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A ROBUST SAMPLED REGULATOR FOR STABLE PROCESSES WITH MONOTONE STEP RESPONSES

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A robust sampled regulator for stable processes with monotone step responses.

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It is shown that a discrete time integrating regulator can always be designed based on a strongly simplified model so that the regulator gives good control for a stable system with monotone step response. The problem is a simple prototype for a general robustness problem.

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Contents

1. INTRODUCTION

It is useful to have simple robust methods for solving simple problems. This paper gives a very simple way of designing a robust digital regulator for stable linear single-input single-output systems with positive impulse responses. The basic idea is that a regulator is designed for a strongly simplified model of the process. The regulator obtained has two tuning parameters, a gain and the sampling period. It is shown that the regulator works for a given class of problems provided that a simple condition is satisfied. Linear time-invariant systems with monotone step responses occur commonly in the process industries. Typical examples are systems which describe flows, levels, concentrations, and temperatures. See e.g. Åström (1976). The regulator obtained has integral action. The settling time of the closed loop system is larger than the settling time of the open loop system.

2. THE ALGORITHM

Consider a stable system with a monotone step response H. See Figure 1.

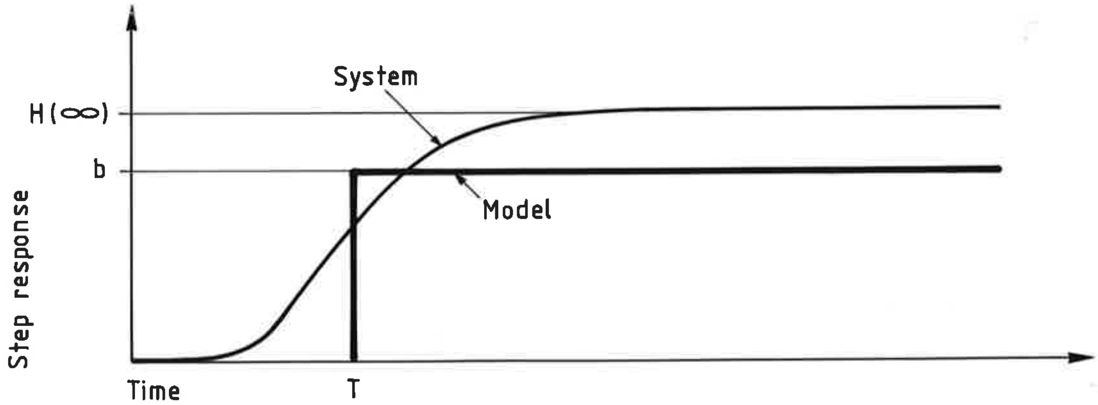

Fig. 1 - Step responses of the system and the approximating model.

The system is approximated with the following crude sampled model

$$y(t) = bu(t-T), \qquad b > 0 \tag{1}$$

where b is the process gain and T the sampling period. See Fig. 1. It is straightforward to derive a dead-beat regulator with integral action for the process (1) as follows. Equation (1) gives

$$y(t) - y(t-T) = b[u(t-T) - u(t-2T)].$$

Requiring that y(t) equals the desired reference signal y_r after one sampling period the following regulator is obtained

$$u(t) = [y_r - y(t)] / b + u(t-T) \tag{2}$$

provided that the model (1) is correct. The regulator (2) is an integrating regulator with dead-time compensation. It has two tuning parameters, b and T.

3. ANALYSIS

It will now be investigated what happens when the sampled regulator (2), which is designed from the simplified model (1), is applied to a real system. The following result holds.

