LUND UNIVERSITY

Overhead Slides for Eight Lectures on Stochastic Control and its Applications

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OVERHEAD SLIDES FOR EIGHT LECTURES ON STOCHASTIC CONTROL

AND ITS APPLICATIONS

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OVERHEAD SLIDES

FOR EIGHT LECTURES ON STOCHASTIC CONTROL THEORY

AND ITS APPLICATIONS

K J Åström

LECTURES 1 & 2 - LINEAR STOCHASTIC CONTROL THEORY

The major results of linear stochastic control theory are reviewed. Emphasis is given to the problem formulation and to the main mathematical models used. The topics covered include minimum variance control, Kalman filtering, and the separation theorem. The problems are discussed both from the input-output and the state space point of view. The presentation is based on references [1], [2], and [3].

LECTURE 3 - MODELING AND SYSTEM IDENTIFICATION

The problem of obtaining the mathematical models used in linear stochastic control from experimental plant data is discussed. Principles of parameter estimation are discussed. The techniques include maximum likelihood and prediction error methods. Theoretical results for large sample properties are discussed together with computational and practical aspects. The presentation is based on references [4] and [5].

LECTURE 4 - APPLICATION OF PARAMETER ESTIMATION METHODS TO DETERMINE SHIP STEERING DYNAMICS

Application of the methods described in lecture 2 to determination of mathematical models for ship steering dynamics. The topics discussed include: mathematical models for ship dynamics, disturbances due to wind and waves, parameter estimation and practical aspects on parameter estimation. The presentation is based on references [6], [7], and [8].

LECTURE 5 - NONLINEAR STOCHASTIC CONTROL THEORY

Some important results in nonlinear stochastic control theory are discussed. The structure of the optimal feedback is derived for processes described by Markov chains using Dynamic programming. Nonlinear filtering and the nature of

the solutions for more general problems are treated. Concepts of certainty equivalence, caution and probing are introduced. The presentation is based on reference [2].

LECTURES 6 & 7 - SELF-TUNING AND ADAPTIVE CONTROL

Principles of adaptive control are introduced based on nonlinear stochastic control theory. Self-tuning regulators are introduced as an approximation to the general nonlinear problem. It is also shown that self-tuning generators can be motivated in more general cases. Relations to model reference adaptive systems is discussed. Theory of adaptive control and its short-comings and applications are reviewed. The presentation is based on reference [9].

LECTURE 8 - DESIGN OF AN ADAPTIVE AUTOPILOT FOR SHIP STEERING

Describes the application of adaptive control to a practical problem. The autopilot includes gain scheduling to eliminate influence of velocity variations, a Kalman filter, and a self-tuning regulator. The design trade-offs are discussed. Results from full-scale experiments on several ships are described. The presentation is based on reference [10].

2

REFERENCES

[1] K J Aström: Introduction to Stochastic Control Theory, Academic Press, 1970.

[2] K J Aström: Stochastic control problems. In W A Coppel (Ed.): Mathematical Control Theory, Springer Verlag, Berlin, 1978.

[3] K J Aström: Algebraic system theory as a tool for regulator design. In Halme et al: Topics in System Theory Publication in honour of Prof Hans Blomberg on the occasion of his sixtieth birthday, Acta Polytechnica Scandinavica, Ma31, Helsinki, 1979.

[4] K J Aström: The role of identification in process modeling. VDE-Berichte Nr 276, 1977.

[5] K J Aström: Maximum likelihood and predidtion error methods. Tutorial 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, 1979.

[6] K J Aström and C G Källström: Identification of ship-steering dynamics, Automatica 12 (1976) 9-22.

[7] C G Källström and K J Aström: Experiences of system identification applied to ship-steering dynamics. Case study 5th IFAC Symposium on Identification and System Parameter Estimation, Darmstadt, 1979.

[8] K J Aström, C G Källström, N H Norrbin, and L Byström: The identification of linear ship-steering dynamics using maximum likelihood parameter estimation. Report 1920-1, The Swedish State Shipbuilding Experimental Tank, Göteborg, Sweden, December 1974.

[9] K J Aström: Self-tuning regulators design principles and applications. To appear in K S Narendra and R V Monopoli: Applications of Adaptive Control, Academic Press.

[10] C G Källström, K J Aström, N E Thorell, J Eriksson, and L Sten: Adaptive autopilots for tankers, Automatica 15 (1979) 241-254.

STOCHASTIC CONTROL THEORY AND APPLICATIONS

STOCHASTIC CONTROL THEORY AND APPLICATIONS

NO DISTURBANCES — NO CONTROL PROBLEM

HOW TO CHARACTERIZE DISTURBANCES?

- CLASSICAL CONTROL THEORY

  STEP, RAMP, SINUSOID, EXPONENTIAL
  (SIGNALS GENERATED BY ODE.)

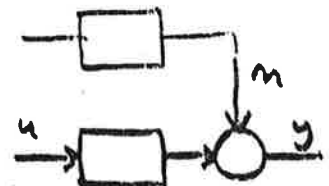- STOCHASTIC CONTROL THEORY

  STOCHASTIC PROCESSES

- MODELS:

  MOSTLY LINEAR → EXTERNAL I/O

  CHARACTERIZED BY COVARIANCE
  FUNCTION & SPECTRAL DENSITY

  CAN BE (NON)LINEAR → INTERNAL (STATE EQUATIONS)

  $$x(t+1) = f(x(t), u(t), t) + v(x(t), u(t), t)$$

  $$\sigma(x(t), u(t), t) \cdot e(t)$$

  CONCEPTUAL IMPLICATION
  PRACTICAL —''—

- IMPORTANT CONCEPTS

  PREDICTION   FILTERING   SMOOTHING

EXAMPLE:

$$\frac{dy}{dt} = u \qquad , \quad y(0) = a$$

$$J = \int_0^\infty [y^2(t) + u^2(t)] \, dt$$

FIND u TO MINIMIZE J!

SOLUTION

(1) $\quad u(t) = -y(0) \cdot e^{-t} \qquad J = a^2$

(2) $\quad u(t) = -y(t) \qquad\qquad J = a^2$

FEEDBACK PROCESS
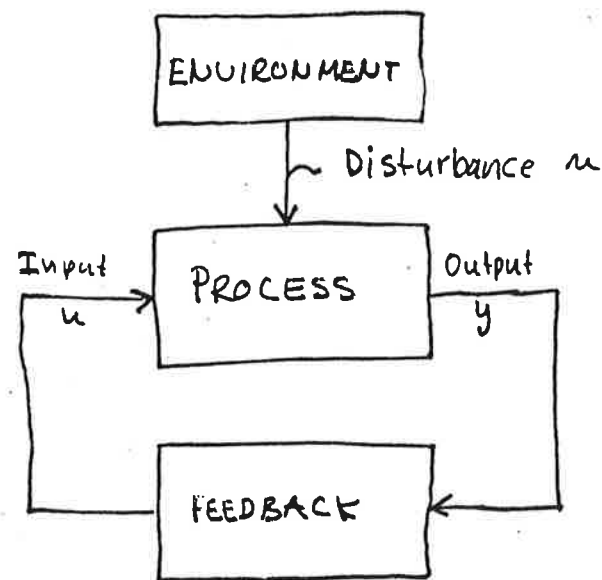
PI · REGULATOR

$$u(t) = u_{ref}(t) + K \left[e(t) + \int^t e(s) \frac{ds}{T} \right]$$

$$e(t) = y_{ref}(t) - y(t)$$

CONTROL ENGINEERS BLESSING

CONTROL THEORETICIANS CURSE

PROBLEM: FIND A SUITABLE
THEORETICAL FRAMEWORK
TO DISCUSS & ANALYSE FEEDBACK

STOCHASTIC CONTROL THEORY AND APPLICATIONS

MINIMUM VARIANCE CONTROL
EXTERNAL DESCRIPTIONS

1. MATHEMATICAL MODELS

Process Dynamics
Environment   — ARMAX
"CARMA" model
Multivariable generalizations

2. OPTIMAL PREDICTION

3. CONTROL

Criteria
Admissible controls
Min variance control
"Minimum Phase" condition
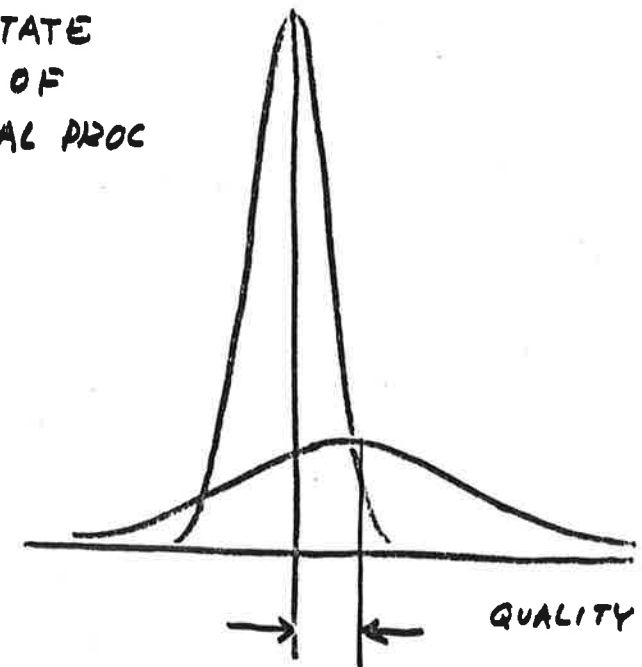
4. APPLICATIONS

REVIEW OF ASSUMPTIONS

o LINEAR DYNAMICS

o STOCHASTIC DISTURBANCES

o QUADRATIC LOSS FUNCTIONS

STEADY STATE
CONTROL OF
INDUSTRIAL PROC

QUALITY

o MODELS OF PROCESS DYNAMICS &
DISTURBANCES KNOWN
(IDENTIFICATION CAN HELP IF
MODELS ARE NOT KNOWN)

✤ MINIMUM VARIANCE CONTROL IS
EASY TO DERIVE BECAUSE
ONE-STEP OPTIMIZATION IS
THE SAME AS N-STEP OPTIMI-
ZATION I:E

$$\min E \frac{1}{N} \sum_1^N y^2(t)$$

$$\min E \, y^2(t)$$

GIVES THE SAME CONTROL LAW
(THIS IS OF INTEREST FOR THE
ADAPTIVE PROBLEM WHERE SIMPLICITY
OF CALCULATIONS IS IMPORTANT)

✤ ONE-STEP AND N-STEP NOT THE
SAME IF THERE IS PENALTY ON
CONTROL

✤ NOTICE

$$A \, y(t) = B \, u(t-k) + \lambda C \, e(t)$$
$$C = AF + q^{-k} G$$
$$u(t) = -\frac{G}{BF} \, y(t) \quad \text{MIN VAR CONTROL LAW}$$

MATHEMATICAL MODELS
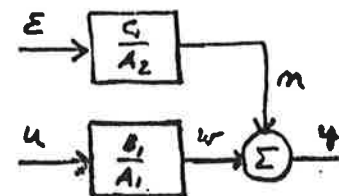
$$w(t) + a'_1 w(t-1) + \ldots + a_m w(t-m) = b'_0 u(t-k) + \ldots + b'_m u(t-k \ldots$$

$$A_1(q^{-1}) \, w(t) = B_1(q^{-1}) \, u(t-k) \qquad q \, y(t) = y(t+1)$$

$$y(t) = \frac{A_2 \, B_1(q^{-1})}{A_2 \, A_1(q^{-1})} \, u(t-k) + \frac{A_1 \, C_1(q^{-1})}{A_1 \, A_2(q^{-1})} \, \varepsilon(t)$$

$$A(q^{-1}) \, y(t) = B(q^{-1}) \, u(t-k) + C(q^{-1}) \, \varepsilon(t)$$

$C(\xi)$ CAN ALWAYS BE CHOSEN
TO HAVE ALL ZEROS OUTSIDE
UNIT DISC OR ON UNIT
CIRCLE

IF C HAS ZEROS ON THE
UNIT CIRCLE THEN OPTIMAL
PREDICTORS & CONTROLS ARE
TIMEVARYING !

