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SIMPLE SELF-TUNERS I

K. J. ÅSTRÖM

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Contents

1. INTRODUCTION

A PI-regulator has only two tuning parameters, which are easy to adjust using simple tuning rules. The PID-regulator, which has three tuning parameters, is more difficult to tune. A consequence of this is that the derivative action is often switched off in industrial applications. One motivation for introducing a self-tuning regulator is to simplify the tuning problem. The self-tuning regulator has, however, also tuning parameters. When using a self-tuner it can thus be said that one set of tuning parameters is replaced by another set. Hopefully the new parameters are easier to choose. In the early applications (Åström et al, 1977) the self-tuners were applied to special problems. Good rules for choosing the parameters could then be found (Wittenmark, 1973). Some parameters are, however, critical. For self-tuners based on minimum variance control and least squares parameter estimation it is crucial to have an upper bound on the time delay of the process.

It has also been proposed to design universal self-tuners which do not have any adjustable parameters. A moments reflection shows that this is an impossible task. It is at least necessary to provide the self-tuner with information about the desired specification. Once this is realized there appear to be many possibilities to design practical regulators for special classes of problems based on self--tuning. These regulators will all have tuning parameters which have to be selected by the user. The tuning parameters may, however, be directly performance-related and consequently also easy to choose.

This report is the first in a series which describes simple self--tuners. The particular self-tuner is intended for simple servo applications where the process can be described by a low order model. The regulator is based on recursive least squares parameter estimation and pole-placement design. See Åström and Wittenmark (1979). The regulator has only *one* tuning parameter, namely the desired bandwidth of the closed loop system. Apart from being of interest in themselves the results also give interesting insights into the properties of self-tuning regulators. The regulator discussed is related to the self-tuning PID regulator discussed in Wittenmark (1979). The main difference is that a PID structure is not imposed on the regulator.

The report is organized as follows. The algorithm is presented in Section 2 together with a discussion of the major trade-offs done in the algorithm. Section 3 presents a few simulations which show the behaviour of the algorithm in typical cases. Problems which appear appropriate for using the algorithm and those which do not are discussed in Section 4. The tuning rules for the algorithm are also given in that section.

2. ALGORITHM DESIGN

The algorithm is intended to solve simple servo problems for systems which can be described by low order models. It is natural to characterize the performance of a simple servo by the bandwidth of the closed loop system. The basic idea is thus to design a self-tuning servo which literally speaking only has one dial, marked bandwidth, on the front panel. A servo problem is conveniently formulated in terms of pole-placement. It is then natural to use the formulation of self-tuning servos discussed in Åström and Wittenmark (1979). A low order process model is thus estimated recursively. Using the pole-placement design method a regulator is then designed which gives a closed loop system with the desired bandwidth. Having specified the desired bandwidth the other system parameters like sampling period are then determined from the desired closed loop bandwidth. Since the low frequency behaviour of many systems can be approximated by low order models it can be expected that the self--tuning regulator will work satisfactorily, provided that the chosen bandwidth is sufficiently small. An indication of this is e.g. that the overshoot of the closed loop system is that specified in the design. This is supported by the sensitivity analysis presented in Åström (1979a), which guarantees that the pole-placement design based on a simplified model will always give a stable closed loop system provided that the process is stable and the desired bandwidth sufficiently small. As the desired bandwidth is increased it can be expected that the closed loop system will respond faster. As the bandwidth of the system is increased it may, however, happen that a low order model is not sufficient. The performance of the self-tuner will then deteriorate. The overshoot may deviate from the specifications and the closed loop system may be unstable. The tuning rule for the regulator is thus very simple. Start with a small bandwidth. Establish the possible range of bandwidths for which the regulator will work for the process by increasing the specified bandwidth until the overshoot deviates from the specified value. If the desired bandwidth is outside the range found it is necessary to use a more complex regulator. Since the tuning involves only one parameter it chould be fairly easy to do.

Problem Formulation

Consider a process described by the model

$$y(t) + a_1 y(t-h) + a_2 y(t-2h) = b_1 u(t-h) + b_2 u(t-2h) + b_3 \qquad (2.1)$$

where h is the sampling period and b_3 is a bias term. Find a feedback such that the closed loop system has poles in

$$z = e^{-\zeta\omega h} (\cos \omega h \sqrt{1-\zeta^2} \pm i \sin \omega h \sqrt{1-\zeta^2}). \qquad (2.2)$$

This means that the closed loop poles correspond to a continuous time system with poles in

$$s = -\zeta\omega \pm i\omega\sqrt{1-\zeta^2}.$$

The characteristic polynomial for the closed loop system is thus given by

$$P(z) = z^2 + p_1 z + p_2$$

where

$$\begin{aligned} p_1 &= -2 e^{-\zeta\omega h} \cos \omega h \sqrt{1-\zeta^2} \\ p_2 &= e^{-2\zeta\omega h}. \end{aligned} \qquad (2.3)$$

The process model has a zero at

$$z = -b_2/b_1.$$

If this zero corresponds to a welldamped mode then the factor $b_1 z + b_2$ can be cancelled by the regulator and the desired closed loop response is characterized by the pulse transfer function

$$G_d(z) = \frac{z\,(1 + p_1 + p_2)}{z^2 + p_1 z + p_2}. \qquad (2.4)$$

The scaling factor is necessary to ensure that the desired closed loop transfer function has unit gain.

If the process zero corresponds to an unstable ($|b_2| > |b_1|$) or poorly damped mode the factor $b_1 z + b_2$ can not be cancelled by the regulator and the desired closed loop transfer function is instead given by

$$G_d(z) = \frac{1 + p_1 + p_2}{b_1 + b_2} \cdot \frac{b_1 z + b_2}{z^2 + p_1 z + p_2} . \tag{2.5}$$

To complete the problem statement it is also necessary to specify the observer. To simplify the problem and to avoid the necessity of introducing extra parameters it is assumed that a Luenberger observer is used and that the observer polynomial is

$$T_1(z) = z. \tag{2.6}$$

This means that the observer settles in one sampling period. It also means that the characteristics of the observer will critically depend on the chosen sampling period.

Control Design for Known Parameters

The calculation of the control law when the process model (2.1) is known is straightforward. See e.g. Åström (1979b). The feedback law is given by

$$Ru = Tu_c - Sy - u_b \tag{2.7}$$

where u_c is the command signal.

When the process zero is cancelled the polynomials R, S, and T are given by

$$\left\{\begin{aligned} r_1 &= -\, b_2/b_1 \\ s_0 &= (p_1-a_1)/b_1 \\ s_1 &= (p_2-a_2)/b_1 \\ t_0 &= (1+p_1+p_2)/b_1 \\ u_b &= b_3/b_1. \end{aligned}\right. \tag{2.8}$$

When the process zero is not cancelled the polynomials R, S satisfy the algebraic equation

$$AR + BS = PT_1 \tag{2.9}$$

where

$$T = t_0\, T_1.$$

The unique solution with deg R = deg S = 1 is chosen. The equation (2.9) can then be written as

$$(z^2 + a_1 z + a_2)(z + r_1) + (b_1 z + b_2)(s_0 z + s_1) = z^3 + p_1 z^2 + p_2 z.$$

The following equations are then obtained:

$$a_1 + r_1 + b_1 s_0 = p_1$$

$$a_2 + a_1 r_1 + b_2 s_0 + b_1 s_1 = p_2$$

$$a_2 r_1 + b_2 s_1 = 0.$$

The solution is

$$\left\{\begin{aligned} r_1 &= [(p_1-a_1)\, b_2^2 - (p_2-a_2)\, b_1 b_2] / N \\ s_0 &= [(p_1-a_1)(a_2 b_1 - a_1 b_2) + (p_2-a_2)\, b_2] / N \\ s_1 &= -\, a_2 r_1 / b_2 \\ t_0 &= (1+p_1+p_2) / (b_1+b_2) \\ N &= b_2^2 - a_1 b_1 b_2 + a_2 b_1^2. \end{aligned}\right. \tag{2.10}$$

The polynomial T is given by

$$T = z(1 + p_1 + p_2) / (b_1 + b_2). \tag{2.11}$$

To find the control law when there is a bias b_3 in the process model, use the design identity (2.9) i.e.

$$PTy = ARy + BSy.$$

Substitute Ay using the process model (2.1) i.e.

$$PTy = RBu + Rb_3 + BSy = B(Ru + Sy) + Rb_3 \equiv BTu_c.$$

Hence

$$Ru = Tu_c - Sy - \frac{R}{B} b_3.$$

Since B is not always a stable operator the operator R/B is replaced by its static gain. Hence

$$u_b = \frac{(1 + r_1)\, b_3}{b_1 + b_2}. \tag{2.12}$$

Common Factors in the Process Model

The linear polynomial equation (2.9) is singular when the polynomials A and B have common factors. In the particular case this happens when $z = -b_2/b_1$ is a zero of the polynomial A i.e.

$$N = b_2^2 - a_1 b_1 b_2 + a_1 b_1^2 = 0.$$

When N is small the equations are also poorly conditioned. This means that the parameters r_1, s_0, and s_1 which define the feedback law may become very large. This is a potential danger since the high feedback gains may result in very large control signals. It is easy to avoid the difficulty simply by canceling the common factor in the estimated transfer function and design the appropriate feedback for the reduced model. The process model (2.1) has a zero at $z = -b_2/b_1$. If there is a corresponding pole we get

$$\frac{b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_1(z + b_2/b_1)}{(z + b_2/b_1)(z + a)} = \frac{b}{z + a}.$$

Hence

$$\begin{cases} b = b_1 \\ a = a_2 b_1/b_2 = a_1 - b_2/b_1 . \end{cases} \tag{2.13}$$

To design a feedback (2.7) for the process model

$$y(t) + ay(t-1) = bu(t-1) \tag{2.14}$$

which gives the closed loop transfer function (2.4) the equation (2.9) has to be solved. Since the model is of first order we can choose $T_1 = 1$. A solution to (2.9) is then given by

$$\begin{cases} r_1 = 0 \\ s_0 = \dfrac{p_1 - a_1}{b} \\ s_1 = \dfrac{p_2}{b} \\ t_0 = \dfrac{1 + p_1 + p_2}{b} \\ u_b = \dfrac{b_3}{b_1} . \end{cases} \tag{2.15}$$

The variable N is used as a test quantity to decide when the estimated process model has common factors. To obtain a dimensionfree test quantity the following test is used

$$N \lesssim \varepsilon [\max(b_1^2, b_2^2)] \tag{2.16}$$

where the number ε determines the maximum size of the feedback gain. An example is used to determine a suitable size of ε.

EXAMPLE 2.1

Consider the continuous time process

$$\frac{dy}{dt} = -y + u.$$

Let this process be controlled by an adaptive controller where it is desired to have the closed loop poles given by (2.2) with

$\omega = 1.5$ and $\zeta = 0.707$. The adaptive regulator is initialized with a second order process model with

$$\hat{a}_1 = -1.5 \qquad \hat{b}_1 = 0$$
$$\hat{a}_2 = 0.7 \qquad \hat{b}_2 = 0$$

The simulation programs are given in Appendix A. Figure 2.1 shows what happens when the process zero is not cancelled when computing the control law.

Notice that the controller performs very poorly although the estimates of the process parameters converge very quickly. The reason for the poor performance is the common factor in the process transfer function. The presence of this common factor implies that the linear equations for the regulator gain are poorly conditioned. The regulator gains will therefore occasionally be very large. See for example the parameter s_0 in Fig. 2.1. Notice the scale! These very large gains imply that small numerical errors will give very large control signals and very erratic behaviour. In the particular case the common factor is $z - 0.736$. The particular value of the common zero, which is obtained, depends on the initial estimates. Figure 2.2 shows what happens when it is attempted to cancel the common factor before calculating the feedback gains. The test quantity used to decide if there is a common factor is given by (2.16).

The value $\varepsilon = 0.0001$ is used in Fig. 2.2. Notice that the closed loop system behaves quite well in this case but that the feedback gains are still quite large during the transient. The gains are almost two orders of magnitude larger in the transient compared to the steady state case.

To ensure that the feedback gains have reasonable values also during the transients it is necessary to use smaller values of ε. Figure 2.3 shows a simulation with $\varepsilon = 0.01$. □

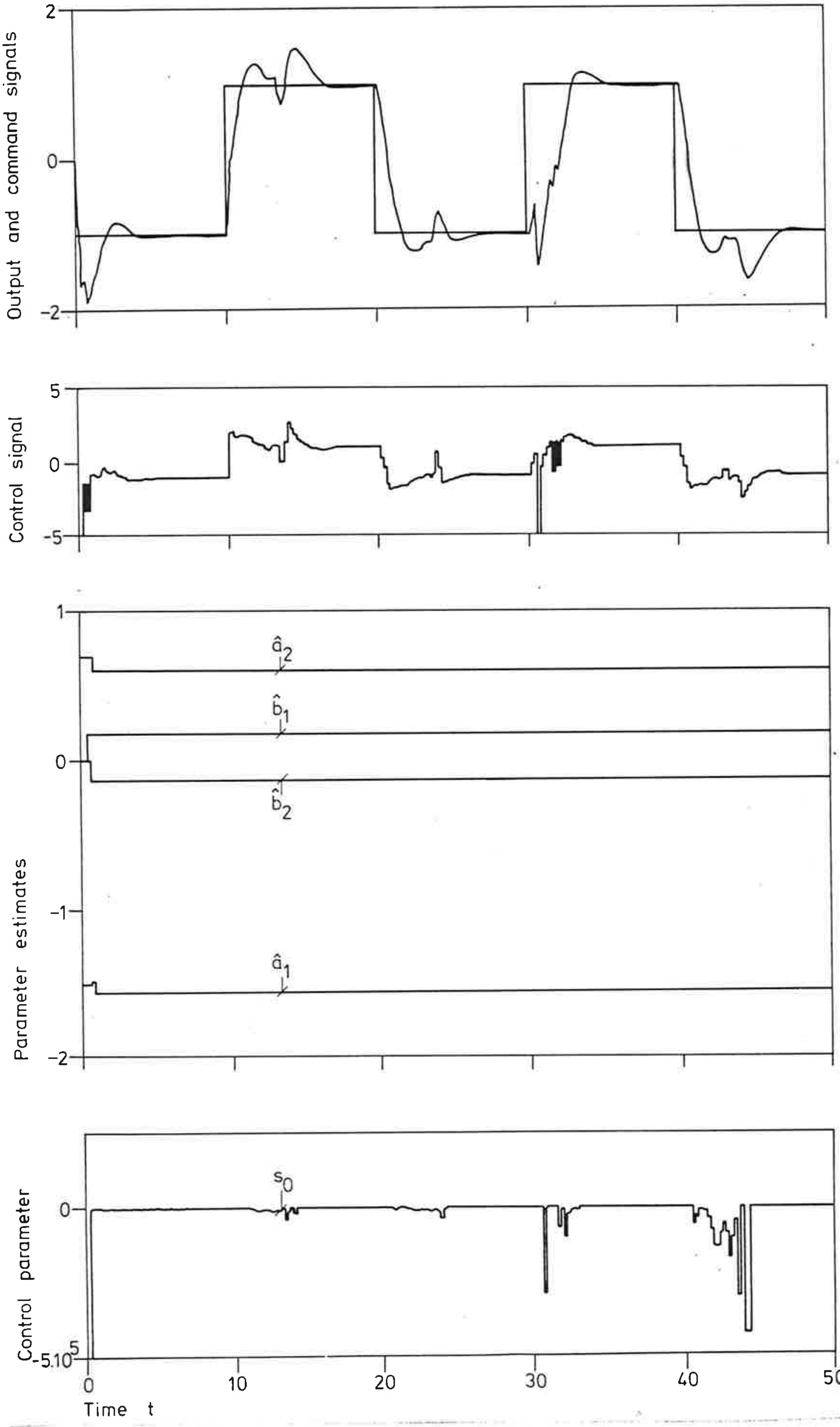


Fig. 2.1 - Simulation of a self-tuning controller based on a second order process model on a first order process.

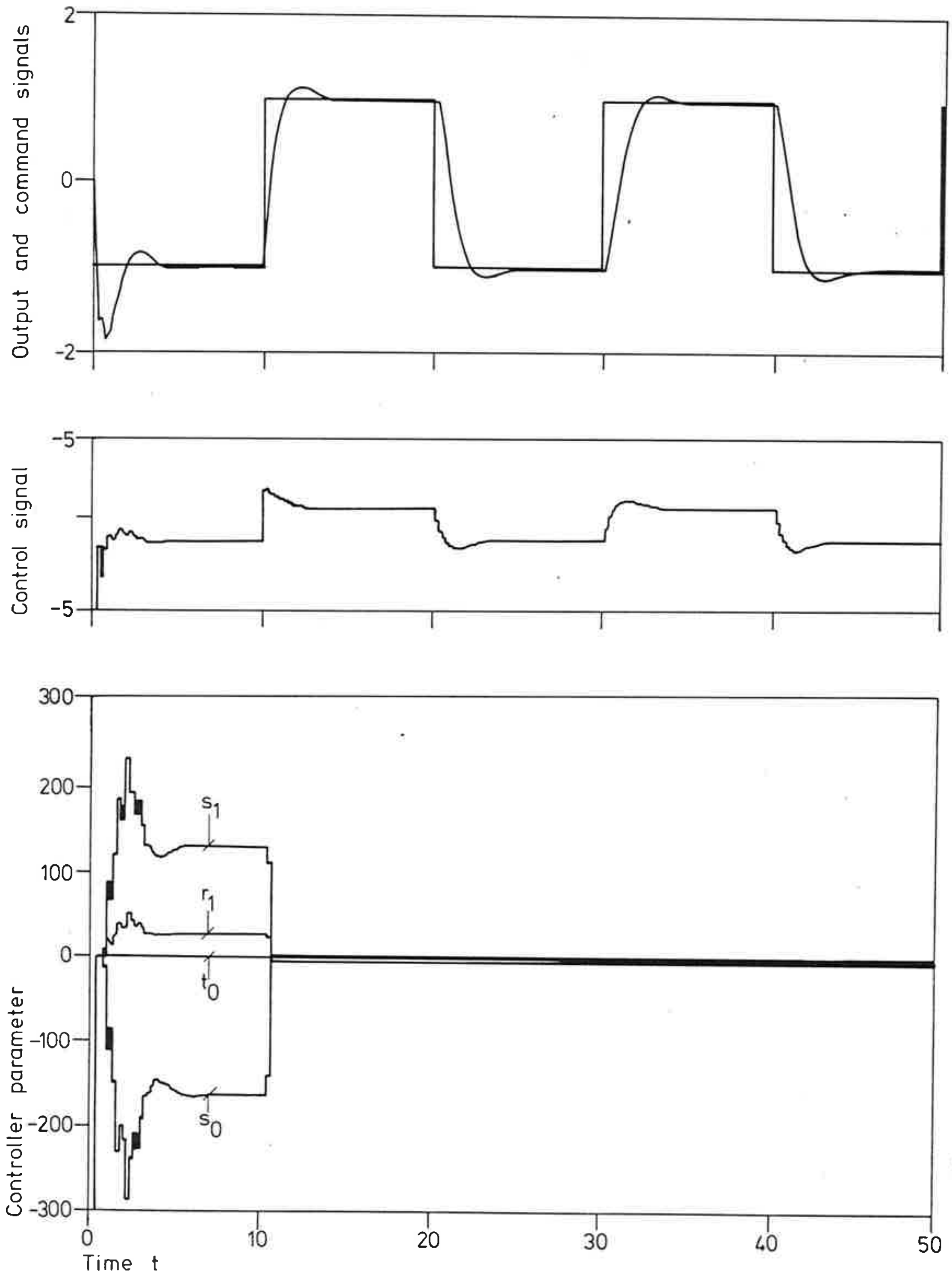


Fig. 2.2 - Simulation of a self-tuning controller based on a second order process model. A possible common factor is cancelled before calculating the control law if the condition (2.16) holds with $\varepsilon = 0.0001$.

