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Åström, Karl Johan; Wittenmark, Björn

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

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SELF-TUNING CONTROLLERS BASED ON POLE-ZERO PLACEMENT

K. J. ÅSTRÖM

B. WITTENMARK

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY
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B Wittenmark

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Self-tuning controllers based on pole-zero placement.

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Self-tuning regulators based on deterministic pole-zero placement design are discussed in this paper. The regulators are designed to handle the servo problem. Two types of algorithms are described and analysed. The first type contains explicit identification of the process model. It is shown that some simplifications of the adaptive algorithms can be achieved by instead estimating parameters in a modified process model. This model contains the parameters in the regulator. This leads to implicit schemes. The properties of the algorithms are illustrated using simulation.

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Contents

1. INTRODUCTION

The simple PI-regulator is unquestionable the most common regulator in industry. In spite of this there are cases where it is advantageous to use more complex control algorithms. More complex regulators have unfortunately often more adjustable parameters. It may thus be costly and time consuming to tune such regulators. Self-tuning control is one possibility to simplify the tuning.

The basic self-tuning regulator described in Åström and Wittenmark (1973) was designed for a situation where the control problem could be characterized as a minimum variance control problem. This means that the criterion is to minimize the variance of the output. The basic self-tuning regulator was designed based on a certainty--equivalence argument. The appropriate model of the process and its environment is thus estimated recursively. The control is determined as if the estimated model is equal to the true model. There are many problems which fit this problem formulation and the basic self-tuning regulator has also been shown to work very well in such cases. There are, however, also stochastic control problems where minimum variance control is not appropriate. One case is a non-minimum phase plant. Another case is when large control signals are required to achieve minimum variance. These cases can, however, be formulated as linear-quadratic-gaussian (LQG) control problems. A self-tuning regulator based on the LQG design technique was described in Åström and Wittenmark (1974). Other versions are given in Peterka and Åström (1973), Åström et al (1977). The self-tuning regulator based on the LQG formulation has the drawback of being more complicated than the basic self-tuning regulator. The reason for this is that the design calculations which are done in each step involve the solution of a steady-state riccati equation or equivalently a spectral factorization. A simpler algorithm was proposed by Clarke and Gawthrop (1975). They proposed to use a LQG formulation with a *one-step* criterion only. This simplifies the algorithm considerably. The algorithm can be made to work well in many cases but it is not foolproof. Further discussions of the algorithm are given in Gawthrop (1977) and Clarke and Gawthrop (1979).

There are many problems which do not fit the stochastic control formulation. Encouraged by the success of the self-tuning regulators for stochastic control problems, it is tempting to try a similar approach in other cases. Using the certainty equivalence argument the design is straightforward. Start with a design method for systems with known parameters. Substitute the parameters of the known system model by estimates which are obtained recursively and recalculate the control parameters in each step. Self-tuning controllers of this type which are based on pole-placement design and least-squares estimation are discussed in this paper. The controllers obtained are useful in many situations. For instance they can be used to tune control loops when the parameters or the controlled system is unknown or slowly time--varying. It is assumed that the main source of disturbances are changes in the reference value or occasionally large disturbances that have to be eliminated. The self-tuning regulator based on minimum variance control is not well suited for this case. The new self-tuning controllers can be used to solve the servo problem and can thus be regarded as useful complements.

Self-tuning regulators based on pole-placement design have been discussed by several other authors. A digital adaptive pole shifting algorithm was discussed in a dissertation by Edmunds (1976). This and similar algorithms are further discussed in Wellstead (1978), Wellstead et al (1978), Wellstead et al (1979), Elliot and Wolovich (1979). In these works the emphasis is, however, on the regulation problem and not on the servo problem. The use of feed-forward is not discussed. Servo self-tuners have been discussed in Åström et al (1977), Åström et al (1978), and Wellstead and Zanker (1979). Self--tuning of PID-controllers based on pole placement is discussed in Wittenmark (1979). Wouters (1977) also proposes a stochastic pole--placement strategy. He also focuses on the stochastic regulation properties of the algorithm. The self-tuning controller proposed by Clarke can also be interpreted in a pole-placement framework. See Gawthrop (1977). Our paper differs from the previous treatments by focusing entirely on the servo problem. In our formulation the links to a deterministic design procedure are also emphasized. This makes it possible to establish links to MRAS. See Egardt (1978). Another feature of this paper is that the notions of algorithms with implicit and explicit identification are introduced. Several of the algorithms proposed in this paper are also new.

The paper is organized as follows. Pole-placement design for systems with known parameters is reviewed in Section 2. The suitability at the pole-placement design as a basis for adaptive control is discussed in Section 3. It is shown that there are some difficulties which are inherent in the problem formulation. Adaptive pole-placement algorithms based on estimation of the parameters in an explicit process model are discussed in Section 4. This leads to the so called *explicit* schemes. In Section 5 it is shown that some simplification of the adaptive algorithms can be achieved by instead estimating parameters in a modified process model. This leads to the *implicit* schemes. Some simulations which illustrate the behaviour of adaptive algorithms based on the pole-placement design are given in Section 6.

2. POLE ZERO PLACEMENT DESIGN

A brief review of the pole-zero placement design method for systems with known parameters will now be given. This material is quite well-known. See e.g. the classic text on sampled data systems by Ragazzini and Franklin (1958). More recent discussions on design of digital control systems based on pole-placement design are found in Andersson (1977), Wittenmark (1976), and Franklin (1977). Due to the algebraic nature of the problem there are strong similarities to the corresponding design procedure for continuous systems. See Åström (1976). The discussion given here is limited to single input systems.

Notations

The systems and regulators are described using a polynomial representation. The following notation is used:

$$A(q^{-1}) = a_0 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a}, \qquad a_0 \neq 0,$$

where q^{-1} is the backward shift operator. If $a_0 = 1$ the polynomial is said to be monic. The degree of a polynomial $A(q^{-1})$ is written either as deg A or as n_a. The argument of the polynomial is dropped if there is no ambiguity.

The input, output and command signals of the process are denoted $u(t)$, $y(t)$, and $u_c(t)$ respectively, and $v(t)$ is a disturbance. Z is a region well outside the unit disc. If the zeros of a polynomial belong to Z this implies that the corresponding modes are sufficiently well damped. This region is called the restricted stability region.

Formulation

Consider a process characterized by the rational operator

$$G(q^{-1}) = \frac{q^{-k} B(q^{-1})}{A(q^{-1})}. \tag{2.1}$$

It is assumed that A and B are coprime, that A is monic, and that the delay in the process is such that

$$k \geq 1. \tag{2.2}$$

It is desired to find a controller such that the closed loop is stable and that the transfer function from the command input u_c to the output is given by

$$G_m(q^{-1}) = \frac{q^{-k} B_m(q^{-1})}{A_m(q^{-1})} \tag{2.3}$$

where A_m and B_m are coprime and A_m is monic. The zeros of A_m are assumed to be inside Z.

For simplicity it is assumed that the time delay in (2.3) is the same as the time delay in (2.1). It is, however, sufficient to assume that the delay in (2.3) is at least as long as the delay in (2.1).

Design procedure

A general linear regulator can be described by

$$R(q^{-1})\, u(t) = T(q^{-1})\, u_c(t) - S(q^{-1})\, y(t). \tag{2.4}$$

The closed loop transfer function relating y to u_c is given by

$$\frac{q^{-k}\, TB}{AR + q^{-k}\, BS} = \frac{q^{-k}\, B_m}{A_m} \tag{2.5}$$

where the right hand side is the desired closed loop transfer function G_m given by (2.3).

The design problem is thus equivalent to the algebraic problem of finding polynomials R, S, and T such that (2.5) holds. It follows from (2.5) that factors of B which are not also factors of B_m must be factors of R. Since factors of B correspond to open loop zeros it means that open loop zeros which are not desired closed loop zeros must be canceled. Factor B as

$$B = B^+B^- \tag{2.6}$$

where all the zeros of B^+ are in the restricted stability region Z and all zeros of B^- outside Z. This means that all zeros of B^+ correspond to well damped modes and all zeros of B^- correspond to unstable or poorly damped modes.

A necessary condition for solvability of the servo problem is thus that the specifications are such that

$$B_m = B_{m_1} B^- . \tag{2.7}$$

Since deg A_m is normally less than deg $(AR + q^{-k}BS)$ it is clear that there are factors in (2.5) which cancel. In state space theory it can be shown that the regulator (2.4) corresponds to a combination of an observer and a state feedback. See Åström (1976). It is natural to assume that the observer is designed in such a way that changes in command signals do not generate errors in the observer. This means that the factor which cancels in the right hand side of (2.5) can be interpreted as the observer polynomial A_0. It is assumed that all zeros of A_0 are in the restricted stability region Z.

