACTER OF RESULTS

NOTIONS

$\mathcal{D}$ DATA GENERATED FROM M_0

$\mathcal{M}$ MODEL SET

$\mathcal{C}$ CRITERIA

INTRODUCE

$$W(\theta) = \lim_{N \to \infty} \frac{1}{N} V_N(\theta) = \left[-\lim_{N \to \infty} \frac{1}{N} \log L(\theta, y_N) \right]$$

SHOW <u>UNIFORM</u> CONVERGENCE

(ERGODIC THEOREMS OR MARTINGALE THEOREMS)

ANALYSE $W(\theta)$ FIND θ_0 WHICH

MINIMIZES $W(\theta)$

UNDER GENERAL BUT MESSY CONDITIONS

$$\hat{\theta}_N \to \theta_0$$

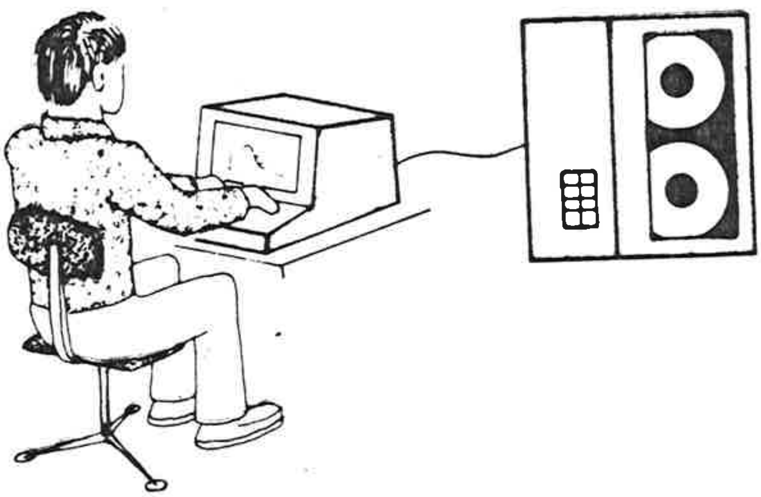
COMPUTER AIDED ANALYSIS AND DESIGN

BACKGROUND

MANY METHODS ARE CONCEPTUALLY SIMPLE
BUT THEIR DETAILS MAY BE MESSY

SOLUTION

COMBINE MAN'S INTUITION WITH THE COMPUTERS
CALCULATING CAPACITY



EXAMPLES

SIMNON
IDPAC
MODPAC
SYNPAC

PRACTICAL EXPERIENCES

PAPER MACHINES

DRUM BOILERS

DISTILLATION COLUMNS

NUCLEAR REACTORS

ACTIVATED SLUDGE PROCESSES

SHIP STEERING DYNAMICS

THERMAL HEAT CONDUCTION

MACROECONOMICS

PHARMACOKINETICS

INSULIN KINETICS

WHERE DOES ML & PE FIT INTO THE MODELING WORK?

- EXPLORATORY PHASE ASSUME A CANONICAL MISO MODEL. FIT TO DATA AND TEST.
- FINAL PARAMETER ESTIMATION PHASE. ASSUME PHYSICAL MODEL WITH ALL AVAILABLE INFORMATION. FIT PARAMETERS AND VALIDATE.

SPECIAL FEATURES OF ML & PRED. ERR.

- GREAT FLEXIBILITY WRT MODEL STRUCTURE
- DISTURBANCES ARE MODELED
- GREAT FLEXIBILITY WRT PARAMETRIZATION.

  "PHYSICAL" PARAMETERS & CONTINUOUS TIME MODELS CAN BE USED
- THEORETICALLY REASONABLY WELL UNDERSTOOD
- WILL OFTEN REQUIRE SUBSTANTIAL CALCULATIONS

ADAPTIVE CONTROL

1. INTRODUCTION
2. DESIGN PRINCIPLES
3. THE MINIMUM VARIANCE SELF-TUNER
4. ANALYSIS (EXAMPLE)
5. ANALYSIS (RESULTS)
6. CONCLUSIONS