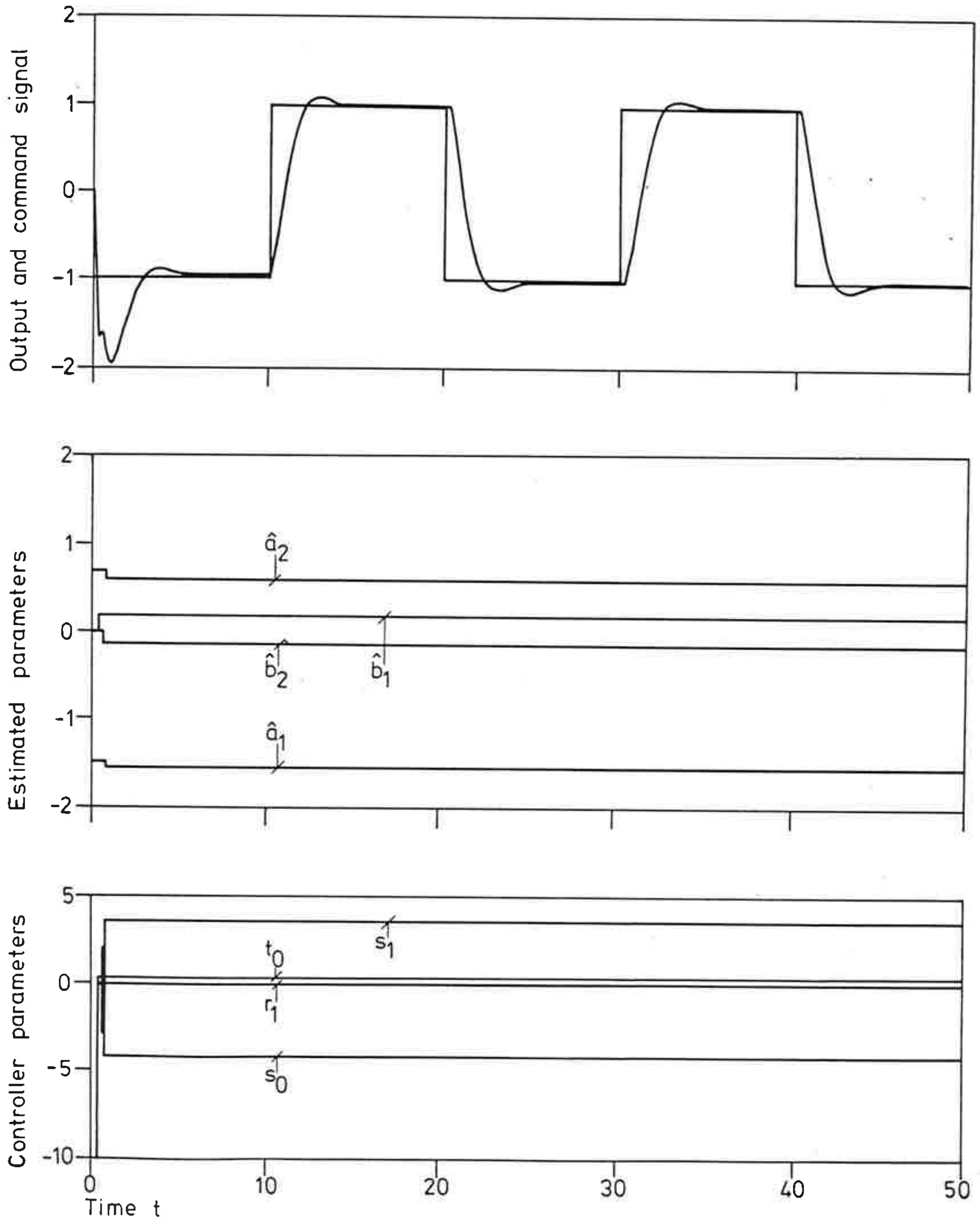


Fig. 2.3 - Simulation of a self-tuning controller based on a second order process model. A possible common factor in the process transfer function is cancelled before calculating the control law if the condition (2.16) holds with $\varepsilon = 0.01$.

It is clearly a practical necessity to cancel common factors in the explicit algorithm before attempting the control design. Based on the simulations in Example 2.1 and several other simulations, which are not reported, it was decided to choose $\varepsilon = 0.01$ to ensure that the feedback gains have reasonable magnitudes.

Initial Values of Parameter Estimates

A self-tuning regulator must be provided with initial values of the parameter estimates. The transient behaviour of the algorithm will depend on the chosen initial values. For an explicit algorithm the parameter estimates are simply the coefficients of a sampled process model. If apriori knowledge is available it is easy to use those. It is also possible to initialize the model with all parameters equal to zero. This means however that the process gain is very small. Consequently the controller gain will be very high and the initial transient may be very large.

In a practical problem it is therefore useful to have reasonable apriori estimates of the parameters. It is easy to find initial estimates of a_1 and a_2. If the process is sampled very fast we get

$$\hat{a}_1(0) = -2$$

$$\hat{a}_2(0) = 1 .$$

For slower sampling a reasonable compromize may be

$$\hat{a}_1(0) = -1.5$$

$$\hat{a}_2(0) = 0.7.$$

It is more difficult to find good universal estimates of the parameters b_1 and b_2, since any initial estimate will require that the process gain is known approximatively. An example is used to illustrate the consequences of different initial conditions.

EXAMPLE 2.2

Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

Let this process be controlled by an adaptive controller where it is desired to have a closed loop system with $\omega = 1.5$ and $\zeta = 0.707$. The programs for simulating the regulator and the process are the same as those used for Example 2.1. The program listings are found in Appendix A. The parameter values used in the simulation are given in Appendix B.

Figure 2.4 shows the results obtained when all initial values are set to zero. The initial behaviour is far from satisfactory. The controller gain is very large initially. It is only the hard bound on the control signal which keeps the control signal bounded. Figure 2.5 shows what happens when the initial estimates $\hat{a}_1(0) = -2$, $\hat{a}_2(0) = 1$, $\hat{b}_1(0) = \hat{b}_2(0) = 0$. The initial performance is clearly much better. Slightly better results are obtained for the initial values $\hat{a}_1(0) = -1.5$, $\hat{a}_2(0) = 0.7$, $\hat{b}_1(0) = \hat{b}_2(0) = 0$. Figure 2.6 finally shows what happens when very good initial values are chosen. □

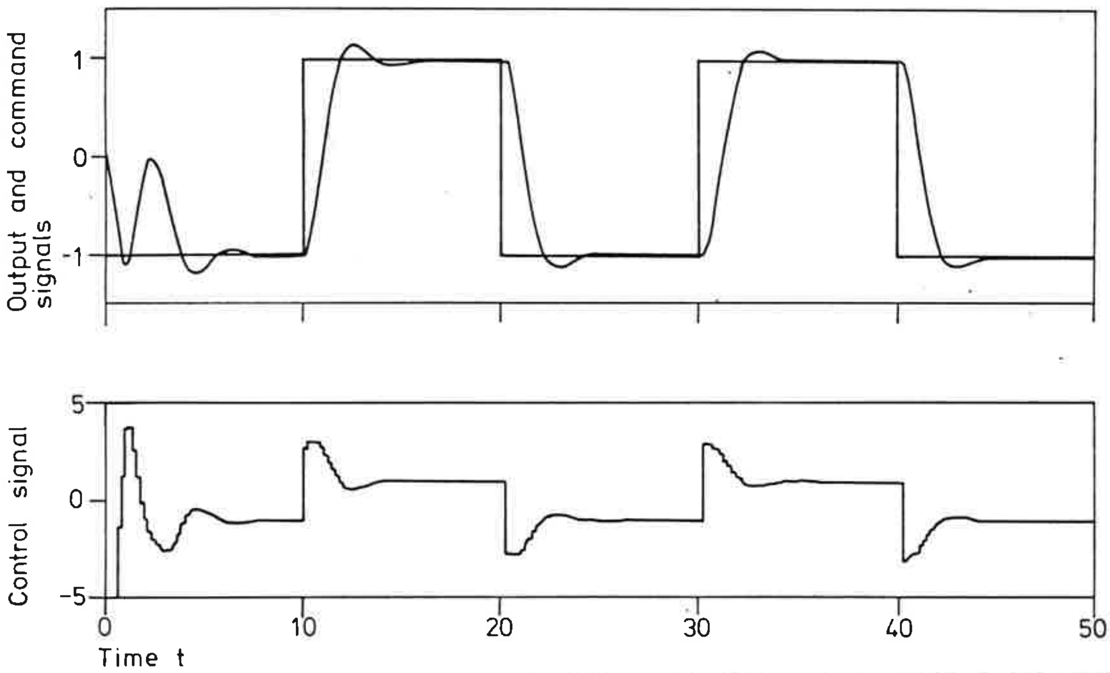


Fig. 2.4 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are $\hat{a}_1(0) = 0$, $\hat{a}_2(0) = 0$, $\hat{b}_1(0) = \hat{b}_2(0) = 0$.

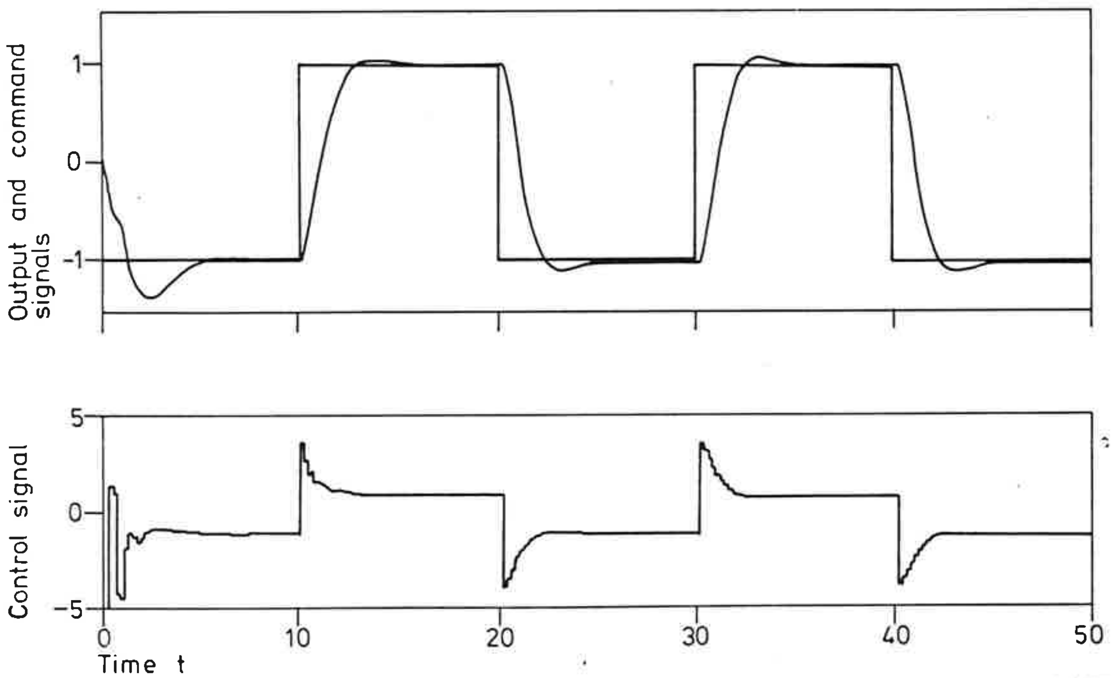


Fig. 2.5 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are $\hat{a}_1(0) = -2$, $\hat{a}_2(0) = 1$, $\hat{b}_1(0) = \hat{b}_2(0) = 0$.

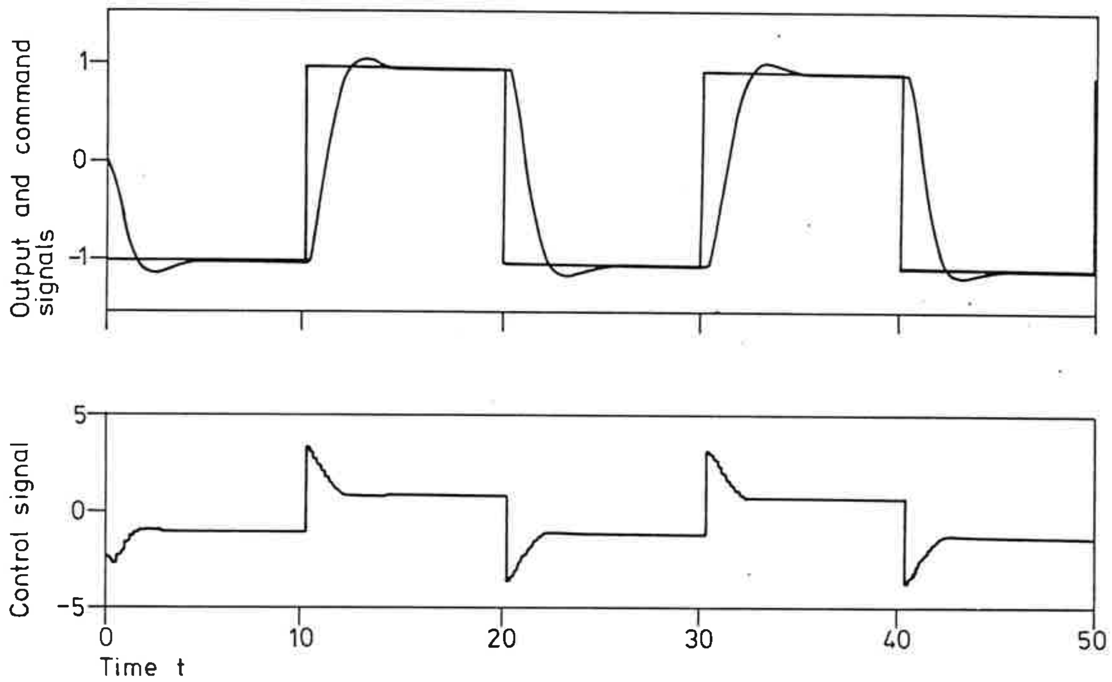


Fig. 2.6 - Simulation which illustrates the transient behaviour of the adaptive controller. The initial values are $\hat{a}_1(0) = -1.6$, $\hat{a}_2(0) = 0.7$, $\hat{b}_1(0) = 0.02$, $\hat{b}_2(0) = 0.01$.

Based on the results of the example it seems reasonable to choose the following initial conditions:

$$\hat{a}_1(0) = -1.5$$

$$\hat{a}_2(0) = 0.7$$

$$\hat{b}_1(0) = \hat{b}_2(0) = 0.$$

Since the estimates $\hat{b}_1$ and $\hat{b}_2$ are zero the controller gain will be very high initially. It is thus necessary to limit the control signal with this choice.

The Sampling Period

To obtain a self-tuning controller with only one knob on the front panel it is necessary to fix the sampling period. Since the setting of the knob gives the desired closed loop bandwidth it is natural to choose the sampling period inversely proportional to the bandwidth. A reasonable choice is

$$h = \frac{2\pi}{N_p \omega \sqrt{1-\zeta^2}} \tag{2.17}$$

where N is the number of samples per period. With $\zeta = \sqrt{2}/2$ we get

$$\omega h = 2\pi \sqrt{2}/N \approx 9/N.$$

An example is used to illustrate the consequences of choosing different values of N.

