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When using digital control it is useful to relate the parameters in the discrete time algorithm to well-known concepts. This helps in discussion with industrialists. It is also useful in order to introduce meaningful constraints on the parameters. This note gives an interpretation of the parameters in a control algorithm which is used in many self-tuners. The interpretation suggests several different parameterization of the algorithm.

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PARAMETERIZATION OF A SPECIAL DIGITAL CONTROL ALGORITHM

Karl Johan Åström

Abstract:

When using digital control it is useful to relate the parameters in the discrete time algorithm to well-known concepts. This helps in discussion with industrialists. It is also useful in order to introduce meaningful constraints on the parameters. This note gives an interpretation of the parameters in a control algorithm which is used in many self-tuners. The interpretation suggests several different parameterization of the algorithm.

1. THE ALGORITHM

The discrete control algorithm is described by the recursive equation

$$\Delta u(t) = s_0 e(t)+s_1 e(t-h)-r_1 \Delta u(t-h)-\dots-r_n \Delta u(t-nh) \tag{1}$$

where e is the output error, u is the control signal and Δ is the difference operator.

By introducing z-transforms the regulator (1) can also be written as

$$U(z) = \frac{s_0+s_1 z^{-1}}{1-z^{-1}} E(z) - (r_1 z^{-1}+\dots+r_n z^{-n})U(z).$$

The algorithm is useful for controlling stable systems with time delays. The algorithm gives proportional and integral control and it provides dead-time compensation as was suggested by Smith (1959).

2. A SPECIAL CASE

It is easy to give an interpretation of the control law (1) if $r_i = 0$, $i = 1,2,\dots,n$. The pulse transfer function of the regulator is

$$H(z) = \frac{U(z)}{E(z)} = \frac{s_0+s_1 z^{-1}}{1-z^{-1}}.$$

This can be rewritten as

$$H(z) = \frac{s_0-s_0 z^{-1} + (s_0+s_1)z^{-1}}{1-z^{-1}} = s_0 \left[1 + \frac{s_0+s_1}{s_0 h} \cdot \frac{hz^{-1}}{1-z^{-1}}\right]. \tag{2}$$

This shows that the algorithm can be interpreted as a PI regulator with the gain

$$k = s_0 \tag{3}$$

and the integration time

$$T_i = \frac{hs_0}{s_0+s_1} \tag{4}$$

where h is the sampling period.

3. THE GENERAL CASE

To give an interpretation of the algorithm (1) in the general case when $r_i \neq 0$ it will be compared with a dead-time compensator of the type proposed by Smith (1959). In the continuous time case such a regulator is characterized by the equation

$$U = G_R \left[E - G_P (1-e^{-sT}) U \right] \tag{5}$$

where E and U are the Laplace transforms of the error signal and the control signal, G_P is the process transfer function, G_R is the regulator transfer function and T is the time delay.

In the discrete time case Smith's compensator is instead characterized by

$$U(z) = G_R(z)\{E(z) - [H(z) - H_T(z)]U(z)\} \tag{6}$$

where H_T is the pulse transfer function of the process with delay and H the corresponding transfer function without the delay.

The pulse transfer function can be expressed in terms of the step response. Let g be the step response of the process without the delay. Then

$$H(z) = \sum_{k=1}^{\infty} [g(kh) - g(kh-h)] z^{-k} . \tag{7}$$

Furthermore let d be the largest integer such that dh is less than the time delay T and let

$\tau = T - dh$

Then

$$H_T(z) = \sum_{k=1}^{\infty} [g(kh+\tau) - g(kh+\tau-h)] z^{-d-k} \tag{8}$$

The function

$$L(z) = H(z) - H_T(z) \tag{9}$$

then has the series expansion

$$L(z) = \sum_{k=1}^{\infty} \ell_k z^{-k}$$

where

$$\ell_k = \begin{cases} g(kh) - g(kh-h) & k = 1,2,\dots,d \\ g(dh+h) - g(dh) - g(h-\tau) & k = d+1 \\ g(kh) - g(kh-h) - g((k-d)h-\tau) & \\ +g((k-d-1)h-\tau) & k > d+1 \end{cases} \tag{10}$$

The first d coefficients are thus identical to the d first values of the pulse response of the process without delay. See Fig. 1 and Fig. 2. Notice that it follows from (9) that

