Adaptive Control with Fault Detection

Hägglund, Tore

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LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

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TORE HÄGGLUND

DEPARTMENT OF AUTOMATIC CONTROL
LUND INSTITUTE OF TECHNOLOGY

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DEPARTMENT OF AUTOMATIC CONTROL
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S 220 07 Lund 7 Sweden

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Author(s)

Supervisor
Karl Johan Åström

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Abstract

The report describes a method to decouple the transient and the stationary properties of recursive parameter estimators applied to self-tuning regulators. The decoupling is achieved by introducing a fault detection procedure and letting the gain in the estimator depend on whether a fault has occurred or not.

A new fault detection method is presented, which fulfils the requirements of this special application. One of these requirements is, that the noise variance is not assumed to be constant. The fault detection method can be applied to ordinary fault detection problems as well.

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ADAPTIVE CONTROL WITH FAULT DETECTION

Tore Hägglund

Department of Automatic Control
Lund Institute of Technology
June 1982

<u>Contents</u>

1. INTRODUCTION

A self-tuning regulator consists of two parts, a parameter estimator and a controller based on some design strategy, see Åström and Wittenmark (1973). The most commonly used estimators are recursive versions of well-known identification methods such as least squares, stochastic approximation and maximum-likelihood algorithms. These estimation algorithms were originally proposed for systems with constant parameters. Since the main reason for using self-tuning regulators is the possibility to track time-varying parameters in the system, the recursive versions have to be modified.

A useful way to modify the algorithms is to introduce a so called forgetting factor. It has the effect that old data are discounted exponentially. If the forgetting factor is small, which means that the data are quickly discounted, the estimated parameters will rapidly converge towards the true values. On the other hand, the accuracy of the estimates will decrease. The choice of forgetting factor is therefore a compromise between the demands on convergence rate and long term quality of the estimates. It is often possible to find an acceptable forgetting factor, but there are also cases where the compromise is not satisfactory.

If it is possible to detect when a fault in the process model has occurred, the estimation algorithm can be modified so that the forgetting factor plays a minor part in the algorithm. The forgetting factor can then be chosen mainly with respect to the demand on accuracy of the estimates. When a fault is detected, the algorithm can be modified so that the convergence rate increases. In that way, the influence of the forgetting factor on the rate of convergence is reduced. It is therefore interesting to study methods for fault detection, and in which way the algorithm is to be modified when a fault is detected.

It should be remarked that the notation "fault" in this report means a change in the process model, which not necessarily has to originate from a physical fault in the process. It can e.g. as well be a parameter change due to a shift of working point in a nonlinear system.

Methods for fault detection are of course of great importance in their own. It is a well-known trend, that the industrial processes are getting more and more sophisticated. The increased performance are often reached at the price of processes that are more sensitive and harder to supervise. Illuminating examples are e.g. aircrafts and the power industry. This progress, together with increasing demands on availability and security, has caused an increased interest in the problem of detecting faults in dynamic systems. This is often solved by hardware redundancy combined with some voting technique. Since this is an expensive solution and since computers are getting cheaper and cheaper, software solutions have received much interest in the past few years.

This report deals with the problem of reducing the hard coupling between the transient and the stationary properties of the identification procedure in a self-tuning regulator. The solution to the problem can be separated into two parts: fault detection and modification of the identification procedure. The problem of parameter tracking in adaptive control systems is first described in Chapter 2. Since none of the presently existing fault detection methods suit the demands required in this special application, a new way to detect faults is proposed here, in Chapter 3. In the fault detection method, the statistical properties of a stochastic difference equation are considered. This equation is investigated in Chapter 4. The fault detection makes it possible to decouple the transient and the stationary properties of the identification procedure. The modifications of the estimation algorithm needed for this decoupling are described in Chapter 5. The proposed way of doing adaptive control with fault detection is illustrated by an example in Chapter 6.

2. PROBLEM FORMULATION

In this chapter, the basic notations and the problem are introduced. The solution to the problem can be separated into two parts, detection of faults in the system and modification of the estimation algorithm. The chapter ends with a list of requirements on the solution to the first part of the problem.

Notations

The structure of a process controlled by a self-tuning regulator is shown in Figure 2.1. The system consists of three parts, namely the process (P), a parameter estimator (E) and a regulator (R). It is assumed that the process can be described by the regression model

$$y(t) = \theta(t-1)^T \varphi(t) + e(t) \tag{2.1}$$

where $y(t)$ is the measured output from the process, $\varphi(t)$ is a vector containing old outputs and inputs of the process, $\{e(t)\}$ is a sequence of independent random variables and $\theta(t)$ is a parameter vector. Furthermore, it will be assumed that the disturbances $\{e(t)\}$ have a symmetrical distribution. The parameter vector $\theta(t)$ is estimated by the recursive least squares method:

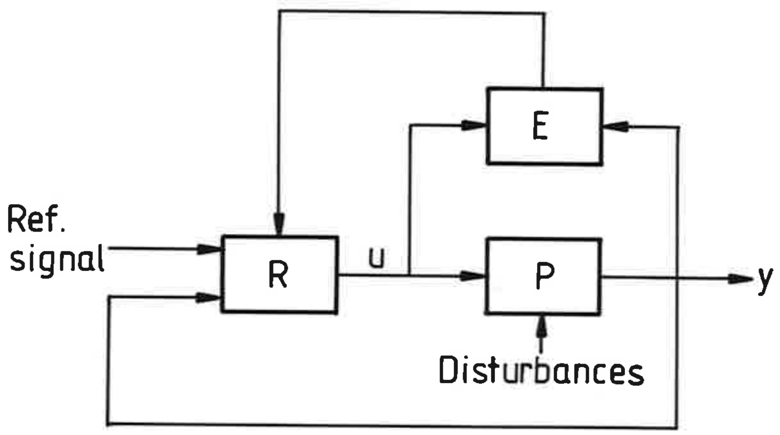


Figure 2.1 - The structure of a process controlled by a self-tuning regulator.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\varphi(t)\varepsilon(t) \tag{2.2a}$$

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)^T P(t-1)}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} \right] \tag{2.2b}$$

$$\varepsilon(t) = y(t) - \hat{y}(t) = (\theta(t-1) - \hat{\theta}(t-1))^T \varphi(t) + e(t) \stackrel{\Delta}{=}$$

$$\stackrel{\Delta}{=} \tilde{\theta}(t-1)^T \varphi(t) + e(t) \tag{2.2c}$$

Here $\hat{\theta}(t)$ is the estimate of $\theta(t)$, $\hat{y}(t)$ is the prediction of $y(t)$ and $\tilde{\theta}(t)$ is the estimation error at time t.

The report is devoted to the least squares algorithm for purpose of convenience, but all what is written in the sequel can easily be applied to other estimation algorithms as well. Processes with colored noise disturbances instead of white noise disturbances can also be treated, when an appropriate identification method is applied.

The control signal is supposed to be a function of the estimated parameters and the measured variables, i.e.

$$u(t) = f(\hat{\theta}(t), \hat{\theta}(t-1), \ldots, \varphi(t), \varphi(t-1), \ldots) \tag{2.3}$$

The notations

$$\Delta\hat{\theta}(t) \stackrel{\Delta}{=} \hat{\theta}(t) - \hat{\theta}(t-1)$$

$$\Delta\tilde{\theta}(t) \stackrel{\Delta}{=} \tilde{\theta}(t) - \tilde{\theta}(t-1)$$

will also be used frequently in the following chapters.

The problem

The parameter λ in equation (2.2) is the forgetting factor, which has the effect that old data are discounted exponentially. The value of λ is to be chosen between 0 and 1. A small λ, which means that old data are discounted rapidly, causes a fast adaptation to new values, when a change in $\theta(t)$ occurs. However, a small λ will also reduce the accuracy of the estimates during normal operation. This is illustrated in Figure 2.2.