EXAMPLE 2.3

Consider the same system as in Example 2.2 i.e. a continuous time system with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

It is desired that the closed loop system has $\omega = 1.5$ and $\zeta = 0.707$. Figure 2.7 shows what happens when different sampling periods are used in the controller. The simulation programs used to generate the curves are listed in Appendix A. The parameter values used are given in Appendix C. In Figure 2.8 are shown the outputs of the regulators with 3, 10, and 30 samples per period at times 30 - 50. It is seen from this Figure that there are no drastic differences between the outputs of the systems. The overshoot is about 5 % higher for the system which has only 3 samples per period. □

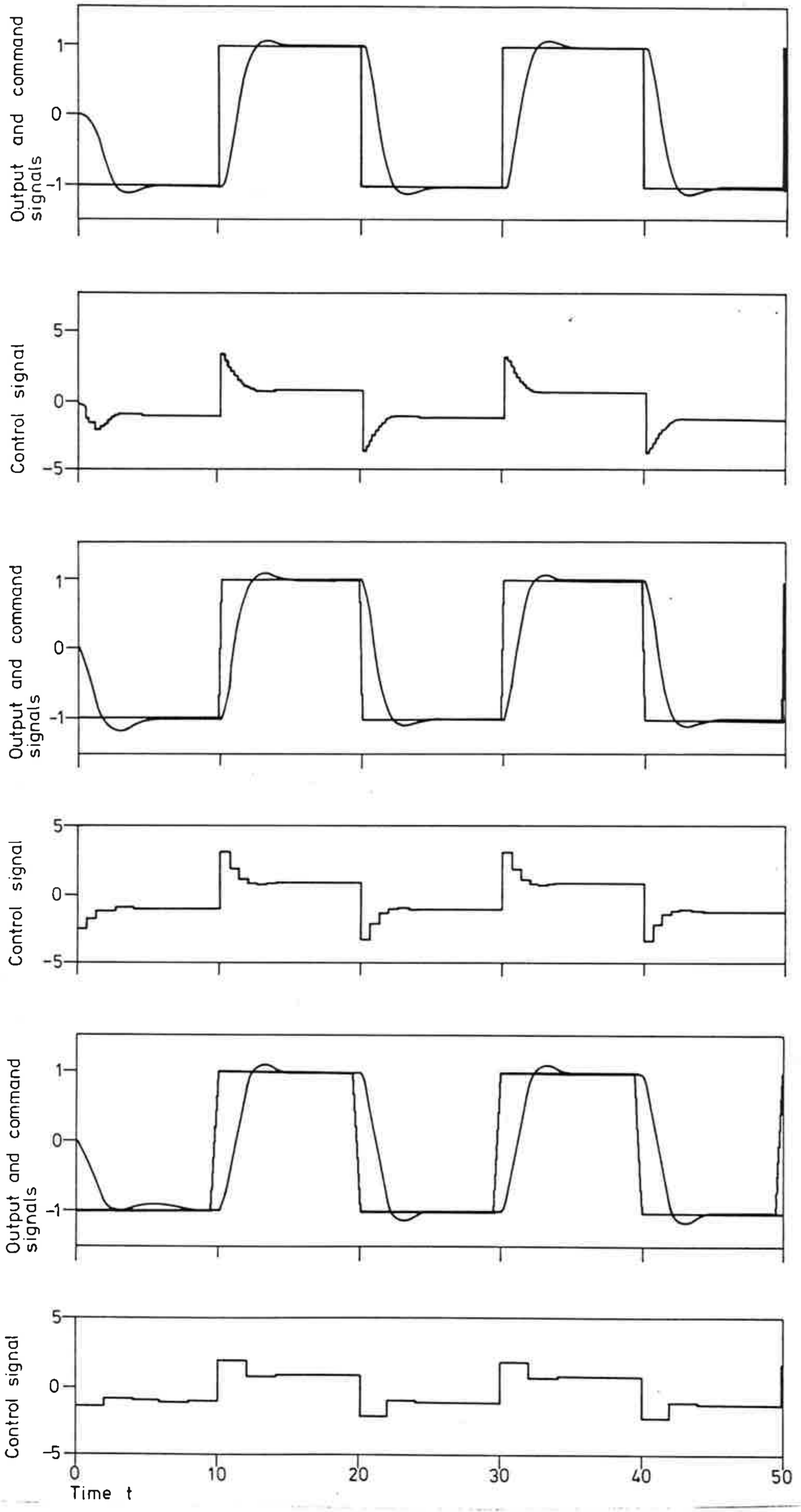


Fig. 2.7 - Illustrates the consequences of different choices of sampling periods in the controller. The curves correspond to 30, 10, and 3 points per period.

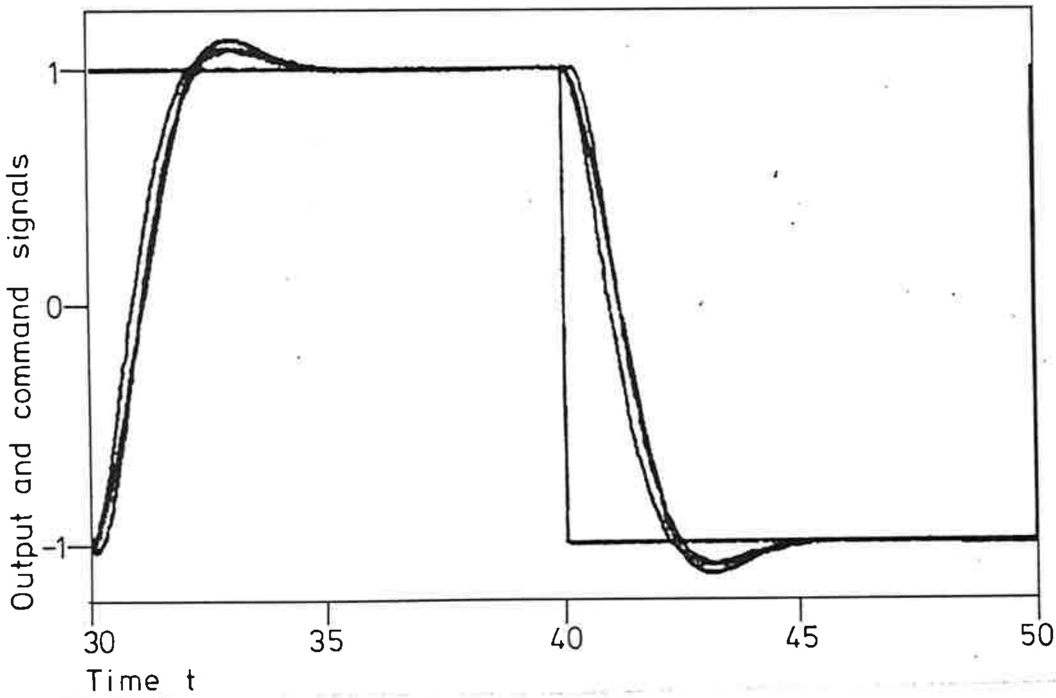


Fig. 2.8 - Outputs at times 30 - 50 of controllers with 3, 10, and 30 samples per period.

The example shows that the sampling period is not critical and that good behaviour is obtained for as slow sampling as 3 samples per period. With a sampled system there will always be an extra delay in response due to the sampling. This delay will on the average be half a sampling period. When N = 3 the delay is an appreciable fraction (≈ 15 %) of the settling time. For this reason it appears reasonable to choose a larger value of N. The sampling period also influences the robustness of the system to time delays and high order process dynamics. This is discussed in Section 3. Based on these considerations the sampling period is chosen so that there are 10 - 20 samples per period. To be specific let us choose

$$\omega h = 0.45.$$

If the sampling period is chosen in this way then it is necessary to change h whenever ω is changed. The parameter estimates will then automatically be modified. It seems, however, useful to speed up the tuning procedure by adjusting the parameter estimates when the sampling period is changed. In principle this is easy to do by first transforming the sampled data model to a continuous time model which is then sampled with the new sampling period. The calculations are, however, involved and it may be useful **to have** simplified approximative procedures which involve less calculations. One possibility is to approximate exp(sh) with the first term in a Taylor series expansion i.e.

$$z = \exp(sh) = 1 + sh.$$

Simple calculations then show that when the sampling period is changed from h to h_1 then the z-transform variable is transformed to

$$z_1 = (h_1/h)\, z + (1 - h_1/h).$$

The inverse transformation is

$$z = (h/h_1)\, z_1 + (1 - h/h_1).$$

Under this transfer function the pulse transfer function

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} \tag{2.18}$$

is transformed to

$$H_1(z_1) = \frac{b_1' z_1 + b_2'}{z_1^2 + a_1' z_1 + a_2'} \tag{2.19}$$

where

$$\left\{\begin{aligned} a_1' &= (2 + a_1)(h_1/h) - 2 \\ a_2' &= (1 + a_1 + a_2)(h_1/h)^2 - (2 + a_1)(h_1/h) + 1 \\ b_1' &= b_1(h_1/h) \\ b_2' &= (b_1 + b_2)(h_1/h)^2 - b_1(h_1/h). \end{aligned}\right. \tag{2.20}$$

When the desired bandwidth and then also the sampling period is changed the process parameters are thus transformed using the above transformation before the updating is started.

Reset

For all practical regulators it is important to incorporate reset action i.e. the elimination of small static errors. For simple controllers reset action is obtained by integrating the error and making the control signal proportional to the integral of the error. For self-tuning controllers reset can be obtained through several mechanisms. One possibility is to get reset indirectly via the parameter estimation algorithm. This is demonstrated by the following theorem.

THEOREM 1

Consider a self-tuning regulator based on estimation of the parameters of the model (2.1) and determination of the control law (2.7) by solving the equation (2.9). Assume that the command signal u_c is constant. If the closed loop system is in steady state with constant parameter estimates then the control error is zero.

Proof

Let the constant signals be denoted by the superscript "^{o}". Let A^o denote $\hat{A}(1)$ where the parameters of the polynomial have constant values. The closed loop system is then described by the equations

$$A^o y^o = B^o u^o + b_3^o \qquad (2.21)$$

$$R^o u^o = T^o u_c^o - s y^o - (R^o/B^o)\, b_3^o \qquad (2.22)$$

$$A^o R^o + B^o S^o = P^o T_1^o \qquad (2.23)$$

$$T^o = T_1^o P^o / B^o. \qquad (2.24)$$

Equations (2.21) and (2.22) give

$$A^o R^o y^o = B^o R^o u^o + R^o b_3^o = B^o T^o u_c - B^o S^o y^o - R^o b_3^o + R^o b_3^o.$$

Hence

$$(A^o R^o + B^o S^o)\, y^o = B^o T^o u_c = T_1^o P^o u_c.$$

It then follows from (2.23) that

$$P^oT_1^oy^o = P^oT_1^ou_c$$

which proves the result.

Corollary

Notice that it follows from the proof that the result holds even if the parameter b_3 is not estimated. □

The corollary indicates that it is not necessary to introduce integral action in order to eliminate the steady state errors. When using the self-tuning regulator the steady state off-set is instead eliminated indirectly via the parameter estimation. A simulation example will provide further insight.

EXAMPLE 2.4

Consider a process with the transfer function

$$G(s) = \frac{1}{(s+1)^2}.$$

Assume that it is desired to have a closed loop system with poles such that $\omega = 1.5$ and $\zeta = 0.707$. A fixed sampled regulator with sampling period h which achieves this is given by

$$u(t) = t_0u_c(t) - s_0y(t) - s_1y(t-h) - r_1u(t-h)$$

where

$$r_1 = 0.0198 \qquad s_0 = 2.060$$

$$t_0 = 2.215 \qquad s_1 = -0.865$$

Figure 2.9 shows what happens with this regulator if the process is subject to a load disturbance.

It is clear from the figure that the constant gain regulator is not capable of dealing with the load disturbance. This is easy to see because the gain is fairly low and there is no integral action.

The corresponding simulation of the self-tuning controller, where the parameter b_3 is kept equal to zero, is shown in Fig. 2.10.

Notice that there is no steady state error as can be expected from the corollary of Theorem 1. Also notice that the step response is unsymmetric and that the parameter estimates change drastically during the transient. □

Example 2.4 shows clearly that although a regulator, where the bias b_3 is not estimated, will give zero steady state error the regulator will have other undesirable properties. It therefore seems useful to estimate the bias b_3. The problem is how the bias should be estimated. A simulation example is used to give insight.

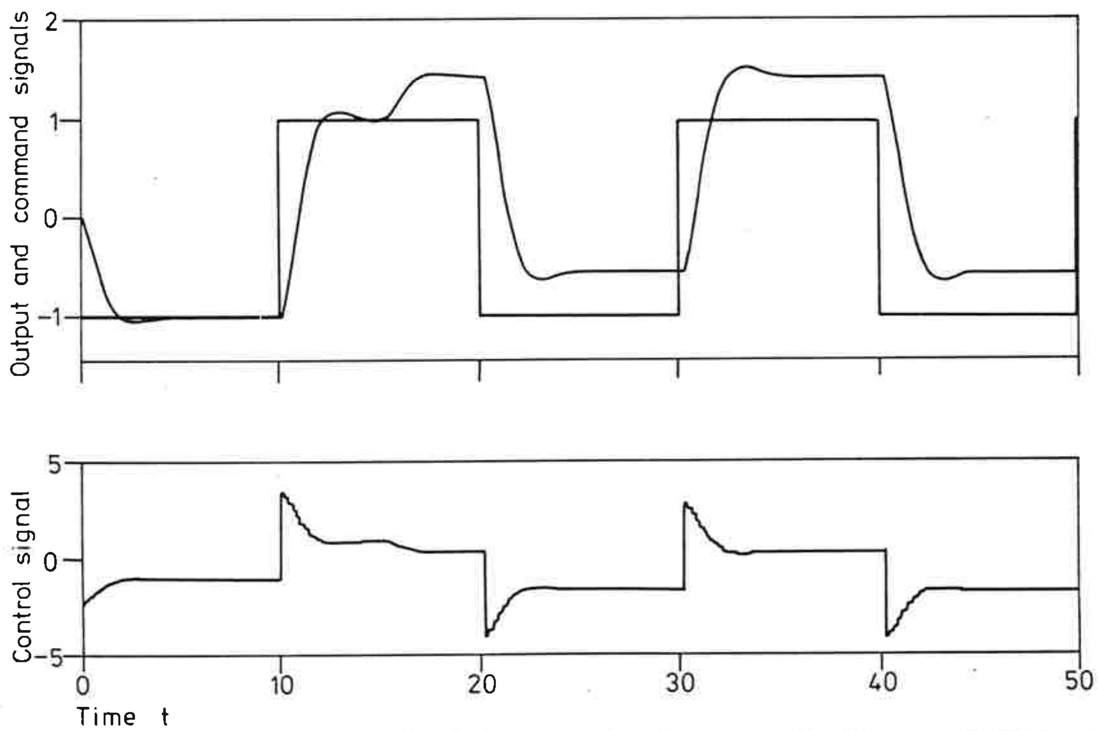


Fig. 2.9 - Simulation of the process $G(s) = (1+s)^{-2}$ with a constant gain regulator. The process is subject to a unit gain load disturbance at time t = 15 s.

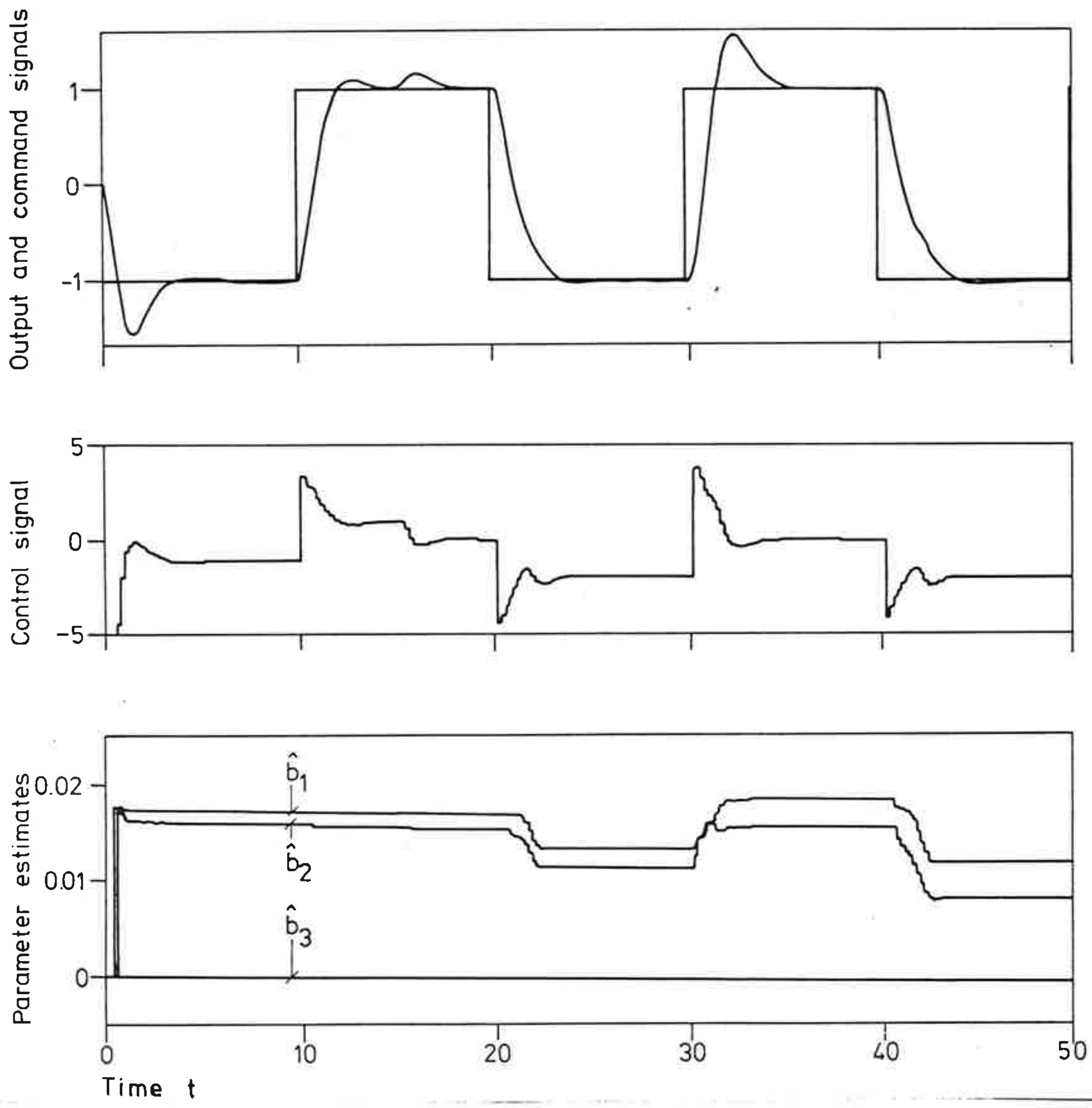


Fig. 2.10 - Simulation of the process $G(s) = (1+s)^{-2}$ with a self-tuning controller where the bias b_3 is not estimated. The process is subject to a unit step load disturbance at time $t = 15$ s.

