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USE OF INSTRUMENTAL VARIABLES IN SELF-TUNING REGULATORS.

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1. INTRODUCTION

A self-tuning regulator consists of two parts, a recursive parameter estimator and a control system design algorithm. Since there are several ways to choose both the parameter estimator [1] and the control strategy, there are also many different ways to construct self-tuning regulators. The basic self-tuning regulator [2] e.g. is based on least squares identification and minimum variance control. There are several examples of other design methods used in self-tuning regulators, e.g. linear quadratic control [3] and pole placement [4].

The most widely used recursive parameter estimators are least squares and stochastic approximation methods. The major disadvantage of these methods is that they give biased estimates if the disturbances are correlated.

Off-line identification using instrumental variables has the advantage of giving unbiased estimates even if the disturbances are correlated. There are many ways to choose the instrumental variables. In [5], some choices are discussed and compared.

In this report it is investigated if a recursive instrumental varible method gives unbiased estimates when it is used in a self-tuning regulator. Unfortunately it turns out that if the regulator is required to be stable, it will give biased estimates in most cases. The usefulness of instrumental variable methods in self-tuning regulators is therefore questionable.

The report is organized as follows. The algorithm is presented in chapter 2. Stability conditions are given in chapter 3. Finally, in chapter 4 the properties of the algorithm are discussed. It is shown that stability and consistency requirements are contradictory.

2. THE ALGORITHM

A brief description of the algorithm is given here.

Assume that the system can be described by the difference equation

$$y(t+1) = b_0(-a_1 y(t) - \ldots - a_n y(t-n+1) + u(t) + b_1 u(t-1) + \ldots + b_m u(t-m)) + v(t+1) \qquad (2.1)$$

where $\{y(t)\}$ is the output and $\{u(t)\}$ is the input of the system. The disturbance $\{v(t)\}$ is a sequence of random variables.

The control objective is to achieve

$$\lim_{t\to\infty} [y(t)-y^*(t)] \equiv \lim_{t\to\infty} \varepsilon(t) = 0 \qquad (2.2)$$

where $\{y^*(t)\}$ is an apriori known reference sequence.

Introduce

$$\theta = (a_1 \; \ldots \; a_n \; b_1 \; \ldots \; b_m \; 1/b_0)^T \qquad (2.3)$$

and

$$\varphi(t+1) = (-y(t) \; \ldots \; -y(t-n+1) \; u(t-1) \; \ldots \; u(t-m) \; -y^*(t+1))^T. \qquad (2.4)$$

The equation (2.1) can then be written

$$y(t+1) = y^*(t+1) + b_0\theta^T\varphi(t+1) + b_0 u(t) + v(t+1). \qquad (2.5)$$

In the instrumental variable identification method, a vector

$$z(t+1) = (z_1(t) \; \ldots \; z_{n+m}(t) \; 0)^T \qquad (2.6)$$

with the property that it is uncorrelated with the noise sequence $\{v(t)\}$, is used. In the off-line version it is also required that $z(t)$ and $\varphi(t)$ are correlated. As will be proved in chapter 3, this is not necessary in the recursive version considered in this report. Stability is instead garanteed by some modifications of the original algorithm. However, it is often favorable to choose instumental variables which are strongly correlated with the vector $\varphi(t)$. There are several ways to do this. The most common choices are either delayed input and output signals of the real system or input and output signals of a model of the system [6], [7]. Various types of filters can also be used to avoid correlation between $z(t)$ and the noise. In [5] some instrumental variables are compared and discussed.

Since the parameter vector θ is unknown, it has to be estimated. The estimated parameters are denoted by

$$\hat{\theta}(t) = (\hat{a}_1(t) \; ... \; \hat{a}_n(t) \; \hat{b}_1(t) \; ... \; \hat{b}_m(t) \; 1/\hat{b}_0)^T . \tag{2.7}$$

The parameter $\hat{b}_0$ is considered as a known constant. It is not updated.