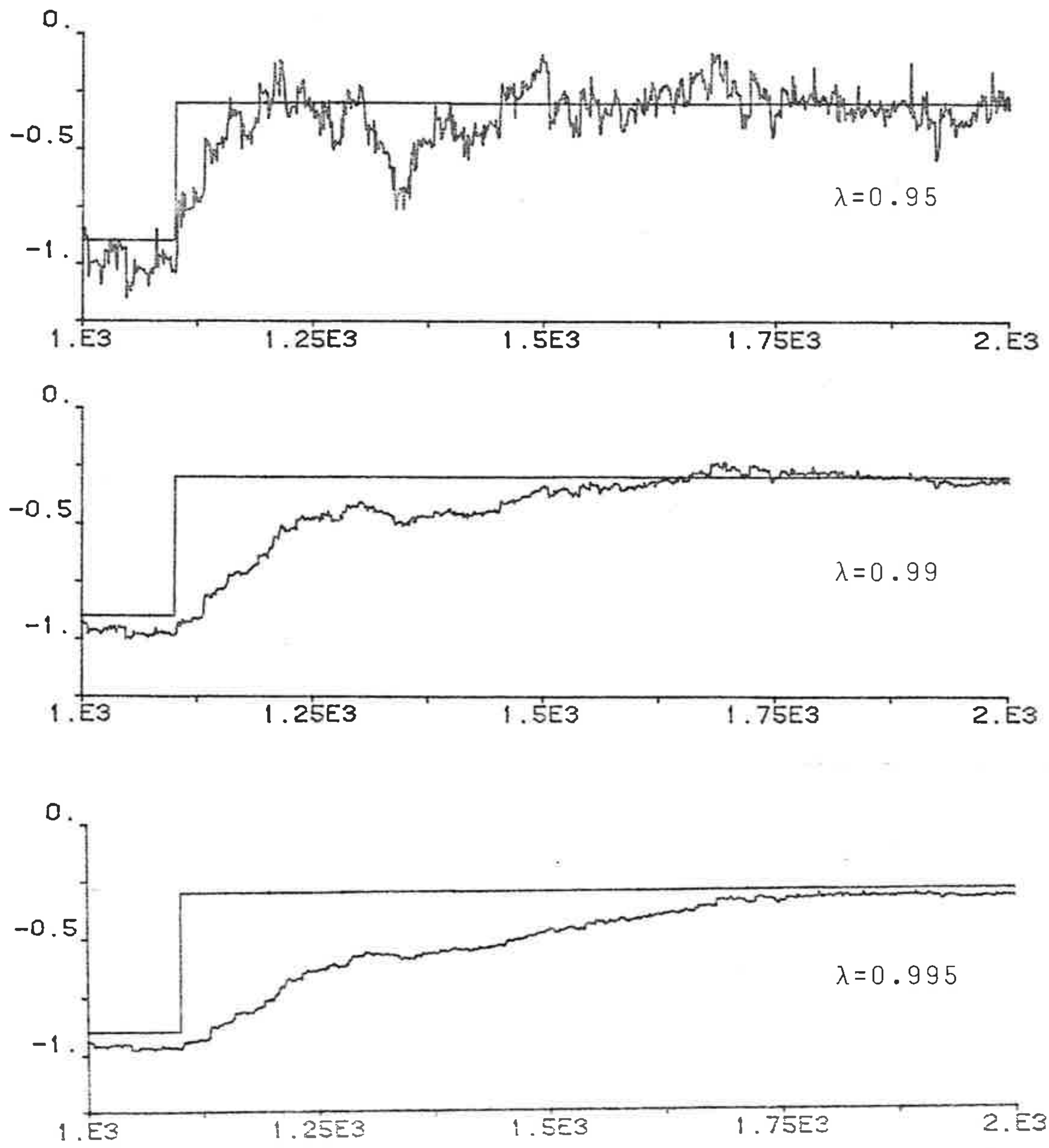


Figure 2.2 - Estimation of the parameter α in the system $y(t+1) = \alpha \cdot y(t) + e(t)$, where α changes from -0.9 to -0.3. Different forgetting factors are used in the different simulations.

The forgetting factor in algorithm (2.2) determines both the transient and the stationary properties of the estimation. This is unpleasant, not only in systems with timevarying parameters. Even in systems with constant parameters, it is valuable to have a higher gain in the estimation algorithm during the transient period. The aim of this study is to reduce the influence of the forgetting factor on the transient behaviours, by modifying the estimation algorithm when a fault occurs. This approach requires an algorithm for detection of the faults.

Requirements on the fault detection

Many techniques have been proposed for the detection of faults in dynamic systems. Some of them are very general, while others use more a priori information. To facilitate the choice of method in the next chapter, some natural requirements for this special application will be stated here.

(R1) The times when the faults occur are not known.

(R2) The nature of the faults is not known.

Since the transformation between the physical parameters in the process and the parameters in the model (2.1) usually is very involved, this is a natural requirement.

(R3) It must be possible to repeat the detection from the new modes of operation.

This means e.g. that there does not exist any "normal mode". As soon as a change in $\theta(t)$ is accepted, the old parameters are forgotten. This requirement is considered to get a general method. In some applications it can be relaxed.

(R4) A change in the noise level must not disturb the detection.

The only assumption made on the noise sequence $\{e(t)\}$ is that it consists of independent symmetrically distributed random variables. Therefore, a change in the noise level does not effect the parameters $\theta(t)$. This is an important requirement, since a change in the noise level is often much more likely than a change in the process parameters.

3. FAULT DETECTION

After a short review of earlier work in the area of fault detection, a new method is presented, which fulfils the requirements given previously.

3.1. Earlier work

In recent years, a great variety of design methods for fault detection has appeared. Some of them are general, while others are more or less devoted to special applications or concerned with voting between some known models. Since the problem described in the previous chapter requires a general method, only such will be considered here.

A survey of methods for fault detection is given in Willsky (1976). Most methods are based on a comparison between the true output signals and the expected output signals derived from a Kalman filter. When the difference is large, a fault is supposed to have occured. These are useful methods if the faults have a large influence on the output signals compared with the noise. On the other hand, this is an easy problem where almost all methods work well. In processes with reasonable noise levels, large faults may often occur without any immediate large influence on the output signals. An example is given in Chapter 6. Looking at the list of requirements in Chapter 2, it is seen that requirement (R4) is violated, since it is assumed that the noise level is known or can be estimated in these methods.

Since the problem of fault detection is concerned with changes in the parameter vector $\theta(t)$, it seems more natural to study $\theta(t)$ via recursive estimation rather than the effects the changes can have on the magnitude of the output noise signal. This is also suggested some times in the literature, but from what is known all these methods do also violate requirement (R4), i.e. they assume that the noise level is known or can be estimated.

Requirement (R4) is important, not only because a change in the noise level often is much more likely than a change in the parameter vector. In real processes, disturbances are often entering at several points, and not only additively to the input or output signal. In the process model, the different disturbance sources are represented by one equivalent disturbance source entering at one point, see Åström (1970). The characteristics of these equivalent disturbances are dependent on the process parameters. This means that a change in the parameter vector usually also causes a change in the equivalent output noise level. Under these circumstances, it does not seem to be appealing to detect faults under the assumption that the noise level in the output is constant.

3.2. A new method

As was noted above, norms of the residuals

$$\varepsilon(t) = y(t) - \hat{y}(t) = \tilde{\theta}(t-1)^T \varphi(t) + e(t)$$

are examined in most fault detection methods, and it is assumed that the sequence $\{e(t)\}$ has a constant variance. This assumption will not be made in the method proposed below.

The idea

The real problem is to detect when $\theta(t)$ changes from its previous value. The vector $\theta(t)$ is not known, and consequently not $\tilde{\theta}(t)$. However, $\Delta\tilde{\theta}(t)$ is known when $\theta(t)$ is constant, since

$$\Delta\tilde{\theta}(t) = \theta(t) - \hat{\theta}(t) - \theta(t-1) + \hat{\theta}(t-1) = - \Delta\hat{\theta}(t)$$

in this case. These innovations of the estimates will yield the information needed for the fault detection. To be able to extract this information, the statistics of $\{\Delta\hat{\theta}(t)\}$ will first be investigated.

In normal operation when no fault has occurred, i.e. when the estimated parameters are close to the true ones, the innovations of the estimates are given by

$$\Delta\hat{\theta}(t) = P(t)\varphi(t)(\varphi(t)^T\tilde{\theta}(t-1) + e(t)).$$

The updating of the estimates at time t is therefore in the direction of the $P(t)\varphi(t)$ vector. The probabilities of positive and negative direction are almost the same. This is intuitively seen from the following arguments.

When $\lambda = 1$, the identification procedure is the ordinary least squares algorithm without any discounting of past data. It is known to be the best linear unbiased estimator, see Goodwin and Payne (1977). This implies that there is no correlation between the innovations of the parameter estimates in normal operation. If there were a correlation, it would be possible to modify the algorithm so that a smaller variance of the estimates were obtained. (If there at time t is any information about how the estimates will be changed at time t+1, all information given at time t is not used, and it is possible to derive a better estimate). This is contradictory to the fact that the least squares algorithm is the best linear unbiased estimator. Hence, when $\lambda = 1$ the probabilities of positive and negative $P(t)\varphi(t)$ direction of the innovations of the estimates are the same, 0.5.

When $\lambda < 1$, a negative correlation between the innovations of the estimates may be expected. If a forgetting factor less than one is used, the gain in the parameter estimator is greater than it should be if the forgetting factor was equal to one. This means that the algorithm in each updating of the estimates has to compensate for the large step taken previously. Hence the expected correlation is negative. However, from continuity arguments this correlation is small when λ is close to one, and the probabilities of positive and negative $P(t)\varphi(t)$ direction of the innovations of the estimates are approximately the same.

The arguments above implies that

$$P\{\Delta\hat{\theta}(t)^T \Delta\hat{\theta}(t-1) > 0\} \approx P\{\Delta\hat{\theta}(t)^T \Delta\hat{\theta}(t-1) < 0\}, \qquad (3.1)$$

where P is the probability measure, under normal operation.