EXAMPLE 2.5

Consider the same process and the same regulator as in Example 2.4 but estimate the bias too. Fig. 2.11 shows what happens. There is no steady state error. The response at the system is, however, unsymmetric and the parameters estimates also change significantly during the transients. As time increases the estimate of the bias will, however, improve and the step responses become more and more symmetric. □

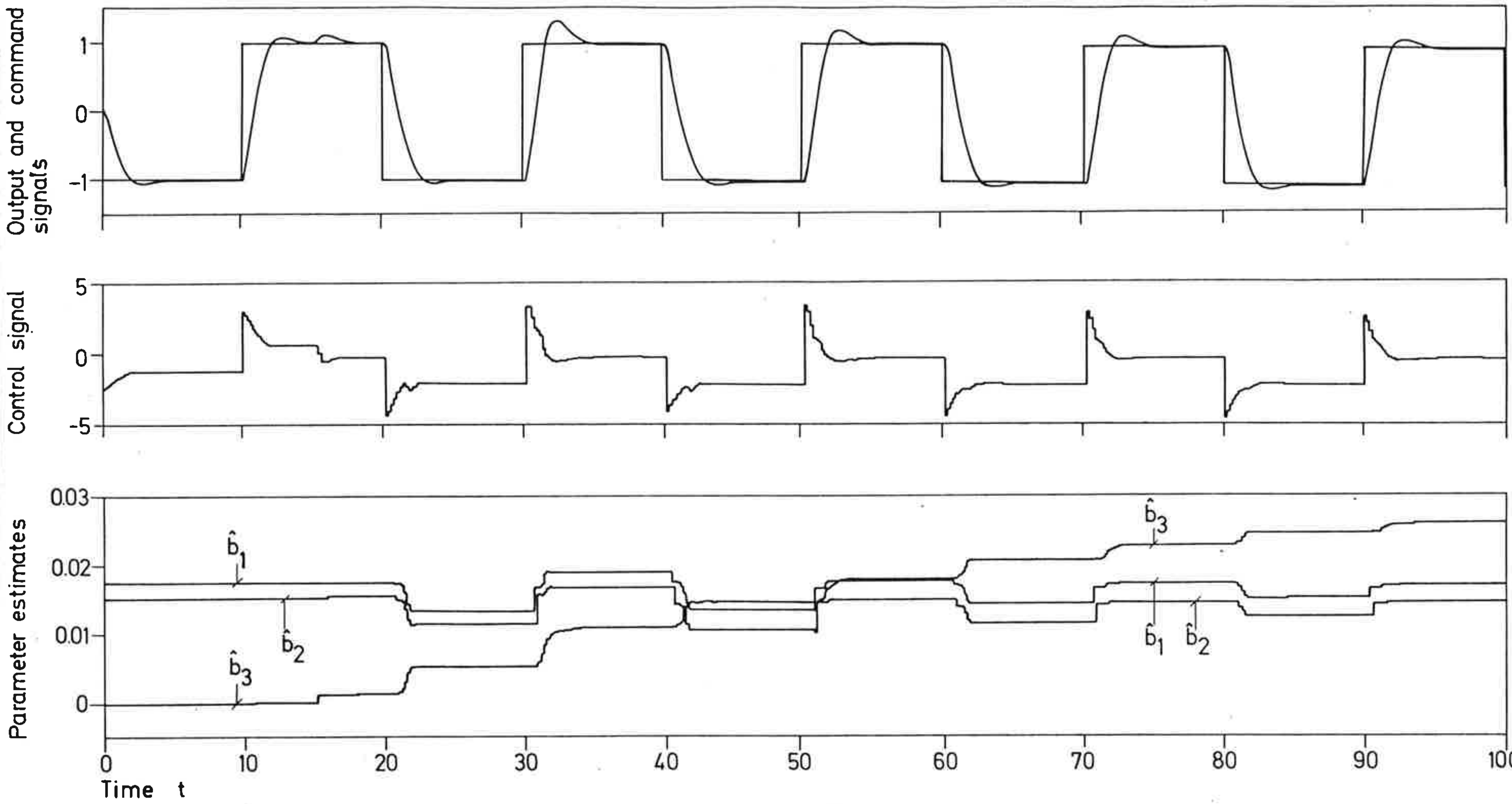


Fig. 2.11 - Simulation of the process $G(s) = (1+s)^{-2}$ with a self-tuning controller. The process is subject to a unit step load disturbance at time 15 s and the bias b_3 is estimated. The forgetting factor is 0.5 for the bias and 0.98 for the remaining parameters.

The behaviour shown in Example 2.5 indicates that it is advantegous to estimate the bias. The performance shown in Fig. 2.11 is, however, far from satisfactory. The convergence is slow and there is too much interaction between the estimates of the bias and of the other parameters. It is possible to improve the convergence rate by choosing a smaller forgetting factor. The estimates will, however, then be more irregular and there will still be a substantial interaction between the estimation of the bias and of the other parameters. The obvious solution is to use different forgetting factors for the bias and for the remaining parameters. It also seems reasonable to simplify the calculations by eliminating the cross covariances between the bias and the remaining parameters. An example illustrates what happens when this is done.

EXAMPLE 2.6

Consider the same process as in Example 2.5 but use now a separate forgetting factor for estimating the bias. The forgetting factor $\lambda = 0.98$ is used for all parameters except for the bias for which $\lambda = 0.5$ is used. The details of the algorithm are given in the program listing in Appendix A.

Figure 2.12 shows what happens. Notice that the transient behaviour of the adaptation loop is now much faster. Also notice that there is much smaller interaction between the estimation of the bias and the other parameters. The estimate of the bias now changes much more rapidly than before. □

The conclusions is thus that it is useful to estimate the bias b_3 in the model and that it is advantageous to use a smaller forgetting factor for the bias.

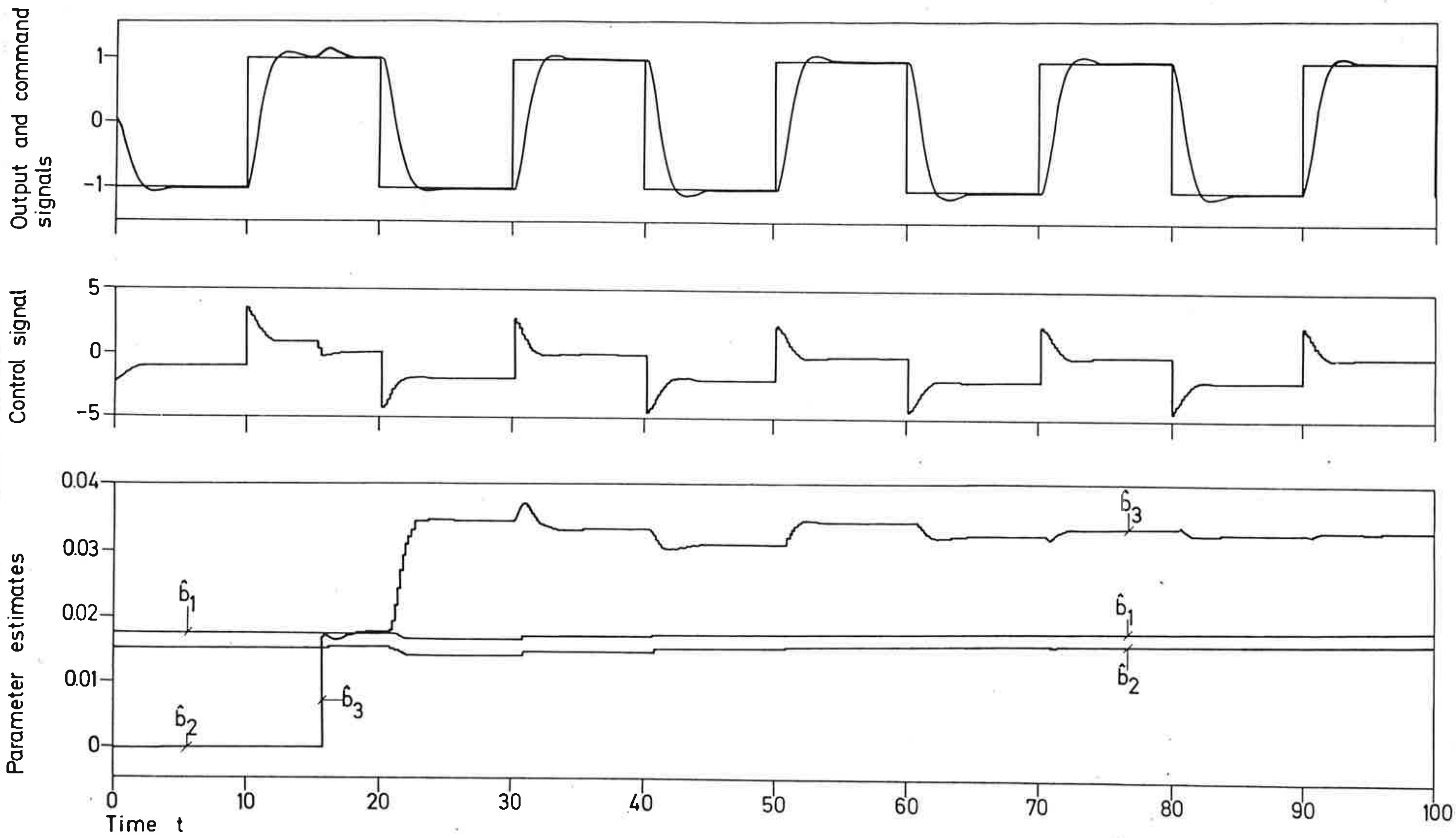


Fig. 2.12 - Simulation of the process $G(s) = (1+s)^{-2}$ with a self-tuning controller. The process is subject to a unit step load disturbance at time 15 and the bias b_3 is estimated.

3. THE ALGORITHM

Based on the discussion in Section 2 it appears possible to design a self-tuning servo which has only one adjustable parameter namely the desired bandwidth of the closed loop system. The basic principles of such a regulator are given below.

Parameter Estimation

The parameters a_1, a_2, b_1, b_2, and b_3 of the process model

$$y(t) + a_1y(t-1) + a_2y(t-2) = b_1u(t-1) + b_2u(t-2) + b_3 \qquad (3.1)$$

are estimated using least squares. The forgetting factors are applied only to the diagonal elements of the P-matrix. Separate forgetting factors are used for the bias b_3 and for the remaining parameters. The cross-coupling between the bias and the remaining parameters are neglected. The Simnon code for the parameter estimation is listed below.

```
"PART 1 PARAMETER ESTIMATION

E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5

"ESTIMATOR GAIN
K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5

"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D

"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
```

```
N13=P13-K1*K3/D
N14=P14-K1*K4/D
N22=(P22-K2*K2/D)/LAM
N23=P23-K2*K3/D
N24=P24-K2*K4/D
N33=(P33-K3*K3/D)/LAM
N34=P34-K3*K4/D
N44=(P44-K4*K4/D)/LAM
N55=(P55-K5*K5/D)/LAM1

"UPDATE F-VECTOR
NF1=-Y
NF2=F1
NF3=U
NF4=F3
```

Regulator Design

The following control law is used in the algorithm

$$u(t) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-n) - r_1 u(t-n) - u_b. \qquad (3.2)$$

The parameters t_0, s_0, s_1, and r_1 are chosen in such a way that the closed loop system has the poles obtained when sampling a continuous time system with poles

$$S = - \zeta\omega \pm \omega\sqrt{1 - \zeta^2}. \qquad (3.3)$$

Common zeros in the process model (3.1) are eliminated before calculating the control law. A normalized test-quantity ε is used to decide if there are common factors. The process model (3.1) has a zero at

$$z_1 = - b_2/b_1.$$

If this zero corresponds to a well damped mode i.e.

$$z_1 \leq - b_2/b_1 \leq z_2$$

then the process zero is cancelled. Otherwise the process zero is also retained as a zero of the closed loop system. In the design it is also postulated to use a Luenberger observer with a zero at the origin. Since the process model and the desired closed loop system are of low order the design equations can be solved analytically. The Simnon code for the design calculations is listed below.

```
"PART 2 - REGULATOR DESIGN

SQ=SQRT(1-Z*Z)
DT=2*PI/(NP*W*SQ)
P1=-2*EXP(-(2*PI/NP)*Z/SQ)*COS(2*PI/NP)
P2=EXP(-2*(2*PI/NP)*Z/SQ
PS=1+P1+P2
A1=TH1
A2=TH2
B1=TH3
B2=TH4
BS=B1+B2
BMAX=MAX(B1,B2)

N=B2*B2-A1*B1*B2+A2*B1*B1
W1=P1-A1
W2=P2-A2
W3=W1*B2-W2*B1
W4=W1*(A2*B1-A1*B2)+W2*B2
TEST=N/(BMAX*BMAX*EPS)
TEST1=IF -B2/B1<Z1 THEN -1 ELSE IF -B2/B1<Z2 THEN 1 ELSE -1

TO=IF TEST<1 THEN PS/B1 ELSE IF TEST1>0 THEN PS/B1 ELSE PS/BS
R1=IF TEST<1 THEN 0 ELSE B2*W3/N
SO=IF TEST<1 THEN P1/B1-A2/B2 ELSE IF TEST1>0 THEN (P1-A1)/B1 ELSE W4/N
S1=IF TEST<1 THEN P2/B1 ELSE IF TEST1>0 THEN (P2-A2)/B1 ELSE -A2*W3/N

UB=IF TEST<1 THEN TH5/B1 ELSE IF TEST1>0 THEN TH5/B1 ELSE (1+R1)*TH5/BS
U1=TO*UC-SO*Y-S1*(-F1)-R1*F3-UB

U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH
```

Parameters

The parameters of the regulator are listed below

ω, ζ	desired closed loop poles
N_p	number of samples per period
λ, λ_1	forgetting factors
ε	test quantity for common factors
z_1, z_2	bounds on welldamped zero

u_{high}, u_{low}	hard limit on control signal
$\hat{a}_1(0)$, $\hat{a}_2(0)$, $\hat{b}_1(0)$, $\hat{b}_2(0)$, $\hat{b}_3(0)$	initial estimates
P_0, P_{0b}	initial covariances

The parameter ω determines the desired closed loop bandwidth. The relative damping of the closed loop poles is normally set to 0.707. The sampling period is specified indirectly by requiring N_p to be 20 samples per period. There are reasonable universal values of the forgetting factor $\lambda = 0.98$ and $\lambda_1 = 0.8$. The test quantities are chosen as $\varepsilon = 0.01$, $z_1 = -0.1$, $z_2 = 0.99$. The hard limits of the output are normally set to values which would correspond to the limits of the DA-converters. The initial values are not crucial. They are chosen as follows: $\hat{a}_1(0) = -1.5$, $\hat{a}_2(0) = 0.7$, $\hat{b}_1(0) = \hat{b}_2(0) = \hat{b}_3(0) = 0$, $P_0 = 100$, $P_{0b} = 0.01$.

Application and Tuning Rules

The controller can be expected to give good results for processes which can be modeled by (3.1) and conveniently controlled by a feedback law like (3.2). Besides the self-tuning controller will provide reset action. The controller can thus be expected to give good results when applied to processes which can be modeled by first or second order models. Since many stable high order systems can also be modeled well by (3.1) if the sampling period is sufficiently large, the controller can thus be expected to work well also when applied to high order systems provided that the desired bandwidth is sufficiently small.

The rule for applying the algorithm is thus very simple. Start with a low value of ω. Subject the system to a step command and observe if the closed loop behaviour is as can be exptected. Increase the bandwidth gradually and observe when the response of the system starts to deviate from the expected (too high overshoot!) and when the control signal becomes so large that the system saturates. If the desired bandwidth can not be achieved it is **necessary to use a** more complex control law.

4. SIMULATIONS

The properties of the self-tuning servo in different situations will now be demonstrated using simulations.

Process Model

In all simulations it is assumed that the process to be controlled has the transfer function

$$G(s) = \frac{k}{(1+sT)^n}.$$

In the simulations a load disturbance in terms of a step on the process input is also added. The parameters k, T, and N are changed in the simulations. A Simnon program for the process model is listed below.

```
CONTINUOUS SYSTEM PROC2

TIME T
INPUT U V
OUTPUT Y
STATE X1 X2 X3 X4 X5 X6
DER D1 D2 D3 D4 D5 D6

OUTPUT
Y1=IF 0<2 THEN X1 ELSE IF 0<3 THEN X2 ELSE IF 0<4 THEN X3 ELSE X4
Y=IF 0<4 THEN Y1 ELSE IF 0<5 THEN X4 ELSE IF 0<6 THEN X5 ELSE X6
DYNAMICS
PA1=IF T<TPCH THEN 1 ELSE PA11
PA2=IF T<TPCH THEN 1 ELSE PA22
D1=-PA1*X1+PA2*U+V
D2=-PA1*X2+X1
D3=-PA1*X3+X2
D4=-PA1*X4+X3
D5=-PA1*X5+X4
D6=-PA1*X6+X5

PA11:1
PA22:1
TPCH:50
FIRST:-1
0:1
END
```

Bandwidth Changes

The case $n = 2$ is first discussed. Since the process model can be described exactly by (3.1) good results can be expected. Figures 4.1, 4.2, and 4.3 shows results obtained for $\omega = 0.5$, 1.5, and 4.5. Notice that the parameter estimates, except the bias, settle quickly in all cases. The irregular estimates of b_3 is the prize to be paid for having the possibility to quickly eliminate load disturbances. Also notice that the relative variations in the controller parameters is somewhat larger than in the process parameters. The regulators obtained for different values of ω are very different. This is seen from the comparison of the controller parameters in Table 4.1.

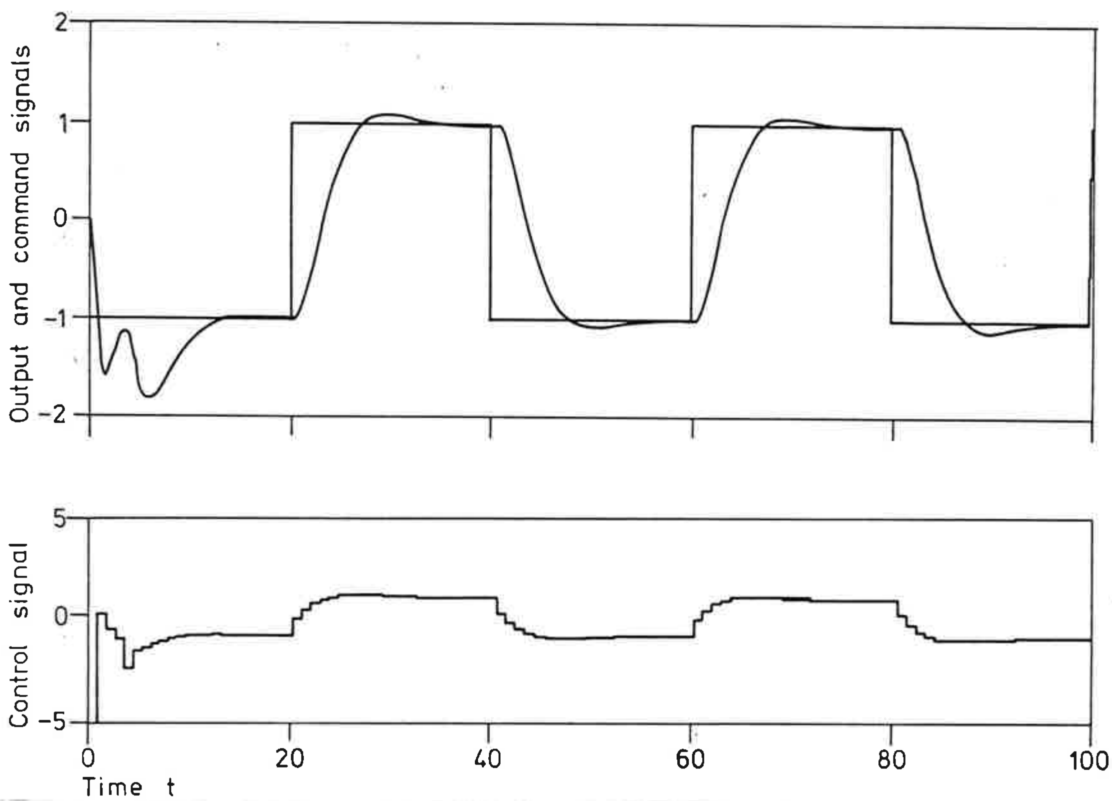


Fig. 4.1 - Simulation of the self-tuning controller applied to the system $(1+s)^{-2}$ with specifications $\omega = 0.5$.