A block diagram of the closed loop system is shown in Fig. 2.1.

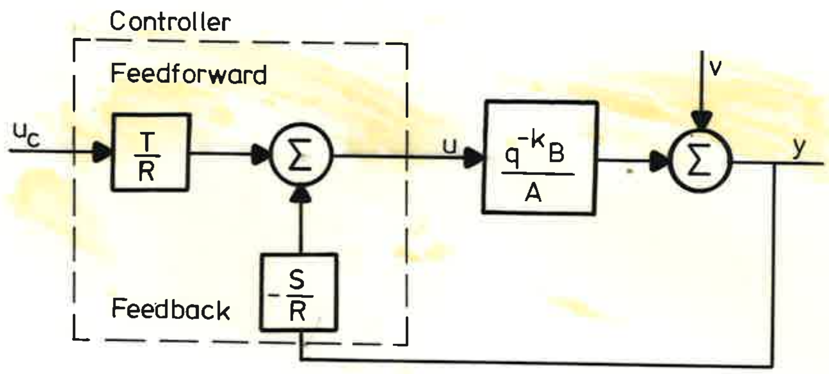


Fig. 2.1 - Block diagram of the closed loop system.

The regulator can be interpreted as being composed of a feedforward path with the transfer function T/R and a feedback path with the transfer function -S/R.

The design method can be described as follows.

Data: Given a mathematical model (2.1) of the process characterized by the polynomials A and B, the desired response (2.3) characterized by the polynomials A_m and B_m and the desired observer polynomial A_0. Assume that the data satisfies the conditions (2.2), (2.6), and (2.7) and that all zeros of A_0 are in Z.

Step 1: Solve the equation

$$AR_1 + q^{-k} B^- S = A_m A_0 \tag{2.8}$$

with respect to R_1 and S.

Step 2: The regulator which gives the desired closed loop response is given by (2.4) with

$$R = R_1 B^+ \tag{2.9}$$

and

$$T = A_0 B_{m_1}. \tag{2.10}$$

The equation (2.8) can always be solved because it was assumed that A and B where coprime. This implies of course that A and B^- are also coprime.

Equation (2.8) has infinitely many solutions. If R_1^0 and S^0 are solutions, then

$$R_1 = R_1^0 + B^- U$$

$$S = S^0 - AU,$$

where U is an arbitrary polynomial, is also a solution. All solutions will give a closed loop system with the desired closed loop transfer function G_m. The different solutions will, however, give systems with different noise rejection properties. The transfer function from the disturbance v acting on the process output to the output, see Fig. 2.1, is given by

$$\frac{AR}{AR + q^{-k}BS} = \frac{AR}{A_m A_0 B^+}.$$

The particular solutions used for the self-tuning regulators in this paper are such that the transfer functions S/R and T/R are causal with no extra delay. The following are natural choices of solutions:

$$\begin{aligned} \deg S &= \deg A - 1 \\ \deg R_1 &= \deg A_m + \deg A_0 - \deg A \end{aligned} \tag{2.11}$$

or

$$\begin{aligned} \deg S &= \deg A_m + \deg A_0 = \deg B^- - k \\ \deg R_1 &= \deg B^- + k - 1. \end{aligned} \tag{2.12}$$

The case (2.11) corresponds to "integral" compensation and the case (2.12) corresponds to "derivative" compensation. There are, however, many other possibilities.

<u>Interpretation as model following</u>

The regulator (2.4) can be interpreted as a model following. It follows from (2.8), (2.9), and (2.10) that

$$\frac{T}{R} = \frac{A_0 B_{m1}}{B^+ R_1} = \frac{(AR_1 + q^{-k}B^-S)\, B_{m1}}{A_m B^+ R_1} = \frac{AB_{m1}}{B^+ A_m} + \frac{q^{-k}SB^-B_{m1}}{B^+ R_1 A_m} =$$

$$= \frac{A}{B} \cdot \frac{B_m}{A_m} + \frac{q^{-k}S}{R} \cdot \frac{B_m}{A_m}.$$

The feedback law (2.4) can thus be written as

$$u(t) = \frac{A}{B}\, y_c(t+k) + \frac{S}{R}\, [y_c(t) - y(t)] \tag{2.13}$$

where

$$y_c(t) = \frac{q^{-k}B_m}{A_m} u_c(t).$$

The signal y_c can be interpreted as the output obtained when the command signal u_c is applied to the model $q^{-k}B_m/A_m$. When the regulator (2.4) is rewritten as (2.13) it is clear that it can be thought of as composed of two parts, one feedforward term $\frac{A}{B} y_c(t+k)$ and one feedback term $(S/R)(y_c(t) - y(t))$. The feedforward is a combination of the ideal model and an inverse of the process model. The feedback term is obtained by feeding the error $y_c - y$ through a dynamical system characterized by the operator S/R. The linke between pole placement and model following design is thus established. Notice, however, that the system q^kA/B is not realizable although the combination $AB_m/(BA_m)$ is.

Special cases

To perform the design it is necessary to have procedures for decomposing a polynomial B into its factors B^+ and B^- and for solving the linear polynomial equation (2.8). The decomposition is essentially a spectral factorization problem. Equation (2.8) can be solved by using Gauss' elimination or by using Euclid's algorithm. In the adaptive algorithms these calculations have to be repeated in each step of the iteration. It is then of interest to see if there are some special cases where the design calculations can be simplified. Two special cases, where the decomposition problem is avoided, are given below.

EXAMPLE 2.1 (All process zeros cancelled)

Assume that all process zeros are cancelled and that no additional zeros are introduced. Further assume that $\deg A_m = \deg A$ and $\deg A_0 = \deg A - 1$. Equation (2.8) then reduces to

$$AR_1 + q^{-k}S = A_mA_0 \tag{2.14}$$

and the controller is then given by (2.4) with

$$R = R_1B$$

$$
\begin{aligned}
B_m &= B_{m_1} = K \\
T &= A_0,
\end{aligned}
$$

where K is a constant. Notice that B appears as a factor of R which is the denominator of the regulator transfer function. Also notice that the specifications are normally such that the desired closed loop transfer function has unit gain at low frequencies. This means that the polynomials A_m and B_m should be normalized such that $B_m(1)/A_m(1) = 1$. Since A is monic the polynomials q^{-k} and A are always relatively prime. The Equation (2.14) can thus always be solved. The solutions corresponding to

$$
\begin{aligned}
\deg S &= \deg A - 1 \\
\deg R_1 &= \deg A_m + \deg A_0 - \deg A
\end{aligned}
\tag{2.15}
$$

or

$$
\begin{aligned}
\deg S &= \deg A_m + \deg A_0 - k \\
\deg R_1 &= k - 1.
\end{aligned}
\tag{2.16}
$$

are chosen. In the special case of Example 2.1 the design calculations thus reduce to the solution of (2.14). Notice that (2.14) is easy to solve for the case (2.16). The coefficients of the polynomials R_1 and S can then be obtained one at the time.

EXAMPLE 2.2 *(No process zeros are cancelled)*

Assume that the specifications are such that the desired closed loop zeros are equal to the process zeros, i.e. $B_m = K \cdot B$, where K is a constant. The specifications are normally such that the low frequency gain of the desired closed loop is unity. The constant factor of the polynomial B_m is then chosen so that $B_m(1) = A_m(1)$. It is assumed that this normalization is made. Equation (2.8) then gives

$$
AR + q^{-k}BS = A_mA_0. \tag{2.17}
$$

The design procedure is thus again to choose the polynomials A_m and A_0. Equation (2.17) is then solved with respect to R and S and the controller is then given by (2.4), where $T = KA_0$. □

Other alternatives

There are many possible variations on the pole-placement design scheme presented in this section. Franklin (1977) has pointed out that the observer poles must not necessarily be cancelled precisely. Almost cancellations lead to an extension of the classical notion of dipole compensation. Similarly Gawthrop (1977) has pointed out that in the case of stable but poorly damped process zeros it is possible to cancel them, provided that the specified closed loop transfer function has zeros close to the process zeros.

3. SELF-TUNING CONTROL

The basic idea when using the separation principle to design self--tuning regulators can be expressed as follows. Start with a design procedure for systems with known parameters. When the parameters are not known they are estimated recursively and the regulator is re-designed in each step, using the estimated parameters instead of the true ones. This means that the certainty equivalence hypothesis is used to determine the controller. A block diagram of a general self--tuning regulator is shown in Fig. 3.1.