Theorem 1. Consider a stable time invariant linear system with monotone nonnegative step response. The closed loop obtained with the regulator (2) is always stable if

$$2H(T) > H(\infty) \tag{3}$$

and

$$2b > H(\infty). \tag{4}$$

Proof. Let h be the impulse response of the system. Then

$$y(t) = \int_0^\infty h(s)\, u(t-s)\, ds.$$

Since the control signal u is constant over the sampling periods, it follows that

$$\begin{aligned} y(t) &= \int_0^T h(s)\, ds\, u(t-T) + \int_T^{2T} h(s)\, ds\, u(t-2T) + \ldots \\ &= H(T)\, u(t-T) + [H(2T) - H(T)]\, u(t-2T) + \ldots \end{aligned} \tag{5}$$

for $t = 0, \pm T, \pm 2T, \ldots$ The closed loop system is thus characterized by equations (2) and (5). Elimination of y between these two equations gives

$$\sum_{n=0}^{\infty} a_n\, u(t-nT) = y_r/b,$$

where

$$a_n = \begin{cases} 1 & n = 0 \\ [H(T) - b]/b & n = 1 \\ [H(nT) - H(nT-T)]/b & n > 1. \end{cases}$$

It follows from a theorem of Wiener (1933) that the closed loop system is asymptotically stable if the function

$$A(z) = \sum_{n=0}^{\infty} a_n z^{-n}$$

has the property

$$\inf |A(z)| > 1 \quad \text{for} \quad |z| \geq 1.$$

See also Desoer and Vidyasagar (1975). Since

$$\left| \sum_{n=1}^{\infty} a_n z^{-n} \right| \leq \sum_{n=1}^{\infty} |a_n| \quad \text{for} \quad |z| \geq 1$$

the system is thus stable if

$$\sum_{n=1}^{\infty} |a_n| < 1.$$

Consider

$$\sum_{1}^{\infty} |a_n| = \left| \frac{H(T) - b}{b} \right| + \frac{H(2T) - H(T)}{b} + \frac{H(3T) - H(2T)}{b} + \dots$$

$$= \left| \frac{H(T) - b}{b} \right| + \frac{H(\infty) - H(T)}{b}.$$

Two different cases are considered separately. First assume that $H(T) \geq b$. It then follows from (4) that

$$\sum_{1}^{\infty} |a_n| = \frac{H(\infty) - b}{b} < 1.$$

Next, assume that $H(T) < b$. Then it follows from (3) that

$$\sum_{1}^{\infty} |a_n| = 1 + \frac{H(\infty) - 2H(T)}{b} < 1$$

and the theorem is proven. □

The bounds (3) and (4) can not be improved as is seen by the following counterexamples. Consider a system described by

$$y(t) = b_0 \, u(t-T_0).$$

To satisfy (3) choose $T_0 \leq T$. At the sampling instants the closed loop system is then described by

$$u(t) = (1 - b_0/b) \, u(t-T).$$

Since $b_0 = H(\infty)$ this equation becomes unstable if (4) is violated.

Similarly, choose $b < b_0 < 2b$, which clearly satisfies (4). Assume that $T \lesssim T_0 < 2T$, which implies that (3) does not hold. At the sampling instants the closed loop system is described by

$$u(t) - u(t-T) + (b_0/b)\, u(t-2T) = 0,$$

which is unstable.

Based on the theorem the following simple design rule can be obtained for the regulator (2). Choose a sampling period such that $H(T) > 0.5\, H(\infty)$. Choose the integrator gain such that $1/b < 2H(\infty)$. It can be seen that the response time of the closed loop system will in general be slower than the response time of the open loop system. This is often acceptable in simple process control application.

The control law (2) can be interpreted as a special case of model algorithmic control (MAC). See Richalet et al (1978) and Mehra and Rouani (1979). Theorem 1 can therefore also be interpreted as a robustness result for MAC. Another approach to robustness for MAC is given in Mehra et al (1979). In MAC the process is modeled by an impulse response. It is therefore of interest to interprete the conditions of Theorem 1 in terms of impulse responses. The assumption that the step response is monotone implies that the impulse response is nonnegative. The model (1) implies that the process is modeled by an impulse at $t = T$. The inequality (3) implies that T is larger than the median of the impulse response and the inequality (4) implies that the area of the model impulse response is larger than half the area under the impulse response of the system.