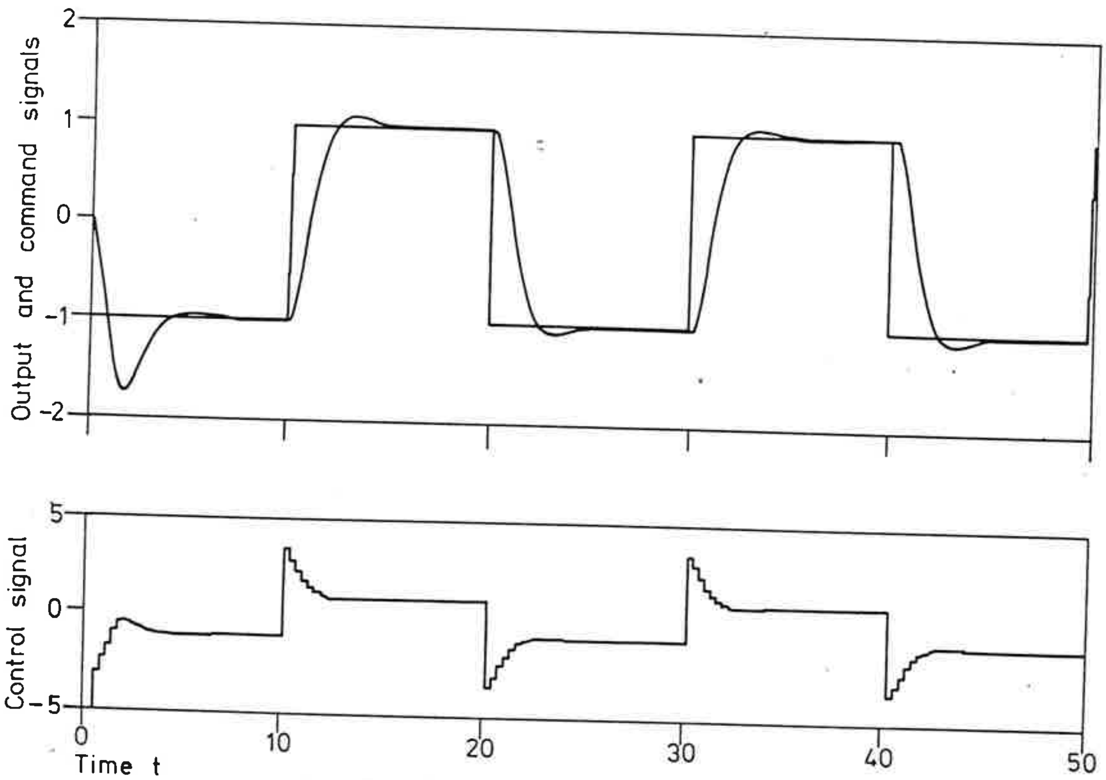


Fig. 4.2 - Simulation of the self-tuning controller applied to the system $(1+s)^{-2}$ with specifications $\omega = 1.5$

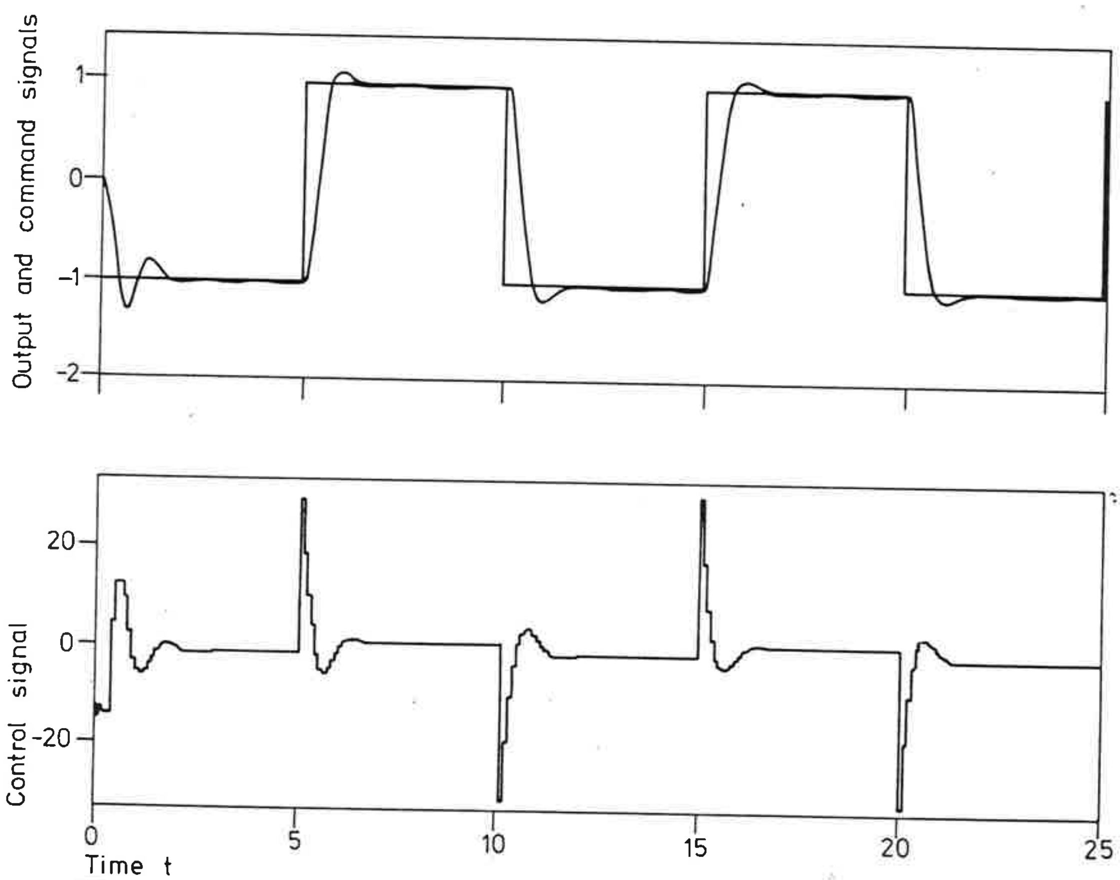


Fig. 4.3 - Simulation of the self-tuning controller applied to the system $(1+s)^{-2}$ with specifications $\omega = 4.5$.

Table 4.1 - Limiting values of the controller parameters.

ω	r_1	s_0	s_1	t_0
0.5	-0.332	-0.849	0.616	0.436
1.5	0.035	1.835	-0.663	2.208
4.5	0.189	51.26	-36.18	16.27

For $\omega = 0.5$ the natural process response is faster than the desired response. The regulator gain is therefore negative. The control signal in Fig. 4.1 also shows a gradual buildup. For $\omega =$ $= 1.5$ the controller gain is positive. In Fig. 4.2 it is seen that the initial value of the control signal at a step change is larger than the steady state value. For $\omega = 4.5$ the controller gain is quite large. It is also seen from Fig. 4.3 that the regulator generates very large control signals during the transients to achieve the desired performance. Notice that the scale is different in Fig. 4.3.

Load Changes

The behaviour of the algorithm subject to load changes were discussed extensively in Section 2. See for example Fig. 2.9. It should be remembered, however, that the properties of the system in this respect depend drastically on the chosen value of ω. In Fig. 4.4 are shown the response to load changes of a system with $\omega = 4.5$

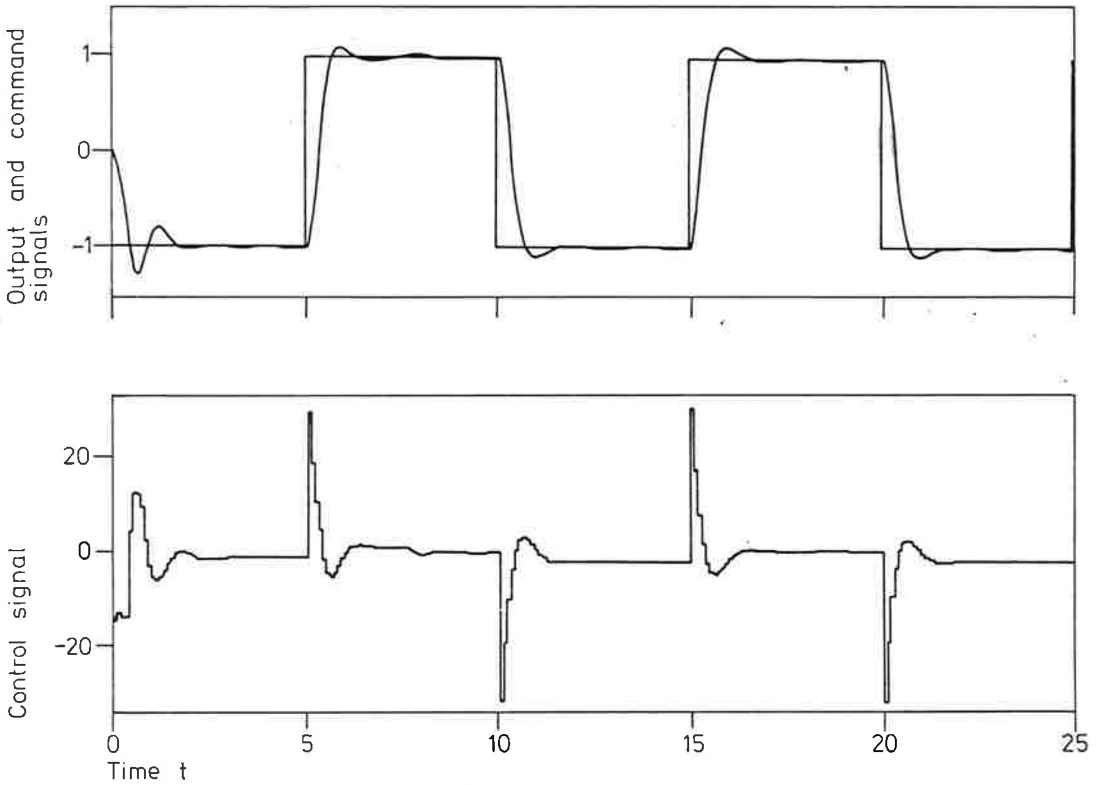


Fig. 4.4 - Simulation of the process $G(s) = (1+s)^{-2}$ with a self-tuning controller. The process is subject to a unit step load disturbance at $t = 15$ s. Compare with Figure 2.9.

Changes in Process Gain

The simulation in Fig. 4.5 shows what happens when the process gain is suddenly increased from $k = 1$ to $k = 4$ at time $t = 15$ s. The increased gain makes the system momentarily unstable. The regulator recovers quickly and the step response is back to normal again after two steps. Also notice that the regulator parameters quickly change to the new values. The speed of response will of course depend on the chosen forgetting factors. The value $\lambda = 0.95$ was used in the simulations. With $\lambda = 0.98$ the convergence is somewhat slower.

The values of the controller gains that are obtained when the process gain is increased or decreased are shown in Table 4.2.

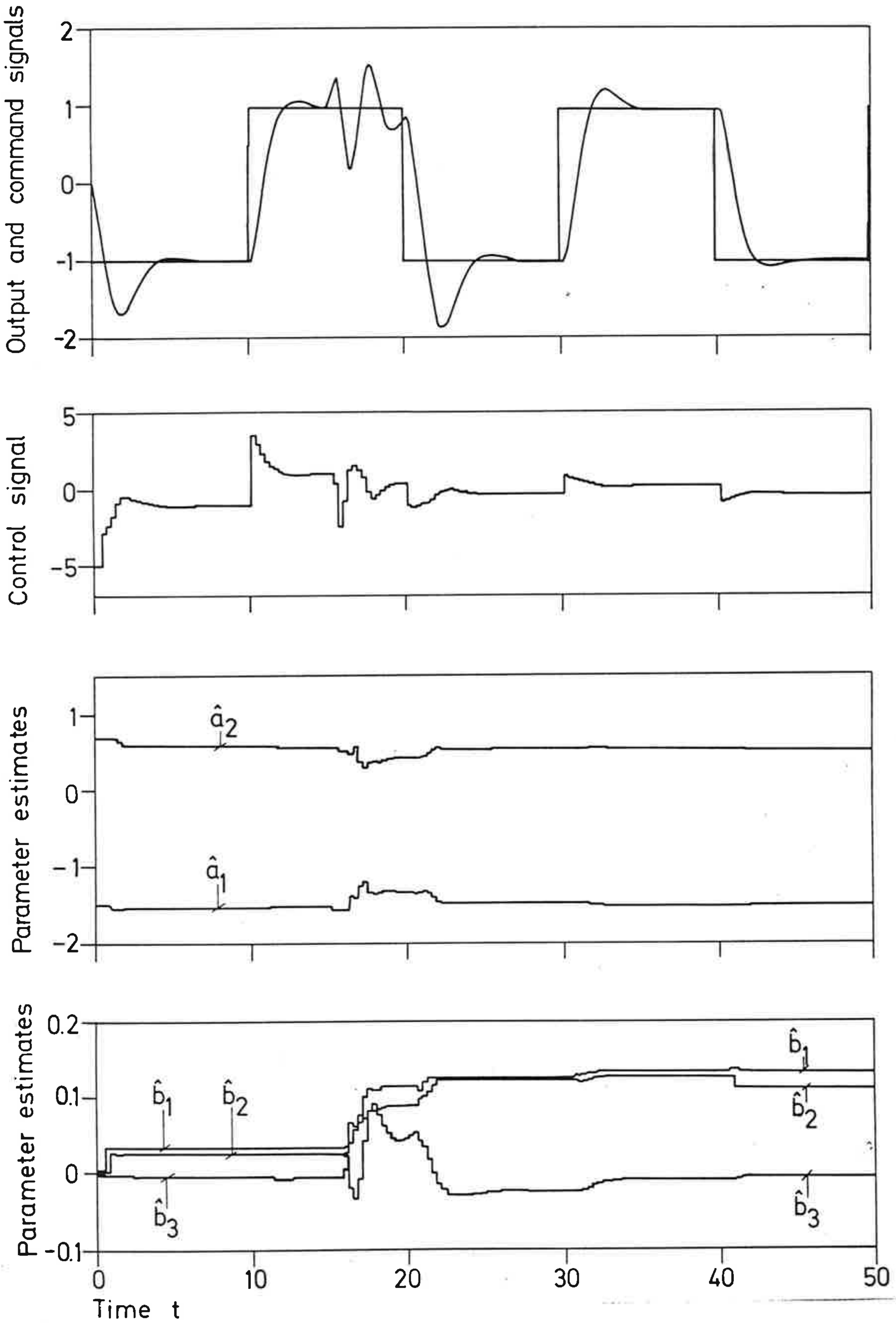


Fig. 4.5 - Simulation of the process $G(s) = (1+s)^{-2}$ with a self-tuning controller. The process gain is increased from 1 to 4 at time $t = 15$ s.

Table 4.2 - Limiting controller gains obtained for different values of process gains.

k	r_1	s_0	s_1	t_0
0.25				
1	0.035	1.835	-0.663	2.208
4	0.004	0.482	-0.179	0.558

The simulations indicate clearly that the controller copes well with changes in the process gain.

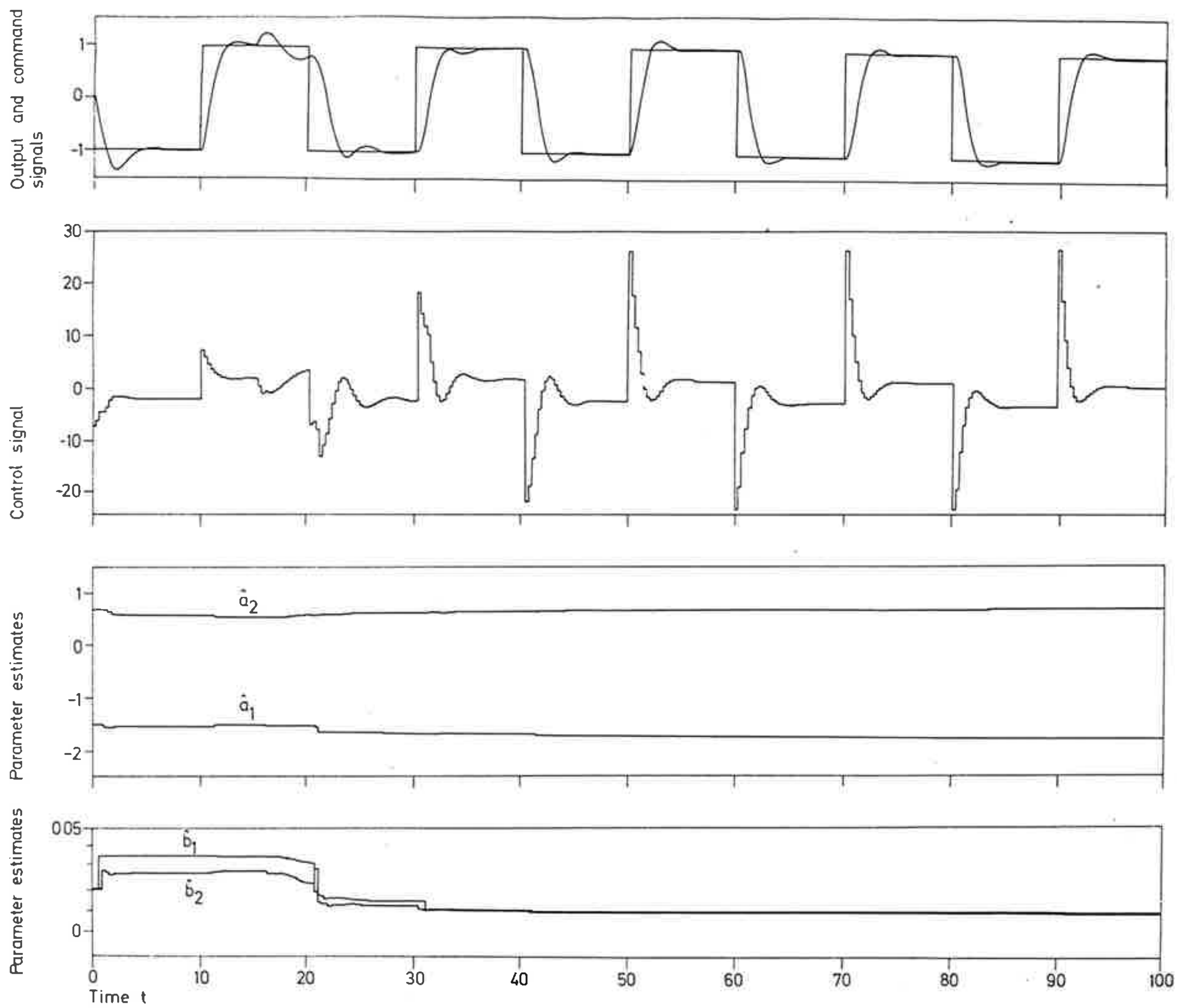
Changes in Process Time Constants

Figure 4.6 shows what happens when the process time constant T is increased from T = 1 s to T = 2 s at time t = 15 s. Since the process model given by (2.1) is of second order this means that the effective process time constant is four times as large. Fig. 4.7 shows similar results when the process time constant is decreased from T = 1 s to T = 0.5 s. Notice that the limiting regulators obtained in the two cases are quite different. This is seen from Table 4.3.