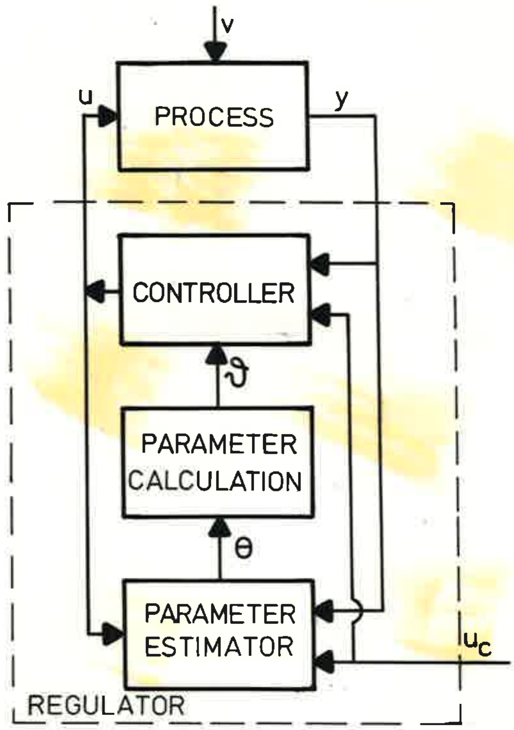


Fig. 3.1 - Schematic diagram of a self-tuning regulator.

The pole-zero placement design method was discussed in Section 2. The parameter estimation will be discussed in this section. A general discussion of some properties of self-tuners based on pole-zero placement is also given in this section. More details are given in the following sections.

Parameter estimation

An overview of methods for recursive parameter estimation is given in Söderström et al (1974). There is unfortunately no recursive parameter estimator which is uniformly best. Least squares, which is one of the simplest recursive estimation schemes will be used here. This procedure will give biased estimates if there are stochastic disturbances which are coloured noise. Since the discussion is focused on the servo problem the major disturbances are, however, command inputs. This is also compatible with the pole-zero placement design procedure which is not suitable to handle trade-offs between measurement noise and process noise quantitatively.

Recursive least squares

Consider the process model

$$Ay(t) = Bu(t-k) \tag{3.1}$$

which can be represented explicitly as

$$y(t) + a_1 y(t-1) + \ldots + a_{n_a} y(t-n_a) = b_0 u(t-k) + \ldots + b_{n_b} u(t-n_b-k).$$

Introduce a vector of parameter estimates

$$\theta = (\hat{a}_1 \ldots \hat{a}_{n_a} \quad \hat{b}_0 \ldots \hat{b}_{n_b})^T \tag{3.2}$$

and a vector of regressors

$$\varphi(t) = (-y(t-1) \ldots -y(t-n_a) \quad u(t-k) \ldots u(t-n_b-k))^T. \tag{3.3}$$

The recursive least squares estimate is then given by

$$\theta(t+1) = \theta(t) + P(t+1)\,\varphi(t+1)\,\varepsilon(t+1) \tag{3.4}$$

where

$$\varepsilon(t+1) = y(t+1) - \varphi(t+1)^T\,\theta(t) \tag{3.5}$$

and

$$P(t+1) = \left[P(t) - P(t)\varphi(t)[1 + \varphi^T(t)P(t)\varphi(t)]^{-1}\varphi^T(t)P(t)\right] / \lambda. \tag{3.5}$$

There are also other possibilities to perform the least squares calculations. Square root algorithms are useful if the problem is poorly conditioned. See e.g. Peterka (1975), Bierman (1977). Fast algorithms can be used if computing time is critical. See e.g. Levinson (1947).

Choice of λ and modifications of the P-equation

The factor λ in Equation (3.6) is introduced to discount past data when performing the least squares. For the regulation problem the estimator is excited by the process disturbances which normally are reasonably uniform in time. It has been found empirically that a value of λ between 0.95 and 0.99 works well in such cases. For the servo problem the major excitation comes from the changes in the command signal. Such changes may be irregular and it has been found that there may be bursts in the process output if Equation (3.6) is used with λ less than one. The presence of bursts can be understood intuitively as follows. The negative term in (3.6) represents the reduction in parameter uncertainty due to the last measurement. When there are no changes in the set point the vector $P(t)\varphi(t)$ will be zero. There will not be any changes in the parameter estimate and the negative term in the right hand side of (3.6) will be zero. The Equation (3.6) then reduces to

$$P(t+1) = \frac{1}{\lambda} P(t)$$

and the matrix P will thus grow exponentially if $\lambda < 1$. If there are no changes for a long time the matrix P may thus become very large. A change in the command signal may then lead to large changes in the parameter estimates and in the process output. The large values of the matrix P may also lead to numerical problems. Examples which illustrate this behaviour are found e.g. in Fortescue et al (1979) and Morris et al (1977).

There are many ways to eliminate bursts. Perturbation signals may be added to ensure that the process is properly excited. The estimation algorithm may be modified. One possibility is to stop the updating of the matrix $P(t)$ when the signal $P(t)\varphi(t)$ or the prediction error is smaller than a given value. Another possibility is to subtract a term like $\alpha P^2(t)$ from the right hand side of (3.6) to ensure that the matrix $P(t)$ stays bounded. A third possibility is to choose the forgetting factor so that a function of P like tr P is constant.

Self-tuners based on pole-zero placement

Before presenting specific self-tuning algorithms, it will be discussed whether the pole-placement design procedures are suitable for design of adaptive regulators. There are several problems to be considered.

It follows from the discussion of the pole-zero assignment method in Section 2 that it is not possible to specify the closed loop transfer function arbitrarily. Firstly it is assumed that the timedelay in the model G_m (2.3) is at least as long as that of the process (2.1). Secondly the specifications must be such that unstable or poorly damped process zeros must also be zeros of the desired closed loop transfer function. This is formally expressed by the condition (2.7). Notice that this does not mean that the poorly damped zeros must be known a priori. These zeros are estimated in the self-tuner. Since the zeros can not be cancelled it means, however, that the properties of the closed loop system will change when the poorly damped process zeros change. In practice it has been found that this is not of great importance if the poorly damped zeros correspond to frequencies which are higher than those of the dominating poles. Poorly damped process zeros within the servo bandwidth will, however, have a very noticable influence on the system. Hence it is not possible to ensure that properties like overshoot, bandwidth, static errors etc. remain invariant for the adaptive system.

It may be too restrictive to specify all closed loop poles at least for high order systems. One possibility to avoid this difficulty for discrete time systems is to specify only the dominant poles and require that the remaining poles are close to the origin. In practice it is often satisfactory to choose A_m as

$$A_m(q^{-1}) = 1 - 2e^{-\zeta\omega h} \cos \omega h\sqrt{1-\zeta^2}\, q^{-1} + e^{-2\zeta\omega h} q^{-2}, \qquad (3.7)$$

which corresponds to a second order continuous time system with damping ζ and frequency ω sampled with period h. It is often easy to determine ζ and ω such that the system gets desired properties. The relative damping is often chosen in the interval 0.5 - 0.8. The resonance frequency ω is chosen based on the demands on the rise time and the solution time.

The pole-placement design procedure requires that the observer poles are specified. The observer poles are not critical. Their choice should, however, reflect the characteristics of the disturbances. If an estimation procedure which gives the disturbance dynamics is used e.g. in the form of a controlled ARMA model it is natural to choose the observer polynomial proportional to the polynomial which characterizes the moving average. In this paper this is not done and the observer polynomial can thus be chosen arbitrarily.

4. ALGORITHMS BASED ON EXPLICIT IDENTIFICATION

Some self-tuning algorithms based on the pole-zero placement design method will now be discussed. The algorithms are first presented. Their properties are then discussed briefly. Some practical aspects are then given.

Algorithms

A self-tuning controller can be obtained by implementing the system in Fig. 3.1 directly. The following algorithm is then obtained

ALGORITHM E1 (*Basic explicit algorithm*)

Data: The polynomials A_m and A_0, both with zeros in Z, and B_{m_1} are given.

Step 1: Estimate the parameters of the model
$Ay(t) = Bu(t-k)$
by least squares.

Step 2: Factor

Step 2: Factor the polynomial B into B^+ and B^-.

Step 3: Solve the linear equation
$AR_1 + q^{-k}B^-S = A_mA_0$
with deg R_1 and deg S chosen as in (2.11) or (2.12).

Step 4: Calculate the control signal from
$Ru = Tu_c - Sy$
with
$R = R_1B^+$
$T = A_0B_{m_1}$.

The steps 1, 2, 3, and 4 are repeated at each sampling period. □

An algorithm of this type is called an algorithm based on *estimation of process parameters* or an algorithm with *explicit identification*, because the estimated parameters are the parameters of the process model in the standard form. In the terminology of model reference adaptive systems (MRAS) the corresponding algorithms are called *indirect* because the controller parameters are updated indirectly via estimation of the process model and design calculations.