OPTIMAL PREDICTION

$$A(q^{-1}) y(t) = C(q^{-1}) \varepsilon(t)$$

All zeros of $\det C(\xi)$ outside unit disc.

$$y(t+k) = A^{-1}(q^{-1}) C(q^{-1}) \varepsilon(t+k) = \mathcal{C}(q^{-1}) \mathcal{A}^{-1}(q^{-1}) \varepsilon(t+k)$$

$$\det A(\xi) = \det \mathcal{A}(\xi)$$

$$\mathcal{C}(q^{-1}) \mathcal{A}^{-1}(q^{-1}) = F(q^{-1}) + q^{-k} \mathcal{G}(q^{-1}) \mathcal{A}^{-1}(q^{-1})$$

$$\underset{\text{degree } k-1}{\Big\uparrow}$$

$$y(t+k) = F(q^{-1}) \varepsilon(t+k) + \mathcal{G}(q^{-1}) \mathcal{A}^{-1}(q^{-1}) \varepsilon(t)$$

$$= F(q^{-1}) \varepsilon(t+k) + \mathcal{G}(q^{-1}) \mathcal{C}^{-1}(q^{-1}) y(t)$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad}$$

$$\varepsilon(t+k) + f_1 \varepsilon(t+k-1) + \ldots + f_{k-1} \varepsilon(t+1)$$

$$\tilde{y} = y(t+k) - \hat{y} = F\varepsilon(t+k) + [\mathcal{G}\mathcal{C}^{-1} y(t) - \hat{y}]$$

$$E\{ a^T F \varepsilon(t+k) \} \{ a^T [\mathcal{G}\mathcal{C}^{-1} y(t) - \hat{y}] \} = 0$$

Independence $\hat{y}$ arbitrary f(n) of $y(t), y(t-1), \ldots$
Uncorrelated $\mathcal{G}$ linear in $y(t), y(t-1), \ldots$

$$\hat{y}(t+k/t) = \mathcal{G}(q^{-1}) \mathcal{C}^{-1}(q^{-1}) y(t)$$

$$\tilde{y}(t+k/t) = F(q^{-1}) \varepsilon(t+k) = \varepsilon(t+k) + f_1 \varepsilon(t+k-1) + \ldots + f_{k-1}\varepsilon(t+1)$$

EXAMPLE:

$$y(t+1) = \varepsilon(t+1) - \varepsilon(t)$$

$$y(t) = \varepsilon(t) - \varepsilon(t-1) \qquad C = 1 - q^{-1}$$

$$\hat{y}(t+1/t) = -\varepsilon(t)$$

$$\varepsilon(t) = \sum_{k=t_0+1}^{t} y(k) + \varepsilon(t_0) = z(t) + \varepsilon(t_0)$$

$$\hat{\varepsilon}(t_0) = -\frac{1}{t-t_0} \sum_{t_0+1}^{t} z(k) = -\frac{1}{t-t_0} \sum_{t_0+1}^{t} (t+1-k) y(k)$$

$$\hat{y}(t+1/t) = -\sum_{i=1}^{t-t_0} \frac{t-t_0-i}{t-t_0} y(t+1-i)$$

$$E(y-\hat{y})^2 \rightarrow E\varepsilon^2$$

$$\hat{y} = -\sum y(k)$$

$$E(y-\hat{y})^2 \rightarrow 2E\varepsilon^2$$

MINIMUM VARIANCE CONTROL

PROCESS MODEL

$$A(q^{-1}) y(t) = B(q^{-1}) u(t-k) + C(q^{-1}) \varepsilon(t)$$

$$A(q^{-1}) = I + A_1 q^{-1} + \ldots + A_n q^{-n}$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \ldots + B_n q^{-n}$$

$$C(q^{-1}) = I + C_1 q^{-1} + \ldots + C_n q^{-n}$$

CRITERION

$$\min E \, y^T(t) \, Q \, y(t)$$

ASSUMPTIONS

det $C(\xi)$ and det $B(\xi)$ all zeros
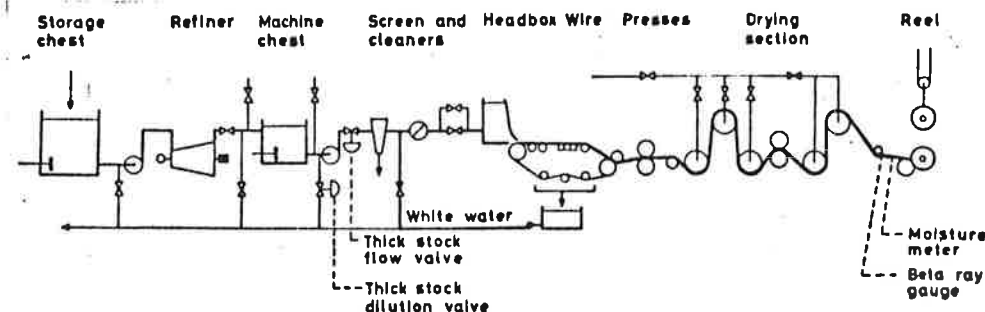outside unit disc, det $B_0 \neq 0$

$$u(t) = -B^{-1}(q^{-1}) G(q^{-1}) F^{-1}(q^{-1}) y(t)$$

$$= -B^{-1}(q^{-1}) G(q^{-1}) \varepsilon(t)$$

$$y(t) = F(q^{-1}) \varepsilon(t)$$

$$A^{-1}(q^{-1}) C(q^{-1}) = F(q^{-1}) + q^{-k} A^{-1}(q^{-1}) G(q^{-1})$$

BASIS WEIGHT CONTROL OF PAPERMACHINE

SECOND ORDER MODEL
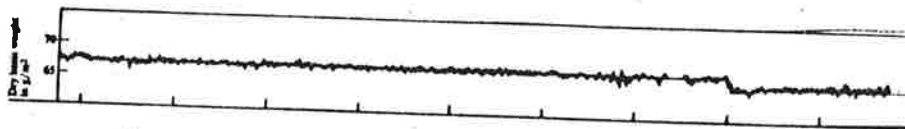TWO TIME DELAYS
SEVEN PARAMETERS

$$\Delta y(t) = \frac{4.61q - 4.05}{q^2 - 1.283q + 0.495} \Delta u(t-2) +$$

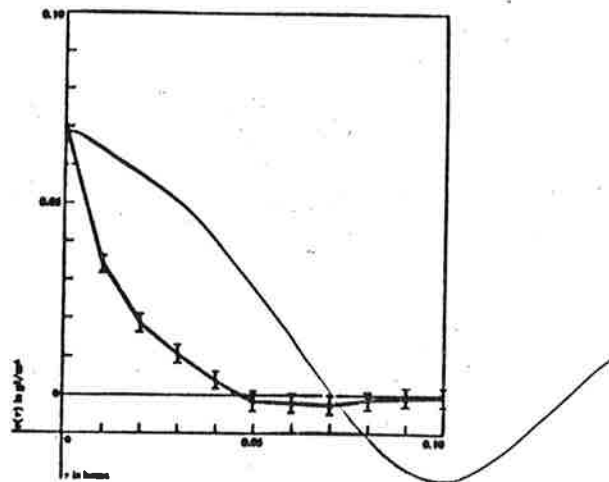$$+ \, 0.382 \, \frac{q^2 - 1.438q + 0.550}{q^2 - 1.283q + 0.495} \, e(t)$$

REF K. J. Å. INTRODUCTION TO STOCHASTIC
CONTROL THEORY

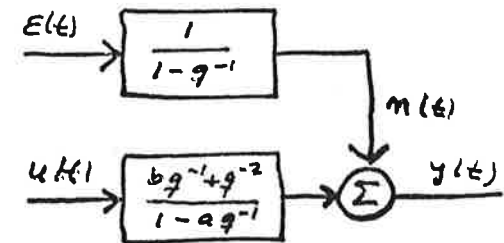ASSESSMENT OF NEED FOR RETUNING OF MINIMUM VARIANCE CONTROL

LOG CONTROLLED OUTPUT DURING NORMAL OPERATION

CALCULATE COVARIANCE FUNCTION OF OUTPUT
(RESID Y)

USE KNOWLEDGE OF PROCESS DEAD TIME AND SAMPLING PERIOD
TO TEST IF COVARIANCE FUNCTION SATISFIES MINIMUM
VARIANCE CONDITIONS

$$y(t) = \frac{b + q^{-1}}{1 - a q^{-1}} \, u(t-1) + \frac{1}{1 - q^{-1}} \, \varepsilon(t)$$

$$E \, \varepsilon^2(t) = 1 \quad , \qquad b \neq 0$$

CASE

		Ey
STABLE MIN. PHASE	$|a|<1, |b|>1$	1
UNSTABLE MIN PHASE	$|a|>1, |b|>1$	a^2
STABLE NON MIN PHASE	$|a|<1, |b|<1$	$\frac{z}{1+b}$
UNSTABLE NON MIN PHASE	$|a|>1, |b|<1$	d

$$d = a^2 \left[1 + \frac{(f_1 - b)^2}{1 - b^2} \right]$$

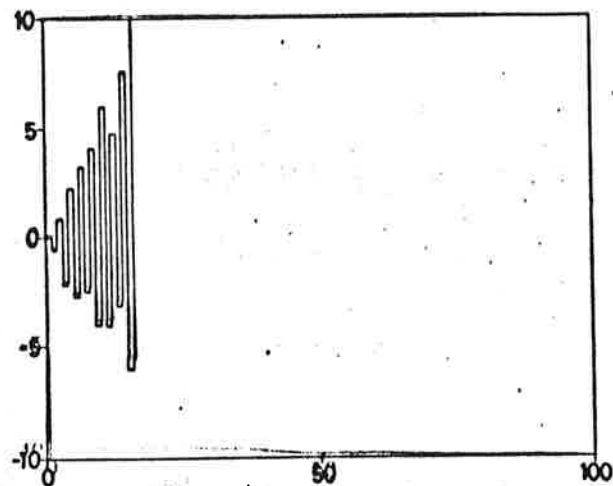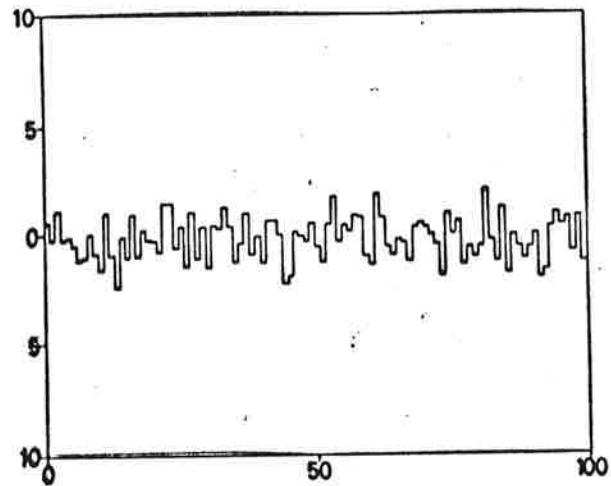$$f_1 = \frac{a + a^2 - 1 + ab(a+1) + b^2}{a[1 + b + ab + ab^2]}$$

STABLE NON MIN PHASE SYSTEM
MIN VAR STRATEGY
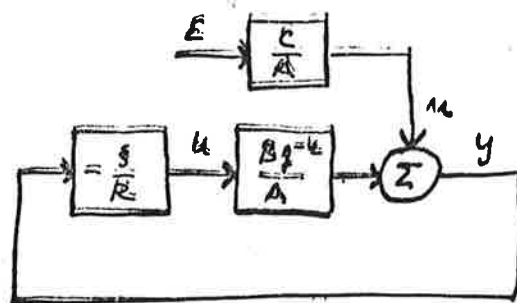
$$A(q^{-1}) = 1 - 1.7q^{-1} + 0.7q^{-2}$$
$$B(q^{-1}) = 0.9 + q^{-1}$$
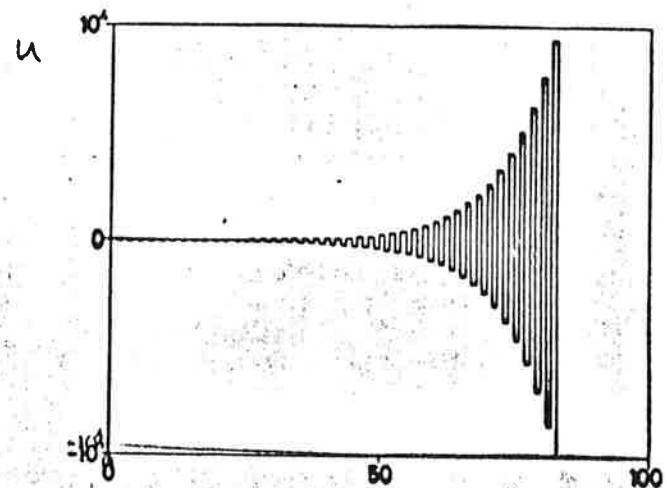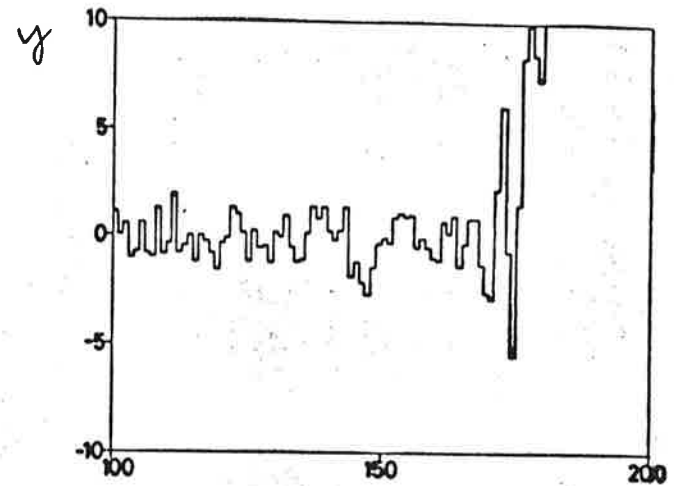$$C(q^{-1}) = 1 - 0.7q^{-1}$$

$$y = \frac{AR}{AR+BSq^{-1}} \cdot \frac{C}{A} \varepsilon = \left[1 - \frac{q^{-k}BS}{AR+q^{-k}BS}\right]\frac{C}{A}\varepsilon$$

$$= F\varepsilon + q^{-k}\frac{G}{A}\varepsilon - q^{-k}\frac{BS}{AR+q^{-k}BS}\cdot\frac{C}{A}\varepsilon$$

$$= F\varepsilon + \left[1 - \frac{BSC}{G(AR+q^{-k}BS)}\right]q^{-k}\frac{G}{A}\varepsilon .$$

$$= F\varepsilon_{(t)} + [1 - BX]\frac{G}{A}\varepsilon_{(t-k)}$$

$$E\,y^2 = E\,(F\varepsilon)^2 + J$$

$$J = \frac{1}{2\pi i}\oint \left\{[1-BX]\frac{G}{A}\right\}(z^{-1})\left\{[1-BX]\frac{G}{A}\right\}(z)\frac{dz}{z}$$

$$= \frac{1}{2\pi i}\oint [1-B(z^{-1})X(z^{-1})]\int Q(z)[1-B(z)X(z)]\frac{dz}{z}$$