For T = 0.5 the process is faster than the desired response and the controller gain is therefore negative. Also notice that a substantial process gain is required when T = 2.0 s. The simulations show clearly that the controller handles variations in process time constants well.

Table 4.3 - Limiting controller gains obtained for different process time constants.

T	r_1	s_0	s_1	t_0
0.5	-0.169	-0.800	0.709	0.739
1.0	0.035	1.835	-0.663	2.208
2.0	0.149	18.73	-12.27	7.604



<u>Fig. 4.6</u> - Simulation of the process $G(s) = (1+sT)^{-2}$ with the self--tuning controller. The time constant T is 1 for $0 \leq t \leq 15$ s and 2 for $t \geq 15$ s. The forgetting factor is $\lambda = 0.98$.

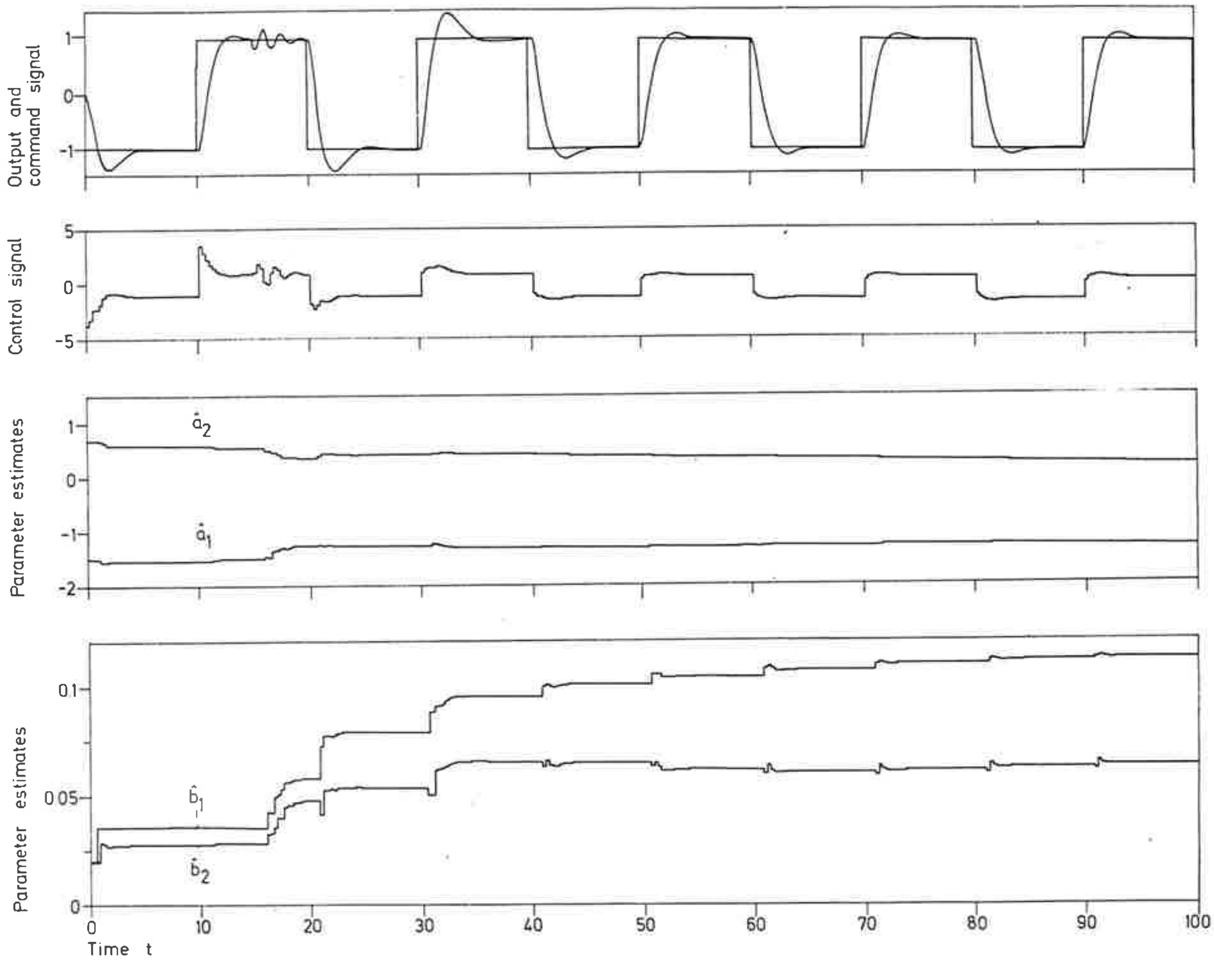


Fig. 4.7 - Simulation of the process $G(s) = (1+sT)^{-2}$ with the self--tuning controller. The time constant T is 1 for $0 \leq t < 15$ s and 0.5 for $t \geq 15$ s. The forgetting factor is $\lambda = 0.98$.

High Order Process Dynamics

The design of the self-tuning controller was based on the assumption that the process could be described by a second order model. It is of interest to investigate how the regulator behaves when it is applied to a process of higher order. A process with the transfer function

$$G(s) = \frac{1}{(1+s)^4}$$

was simulated. Figure 4.8 shows what happens when it is demanded that the closed loop system has $\omega = 0.3$. The result shows that the desired results are obtained. The overshoot and the response time are as expected. When the closed loop bandwidth is increased to $\omega = 0.6$ the results shown in Fig. 4.9 are obtained. These results are far from satisfactory.

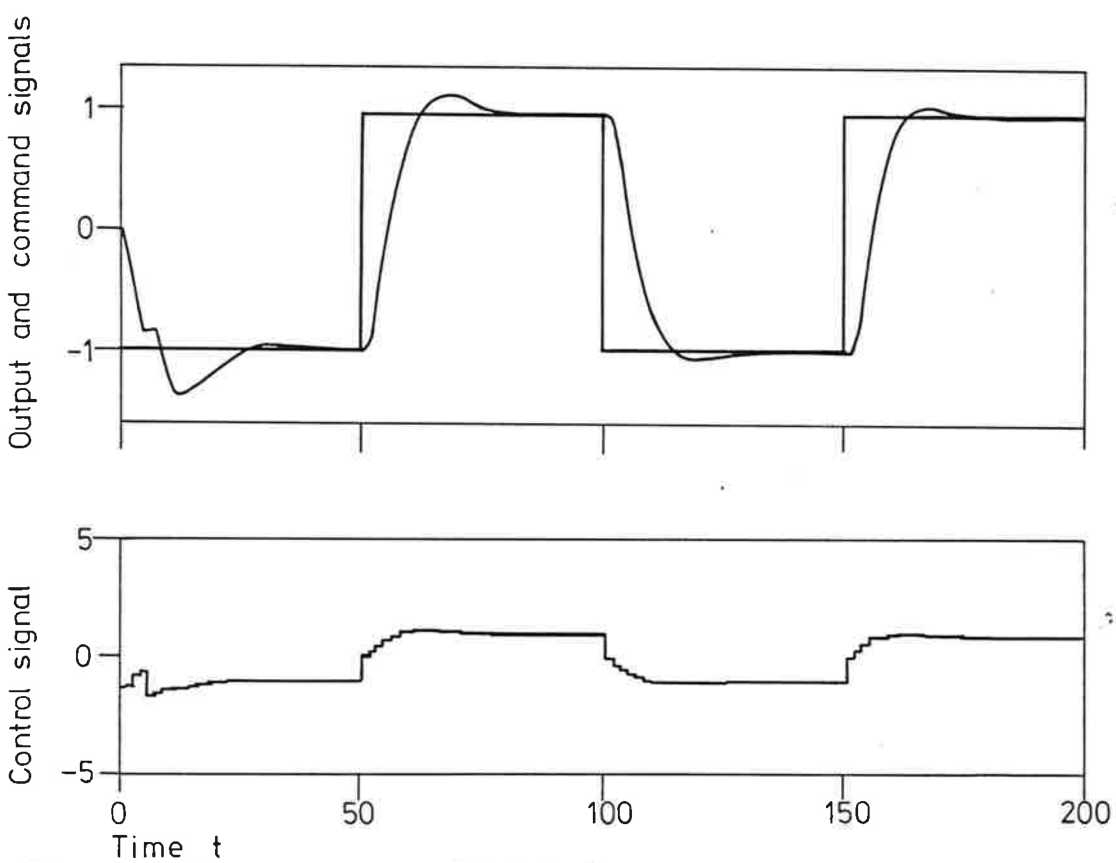


Fig. 4.8 - Simulation of the process $G(s) = (1+s)^{-4}$ with the self-tuning controller. The specified ω is 0.3.

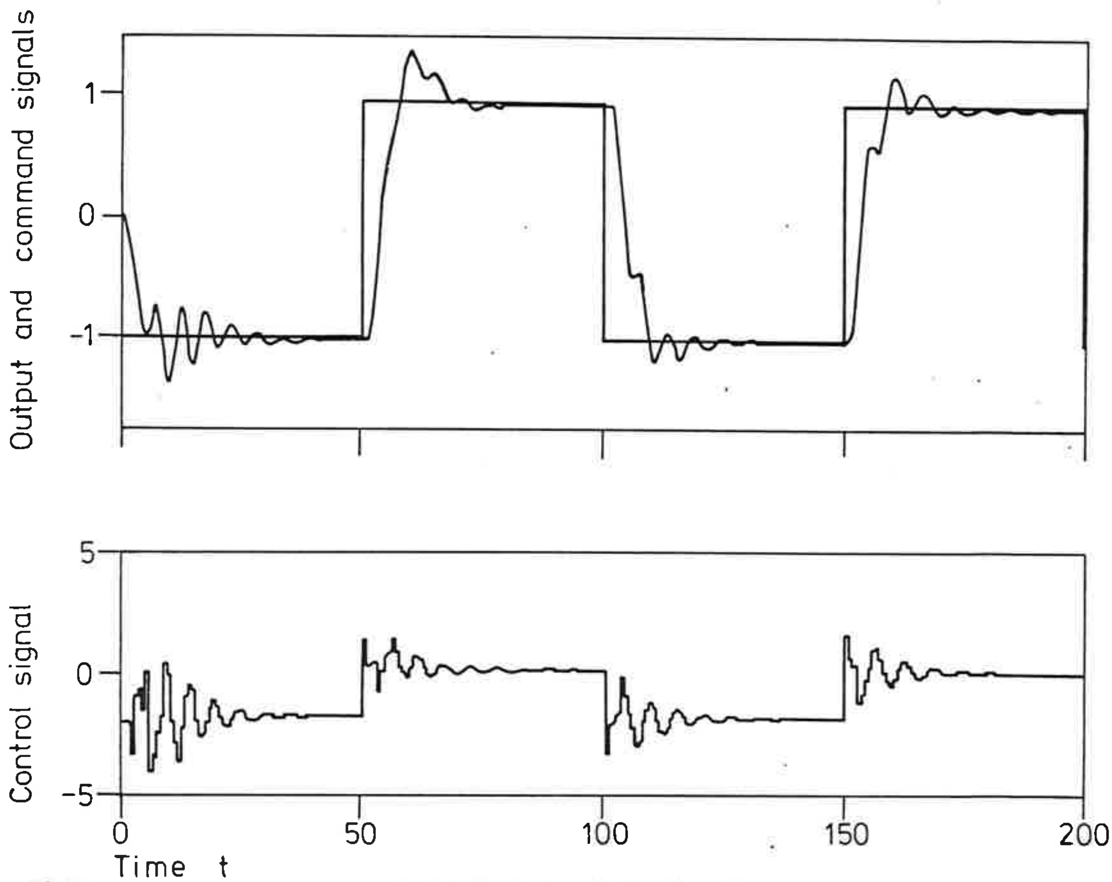


Fig. 4.9 - Simulation of the process $G(s) = (1+s)^{-4}$ with the self--tuning controller. The specified ω is 0.6.

The estimates of the process parameters obtained for some different values of ω are shown in Table 4.4. The stepresponses of the estimated process models for $\omega = 0.3$ and $\omega = 0.6$ are shown in Fig. 4.10. The Nyquist-diagrams of the systems are shown in Fig. 4.11.

Table 4.4 - Process parameter estimates obtained when applying the self-tuning controller to the process $(1+s)^{-4}$.

ω	h	a_1	a_2	b_1	b_2
0.30	1.48	-1.079	0.351	0.0567	0.2126
0.40	1.11	-1.333	0.496	0.0244	0.1347
0.50	0.89	-1.497	0.609	0.0136	0.0926
0.60	0.74	-1.624	0.707	0.0075	0.0717
0.80	0.56	-1.886	0.944	-0.0014	0.0551

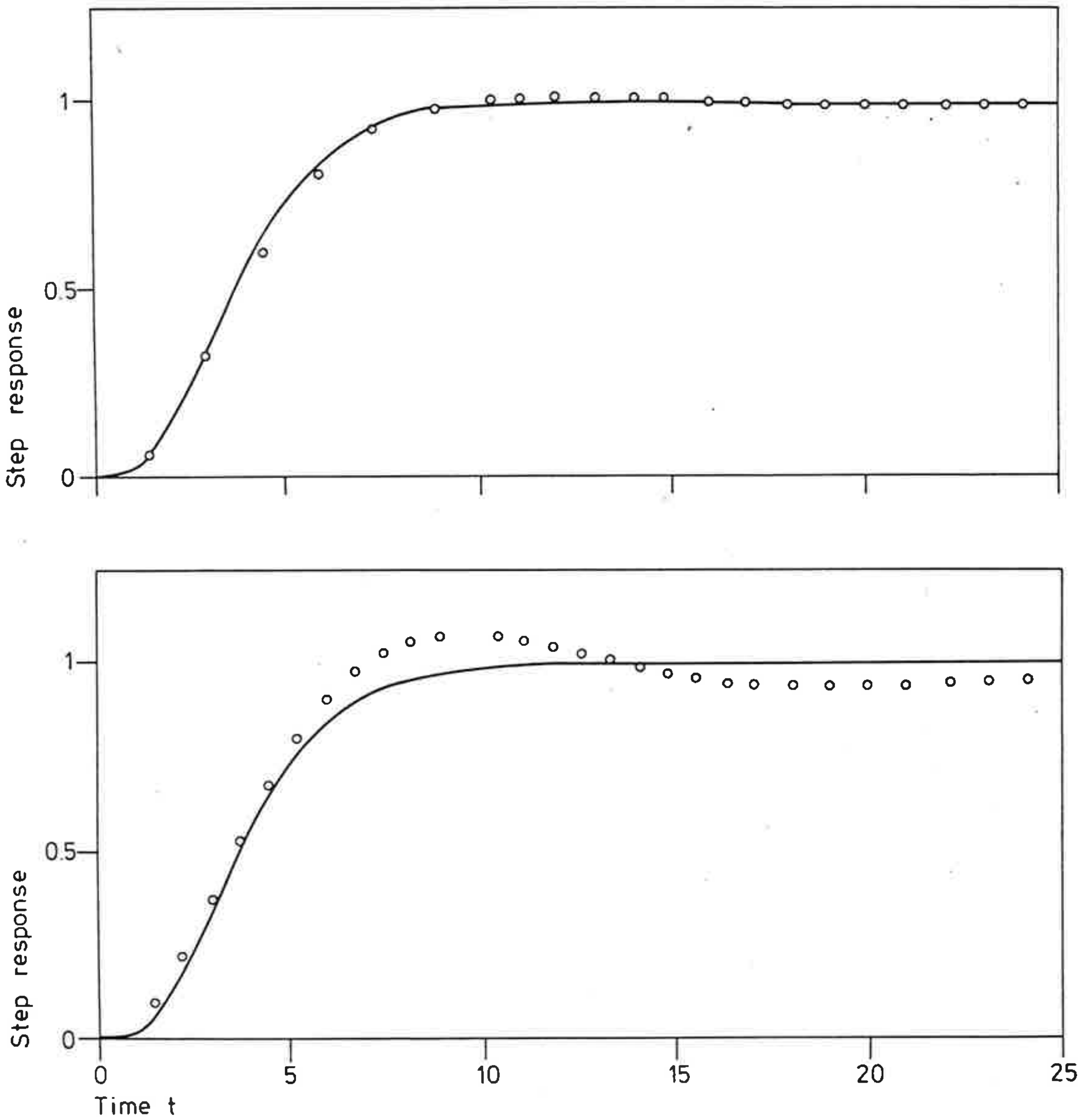


Fig. 4.10 - Step responses of the system $G(s) = (1+s)^{-4}$ and of the estimated process models for $\omega = 0.3$ (above) and $\omega = 0.6$ (below).