EXAMPLE OF SYSTEM IDENTIFICATION

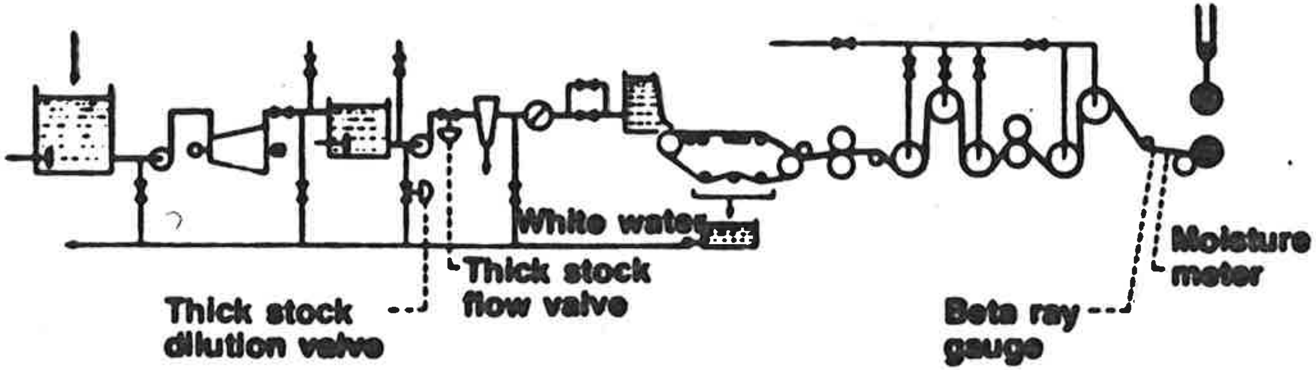
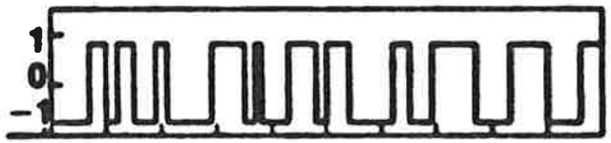


Figure 1 Simplified diagram of a kraft paper machine.



MODEL OF PROCESS DYNAMICS AND DISTURBANCES

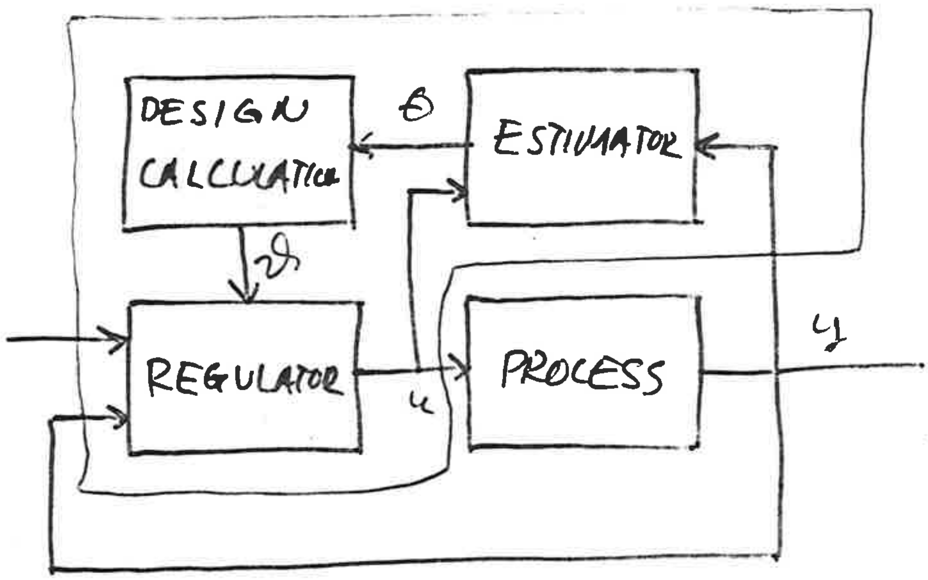
CONTROL LAW

CONTROL PERFORMANCE (UNDER THE CONDITIONS OF THE EXPERIMENT)

INTRODUCTION

- HOW DO THE REGULATORS WORK ?
- HOW CAN THEY BE CHANGED TO WORK BETTER
- KEY PROBLEMS
  - STABILITY
  - CONVERGENCE
  - PERFORMANCE
- NATURE OF MATHEMATICAL PROBL.
  - NONLINEAR
  - STOCHASTIC
- KEY ASSUMPTIONS
  - $\varphi \in \mathcal{M}$
  - SELF-TUNING NOT ADAPTIVE
- ROLE OF SIMULATION