4. AN EXAMPLE

The design procedure is illustrated by an example. Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^6} .$$

A step response of the process is shown in Fig. 2. It is seen from this step response that the condition (3) is satisfied for the sampling period T = 7.5. The parameter b was chosen to b = 1.5.

The response of the closed loop system to a step command signal and a step load disturbance at time 100 is shown in Fig. 3

The Simnon programs sued to generate the curves are given in the Appendix.

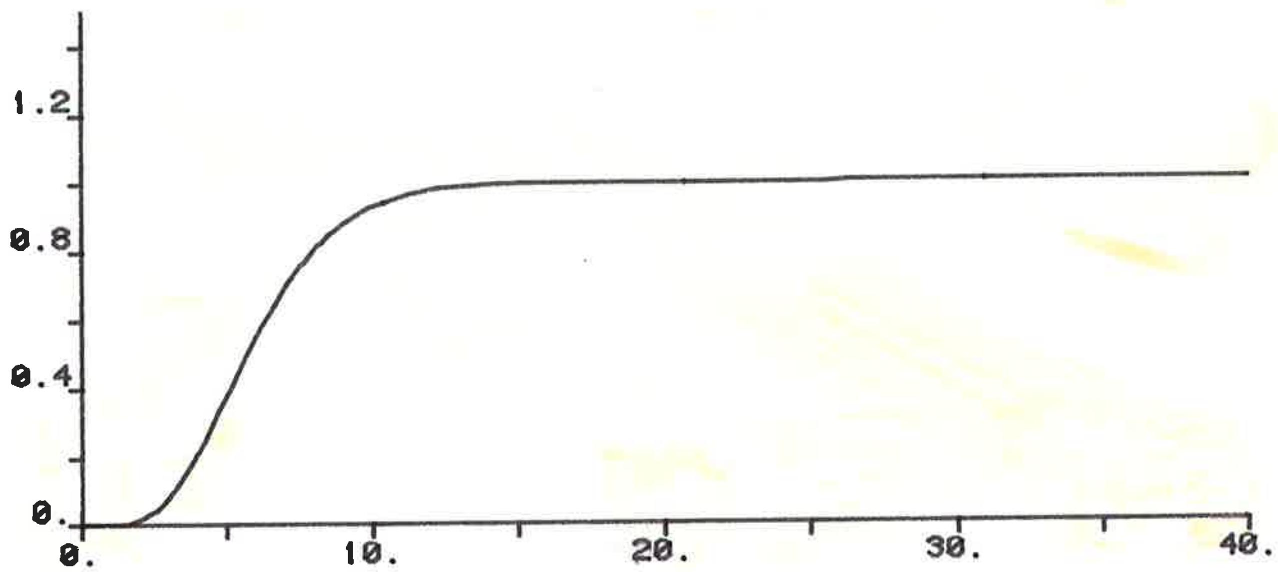

Fig. 2 - Step response of the system.