Notice that the closed loop transfer function obtained with this algorithm is

$$G = \frac{q^{-k} B_{m1} B^-}{A_m}$$

where B^- is the polynomial which correspond to unstable or poorly damped process zeros. When these zeros change the closed loop response will also change.

One difficulty with the Algorithm E1 is that the equation to be solved in Step 3 is poorly conditioned for parameter values such that A and B almost have a common factor.

The factorization in Step 2 may also be difficult and timeconsuming. There are two special cases where the factorization can be avoided. One case is when all process zeros are well damped. It is then reasonable to have a pole-placement design where all the process zeros are cancelled. Under this hypothesis the pole-placement procedure can be simplified as shown in Example 2.1. The corresponding self-tuning pole-placement algorithm then becomes

ALGORITHM E2 *(Explicit algorithm with all process zeros cancelled)*

Data: Given specifications in the form of the desired closed loop poles and desired observer poles specified by the polynomials A_m and A_0 with zeros in Z. Further B_m is a constant. The polynomial B_m is normalized so that $B_m(1)/A_m(1) = 1$. The polynomial A_0 is normalized arbitrarily.

Step 1: Estimate the parameters of the model

$$Ay(t) = Bu(t-k)$$

by least squares.

Step 2: Determine the polynomials R_1 and S such that

$$AR_1 + q^{-k}S = A_m A_0$$

with deg R_1 and deg S chosen as (2.15) or (2.16).

Step 3: Use the control law

$$BR_1 u = Tu_c - Sy,$$

where $T = A_0 B_m$.

The steps 1, 2, and 3 are repeated for each sampling period. As safe--guards it should be tested that A and B do not have common factors and that B is a stable polynomial. □

This algorithm cannot be expected to work well unless the corresponding design procedure for systems with known parameters work well. Since all process zeros are cancelled the regulator will not be satisfactory for non-minimum phase systems or for systems with zeros having poor damping. Such systems can, however, be handled using the design procedure in Example 2.2. The corresponding self-tuning control algorithm is given by

ALGORITHM E3 *(Explicit algorithm with no process zeros cancelled)*

Data: Given specifications in the form of the desired closed loop poles and the desired observer poles specified by the polynomials A_m and A_0 with zeros in Z. A_0 is normalized arbitrarily.

Step 1: Estimate the parameters of the model

$$Ay(t) = Bu(t-k)$$

by least squares.

Step 2: Introduce $B_m = K \cdot B$ and choose K such that $B_m(1) = A_m(1)$. Then determine the polynomials R and S such that

$$AR + q^{-k}BS = A_m A_0.$$

Deg S and deg R are chosen as in (2.11) or (2.12) with deg R = deg R_1 and deg B = deg B^-.

Step 3: Use the control law

$$Ru = Tu_c - Sy,$$

where $T = K \cdot A_0$.

The steps 1, 2, and 3 are repeated for each sampling period. □

REMARK

Possible common factors of A and B should be eliminated after the first step to ensure that the equation in Step 2 has a solution. Notice that the polynomial A_m cannot be normalized a priori because the normalization requires knowledge of the polynomial B in the process model. □

Notice that with Algorithm E3 the properties of the closed loop system will change even if A_m and A_0 are fixed because the closed loop zeros will change if the process zeros change.

Properties

The properties of the closed loop system obtained when the self-tuning regulator is applied to a given process will first be discussed. It is first assumed that the process to be controlled is described by the difference equation

$$A_s\, y(t) = B_s\, u(t-k). \tag{4.1}$$

It is assumed that this equation is of the form (3.1) and that deg A = = deg A_s and deg B = deg B_s. The Equations (3.1) and (4.1) are then said to be compatible.

Using the notation (3.3), the Equation (4.1) can also be written as

$$y(t) = \theta_s^T \varphi(t) = \varphi^T(t)\, \theta_s,$$

where the components of θ_s are the coefficients of the polynomials.

The closed loop system obtained with the algorithm E1 can then be described by (4.1) and the equations

$$\begin{cases} \theta(t+1) = \theta(t) + P(t+1)\ \varphi(t+1)\ \varepsilon(t+1) \\ \varepsilon(t+1) = \varphi^T(t+1)\ [\theta_s - \theta(t)] \\ P(t+1) = \left[P(t) - P(t)\varphi(t)[1+\varphi(t)^{-T}P(t)\varphi(t)]^{-1}\varphi^T(t)P(t)\right]/\lambda \\ R\ u(t) = T\ u_c(t) - S\ y(t) \\ R = R_1B^+ \\ T = A_0B_{m_1} \\ AR_1 + q^{-k}B^-S = A_mA_0 \end{cases} \tag{4.2}$$

The states of the closed loop system can be chosen as θ, P the state of a representation of

$$\begin{cases} A_s\ y = q^{-k}\ B_s\ u \\ R\ u = T\ u_c - S\ y \end{cases}$$

and possibly some additional delayed values of u and y, which are needed to represent the vector φ given by (3.3). To obtain a complete description it is also necessary to specify the command signal. The equations describing the closed loop system are nonlinear. Their global properties are not yet fully explored. A difficulty of the equations is that the mapping from the coefficients of the polynomials A, B to those of R and S is discontinuous at those points where A and B have common factors.

There are no proper stationary solutions to the Equations (4.1) and (4.2) in the sense that all state variables are constant unless the command signal is constant. There are, however, solutions such that the parameters estimates $\theta(t)$ assume constant for arbitrary command signals. Assuming that the matrix P(t) is positive definite for all t. It follows from (4.2) that $\theta(t)$ is constant if $\varphi(t)\varepsilon(t)$ is zero i.e.

$$y(t-i)\ \varepsilon(t) = 0, \qquad i = 1, 2, \ldots, n_a,$$

$$u(t-i)\ \varepsilon(t) = 0, \qquad i = k, k+1, \ldots, k+n_b.$$

These equations imply that $\varepsilon(t) = 0$. Assume on the contrary that

$\varepsilon(t) \neq 0$. Then

$$y(t-i) = 0, \qquad i = 1, 2, \ldots, n_a,$$
$$u(t-i) = 0, \qquad i = k, k+2, \ldots, k+n_b+1.$$

Equation (4.1) then implies that $y(t) = 0$. Since

$$\varepsilon(t) = y(t) - a_1 y(t-1) - \ldots - a_n y(t-n_a)$$
$$- b_0 u(t-k) - \ldots - b_m u(t-k-n_b)$$

we get $\varepsilon(t) = 0$ which is a contradiction. When the parameters $\theta(t)$ are constant it follows that

$$y = B_s v$$
$$u = A_s v$$

where

$$(A_s R + q^{-k} B_s)\ v = T u_c.$$

Hence

$$\varepsilon(t) = Ay - Bu = (AB_s - BA_s)\ v.$$

Under modest requirements on u_c (e.g. piecewise deterministic with arbitrary generator, Åström (1979a)) it now follows that the condition $\varepsilon(t) = 0$ implies that

$$AB_s = BA_s.$$

The correct estimates are thus the only parameter values such that the estimates remain constant.

To investigate the local stability at the stationary solution $\theta(t) = \theta_s$ the equations are linearized. The linearized equation for $\theta(t)$ is decoupled from the rest of the equation. We get

$$\delta\theta(t+1) = [I - P_s(t+1)\ \varphi_s(t+1)\ \varphi_s^T(t+1)]\ \delta\theta(t), \tag{4.2}$$

where the subscript "s" indicate that the quantities have been evaluated at $\theta(t) = \theta_s$. The Equation (4.2) is stable if

$$\sum_{t_0}^{t+r} \varphi_s(k)\ \varphi_s^{k}(k)$$

is positive definite. A proof of local stability for a similar algorithm is given by Goodwin and Sin (1979).

A more general model than (4.1) is

$$A_s y = B_s u + C_s e \tag{4.3}$$

where $\{e(t)\}$ is a sequence of indpendent random variables. If $C_s = 1$ then the parameter θ_s a possible convergence point, which is locally stable. If $C_s \neq 1$ the parameter θ_s is not a possible convergence point because

$$E\ \varphi(t)\ \varepsilon(t) \neq 0.$$

A pole-placement algorithm which has a self-tuning property for the process (4.3) if the reference value is zero is described in Wellstead, Prager and Zanker (1979).