When $\tilde{\theta}(t)$ is not close to the true values, i.e. when a fault has occurred, the approximation above is no longer valid. Since the estimated parameters then will be driven towards the new values, the following inequality holds

$$P\{\Delta\hat{\theta}(t)^T \Delta\hat{\theta}(t-1) > 0\} > P\{\Delta\hat{\theta}(t)^T \Delta\hat{\theta}(t-1) < 0\}. \qquad (3.2)$$

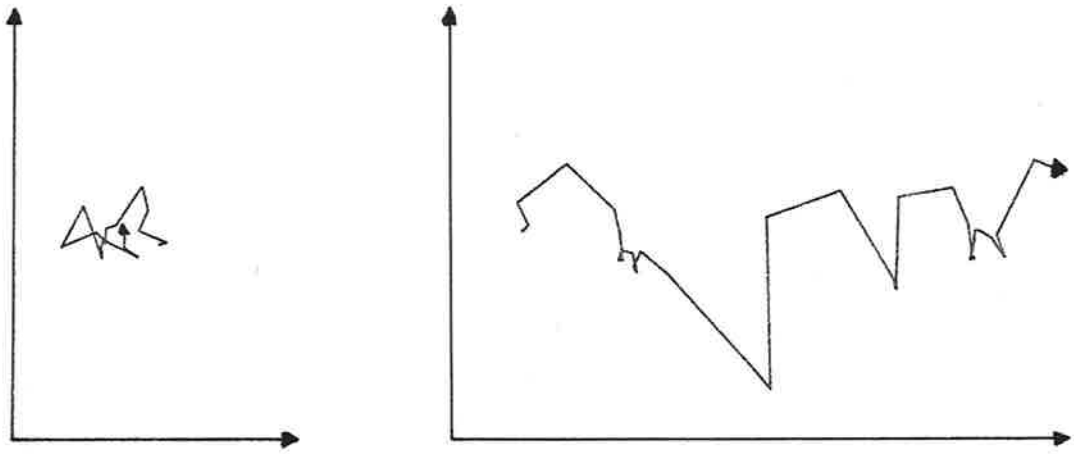
The intuitive arguments above are illustrated in Figure 3.1, where the estimated parameters in a two parameter model are shown in the parameter plane in case of stationarity (a) and when a fault has occurred (b). The difference described by equations (3.1) and (3.2) will be used in the sequel to derive the fault detection method.

<u>Implementation</u>

Instead of observing the scalar products

$$\Delta\hat{\theta}(t)^T \Delta\hat{\theta}(t-1)$$

it will often be more useful to study the scalar products between $\Delta\hat{\theta}(t)$ and a sum of the latest innovations of the estimates. To simplify the algorithm, an exponential



<u>Figure 3.1</u> - The estimated parameters in case of stationarity (a) and when a fault has occurred (b).

filtering of the innovations of the estimates will be used instead of an ordinary sum. For this purpose, introduce v(t) as

$$v(t) = \gamma_1 v(t-1) + \Delta\hat{\theta}(t) \qquad 0 \le \gamma_1 < 1 \tag{3.3}$$

In the case when a fault has occurred, v(t) can be viewed as an estimate of the direction of the parameter change. Equations (3.1) and (3.2) with v(t-1) substituted for $\Delta\hat{\theta}(t-1)$ are still valid. The test quantity that will be studied is

$$s(t) \overset{\Delta}{=} \text{sign}(\Delta\hat{\theta}(t)^T v(t-1)) \tag{3.4}$$

It is now intuitively clear how the fault detection should be carried out:

> "Inspect the latest values of s(t). If s(t) is +1 unlikely many times, conclude that a fault has occurred."

<u>Testing method</u>

Most fault detection methods end up with a statistical test of a time sequence, e.g. $\{\varepsilon(t)\}$, $\{\varepsilon(t)^2\}$ or as in this case {s(t)}. A common way of doing this is to use the Sequential Probability Ratio Test (SPRT), see Wald (1947). The SPRT is designed to decide between two hypotheses, which in this application means between two different parameter vectors. The SPRT is usually efficient in terms of short times to detect the faults, if the values of the true parameters occurring in practice are close to the hypothesized ones. However, if the hypothesized values are taken merely to obtain a SPRT, and do not represent the most frequently occurring values, the SPRT may not lead to any time saving compared with other methods, see Wetherill (1966). Requirement (R2) implies that no a priori information about the parameter changes is known. Since requirement (R1) furthermore implies that a new sequence to be tested must be introduced every sample instant if the SPRT shall reach the expected efficiency, a traditional Bayesian approach will instead be used here.

Under normal operation, i.e. when the parameter estimates are fluctuating close to the true values, s(t) has approximately a symmetric Bernoullian distribution with mass 0.5 each at +1 and −1. When a fault has occurred, the distribution is no longer symmetric, but the mass at +1 is larger than the mass at −1. Summing up the latest values of s(t), for computational simplicity by exponential filtering, the stochastic variable r(t) is defined as

$$r(t) = \gamma_2 r(t-1) + (1-\gamma_2)s(t) \qquad 0 \le \gamma_2 < 1 \qquad (3.5)$$

When the parameter estimates are close to the true ones, r(t) has a mean value close to zero. When a fault has occurred, a positive mean is expected.

The parameter γ_2 determines, roughly speaking, how many s(t) that shall be taken under consideration. E.g. $\gamma_2 = 0.95$ corresponds to about 20 values, which is a reasonable choice in many applications. A small γ_2 allows a fast fault detection, although at the price of a minor security against false alarms. This weighting is typical for all fault detection methods. When the signal to noise ratio is small, it is not possible to detect the faults as fast as otherwise. It is then necessary to have more information available to be able decide whether a fault is present. This can be achieved by increasing γ_2.

The stochastic properties of r(t) are investigated in the next chapter. For values of γ_2 close to one, r(t) will approximately have a Gaussian distribution with variance

$$\sigma^2 = \frac{1 - \gamma_2}{1 + \gamma_2} \ . \tag{3.6}$$

Since γ_2 is generally chosen in this region, it will in the sequel be assumed that $r(t)$ has a Gaussian distribution. It is possible to specify a limit of how frequently false alarms occur. If it is acceptable to get a false alarm every n:th sample instant, a fault detection should be given every time $r(t)$ is greater than a threshold r_0, defined by

$$P\{r(t) \geq r_0\} = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{r_0}^{\infty} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx = \frac{1}{n} \tag{3.7}$$

If a small value of the threshold is chosen to make it possible to detect faults quickly, the frequency of false detections will be high. This is seen in equation (3.7), where there is an inverse relation between r_0 and $1/n$. As was said before, this compromise between fast detection and security against false alarms must be made in all fault detection methods. The determination of r_0 in this method has the advantage that it is formulated in terms of the expected frequency of false detections, which can be chosen to suit any particular application. The relations between r_0 and $1/n$, for some different values of γ_2, are shown in Figure 3.2.

The fault detection method described above fulfils the requirements stated in Chapter 2.

Finally, it should be remarked that if the requirement (R4) is eliminated, an extremely powerful fault detection method can probably be derived by studying the quantity $v(t)^T v(t)$. This variable is usually much more sensitive to parameter changes than the generally studied $\varepsilon(t)^2$. This fact is illustrated in Chapter 6.

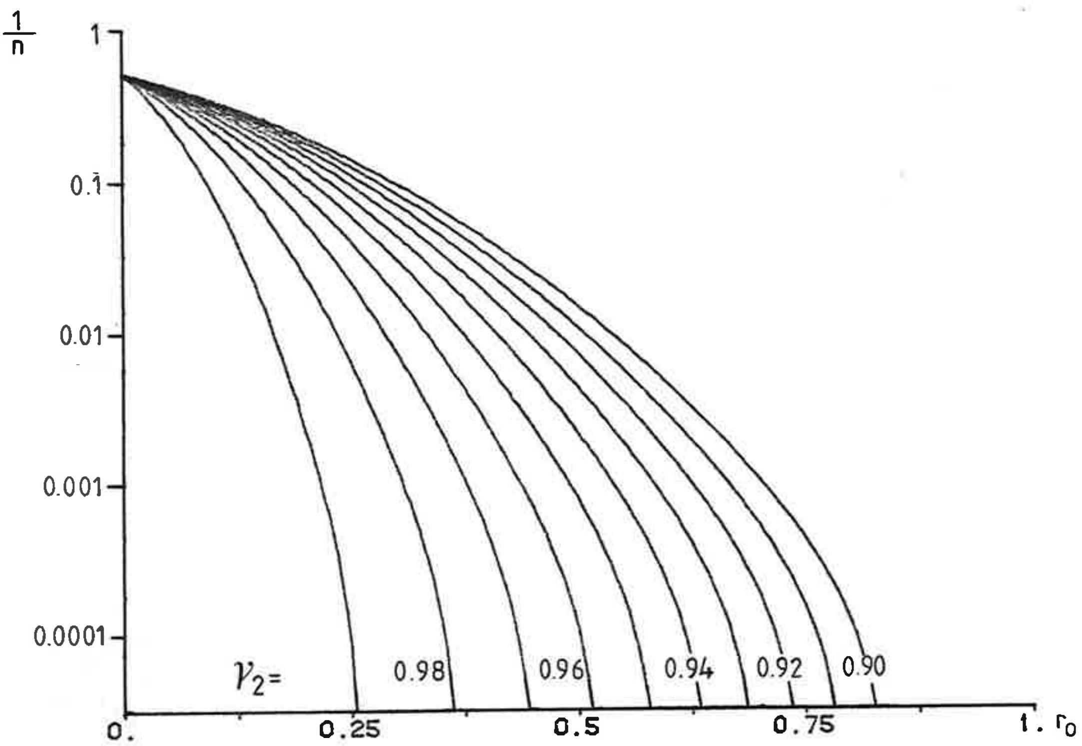


Figure 3.2 - The relations between r_o and $1/n$.