$L(1) = 0$

The function L is thus divisible by $(1-z^{-1})$. Carrying out the division we get after simple calculations

$$\frac{L(z)}{1-z^{-1}} = \sum_{k=1}^{\infty} p_k z^{-k} \tag{11}$$

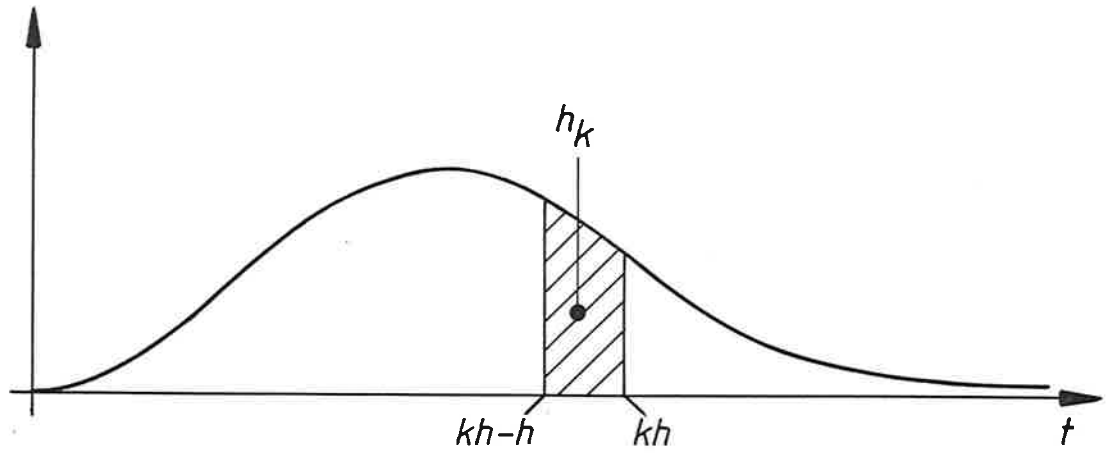


Fig. 1 Shows how the pulse response is obtained from the impulse response. The number h_k represents the shaded area.

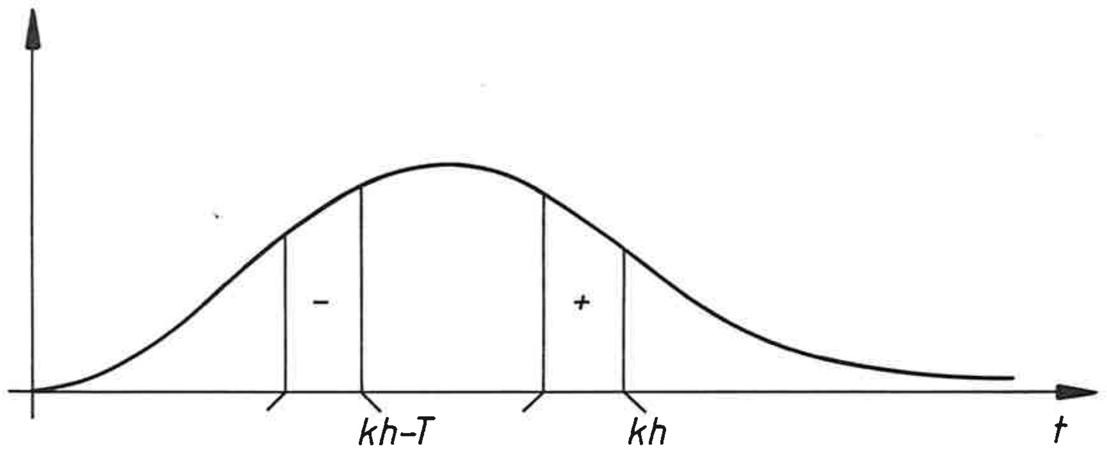


Fig. 2 Shows how the numbers ℓ_k are computed from the impulse response. The number ℓ_k is equal to the difference between the areas marked + and -.

where

$$p_k = \begin{cases} g(kh) & k = 1,2,\ldots,d \\ g(kh) - g((k-d)h-\tau) & k > d \end{cases} \tag{12}$$

The numbers p_k are easily obtained from the step response. See Fig. 3. It is clear that the numbers p_k will go to zero as k increases if the system is stable. The number of significant parameters will depend on the shape of the step response, the time delay and the sampling period. It is seen from Fig. 3 that it can easily happen that a large number of parameters may be needed.