The identification algorithm is now given by

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{z(t+1)\varepsilon(t+1)}{\hat{b}_0 S(t+1)} \tag{2.8}$$

$$S(t+1) = \bar{S}(t+1) \; \mathrm{sign}(z(t+1)^T \varphi(t+1)) \tag{2.9}$$

$$\bar{S}(t+1) = \lambda(t+1)\bar{S}(t) + |z(t+1)^T \varphi(t+1)| \tag{2.10}$$

where

$$0 < \lambda(t) \leq 1 \tag{2.11}$$

and $\lambda(t) = 1$ if $\bar{S}$ is less than a given constant. If $z(t)$ is equal to $\varphi(t)$ this algorithm is exactly the stochastic approximation method. The reason why a method corresponding to the least squares is not used will be motivated in chapter 4.

Finally the minimum variance control law is given by

$$u(t) = -\hat{\theta}(t)^T \varphi(t+1). \tag{2.12}$$

3. PROOF OF STABILITY IN THE DETERMINISTIC CASE

A stability proof for the algorithm given in chapter 2 will be given here. Only the deterministic case, i.e. $v(t+1) = 0$ in (2.1), is considered. For convenience, the algorithm is first summarized.

$$y(t+1) = y^*(t+1) + b_0\theta^T\varphi(t+1) + b_0u(t) \tag{3.1}$$

$$u(t) = -\hat{\theta}(t)^T\varphi(t+1) \tag{3.2a}$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{z(t+1)\varepsilon(t+1)}{\hat{b}_0S(t+1)} \tag{3.2b}$$

$$S(t+1) = \bar{S}(t+1)\ \mathrm{sign}(z(t+1)^T\varphi(t+1)) \tag{3.2c}$$

$$\bar{S}(t+1) = \lambda(t+1)\bar{S}(t) + |z(t+1)^T\varphi(t+1)| \tag{3.2d}$$

$$\varepsilon(t+1) = y(t+1) - y^*(t+1) \tag{3.2e}$$

The following assumptions are needed for the proof.

1. Upper limits for n and m, see (2.1), are known.
2. The system is minimum phase.
3. The reference sequence is bounded, i.e. $|y^*(t)| < \infty$ for all t.
4. $0 < b_0/\hat{b}_0 < 2$
5. $z(t)$ is never perpendicular to $\varphi(t)$.

The first assumption is restrictive and common in stability proofs for self-tuning regulators. The second one has to do with the design method used in the algorithm. If the signals in the system are required to be bounded, it is also reasonable to assume that the reference sequence is bounded. Thus the third assumption is weak. Since the the parameter $\hat{b}_0$ can be estimated, assumption 4 is not restrictive either.

The last assumption is more restrictive than required in the proof. In fact, it is sufficient that there exists a subsequence of the time sequence $\{t_i\}$ given in the proof, such that $z(t)$ and $\varphi(t)$ never are perpendicular for this subsequence.

The proof is based on the following lemma.

<u>Lemma</u>: Suppose that the assumptions 2 and 3 above hold. Then there exist constants $0 \le C1 < \infty$ and $0 \le C2 < \infty$ such that

$$\|\varphi(t)\| \le C1 + C2 \max_{1 \le \tau \le t} |\varepsilon(\tau)| \tag{3.3}$$

A proof of the lemma is given in [8].

<u>Theorem</u>: Subject to assumptions 1, 2, 3 ,4 and 5 above; if the algorithm (3.2) is applied to the system (3.1), then

a. The sequence $\{\tilde{\theta}(t)\} \equiv \{\hat{\theta}(t) - \theta\}$ is bounded and converges monotonely.

b. The sequences $\{y(t)\}$ and $\{u(t)\}$ are bounded.

c. $\lim_{t \to \infty} \varepsilon(t) = 0$.