5. ALGORITHMS BASED ON IMPLICIT IDENTIFICATION

The design calculations for the algorithms discussed in the previous section may be time-consuming. It is possible to obtain different algorithms where the design calculations are simplified considerably. The self-tuning regulator in Åström and Wittenmark (1973) is a prototype for algorithms of this type. The basic idea is to rewrite the process model in such a way that the design step is trivial. For minimum variance control the process model can be rewritten so that the parameters of the minimum variance regulator are the parameters of the rewritten model. By a proper choice of model structure the regulator parameters are thus updated directly and the design calculations are thus eliminated. With reference to Fig. 3.1 it means that $\vartheta = \theta$ and the block marked design can be eliminated. Algorithms of this type are called algorithms based on *implicit identification* of a process model. In the terminology of MRAS the algorithms are also called *direct methods* because the parameters of the regulators are updated directly. Implicit algorithms and some of their properties will be discussed.

Algorithms

Consider a process described by

$$Ay = q^{-k}Bu. \tag{5.1}$$

The regulator (2.4) gives a closed loop system with the transfer function

$$G = \frac{B^{-}B_{m1}}{A_m}. \tag{5.2}$$

Equation (2.8) gives

$$A_mA_0y = AR_1y + q^{-k}B^{-}Sy.$$

Combination of this with (5.1) gives

$$A_m A_0 y = q^{-k} R_1 Bu + q^{-k} B^- Sy = q^{-k} B^- \,(Ru + Sy). \tag{5.3}$$

If the control signal is chosen such that

$$Ru = Tu_c - Sy.$$

where $T = A_0 B_{m_1}$, then it follows from (5.3) that the closed loop transfer function (5.2) is obtained. Notice that Equation (5.3) can be regarded as a process model. The polynomials R and S of the feedback law appear directly in the model. The design problem is also trivial for the model (5.3).

The following self-tuning algorithm is now obtained.

ALGORITHM I1 (*Basic implicit algorithm*)

Data: The polynomials A_m and A_0, both with zeros in Z, and B_{m_1} are given.

Step 1: Estimate the parameters of the model

$$A_m A_0 y = q^{-k} B^- (Ru + Sy) \tag{5.4}$$

i.e. estimate B^-, R, and S.

Step 2: Calculate the control signal from

$$Ru = Tu_c - Sy$$

where

$$T = A_0 B_{m_1}.$$

The steps 1 and 2 are repeated at each sampling period. □

Notice that the model (5.4) is bilinear in the parameters. This means that the estimation problem is not trivial. For example the parametrization is not unique unless it is required that R and S do not have common factors. The polynomial B^- must also be such that it has all its zeros outside the stable region Z. A recursive estimation procedure for (5.4) is proposed in Åström (1979b). Because of the difficulties of estimating the parameters of (5.4) it is of interest to consider special cases which lead to simpler calculations.

The special case when all process zeros were cancelled was discussed in Example 2.1 for the case of known parameters. In that case $B^- = 1$ and the self-tuning algorithm I1 reduces to

ALGORITHM

ALGORITHM I2 (Implicit algorithm with all process zeros cancelled)

Data: The polynomials A_m and A_0 with zeros in Z are given. Further $B_{m_1} = K = A_m(1)$.

Step 1: Estimate the parameters of the polynomials R and S in the model

$$A_m A_0 \, y = q^{-k}(Ru + Sy) \tag{5.5}$$

by least squares. The degrees of the polynomials S and R are chosen as

$$\begin{aligned} \deg S &= \deg A_m + \deg A_0 - k \\ \deg R &= \deg B + k - 1 \end{aligned} \tag{5.6}$$

or

$$\begin{aligned} \deg S &= \deg A \\ \deg R &= \deg A_m + \deg A_0 + \deg B - \deg A. \end{aligned} \tag{5.7}$$

Step 2: Compute the control signal from

$$R\, u(t) = T\, u_c(t) - S\, y(t) \tag{5.8}$$

where $T = A_0 K$.

The steps 1 and 2 are repeated at each sampling interval. □

This algorithm is identical to the self-tuning controller proposed by Clarke and Gawthrop (1975). The algorithm has also been explored by Kurz, Isermann and Schumann (1978).

A difficulty with the Algorithm I1 is that it may conceivably happen that the estimate of the leading coefficient r_0 of the polynomial R is zero. The feedback law (5.8) then is no longer causal. There are various ways to overcome this difficulty. One possibility is to fix the value of the coefficient. Another possibility is to reparametrize the polynomial as

$$r_0\,[1+r_1q^{-1}+\ldots]$$

and use special techniques to estimate r_0. This is done in the MRAS systems. See Egardt (1978). Another possibility which is often used in practice is to increase the number k in the model.

Properties

It is assumed that the process to be controlled is described by the difference equation (4.3). The closed loop system obtained when the implicit algorithms are applied to the process (4.3) is governed by a set of nonlinear difference equations. These equations are similar to the ones obtained for the explicit algorithms. The equations obtained for the implicit algorithms are somewhat simpler because the regulator parameters are updated directly. There is no complete analysis for the general case. The special case of the Algorithm I2 is, however, reasonably well understood. The key results on stability are due to Egardt (1978) and Goodwin et al (1978). A main result is that the closed loop system is stable and that the output of the system converges to the desired output. The assumptions required are that the time-delay k is known, that upper bounds on the degrees of the polynomials and that the system (4.3) is minimum phase. The result is proven for the special case $C_se(t)=0$ and $k=1$ in Goodwin et al (1978). In Egardt (1978) it is shown that the output is bounded even if there are disturbances $C_se(t)\neq 0$ provided that the disturbance is bounded.

If $C_S=1$ in (4.3) and if $B^-=1$ it is easy to see that the process model can be written as (compare (5.3))

$$A_mA_0\;y(t) = R\;u(t-k) + S\;y(t-k) + R_1\;e(t).$$

Using the method of least squares the estimates of R and S will be unbiased if the degrees are chosen as in (5.6). The degree of R_1 is then k and the regressors will be independent of $R_1e(t)$.

Modifications

There are several modifications of the algorithms that are useful. When there are stochastic disturbances in the process it is shown in Åström (1979b) that it is advantageous to replace the model (5.4) by

$$A_m y = q^{-k} B^{-}(R\bar{u} + S\bar{y}) \tag{5.9}$$

where

$$\begin{aligned} \bar{u} &= \frac{1}{A_0} u, \\ \bar{y} &= \frac{1}{A_0} y. \end{aligned} \tag{5.10}$$

Otherwise the parameters will not converge to the correct values even if the observer polynomial is known. Similarly it is sometimes useful to replace (5.5) by

$$A_m y = q^{-k}(R\bar{u} + S\bar{y}), \tag{5.11}$$

where $\bar{u}$ and $\bar{y}$ are given by (5.10).

6. SIMULATIONS

Some of the properties of the algorithms are illustrated through simulations in this section. The simulations have been done using the simulation program SIMNON, see Elmqvist (1975). The special SIMNON system for simulation of general adaptive controllers described in Gustavsson (1978) was used. More examples are found in Åström, Westerberg and Wittenmark (1978), Westerberg (1977), and Åström (1978).

Choice of parameters

There are several parameters which have to be selected in the algorithms. Unless otherwise stated the following parameters have been used. Initial values of the parameters are chosen as zero except for $r_0 = 1$ in the implicit algorithm and $b_{n_b} = 1$ in the explicit algorithm. The initial value of the covariance matrix is chosen as hundred times the unit matrix and the forgetting factor was equal to one. Further $A_0(q^{-1}) = 1$ has been used in the simulations. The reference signal was a square wave with amplitude ±1 and a period of 100 samples.

EXAMPLE 6.1

A continuous time system with the transfer function

$$G(s) = \frac{0.15\, e^{-0.45\, s}}{s + 0.15}$$

sampled with a sampling time of $T = 1$ gives the discrete time system

$$y(t) - 0.8607\, y(t-1) = 0.0792\, u(t-1) + 0.0601\, u(t-2). \qquad (6.1)$$

Notice that the continuous time system has a time delay which is not a multiple of the sampling time. The sampled model has a zero $z = -0.759$, which corresponds to a mode with damping $\zeta = 0.087$. The solution time of the open loop system is 20-25 seconds. The specifications for the closed loop system have been chosen as a solution time of about 10 seconds and a damping of about $\zeta = 0.7$. The desired characteristic equation has been chosen as

$$A_m(q^{-1}) = 1 - 1.5\, q^{-1} + 0.6\, q^{-2}.$$

The behaviour of the implicit algorithm I2 with deg $R = \deg S = 1$ (i.e. 4 estimated parameters) and with the forgetting factor $\lambda = 0.95$ is shown in Fig. 6.1. The behaviour of the closed loop system is good already in the second transient. The parameters have converged at the fourth transient. The control signal has an oscillatory tendency. That is due to cancellation of the zero at -0.759.