J MAY HAVE SEVERAL LOCAL MIN

EXAMPLE

$$J[f] = \frac{1}{2\pi i}\oint [1-(z-a)f(z)][1-(z^{-1}-a)f(z)]\frac{dz}{z}$$

J min for

$$f(z) = \frac{1}{z-a} \qquad J[f] = 0$$

But if $|a| > 1$ local minimum
also for

$$f(z) = \frac{1}{a^2}\cdot\frac{1}{z-1/a} \qquad J[f] = 1-\frac{1}{a^2}$$

POLYNOMIAL B, UNSTABLE

$$B = B^+ . B^-$$

↑ ALL ZEROS INSIDE UNIT DISC
↳ ALL ZEROS OUTSIDE UNIT DISC

$$y = \frac{B}{A} u + \frac{C}{A} e = \frac{B^-}{\tilde{B}^-} \left[\frac{B^+ \tilde{B}^-}{A} u + \frac{C \tilde{B}^-}{AB^-} e \right]$$

$\underbrace{}_{w}$

$$E y^2 = E w^2$$

$$w = \frac{B^+ \tilde{B}^-}{A} u + \frac{F}{B^-} e + \frac{G}{A} e$$

$$\deg F < \deg B^-$$

$$\frac{F}{B^-} e = f_0 e(t+1) + f_1 e(t+2) + \ldots$$

$$w = \frac{F}{B^-} e + \frac{1}{A} \left[B^+ \tilde{B}^- u + G e \right]$$

$$w = \frac{F}{B^-} e \quad \Rightarrow \quad y = \frac{F}{\tilde{B}^-} e$$

$$u = - \frac{G}{B^+ \tilde{B}^-} e = - \frac{G}{B^+ F} y$$

$$C \tilde{B}^- = A F + B^- G \qquad \deg F < \deg B^-$$
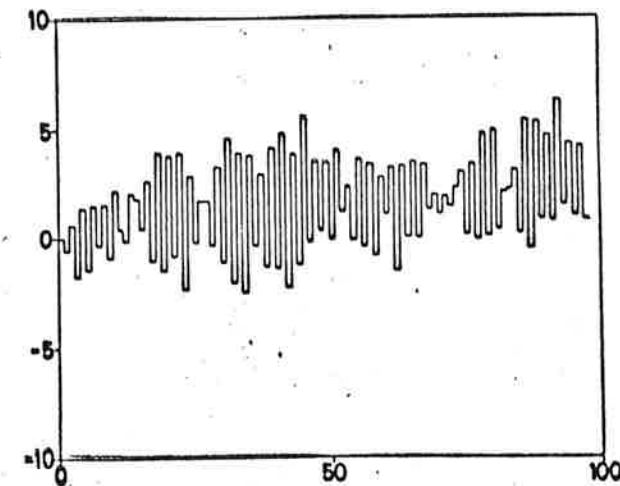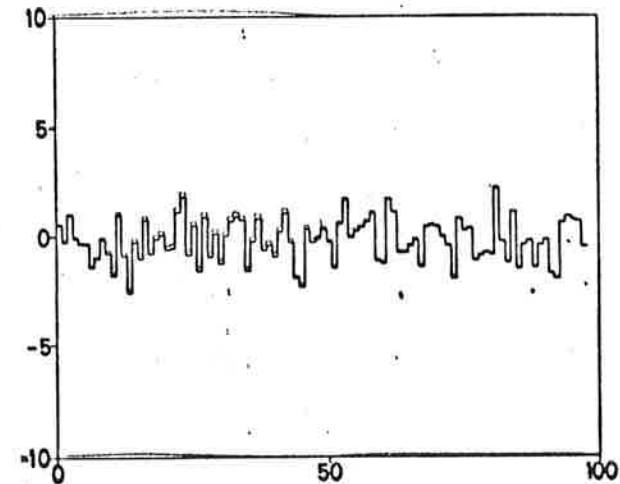
DATALOGGING:
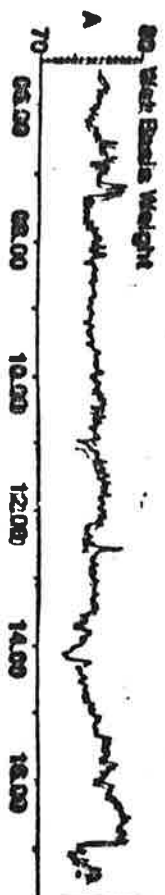
PROCESS IDENTIFICATION: PROCESS MODEL

Wet Blade Weight

70 65.00 68.00 10.00 12.00 14.00 16.00

PREDICTION
ERROR ANALYSIS
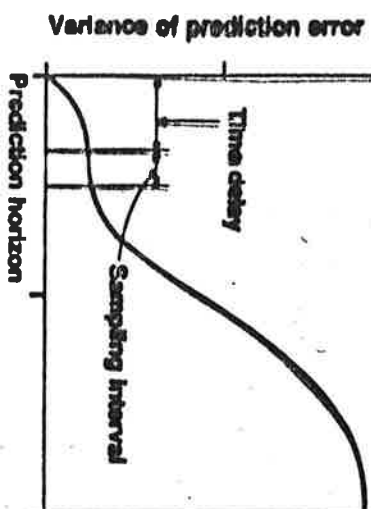
Variance of prediction error

Prediction horizon

Time delay

Sampling interval

HEDGE

STOCHASTIC CONTROL THEORY AND APPLICATIONS

THE LGG PROBLEM
INTERNAL DESCRIPTIONS

1. MATHEMATICAL MODELS

2. OPTIMAL PREDICTION
   Kalman Filtering
   Innovations
   Duality

3. OPTIMAL CONTROL
   Loss Functions
   The Separation Principle

4. RELATIONS TO MIN. VAR. CONTROL
   Internal vs external descriptions
   Matrices vs rational functions
   Riccati Equations vs spectral factorizability

5. APPLICATIONS

2. LINEAR QUADRATIC CONTROL

$$x(t + 1) = A\,x(t) + B\,u(t) + v(t)$$
$$y(t) = C\,x(t) + e(t)$$

$$J = \min E\left[x^T(t_1)\, Q_0\, x(t_1) + \sum x^T(t)\, Q_1\, x(t) + u^T(t)\, Q_2\, u(t) \right]$$
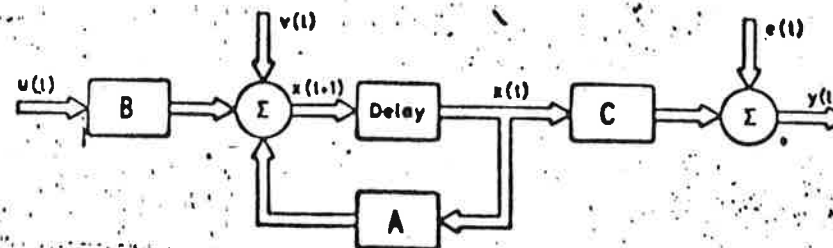
TWO SETS OF HYPOTHESES

▷ {e(t)} {v(t)}   NORMAL RANDOM PROCESSES
   u(t) AN ARBITRARY FUNCTION OF y(t − 1), y(t − 2), ...

▷ {e(t)} {v(t)}   SECOND ORDER RANDOM PROCESSES
   u(t) A LINEAR FUNCTION OF y(t − 1), y(t − 2), ...

KALMAN FILTERING

$$X(t+1) = A X(t) + v(t) \qquad (+ F(y_t))$$
$$y(t) = C x(t) + e(t)$$

$\begin{Bmatrix} v(t) \\ e(t) \end{Bmatrix}$ independent gaussian $\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \right)$

$x(t_0)$ gaussian (m, R_0)

Th. (Kalman 1960)

The conditional distribution of $x(t+1)$
given Y_t is gaussian $[\hat{x}(t+1), P(t+1)]$
where

$$\hat{x}(t+1) = A\hat{x}(t) + K(t)[y(t) - C\hat{x}(t)] \quad (+F(y_t))$$

$$K(t) = A P(t) C^T [C P(t) C^T + R_2]^{-1}$$

$$P(t+1) = A P(t) A^T + R_1 - A P(t) C^T [C P(t) C^T + R_2]^{-1} C P(t) A^T$$

$$= [A - K(t)C] P(t) A^T + R_1$$

$$= [A - K(t)C] P(t) [A - K(t)C] + R_1 + K(t) R_2 K^T(t)$$

$$P(t_0) = R_0 , \quad \hat{x}(t_0) = m$$

Proof.

$\begin{bmatrix} x \\ y \end{bmatrix}$ gaussian $\begin{bmatrix} m_x \\ m_y \end{bmatrix} \begin{bmatrix} R_x & R_{xy} \\ R_{yx} & R_y \end{bmatrix}$

$\Rightarrow E\{x|y\} = m_x + R_{xy} R_y^{-1} (y - m_y)$

(left margin, rotated text): firms, ... independent, ... computations, sufficient stat., statistical base, Practical base, Asymptotic properties

---

INNOVATIONS REPRESENTATION
OF $\{y(t)\}$

$$\begin{cases} x(t+1) = A x(t) + v(t) \\ y(t) = C x(t) + e(t) \end{cases}$$

Introduce

$$E(t) = y(t) - C\hat{x}(t) = y(t) - \hat{y}(t)$$

$E(t)$ is independent of $E(s)$ for $t \neq s$

$$\hat{x}(t+1) = A\hat{x}(t) + K[y(t) - C\hat{x}(t)]$$

$$\begin{cases} \hat{x}(t+1) = A\hat{x}(t) + K E(t) \\ y(t) = C\hat{x}(t) + E(t) \end{cases}$$

Alternative representation for $\{y(t)\}$

$$E\, E^2(t) = R_2 + C P(t) C^T$$

DUALITY

$$x(t+1) = Ax(t) + v(t)$$
$$y(t) = Cx(t) + e(t)$$

(arrow labeled R_1)
(R_2)

Predict $a^T x(t_1+1)$ linearly from $y(t_1), \ldots, y(t_0)$

$$a^T x(t_1+1) = -\sum_{t_0}^{t_1} u^T(t) y(t) + b^T m$$

Introduce

$$z(t) = A^T z(t+1) + C^T u(t+1), \quad z(t_1) = a$$

$$a^T x(t_1+1) = z^T(t_1) x(t_1+1) = z^T(t_0-1) x(t_0)$$
$$+ \sum_{t_0}^{t_1} [z^T(t) x(t+1) - z^T(t-1) x(t)]$$

$$= z^T(t_0-1) x(t_0) + \sum_{t_0}^{t_1} z^T(t) v(t) - u^T(t) Cx(t)$$

$$a^T x(t_1+1) - a^T \hat{x}(t_1+1) = z^T(t_0-1) x(t_0) - b^T m$$
$$+ \sum_{t_0}^{t_1} z^T(t) v(t) + u^T(t) \underbrace{[y(t) - Cx(t)]}_{e(t)}$$

$$E\{a^T x(t_1+1) - \hat{x}(t_1+1)\}^2 = z^T(t_0-1) R_0 z(t_0-1)$$
$$+ \sum_{t_0}^{t_1} z^T(t) R_1 z(t) + u^T(t) R_2 u(t)$$

$$x(t+1) = Ax(t) + Bu(t) + v(t) \qquad x(t_0) \in N(m, R_0)$$
$$y(t) = Cx(t) + e(t)$$

$$J = E\left\{ x^T(N) Q_0 x(N) + \sum_{t_0}^{N-1} x^T(s) Q_1 x(s) + u^T Q_2 u(s) \right\}$$

$u(t) =$ fcn of $y(t-1), \ldots$ [measurable w.r.t y_{t-1}

THEOREM (SEPARATION THEOREM)

Let the difference equation

$$S(t) = A^T S(t+1) A + Q_1 - A^T S(t+1) B [Q_2 + B^T S(t+1) B]^{-1} B^T S(t+1) A$$
$$S(N) = Q_0$$

have a solution which is positive semidef't
for $t_0 \le t \le N$. Let $L(t)$ be defined by

$$L(t) = [Q_2 + B^T S(t+1) B]^{-1} B^T S(t+1) A$$

then the control law

$$u(t) = -L(t) E[x(t)|y_{t-1}] = -L(t) \hat{x}(t|t-1)$$

minimizes J. The minimum is
given by

$$\min J = m^T S(t_0) m + tr\, S(t_0) R_0 + \sum_{t=t_0}^{N-1} tr\, S(t+1) R_1$$
$$+ \sum_{t=t_0}^{N-1} tr\, P(t) L^T(t) B^T S(t+1) A$$

LEMMA

Assume that the difference equation

$$S(t) = A^T S(t+1) A + Q_1 - A^T S(t+1) B [Q_2 + B^T S(t+1) B]^{-1} B^T S(t+1) A$$

$$S(N) = Q_0$$

has a solution which is pos. semi def.

for $t_0 \leq t \leq N$. Then

$$J = x^T(N) Q_0 x(N) + \sum_{t=t_0}^{N-1} x^T(t) Q_1 x(t) + u^T(t) Q_2 u(t)$$

$$u(t) = -L E[x(t) | y_{t \leq N}]$$

$$= x^T(t_0) S(t_0) x(t_0) + \sum_{t=t_0}^{N-1} [u(t) + L x(t)]^T [Q_2 + B^T S(t+1) B][u(t) + L x(t)]$$

$$+ \sum_{t=t_0}^{N-1} \left\{ v^T(t) S(t+1) [A x(t) + B u(t)] + \right.$$

$$\left. [A x(t) + B u(t)]^T S(t+1) v(t) + v^T(t) S(t+1) v(t) \right\}$$

Proof:

$$x^T(N) Q_0 x(N) = x^T(N) S(N) x(N) \equiv x^T(t_0) S(t_0) x(t_0)$$

$$+ \sum_{t=t_0}^{N-1} \left] x^T(t+1) S(t+1) x(t+1) = x^T(t) S(t) x(t) \right]$$