In the case when the function L can be truncated after n terms we get

$$r_1 z^{-1} + r_2 z^{-2} + \ldots + r_n z^{-n} \approx \frac{(s_0 + s_1 z^{-1})}{1 - z^{-1}} (\ell_1 z^{-1} + \ldots + \ell_n z^{-n})$$

$$= (s_0 + s_1 z^{-1})(p_1 z^{-1} + \ldots + p_n z^{-n})$$

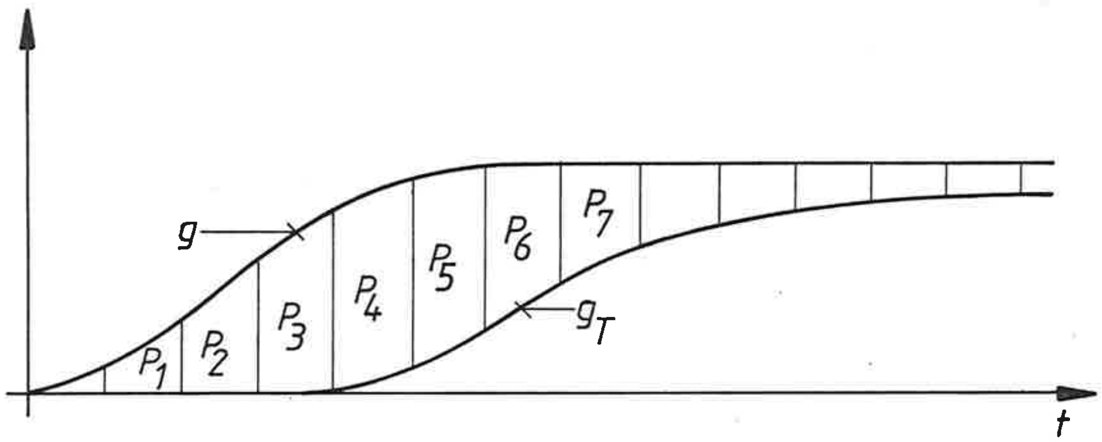


Fig. 3 Shows how the parameters p_k can be obtained from the step response.

Hence

$$
\begin{aligned}
r_1 &= s_0 p_1 \\
r_2 &= s_0 p_2 + s_1 p_1 \\
r_3 &= s_0 p_3 + s_1 p_2 \\
&\vdots \\
r_{n-1} &= s_0 p_{n-1} + s_1 p_{n-2} \\
r_n &= s_1 p_{n-1}
\end{aligned}
\tag{13}
$$

or

$$
\begin{aligned}
p_1 &= r_1/s_0 \\
p_2 &= (r_2 - s_1 \ell_1)/s_0 \\
&\vdots \\
p_{n-1} &= (r_{n-1} - s_1 \ell_{n-2})/s_0
\end{aligned}
\tag{14}
$$

It seems reasonable to approximate the function $H(z) - H_T(z)$ with a finite polynomial. The equations (2) and (6) are then identical and the following equivalence is obtained

$$G_R = \frac{s_0 + s_1 z^{-1}}{1 - z^{-1}}, \tag{15}$$

and

$$r_1 z^{-1} + r_2 z^{-2} + \ldots + r_n z^{-n} \approx \frac{(s_0 + s_1 z^{-1})}{1 - z^{-1}} [H(z) - H_T(z)]. \tag{16}$$

The control algorithm (1) can thus be interpreted as a PI regulator with dead time compensation. The gain and the integration time are given by (3) and (4). It follows from (16) that the coefficients r_i are related to both the process model and the regulator settings s_0 and s_1.

4. APPLICATIONS TO SELF-TUNERS

Assume that a self-tuning regulator is designed based on the algorithm (1). The coefficients given can then be interpreted in several different ways.

First observe that it may be useful to talk about gain and integration time in order to explain the behaviour of the self-tuner to instrument engineers. The corresponding formulas are given by (3) and (4). The interpretation of the coefficients can also be used to supervise the estimated parameters. In many cases it is possible to give reasonable values of the process gain. It is then straight forward both to give good starting values for s_0 and also limits to safeguard that the algorithm will not misbehave. The same arguments also apply to the integration time T_i. Prior information about T_i thus gives bounds on s_1.

From the interpretation of the algorithm (1) as a PI controller with dead time compensation, it was also possible to relate the coefficients r_i to the pulse response of the system. If bounds on the gain and time constants are available it is then also possible to give bounds on the parameters.