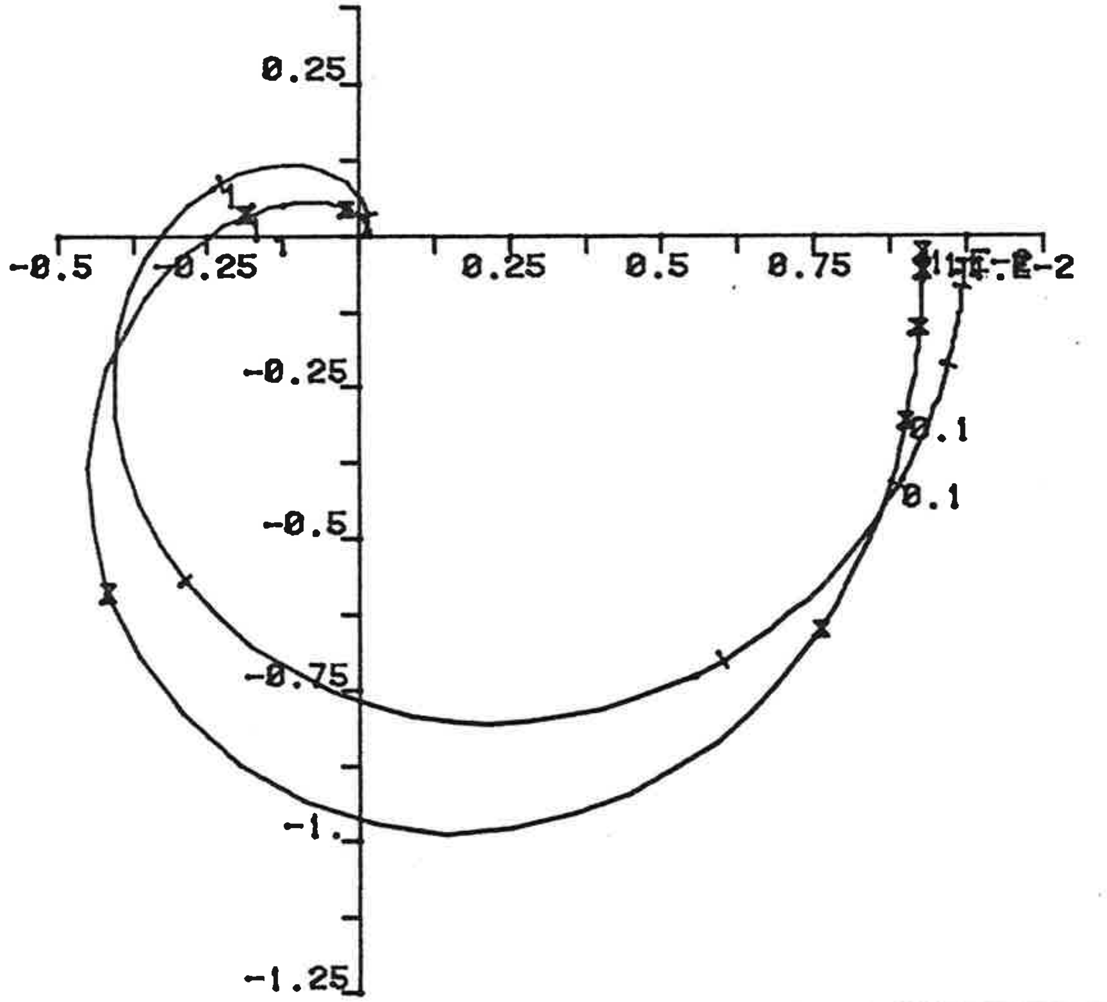


Fig. 4.11 - Nyquist-diagrams for the systems shown in Fig. 4.10.

It is seen from Fig. 4.11 that the estimated model and the sampled system model are closer for $\omega = 0.3$ than for $\omega = 0.6$. According to the sensitivity results given in Åström (1979b) it is shown that the closed loop system designed on the basis of an approximative model G will be stable if

$$|G - G_0| \leq G_s = \left|\frac{G}{G_M}\right| \cdot \left|\frac{G_{FF}}{G_{FB}}\right|$$

where G_0 is the true transfer function. In this particular case we have

$$G_s = \frac{z(z^2 + p_1 z + p_2)}{(s_0 z + s_1)(z^2 + a_1 z + a_2)}.$$

The magnitude of the transfer function G_s for the two cases are shown in Fig. 4.12. It is seen from this figure that a much more accurate model is given for the design corresponding to $\omega = 0.6$.

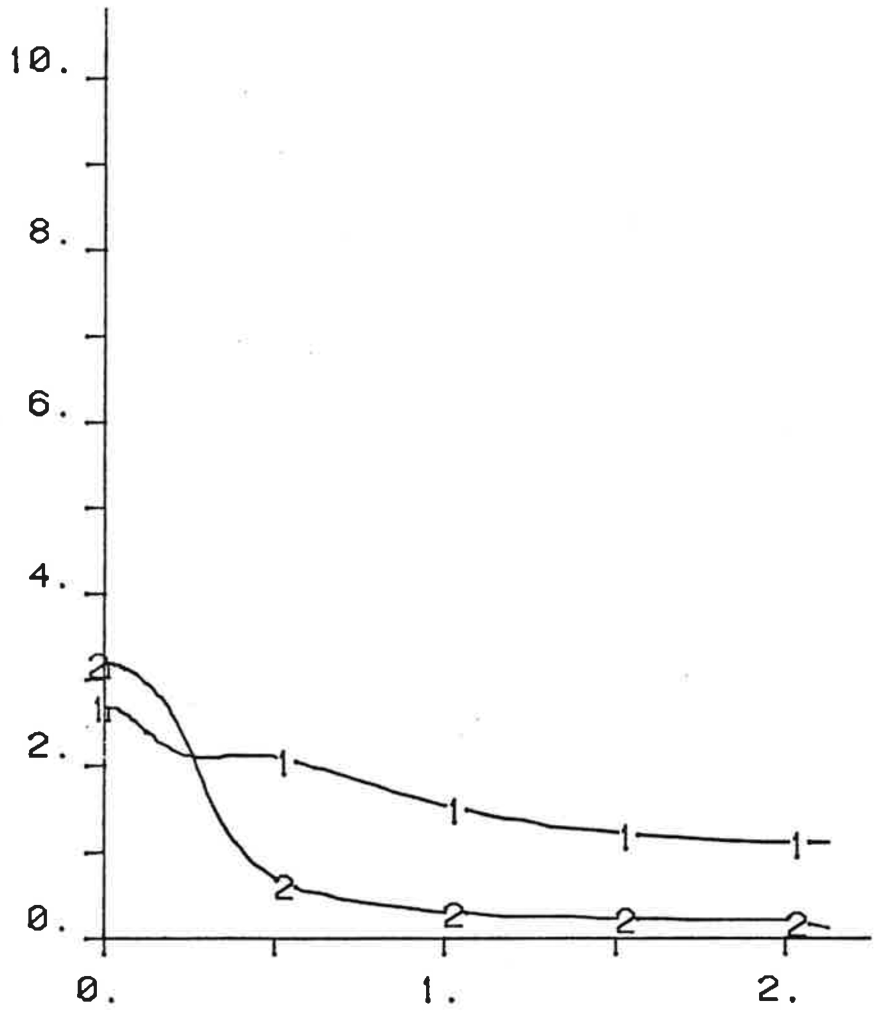


Fig. 4.12 - Magnitude of the transfer function G_s for $\omega = 0.3$ and $\omega = 0.6$.

5. CONCLUSIONS

In modern DDC-packages there are frequently a large number of different versions of the PID-algorithm. It therefore does not seem unreasonable to have a number of different self-tuners for different types of problems. This report presents a self-tuner for typical servo problems. The self-tuner has one major adjustable parameter which is proportional to the desired bandwidth of the closed loop system. All other parameters are fixed or related to the bandwidth. It is shown by simulation that the algorithm will work well in many circumstances. When the estimated parameters are constant the regulator corresponds to a feedback law of the form

$$u(t) = t_0 u_c(t) - s_0 y(t) - s_1 y(t-h) - r_1 u(t-h) - u_D.$$

This feedback law corresponds to a proportional and derivative feedback. There is no explicit integrator in the feedback. In spite of this off sets are eliminated indirectly via the parameter estimation. It is in fact demonstrated that the self-tuning regulator will give zero steady state error.

There are several improvements of the regulator that could be interesting to investigate. The selection of forgetting factors is a compromize. It would thus be worthwhile to consider other ways to eliminate the bias and to choose the forgetting factors.

6. ACKNOWLEDGEMENT

The idea of designing a servo with one knob for the desired closed loop bandwidth was triggered during a discussion with my graduate student Carl Fredrik Mannerfelt. When assigned to do a self-tuner which could automatically tune its sampling period Carl Fredrik responded by saying: "Why should you?". I am also grateful to Dr Björn Wittenmark, with whom I have had many discussions on this topic.

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APPENDIX A - SIMNON PROGRAMS STRE3, PROC1

```
DISCRETE SYSTEM STRE3
"SELF-TUNING REGULATOR BASED ON POLE AND ZERO PLACEMENT
"       SPECIAL VERSION FOR SECOND ORDER SYSTEMS
"       THE PROCESS ZERO IS RETAINED
"       EXPLICIT IDENTIFICATION
"       AUTHOR KJ ASTROM 790820


INPUT Y UC
OUTPUT U
STATE F1 F2 F3 F4 TH1 TH2 TH3 TH4 TH5
STATE P11 P12 P13 P14 P22 P23 P24 P33 P34 P44 P55
NEW NF1 NF2 NF3 NF4 NT1 NT2 NT3 NT4 NT5
NEW N11 N12 N13 N14 N22 N23 N24 N33 N34 N44 N55
TIME T
TSAMP TS

INITIAL
TH1:-1.5
TH2:0.7
P11=P0
P22=P0
P33=P0
P44=P0
P55=P0B

OUTPUT

"PART 2 - REGULATOR DESIGN

P1=-2*EXP(-Z*W*DT)*COS(W*DT*SQRT(1-Z*Z))
P2=EXP(-2*Z*W*DT)
PS=1+P1+P2
A1=TH1
A2=TH2
B1=TH3
B2=TH4
BS=B1+B2
BMAX=MAX(B1,B2)

N=B2*B2-A1*B1*B2+A2*B1*B1
W1=P1-A1
W2=P2-A2
W3=W1*B2-W2*B1
TEST=N/(BMAX*BMAX*EPS)

T0=IF TEST<1 THEN PS/B1 ELSE PS/BS
R1=IF TEST<1 THEN 0 ELSE B2*W3/N
S0=IF TEST<1 THEN P1/B1-A2/B2 ELSE (W1*(A2*B1-A1*B2)+W2*B2)/N
S1=IF TEST<1 THEN P2/B1 ELSE -A2*W3/N
```

```
UB=IF TEST<1 THEN TH5/R1 ELSE (1+R1)*TH5/BS
U1=T0*UC-S0*Y-S1*(-F1)-R1*F3-UB

U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH

DYNAMICS

"PART 1 PARAMETER ESTIMATION

E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5

"ESTIMATOR GAIN

K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5

"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D

"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
N13=P13-K1*K3/D
N14=P14-K1*K4/D
N22=(P22-K2*K2/D)/LAM
N23=P23-K2*K3/D
N24=P24-K2*K4/D
N33=(P33-K3*K3/D)/LAM
N34=P34-K3*K4/D
N44=(P44-K4*K4/D)/LAM
N55=(P55-K5*K5/D)/LAM1

"UPDATE F-VECTOR
NF1=-Y
NF2=F1
NF3=U
NF4=F3

"UPDATE SAMPLING TIME
TS=T+DT

P0:100
P0B:0
W:1.5
Z:0.707
DT:0.2
```

```
LAM:0.98
LAM1:5
UHIGH:5
ULOW:-5
F5:1
EPS:1.E-5

END
```

- -

```
CONTINUOUS SYSTEM PROC1

TIME T
INPUT U V
OUTPUT Y
STATE X1 X2 X3
DER D1 D2 D3

OUTPUT
Y=IF O<2 THEN X1 ELSE IF O<3 THEN X2 ELSE X3

DYNAMICS
PA1=IF T<TPCH THEN 1 ELSE PA11
PA2=IF T<TPCH THEN 1 ELSE PA22
D1=-PA1*X1+PA2*U+V
D2=-PA1*X2+X1
D3=-PA1*X3+X2

PA11:1
PA22:1
TPCH:50
FIRST:-1
O:1
END
```

APPENDIX B - EXAMPLE 2.2

Data for Figure 2.4

DISCRETE SYSTEM STRE3

TIME :	T	50.0000				
STATE :	F1	1.00004	F2	1.00007	F3	-0.999980
	F4	-0.999935	TH1	-1.61450	TH2	0.649384
	TH3	1.763140E-02	TH4	1.725639E-02	TH5	0.000000
	P11	10.8859	P12	-9.71358	P13	0.210452
	P14	0.499973	P22	9.68449	P23	-9.317899E-02
	P24	-0.345708	P33	0.160839	P34	-3.679857E-02
	P44	0.199465	P55	0.000000		
INIT :	F1	0.000000	F2	0.000000	F3	0.000000
	F4	0.000000	TH1	0.000000	TH2	0.000000
	TH3	0.000000	TH4	0.000000	TH5	0.000000
	P11	0.000000	P12	0.000000	P13	0.000000
	P14	0.000000	P22	0.000000	P23	0.000000
	P24	0.000000	P33	0.000000	P34	0.000000
	P44	0.000000	P55	0.000000		
NEW :	NF1	1.00001	NF2	1.00004	NF3	-1.00001
	NF4	-0.999980	NT1	-1.61450	NT2	0.649384
	NT3	1.763140E-02	NT4	1.725639E-02	NT5	0.000000
	N11	10.9928	N12	-9.81392	N13	0.212107
	N14	0.502027	N22	9.79117	N23	-9.170840E-02
	N24	-0.343883	N33	0.164097	N34	-3.682867E-02
	N44	0.203497	N55	0.000000		
INPUT :	Y	-1.00001	UC	1.00000		
OUTPUT:	U	-1.00001				
TSAMP :	TS	50.2000				
PAR :	PO	100.000	POB	0.000000	Z	0.707000
	W	1.50000	DT	0.200000	EPS	1.000000E-04
	ULOW	-5.00000	UHIGH	5.00000	F5	1.00000
	LAM	0.980000	LAM1	5.00000		
VAR :	P1	-1.58149	P2	0.654293	PS	7.279973E-02
	A1	-1.61450	A2	0.649384	R1	1.763140E-02
	B2	1.725639E-02	BS	3.488779E-02	RMAX	1.763140E-02
	N	9.908721E-04	W1	3.300267E-02	W2	4.908860E-03
	W3	4.829569E-04	TEST	31874.5	T0	2.08668
	R1	8.410866E-03	S0	1.39477	S1	-0.316514
	UB	0.000000	U1	-1.00001	E	2.500601E-07
	K1	0.461620	K2	0.410138	K3	-6.765923E-03
	K4	-8.395918E-03	K5	0.000000	D	1.88697

- -

CONTINUOUS SYSTEM PROC1

TIME :	T	50.0000				
STATE :	X1	-0.999891	X2	-1.00001	X3	-0.999794
INIT :	X1	0.000000	X2	0.000000	X3	0.000000
DER :	D1	-1.220852E-04	D2	1.166612E-04	D3	-2.128631E-04
INPUT :	U	-1.00001	V	0.000000		
OUTPUT:	Y	-1.00001				
PAR :	O	2.00000	TPCH	50.0000	PA11	1.00000
	PA22	1.00000	FIRST	-1.00000		
VAR :	PA1	1.00000	PA2	1.00000		

```
                          CONNECTING SYSTEM EX1
TIME    : T        50.0000
PAR     : PER      20.0000         U1      -1.00000         U2       1.00000
          TV       50.0000         V1      0.000000         V2      0.000000
          AP      0.200000         BP      -3.00000
VAR     : UP      -3.20000
```

Data for Figure 2.5

```
                          DISCRETE SYSTEM STRE3
TIME    : T        50.0000
STATE   : F1       1.00011         F2       1.00012         F3      -0.999890
          F4     -0.999870         TH1     -1.64354         TH2      0.675878
          TH3    1.748443E-02      TH4     1.485085E-02     TH5      0.000000
          P11      8.78099         P12     -7.84062         P13      0.164219
          P14     0.375031         P22      7.86464         P23     -7.588292E-02
          P24    -0.259255         P33     0.117339         P34     -2.368760E-02
          P44     0.146084         P55     0.000000
INIT    : F1      0.000000         F2      0.000000         F3       0.000000
          F4      0.000000         TH1     -2.00000         TH2      1.00000
          TH3     0.000000         TH4     0.000000         TH5      0.000000
          P11     0.000000         P12     0.000000         P13      0.000000
          P14     0.000000         P22     0.000000         P23      0.000000
          P24     0.000000         P33     0.000000         P34      0.000000
          P44     0.000000         P55     0.000000
NEW     : NF1      1.00009         NF2      1.00011         NF3     -0.999911
          NF4    -0.999890         NT1     -1.64354         NT2      0.675878
          NT3    1.748443E-02      NT4     1.485085E-02     NT5      0.000000
          N11      8.86753         N12     -7.92193         N13      0.165418
          N14     0.376524         N22      7.95085         N23     -7.480941E-02
          N24    -0.257919         N33     0.119717         N34     -2.370731E-02
          N44     0.149041         N55     0.000000
INPUT   : Y       -1.00009         UC       1.00000
OUTPUT  : U      -0.999911
TSAMP   : TS       50.2000
PAR     : P0       100.000         P0B     0.000000         Z        0.707000
          W        1.50000         DT      0.200000         EPS      1.000000E-04
          ULOW    -5.00000         UHIGH    5.00000         F5       1.00000
          LAM     0.980000         LAM1     5.00000
VAR     : P1      -1.58149         P2      0.654293         PS       7.279973E-02
          A1      -1.64354         A2      0.675878         R1       1.748443E-02
          R2     1.485085E-02      BS      3.233527E-02     BMAX     1.748443E-02
          N      8.539271E-04      W1      6.204939E-02     W2      -2.158515E-02
          W3     1.298890E-03      TEST     27933.0         T0       2.25140
          R1     2.258930E-02      S0       2.25687         S1      -1.02806
          UB      0.000000         U1     -0.999911         E       -1.587905E-07
          K1      0.401193         K2      0.359223         K3      -5.296686E-03
          K4    -6.595507E-03      K5      0.000000         D        1.77240
```

```
                          CONTINUOUS SYSTEM PROC1
TIME   : T        50.0000
STATE  : X1      -1.00001          X2      -1.00009          X3     -0.999776
INIT   : X1       0.000000         X2       0.000000         X3      0.000000
DER    : D1       9.655952E-05     D2       8.714199E-05     D3     -3.178418E-04
INPUT  : U       -0.999911         V        0.000000
OUTPUT : Y       -1.00009
PAR    : O        2.00000          TPCH     50.0000          PA11    1.00000
         PA22     1.00000          FIRST   -1.00000
VAR    : PA1      1.00000          PA2      1.00000

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                          CONNECTING SYSTEM EX1
TIME   : T        50.0000
PAR    : PER      20.0000          U1      -1.00000          U2      1.00000
         TV       50.0000          V1       0.000000         V2      0.000000
         AP       0.200000         BP      -3.00000
VAR    : UP      -3.19998
```