4. A STOCHASTIC DIFFERENCE EQUATION

In the previous chapter, the derivation of the new fault detection method resulted in a proposed test of a sequence, $\{r(t)\}$. To be able to perform this test, the statistical properties of $r(t)$ must first be explored. This is done in the present chapter.

According to equation (3.5), $r(t)$ is generated by the stochastic difference equation

$$r(t) = \gamma r(t-1) + (1-\gamma)s(t) \qquad 0 \le \gamma < 1$$

where the subscript "2" of γ is dropped for convenience. The sequence $\{s(t)\}$ consists of independent random variables with the symmetric Bernoullian distribution

$$s(t) = \begin{cases} -1 & \text{with probability } 0.5 \\ 1 & \text{with probability } 0.5 \end{cases}$$

The distribution of $r(t)$ is highly dependent on the value of γ. Even if only γ's with values close to one are considered in this special application, it is interesting to investigate the behaviour of $r(t)$ in the whole interval $0 \le \gamma < 1$. This will be done below, starting with $\gamma = 0$.

<u>$\gamma = 0$.</u>
When $\gamma = 0$, $r(t)$ is equal to $s(t)$ and has consequently a symmetric Bernoullian distribution.

<u>$0 < \gamma < 0.5$.</u>
For γ's in this interval, a distribution of Cantor-type occurs, see e.g. Chung (1968). To see this, it is first noted that $r(t)$ can take any of the following values asymptotically

$$r(t) \in \{ \pm(1-\gamma)(1 \pm \gamma \pm \gamma^2 \pm \ldots) \}$$

Arrange the asymptotic values of $r(t)$ in groups according to

1. $r(t) = \pm(1-\gamma)(1+(\gamma+\gamma^2+\gamma^3+ \ldots)) = \pm 1$

   $r(t) = \pm(1-\gamma)(1-(\gamma+\gamma^2+\gamma^3+ \ldots)) = \pm(1-2\gamma)$

2. $r(t) = \pm(1-\gamma)(1+\gamma-(\gamma^2+\gamma^3+\gamma^4+ \ldots)) = \pm(1-2\gamma^2)$

$r(t) = \pm(1-\gamma)(1-\gamma+(\gamma^2+\gamma^3+\gamma^4+ \ldots)) = \pm(1-2\gamma+2\gamma^2)$

3. $r(t) = \pm(1-\gamma)(1+\gamma+\gamma^2-(\gamma^3+\gamma^4+\gamma^5+ \ldots)) = \pm(1-2\gamma^3)$

$r(t) = \pm(1-\gamma)(1-\gamma-\gamma^2+(\gamma^3+\gamma^4+\gamma^5+ \ldots)) = \pm(1-2\gamma+2\gamma^3)$

$r(t) = \pm(1-\gamma)(1+\gamma-\gamma^2+(\gamma^3+\gamma^4+\gamma^5+ \ldots)) = \pm(1-2\gamma^2+2\gamma^3)$

$r(t) = \pm(1-\gamma)(1-\gamma+\gamma^2-(\gamma^3+\gamma^4+\gamma^5+ \ldots)) = \pm(1-2\gamma+2\gamma^2-2\gamma^3)$

and so on.

The groups are formed in the following way. In group number i, the expressions in all the previous groups are rewritten, but with changed sign in front of all terms of order greater or equal to i in the right parenthesis. These values, defining the possible values of r, can also be derived in the following way:

1. From the closed interval [-1,1], the open interval $(1-2\gamma)$ times the interval length is removed in the middle. This results in two disjoint closed intervals with endpoints equal to the values in group 1.

2. From the two disjoint intervals obtained under 1, remove $(1-2\gamma)$ times the interval length in each middle. This operation results in four closed disjoint intervals, with end points equal to the values in group 1 and 2 above.

and so on.

Dividing the closed intervals like this into smaller and smaller intervals, a Cantor set is obtained. For values of γ in the interval $0 < \gamma < 0.5$, the distribution is consequently singular. (It can be shown that the distribution function in spite of this is continuous, see Chung (1968)).

$\gamma = 0.5$.

When $\gamma = 0.5$, r(t) will asymptotically have a uniform distribution in the interval [-1,1]. This can be shown by investigating the characteristic function. The stochastic variable r(t) can be decomposed into a sum of stochastic variables

$$r(t) = \sum_{n=1}^{t} X_n \qquad X_n = \begin{cases} 0.5^n & \text{with probability } 0.5 \\ -0.5^n & \text{with probability } 0.5 \end{cases}$$

The characteristic function of X_n is

$$\varphi_n(t) = \int_{-\infty}^{\infty} e^{itx}\, 0.5(\delta(-0.5^n) + \delta(0.5^n))dx = \cos(0.5^n t)$$

The asymptotic characteristic function of $r(t)$ is now given as the product

$$\varphi(t) = \prod_{n=1}^{\infty} \varphi_n(t) = \prod_{n=1}^{\infty} \cos(0.5^n t) = \frac{\sin(t)}{t}$$

The second equality can be obtained from ordinary mathematical tables, e.g. Gradshteyn and Ryzik (1965). The characteristic function $\varphi(t)$ is the characteristic function of a uniformly distributed stochastic variable in the interval [-1,1]. It is therefore proved that $r(t)$ is uniformly distributed when $\gamma = 0.5$.

<u>$0.5 < \gamma < 1.$</u>

The distribution of $r(t)$ for these values of γ is not easy to determine. The density function f of $r(t)$ satisfies the functional equation

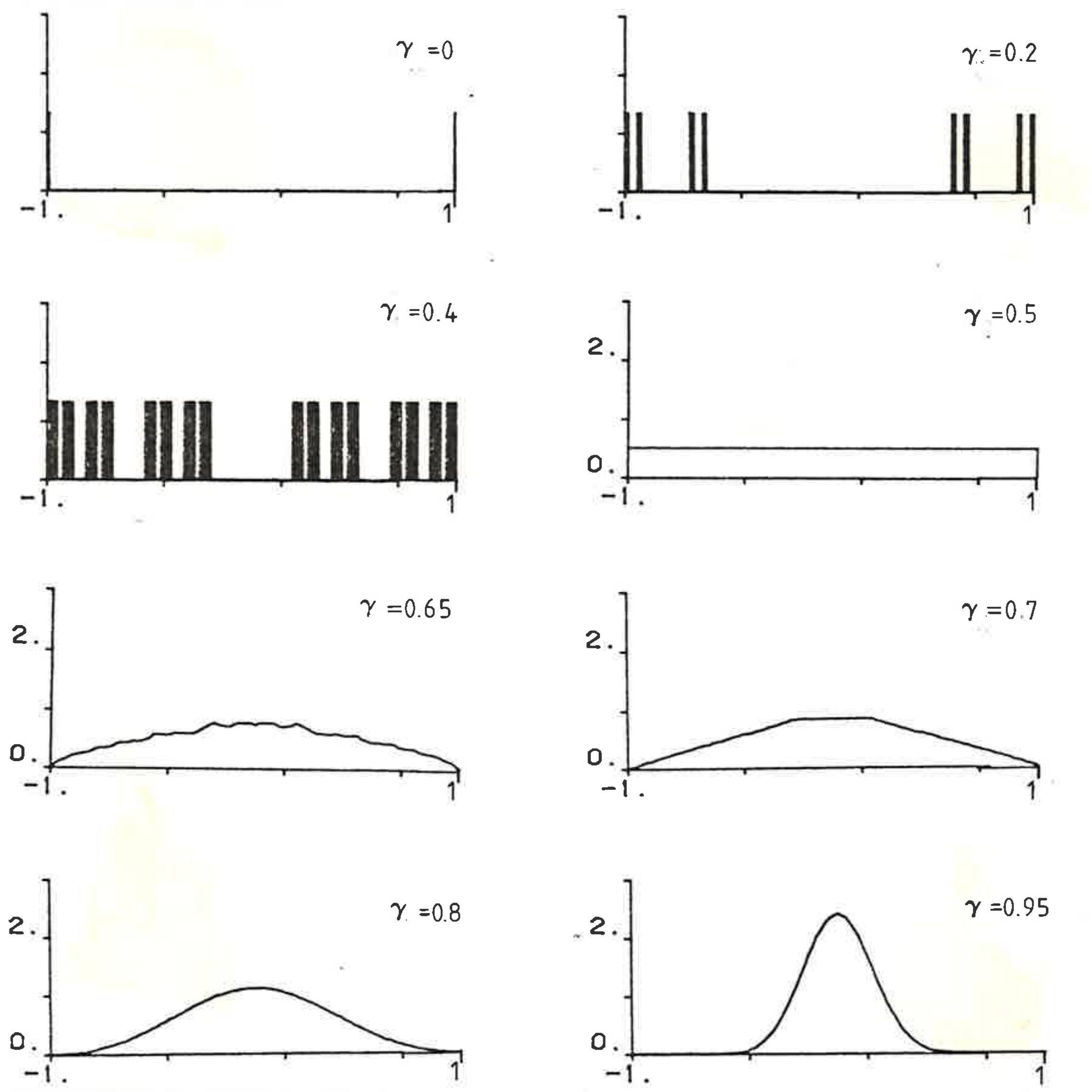
$$f(r) = \frac{1}{2\gamma} f\left[\frac{r - 1 + \gamma}{\gamma} \right] + \frac{1}{2\gamma} f\left[\frac{r + 1 - \gamma}{\gamma} \right] \qquad (4.1)$$