For the Smith compensator it follows from (5) that

$$u = \frac{G_R}{1+(G_p-e^{-sT}G_p)}\, e.$$

To guarantee that u is bounded the function

$$1 + (G_p-e^{sT}G_p)$$

must thus not have poles in the RHP. Similarly for the discrete time case the function

$$1 + G_R(H-H_T) \approx \frac{1+r_1z^{-1}+\dots+r_nz^{-n}}{1-z^{-1}}$$

must not have zeros outside the unit disc. This indicates that it could be useful to monitor the polynomial

$$R(z) = z^{n} + r_{1}z^{n-1} + \dots + r_{n}$$

and test for stability in a self-tuner.

Another interesting observation can also be made. It has been remarked that many parameters p_k may be needed. The number of parameters can be reduced if the control law is reparameterized as

$$\Delta u(t) = s_{0}e(t) + s_{1}e(t-1) - r'_{1}u(t-1) - \dots - r'_{n}u(t-n), \qquad (17)$$

where

$$r'_{k} = r_{k} - r_{k-1}. \qquad (18)$$

The relations between the sequences $\{r'_k\}$ and $\{r_k\}$ are illustrated in Fig. 4. Equation (16) suggests the following representation

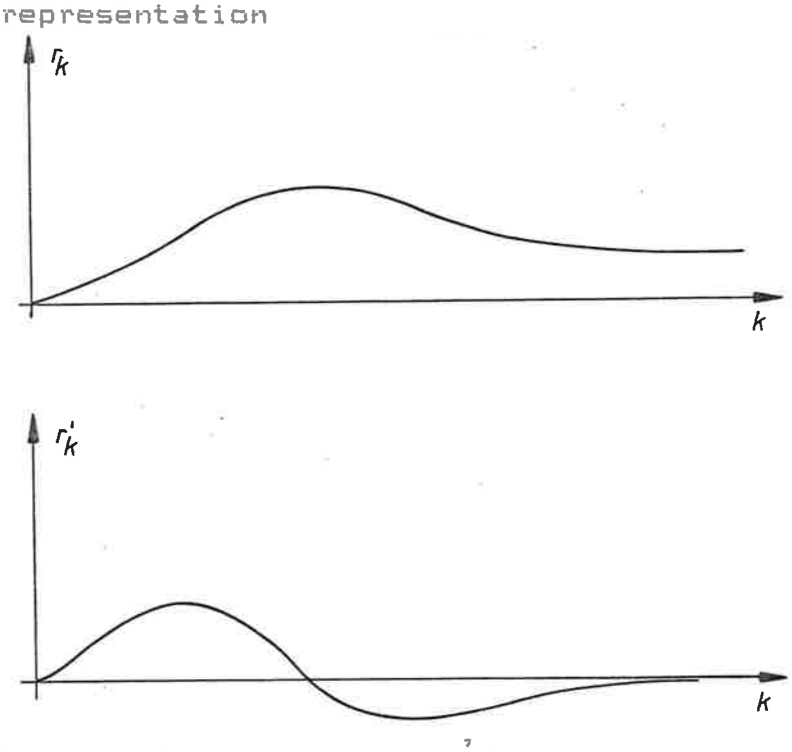


Fig. 4 The sequences $\{r'_k\}$ and $\{r_k\}$.

$$\Delta u(t) = (s_0 + s_1 q^{-1})[e(t) - r_1'' u(t-1) - \ldots - r_n'' u(t-n)].$$

With this parameterization the parameters r_k'' will not depend on the setting of the regulator problem but only on the process dynamics. It can thus be expected that they would be easier to estimate. From a comparison with the Smith predictor we can expect

$$r_n'' = \ell_n \qquad (19)$$

where ℓ_n are given by (10).

It also appears that apart from the first parameters of the sequence $\{\ell_n\}$ we could find other functionals of u to correlate with e.g.

$$\Delta u(t) = (s_0 + s_1 q^{-1})[e(t) - r_1'' u(t-h) - \ldots - r_d'' u(t-dh) - t_0 v(t)] \qquad (20)$$

where

$$v(t) = u(t-dh-h) + u(t-dh-2h) + \ldots . \qquad (21)$$

It would be interesting to explore if there are any differences between these structures.

5. CONCLUSIONS

A physical interpretation of the parameters in the control law (1) has been given. The control law may be regarded as a PI regulator with the gain

$$k = s_0,$$

and the integration time

$$T_i = \frac{hs_0}{s_0 + s_1}$$

combined with a Smith predictor to compensate for dead time. The parameters $r_1, \ldots, r_n$ are related to the dead time compensation.

The analysis also suggests two other parameterizations given by equations (19) and (20).