REGULATOR STRUCTURE



NOTICE

1. CAN BE VIEWED AS A NONLINEAR REGULATOR

2. TWO SIGNAL PATHS "PARAMETERS" AND "STATES"

DESIGN METHODS

- MINIMUM VARIANCE
- LINEAR QUADRATIC GAUSSIAN

FREQUENCY RESPONSE
POLE PLACEMENT
ETC

DESIGN PHILOSOPHY

- LOOK AT THE PARTICULAR PROBLEM
  - SERVO
  - REGULATOR
  - MAJOR DISTURBANCES
  - CRITERIA
- WHAT WOULD YOU DO IF THE PROCESS & ENVIROMENT KNOWN
- HOW WOULD YOU ESTIMATE THE MISSING DATA?

⇒ THEN DECIDE!

WARNING!

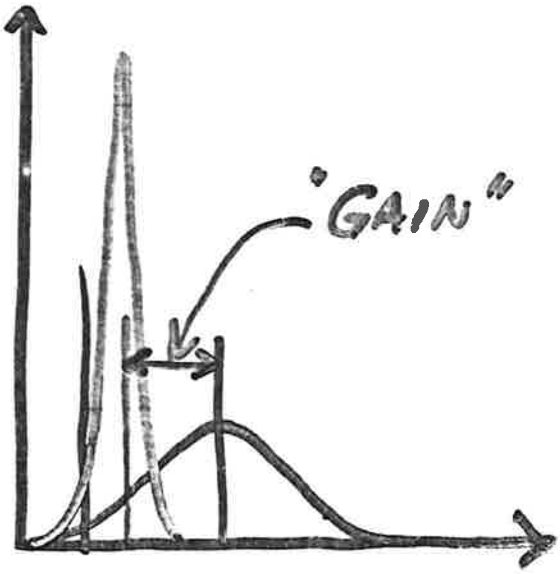
DO SIMPLE THINGS FIRST

- PI, PID
- OUTPUT FEEDBACK
- STATE FEEDBACK W. OBSERVER
- NONLINEAR
- FIXED GAIN
- GAIN SCHEDULE
- ADAPTIVE

SPARSAMKEIT!

MINIMUM VARIANCE CONTROL

$$A y_t = B u_{t-k} + C \xi_t$$



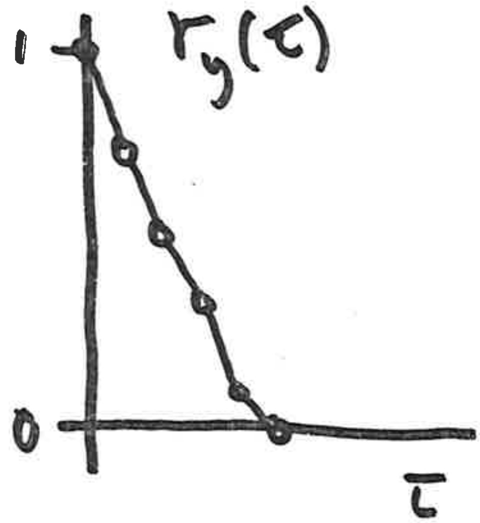
$$\lim_{N \to \infty} \frac{1}{2N} \sum_{-N}^{N} y_t^2 = E y_t^2$$

MINIMAL

SOLUTION

$$C = AF + q^{-k} G$$

$$u_t = -\frac{G}{BF} y_t$$



PROPERTY

$$y_t = F \xi_t$$

REMARKS

- B IS CANCELLED $\quad u_t = -\frac{G}{B} \xi_t$
- SAMPLING PERIOD IS THE MAJOR DESIGN VARIABLE