STRUCTURE OF FEEDBACK
GIVEN BY SEPARATION THEOREM
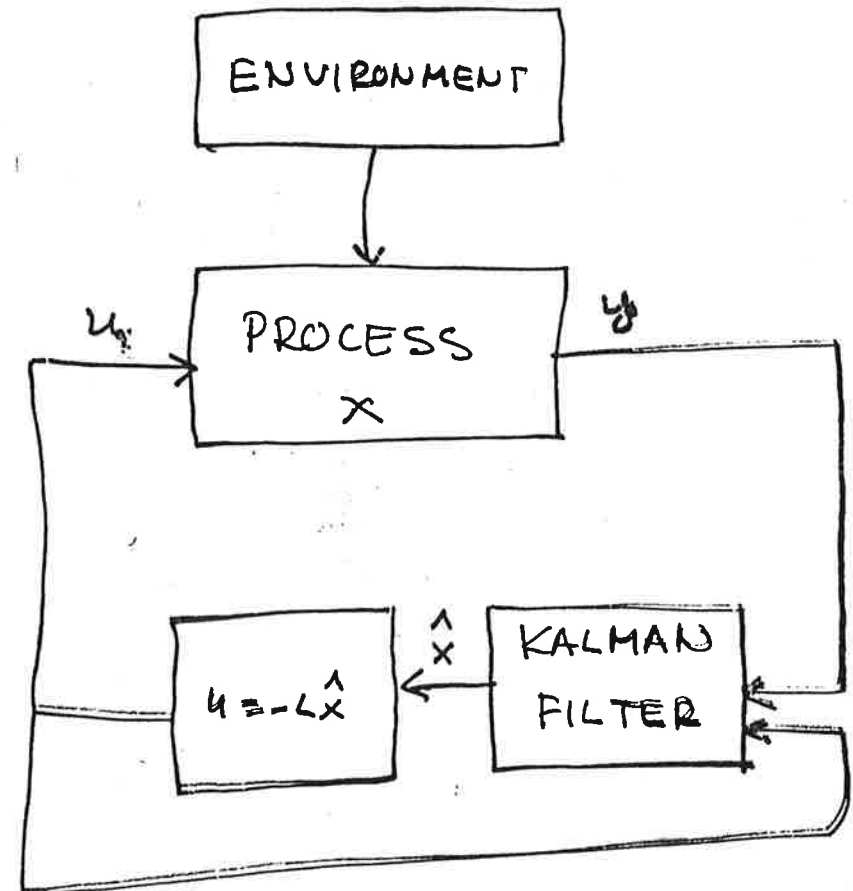
$$\begin{cases} \hat{x}(t+1) = A \hat{x}(t) + B u(t) + K [y(t) - C \hat{x}(t)] \\ u(t) = -L \hat{x}(t) \end{cases}$$

SEPARATION THEOREM   CERTAINTY EQUIVALENCE PRINCIPLE

$x(t+1) = A x(t) + B u(t) + v(t)$

$y(t) = C x(t) + e(t)$

$J = \min E \left\{ x^T(t_1) Q_0 x(t_1) + \sum_{t=t_0}^{t_1-1} \left(x^T(t) Q_1 x(t) + u^T(t) Q_2 u(t) \right) \right\}$

$u(t) = - L \hat{x}(t)$

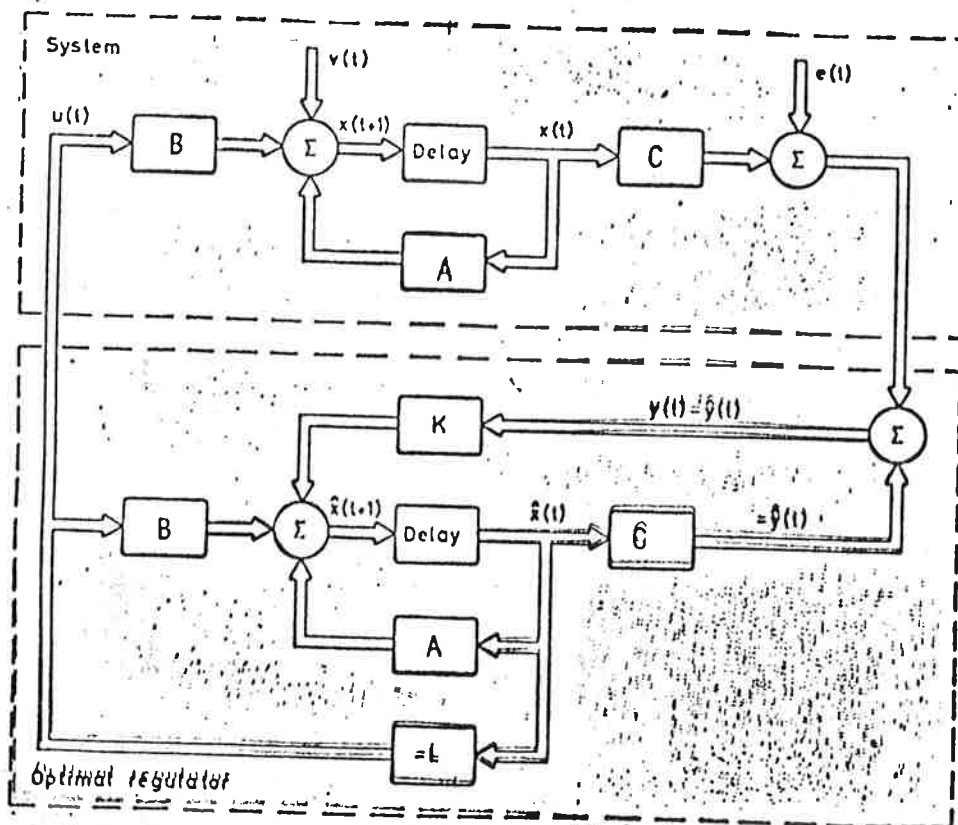$\hat{x}(t+1) = A \hat{x}(t) + B u(t) + K [y(t) - C \hat{x}(t)]$

Optimal regulator

3. A SPECIAL CASE

$$x(t+1) = \begin{bmatrix} -a_1 & 1 \ldots 0 \\ -a_2 & 0 \ldots 0 \\ \vdots & \\ -a_{n-1} & 0 \ldots 1 \\ -a_n & 0 \ldots 0 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} u(t) + \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n-1} \\ k_n \end{bmatrix} e(t)$$

$y(t) = x_1(t) + e(t)$

$J = \min E \left\{ \dfrac{1}{N} \sum_{t=1}^{N} y^2(t) \right\} \longrightarrow \min E\, y^2(t)$

ONE STEP OPTIMIZATION THE SAME AS N STEP OPTIMIZATION

$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) = b_1 u(t-1) + \ldots + b_n u(t-n) + \lambda [e(t) + c_1 e(t-1) + \ldots + c_n e(t-$

$A(q)\, y(t) = B(q)\, u(t-k) + \lambda\, C(q)\, e(t)$

$q\, y(t) = y(t+1)$
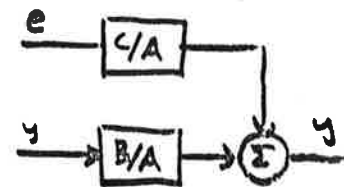
EXAMPLE

$y(t) + a y(t-1) = b u(t-1) + \lambda [e(t) + c e(t-1)]$

$u(t) = \dfrac{a - c}{b}\, y(t)$

EXAMPLE

$$x(t+1) = \begin{bmatrix} -a & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

$$y(t+1) + a y(t) = b_1 u(t) + b_2 u(t-1)$$

$$J = \sum_1^N y^2(t) \quad \min \quad N \to \infty$$

$$\begin{cases} S(t) = A^T S(t+1) A^T + Q_1 - A^T S(t+1) B [Q_2 + B^T S(t+1) B]^{-1} \overbrace{B^T S(t+1) A} \\ S(N) = Q_1 \end{cases}$$

Solution 1

$$S(t=\infty) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad, \quad L = \frac{1}{b_1}[-a, 1]$$

Solution 2

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix}$$

$$S_1 = 1 + \frac{a^2(b_2^2 - b_1^2)}{(ab_1 - b_2)^2}$$

$$S_2 = -\frac{a(b_2^2 - b_1^2)}{(ab_1 - b_2)^2}$$

$$S_3 = \frac{b_2^2 - b_1^2}{(ab_1 - b_2)^2}$$

$$L = \frac{ab_2 - b_1}{b_2(ab_1 - b_2)}[-a, 1]$$

LQG EXTERNAL (INPUT-OUTPUT MODELS

GIVEN THE MODEL

$$A(q^{-1}) y(t) = B(q^{-1}) u(t) + C(q^{-1}) e(t)$$

FIND CONTROL WHICH MINIMIZ

$$J = \lim_{N \to \infty} \frac{1}{N} \sum_1^N \left[y^2(t) + \rho u^2(t) \right]$$

1. SOLVE FACTORIZATION P

$$P \tilde{P} = \rho A \tilde{A} + B \tilde{B}$$

2. SOLVE DIOPHANTINE EQUATION

$$AR + BS = CP$$

3. OPTIMAL CONTROL LAW IS

$$R u(t) = C y_{ref}(t) - S y(t)$$

EXAMPLE SHIP STEERING

SWAY VELOCITY v
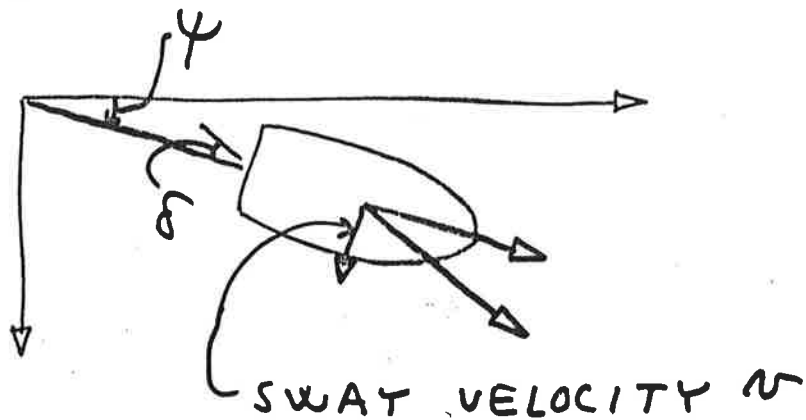
$$\frac{d}{dt}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix}\delta$$

$$\frac{\Delta R}{R} = k[\bar{\psi}^2 + \lambda\bar{\delta}^2]$$

TANKER $\qquad k = 0.014, \quad \lambda = 0.1$

$$J = \lim_{T \to \infty} \frac{1}{T}\int_0^T [\psi^2(t) + \lambda\delta^2(t)]\, dt$$

STOCHASTIC CONTROL THEORY AND APPLICATIONS

MODELING AND IDENTIFICATION

1. INTRODUCTION

2. CRITERIA

3. DYNAMICAL SYSTEMS

4. MODEL STRUCTURES

5. ESTIMATION THEORY

6. INTERACTIVE COMPUTING

7. EXAMPLE

8. CONCLUSIONS

IDENTIFICATION
MODELING

THEORY

MATH. MODEL

PROCESS
ENVIRONMENT
CRITERIA

⇓

CONTROL LAW

REAL WORLD

PROCESS

REGULATOR

IMPLEMENTATION
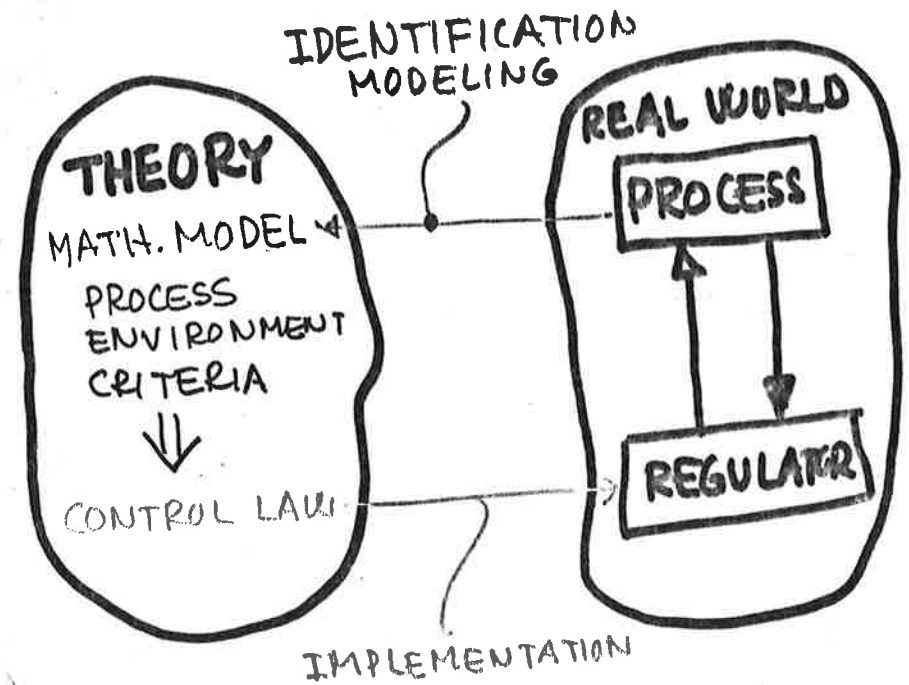
CLASSICAL:

TRANSFER FUNCTION
IMPULSE & FREQUENCY RESPONSE

"MODERN":

PARAMETRIC STATE SPACE MODELS

LEAST SQUARES
MAXIMUM LIKELIHOOD

TRADE EXPERIMENTAL SIMPLICITY
FOR COMPUTATIONS!