Conversely the parameter s_0 and s_1 may be expressed as follows

$$s_0 = k$$

$$s_1 = -k \frac{T_i - h}{h}$$

in terms of the gain and the integration time.

6. REFERENCE

Smith, O.J.M. (1959): A controller to overcome Dead Time. ISA Journal 6, 28-33.

Fig. 1

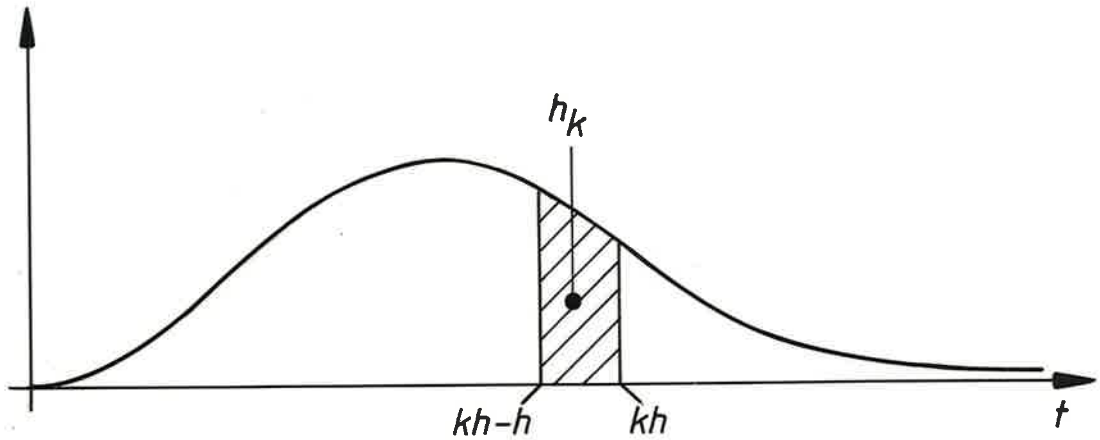
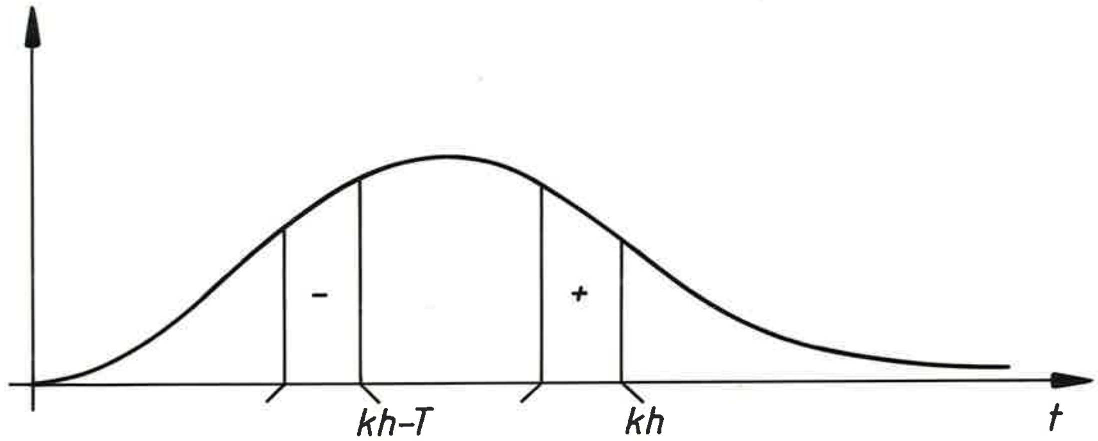