Data for Figure 2.6

```
                          DISCRETE SYSTEM STRE3
TIME   : T        50.0000
STATE  : F1       1.00011          F2       1.00013          F3     -0.999893
         F4      -0.999865         TH1     -1.63740          TH2     0.670267
         TH3      1.752368E-02     TH4      1.534014E-02     TH5     0.000000
         P11      12.6925          P12     -11.3524          P13     0.262432
         P14      0.566340         P22      11.2686          P23    -0.136367
         P24     -0.401051         P33      0.163536         P34    -3.091533E-02
         P44      0.204404         P55      0.000000
INIT   : F1       0.000000         F2       0.000000         F3      0.000000
         F4       0.000000         TH1     -1.60000          TH2     0.700000
         TH3      2.000000E-02     TH4      1.000000E-02     TH5     0.000000
         P11      0.000000         P12      0.000000         P13     0.000000
         P14      0.000000         P22      0.000000         P23     0.000000
         P24      0.000000         P33      0.000000         P34     0.000000
         P44      0.000000         P55      0.000000
NEW    : NF1      1.00009          NF2      1.00011          NF3    -0.999919
         NF4     -0.999893         NT1     -1.63740          NT2     0.670267
         NT3      1.752368E-02     NT4      1.534014E-02     NT5     0.000000
         N11      12.8168          N12     -11.4696          N13     0.264118
         N14      0.568449         N22      11.3925          N23    -0.134870
         N24     -0.399179         N33      0.166851         N34    -3.094227E-02
         N44      0.208542         N55      0.000000
INPUT  : Y       -1.00009          UC       1.00000
OUTPUT : U       -0.999919
TSAMP  : TS       50.2000
PAR    : PO       100.000          POB      0.000000         Z       0.707000
         W        1.50000          DT       0.200000         EPS     1.000000E-04
         ULOW    -5.00000          UHIGH    5.00000          F5      1.00000
         LAM      0.980000         LAM1     5.00000
```

```
VAR    : P1     -1.58149        P2      0.654293        PS      7.279973E-02
         A1     -1.63740        A2      0.670267        R1      1.752368E-02
         B2     1.534014E-02    BS      3.286383E-02    RMAX    1.752368E-02
         N      8.813052E-04    W1      5.590987E-02    W2     -1.597397E-02
         W3     1.137588E-03    TEST     28699.6        T0       2.21519
         R1     1.980105E-02    S0       2.06057        S1     -0.865180
         UB     0.000000        U1     -0.999919        E      -1.699664E-08
         K1     0.511264        K2      0.453776        K3     -6.532279E-03
         K4    -8.166708E-03    K5      0.000000        D        1.97985

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                   CONTINUOUS SYSTEM PROC1
TIME   : T        50.0000
STATE  : X1     -0.999977       X2     -1.00009         X3     -0.999790
INIT   : X1      0.000000       X2      0.000000        X3      0.000000
DER    : D1     5.820394E-05    D2      1.079738E-04    D3     -2.957433E-04
INPUT  : U      -0.999919       V       0.000000
OUTPUT : Y      -1.00009
PAR    : O       2.00000        TPCH     50.0000        PA11     1.00000
         PA22    1.00000        FIRST  -1.00000
VAR    : PA1     1.00000        PA2      1.00000

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

                   CONNECTING SYSTEM EX1
TIME   : T        50.0000
PAR    : PER      20.0000       U1     -1.00000         U2       1.00000
         TV       50.0000       V1      0.000000        V2      0.000000
         AP      0.200000       BP     -3.00000
VAR    : UP     -3.19998
```

APPENDIX C - EXAMPLE 2.3

Data for Figure 2.7 A

DISCRETE SYSTEM STRE3

TIME :	T	50.0000				
STATE :	F1	1.00011	F2	1.00013	F3	-0.999689
	F4	-0.999859	TH1	-1.60198	TH2	0.641634
	TH3	2.130907E-02	TH4	1.834659E-02	TH5	0.000000
	P11	9.65710	P12	-8.53877	P13	0.200020
	P14	0.490416	P22	8.46142	P23	-0.100704
	P24	-0.351379	P33	0.133592	P34	-2.839753E-02
	P44	0.175146	P55	0.000000		
INIT :	F1	0.000000	F2	0.000000	F3	0.000000
	F4	0.000000	TH1	-1.60000	TH2	0.700000
	TH3	0.200000	TH4	0.200000	TH5	0.000000
	P11	0.000000	P12	0.000000	P13	0.000000
	P14	0.000000	P22	0.000000	P23	0.000000
	P24	0.000000	P33	0.000000	P34	0.000000
	P44	0.000000	P55	0.000000		
NEW :	NF1	1.00009	NF2	1.00011	NF3	-0.999918
	NF4	-0.999689	NT1	-1.60198	NT2	0.641634
	NT3	2.130907E-02	NT4	1.834659E-02	NT5	0.000000
	N11	9.75132	N12	-8.62708	N13	0.201400
	N14	0.492226	N22	8.55517	N23	-9.949481E-02
	N24	-0.349794	N33	0.136299	N34	-2.842231E-02
	N44	0.178687	N55	0.000000		
INPUT :	Y	-1.00009	UC	1.00000		
OUTPUT:	U	-0.999918				
TSAMP :	TS	50.2222				
PAR :	PO	100.000	POB	0.000000	Z	0.707000
	W	1.50000	DT	0.222222	EPS	1.000000E-04
	ULOW	-5.00000	UHIGH	5.00000	F5	1.00000
	LAM	0.980000	LAM1	5.00000		
VAR :	P1	-1.53639	P2	0.624170	PS	8.778299E-02
	A1	-1.60198	A2	0.641634	R1	2.130907E-02
	R2	1.834659E-02	BS	3.965566E-02	RMAX	2.130907E-02
	N	1.254240E-03	W1	6.559160E-02	W2	-1.746432E-02
	W3	1.575531E-03	TEST	27621.8	T0	2.21363
	R1	2.304632E-02	S0	1.99658	S1	-0.805997
	UB	0.000000	U1	-0.999918	E	-1.504086E-07
	K1	0.427916	K2	0.374857	K3	-5.857613E-03
	K4	-7.682636E-03	K5	0.000000	D	1.81641

- -

CONTINUOUS SYSTEM PROC1

TIME :	T	50.0000				
STATE :	X1	-0.999982	X2	-1.00009	X3	-0.999782
INIT :	X1	0.000000	X2	0.000000	Y3	0.000000
DER :	D1	6.382167E-05	D2	1.067519E-04	D3	-3.065020E-04
INPUT :	U	-0.999918	V	0.000000		
OUTPUT:	Y	-1.00009				
PAR :	O	2.00000	TPCH	50.0000	PA11	1.00000
	PA22	1.00000	FIRST	-1.00000		
VAR :	PA1	1.00000	PA2	1.00000		

```
                    CONNECTING SYSTEM EX1
TIME    : T        50.0000
PAR     : PER      20.0000      U1       -1.00000       U2       1.00000
          TV       50.0000      V1       0.000000       V2       0.000000
          AP       0.200000     BP       -3.00000
VAR     : UP       -3.19998
```

Data for Figure 2.7 B

```
                    DISCRETE SYSTEM STPE3
TIME    : T        50.0000
STATE   : F1       1.00016      F2       0.999913       F3       -0.999802
          F4       -0.999842    TH1      -1.02687       TH2      0.263626
          TH3      0.144295     TH4      9.246067E-02   TH5      0.000000
          P11      1.30879      P12      -0.955527      P13      1.781061E-02
          P14      0.226115     P22      0.911915       P23      -2.511634E-03
          P24      -0.153217    P33      3.338497E-02   P34      -1.223567E-02
          P44      9.913663E-02 P55      0.000000
INIT    : F1       0.000000     F2       0.000000       F3       0.000000
          F4       0.000000     TH1      -1.00000       TH2      0.300000
          TH3      0.100000     TH4      0.100000       TH5      0.000000
          P11      0.000000     P12      0.000000       P13      0.000000
          P14      0.000000     P22      0.000000       P23      0.000000
          P24      0.000000     P33      0.000000       P34      0.000000
          P44      0.000000     P55      0.000000
NEW     : NF1      1.00014      NF2      1.00016        NF3      -0.999890
          NF4      -0.999802    NT1      -1.02687       NT2      0.263626
          NT3      0.144295     NT4      9.246068E-02   NT5      0.000000
          N11      1.32562      N12      -0.965410      N13      1.832678E-02
          N14      0.227347     N22      0.920239       N23      -1.985115E-03
          N24      -0.151960    N33      3.403823E-02   N34      -1.230129E-02
          N44      0.101000     N55      0.000000
INPUT   : Y        -1.00007     UC       1.00000
OUTPUT  : U        -0.999890
TSAMP   : TS       50.0002
PAR     : PO       100.000      POB      0.000000       Z        0.707000
          W        1.50000      DT       0.666670       FPS      1.000000E-04
          ULOW     -5.00000     UHIGH    5.00000        F5       1.00000
          LAM      0.980000     LAM1     5.00000
VAR     : P1       -0.749712    P2       0.243167       PS       0.493455
          A1       -1.02687     A2       0.263626       B1       0.144295
          B2       9.246067E-02 BS       0.236756       BMAX     0.144295
          N        2.773812E-02 W1       0.277157       W2       -2.045944E-02
          W3       2.857831E-02 TEST     13322.1        T0       2.08423
          R1       9.526130E-02 S0       1.26058        S1       -0.271612
          UB       0.000000     U1       -0.999890      E        -1.138076E-06
          K1       0.109665     K2       0.111863       K3       -5.842642E-03
          K4       -1.394033E-02 K5      0.000000       D        1.24131
```

```
                         CONTINUOUS SYSTEM PROC1
TIME   : T       50.0000
STATE  : X1     -0.999965        X2     -1.00007         X3    -0.999844
INIT   : X1      0.000000        X2      0.000000        X3     0.000000
DER    : D1      7.474422E-05    D2      1.029819E-04    D3    -2.236664E-04
INPUT  : U      -0.999890        V       0.000000
OUTPUT : Y      -1.00007
PAR    : O       2.00000         TPCH    50.0000         PA11   1.00000
         PA22    1.00000         FIRST  -1.00000
VAR    : PA1     1.00000         PA2     1.00000

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                         CONNECTING SYSTEM EX1
TIME   : T       50.0000
PAR    : PER     20.0000         U1     -1.00000         U2     1.00000
         TV      50.0000         V1      0.000000        V2     0.000000
         AP      0.200000        BP     -3.00000
VAR    : UP     -3.19998
```

Data for Figure 2.7 C

```
                         DISCRETE SYSTEM STRE3
TIME   : T       50.0000
STATE  : F1      0.999696        F2      0.996236        F3    -0.999655
         F4     -1.00161         TH1    -0.270321        TH2    1.815424E-02
         TH3     0.593990        TH4     0.153863        TH5    0.000000
         P11     7.58123         P12    -3.14287         P13    0.152110
         P14     4.33317         P22     1.60430         P23   -6.453098E-02
         P24    -1.80747         P33     5.971441E-02    P34    5.926339E-02
         P44     2.85859         P55     0.000000
INIT   : F1      0.000000        F2      0.000000        F3     0.000000
         F4      0.000000        TH1    -0.300000        TH2    0.000000
         TH3     0.600000        TH4     0.200000        TH5    0.000000
         P11     0.000000        P12     0.000000        P13    0.000000
         P14     0.000000        P22     0.000000        P23    0.000000
         P24     0.000000        P33     0.000000        P34    0.000000
         P44     0.000000        P55     0.000000
NEW    : NF1     1.00004         NF2     0.999696        NF3    2.04846
         NF4    -0.999655        NT1    -0.270321        NT2    1.815679E-02
         NT3     0.593990        NT4     0.153860        NT5    0.000000
         N11     7.73477         N12    -3.13428         N13    0.151299
         N14     4.32302         N22     1.57152         N23   -5.846820E-02
         N24    -1.73163         N33     6.034892E-02    N34    5.210208E-02
         N44     2.82552         N55     0.000000
INPUT  : Y      -1.00004         UC      1.00000
OUTPUT : U       2.04846
TSAMP  : TS      52.0000
```

```
PAR    : PO      100.000        POB     0.000000        Z       0.707000
         W       1.50000        DT      2.00000         EPS     1.000000E-04
         ULOW   -5.00000        UHIGH   5.00000         F5      1.00000
         LAM     0.980000       LAM1    5.00000
VAR    : P1      0.125525       P2      1.437881E-02    PS      1.13990
         A1     -0.270321       A2      1.815424E-02    B1      0.593990
         B2      0.153863       BS      0.747853        BMAX    0.593990
         N       5.478432E-02   W1      0.395846        W2     -3.775431E-03
         W3      6.314843E-02   TEST    1552.74         T0      1.52424
         R1      0.177353       S0      0.367839        S1     -2.092590E-02
         UB      0.000000       U1      2.04846         E       1.314655E-05
         K1     -4.434407E-02   K2      0.331242        K3     -3.127683E-02
         K4     -0.391261       K5      0.000000        D       1.70882

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                  CONTINUOUS SYSTEM PROC1
TIME   : T       50.0000
STATE  : X1     -0.999840       X2     -1.00004         X3     -0.999782
INIT   : X1      0.000000       X2      0.000000        X3      0.000000
DER    : D1      3.04830        D2      1.955628E-04    D3     -2.538264E-04
INPUT  : U       2.04846        V       0.000000
OUTPUT : Y      -1.00004
PAR    : O       2.00000        TPCH    50.0000         PA11    1.00000
         PA22    1.00000        FIRST  -1.00000
VAR    : PA1     1.00000        PA2     1.00000

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

                  CONNECTING SYSTEM EX1
TIME   : T       50.0000
PAR    : PER     20.0000        U1     -1.00000         U2      1.00000
         TV      50.0000        V1      0.000000        V2      0.000000
         AP      0.200000       BP     -3.00000
VAR    : UP     -2.59031
```

APPENDIX D - SIMNON PROGRAMS STSE2, PROC2

```
DISCRETE SYSTEM STSE2
"SELF-TUNING REGULATOR BASED ON POLE AND ZERO PLACEMENT
"       SPECIAL VERSION FOR SECOND ORDER SYSTEMS
"       A SECOND ORDER PROCESS MODEL IS ESTIMATED EXPLICITELY
"       COMMON FACTORS IN THE ESTIMATED TRANSFER FUNCTION ARE CANCELLED
"       THE PROCESS ZERO IS RETAINED IF IT IS POORLY DAMPED
"       AUTHOR KJ ASTROM 791012

INPUT Y UC
OUTPUT U
STATE F1 F2 F3 F4 TH1 TH2 TH3 TH4 TH5
STATE P11 P12 P13 P14 P22 P23 P24 P33 P34 P44 P55
NEW NF1 NF2 NF3 NF4 NT1 NT2 NT3 NT4 NT5
NEW N11 N12 N13 N14 N22 N23 N24 N33 N34 N44 N55
TIME T
TSAMP TS

INITIAL
TH1=A10
TH2=A20
P11=P0
P22=P0
P33=P0
P44=P0
P55=P0B

OUTPUT

"PART 2 - REGULATOR DESIGN

SQ=SQRT(1-Z*Z)
DT=2*PI/(NP*W*SQ)
P1=-2*EXP(-(2*PI/NP)*Z/SQ)*COS(2*PI/NP)
P2=EXP(-2*(2*PI/NP)*Z/SQ)
PS=1+P1+P2
A1=TH1
A2=TH2
B1=TH3
B2=TH4
BS=B1+B2
BMAX=MAX(B1,B2)

N=B2*B2-A1*B1*B2+A2*B1*B1
W1=P1-A1
W2=P2-A2
W3=W1*B2-W2*B1
W4=W1*(A2*B1-A1*B2)+W2*B2
TEST=N/(BMAX*BMAX*EPS)
TEST1=IF -B2/B1<Z1 THEN -1 ELSE IF -B2/B1<Z2 THEN 1 ELSE -1
```

```
T0=IF TEST<1 THEN PS/B1 ELSE IF TEST1>0 THEN PS/B1 ELSE PS/BS
R1=IF TEST<1 THEN 0 ELSE IF TEST1>0 THEN B2/B1 ELSE B2*W3/N
S0=IF TEST<1 THEN P1/B1-A2/B2 ELSE IF TEST1>0 THEN (P1-A1)/B1 ELSE W4/N
S1=IF TEST<1 THEN P2/B1 ELSE IF TEST1>0 THEN (P2-A2)/B1 ELSE -A2*W3/N

UB=IF TEST<1 THEN TH5/B1 ELSE IF TEST1>0 THEN TH5/B1 ELSE (1+R1)*TH5/BS
U1=T0*UC-S0*Y-S1*(-F1)-R1*F3-UB

U=IF U1<ULOW THEN ULOW ELSE IF U1<UHIGH THEN U1 ELSE UHIGH

DYNAMICS

"PART 1 PARAMETER ESTIMATION

E=Y-TH1*F1-TH2*F2-TH3*F3-TH4*F4-TH5

"ESTIMATOR GAIN
K1=P11*F1+P12*F2+P13*F3+P14*F4
K2=P12*F1+P22*F2+P23*F3+P24*F4
K3=P13*F1+P23*F2+P33*F3+P34*F4
K4=P14*F1+P24*F2+P34*F3+P44*F4
K5=P55*F5
D=1+F1*K1+F2*K2+F3*K3+F4*K4+F5*K5

"UPDATE ESTIMATE
NT1=TH1+K1*E/D
NT2=TH2+K2*E/D
NT3=TH3+K3*E/D
NT4=TH4+K4*E/D
NT5=TH5+K5*E/D

"UPDATE COVARIANCES
N11=(P11-K1*K1/D)/LAM
N12=P12-K1*K2/D
N13=P13-K1*K3/D
N14=P14-K1*K4/D
N22=(P22-K2*K2/D)/LAM
N23=P23-K2*K3/D
N24=P24-K2*K4/D
N33=(P33-K3*K3/D)/LAM
N34=P34-K3*K4/D
N44=(P44-K4*K4/D)/LAM
N55=(P55-K5*K5/D)/LAM1

"UPDATE F-VECTOR
NF1=-Y
NF2=F1
NF3=U
NF4=F3

"UPDATE SAMPLING TIME
TS=T+DT
```

```
W:1.5
Z:0.707
NP:20
LAM:0.98
LAM1:0.8
EPS:0.01
UHIGH:5
ULOW:-5

A10:-1.5
A20:0.7
P0:100
P0B:0.1

Z1:0.1
Z2:0.99

F5:1
PI:3.1415926

END
```

- -

```
CONTINUOUS SYSTEM PROC2

TIME T
INPUT U V
OUTPUT Y
STATE X1 X2 X3 X4 X5 X6
DER D1 D2 D3 D4 D5 D6

OUTPUT
Y1=IF O<2 THEN X1 ELSE IF O<3 THEN X2 ELSE IF O<4 THEN X3 ELSE X4
Y=IF O<4 THEN Y1 ELSE IF O<5 THEN X4 ELSE IF O<6 THEN X5 ELSE X6
DYNAMICS
PA1=IF T<TPCH THEN 1 ELSE PA11
PA2=IF T<TPCH THEN 1 ELSE PA22
D1=-PA1*X1+PA2*U+V
D2=-PA1*X2+X1
D3=-PA1*X3+X2
D4=-PA1*X4+X3
D5=-PA1*X5+X4
D6=-PA1*X6+X5

PA11:1
PA22:1
TPCH:50
FIRST:-1
O:1
END
```