This equation has been solved numerically for some values of γ, see figure 4.1. For values of γ close to one, $r(t)$ will approximately have a Gaussian distribution with variance

$$\sigma^2 = \frac{1 - \gamma}{1 + \gamma}$$

since $r(t)$ in this case is a sum of almost equally distributed random variables.

It has been shown above that the distribution of $r(t)$ varies considerably as γ varies. To illustrate this, the density functions for a couple of values of γ are shown in Figure 4.1. For values of $\gamma < 0.5$, the density function is singular. Therefore the peaks in the corresponding diagrams represent Dirac-impulses of appropriate hights.



<u>Figure 4.1</u> – The density function of $r(t)$ for different values of γ.

5. MODIFICATION OF THE ESTIMATION ALGORITHM

The basic problem with the estimation algorithm (2.2) was that a compromise had to be made such that the gain was higher than desired under normal operation and lower than desired when a fault occurred. In Chapter 3, a method to detect faults in the system was derived. In the following chapter, the estimation algorithm is modified by taking this additional information under consideration.

To motivate the modification of the estimation algorithm, this chapter begins with an investigation, in the parameter space, of the effects of an innovation on the estimates.

The least squares algorithm gives, at each sample instant, a solution to the minimization problem

$$\min_{\hat{\theta}} V(\hat{\theta},t) \overset{\Delta}{=} \min_{\hat{\theta}} \sum_{i=0}^{t} \lambda^{t-i} \varepsilon(i)^2$$

Figure 5.1 shows an example of contours of constant values of $V(\hat{\theta},t)$ in the parameter space, in the noise-free case. At the point $\hat{\theta}(t-1)$, $\varphi(t)$ is orthogonal to the contour. In the least squares method, the estimate updating is not done in the $\varphi(t)$ direction, but in the $P(t)\varphi(t)$ direction. Near the

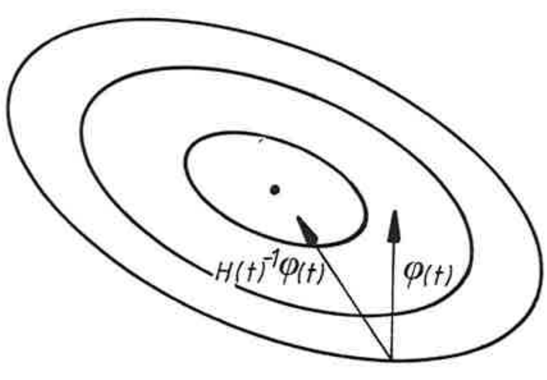


Figure 5.1 - Contours of constant values of $V(\hat{\theta},t)$ in the parameter space.

correct parameter values, $P(t)$ is supposed to be approximately proportional to the inverse of the Hessian

$$H(t) = \nabla^2 V(\hat{\theta},t)|_{\hat{\theta}=\theta}$$

By updating in the $H(t)^{-1}\varphi(t)$ direction, the order of convergence is two instead of one, which would be the case if the updating was done along the $\varphi(t)$ vector, see Luenberger (1973). If the updating is done in the $P(t)\varphi(t)$ direction, the order of convergence is supposed to be somewhere between one and two, and close to two near the correct solution.

When a fault is detected, the gain in the estimation algorithm is to be increased. It means that the norm of $P(t)$ should be greater. This can be achieved in many ways, but there are mainly two methods that have been used previously. The first is to decrease the forgetting factor λ. It will have the effect that $P(t)$ is scaled with almost maintained eigenvectors. The growth of $P(t)$ is nearly exponential. The second method is to add a constant times the unity matrix to the $P(t)$ matrix, in which case $P(t)$ grows instantaneously.

When a fault has occurred, it is likely that the $P(t)$ matrix is no longer a good approximation of the inverse Hessian. If the Hessian is unknown, the most reasonable direction of the parameter updating is along the $\varphi(t)$ vector. The gain in the estimation algorithm will therefore be increased according to the second method, i.e. equation (2.2b) will be substituted by

$$P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t)\varphi(t)^T P(t-1)}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} \right] + \beta(t)\cdot I \qquad (5.1)$$

where $\beta(t)$ is a nonnegative scalar and I is the unity matrix. The variable $\beta(t)$ is zero except when a fault is detected. When a fault is detected, a positive $\beta(t)$ has the effect that the $P(t)$ matrix obtains a greater norm than otherwise and that the parameter updating is made in a direction closer to $\varphi(t)$ than otherwise.

The final problem is to choose a suitable $\beta(t)$. When no fault is detected, $\beta(t)$ is zero. When a fault is detected, it is reasonable to let $\beta(t)$ depend on the actual value of $P(t)$ and on how significant the alarm is, i.e. on the value of $r(t)$. This may of course be done in many ways, and the following proposal is just one of those.

In the noise-free case, the progress of the estimation error, when $\theta(t)$ is constant, is given by

$$\tilde{\theta}(t) = \tilde{\theta}(t-1) - P(t)\varphi(t)\varepsilon(t) = [I - P(t)\varphi(t)\varphi(t)^T]\,\tilde{\theta}(t-1) \overset{\Delta}{=}$$

$$\overset{\Delta}{=} U(t)\tilde{\theta}(t-1) \qquad (5.2)$$

All eigenvalues of $U(t)$ are one, except the one corresponding to the eigenvector $P(t)\varphi(t)$. This eigenvalue determines the step length in the algorithm. Using equation (5.1), the eigenvalue can be written as

$$1 - \varphi(t)^T P(t)\varphi(t) = 1 - \frac{\varphi(t)^T P(t-1)\varphi(t)}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} -$$

$$- \beta(t)\varphi(t)^T\varphi(t) = \frac{\lambda}{\lambda + \varphi(t)^T P(t-1)\varphi(t)} - \beta(t)\varphi(t)^T\varphi(t)$$

In the original algorithm, the eigenvalue is thus

$$v_0(t) = \frac{\lambda}{\lambda + \varphi(t)^T P(t-1)\varphi(t)}$$

The eigenvalue is obviously between zero and one as long as $P > 0$. A small eigenvalue causes large steps, while an eigenvalue close to one means that the step length in the algorithm is small. Suppose now, that an eigenvalue equal to $v(t)$ is wanted when a fault is detected. Then $\beta(t)$ has to be chosen as

$$\beta(t) = \frac{1}{\varphi(t)^T\varphi(t)}\,(v_0(t) - v(t)) \qquad (5.3)$$

The eigenvalue $v(t)$ should lie in the interval

$$0 < v(t) \le v_0(t)$$

in order to keep the $P(t)$ matrix positiv definite. In practice, this choice of $\beta(t)$ must also be combined with a test for nonsingularity of $\varphi(t)^T\varphi(t)$.

It remains to determine a suitable $v(t)$. This can be done in many ways. In the example presented in the next chapter, $v(t)$ is a piecewice linear function of the significance of the fault alarm, see Figure 5.2.

Combining the fault detection in Chapter 3 with the modification of the estimation algorithm proposed in this chapter, a method to increase the gain in the estimation algorithm in case of faults is derived. The method is summarized in a block diagram in Figure 5.3.