ASSESSMENT OF NEED FOR RETUNING OF MINIMUM VARIANCE CONTROL

LOG CONTROLLED OUTPUT DURING NORMAL OPERATION
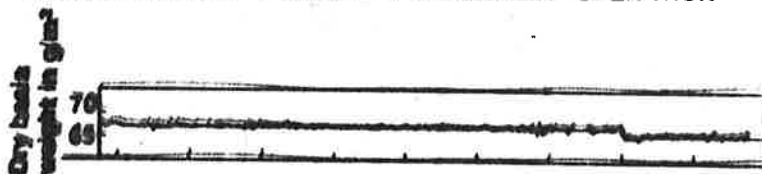
**CALCULATE COVARIANCE FUNCTION OF OUTPUT
(COV y)**
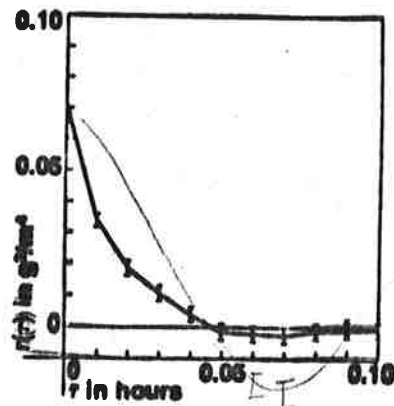
USE KNOWLEDGE OF PROCESS DEAD TIME AND SAMPLING PERIOD TO TEST IF COVARIANCE FUNCTION SATISFIES MINIMUM VARIANCE CONDITIONS

MOTIVATION

PROCESS MODELLING

DESIGN OF CONTROL LAWS

Ex: Given the system

$$x(t+1) = Ax(t) + Bu(t) + V(t)$$
$$y(t) = Cx(t) + e(t)$$

Find control which minimizes

$$E \sum_{t=1}^{N} x^T(t)Q_1 x(t) + u^T(t)Q_2 u(t) \}$$

THEORIA
MOTVS CORPORVM
COELESTIVM

IN

SECTIONIBVS CONICIS SOLEM AMBIENTIVM

AVCTORE

CAROLO FRIDERICO GAVSS.

GÖTTINGEN 1809
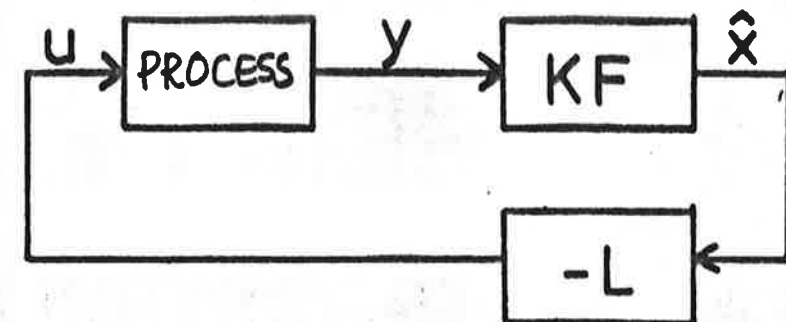
The probability of the errors

$$\Omega = h^{\mu} \pi^{-\frac{1}{2}\mu} e^{-hh(vv+v'v'+v''v''+\ldots)}$$

must become a minimum.

"Therefore, that will be the most probable system of values of the unknown quantities p, q, r, s, etc., in which the sum of the squares of the differences between the observed and computed values of the functions V, V', V'', etc. is a minimum, ..."

PRINCIPLE OF LEAST SQUARES

"In conclusion, the <u>principle</u> that the sum of the squares of the differences between the observed and computed quantities must be a minimum may, in the following manner, be <u>considered independently of the calculus of probabilities.</u>"

"Denoting the differences between observation and calculation by Δ, Δ', Δ'', etc., the first condition will be satisfied not only if $\Delta\Delta + \Delta'\Delta' + \Delta''\Delta'' +$ etc., is a minimum (which is our principle), but also if $\Delta^4 + \Delta'^4 + \Delta''^4 +$ etc., or $\Delta^6 + \Delta'^6 + \Delta''^6 +$ etc., or in general, if the sum of any of the powers with an even exponent becomes a minimum. But of all these principles <u>ours is the most simple;</u> by the others we should be led into the most complicated calculations."

THE MAXIMUM LIKELIHOOD PRINCIPLE   Fisher 1912

RULE

Let Y be a random variable with probability density $p(y,\theta)$. To estimate θ from an observation y choose θ such that

$$L(y,\hat{\theta}) \geq L(y,\theta) \quad \forall \theta$$

where L is the likelihood function defined by $L(y,\theta) = p(y,\theta)$.

INDEPENDENT SAMPLES

$$L(y_1, y_2, \dots, y_n, \theta) = p(y_1,\theta)\, p(y_2,\theta) \dots p(y_n,\theta)$$

PROPERTIES

Consistency

Asymptotic normality

Efficiency

THE LIKELIHOOD FUNCTION

INTRODUCE

$$y_{t_k}^T = \left[y^T(t_0)\ y^T(t_1) \dots y^T(t_k) \right]$$

USING BAYES RULE THE LIKELIHOOD FUNCTION BECOMES

$$L = p(y_{t_k}) = p(y(t_k)|y_{t_{k-1}})\, p(y_{t_{k-1}})$$

$$p(y(t_k)|y_{t_{k-1}})\, p(y(t_{k-1})|y_{t_{k-2}}) \dots p(y(t_1)|y(t_0))\, p(y(t_0))$$

FOR LINEAR GAUSSIAN PROBLEMS

$$p(y(t_k)|y_{t_{k-1}}) = N(\hat{y}(t_k|t_{k-1}),\ R(t_k))$$

$$= (1/2)(2\pi)^{-m/2}(\det R(t_k))^{-1/2} \exp -(1/2)\varepsilon^T(t_k)R^{-1}(t_k)\varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k)$$

THE LIKELIHOOD FUNCTION CAN THEN BE WRITTEN AS

$$-\log L = (1/2)\left[\sum \log \det R(t_k) + \sum \varepsilon^T(t_k)R^{-1}(t_k)\varepsilon(t_k)\right] + const.$$

NOTICE RELATIONS TO FILTERING THEORY !

INTERPRETATION FOR NON GAUSSIAN PROCESSES

OTHER PREDICTION ERROR CRITERIA

ML:

$$V(\theta) = -\log L = \frac{1}{2}\sum_{k=1}^{N}\log\det R(t_k) + \frac{1}{2}m_y N \log 2\pi$$

$$+ \frac{1}{2}\sum_{k=1}^{N}\varepsilon^T(t_k) R^{-1}(t_k)\varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$
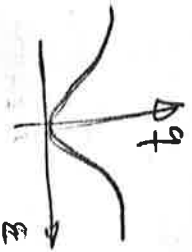
MORE GENERAL

$$V(\theta) = g\,(G(\theta))$$

$$G(\theta) = \sum_{k=1}^{N} F(\varepsilon(t_k), \theta, t_k)$$

LONGER PREDICTION HORIZON

$$V(\theta) = g\,(G_1(\theta), G_2(\theta), \ldots, G_s(\theta))$$

$$G_i(\theta) = \sum_{k=1}^{N} F_i(\varepsilon(t_k|t_{k-i}), \theta, t_k)$$

$$\varepsilon(t_k|t_{k-i}) = y(t_k) - \hat{y}(t_k|t_{k-i})$$

PREDICTION ERROR INTERPRETATION

Notice that the ML-criterion gives a loss function N of the form

$$V(\theta) = \sum_{i=1}^{N} q(\varepsilon(t_k))$$

where

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$

is the prediction error.

Alternative: Postulate prediction model and error criterion

- Dynamical systems
- Consequences for modeling
- Compare Gauss

NOTICE q QUADRATIC FOR GAUSSIAN DISTURBANCES

ROBUSTNESS

ESTIMATING PARAMETERS OF DYNAMICAL SYSTEMS

Properties of the ML-estimate

The minimization problem

Computational aspects

How to obtain the likelihood function

Example

$$\dot{x} = Ax + Bu + v$$

$$y(t_k) = Cx(t_k) + e(t_k)$$

R_1

R_2

THE LIKELIHOOD FUNCTION

INTRODUCE

$$y^T_{t_k} = \left| y^T(t_0) \; y^T(t_1) \ldots y^T(t_k) \right|$$

USING BAYES RULE THE LIKELIHOOD FUNCTION BECOMES

$$L = p(y_{t_k}) = p(y(t_k)|y_{t_{k-1}}) \, p(y_{t_{k-1}})$$

$$p(y(t_k)|y_{t_{k-1}}) \, p(y(t_{k-1})|y_{t_{k-2}}) \ldots p(y(t_1)|y(t_0)) \, p(y(t_0))$$

FOR LINEAR GAUSSIAN PROBLEMS

$$p(y(t_k)|y_{t_{k-1}}) = N(\hat{y}(t_k|t_{k-1}), \; R(t_k))$$

$$= (1/2)(2\pi)^{-m/2}(\det R(t_k))^{-1/2} \exp -(1/2) \, \varepsilon^T(t_k) R^{-1}(t_k)\varepsilon(t_k)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k)$$

THE LIKELIHOOD FUNCTION CAN THEN BE WRITTEN AS

$$-\log L = (1/2) \left| \sum \log \det R(t_k) + \sum \varepsilon^T(t_k) R^{-1}(t_k)\varepsilon(t_k) \right| + \text{const.}$$

NOTICE RELATIONS TO FILTERING THEORY !

INTERPRETATION FOR NON GAUSSIAN PROCESSES

EXAMPLE

$$\dot{x} = Ax + Bu + v$$
$$y(t_k) = Cx(t_k) + e(t_k)$$

THE KALMAN BUCY THEORY GIVES:

$$\hat{y}(t_k|t_{k-1}) = C\hat{x}(t_k|t_{k-1})$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$

$$R(t_k) = R_2 + C P(t_k|t_{k-1}) C^T$$

$$\hat{x}(t_k|t_k) = \hat{x}(t_k|t_{k-1}) + K(t_k) \cdot \varepsilon(t_k)$$

$$K(t_k) = P(t_k|t_{k-1}) C^T R^{-1}(t_k)$$
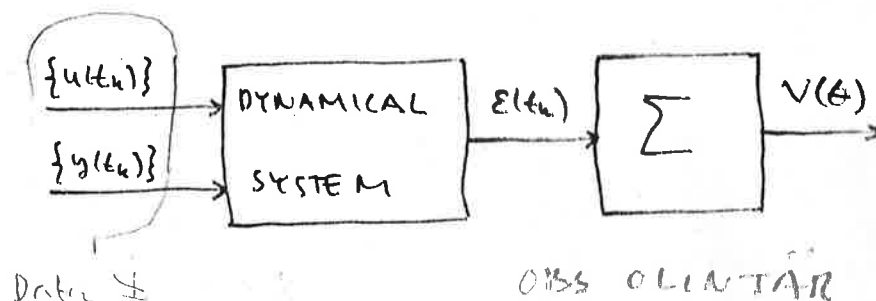
$$P(t_k|t_k) = P(t_k|t_{k-1}) - K(t_k) C P(t_k|t_{k-1})$$

$$\frac{d\hat{x}(t|t_k)}{dt} = A\hat{x}(t|t_k) + Bu(t) \qquad t_k \leq t \leq t_{k-1}$$

$$\frac{dP(t|t_k)}{dt} = AP(t|t_n) + P(t|t_k)A^T + R_1 \qquad t_k \leq t \leq t_{k+1}$$

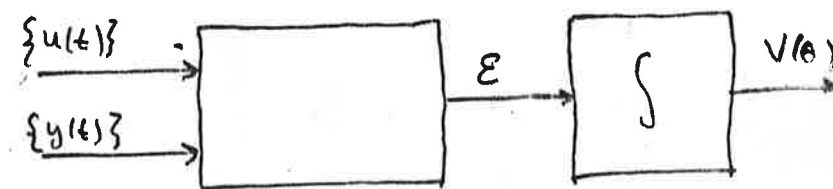**THE LIKELIHOOD FUNCTION**

$$(-2\log L)_{t_k} = (-2\log L)_{t_{k-1}} + \varepsilon^T(t_k) R^{-1}(t_k) e(t_k) + \log \det R(t_k)$$

NOTICE THE STRUCTURE OF THE LIKELIHOOD FUNCTION

Data $\mathcal{L}$ OBS OLINTÄR

CONTINUOUS TIME DATA

$$\frac{dz}{dt} = F(z, u(t), y(t), t) = G(z, t)$$

$$\varepsilon = H(z, t)$$

$$V(\theta) = \frac{1}{2} \int_0^T \kappa(\varepsilon, t, \theta) \, dt$$

COMPUTATIONAL ASPECTS

What must be done?