Fig. 6.2 shows the behaviour when the explicit algorithm E3 is used with deg A = deg B = 1 (i.e. 3 estimated parameters). The behaviour of the two algorithms is in this case very much the same.

Assume that u in (6.1) is replaced by $u + \delta$, where δ is a constant bias. The adaptive controller does not know this bias and only u and y are available for the controller. Figs. 6.3 and 6.4 show the behaviour of the closed loop system when the implicit I2 and the explicit algorithms E3 respectively are used with the same parameters as before. The implicit algorithm handles the bias by introducing an integrator in the controller. The R-polynomial after 250 steps is

$$R(q^{-1}) = 0.116 - 0.115\, q^{-1}.$$

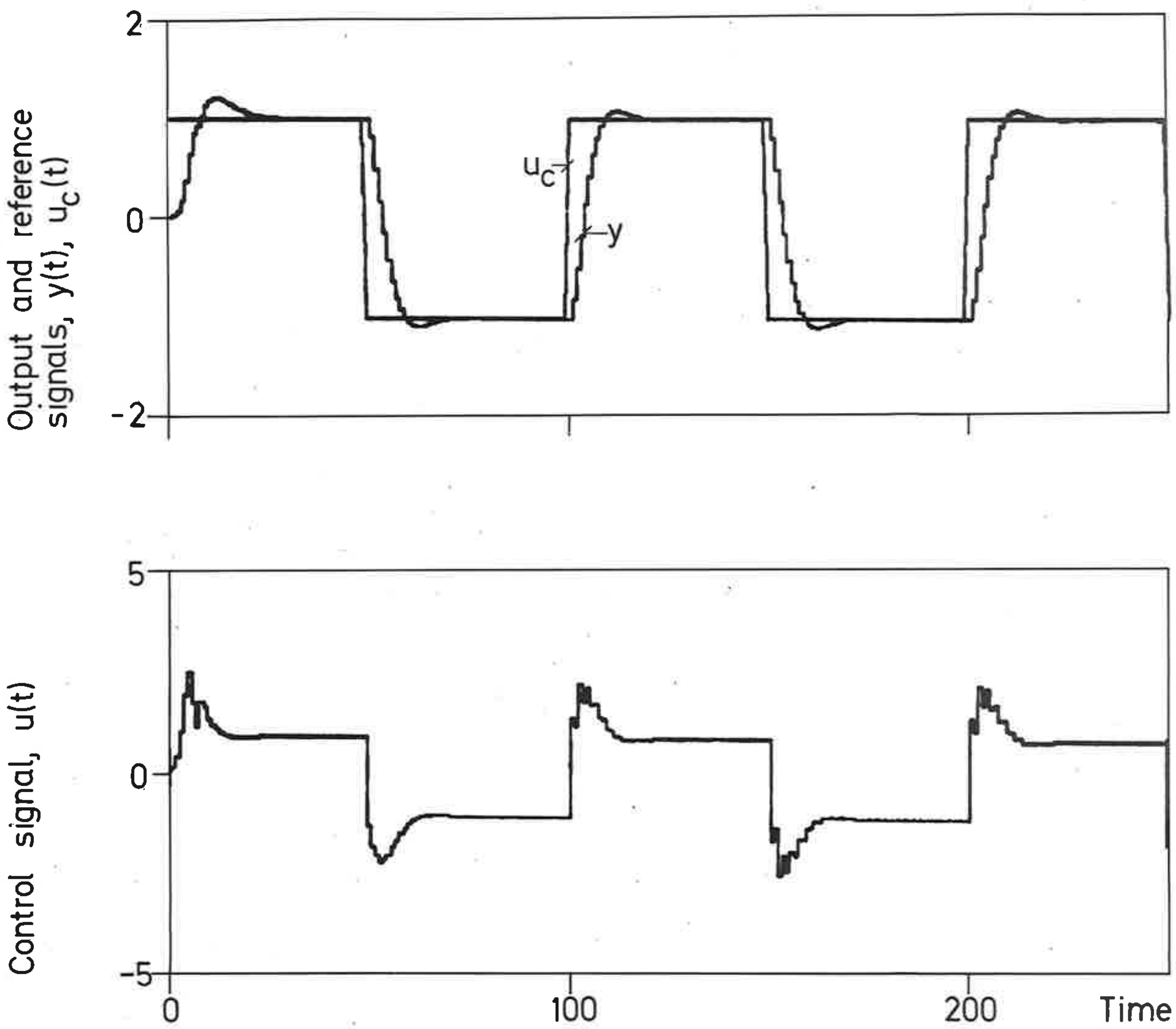
As seen from Fig. 6.3 the system will not converge to the desired closed loop transfer function. By increasing the order of the R-polynomial it is possible to get the same closed loop performance as before. The identification in the explicit algorithm is disturbed by the bias term which explains the bad behaviour. It is, however, easy to also estimate the bias term and take it into consideration when computing the control signal, compare Clarke and Gawthrop (1975). This is done in Fig. 6.5 and it is seen that it is possible to eliminate the bias. □

EXAMPLE 6.2

In this example the adaptive regulator controls a time continuous system. The system has the transfer function

$$G(s) = \frac{1}{(1+s)\,(1+0.5\,s)\,(1+5\,s)} \,. \tag{6.5}$$

Using a sampling interval of $T = 1$ we get a discrete time model which is non-minimum phase. The zeros of the model are $z_1 = -1.798$ and $z_2 = -0.114$. This system was not possible to control with the implicit algorithm since this algorithm cancels all the zeros of the process. The computation of the control signal will then be unstable. The explicit algorithm could easily be used. Fig. 6.6 shows the behaviour when deg A = 3 and deg B = 2 and $A_m(q^{-1}) = 1 - q^{-1} + 0.35\,q^{-2}$. Again, the behaviour of the closed loop system is very good already in the second transient.



<u>Fig. 6.1</u> - The output, y, the reference, u_c, and the control, u, signals when the process (6.1) is controlled using the implicit algorithm I2.