<u>Proof</u>: Substituting (3.1) and (3.2a) into (3.2e) gives

$$\varepsilon(t+1) = b_0(\theta^T - \hat{\theta}(t)^T)\varphi(t+1). \tag{3.4}$$

Introducing $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, (3.2b) can be written as

$$\tilde{\theta}(t+1) = \tilde{\theta}(t) + \frac{z(t+1)}{\hat{b}_0 S(t+1)} b_0(\theta^T - \hat{\theta}(t)^T)\varphi(t+1) =$$

$$= \left[I - \frac{b_0 z(t+1)\varphi(t+1)^T}{\hat{b}_0 S(t+1)} \right] \tilde{\theta}(t) \tag{3.5}$$

Let the matrix inside the brackets in (3.5) be denoted by U, i.e.

$$U = \left[I - \frac{b_0 z(t+1)\varphi(t+1)^T}{\hat{b}_0 S(t+1)} \right]. \tag{3.6}$$

Equation (3.5) can then be abbreviated to

$$e_i^T(I - U^TU)e_i = e_i^Te_i - e_i^Te_i = 0. \tag{3.13}$$

Consider

$$U \frac{z(t+1)}{\|z(t+1)\|} = \left[I - \frac{b_0 z(t+1)\varphi(t+1)^T}{\hat{b}_0 S(t+1)} \right] \frac{z(t+1)}{\|z(t+1)\|} =$$

$$= \left[1 - \frac{b_0 \varphi(t+1)^T z(t+1)}{\hat{b}_0 S(t+1)} \right] \frac{z(t+1)}{\|z(t+1)\|}. \tag{3.14}$$

The matrix $(I - U^TU)$ has thus the last eigenvector

$$e_{n+m+1} = \frac{z(t+1)}{\|z(t+1)\|} . \tag{3.15}$$

For the eigenvector e_{n+m+1} we have

$$e_{n+m+1}^T(I - U^TU)e_{n+m+1} = 1 - \left[1 - \frac{b_0 \varphi(t+1)^T z(t+1)}{\hat{b}_0 S(t+1)} \right]^2 . \tag{3.16}$$

Hence the matrix $(I - U^TU)$ is nonnegative definite if

$$0 \le \frac{b_0 \varphi(t+1)^T z(t+1)}{\hat{b}_0 S(t+1)} \le 2. \tag{3.17}$$

Making use of (3.2c) and (3.2d), the condition can be rewritten as

$$0 \le \frac{b_0}{\hat{b}_0} \cdot \frac{|\varphi(t+1)^T z(t+1)|}{\lambda(t+1)\bar{S}(t) + |\varphi(t+1)^T z(t+1)|} \le 2. \qquad (3.18)$$

Since $\lambda(t+1)\bar{S}(t)$ is always positive, the condition (3.18) follows from assumption 4.

The matrix $(I - U^T U)$ is thus nonnegative definite and it follows from (3.9) that $\{\tilde{\theta}\}$ is a bounded sequence which converges monotonely. Hence statement a in the theorem is proved. Note that only the assumptions 1 and 4 are used for this part of the proof.

Assume for a moment that statement c in the theorem doesn't hold, i.e.

$$\lim_{t\to\infty} \varepsilon(t) \neq 0. \qquad (3.19)$$

This means that there exists an infinite time sequence $\{t_i\}_{i=1}^{\infty}$ such that

$$\varepsilon(t_i) \neq 0 \qquad i = 1, 2, \ldots \qquad (3.20)$$

Substituting (3.4) into (3.20) gives

$$\varepsilon(t_i) = -b_0 \tilde{\theta}(t_i)^T \varphi(t_i+1) \neq 0, \qquad (3.21)$$

and therefore $\tilde{\theta}(t_i)$ and $\varphi(t_i)$ are not perpendicular for any i. Hence there exists a δ such that

$$|\tilde{\theta}(t_i)^T \varphi(t_i+1)| \ge \delta > 0 \qquad i = 1, 2, \ldots \qquad (3.22)$$

Since assumption 5 holds, the egenvalue of U corresponding to the eigenvector e_{n+m+1} is positive. This means that

4. DISCUSSION

In this chapter, the properties of the algorithm are discussed. The algorithm described in chapter 2 is a modified version of the original recursive instrumental variable method. The motivations for theese modifications are discussed in the first part of the chapter. The chapter ends with a consistency analysis.