MINIMUM VARIANCE SELF-TUNERS

$A y_t = B u_{t-k} + C \xi_t$, $E y_t^2 = \text{MIN}$

$C = AF + q^{-k} G$, $u_t = -\frac{G}{BF} y_t$

EXPLICITE ALGORITHM

1° ESTIMATION

FIND A & B IN $A y_t = B u_{t-k}$ BY LS

2° DESIGN

SOLVE $1 = A\underline{F} + q^{-k}\underline{G}$ FOR F & G

3° CONTROL

USE CONTROL LAW $u_t = -\frac{G}{BF} y_t$

IMPLICITE ALGORITHM

$$y_{t+k} = (AF + q^{-k}G)\, y_{t+k} = G y_t + BF u_t + F \xi_{t+k}$$

1° ESTIMATION

FIND G & BF IN $y_{t+k} - G y_t = BF u_t$

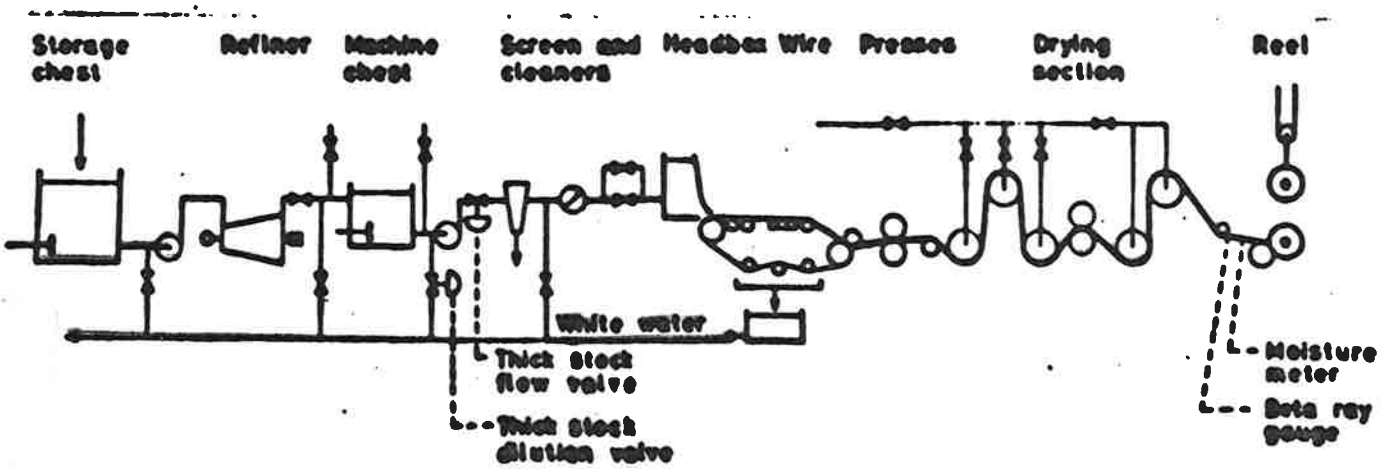
BY LEAST SQUARES

2° CONTROL

USE CONTROL LAW $u_t = -\frac{G}{BF} y_t$

EXAMPLE 4

BASIS WEIGHT CONTROL OF PAPERMACHINE



SECOND ORDER MODEL