Minimization algorithms $\longrightarrow$ FUNCTION EVALUATION
GRADIENT   -"-
HESSIAN   -"-

Simplifications

constant sampling rate

special model structures

EXAMPLE $\varepsilon_{k+1} - \varepsilon_k - 1$

$x(t+1) = A x(t) + B u(t) + K \varepsilon(t)$

$y(t) = C x(t) + \varepsilon(t)$

$-2 \log L = \sum\limits_{1}^{N} \varepsilon^T(t) R^{-1} \underline{\varepsilon(t)} + N \log \det R + C$

MINIMIZE   **W.R.T.**   **R!**

$-2 \log L = N \log \det \frac{1}{N} \sum\limits_{1}^{N} \varepsilon^T(t) \varepsilon(t) + r N + const.$

$\begin{cases} x(t+1) = A x(t) + B u(t) + K [y(t) - C x(t)] \\ \varepsilon(t) = y(t) - C x(t) \end{cases}$

$\hat{R} = \frac{1}{N} \sum\limits_{1}^{N} \varepsilon(t) \varepsilon^T(t)$

EXAMPLE (ARMAX MODEL)

$$y(t) + a_1 y(t-1) + \ldots + a_n y(t-n) =$$

$$= b_1 u(t-1) + \ldots + b_n u(t-n) +$$

$$+ \lambda \left(e(t) + c_1 e(t-1) + \ldots + c_n e(t-n) \right)$$
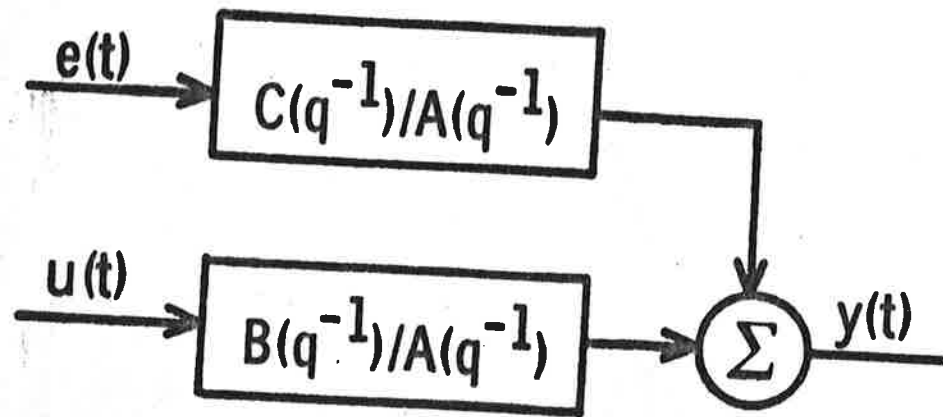
$$A(q^{-1}) y(t) = B(q^{-1}) u(t) + C(q^{-1}) e(t)$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_n q^{-n}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_n q^{-n}$$

MINIMIZATION

$$-\log L = \frac{1}{\lambda} V(\theta) + \frac{N}{2} \log \lambda + const$$

$$V(\theta) = \frac{1}{2} \sum_{t=1}^{N} e^2(t)$$

$$C(q^{-1}) \varepsilon(t) = A(q^{-1}) y(t) - B(q^{-1}) u(t)$$

$$\theta^{k+1} = \theta^k - \left[V_{\theta\theta}(\theta^k) \right]^{-1} V_\theta(\theta^k)$$

$$V_\theta = \sum_{t=1}^{N} \varepsilon(t) \varepsilon_\theta(t)$$

$$V_{\theta\theta} = \sum_{t=1}^{N} \varepsilon_\theta(t) \varepsilon_\theta(t) + \sum_{t=1}^{N} \varepsilon(t) \varepsilon_{\theta\theta}(t)$$

$$C(q^{-1}) \frac{\partial \varepsilon(t)}{\partial a_i} = y(t-i)$$

$$C(q^{-1}) \frac{\partial \varepsilon(t)}{\partial b_i} = -u(t-1)$$

$$C(q^{-1}) \frac{\partial \varepsilon(t)}{\partial c_i} = -\varepsilon(t-1)$$

THE ARMAX MODEL

CANONICAL FORM FOR LINEAR TIME INVARIANT SYSTEM WHOSE DYNAMIS IS RATIONAL TRANSFER FUNCTION + TIME DELAY

DISTURBANCES ARE STATIONARY WITH RATIONAL SPECTRAL DENSITY

CAN BE EXTENDED TO MISO?

$$Ay = B_1 u_1 + B_2 u_2 + \dots + B_r u_r + Ce$$

USING ADJOINT VARIABLES TO CALCULATE GRADIENTS

$$\frac{dx}{dt} = f(x, \theta, t)$$

$$V(\theta) = \int_0^T g(x, s)\, ds$$

$$V_\theta(\theta) = \int_0^T g_x x_\theta\, ds \quad = -\int_0^T p^T(s) f_\theta\, ds$$

$$\begin{cases} \dfrac{dp}{dt} = -\left(\dfrac{\partial f}{\partial x}\right)^T p + g_x^T \\[2mm] p(T) = 0 \end{cases}$$

PROOF:

$$\frac{dx_\theta}{dt} = f_x x_\theta + f_\theta$$

$$V_\theta = \int_0^T [g_x x_\theta + p^T \dot{x}_\theta - p^T f_x x_\theta - p^T f_\theta]\, ds$$

$$= p^T x_\theta \Big]_0^T + \int_0^T [g_x x_\theta - \dot{p}^T x_\theta - p^T f_x x_\theta - p^T f_\theta]\, ds$$

$$= p^T x_\theta \Big]_0^T - \int_0^T [g_x - p^T f_x - \dot{p}^T] x_\theta - \int_0^T p^T f_\theta\, ds$$

NONLINEAR MODELS

$$\frac{d\hat{x}(t|t_k)}{dt} = f\left(\hat{x}(t|t_k), u(t)\right)$$

$$\hat{y}(t_k|t_{k-1}) = g\left(\hat{x}(t_k|t_{k-1})\right) +$$

$$\hat{x}(t_k|t_k) = h\left(\hat{x}(t_k|t_{k-1}), \varepsilon(t_k)\right)$$

$$\varepsilon(t_k) = y(t_k) - \hat{y}(t_k|t_{k-1})$$

MODEL STRUCTURES

$$dx = Ax\,dt + Bu\,dt + dw$$

$$dy = Cx\,dt + de \leftarrow R_2 \quad (R_1)$$

$$y(t) + A_1 y(t-1) + \ldots + A_n y(t-n) =$$
$$= B_1 u(t) + \ldots + B_n u(t-n) + e(t) + C_1 e(t-1) + \ldots + C_n e(t-n)$$

$$y(t) = H(s) u(t) + G(s) e(t)$$

ESTIMATION THEORY

HOW WILL THE METHODS WORK
UNDER IDEAL CIRCUMSTANCES

HOW ARE THE RESULTS INFLUENCED
BY DIFFERENT CHOICES OF THE
PROBLEM ELEMENTS $\mathcal{D}$, $\mathcal{M}$, $\mathcal{C}$

CLASSICAL STATISTICS

CONSISTENCY

ASYMPTOTIC DISTRIBUTIONS

EFFICIENCY

GENERAL COMMENT ON RESULTS

LARGE SAMPLE PROPERTIES $N \to \infty$

CHARACTER OF RESULTS

NOTIONS

$\mathcal{D}$ DATA GENERATED FROM M_0

$\mathcal{M}$ MODEL SET

$\mathcal{C}$ CRITERIA

INTRODUCE

$$W(\theta) = \lim_{N \to \infty} \frac{1}{N} V_N(\theta) = \left[-\lim_{N \to \infty} \frac{1}{N} \log L(\theta, y_N) \right]$$

SHOW UNIFORM CONVERGENCE
(ERGODIC THEOREMS OR MARTINGALE THEOREMS)

ANALYSE $W(\theta)$ FIND θ_0 WHICH

MINIMIZES $W(\theta)$

UNDER GENERAL BUT MESSY CONDITIONS

$$\hat{\theta}_N \to \theta_0$$

AVAILABLE COMMANDS IDPAC

1. INPUT & OUTPUT
   CONV, EDIT, MOVE, LIST
2. DISPLAY
   PLOT, BODE, PLMAG, FHEAD
3. DATA OPERATIONS
   INSI, CUT, CONC, PICK, SLIDE, STAT
   SCLOP, VECOP, TREND, ACOF, CCOF
4. FREQUENCY RESPONSE
   FROP, ASPEC, CSPEC, SPTRF
5. SIMULATION & MODEL ANALYSIS
   FILT, DSIM, DETER, RESID, RAMPA
6. IDENTIFICATION
   ML, STRUC, SQR, LS
   ;
9. MISC
   DELETE, TURN

AVAILABLE COMMANDS INTRAC

MACRO NAME [FARG1 FARG2 ...]
FORML FARG1 [FARG2 ...]
END
EXEC 'ON'/'OFF'
LABEL LNAM
GO TO LNAM
IF ARG1 RELOP ARG2 GOTO LAM
FOR COUNT = BEGIN TO FINISH
                    [STEP INCR]
NEXT COUNT
SWITCH
LET VAR1 = ARG1 [OP ARG2]
READ VAR1 TYPE1 [VAR2 TYPE2]
WRITE [DEV] [STRING1/ARG1] ...
FREE
STOP

LS

$$A(q^{-1}) y(t) = B_1(q^{-1}) u_1(t-k_1) + \ldots + e(t)$$

$$y(t) - \phi(t)\theta = e(t)$$

$$Y - \phi\theta = E \quad \Rightarrow \quad [\phi \ Y]\begin{bmatrix} -\theta \\ 1 \end{bmatrix} = E$$

$$Q[\phi \ Y] = \begin{bmatrix} R \\ 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} r_{11} & r_{12} & \ldots & r_{1n+1} \\ 0 & r_{22} & \ldots & r_{2n+1} \\ \vdots & & & \\ 0 & & & r_{n\,n+1} \\ 0 & \ldots & 0 & r_{n+1\,n+1} \end{bmatrix} \begin{bmatrix} -\theta_1 \\ -\theta_2 \\ -\theta_n \\ 1 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_n \\ e_{n+1} \end{bmatrix}$$

SQR RMAT ← DATA STRF

LS [(SW)] SYST [(NAME)] ← STRF

SUBCOMMANDS

    SAVE STDEV

    SAVE COMAT

ML

IDENTIFICATION OF MODEL

$$A(q^{-1}) y(t) = B_1(q^{-1}) u_1(t) + \ldots + B_m(q^{-1}) u_m(t)$$
$$+ \lambda C(q^{-1}) e(t)$$

ML [(SW)] SYST [(NAME)] ← DATA NO

SUBCOMMANDS

    INVAL ABC/C SYST [(NAME)]

    FIX A(1) [VAL] ...

    SAVE [COMAT] [STDEV] [GRAD] [EVALS]

    EXIT

PARAMETER ESTIMATES

    -"-   UNCERTAINTIES

         LOSS

AKAIKE $AIC = -2 \cdot \log ML + 2 p$

CREATION OF NEW COMMAND

```
> MACRO LSID MODEL ← DATA N
> STRUC   STRF
      > NA    MAX N
      > NU    MAX 1
      > NB    MAX N      @
> SQR RMAT ← DATA STRF
> DELET MODEL
> LS MODEL ← STRF
> END
```

THE NEW COMMAND CAN BE
USED AS FOLLOWS

```
      LSID ADAM ← DATA 2.
```

GENERATION OF COMMAND
WITH A LOT OF GUIDANCE

```
> EDIT FILTR.
    NOT FOUND: FILTR
    INPUT
MACRO FILTR.
WRITE (TP) 'ENTER NAME OF FILTER'
READ FILTN NAME
WRITE (TP) 'WHAT TYPE (HP, BP, LP)
            IS 'FILTN' ?'
READ FTYP NAME
WRITE (TP) 'WHAT FILTER ORDER?'
READ N INT
IF FTYP EQ BP GOTO L1
:
LABEL L1
WRITE (TP) 'ENTER LOW AND
    HIGH CUT-OFF FREQUENCIES (RAD/S)'
READ LCF  REAL HCF REAL
FILT FILTN ← FTYP N LCF HCF
END
    EDIT
```

```
> FILTR
  ENTER NAME OF FILTER.
  TEST
  WHAT TYPE (HP, BP, LP) IS TEST?
  BP
  WHAT FILTER ORDER?
  6
  ENTER LOW AND HIGH CUT-OFF
  FREQUENCIES (RAD/S)
   5   30
>
```

PROGRAM DETAILS

52 SUBROUTINES 9200 STATEMENTS

64k UNIVAC 1108

25k USING SEGMENTATION

OPTIMIZATION ALGORITHMS

    Quasi-Newton Fletcher

    Brent

APPLICATIONS

    SHIP STEERING DYNAMICS

    INSULIN KINETICS

    GLUCOSE KINETICS

    ECONOMETRICS

CONCLUSIONS

o COMMANDS THAT ARE
   NATURAL FROM THEORY

o NEW COMMANDS CAN
   EASILY BE CREATED
   E.G. 2LS GLS ETC

• FLEXIBILITY

o SPECIAL PURPOSE TOOLS CAN
   EASILY BE IMPLEMENTED

• NOT ALL PROBLEMS ARE
   SUITABLE FOR INTERACTION

• COPY OF DIALOGUE

• DIRECT DOCUMENTATION

o "IDENTIFICATION LANGUAGE"