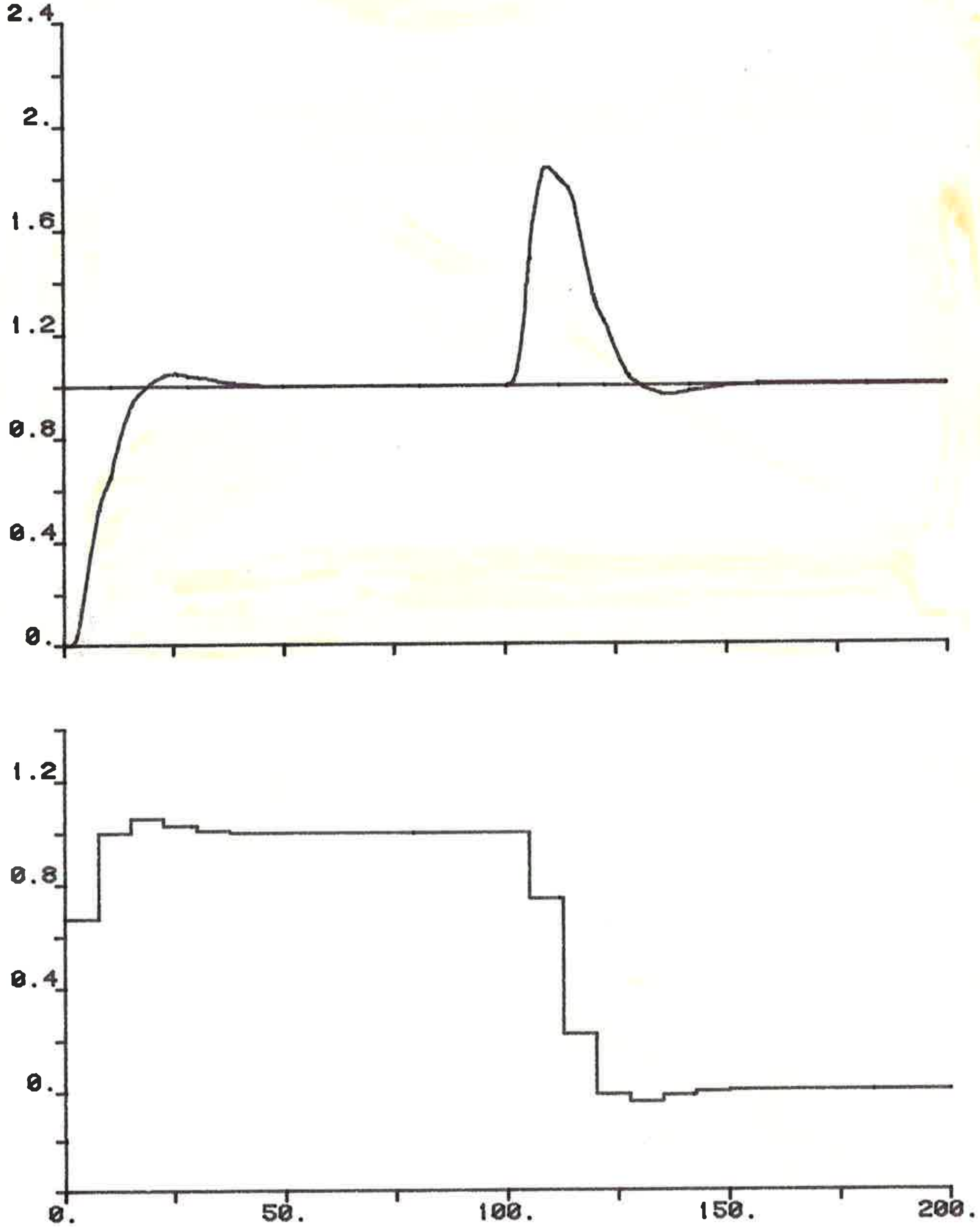

Fig. 3 - Response of a system to a step change in the command signal a load disturbance in the form of a unit step at time 100.

5. CONCLUSIONS

It has been demonstrated in a simple case that good control can be obtained by designing a regulator for a simplified process model. It would be of interest to extend this result to other design procedures and other model classes. Results in this direction are given in Åström (1979).

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APPENDIX

Simnon Programs

The Simnon programs used to generate the curves are listed below.

```
CONTINUOUS SYSTEM PROC
INPUT U D
OUTPUT Y
STATE X1 X2 X3 X4 X5 X6
DER DX1 DX2 DX3 DX4 DX5 DX6
Y=X6
DX1=-X1+U+D
DX2=-X2+X1
DX3=-X3+X2
DX4=-X4+X3
DX5=-X5+X4
DX6=-X6+X5
END
```

```
DISCRETE SYSTEM REG
INPUT Y YR
OUTPUT U
STATE U0
NEW NU0
TIME T
TSAMP TS
U=(YR-Y)/B+U0
NU0=U
TS=T+DT
B:0.5
DT:5
END
```

```
CONNECTING SYSTEM ROBUST
TIME T
YR[REG]=1
U[PROC]=U[REG]
D[PROC]=IF T<T1 THEN 0 ELSE D1
Y[REG]=Y[PROC]
T1:20
D1:1
END
```