Fig 2



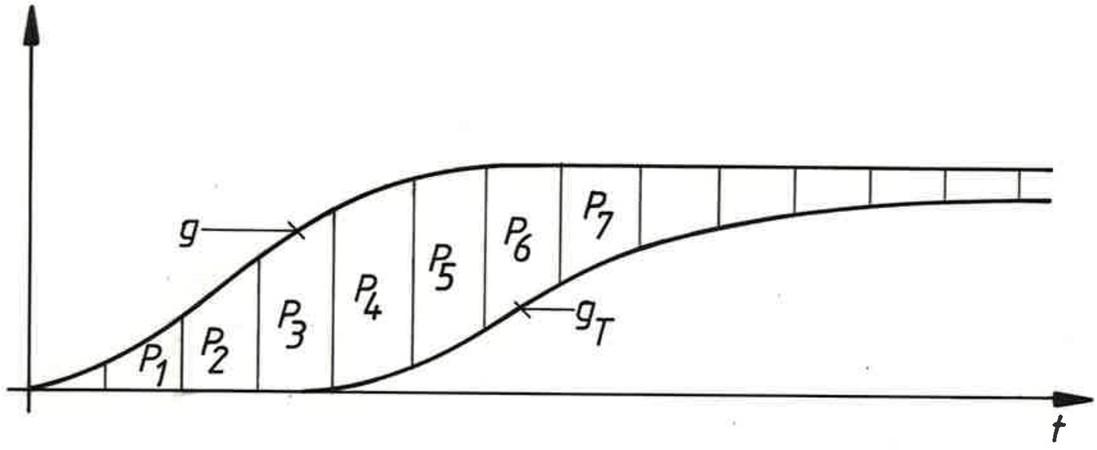


Fig. 3.

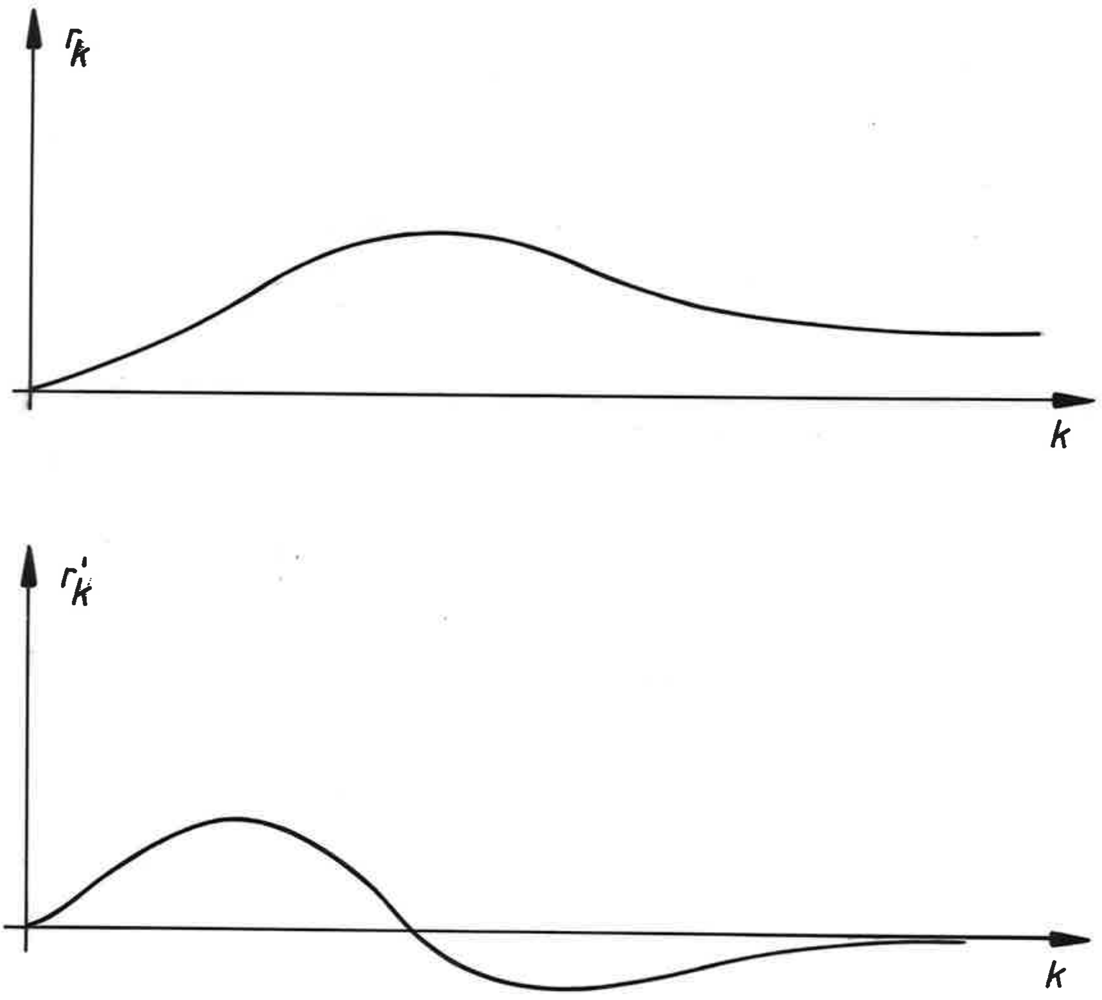


Fig 4