EXAMPLE ARMAX

SCENARIO : EXPLORATORY PHASE

$$A(q)\,y(t) = B(q)\,u(t) + \lambda\,C(q)\,e(t)$$

$$A(q) = (q^2 - 1.95q + 0.96)(q - 0.9)$$
$$\omega h = 0.1,\ \xi = 0.2$$

$$B(q) = q^2 + q + 1$$

$$C(q) = q^2 + 0.7q + 0.2$$

PLOT (30) YS/CHP) U
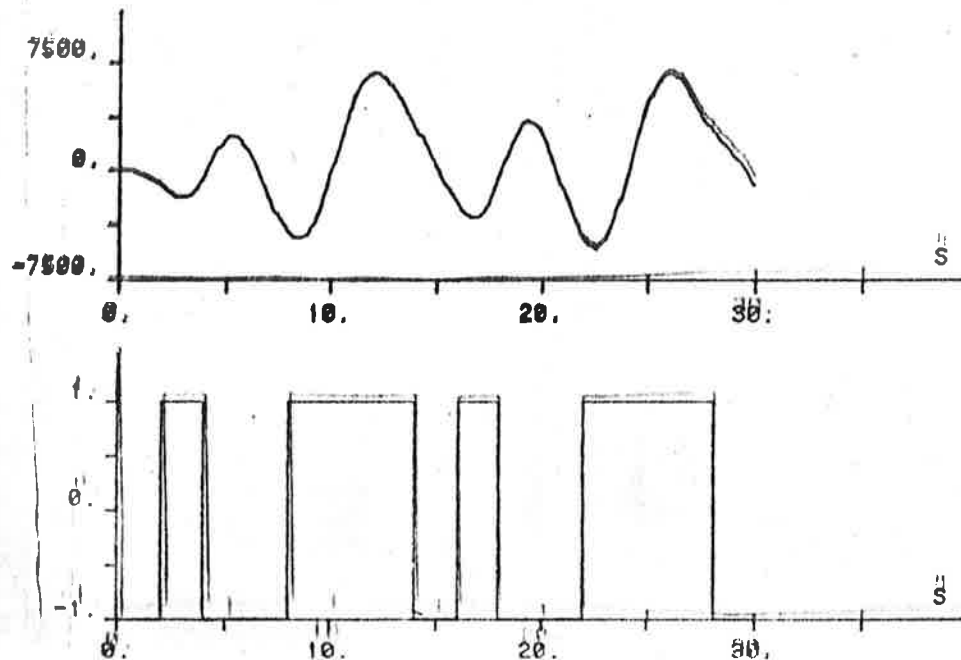
INTERACTIVE COMPUTING
IDPAC

MOVE WRK(1) ← U
MOVE WRK(2) ← YS
ML P1 ← WRK 1
ML P2 ← WRK 2
ML P3 ← WRK 3
ML P4 ← WRK 4

n	V	AIC	A	B	C
1	2.4E6	3768	-0.995	62.1	1.00
2	172.8	1597	-1.979	4.90	1.66
			0.985	4.37	1.79
3	139.3	(847)	-2.851	1.06	0.72
			2.717	0.81	0.20
			-0.865	1.05	0.03
4	198.0	850			

NOISEFREE OUTPUT
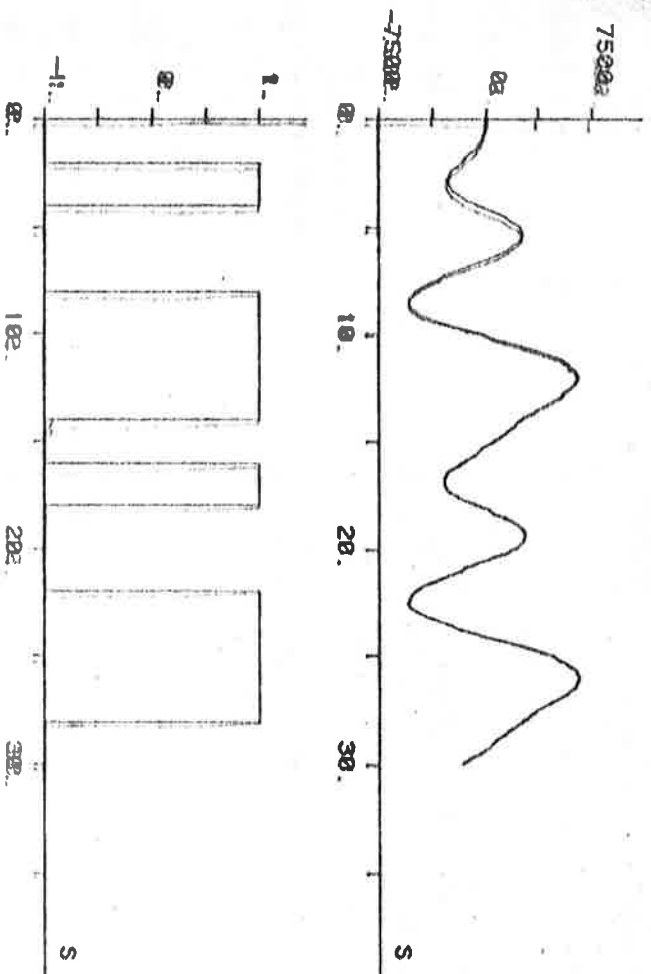
MEASURED DATA
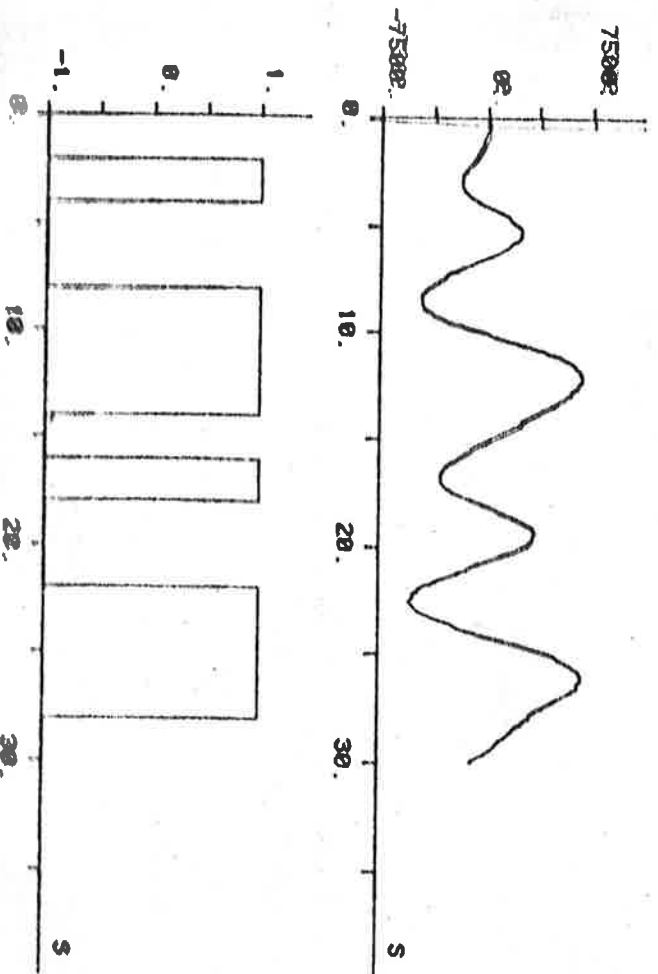
CALCULATION OF RESIDUALS & CORRELATIONS
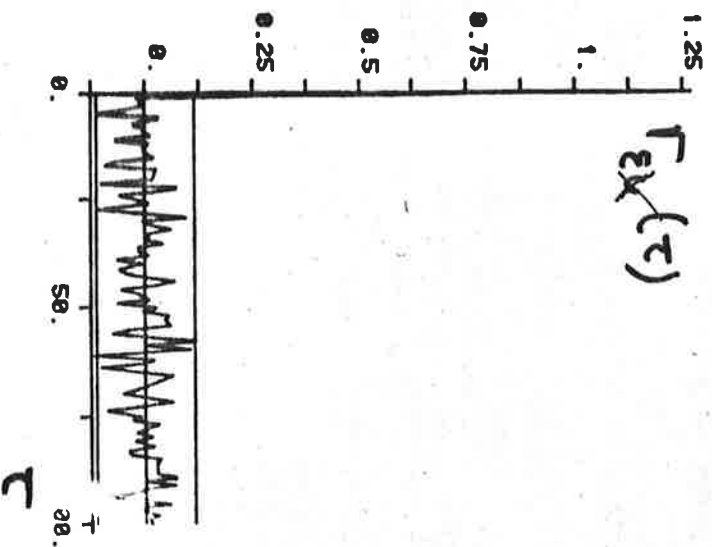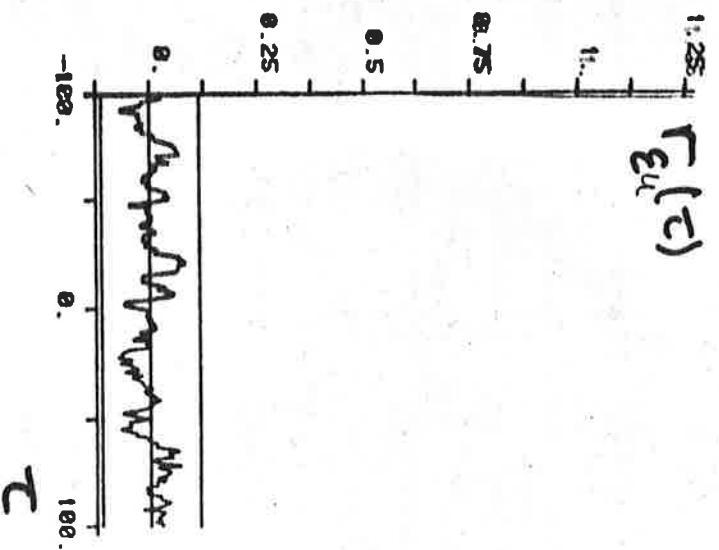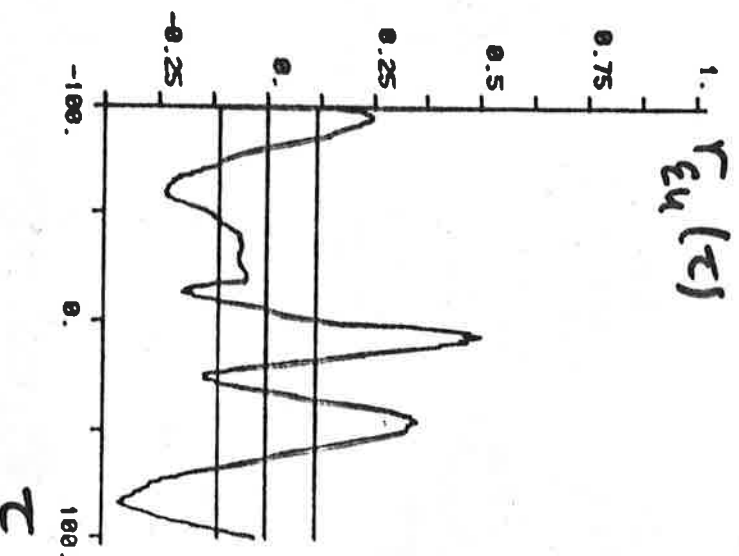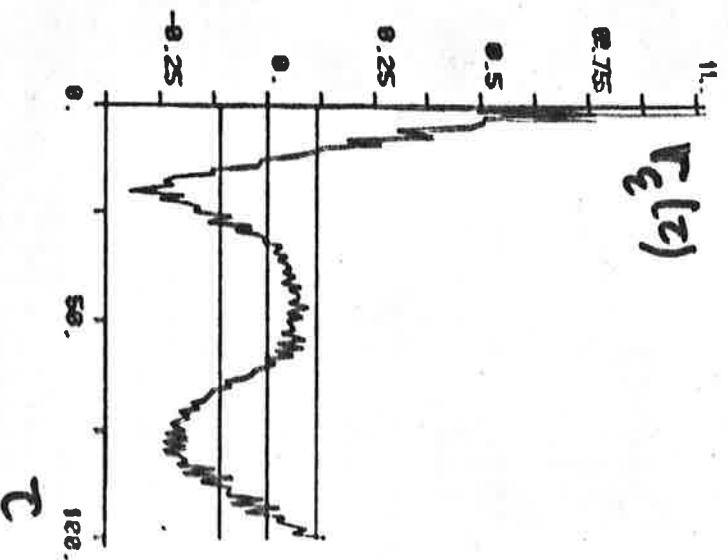
EXAMPLE OF MODEL VALIDATION