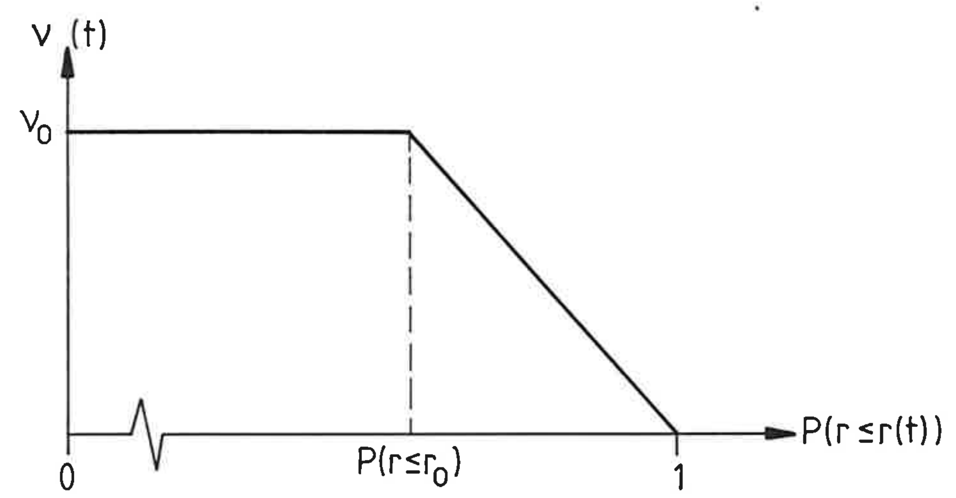


Figure 5.2 - An example of a choice of $v(t)$.

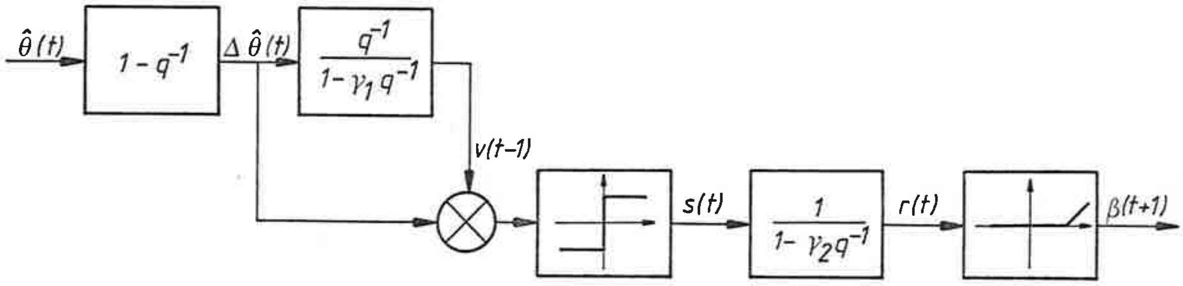


Figure 5.3. - A block diagram describing the method.

6. AN EXAMPLE

To illustrate the new fault detection method and the modified estimation algorithm, a simulation study is presented in this chapter.

The system considered is shown in Figure 6.1. The purpose of control is to keep the level in the tank constant. This is done by measuring the tank level and controlling the inlet valve. The dynamics of the tank is described by the equations

$$\frac{dh}{dt} = \frac{1}{10} (q_{in} - q_{out}) + 0.005\ e(t) \qquad (6.1a)$$

$$q_{out} = a_{out} \sqrt{2gh} \qquad (6.1b)$$

where $\{e(t)\}$ is a disturbance sequence and a_{out} is the outlet area. The sequence $\{e(t)\}$ is generated as discrete Gaussian N(0,1) random variables with a sampling period equals to 1/10:th of the controller sampling period. The stochastic part of the equations can e.g. be viewed as originating from turbulence in the flow or variations in the

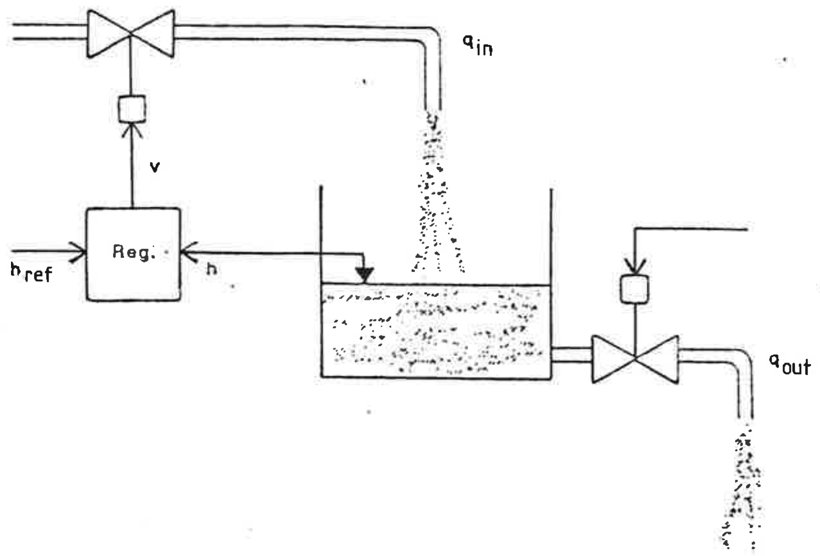


Figure 6.1 - The tank system

valves.

The model of the tank used in the estimation algorithm is

$$h(t+1) = a(t)\cdot h(t) + u(t) + \xi(t) \tag{6.2}$$

where $\{\xi(t)\}$ is a sequence of independent random variables. The parameter $a(t)$ is estimated by the recursive least squares method according to equations (2.2a), (2.2c) and (5.1). The equations become

$$\hat{a}(t) = \hat{a}(t-1) + P(t)h(t-1)\varepsilon(t)$$

$$\varepsilon(t) = h(t) - h_{ref}$$

$$P(t) = \frac{P(t-1)}{\lambda + P(t-1)h(t-1)^2} + \beta(t)$$

The forgetting factor λ is chosen to 0.995. The equations for the fault detection procedure become

$$v(t) = \gamma_1 v(t-1) + (\hat{a}(t) - \hat{a}(t-1))$$

$$s(t) = \mathrm{sign}[(\hat{a}(t) - \hat{a}(t-1))v(t-1)]$$

$$r(t) = \gamma_2 r(t-1) + (1 - \gamma_2)s(t)$$

$$\beta(t) = \begin{cases} 0 & \text{if } r(t-1) < r_0 \\ \dfrac{1}{h(t-1)^2}\left[\dfrac{\lambda}{\lambda + P(t-1)h(t-1)^2} - v(t)\right] & \text{if } r(t-1) \geq r_0 \end{cases}$$

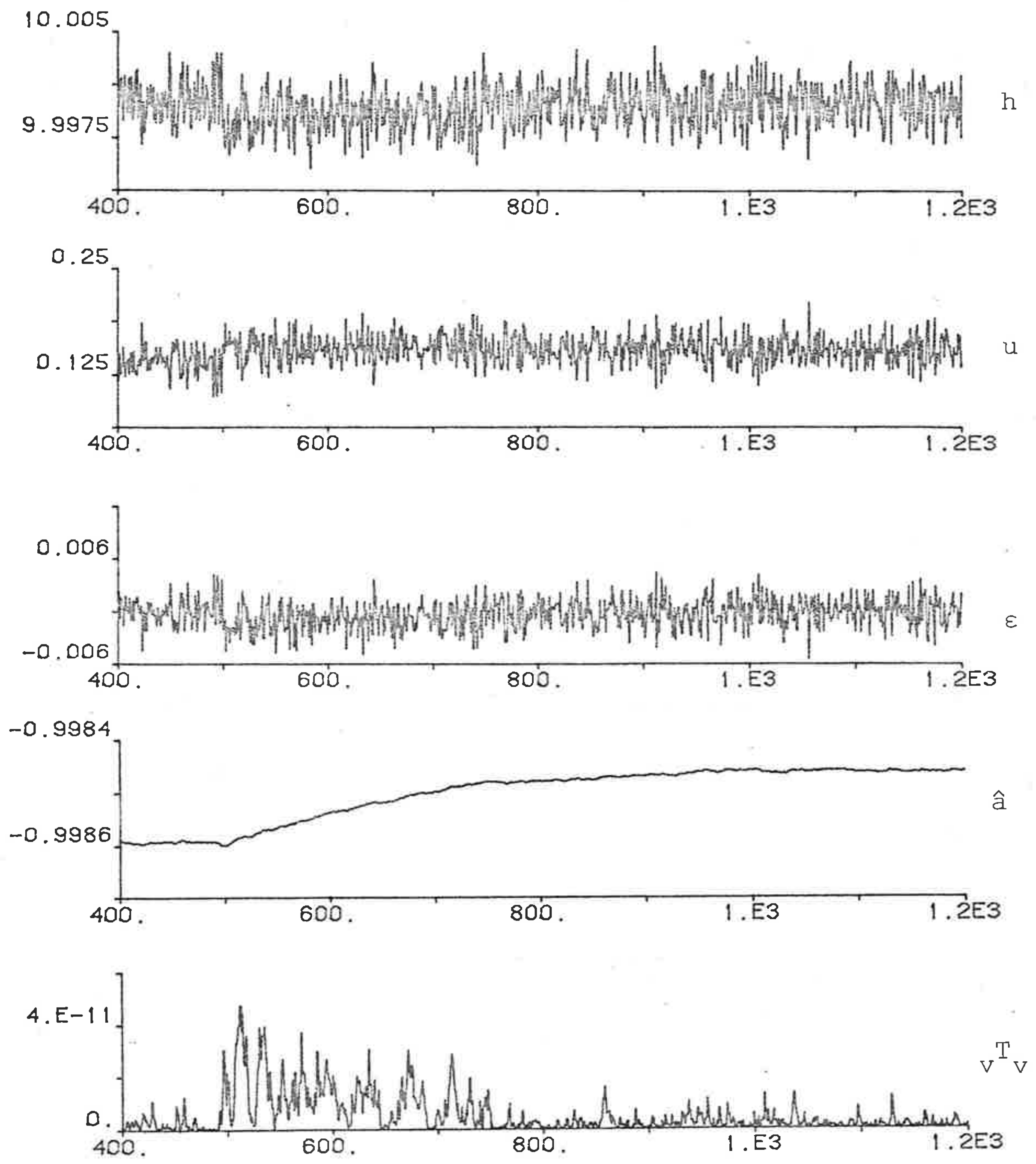
where the two discounting factors γ_1 and γ_2 are 0.85 and 0.95 respectively. The choise of $v(t)$ was presented in Figure 5.2. The value of the threshold is $r_0 = 0.5$, which corresponds to an expected false alarm every 1000:th sample instant. The tank is controlled by a minimum variance regulator with set-point