The modifications of the original algorithm.

If the off-line instrumental variable identification method is made recursive, the algorithm will not be exactly the one given in chapter 2. Apart from the normal introduction of the forgetting factor λ, the difference is that the equations (2.9) and (2.10) would instead be replaced by

$$S(t+1) = \lambda(t+1)S(t) + z(t+1)^T\varphi(t+1). \qquad (4.1)$$

If (4.1) is used, the quantity $S(t)$ can however be zero. In equation (2.8) it is required that $S(t)$ is nonzero. It is also reasonable that the amount of $S(t)$ increases when $|z^T\varphi|$ is large. These are the motivations for introducing the modulus in (2.10). Obviously the algorithm may be unstable if this modification is not used.

To make the direction of the changes of the parameter estimates correct, $\bar{S}(t+1)$ is multiplied by $\mathrm{sign}(z(t+1)^T\varphi(t+1))$ in (2.9) before the usage in (2.8). The following simple example shows that this modification also is necessary if stability is required.

<u>Example 1:</u> The following system and control law are given

$$y(t+1) = 0.9y(t) + u(t) \qquad (4.2)$$

$$u(t) = \hat{a}(t)y(t) \qquad (4.3)$$

Assume that the instrumental variables have the same sign during some time intervals. For simplicity, let $z(t) = -1$. The identification is then given by the equations

$$\hat{a}(t+1) = \hat{a}(t) - \frac{y(t+1)}{S(t+1)} \qquad (4.4)$$

$$S(t+1) = \lambda(t+1)S(t) + |y(t)|. \tag{4.5}$$

Let the initial values be $y(0)=-0.2$, $\hat{a}(0)=0.2$ and $S(0)=10$ and let the forgetting factor be $\lambda(t)=0.99$. In figure 1, the results of a simulation are given. The algorithm is obviously unstable. This originates from the fact that the changes in the estimates in (4.4) have wrong direction. This would not be the case with the algorithm in chapter 2.

□

It is thus shown, that the modifications in the algorithm in chapter 2 are reasonable.

The reason why an instrumental variable method corresponding to the least squares is not used, can also be seen from the discussion above. This algorithm should be like the one in chapter 2, except that equations (2.9) and (2.10) would instead be replaced by