TWO TIME DELAYS

SEVEN PARAMETERS

$$\Delta y(t) = \frac{4.61q - 4.05}{q^2 - 1.283q + 0.495}\,\Delta u(t-2) +$$

$$+\, 0.382\,\frac{q^2 - 1.438q + 0.550}{q^2 - 1.283q + 0.495}\, e(t)$$

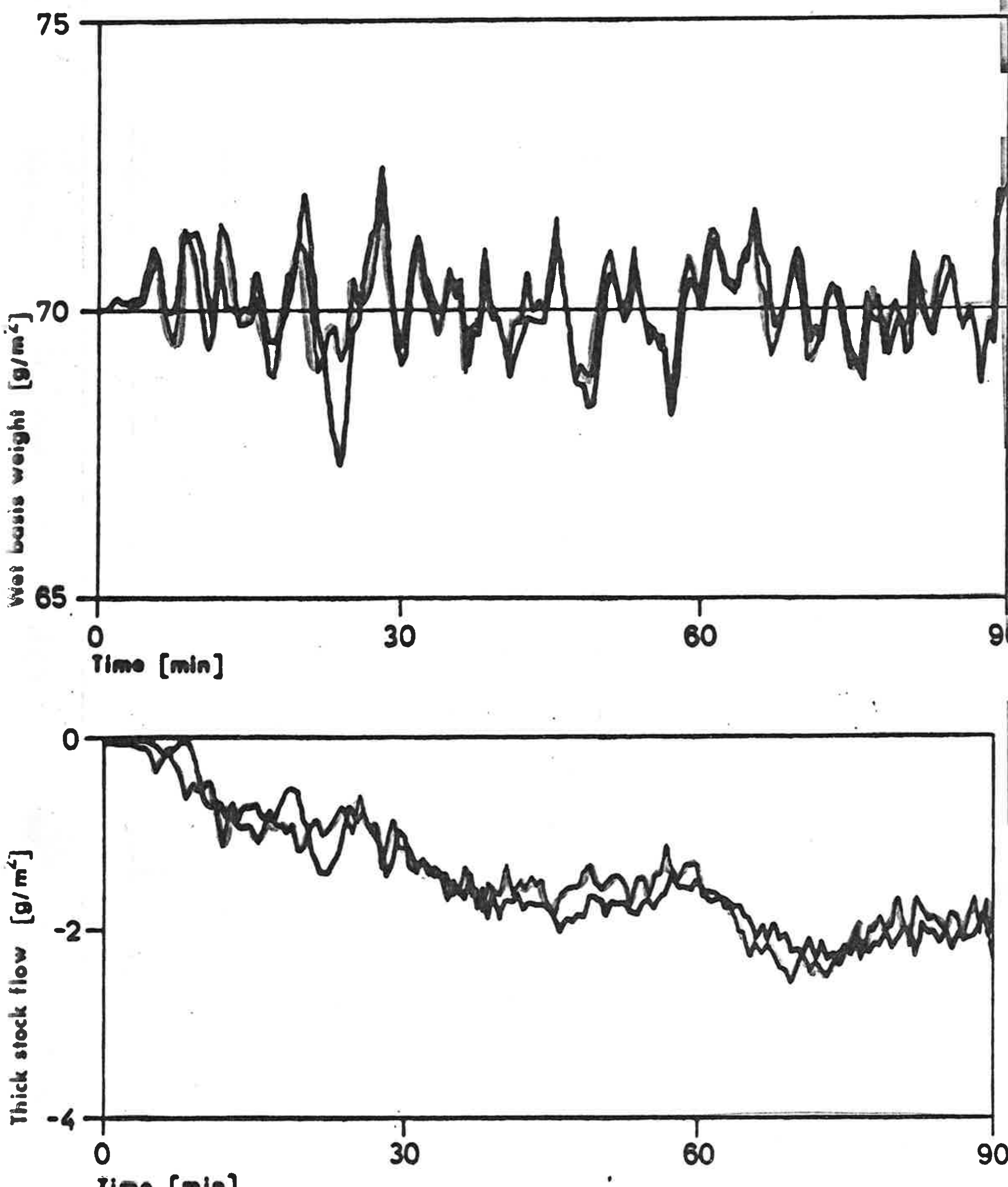
REF K. J. Å. INTRODUCTION TO STOCHASTIC CONTROL THEORY

$\sigma = 1.3$ ~ m ≈ 70 $3\sigma = 3.9$ 5.6%

$\sigma_e = 0.5$ $3\sigma = 1.5 \rightarrow 2.1\%$ ⇒ $\boxed{\text{3.5\% reduction}}$

Figure 9

42%
55



EXAMPLE

$$y(t+1) + a y(t) = b u(t) + e(t+1) + c e(t)$$

$$\min \; E\, y^2(t) \Rightarrow \boxed{u(t) = \frac{a-c}{b} y(t)}$$

ALGORITHM:

1. Estimation:

   Determine α in

   $$\hat{y}(t+1) + \alpha y(t) = 1 \cdot u(t)$$

   by least squares i.e.