$$u(t) = h_{ref} - \hat{a}(t)\cdot h(t) \tag{6.3}$$

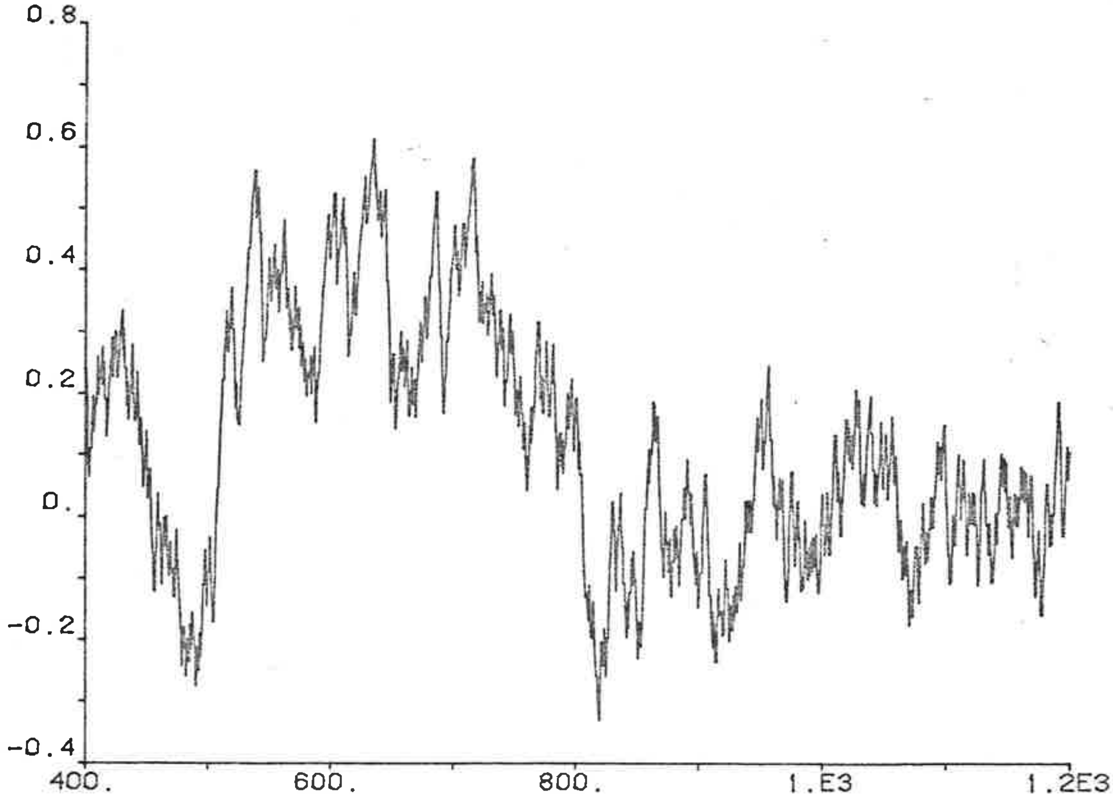
For comparison, the problem is first simulated without any fault detection. The result is shown in Figures 6.2 and 6.3. At t=500, the outlet area is increased from 0.01 to 0.011, corresponding to a sudden increase in the outlet flow or a small leak in the tank. This fault is hard to see in the output-, input- or residual sequences. However, by looking at the estimated parameter $\hat{a}(t)$, it is obvious that something has happened. For comparison, the sequence $v(t)^T v(t)$ is also included. It reacts also very clearly on the fault. In Figure 6.3, the test sequence r(t) is shown. The values of the highest peaks are very unlikely in normal operation, and a fault would have been detected. (Note that r(t) approximately has a Gaussian distribution with a standard deviation of 0.16 in case of no fault).

In Figures 6.4 and 6.5, the result of the simulation is given when the fault detection and the modified estimation algorithm are applied. The increased convergence rate is obvious. Finally, the loss functions in the two simulations are compared in Figure 6.6.

This simple example has shown that it is possible to improve the estimator by including a device for fault detection. It has also been shown, that the proposed fault detection method is able to detect faults, which have a very small influence on the output signal.



<u>Figure 6.2</u> - **Result of the simulation without fault detection.**



<u>Figure 6.3</u> - The sequence $r(t)$ when no modification of the estimation algorithm is done.

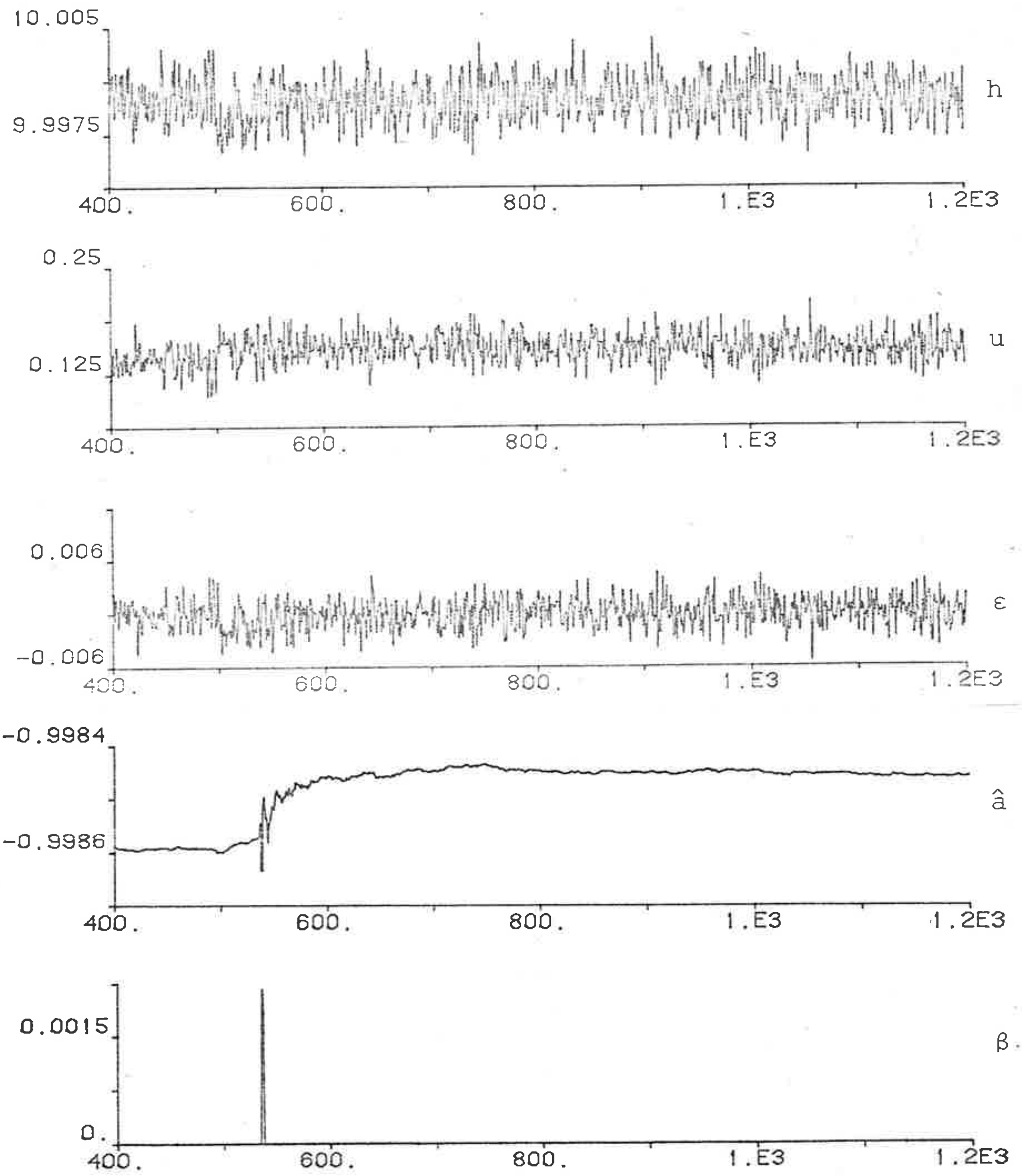


Figure 6.4 – Result of the simulation when the fault detection and the modified estimation algorithm are applied.