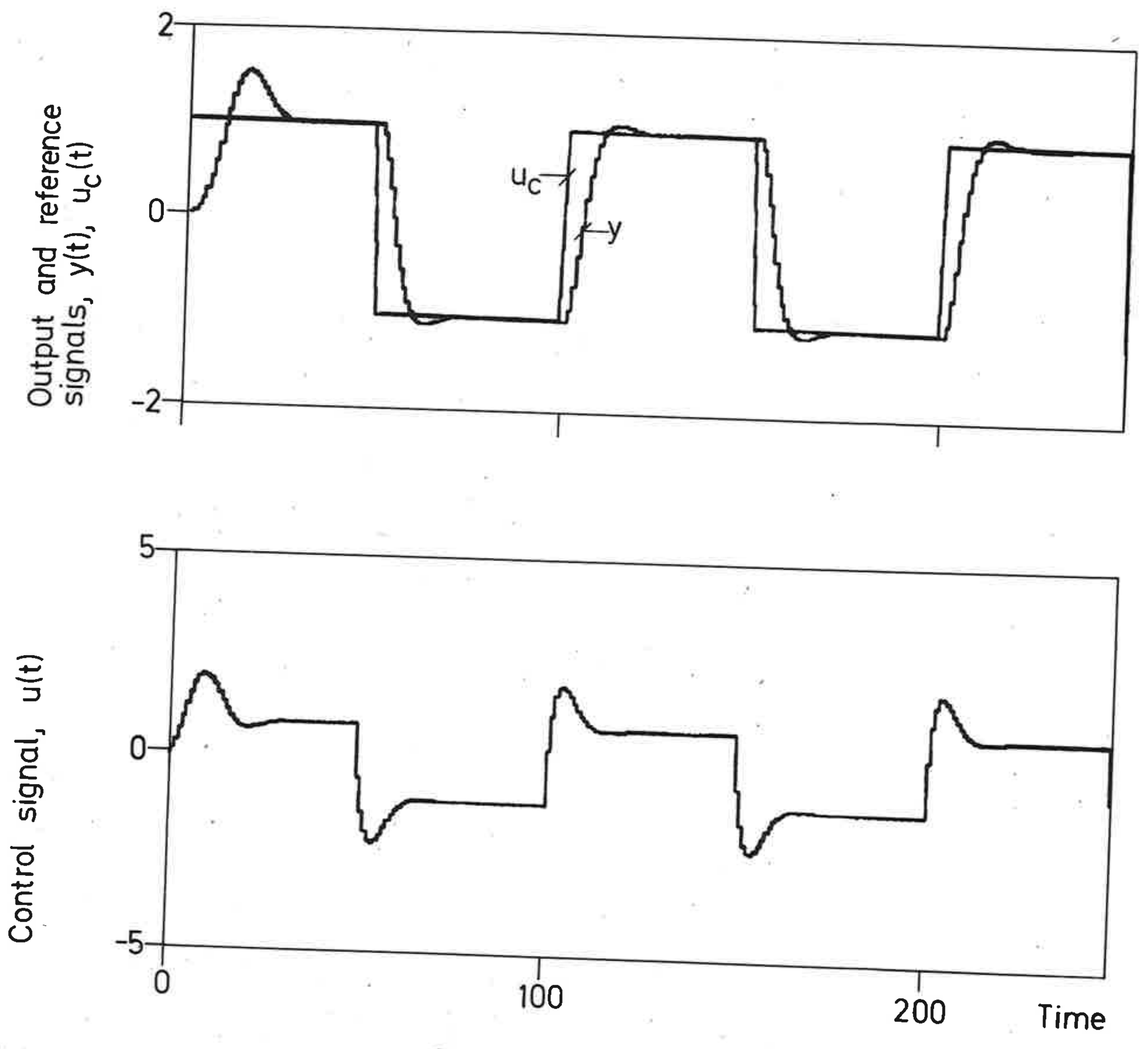
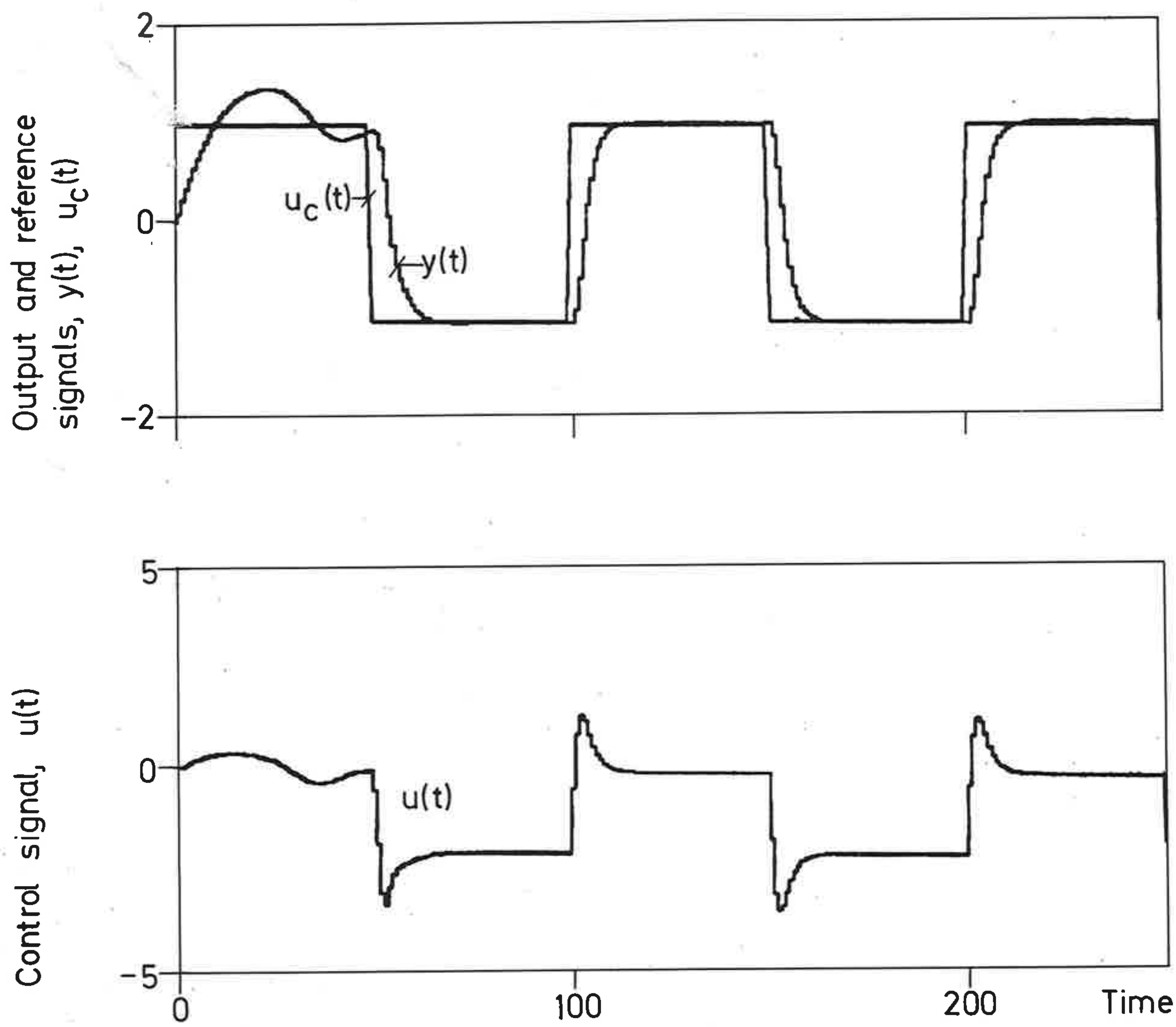


Fig. 6.2 - The output, y, reference, u_c, and input, u, signals when the process (6.1) is controlled with the explicit algorithm E3.



<u>Fig. 6.3</u> - The result when the process (6.1) is controlled with the the implicit algorithm I2 with the same parameters as in Fig. 6.1 but with a constant bias $\delta = 1$ on the input to the process.

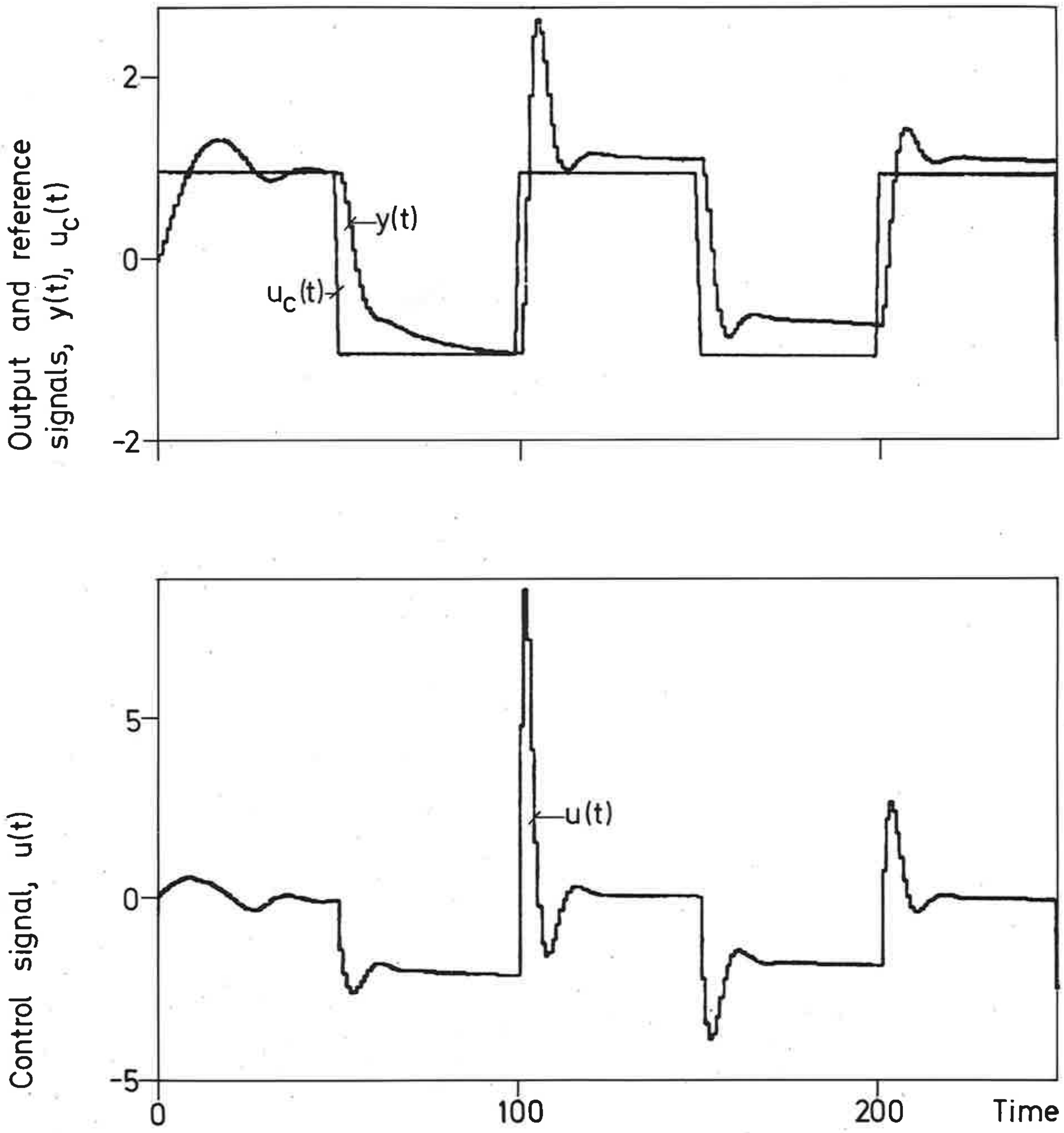


Fig. 6.4 - The same as in Fig. 6.2 but when there is a constant bias $\delta = 1$ on the input.