   $$\frac{1}{t} \sum_{k=1}^{t} [y(t) - \hat{y}(t)]^2 \quad \min.$$

2. Control:

   At each time t choose control

   $$u(t) = \hat{\alpha}(t) y(t)$$

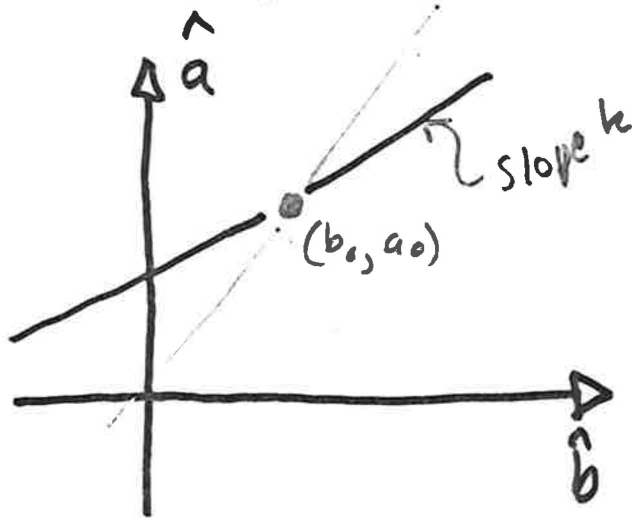
WHY NOT ESTIMATE b ?

$$\mathcal{S}: \quad y(t+1) + a_0 y(t) = b_0 u(t) + v(t) \qquad (1)$$

$$\mathcal{F}: \quad u(t) = \ell \, y(t) \qquad (2)$$

BUT $\mathcal{S} + k \cdot \mathcal{F}$ GIVES

$$y(t+1) + (a_0 + k\ell) y(t) = (b_0 + k) u(t) + v(t)$$



THE PARAMETERS a & b IN $\mathcal{S}$ ARE NOT IDENTIFIABLE WITH THE FEEDBACK $\mathcal{F}$ IF ℓ IS CONSTANT!

IN THE <u>ADAPTIVE CASE</u> THE FEEDBACK IS TIMEVARYING & THE SYSTEM DOES INDEED BECOME IDENTIFIABLE

CONVERGENCE IS HOWEVER SLOWER!