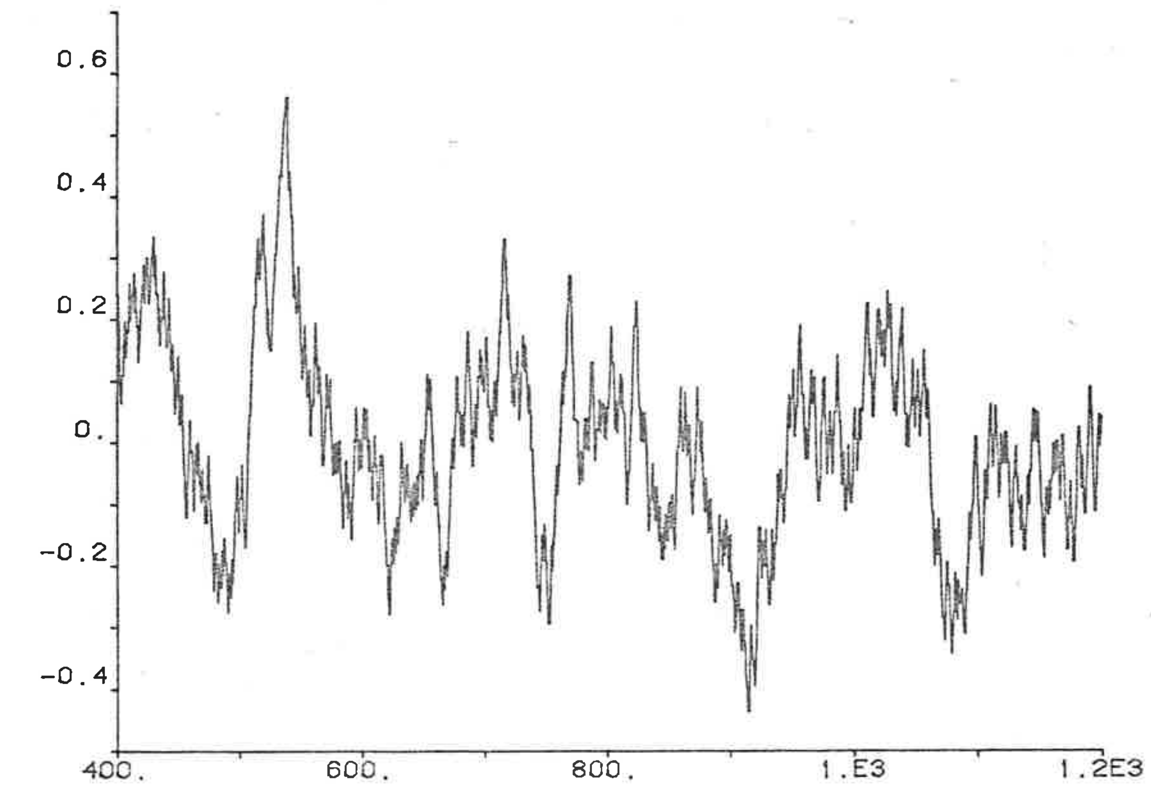


__Figure 6.5__ – The r(t) sequence when the fault detection and the modified estimation algorithm are applied.

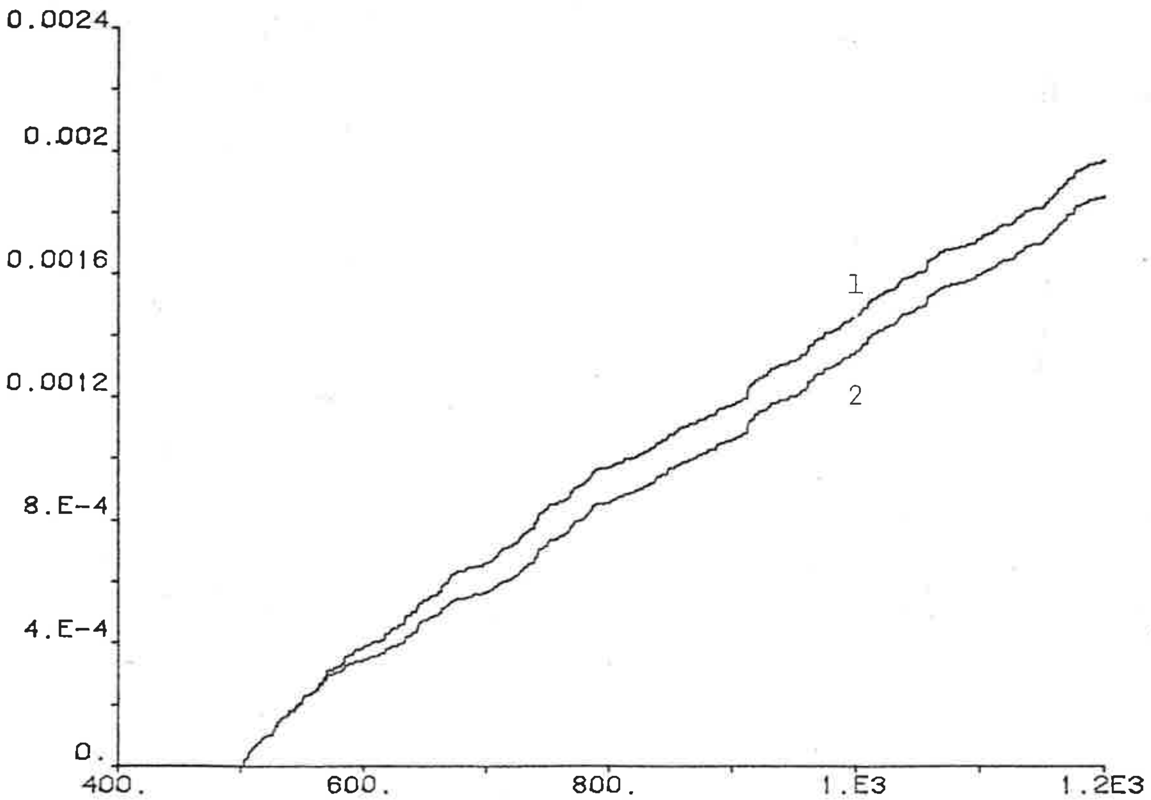


__Figure 6.6__ – The loss functions in the two simulations.

7. CONCLUSIONS

This report deals with the problem of parameter tracking in adaptive timevarying control systems. The underlying, more general, problem is how the information shall be condensed in the estimator and how this condensed information shall be handled to produce as good estimates as possible. In the original estimators, the information is usually condensed in the form of an estimate of the parameters and an estimate of the estimation error covariance matrix. It has here been proposed to extend the stored information with a variable that keeps track of whether a fault in the process model has occurred or not.

The problem of parameter tracking in timevarying systems has of course been studied before. It is often proposed to let the forgetting factor λ be a function of the magnitude of the residuals $\varepsilon(t)$, see e.g. Kershenbaum et al (1981). As said before, the residuals consist of one part corresponding to the estimation error and one part corresponding to the noise, i.e.

$$\varepsilon(t) = \tilde{\theta}(t)^T\varphi(t) + e(t)$$

By assuming a detailed a priori information about the noise sequence $\{e(t)\}$, a great deal of information about $\tilde{\theta}(t)$ can be achieved from $\{|\varepsilon(t)|\}$. It is generally assumed that $\{e(t)\}$ is a sequence of Gaussian distributed random variables with constant variance. The unpleasant assumption of constant variance is discussed earlier in the report. The assumption of Gaussian distribution can to some extent be motivated by the central limit theorem. However, since the loss function is quadratic, a slight deviation from the Gaussian distribution can drastically change the properties of the estimator. From robustness considerations it is therefore often suggested to pay less attention to large values of $|\varepsilon(t)|$ compared with the quadratic loss, see e.g. Huber (1964). In the proposed methods of timevarying forgetting factor, the forgetting factor is decreased, causing a higher gain in the estimator, when the magnitude of $\varepsilon(t)$ is large.

Here is a conflict. From robustness considerations, the gain in the estimator shall be small for lagre values of $|\varepsilon(t)|$. The methods that use the magnitude of $\varepsilon(t)$ as a measure of the accuracy of the parameter estimates, want to have a high gain for large values of $|\varepsilon(t)|$.

In this report, a new way to improve the estimation algorithms with respect to ability to track time-varying parameters, without any deterioration of the stationary properties, is presented. The method does not assume that the noise is Gaussian or has a constant variance. The improvement is achieved by including a fault detection procedure in the estimator and varying the gain in the estimator depending on whether a fault has occurred or not. The method of fault detection is of interest in its own. The problem of fault detection is an area of great research interest. Unfortunately, almost all methods relies on the assumption of Gaussian noise with constant variance, which this method does not do. It has been shown in an example that it is possible to detect faults by the proposed method, even if the faults do not considerably influence the magnitude of the residuals $\varepsilon(t)$.

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