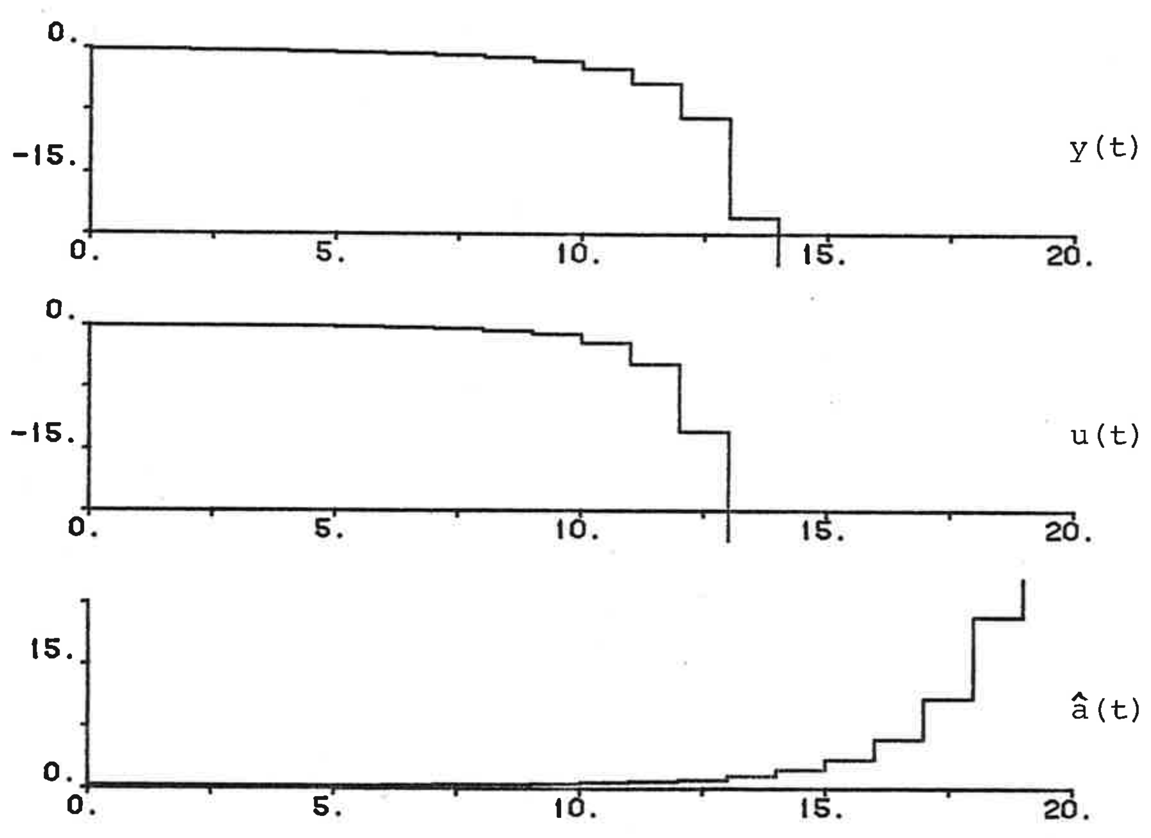

Figure 1: Results of the simulation in example 1.

$$S(t+1) = \lambda(t+1)S(t) + z(t+1)\varphi(t+1)^T . \qquad (4.6)$$

Here the problem of keeping the matrix S(t) regular arises. This is far more complicated than the problem of keeping the scalar S(t) in the other method nonzero.

Convergence properties.

The reason for choosing the instrumental variable identification method instead of the stochastic approximation, is to avoid bias in the estimates, when the process noise is coloured. This chapter ends with a consistency analysis of the algorithm in chapter 2. The differential equations [9], [10] for the algorithm will be set up for a simple example.

Example 2: The following system and control law are given

$$y(t+1) = -ay(t) + u(t) + e(t+1) + ce(t) \qquad |c| < 1 \qquad (4.7)$$

$$u(t) = \hat{a}(t)y(t) + y^* . \qquad (4.8)$$

$\{e(t)\}$ is a sequence of independent normally distributed random variables. y^* is a known constant reference value. (4.7) and (4.8) can be unified to

$$y(t+1) = \tilde{a}(t)y(t) + y^* + e(t+1) + ce(t) \qquad (4.9)$$

where $\tilde{a}(t) = \hat{a}(t) - a$. If $z = -1$, the estimation is given by the following equations.

$$\hat{a}(t+1) = \hat{a}(t) - \frac{y(t+1) - y^*}{S(t+1)} \qquad (4.10)$$

$$S(t+1) = \bar{S}(t+1)\ \mathrm{sign}(y(t)) \qquad (4.11)$$

$$\bar{S}(t+1) = \bar{S}(t) + |y(t)| \qquad (4.12)$$

The forgetting factor λ is equal to 1 in this analysis. Some calculations yield the following differential equations for the algorithm.

very little bias in the estimates.

□

This chapter has shown, that the changes that were required to make the recursive algorithm stable generally causes bias in the estimates when the noise is coloured. The only exception is when $\mathrm{sign}(z(t)^T\varphi(t))$ is constant for all t, and the order of the noise polynomial is small. This is the case e.g. in the servo problem, when there is a large signal-to-noise ratio, provided that the choice of instrumental variables is good. In this case however, the least squares and the stochastic approximation methods give very little bias. It seems therefore seldom be any great advantage to use this instrumental variable method.

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