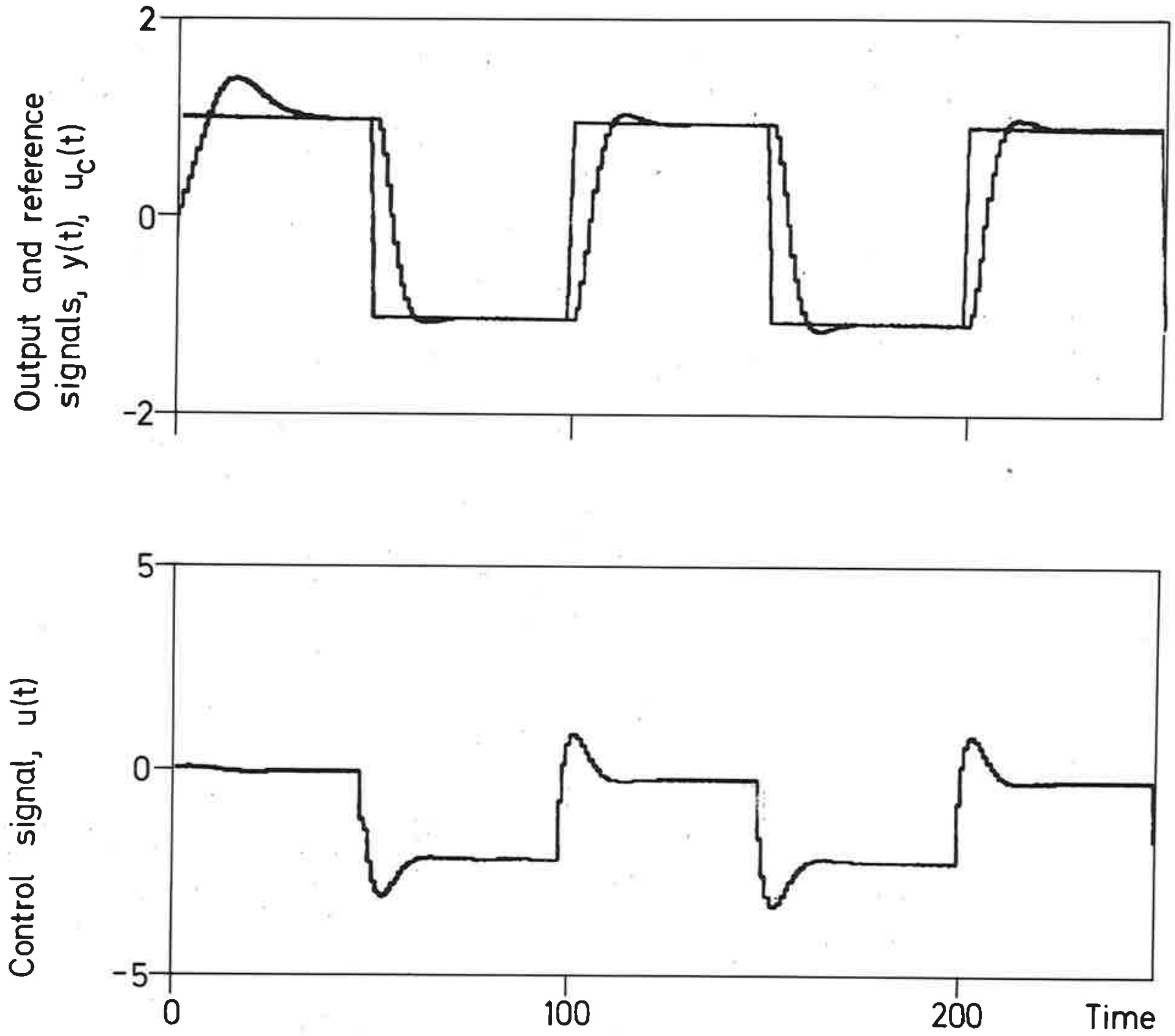


Fig. 6.5 - The result when using the explicit algorithms with compensation for the unknown bias $\delta = 1$ on the input.

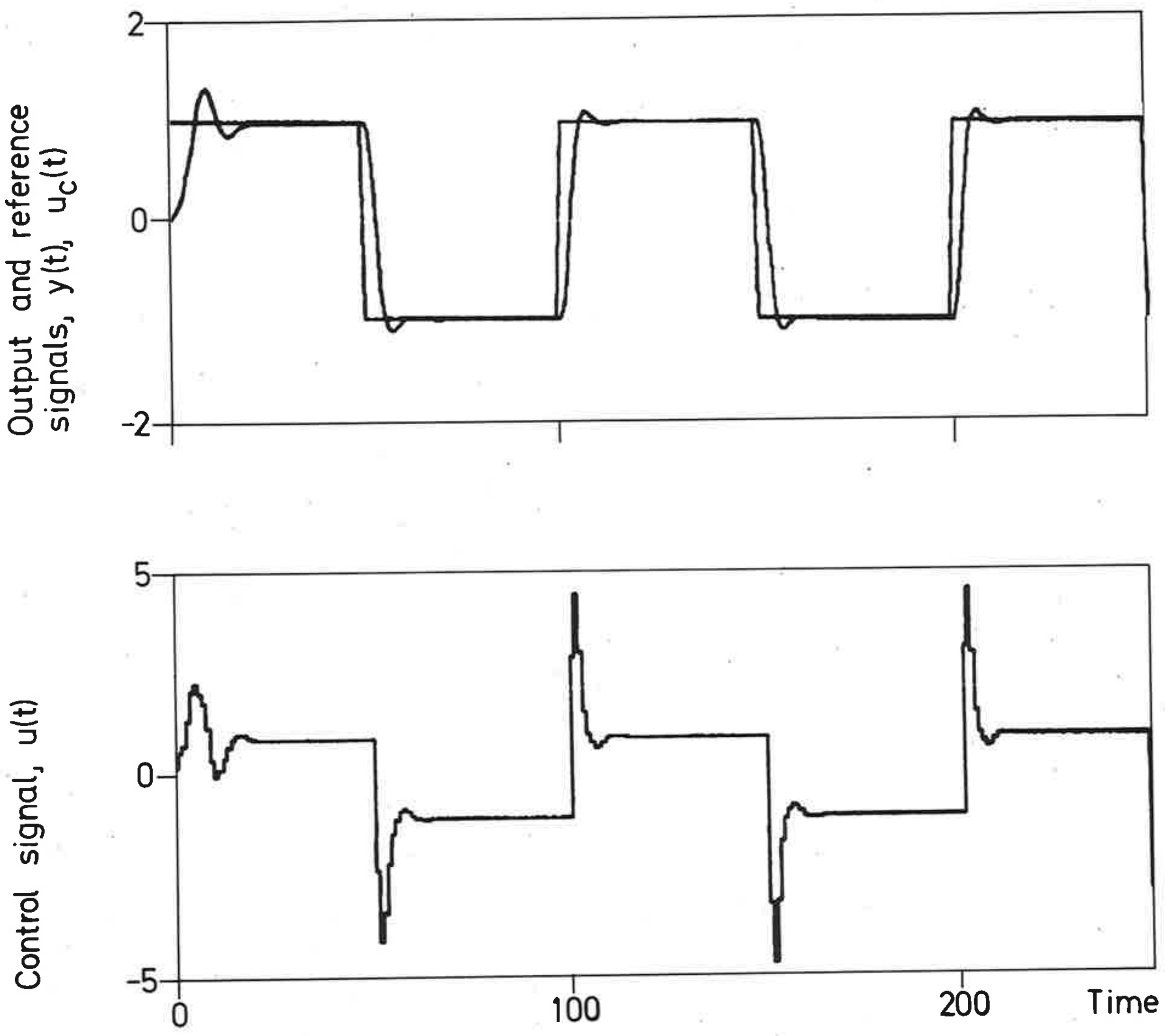


Fig. 6.6 - The results when the explicit algorithm E3 is used to control the continuous time system (6.2).

7. ACKNOWLEDGEMENTS

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