RESID E1 ← PI WR K 100
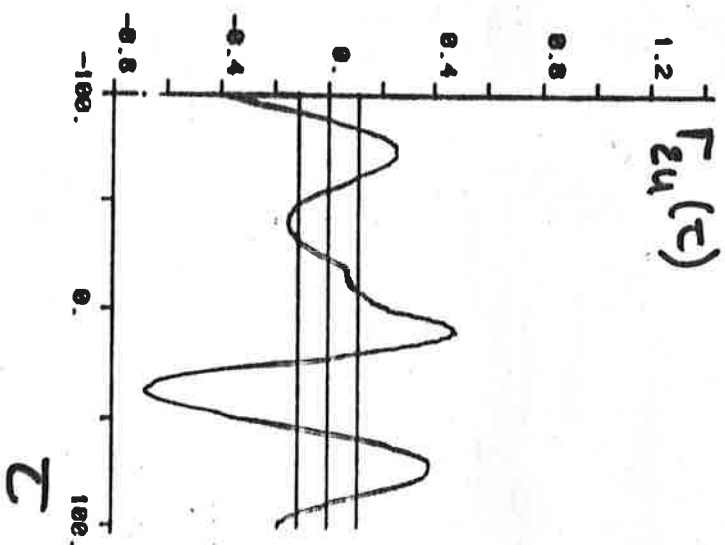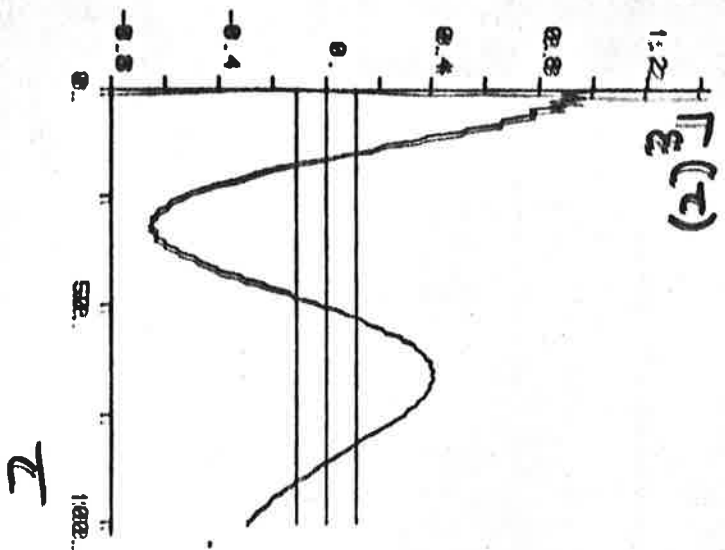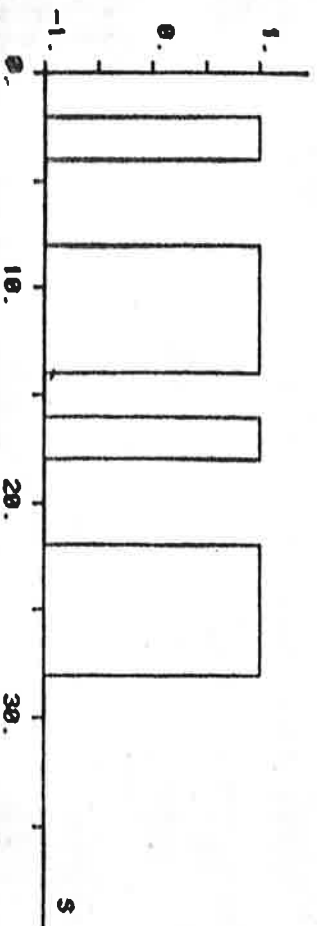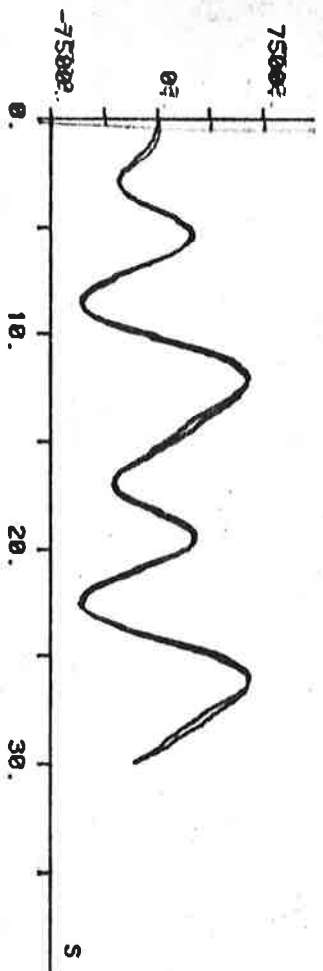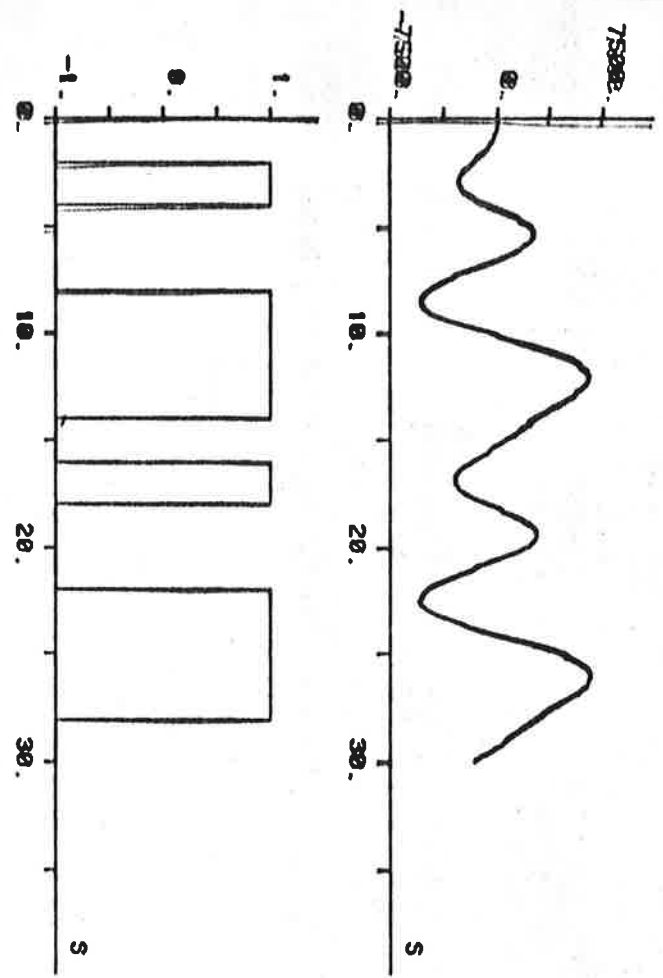
TIME & COST

300 INPUT/OUTPUT PAIRS

GENERATION OF DATA

PARAMETER ESTIMATION

PLOTTING OF CURVES

1.5 h TIME AT TERMINAL

TOTAL COST 82 skr ≈ 27 YUAN

SPECIAL FEATURES OF ML & PRED. ERR.

- ❀ GREAT FLEXIBILITY WRT MODEL STRUCTURE

- ❀ DISTURBANCES ARE MODELED

- ❀ GREAT FLEXIBILITY WRT PARAMETRIZATION.

  "PHYSICAL" PARAMETERS & CONTINUOUS TIME MODELS CAN BE USED

- ❀ THEORETICALLY REASONABLY WELL UNDERSTOOD

- ❀ WILL OFTEN REQUIRE SUBSTANTIAL CALCULATIONS

PRACTICAL EXPERIENCES

PAPER MACHINES
DRUM BOILERS
DISTILLATION COLUMNS
NUCLEAR REACTORS
ACTIVATED SLUDGE PROCESSES
SHIP STEERING DYNAMICS
THERMAL HEAT CONDUCTION
MACROECONOMICS
PHARMACOKINETICS
INSULIN KINETICS

WHERE DOES ML & PE FIT INTO THE MODELING WORK?

- EXPLORATORY PHASE ASSUME A CANONICAL MISO MODEL. FIT TO DATA AND TEST?

- FINAL PARAMETER ESTIMATION PHASE. ASSUME PHYSICAL MODEL WITH ALL AVAILABLE INFORMATION. FIT PARAMETERS AND VALIDATE?

STOCHASTIC CONTROL THEORY AND APPLICATIONS

EXPERIENCES OF SYSTEM IDENTIFICATION
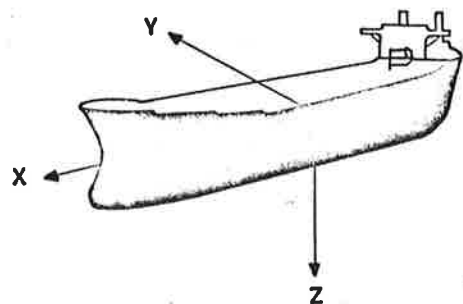# APPLIED TO SHIP STEERING DYNAMICS

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(SSPA)

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(LTH)

1. INTRODUCTION

2. SHIP STEERING DYNAMICS

3. IDENTIFICATION METHODS

4. EXPERIMENTS

5. INTERESTING OBSERVATIONS

6. CONCLUSIONS

- ARE IDENTIFICATION METHODS OF ANY USE FOR DETERMINING SHIP STEERING DYNAMICS ?

- HOW CAN IDENTIFICATION METHODS BE USED IN AN EXERCISE IN DYNAMICAL MODELING ?

- WHAT IDENTIFICATION METHODS ARE APPROPRIATE ?

EXPERIMENTS

	69	70	71	72	73	74	75	76	77	78	79
	▽ Atlantic Song			▽ Sea Splendour	▽ Sea Scout	▽ Sea Swift		▽ Sea Stratus			

ARMAX MODELS

STATE SPACE MODELS:

LINEAR   ▽ the Hague

NONLINEAR   ▽ Tbilisi   ▽ Darmstadt

SIX DEGREES OF FREEDOM:

SURGE, SWAY, HEAVE

ROLL, PITCH, YAW
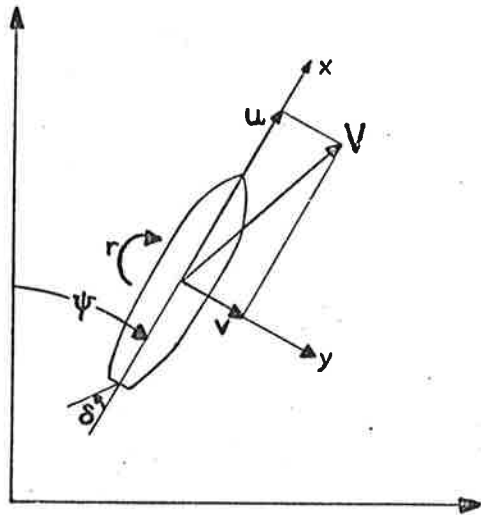
SENSOR AND ACTUATOR DYNAMICS

DISTURBANCES

WIND, WAVES, CURRENTS

DIFFICULTIES

SIMPLIFICATIONS

SEPARATION

NORMALIZATION

EQUATIONS OF MOTION:

$$m(\dot{u} - vr - x_G r^2) \quad = X + X_{disturbance}$$

$$m(\dot{v} + ur + x_G \dot{r}) \quad = Y + Y_{disturbance}$$

$$I_z \dot{r} + m x_G (\dot{v} + ur) \quad = N + N_{disturbance}$$

THE HYDRODYNAMIC FORCES X AND Y
AND THE TORQUE N ARE FUNCTIONS
OF THE MOTION, E.G.:

$$X = X(u, v, r, \delta, \dot{u}, \dot{v}, \dot{r})$$

EQUATIONS OF MOTION:

$$\begin{bmatrix} m - Y_{\dot{v}} & m x_G - Y_{\dot{r}} \\ m x_G - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - m u_o \\ N_v & N_r - m x_G u_o \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \end{bmatrix} \delta$$

STATE EQUATIONS:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & o \\ a_{21} & a_{22} & o \\ o & I & o \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ o \end{bmatrix} \delta$$

TRANSFER FUNCTIONS:

$$G_{\psi\delta}(s) = K \frac{(s + 1/T_3)}{s(s + 1/T_1)(s + 1/T_2)} \approx$$

$$\approx \frac{K}{s(s + 1/T)} \quad \text{(Nomoto's model)}$$

$$G_{v\delta}(s) = K_v \frac{(s + 1/T_{3v})}{(s + 1/T_1)(s + 1/T_2)}$$

MODEL STRUCTURES:

DISCRETE TIME INPUT - OUTPUT (ARMAX)
MODELS [IDPAC]:

$$y(t) + a_1 y(t-1) + ... + a_n y(t-n) = b_1 u(t-k-1) + ... + + b_n u(t-k-n) + \lambda [e(t) + c_1 e(t-1) + ... + c_n e(t-n)]$$

CONTINUOUS TIME STRUCTURAL
MODELS [LISPID]:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{23} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix} \delta$$

CRITERIA:

OUTPUT ERROR
MAXIMUM LIKELIHOOD
PREDICTION ERROR

$$\sum_i \sum_k y^T(t_i + \tau_k) Q y(t_i + \tau_k)$$

PARAMETER ESTIMATION

PROBLEM:
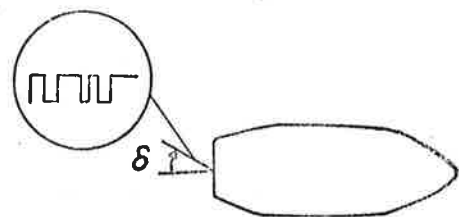
CONSIDER THE MODEL

$$dx = Ax\, dt + Bu\, dt + dw$$

WHERE THE OUTPUT

$$y(t_k) = Cx(t_k) + Du(t_k) + e(t_k)$$

IS MEASURED AT DISCRETE TIMES

$$t_k, \quad k = 0, 1, ..., N$$

DETERMINE THE PARAMETERS
OF THE MODEL!

HEADING ψ
YAW RATE r
SWAY VELOCITY v

SIMPLE EQUIPMENT:

COMPASS
MANUAL EXCITATION AND LOGGING

COMPLEX EQUIPMENT:

COMPASS, RATE GYRO, DOPPLER SONAR,
INERTIAL NAVIGATION
COMPUTER - CONTROLLED EXCITATION
AND LOGGING

25 EXPERIMENT CORRESPONDING TO
18 HOURS PERFORMED WITH
12 SHIPS HAVE BEEN ANALYSED

TANKERS:	SEA SPLENDOUR
	SEA SCOUT
	SEA SWIFT
	SEA STRATUS
	SEA SCAPE
	AK FERNSTRÖM
	NORSEMAN
	THORSHAMMER
CARGO SHIPS:	ATLANTIC SONG
	COMPASS ISLAND
FERRY:	BORE I
NAVAL CRAFT:	HIGH-SPEED PATROL BOAT

7 FREE-SAILING TESTS WITH
2 SCALE MODELS HAVE ALSO BEEN
ANALYSED