SIMULATIONS

EXAMPLE 1

$$y(t) + a\,y(t-1) = b\,u(t-1) + e(t) + c\,e(t-1)$$

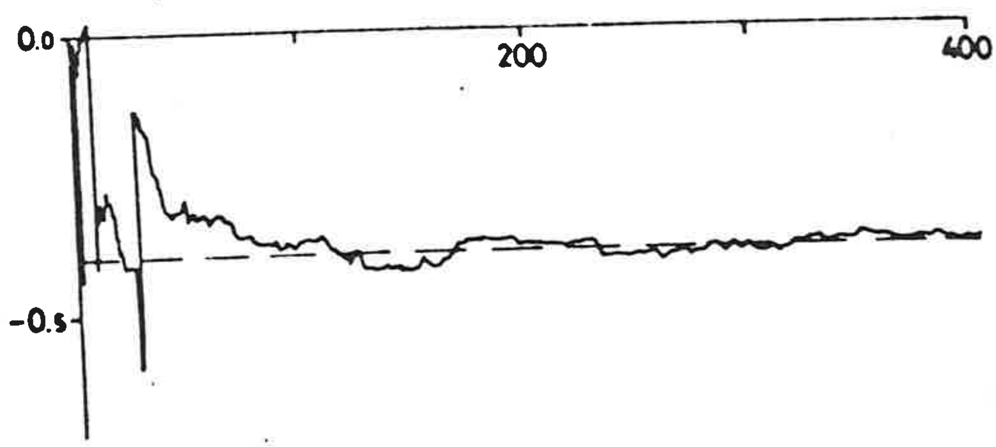
$a = -0.5, \quad b = 3, \quad c = 0.7$

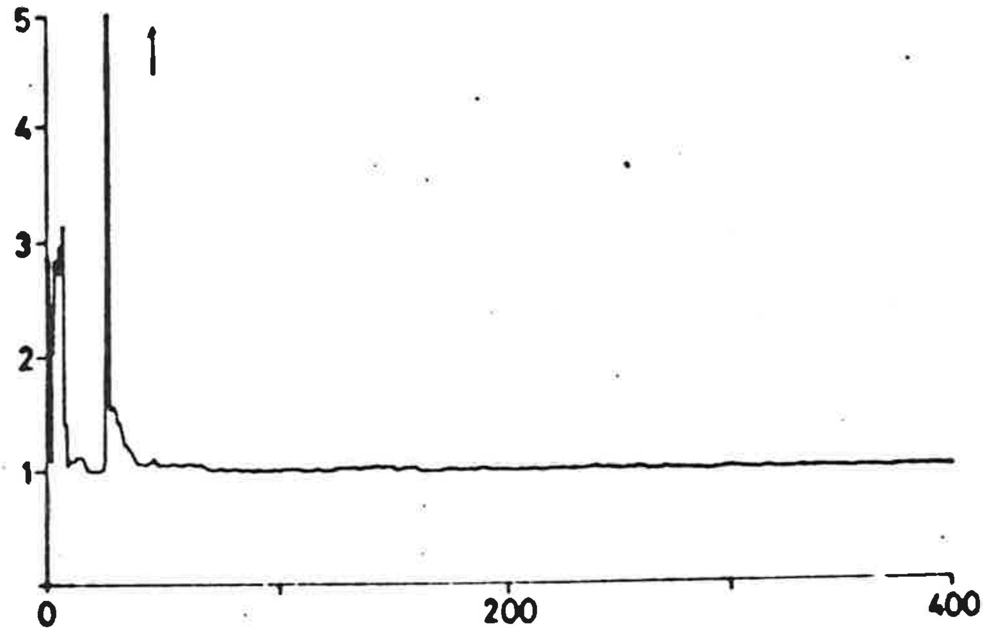
MIN VARIANCE REGULATOR

$$u(t) = \frac{a-c}{b}\, y(t) = -0.4\, y(t)$$

MODEL

$$y(t) + \alpha y(t-1) = \beta_0\, u(t-1) + e(t)$$





STABILITY

$b = 1$

PROCESS:

$$y(t+1) + a\,y(t) = b\,u(t) + n(t)$$

ESTIMATION MODEL:

$$\hat{y}(t+1) + \alpha\, y(t) = u(t)$$

LS ESTIMATE:

$$\hat{\alpha}(t) = -\frac{\sum^{t-1} [y(k+1) - u(k)]\, y(k)}{\sum^{t-1} y^2(k)}$$

$$= a - \frac{\sum^{t-1} n(k)\, y(k)}{\sum^{t-1} y^2(k)}$$

$$|\hat{\alpha}(t) - a| \leq \sqrt{\frac{\frac{1}{t}\sum n^2(k)}{\frac{1}{t}\sum y^2(k)}}$$

$$\frac{1}{t}\sum^{t-1} n^2(k) \text{ bdd} \Rightarrow \frac{1}{t}\sum^{t-1} y^2(k) \text{ bdd.}$$

ALSO TRUE FOR $0 < b < 2$

INTUITIVE INTERPRETATION?

POSSIBLE CONVERGENCE POINTS

ESTIMATE IS GIVEN BY THE NORMAL EQ

$$\frac{1}{t}\sum y(k+1)y(k) + \alpha_t \frac{1}{t}\sum y^2(k) = \frac{1}{t}\sum y(k)\beta_0 u(k)$$

CONTROL LAW:

$$\beta_0 u(k) = \alpha_k y(k)$$

$$\frac{1}{t}\sum y(k+1)y(k) = \frac{1}{t}\sum [\alpha_k - \alpha_t]\, y^2(k)$$

ASSUME $\alpha_t \to \alpha$ $\quad \frac{1}{t}\sum y^2(k)$ BDD.

$$\Rightarrow \boxed{\frac{1}{t}\sum_{k=1}^{t} y(k+1)\,y(k) \to 0} \qquad (*)$$

NOTICE NO ASSUMPTION ON PROCESS!
COMPARE INTEGRAL ACTION! IF

$$y(t+1) + a\,y(t) = b\,u(t) + e(t+1) + c\,e(t)$$

FEEDBACK $u(t) = \alpha/\beta_0\, y(t)$ GIVES

$$y(t+1) + [a - \alpha b/\beta_0]\, y(t) = e(t+1) + c\,e(t)$$

$$(*) \Rightarrow \boxed{\frac{\alpha}{\beta_0} = \frac{a-c}{b}} \qquad \frac{\alpha}{\beta_0} = \frac{a - 1/c}{b}$$

CONVERGENCE ANALYSIS

$$\mathcal{M}: \quad y(t+1) + \alpha y(t) = \beta_0 u(t) + e(t)$$

$$\hat{\alpha}(t+1) = - \frac{\sum^{t} [y(k+1) - \beta_0 u(k)]\, y(k)}{\sum^{t} y^2(k)}$$

$$= \hat{\alpha}(t) - \frac{1}{\sum^{t} y^2(k)} [y(t+1) + \hat{\alpha}(t) y(t) - \beta_0 u(t)]\, y(t)$$

$$= \hat{\alpha}(t) - \frac{1}{\sum^{t} y^2(k)} y(t+1) \cdot y(t)$$

KEY PROBLEM IS TO ANALYSE

$$\mathcal{S}: \quad y(t+1) + a y(t) = b u(t) + e(t+1) + c e(t)$$

$$\mathcal{F}: \quad u(t) = \frac{\hat{\alpha}(t)}{\beta_0} y(t)$$

$$\mathcal{E}: \quad \hat{\alpha}(t+1) = \hat{\alpha}(t) - \frac{y(t+1)\, y(t)}{\sum^{t} y^2(t)}$$

NOTICE $y(t)$ DEPENDS ON ALL PAST $\alpha(k)$.

IT HAS BEEN SHOWN THAT $\frac{1}{t} \sum^{t} y^2(k)$ IS BOUNDED

HEURISTIC DISCUSSION

$$\hat{\alpha}(t+1) = \hat{\alpha}(t) - \frac{y(t+1)\,y(t)}{\sum^{t} y^2(k)}$$

$$\frac{1}{t}\sum y^2(k) \rightarrow 1/P_0$$

$$\hat{\alpha}(t+1) \approx \hat{\alpha}(t) - P_0 \frac{1}{t}\, y(t+1)\,y(t)$$

$$\hat{\alpha}(t_{k+1}) = \hat{\alpha}(t_k) - P_0 \sum_{t_k}^{t_{k+1}} \frac{1}{k}\, y(k+1)\,y(k)$$

$$\approx \hat{\alpha}(t_k) - P_0 \underbrace{\left(\sum_{t_k}^{t_{k+1}} \frac{1}{k}\right)}_{\Delta\tau} \underbrace{\frac{1}{t_{k+1}-t_k} \sum_{t_k}^{t_{k+1}} y(k+1)\,y(k)}_{E\,y(t+1)\,y(t)}$$

$$\tau = \sum^{t} \frac{1}{k} \approx \log t$$

$$\hat{\alpha}(\tau + \Delta\tau) = \hat{\alpha}(\tau) + P_0\,\Delta\tau\, f(\alpha)$$

$$\frac{d\alpha}{d\tau} = P_0\, f(\alpha) \qquad \text{L. LJUNG}$$

$$f(\alpha) = -E\,y(t+1)\,y(t)$$

$\mathcal{S}$: $y(t+1) + a\,y(t) = b\,u(t) + e(t+1) + c\,e(t)$

$\mathcal{F}$: $u(t) = \alpha\, y(t)$

CLOSED LOOP SYSTEM

$y(t+1) + [a - \alpha b]\,y(t) = e(t+1) + c\,e(t)$

$$f(\alpha) = -E\,y(t+1)\,y(t) = -\frac{(c - a + \alpha b)(1 - ac + \alpha bc)}{1 - (a - \alpha b)^2}$$

$$\frac{d\alpha}{d\tau} = P_0\, f(\alpha)$$

$$f(\alpha) = 0 \Rightarrow \alpha_1 = \frac{a-c}{b} \; ; \; \alpha_2 = \frac{a - 1/c}{b}$$

$$f'(\alpha_1) = -\frac{b}{1 - (a - \alpha_1 b)^2} = -\frac{b}{1 - c^2}$$

THE ROLE OF β_0?

$b/\beta_0 < 0$	REGULATOR GIVES UNSTABLE SYSTEM
$0 < b/\beta_0 < 2$	ESTIMATES CONVERGE WP1
$b/\beta_0 \geq 2$	ESTIMATE CONVERGES IF CLOSED LOOP IS STABLE BUT THERE IS A NONZERO PROBABILITY FOR DIVERGENCE

STABILITY PROBLEM

ARE y & u BOUNDED ?

RELATION TO MRAS

PARKS

MONOPOLI

GOODWIN, RAMAGE, CAINES (NOV 78)

EGARDT (DEC 78)

NARENDRA (MARCH 79)

MORSE (APRIL 79)